Accidental Peccei-Quinn symmetry and composite axions from chiral gauge theories

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Based on:
R. Contino, AP and F. Revello — 2112.09635 , *JHEP* 04 (2022) 180
AP and F. Revello — 2205.03428 , *PRD* 106 (2022) 11

Plan of the talk

- Introduction and motivations
- Models of composite axions
- Anomaly cancellation conditions and their general solution
- Classification of high-quality QCD axion models
- Conclusions

The axion solution to the strong CP problem

- Experimental fact: absence of CP violation in the QCD sector
- Theoretical puzzle: QCD theta angle must be extremely small: $\theta < 10^{-10}$

nEDM - PRL 124 (2020) 8, 081803

- Axion solution: Peccei and Quinn, Wilczek, Weinberg (1977) + [...]
  - there is a spontaneously broken <u>anomalous</u> $U(1)_{PQ}$ symmetry
  - the axion is the Nambu-Goldstone boson associated to this symmetry
  - the axion potential generated by QCD effects relaxes θ to 0

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a} - \frac{\theta}{2}\right)}$$

  - θ is dynamically relaxed to <u>exactly</u> 0

Vafa, Witten (1984)

The axion quality problem

- The $U(1)_{PQ}$ symmetry is a global symmetry
- But global symmetries are explicitly broken in quantum gravity
- PQ violating effects generated at the Planck scale could spoil the axion solution

UV effects : local PQ violating operator with dimension $\Delta_{\not{PQ}}$

↓

IR effect

$$\Delta\theta \approx |c|\,\phi_{CP}\left(\frac{M_{\rm Pl}}{\Lambda_{\rm QCD}}\right)^4\left(\frac{f_a}{M_{\rm Pl}}\right)^{\Delta_{\not{PQ}}}$$

- neutron EDM experiment: $\Delta\theta \lesssim 10^{-10}$

high quality for $f_a \lesssim 10^{12}\,\mathrm{GeV}$ → $\Delta_{\not{PQ}} \gtrsim 12$

Kamionkowski and March-Russell, Barr and Seckel (1992) + […]

Some proposed solutions

- Axions in string theory with exponentially small corrections
    - axions and axion-like particles arise naturally in string theory — Witten (1984)
    - $\delta V(a) \sim \Lambda_{UV}^4 \, e^{-S_{UV}} \; \cos\left(a/f_a + \delta\right)$
    - The size of the breaking effects depends on properties of the UV completion

  Kallosh et al. (1995), Svrcek and Witten, Conlon (2006), [...]

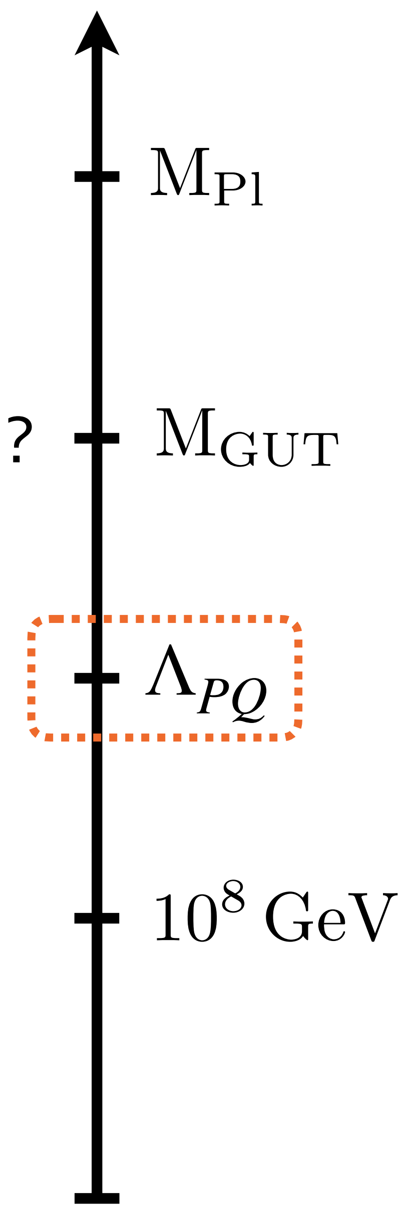
- Quantum field theories with fundamental scalar fields, extra-dim, SUSY, etc.

  → see talk by L. di Luzio for an overview of axion models

- Quantum field theories with fundamental fermions and gauge dynamics in 4D

we focus on this class of models

Models of composite axions



- $SU(N) \times G_W \times G_{SM}$

  - G_W is a possible gauge group assumed to be weak at Λ_{PQ}
  - ψ_i , χ_i are left-handed Weyl fermions
  - simplest example:

	$SU(N)$	$SU(3)_c$	$U(1)_{PQ}$
ψ_1	$\square$	$\square$	$+1$
ψ_2	$\square$	$\mathbf{1}$	-3
χ_1	$\overline{\square}$	$\overline{\square}$	$+1$
χ_2	$\overline{\square}$	$\mathbf{1}$	-3

In this example:
fermionic mass terms
set to zero by hand

Kim - PRD 31 (1985) 1733

can we have accidental (high-quality) PQ symmetry?

Selection rules on PQ violating operators

- Not every PQ violating operator is dangerous !
- A generic PQ violating operator generates a potential only if it has non vanishing matrix element with a state containing axions:

$$\langle \psi_a | \mathcal{O}_{\not{PQ}} | 0 \rangle \neq 0$$

- The operator must be an interpolating operator for the axion, with vanishing vectorial charges.

→ $\mathcal{O}_{\not{PQ}}$ polynomial in $(\psi_r \chi_{\bar{r}}),\ (\psi_r \chi_{\bar{r}})^*, (\psi_r^\dagger \psi_r)$ and $(\chi_{\bar{r}}^\dagger \chi_{\bar{r}})$

important!

- It can be a composite operator built from the insertion of N local operators

$$d_{\rm eff} = \sum_{i=1}^{N} d_i - 4(N-1)$$

→ recently discussed also in Bonnefoy - 2212.00102

Chiral models with non-abelian factors

		$SU(N)$	$SU(m)$	$SU(3)_c$	$U(1)_{PQ}$
	ψ_1	$\square$	$\mathbf{m}$	$\square$	$+1$
$3\times$	ψ_2	$\square$	$\overline{\mathbf{m}}$	$\mathbf{1}$	-1
$m\times$	χ_1	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$+1$
$3m\times$	χ_2	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	-1

- accidental
- anomalous
- spont. broken

Randall - PLB 284 (1992) 77

- The leading PQ violating effects generated at dimension $\Delta_{\not{PQ}} = 3m$

$$\mathcal{O}_{\not{PQ}} = (\psi_1\chi_1)^{m-n}(\bar{\psi}_2\bar{\chi}_2)^n \qquad n = 0,\ldots,m \qquad (n \neq m/2)$$

- Perturbativity of gauge couplings restricts the possible values of m

→ $m > 3$ seems disfavored   Dobrescu - PRD 55 (1997) 5826

- More elaborate models with a moose structure have been proposed

Redi and Sato '16, Vecchi '21, Yanagida et al. [...]

Chiral models with an abelian factor

$$SU(N) \times U(1) \times G_{\rm SM}$$

$$\psi_i \sim (\Box, p_i, r_i), \qquad \chi_i \sim (\bar{\Box}, q_i, \bar{r}_i), \qquad i = 1, \ldots, n_f$$

Harigaya, Nomura '16

- r_i is a (possibly reducible) rep of $G_{\rm SM}$
- p_i and q_i are integer charges for $U(1)_D$ with:

$$p_i \neq -q_i \qquad (i \neq j)$$

→ chiral charge assignment

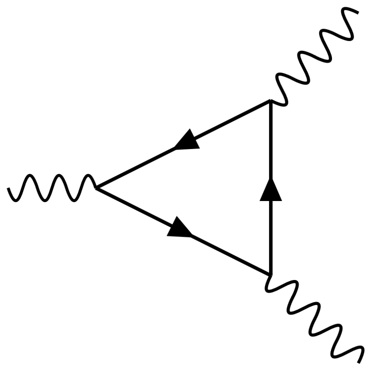
- The classification of PQ violating operators requires a knowledge of the U(1) charges

The anomaly cancellation equations

- Local gauge anomalies for $U(1)$ impose non-trivial constraints

$$
\begin{cases}
\sum_{i=1}^{n_f}(p_i+q_i)T_i=0, & U(1)\times[G_{\rm SM}]^2 \\
\sum_{i=1}^{n_f}(p_i+q_i)d_i=0, & U(1)\times[SU(N)]^2 \\
\sum_{i=1}^{n_f}(p_i^3+q_i^3)d_i=0, & [U(1)]^3
\end{cases}
$$

(zero hypercharge for simplicity)



Dimension of SM rep:

$$d_i=\dim(r_i)=\sum_\alpha \dim(r_i^{(\alpha)}),$$

Dynkin index of SM rep:

$$T_i=\sum_\alpha T(r_i^{(\alpha)}),$$

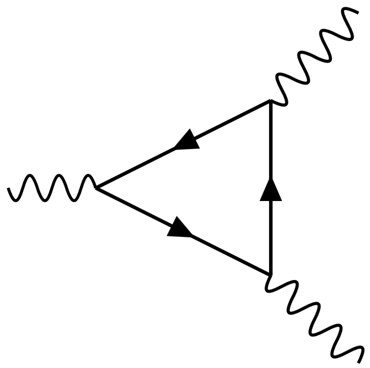
In fact the same equations apply if: $SU(N)\to G_{\rm semisimple}$, $(\square,\overline{\square})\to(R,\overline{R})$

The anomaly cancellation equations

- Local gauge anomalies for $U(1)$ impose non-trivial constraints

$$
\begin{cases}
\sum_{i=1}^{n_f} (p_i + q_i) T_i = 0, & U(1) \times [G_{\mathrm{SM}}]^2 \\
\sum_{i=1}^{n_f} (p_i + q_i) d_i = 0, & U(1) \times [SU(N)]^2 \\
\sum_{i=1}^{n_f} (p_i^3 + q_i^3) d_i = 0, & [U(1)]^3
\end{cases}
$$

(zero hypercharge for simplicity)



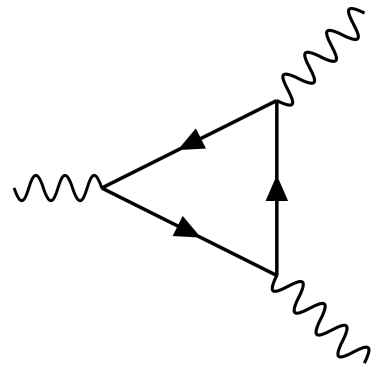
- System of polynomial equations over the integers
  - cubic equations: in general it is very hard to find all the integer solutions

The anomaly cancellation equations

- Local gauge anomalies for $U(1)$ impose non-trivial constraints

$$\begin{cases} \sum_{i=1}^{n_f} (p_i + q_i) T_i = 0, & U(1) \times [G_{\rm SM}]^2 \\ \sum_{i=1}^{n_f} (p_i + q_i) d_i = 0, & U(1) \times [SU(N)]^2 \\ \sum_{i=1}^{n_f} (p_i^3 + q_i^3) d_i = 0, & [U(1)]^3 \end{cases}$$

(zero hypercharge for simplicity)



- System of polynomial equations over the integers
    - cubic equations: in general it is very hard to find all the integer solutions
    - The case of a pure U(1) gauge theory has been solved recently

      Costa, Dobrescu, Fox '19, Allanach, Gripaios, Tooby-Smith '19

General solution for $n_f = 2$

- For $n_f = 2$ and chiral assignments $p_i \neq -q_i$

  - the two linear equations have to be equivalent (so $T_1 d_2 = T_2 d_1$).

$$\begin{cases} (p_1 + q_1)d_1 + (p_2 + q_2)d_2 = 0 \\ (p_1^3 + q_1^3)d_1 + (p_2^3 + q_2^3)d_2 = 0 \end{cases}$$

  - combining the two equations we obtain an homogeneous quadric

$$Q(X, Y, Z) \equiv (d_2^2 - d_1^2)X^2 + 3d_2^2 Y^2 - 3d_2^2 Z^2 = 0$$

Conic in projective space!

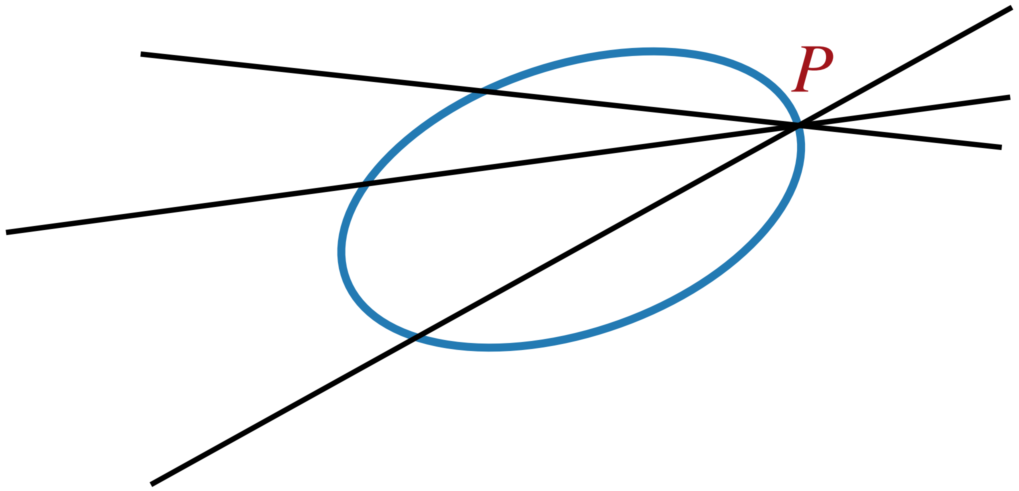
where $X = (p_1 + q_1), \quad Y = (p_1 - q_1), \quad Z = (p_2 + q_2)$

  - Integers charges correspond to the zero locus of $Q(X, Y, Z)$ in $\mathbb{PQ}^2$

General solution for $n_f = 2$

- Theorem 1 (see e.g. Mordell):

Given a rational point P on a conic, there are in fact infinitely many rational points and they can all be found as the intersection of a rational line through P and the conic itself.



- In our case $P = [0:1:1]$, and we find:

$$p_1 = \frac{n}{\mu_2}\tilde{p}_1 = \frac{n}{\mu_2}\Big[d_1^2\ell^2 + d_2^2(3k^2 + 6k\ell - \ell^2)\Big],$$

$$q_1 = \frac{n}{\mu_2}\tilde{q}_1 = \frac{n}{\mu_2}\Big[-d_1^2\ell^2 + d_2^2(\ell^2 + 6k\ell - 3k^2)\Big],$$

$$p_2 = \frac{n}{\mu_2}\tilde{p}_2 = \frac{n}{\mu_2}\Big[d_1^2\ell^2 - 6d_1d_2k\ell - d_2^2(3k^2 + \ell^2)\Big],$$

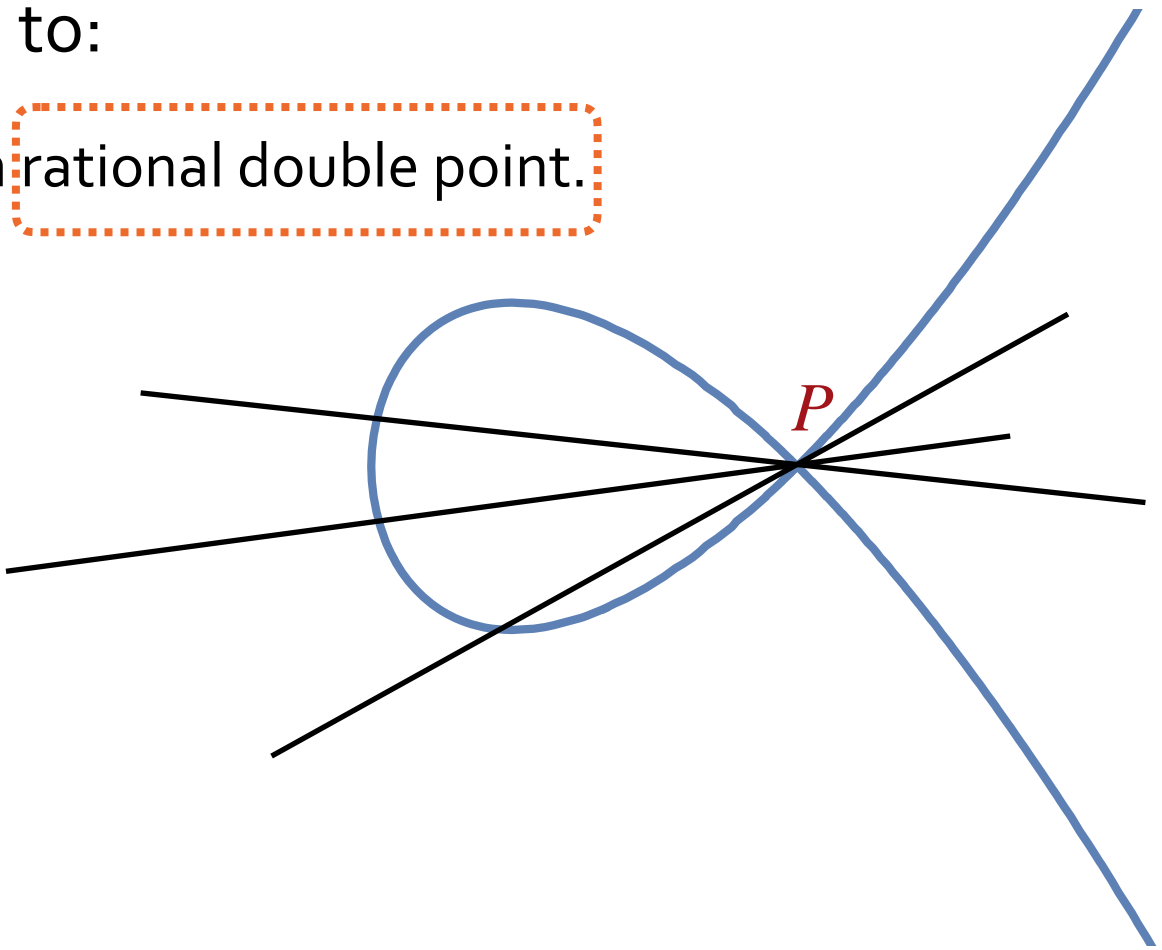
$$q_2 = \frac{n}{\mu_2}\tilde{q}_2 = \frac{n}{\mu_2}\Big[-d_1^2\ell^2 - 6d_1d_2k\ell + d_2^2(3k^2 + \ell^2)\Big],$$

$$n, k, \ell \in \mathbb{Z} \setminus \{0\}$$

$$\mu_2 = \gcd\Big(\tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2\Big)$$

General solution for arbitrary n_f

- In the general case we cannot reduce the system to a quadric.
- However, for our specific system of equations, by using one of the linear equations, we can reduce the system to:
  - a singular cubic hypersurface, with a known rational double point.
  - plus an additional linear equation.



- Theorem 2 (see e.g. Mordell):

Given a rational double point P on a cubic hypersurface, there are infinitely many rational points and they can all be found as the intersection of a rational line through P and the cubic itself.

General solution for arbitrary n_f

- For $n_f = \nu$ and chiral assignments $p_i \neq -q_i$
  - first consider the equations

$$\sum_{i=1}^{\nu} (p_i + q_i) d_i = 0, \qquad \rightarrow \qquad \text{solve for } q_\nu$$

$$\sum_{i=1}^{\nu} (p_i^3 + q_i^3) d_i = 0,$$

  - combining the two equations we obtain a cubic in $\mathbb{PQ}^{2\nu-2}$

$$F\left(p_i, q_i\right) = d_\nu^2 \sum_{i=1}^{\nu-1} d_i \left(p_i^3 + q_i^3\right) + d_\nu^3\, p_\nu^3 - \left(\sum_{i=1}^{\nu-1} d_i (p_i + q_i) + d_\nu\, p_\nu \right)^3 .$$

  - The point $\Pi_0 = [1 : -1 : \ldots : 1 : -1 : 1]$ is a rational double point

→ Find all rational points as intersections of cubic and rational lines through Π_0

General solution for arbitrary n_f

- Setting $\nu = n_f$ for notational convenience, we find

$$
\begin{aligned}
p_i &= \frac{n}{\mu_\nu}\tilde{p}_1 = \frac{n}{\mu_\nu}\Big[A_2 - A_1\,\ell_i\Big], \qquad\qquad i = 1,\dots,\nu-1,\\
q_i &= \frac{n}{\mu_\nu}\tilde{q}_1 = \frac{n}{\mu_\nu}\Big[-A_2 - A_1\,m_i\Big],\\
p_\nu &= \frac{n}{\mu_\nu}\tilde{p}_\nu = \frac{n}{\mu_\nu}\Big[A_2 - A_1\,\ell_\nu\Big],\\
q_\nu &= \frac{n}{\mu_\nu}\tilde{q}_\nu = \frac{n}{\mu_\nu}\left[-A_2 + \frac{A_1}{d_\nu}\left(\sum_{i=1}^{\nu-1} d_i(\ell_i + m_i) + d_\nu\,\ell_\nu\right)\right],
\end{aligned}
$$

  - charges parametrized by a set of arbitrary integers subject to a linear constraint:

$$
\sum_{i=1}^{\nu-1} D_{i\nu}(\ell_i + m_i) = 0. \qquad\qquad \ell_i, m_i, n \in \mathbb{Z}
$$

where $D_{ij} = d_i T_j - d_j T_i$, $\mu_\nu = \gcd(\tilde{p}_1, \dots, \tilde{q}_\nu)$, A_I are polynomials in (ℓ_i, m_i)

General solution for arbitrary n_f

$$A_1 = 3d_\nu^2 \sum_{i=1}^{\nu-1} d_i(\ell_i^2 - m_i^2) + 3d_\nu^3\,\ell_\nu^2 - 3d_\nu \left(\sum_{i=1}^{\nu-1} d_i(\ell_i + m_i) + d_\nu\,\ell_\nu\right)^2,$$

$$A_2 = d_\nu^2 \sum_{i=1}^{\nu-1} d_i(\ell_i^3 + m_i^3) + d_\nu^3\,\ell_3^3 - \left(\sum_{i=1}^{\nu-1} d_i(\ell_i + m_i) + d_\nu\,\ell_\nu\right)^3.$$

- The analysis is valid for *arbitrary* semi-simple compact $G \times G_{\rm SM}$ with new fermions vector-like under $G \times G_{\rm SM}$:

$$\psi_i = (R, p_i, r_i), \qquad \chi_i = (\overline{R}, q_i, \overline{r}_i) \qquad i = 1, \ldots, n_f$$

$G_{\rm SM}$: SM gauge group or a semi-simple extension

- The theory is chiral but the infrared dynamics is controlled by the vector-like G
- Assignment of SM hypercharge can be included straightforwardly

Back to composite axion models

- the knowledge of the $U(1)$ charges allows us to identify the form

  of the gauge invariant PQ violating local operators that generate a potential.

  - For $\nu = 2$ and $\nu = 3$ the gauge invariance of PQ violating operators of the form:

  $$\mathcal{O}_{\not{PQ}} = (\psi_1 \chi_1)^{\kappa_1} \ldots (\psi_\nu \chi_\nu)^{\kappa_\nu}$$

  is independent from the charge assignments — depends only on SM representations

Single insertions can be easily characterised → useful for phenomenology

- The effect of multiple insertions depends on the specific charges.

  - we are still able to find charge assignments with

    the desired (maximal) level of PQ protection

Models with $n_f = 3$

	$\mathrm{SU}(N_{\mathrm{DC}})$	$\mathrm{U}(1)_{\mathrm{D}}$	G_{SM}	$\mathrm{U}(1)_{\mathrm{PQ}}$
ψ_1	$\square$	p_1	r_1	α
ψ_2	$\square$	p_2	r_2	β
ψ_3	$\square$	p_3	r_3	γ
χ_1	$\bar{\square}$	q_1	$\bar{r}_1$	α
χ_2	$\bar{\square}$	q_2	$\bar{r}_2$	β
χ_3	$\bar{\square}$	q_3	$\bar{r}_3$	γ

- We classify the models that satisfy the following conditions:
  - anomaly cancellation
  - confining $SU(N)$ dynamics (bound on the multiplicity of fermions)
  - perturbativity of the SM couplings up to M_{Pl} (no low-scale Landau poles)

Models with $n_f = 3$

	$SU(N_{DC})$	$U(1)_D$	G_{SM}	$U(1)_{PQ}$
ψ_1	$\square$	p_1	r_1	α
ψ_2	$\square$	p_2	r_2	β
ψ_3	$\square$	p_3	r_3	γ
χ_1	$\bar{\square}$	q_1	$\bar{r}_1$	α
χ_2	$\bar{\square}$	q_2	$\bar{r}_2$	β
χ_3	$\bar{\square}$	q_3	$\bar{r}_3$	γ

- The PQ violating **local** operators depend **only** on the choice of SM reps

$$\mathcal{O}_{\not{PQ}} = (\psi_1\chi_1)^{\kappa_1}(\psi_2\chi_2)^{\kappa_2}(\psi_3\chi_3)^{\kappa_3}$$

- Multiple insertions of local operators depend on charges

Classification with irreducible SM reps

G_{SM}	r_1	r_2	r_3	$\Delta^{max}_{\not{PQ}}$	N_{DC}	f_a^{min}
$SU(5)_{GUT}$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{10}$	12	$4, 5, \ldots, 11$	$4 \cdot 10^8$ GeV
	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{15}$	15	$6, 7$	10^{11} GeV
	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{15}$	18	7	10^{12} GeV
$SU(3)_c$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{6}$	12	$3, 4, \ldots, 9$	$4 \cdot 10^8$ GeV
	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{6}$	15	5	$5 \cdot 10^{11}$ GeV
	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{8}$	15	$3, 4, \ldots, 7$	10^9 GeV
	$\mathbf{1}$	$\mathbf{6}$	$\mathbf{8}$	12	$4, 5$	10^{11} GeV

- PQ symmetry can be protected up to dim 18
- With the explicit solution we were able to prove that such charge assignments exist

cfr: Randall (1992) , Dobrescu (1997), Redi and Sato (2016), Vecchi (2021)

Classification with irreducible SM reps

G_{SM}	r_1	r_2	r_3	$\Delta_{\not{PQ}}^{max}$	N_{DC}	f_a^{min}
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	$\mathbf{1}$	$\mathbf{6}$	$\mathbf{8}$	12	$4, 5$	10^{11} GeV

We classified all the high-quality charge assignments with charges $|p_i|, |q_i| \leq 10$

↓

In order to do so we need to check if multiple insertions can induce PQ violating effects

High quality axion - explicit GUT model

Robust model: high quality irrespectively of GUT scalar sector → GUT scalars in arbitrary reps

$N_{\rm DC} = 5$

	SU(N$_{\rm DC}$)	U(1)$_{\rm D}$	SU(5)$_{\rm GUT}$	U(1)$_{\rm PQ}$
ψ_1	$\square$	$+2$	$\mathbf{1}$	$+5$
ψ_2	$\square$	$+3$	$\bar{\mathbf{5}}$	$+1$
ψ_3	$\square$	-5	$\mathbf{10}$	-1
χ_1	$\overline{\square}$	$+3$	$\mathbf{1}$	$+5$
χ_2	$\overline{\square}$	-6	$\mathbf{5}$	$+1$
χ_3	$\overline{\square}$	$+6$	$\overline{\mathbf{10}}$	-1

+ others!

dim 12 PQ violating operator: $\mathcal{O}_1 = \psi_2\chi_2\big(\psi_3\chi_3\big)^3$

Phenomenological features

- The number of light NGBs is $n_{irr} - 2$
  - n_{irr} is the number of irreps in the set $\{\psi_i\}$
  - 1 axion + possibly additional ultralight scalars
- Models with GUT dynamics:

  additional parametrically light GUT states: $$\delta m_{\tilde{a}}^2 \sim \frac{g_5^2}{16\pi^2} \frac{\Lambda_{PQ}^4}{M_{\rm GUT}^2}$$

- The properties of all the (pseudo)-NGBs can be studied in detail systematically using an effective chiral lagrangian
- Cosmological species:
  - the QCD axion can be a component of dark matter
  - all the light scalars can constitute a component of dark radiation

QCD axion low energy couplings

- Axion low energy couplings

  - well-predicted but not distinctive low energy couplings — see talk by G. Villadoro

    → common to “hadronic axion” models

$$m_a = 5.70(7)\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)\mu\mathrm{eV},$$

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right),$$

$$c_p = -0.47(3), \quad c_n = -0.02(3),$$

GUT models have fixed E/N = 8/3

Grilli di Cortona, Hardy, Vega, Villadoro - JHEP 05 (2016) 104

Cosmological evolution

- Cosmological evolution:
  - PQ breaking after the end of inflation is disfavored for composite models
    → accidentally stable heavy resonances + domain walls ✗
  - PQ broken during inflation ✓
    → axion populated with misalignment mechanism (possibly DM)
  - in the GUT scenario, parametrically light metastable NGBs

$$\delta m_{\tilde{a}}^2 \sim \frac{g_5^2}{16\pi^2} \frac{\Lambda_{PQ}^4}{M_{\rm GUT}^2}$$

→ can leave imprints in cosmological observables

Summary

Anomaly cancellation

- General solution for the U(1) charges: $G \times U(1) \times G_{\rm SM}$

  with *new* fermions vector-like under $G \times G_{\rm SM}$

High-quality axion:

- Chiral gauge theory with dynamical generation of scales
- Strong CP solved by the QCD axion
- PQ is an high quality accidental symmetry through gauge protection
- Possibly DM
- Compatibility with SU(5) unified dynamics

Bonus

Chiral models with non-abelian factors — SU(5)

GUT extension

		$SU(N)$	$SU(m)$	$SU(5)_{GUT}$	$U(1)_{PQ}$
	ψ_1	$\square$	$\mathbf{m}$	$\square$	$+1$
$3\times$	ψ_2	$\square$	$\overline{\mathbf{m}}$	$\mathbf{1}$	-1
$m\times$	χ_1	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$+1$
$3m\times$	χ_2	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	-1

- The leading PQ violating effect comes from a double insertion

$$\mathcal{O}_1 = \psi_1\psi_2\chi_1^2\Sigma \quad , \quad \mathcal{O}_2 = \psi_1\psi_2\chi_2^2\Sigma^\dagger$$

Σ is the GUT scalar $\square$

- The axion potential is generated at the level of dimension 10

$$\mathcal{O}_{\not{PQ}} = \mathcal{O}_1\mathcal{O}_2 \quad , \quad d_{eff} = 7 + 7 - 4 = 10$$

An attractive model that **doesn't work**

	$SU(N_{DC})$	$SO(10)_L$	$SO(10)_R$
ψ	$\square$	$\mathbf{16}$	$\mathbf{1}$
χ	$\overline{\square}$	$\mathbf{1}$	$\overline{\mathbf{16}}$

SO(10) x SO(10) breaking to SU(5) x U(1)

	$SU(N_{DC})$	$U(1)_D$	$SU(5)_{GUT}$	$U(1)_{PQ}$
ψ_1	$\square$	$+5$	$\mathbf{1}$	$+5$
ψ_2	$\square$	-3	$\bar{\mathbf{5}}$	$+1$
ψ_3	$\square$	$+1$	$\mathbf{10}$	-1
χ_1	$\overline{\square}$	$+5q$	$\mathbf{1}$	$+5$
χ_2	$\overline{\square}$	$-3q$	$\mathbf{5}$	$+1$
χ_3	$\overline{\square}$	$+q$	$\overline{\mathbf{10}}$	-1

An attractive model that **doesn't work**

	$SU(N_{DC})$	$SO(10)_L$	$SO(10)_R$
ψ	$\square$	$\mathbf{16}$	$\mathbf{1}$
χ	$\overline{\square}$	$\mathbf{1}$	$\overline{\mathbf{16}}$

SO(10) x SO(10) breaking to SU(5) x U(1)

	$SU(N_{DC})$	$U(1)_D$	$SU(5)_{GUT}$	$U(1)_{PQ}$
ψ_1	$\square$	$+5$	$\mathbf{1}$	$+5$
ψ_2	$\square$	-3	$\bar{\mathbf{5}}$	$+1$
ψ_3	$\square$	$+1$	$\mathbf{10}$	-1
χ_1	$\overline{\square}$	$+5q$	$\mathbf{1}$	$+5$
χ_2	$\overline{\square}$	$-3q$	$\mathbf{5}$	$+1$
χ_3	$\overline{\square}$	$+q$	$\overline{\mathbf{10}}$	-1

dim 6 PQ violating operators:

$$O_L = \psi_1\psi_2(\psi_3^*)^2$$

$$O_R = \chi_1\chi_2(\chi_3^*)^2$$

single insertions are innocuous

but a double insertion $O_L O_R$

generates a potential for the axion

$$d_{\rm eff} = 12 - 4 = 8$$

Models with $n_f = 2$

	$\mathrm{SU}(N_{\mathrm{DC}})$	$\mathrm{U}(1)_{\mathrm{D}}$	G_{SM}
ψ_1	$\square$	p_1	r_1
ψ_2	$\square$	p_2	r_2
χ_1	$\bar{\square}$	q_1	$\bar{r}_1$
χ_2	$\bar{\square}$	q_2	$\bar{r}_2$

Models with $n_f = 2$

- We classify the models that satisfy the following conditions:
  - anomaly cancellation
  - confining $SU(N)$ dynamics (bound on the multiplicity of fermions)
  - perturbativity of the SM couplings up to M_{Pl} (no low-scale Landau poles)

- From the explicit solutions for the $U(1)$ charges
  - PQ violating local operators that generate a potential have the form:

$$O_{\not{PQ}} = (\psi_1\chi_1)^{\kappa_1}(\psi_2\chi_2)^{\kappa_2}$$

$$\kappa_i = d_i/\mathrm{gcd}(d_1, d_2)$$

$\longrightarrow$

$$\Delta_{\not{PQ}} = 3(d_1 + d_2)/\mathrm{gcd}(\mathrm{d}_1, \mathrm{d}_2)$$

General classification for $n_f = 2$

G_{SM}	r_1	r_2	d_1	d_2		$\Delta_{\not{PQ}}$
$SU(3)_c$	$(\mathbf{3} \oplus m\,\mathbf{1})$	$(\mathbf{3} \oplus m\,\mathbf{1})$	$3+m$	$3+m$	$1 \leq m \leq 23$	6
	$(\mathbf{3} \oplus m\,\mathbf{1})$	$2\,(\mathbf{3} \oplus m\,\mathbf{1})$	$3+m$	$2(3+m)$	$1 \leq m \leq 8$	9
	$(\mathbf{3} \oplus m\,\mathbf{1})$	$3\,(\mathbf{3} \oplus m\,\mathbf{1})$	$3+m$	$3(3+m)$	$1 \leq m \leq 3$	12
	$2\,(\mathbf{3} \oplus m\,\mathbf{1})$	$2\,(\mathbf{3} \oplus m\,\mathbf{1})$	$2(3+m)$	$2(3+m)$	$1 \leq m \leq 3$	6
	$(\mathbf{3} \oplus \mathbf{1})$	$4\,(\mathbf{3} \oplus \mathbf{1})$	4	16		15
	$2\,(\mathbf{3} \oplus \mathbf{1})$	$3\,(\mathbf{3} \oplus \mathbf{1})$	8	12		15
$SU(5)_{GUT}$	$(\mathbf{5} \oplus m\,\mathbf{1})$	$(\mathbf{5} \oplus m\,\mathbf{1})$	$5+m$	$5+m$	$1 \leq m \leq 21$	6
	$(\mathbf{5} \oplus m\,\mathbf{1})$	$2\,(\mathbf{5} \oplus m\,\mathbf{1})$	$3+m$	$2(3+m)$	$1 \leq m \leq 6$	9
	$(\mathbf{5} \oplus \mathbf{1})$	$3\,(\mathbf{5} \oplus \mathbf{1})$	6	18		12
	$2\,(\mathbf{5} \oplus \mathbf{1})$	$2\,(\mathbf{5} \oplus \mathbf{1})$	12	12		6
	$\mathbf{5}$	$(\mathbf{10} \oplus 5\,\mathbf{1})$	5	15		12

- Using the explicit solutions we are able to easily find charge assignments with no PQ violating effects below $\Delta_{\not{PQ}}$

General classification for $n_f = 2$

G_{SM}	r_1	r_2	d_1	d_2		$\Delta_{\not{PQ}}$
$SU(3)_c$	$(\mathbf{3} \oplus m\,\mathbf{1})$	$(\mathbf{3} \oplus m\,\mathbf{1})$	$3+m$	$3+m$	$1 \leq m \leq 23$	6
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- Using the explicit solutions we are able to easily find charge assignments with no PQ violating effects below $\Delta_{\not{PQ}}$

Consider a massless vector-like theory

	$\mathrm{SU(N_{DC})}$
ψ_1	$\square$
ψ_2	$\square$
χ_1	$\overline{\square}$
χ_2	$\overline{\square}$

Symmetry Breaking Pattern:

$$\mathrm{SU(2)} \times \mathrm{SU(2)} \times \mathrm{U(1)_V} \longrightarrow \mathrm{SU(2)_V} \times \mathrm{U(1)_V}$$

spontaneous

Phenomenology of models with $n_f = 2$

and consider the weak gauging of a subgroup $U(1)_D$

	$\mathrm{SU(N_{DC})}$	$\mathrm{U(1)_D}$
ψ_1	$\square$	$+1$
ψ_2	$\square$	-1
χ_1	$\overline{\square}$	$-a$
χ_2	$\overline{\square}$	$+a$

chiral theory for $0 \leq a < 1$

Harigaya, Nomura - PRD 94 (2016) 035013

Co, Harigaya, Nomura - PRL 118 (2017) 101801

Symmetry Breaking Pattern:

$$\mathrm{SU(2)} \times \mathrm{SU(2)} \times \mathrm{U(1)_V} \longrightarrow \mathrm{SU(2)_V} \times \mathrm{U(1)_V}$$

explicit

$$\longrightarrow \mathrm{U(1)_{3V}} \times \mathrm{U(1)_V}$$

$\mathrm{U(1)_D}$ acquires a mass from the bilinear condensate $m_{\gamma_D} \sim f\, e_D$

Phenomenology of models with $n_f = 2$

- chiral fermions should come in complete GUT multiplets

	$\mathrm{SU(N_{DC})}$	$\mathrm{U(1)_D}$	$\mathrm{SU(5)_{GUT}}$
ψ_1	$\square$	$+1$	$\square$
ψ_2	$\square$	-1	$\square$
χ_1	$\overline{\square}$	$-a$	$\overline{\square}$
χ_2	$\overline{\square}$	$+a$	$\overline{\square}$

Contino, AP, Revello - JHEP 02 (2021) 091

99 NGBs :

pseudo — eaten by γ_D

$$24^{\pm} \oplus 24^{0} \oplus 24^{0\prime} \oplus 1^{\pm} \oplus 1^{0}$$

GUT breaking

$$\supset 1^{0} \oplus 1^{0\prime}$$ SM singlets

one (combination) of the two singlets has anomalous couplings to SM

→ axion-like particle $\qquad \delta m^2_{\tilde{a}} \sim \frac{g^2_{\mathrm{GUT}}}{16\pi^2}\frac{\Lambda^4_{\mathrm{DC}}}{M^2_{\mathrm{GUT}}}$ ✗ not QCD axion

GUT contribution

Phenomenology of models with $n_f = 2$

- Models with 2 or more different $G_{\rm SM}$ reps have an axion

	$\mathrm{SU(N_{DC})}$	$\mathrm{U(1)_D}$	$\mathrm{SU(5)_{GUT}}$
ψ_1	$\square$	$+1$	$\square \oplus \mathbf{1}$
ψ_2	$\square$	-1	$\square \oplus \mathbf{1}$
χ_1	$\overline{\square}$	$-a$	$\overline{\square} \oplus \mathbf{1}$
χ_2	$\overline{\square}$	$+a$	$\overline{\square} \oplus \mathbf{1}$

- 2 light Goldstone bosons
  - one anomalous → axion
  - one exact → massless scalar up to UV effects

Some open questions

- We found all the charge assignments such that local gauge anomalies cancel
  - what about global (non-perturbative) anomalies?
- Are there more efficient ways to classify charge assignments such that operators with a given property (e.g. PQ violating operators) are forbidden up to a given dimension?
- Do all the consistent charge assignments admit an embedding in string theory?
- Observational distinctive signatures from pseudo-NGB ?