Axion hot dark matter bound, reliably

New Physics Signals 2023

NeΨ 23

Gioacchino Piazza

IJCLab, Pôle Théorie, CNRS and Paris Saclay U.

Based on:

L. Di Luzio, G. Martinelli, **GP** *PRL*126 (2021) 24, 241801 [arXiv: 2101.10330]
L. Di Luzio, **GP** [arXiv: 2206.04061]
L. Di Luzio, G. Martinelli, J.M. Camalich, J.A. Oller, **GP** [arXiv: 2211.05073]

HIDDeν

cnrs

IJCLab Irène Joliot-Curie

LPT Orsay

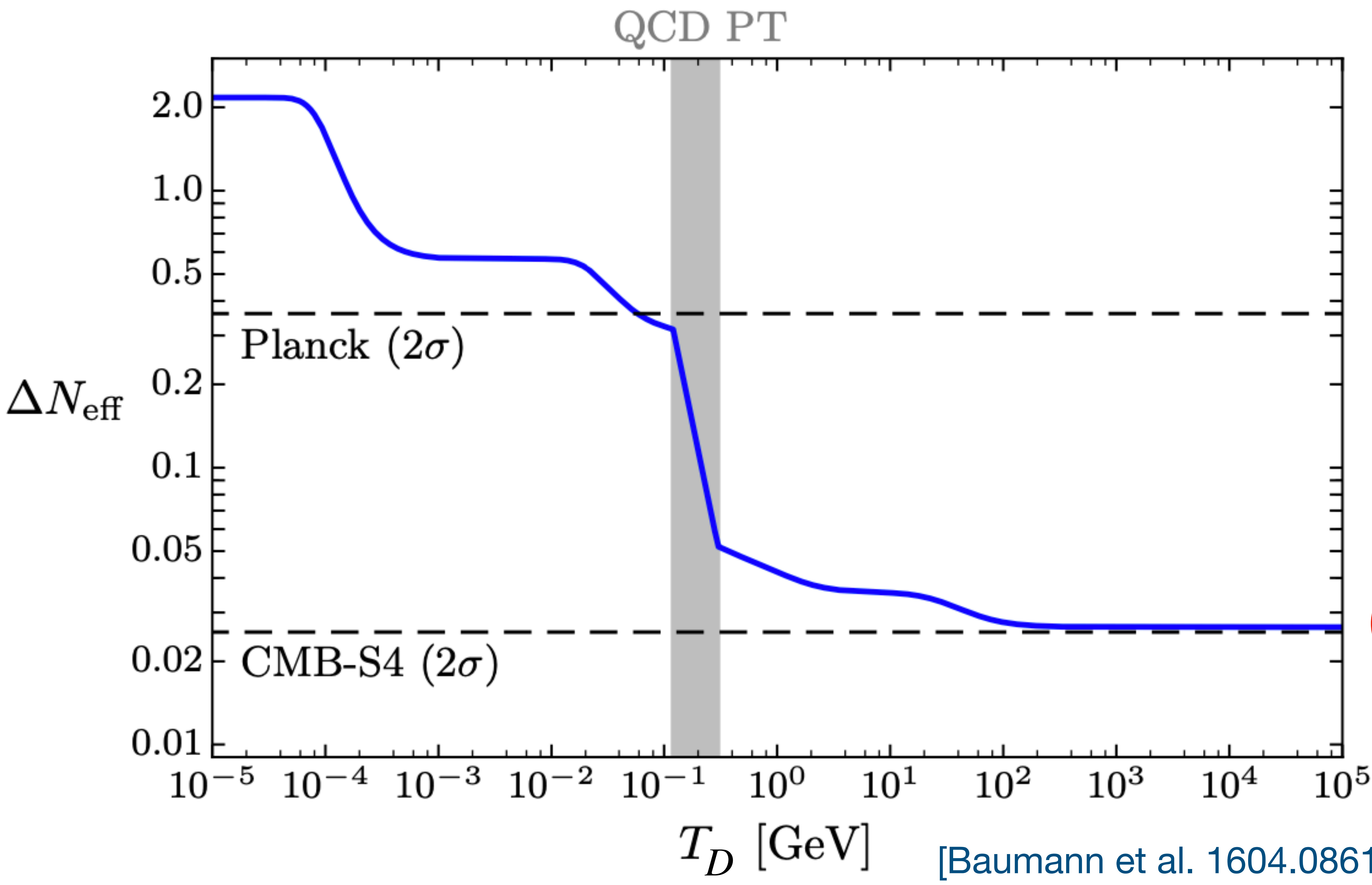
Pisa 17/02/2023

A possible discovery channel for the axion!

- Axions once in equilibrium with SM thermal bath contribute to the radiation density of the Universe ($\Delta N_{\rm eff}$)

- T_D depends on the strength of the axion interactions set by f_a

- Full range of allowed $\Delta N_{\rm eff}$ will be covered by CMB-S4



(2029 ?)

[Baumann et al. 1604.08614]

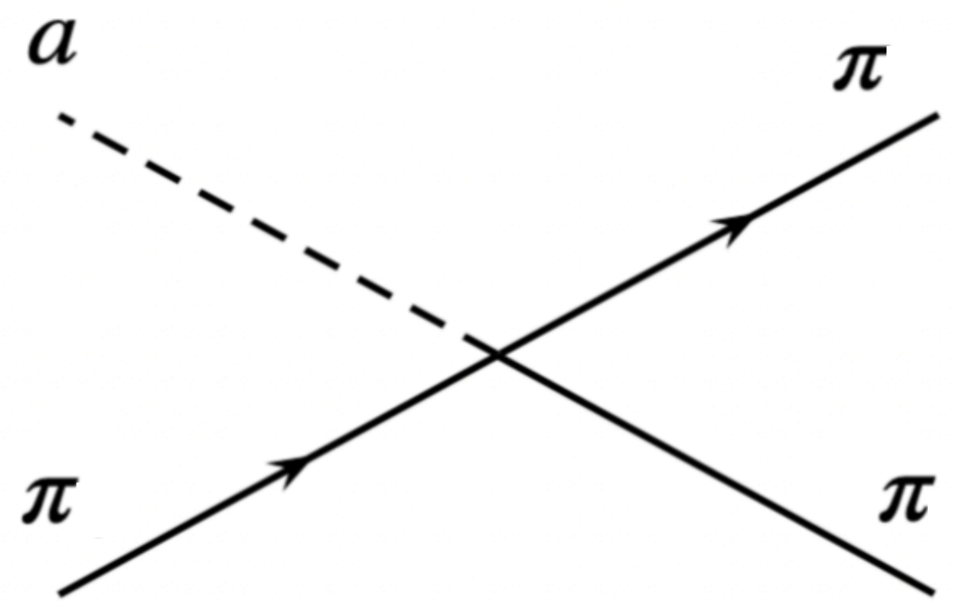
A possible discovery channel for the axion!

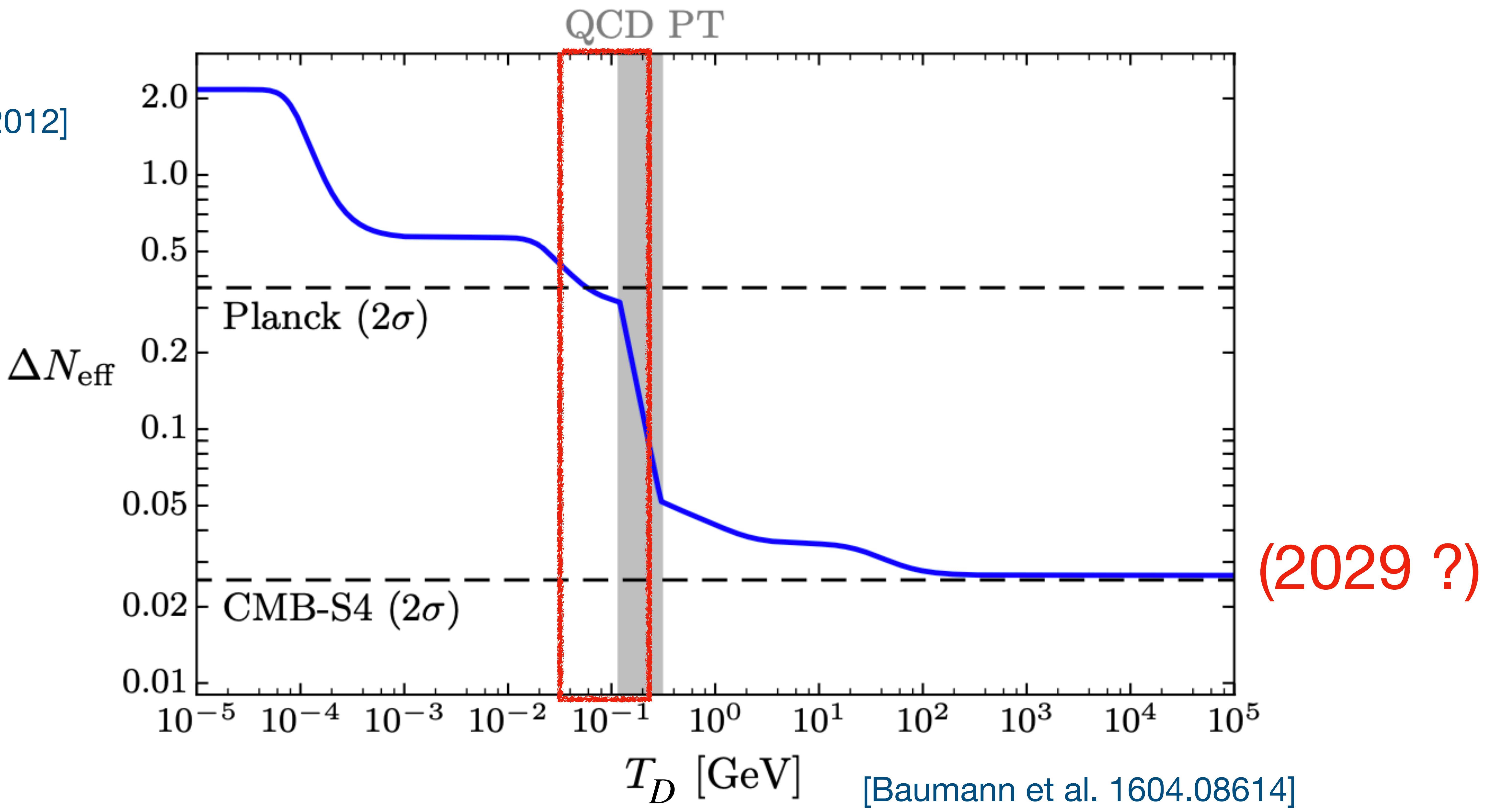
- Axions once in equilibrium with SM thermal bath contribute to the radiation density of the Universe (ΔN_{eff})

$$T_{\text{Decoupling}} \lesssim 155\ \text{MeV} = \text{T}_\text{c}$$

[Bazavov et al. 2012]

- Below QCD deconfinement-T the main thermalization channel is





[Baumann et al. 1604.08614]

Outline:

1. Thermal axion production from LO ChPT —> ΔN_{eff} and HDM bound
2. NLO corrections to assess the (bad) convergence of the chiral expansion;
3. Goal: extend the validity of ChPT up to T_c, via unitarization technique.

Breakdown of chiral perturbation theory for the axion hot dark matter bound

Luca Di Luzio,[1,2,3,*] Guido Martinelli,[4,†] and Gioacchino Piazza[5,‡]

Axion-pion thermalization rate in unitarized NLO chiral perturbation theory

Luca Di Luzio,[1,2,*] Jorge Martin Camalich,[3,4,†] Guido Martinelli,[5,‡] José Antonio Oller,[6,§] and Gioacchino Piazza[7,¶]

Axion-Pion Effective Lagrangian: Leading Order

$$\frac{m_\pi^2}{\Lambda_{\rm QCD}^2} \ll 1$$

$$\mathcal{L}_a^\chi = \frac{f_\pi^2}{4} Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2} Tr\left[c_q \sigma^a\right] J_\mu^a$$

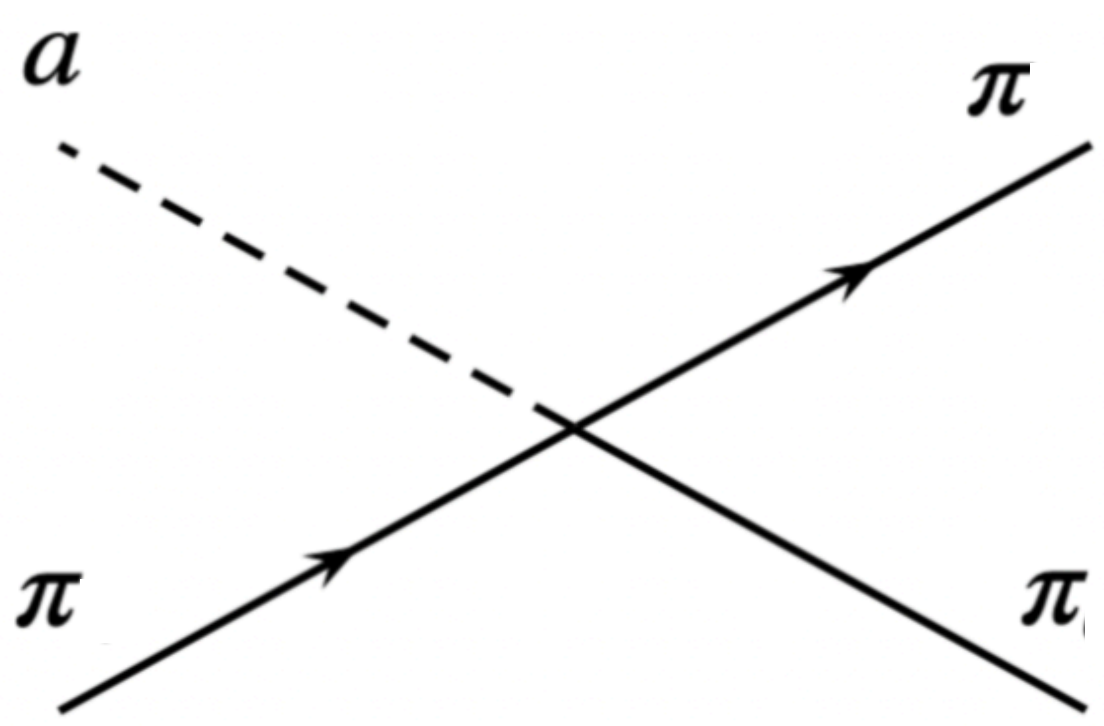
[Georgi, Kaplan, Randall, Phys. Lett. B **169** (1986)]

$$\begin{cases} U = e^{i\pi^a\sigma^a/f_\pi} \\ \chi = 2B_0\, e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \end{cases}$$

$$J_\mu^a = \frac{i}{4} f_\pi^2 Tr\left[\sigma^a\{U, (D^\mu U)^\dagger\}\right]$$

$$\mathcal{L}_{a\pi}^{\rm (LO)} = \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a\left(2\partial_\mu\pi_0\pi_+\pi_- - \pi_0\partial_\mu\pi_+\pi_- - \pi_0\pi_+\partial_\mu\pi_-\right)$$

$$C_{a\pi} = \frac{1}{3}\left(\frac{m_d - m_u}{m_u + m_d} + c_d^0 - c_u^0\right)$$



Axion thermal production in the Early Universe

To extract the HDM bound we compute the axion decoupling temperature T_D via the freeze-out condition*

$$\Gamma_a(T_D) = H(T_D)$$

Rate of reactions that keep the axions in thermal equilibrium

$$\Gamma = \frac{1}{n_a^{\rm eq}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \sum |\mathcal{M}|^2 \\ (2\pi)^4 \delta^4 \left(p_1 + p_2 - p_3 - p_4\right) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

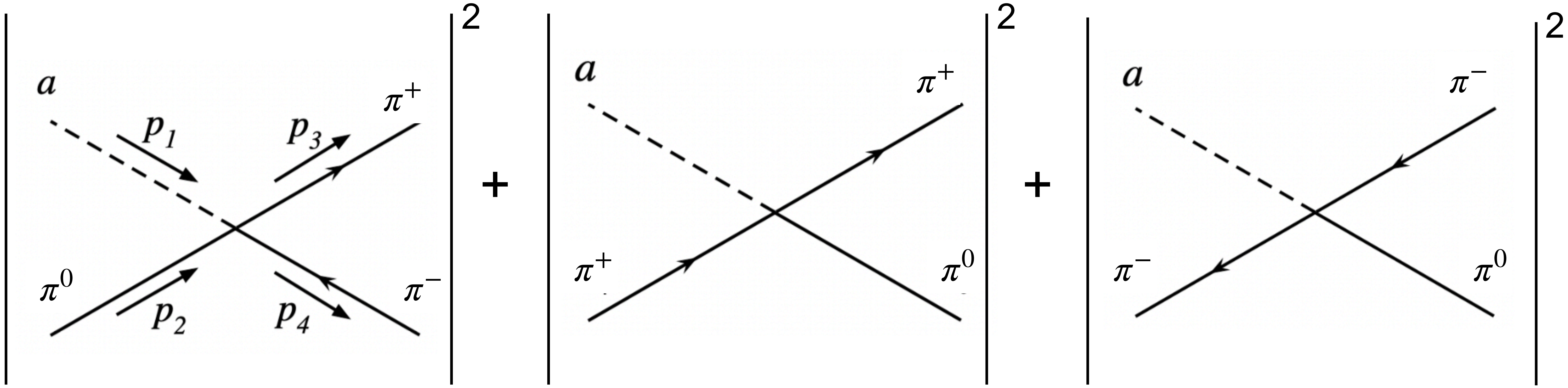
Hubble Rate

$$H(T) = \sqrt{4\pi^3 g_\star(T)/45}\, T^2/m_{\rm pl}$$

* For improved treatment of axion freeze-out see [Notari, Rompineve, Villadoro 2211.03799]

Leading order scattering amplitude

$$\mathscr{L}_{a\pi}^{\mathrm{LO}} = \frac{C_{a\pi}}{f_a f_\pi}\partial^\mu a\left(2\partial_\mu\pi^0\pi^+\pi^- - \pi^0\partial_\mu\pi^+\pi^- - \pi^0\pi^+\partial_\mu\pi^-\right)$$



$$\sum |\mathcal{M}|^2_{\mathrm{LO}} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

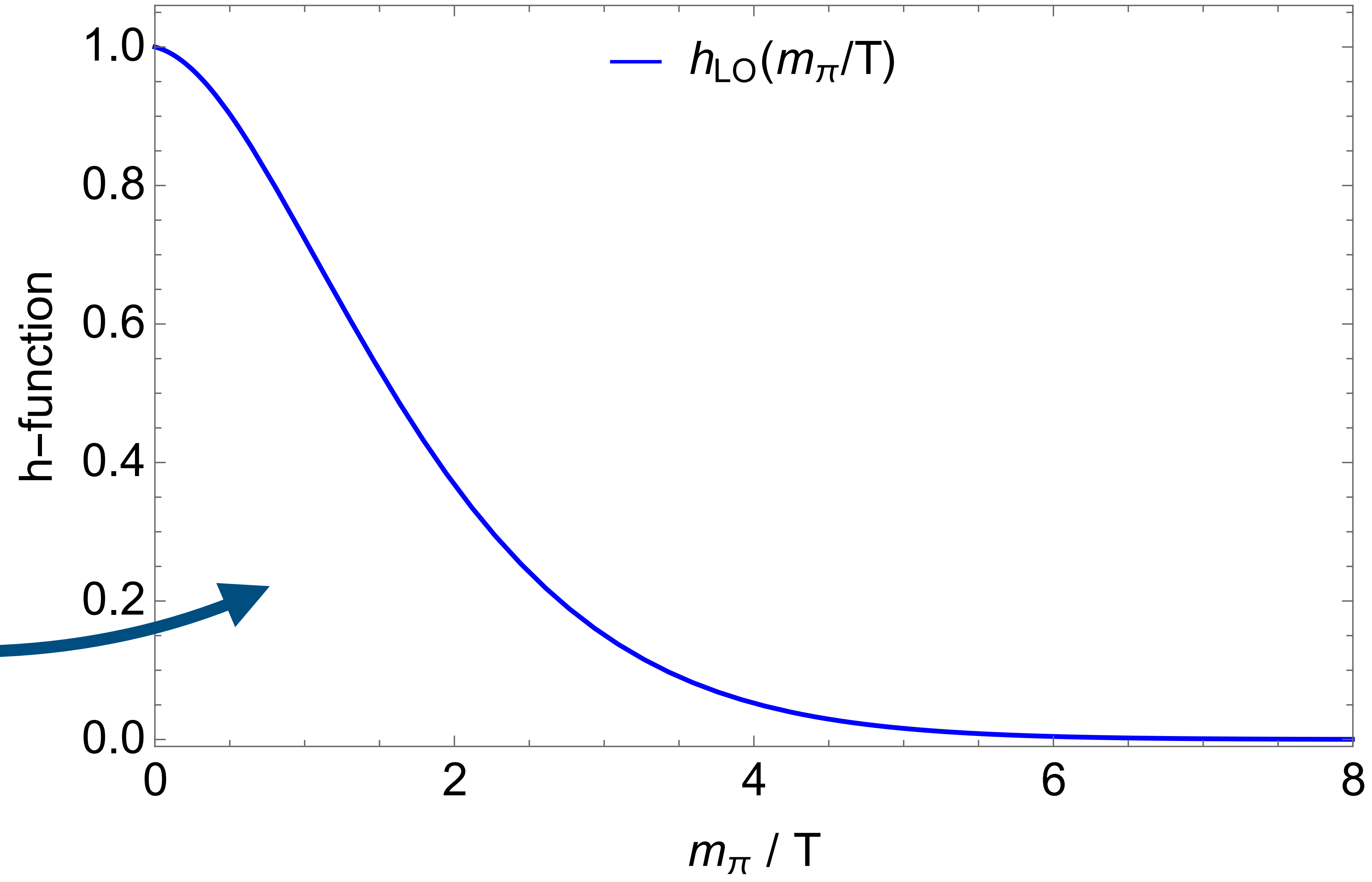
Thermal scattering rate

$$\Gamma = \frac{1}{n_a^{\rm eq}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \sum |\mathcal{M}|^2$$
$$(2\pi)^4 \delta^4 \left(p_1 + p_2 - p_3 - p_4\right) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4} \left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

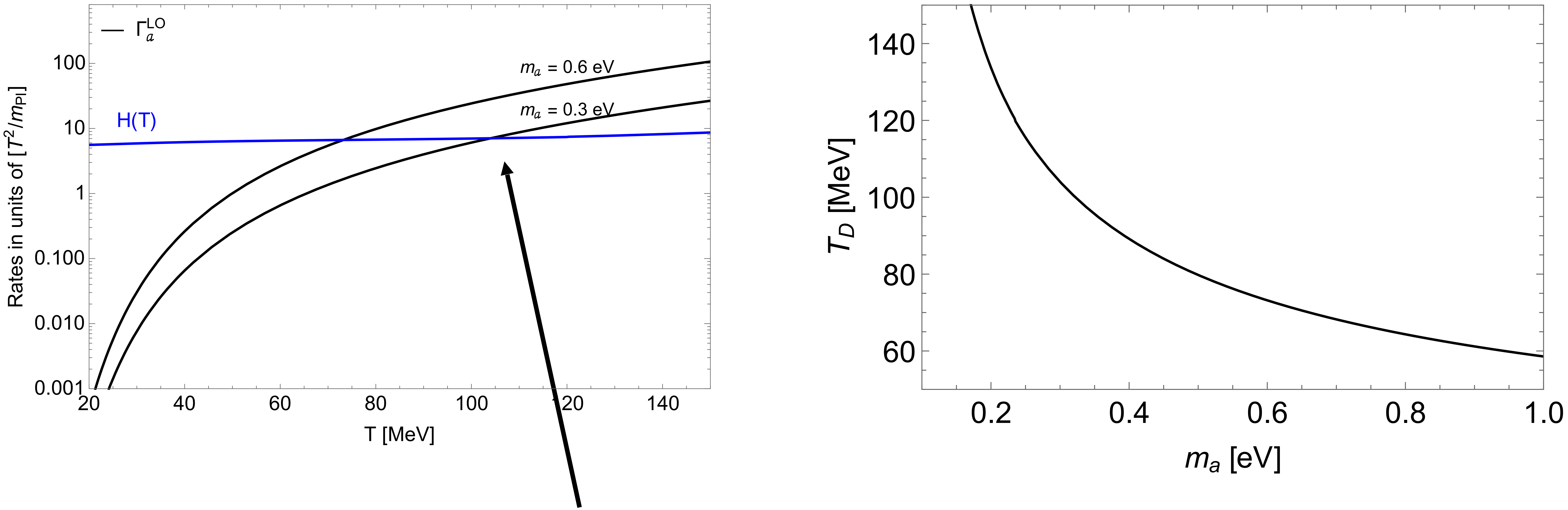
Integrating:

$$\Gamma(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\rm LO}(m_\pi/T)$$



Di Luzio, Martinelli, **GP** *PRL*126 (2021) 24, 241801 [arXiv: 2101.10330]

Decoupling T

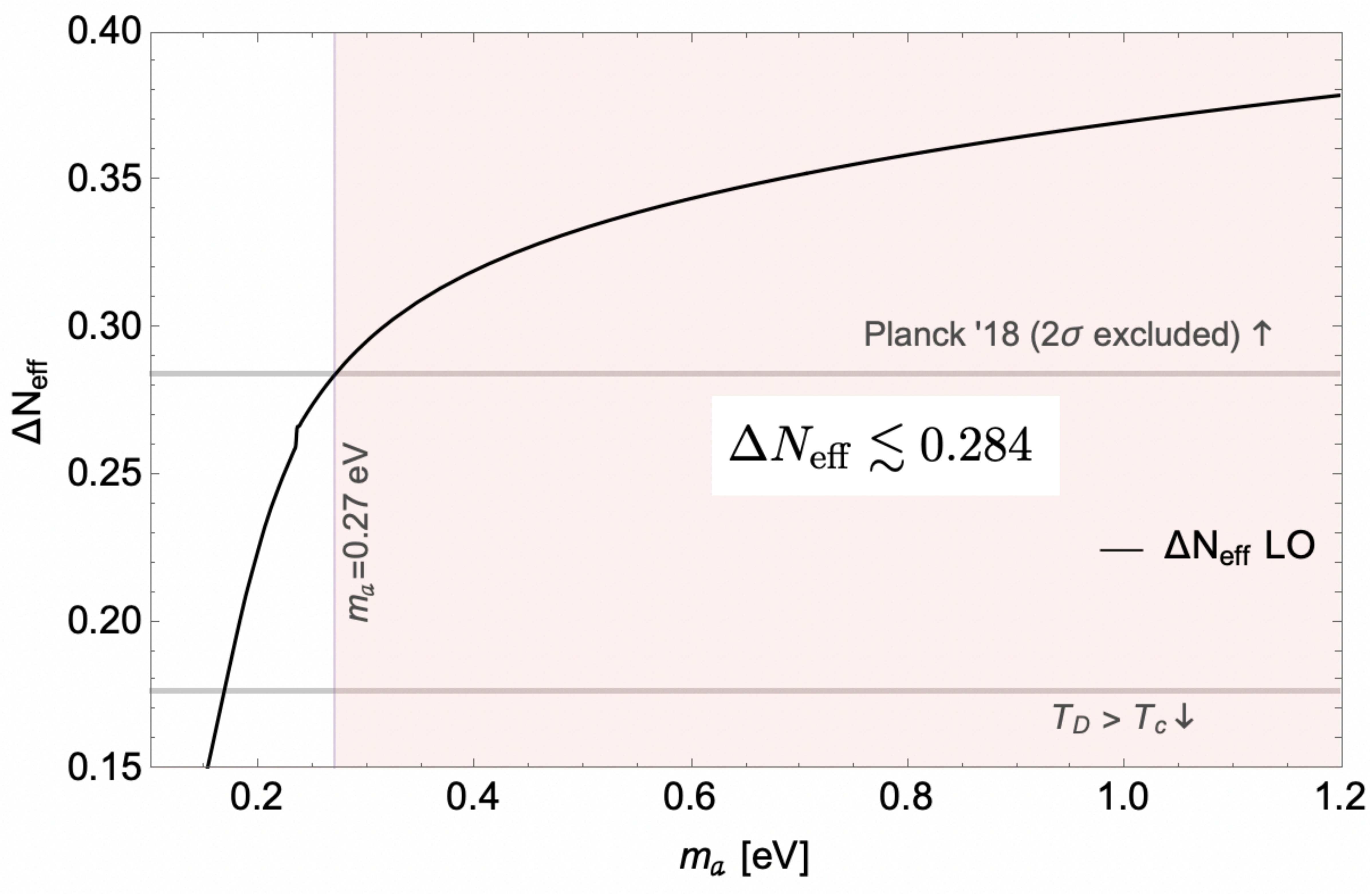


Decoupling Temperature depends on m_a

ΔN_{eff} at LO in ChPT

Axion contribution to the Number of relativistic species

$$\Delta N_{\text{eff}} = 0.027 \left(\frac{106.75}{g_S(T_D)} \right)^{4/3}$$



LO ChPT set the bound

$$m_a < 0.27 \text{ eV}$$

See further:

[Melchiorri, Mena, Slosar, arXiv:0705.2695]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:0803.1585]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:1004.0695]
[Di Valentino, Giusarma, Lattanzi, Mena, Melchiorri, Silk, arXiv:1507.08665]

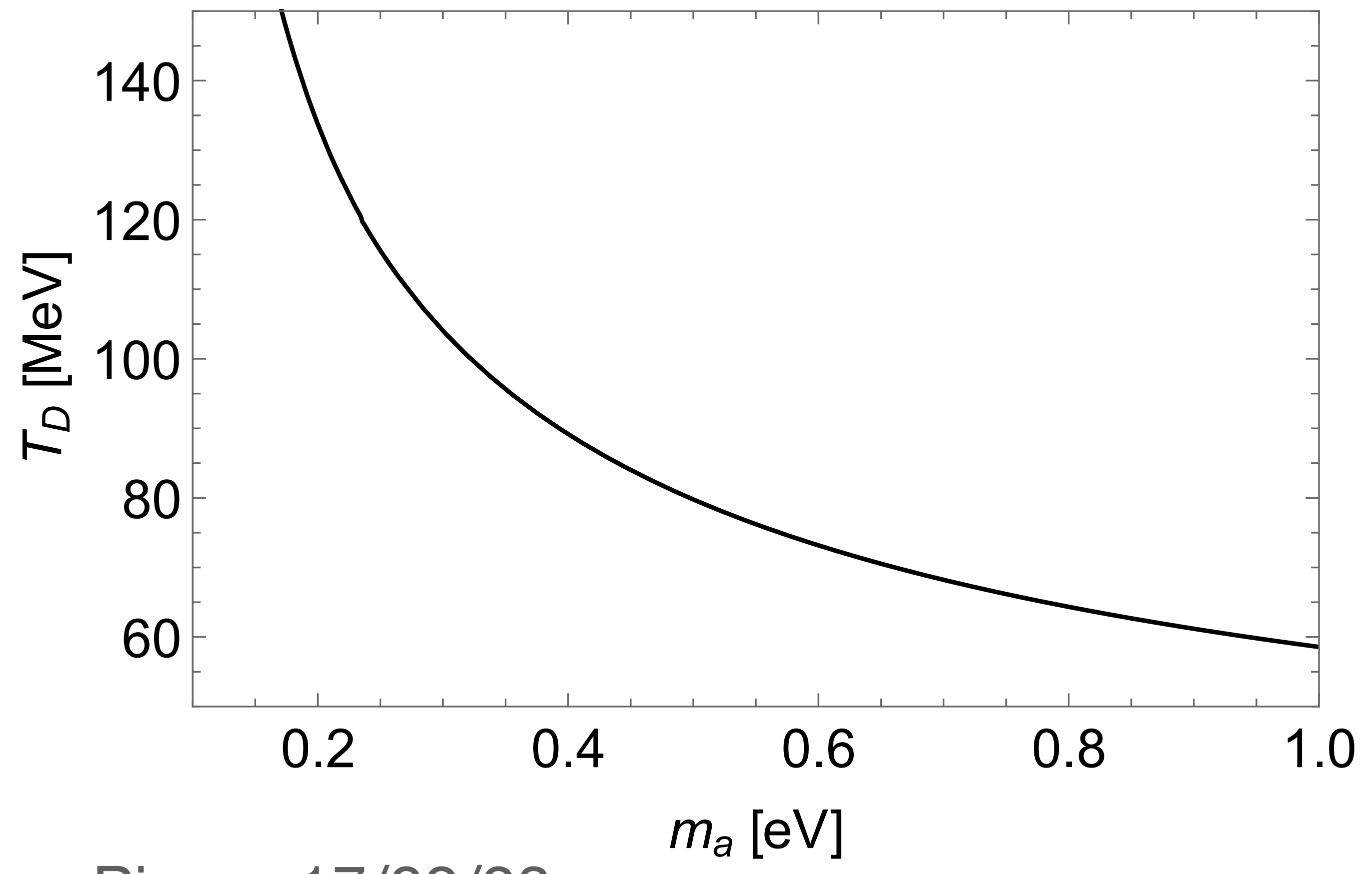
But… is ChPT valid?

The mean energy of π, a at $T \simeq 80$ MeV is $\langle E \rangle \equiv \rho/n \simeq 305$ MeV, 220 MeV

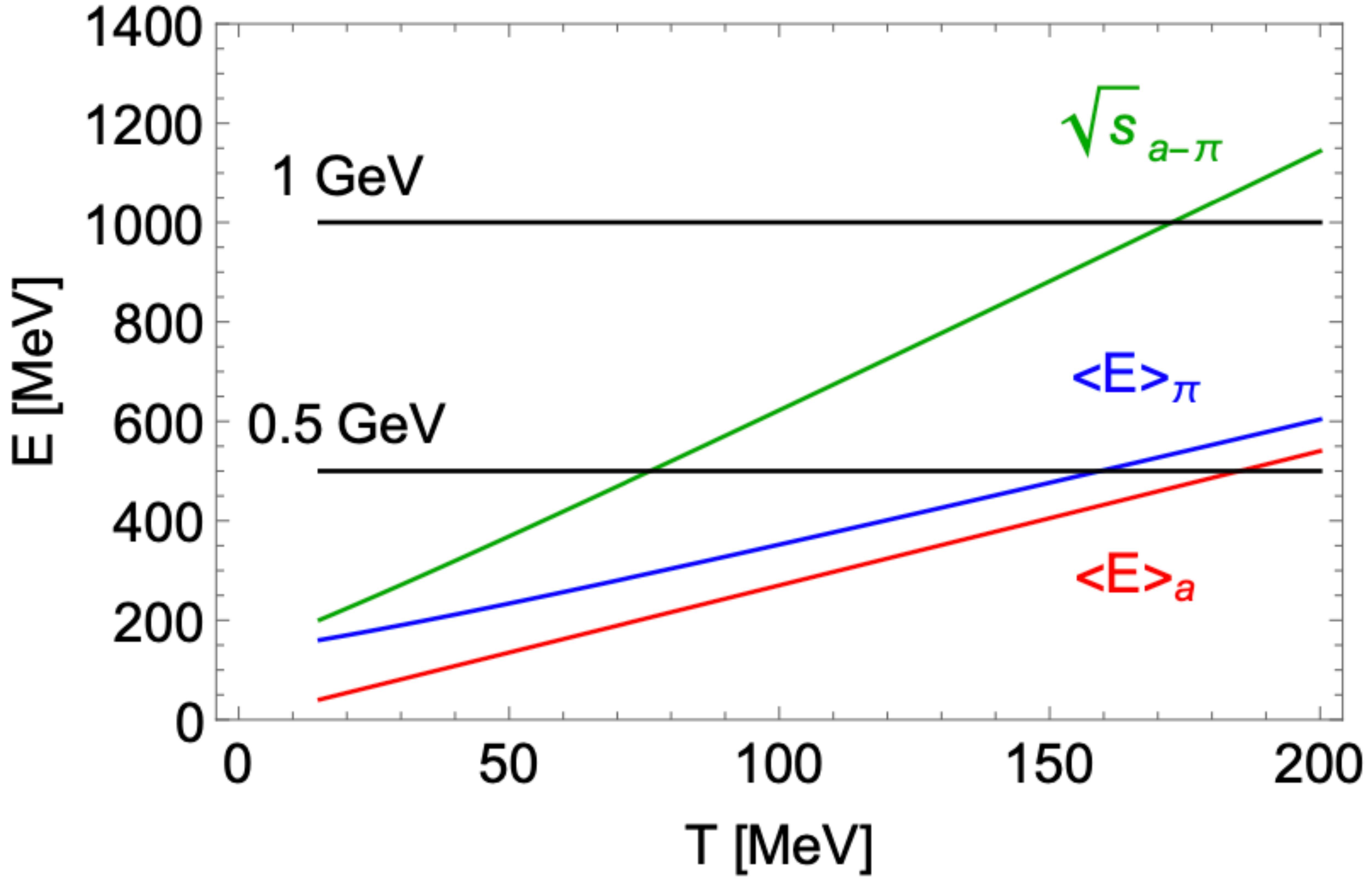
BUT

ChPT violates unitarity for $E \gtrsim 460$ MeV

see e.g. [Donoghue et al., PhysRevD.86.014025]



$$\langle E \rangle (T) \sim \frac{\rho(T)}{n(T)}$$



Is ChPT reliable?

NLO axion production rate

Axion-Pion scattering: Next-to-Leading Order

Ingredients

Tree-level graph from NLO Lagrangian and loop amplitudes from LO Lagrangian contributes to the same Order

$$
\begin{aligned}
\mathcal{L}_{\mathrm{NLO}} =\ & \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\
& + \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right] \\
& + l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu} U f_L^{\mu\nu} U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)\right] \\
& + i\frac{l_6}{2}\mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\
& - \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{4}\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1-h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\
& \left. + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)
\end{aligned}
$$

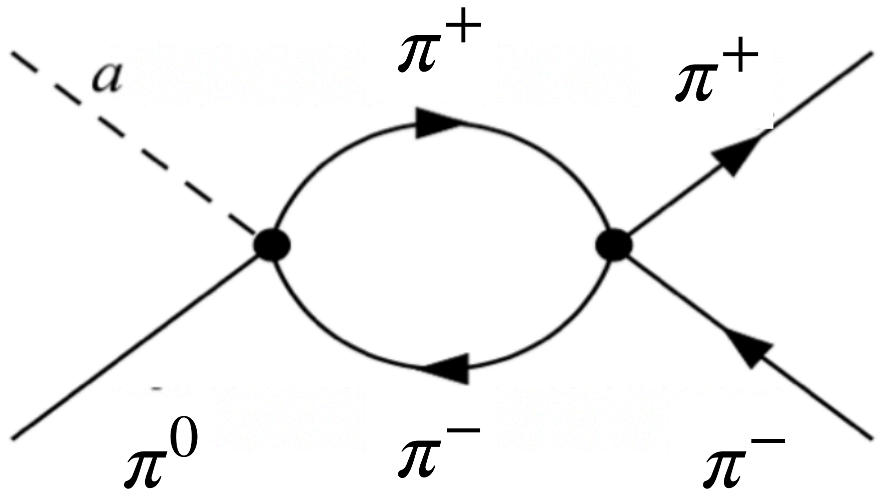
NLO Lagrangian

+

NLO chiral axial current J^a_μ

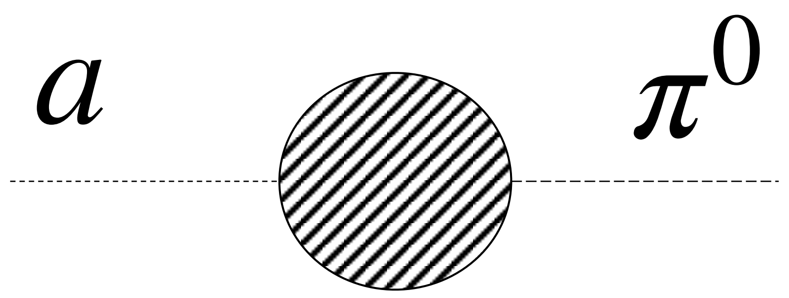
$$\mathcal{L}_a^\chi \supset \frac{\partial^\mu a}{f_a}\mathrm{Tr}\frac{1}{2}\left[c_q \sigma^a\right] J^a_\mu$$

+



1-loop amplitudes from LO

+



$a - \pi_0$ diagonalization

Amplitudes

- After renormalization & including NLO corrections to f_π

$$
\begin{aligned}
\mathcal{M}^{\rm NLO}_{a\pi_0\to\pi_+\pi_-} = \frac{C_{a\pi}}{192\pi^2 f_\pi^3 f_a}\Bigg\{ & 15m_\pi^2(u+t) - 11u^2 - 8ut - 11t^2 - 6\overline{\ell_1}\left(m_\pi^2 - s\right)\left(2m_\pi^2 - s\right) \\
& - 6\overline{\ell_2}\left(-3m_\pi^2(u+t) + 4m_\pi^4 + u^2 + t^2\right) + 18\overline{\ell_4} m_\pi^2 (m_\pi^2 - s) \\
& + 3\Bigg[3\sqrt{1-\frac{4m_\pi^2}{s}}\, s\left(m_\pi^2 - s\right)\ln\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right) \\
& + \sqrt{1-\frac{4m_\pi^2}{t}}\left(m_\pi^2(t-4u) + 3m_\pi^4 + t(u-t)\right)\ln\left(\frac{\sigma(t)-1}{\sigma(t)+1}\right) \\
& + \sqrt{1-\frac{4m_\pi^2}{u}}\left(m_\pi^2(u-4t) + 3m_\pi^4 + u(t-u)\right)\ln\left(\frac{\sigma(u)-1}{\sigma(u)+1}\right)\Bigg]\Bigg\} \\
& - \frac{4\ell_7 m_\pi^2 m_d\left(s - 2m_\pi^2\right) m_u\left(m_d - m_u\right)}{f_\pi^3 f_a\left(m_d + m_u\right)^3},
\end{aligned}
$$

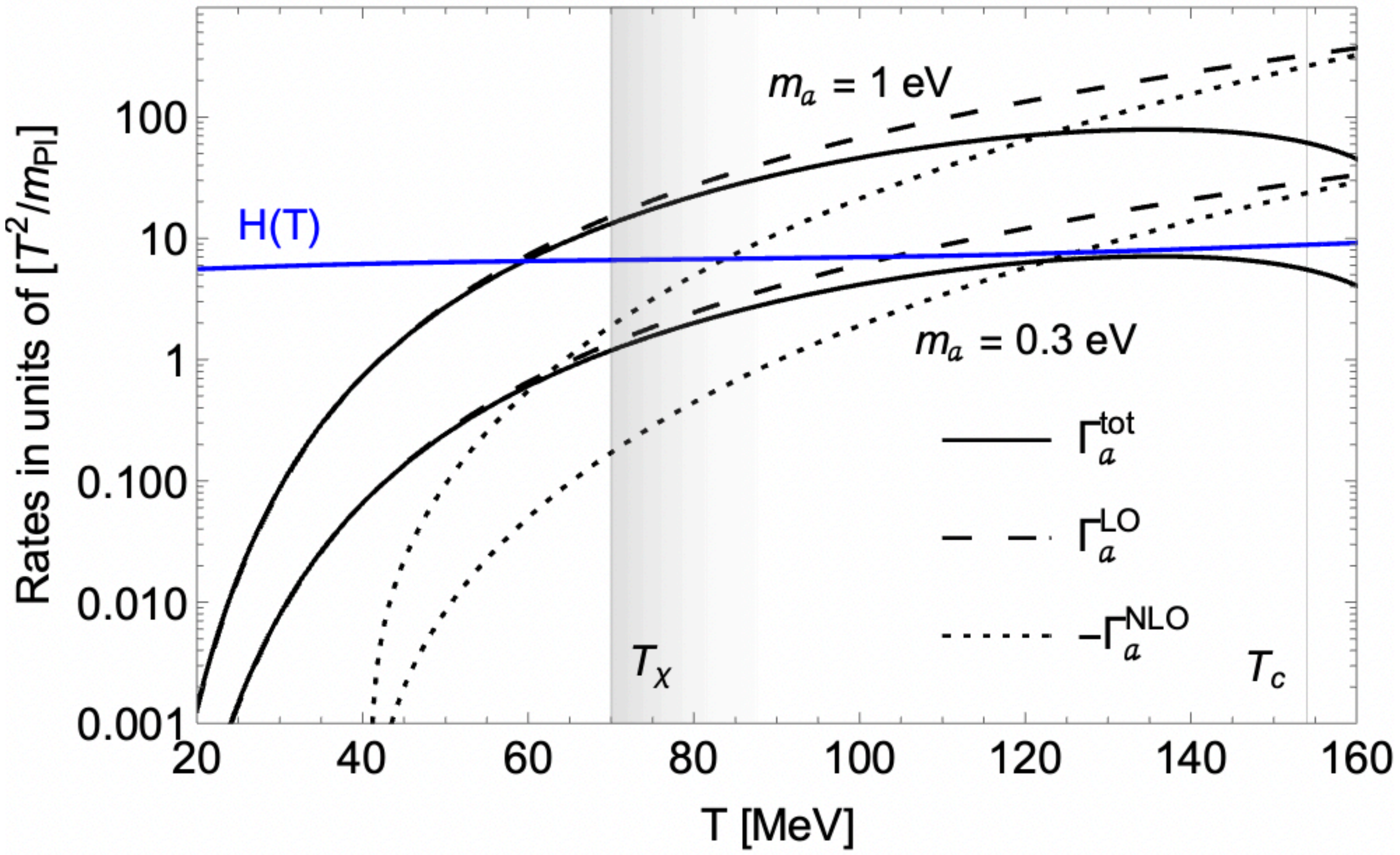
- Other pionic channels obtained via $s \to t, u$

L. Di Luzio, G. Martinelli, **GP** [2101.10330]

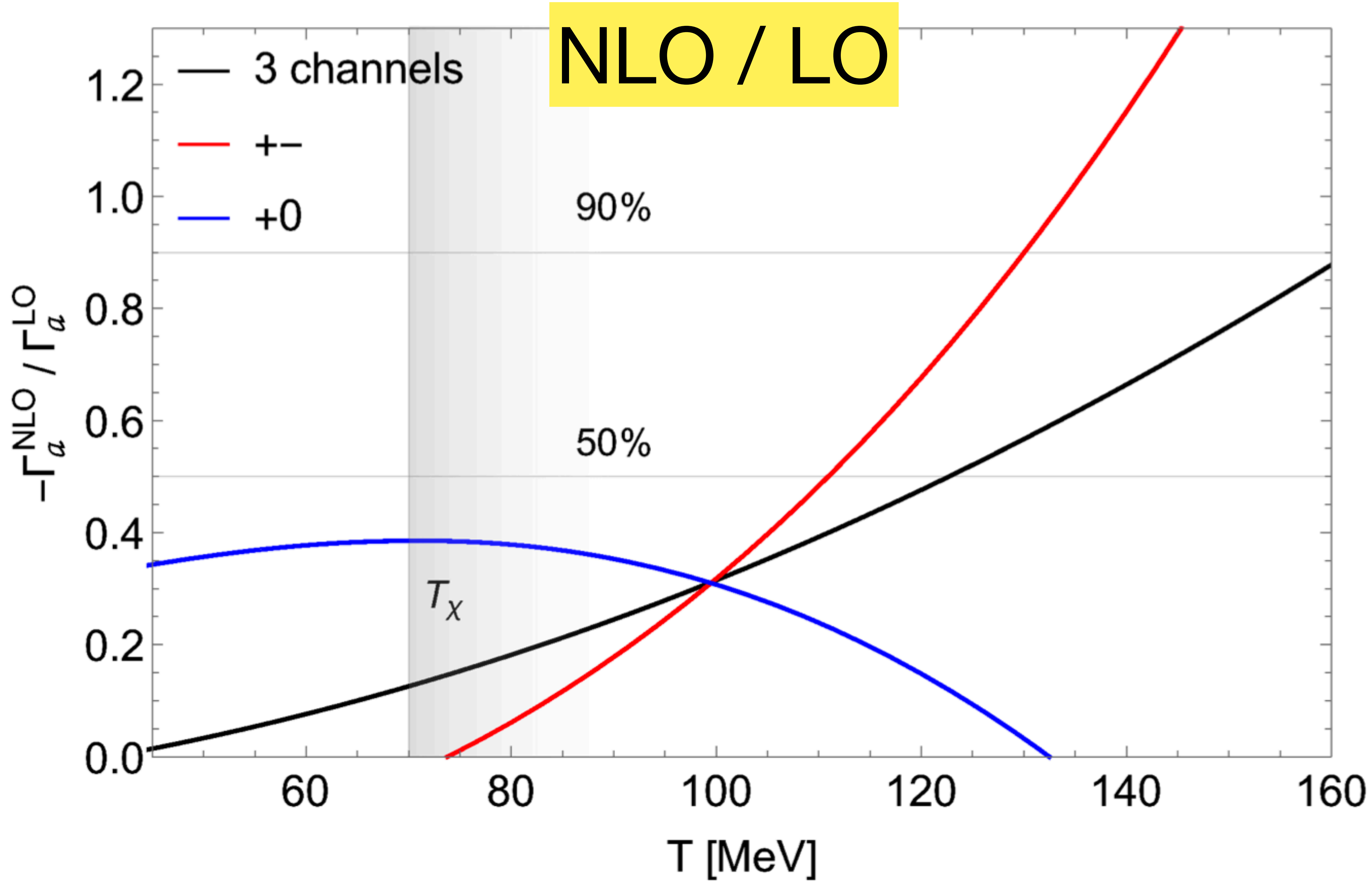
NLO Thermalization rate

$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO}]$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.163\, T^5 \left[h_{\rm LO}(m_\pi/T) - 0.290 \frac{T^2}{f_\pi^2}\, h_{\rm NLO}(m_\pi/T)\right]$$



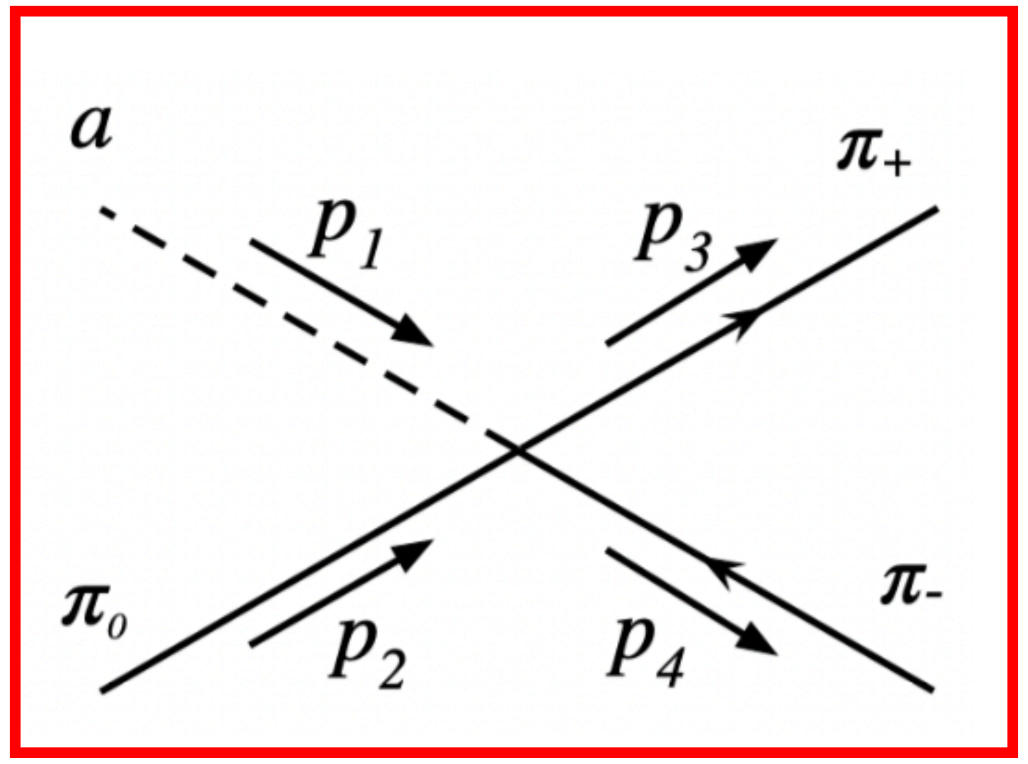
Breakdown of ChPT : Γ

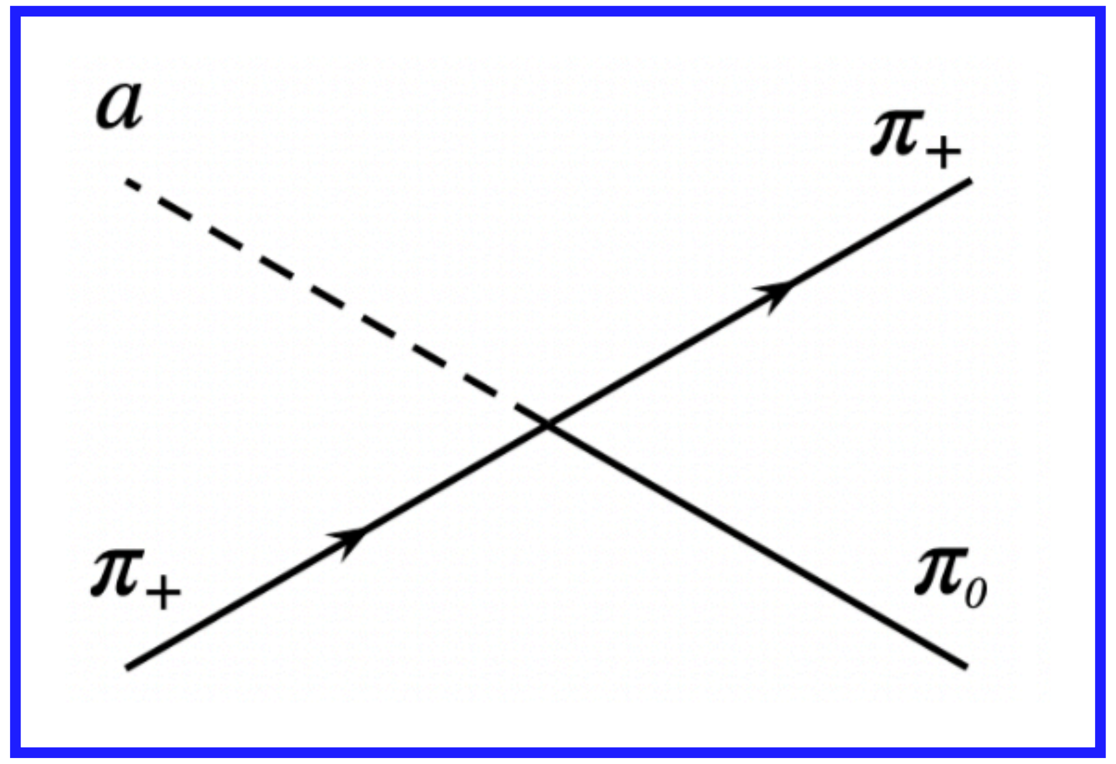


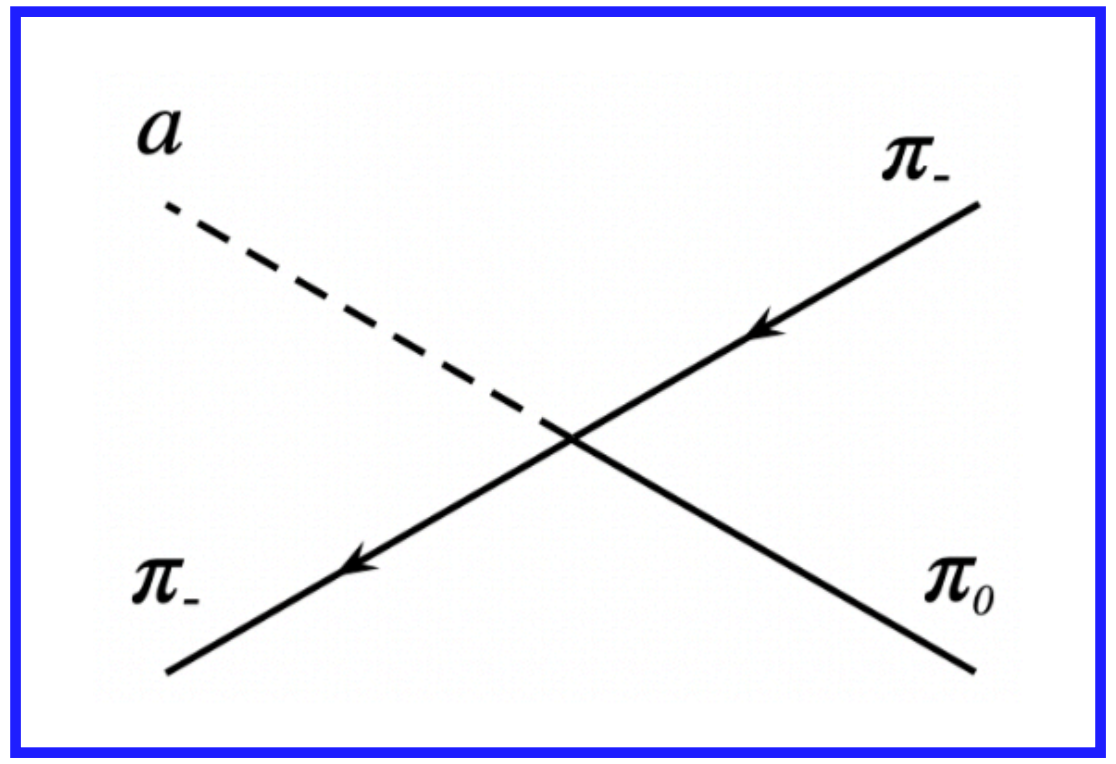
✦ The NLO corrections to total Γ reach 50% x LO at $T \simeq 120$ MeV, due to accidental cancellations;

✦ A more realistic estimate of T_χ by looking at the first exclusive channels with large NLO correction.

➡ In $\pi^+\pi^0$ big corrections at $T_\chi \simeq 70$ MeV

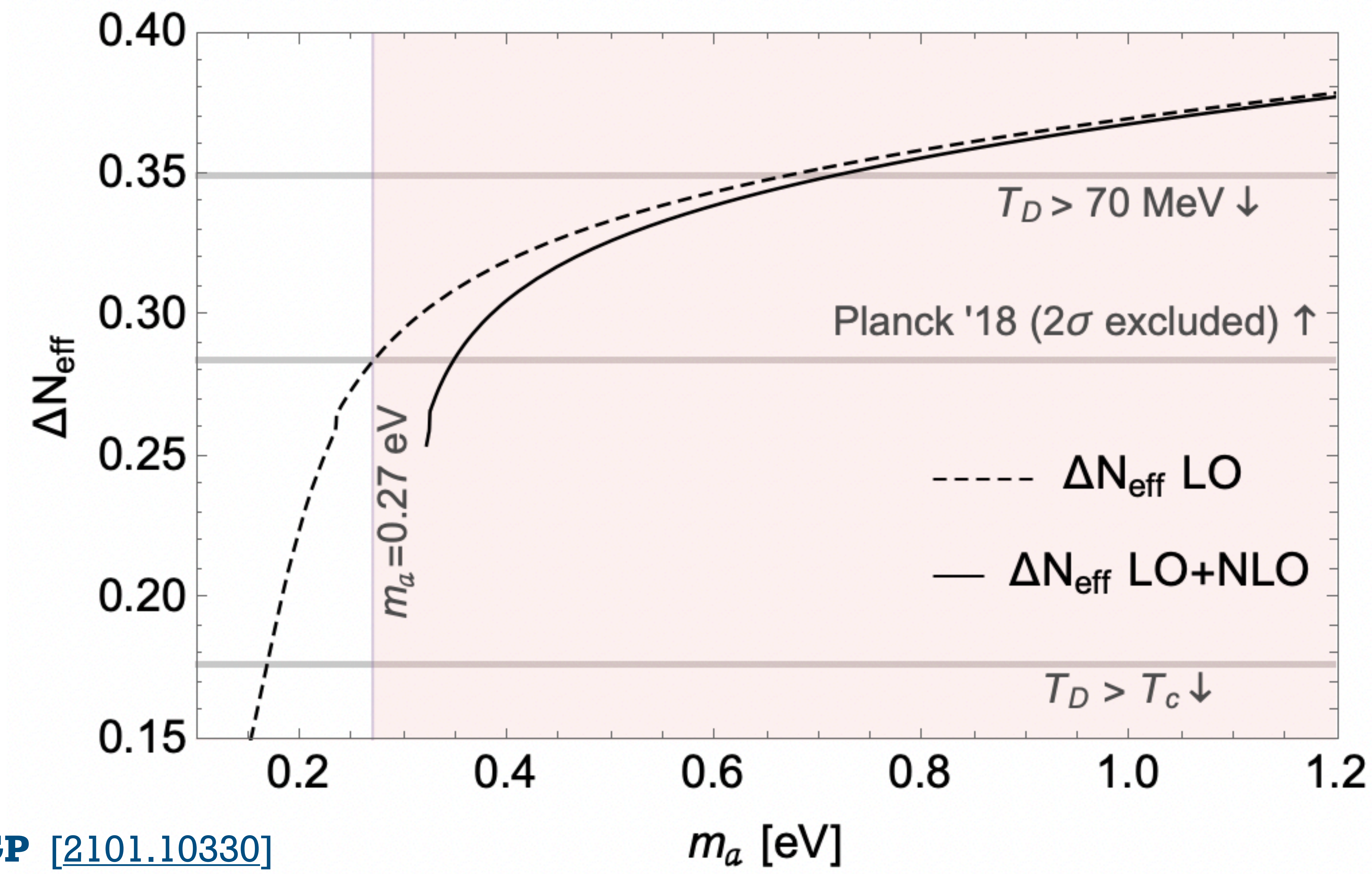






$\Delta N_{\rm eff}$ including NLO correction

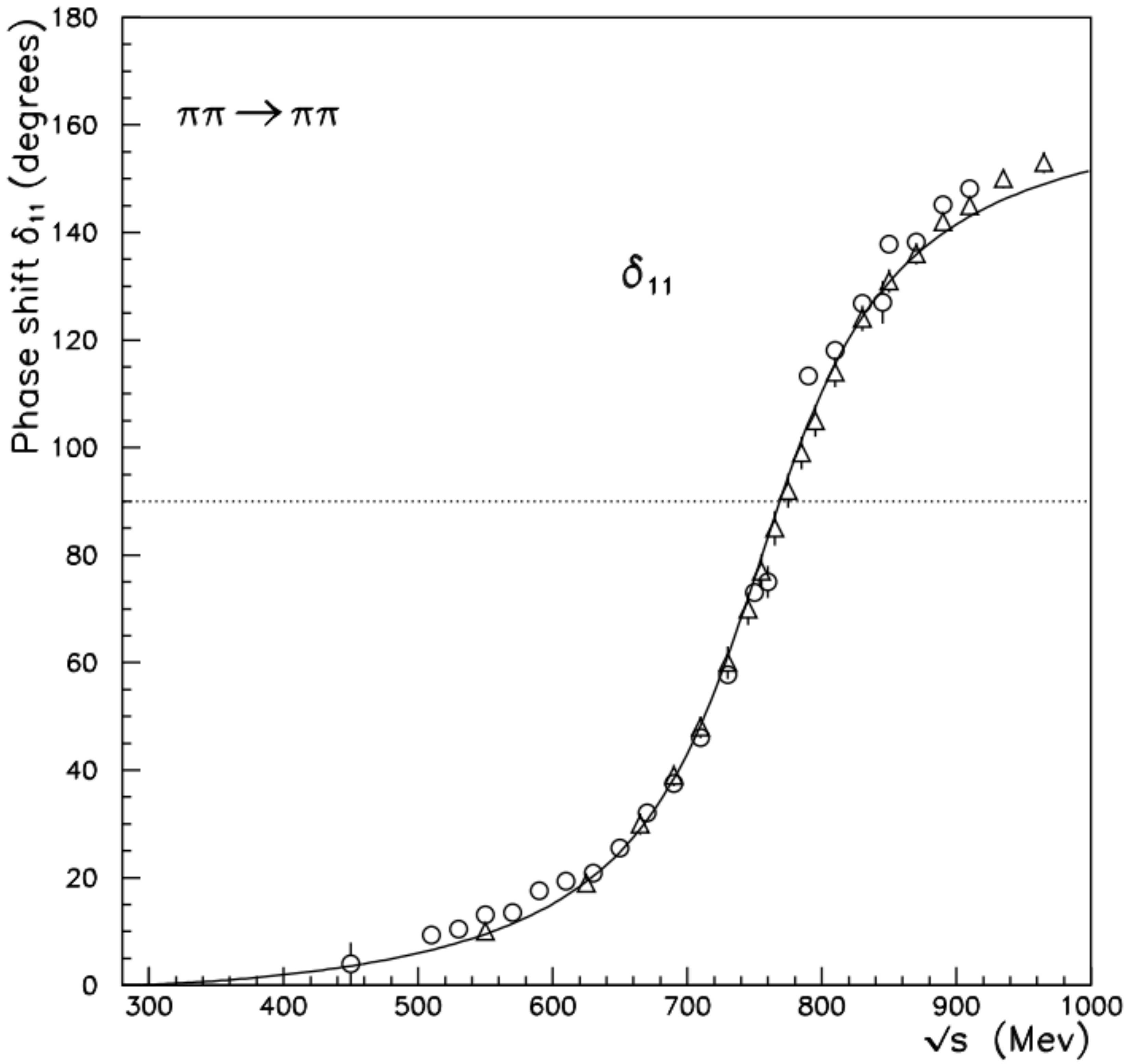
$T_{\rm D}$ cannot be extracted in the region of interest since the *NLO ~ LO for* $T > 70\ {\rm MeV}$

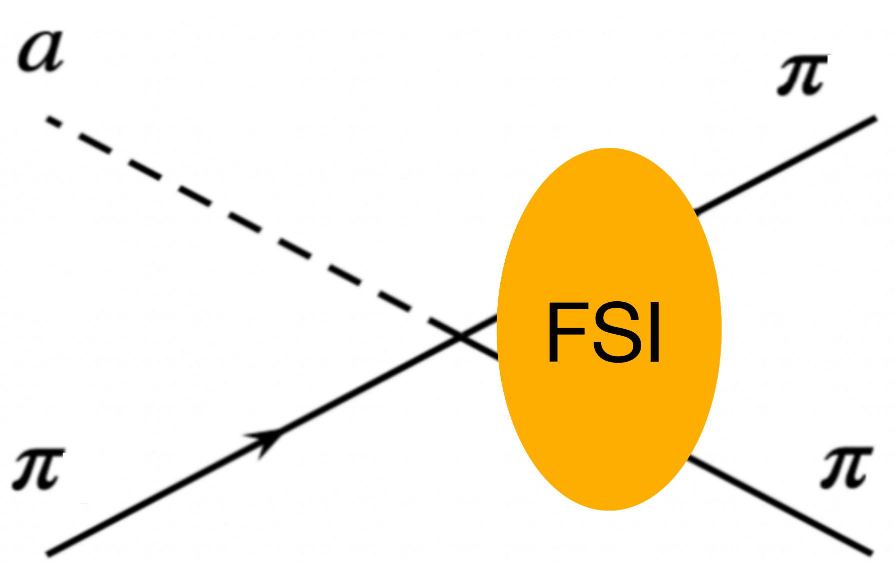


L. Di Luzio, G. Martinelli, **GP** [2101.10330]

Unitarization of Axion-Pion scattering

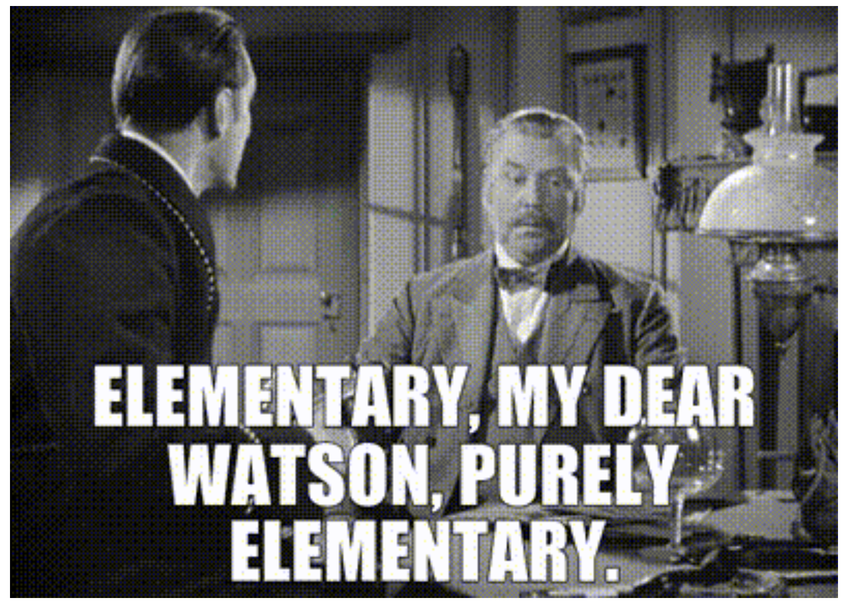
Di Luzio, Martinelli, Camalich, Oller, **GP** [arXiv: 2211.05073]





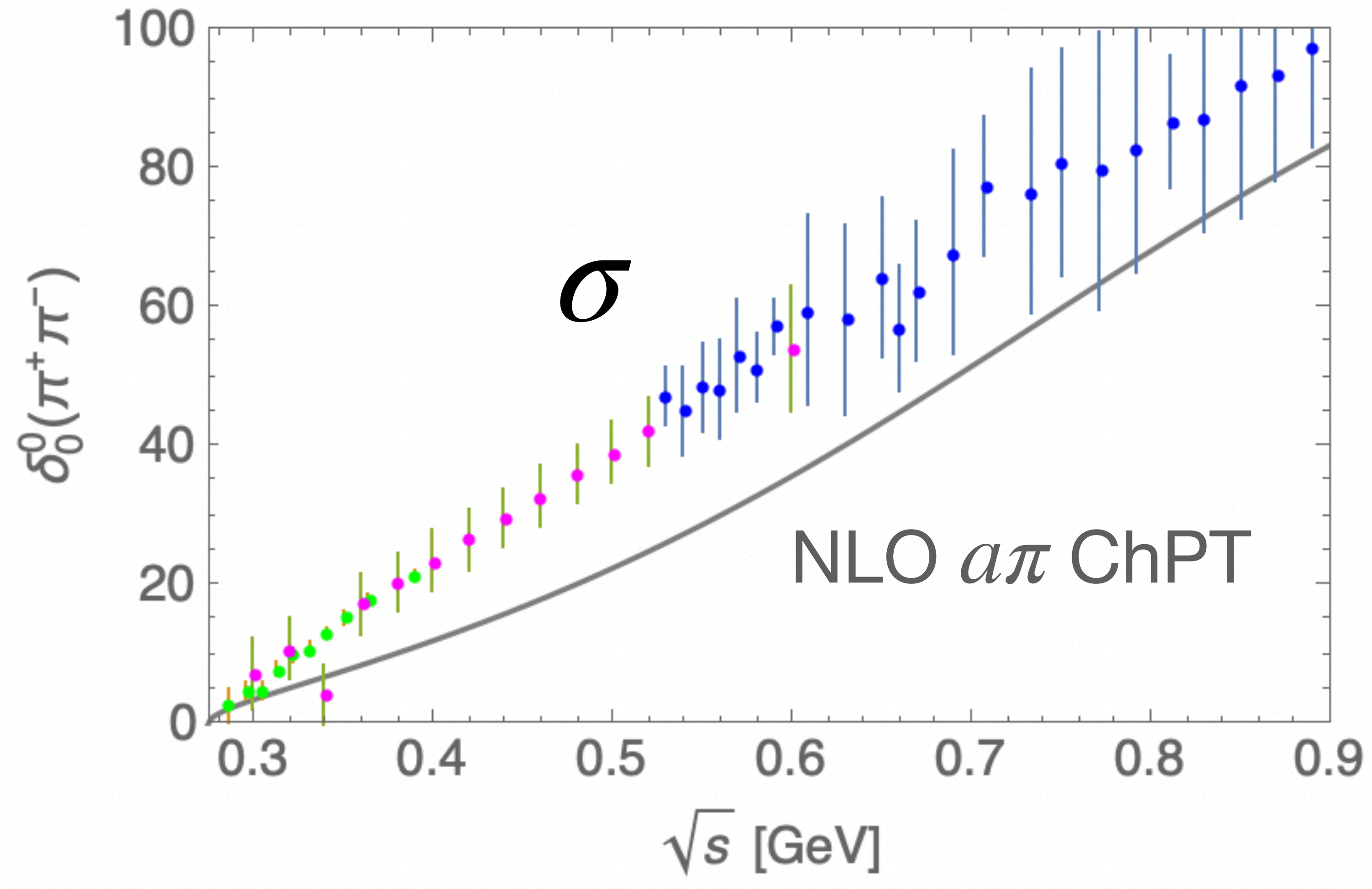
$\pi\pi$ final-state interactions (FSI) are resonant $\begin{cases} \sigma \text{ or } f_0(500) \text{ in } I = L = 0 \\ \rho(770) \text{ in } I = L = 1 \end{cases}$

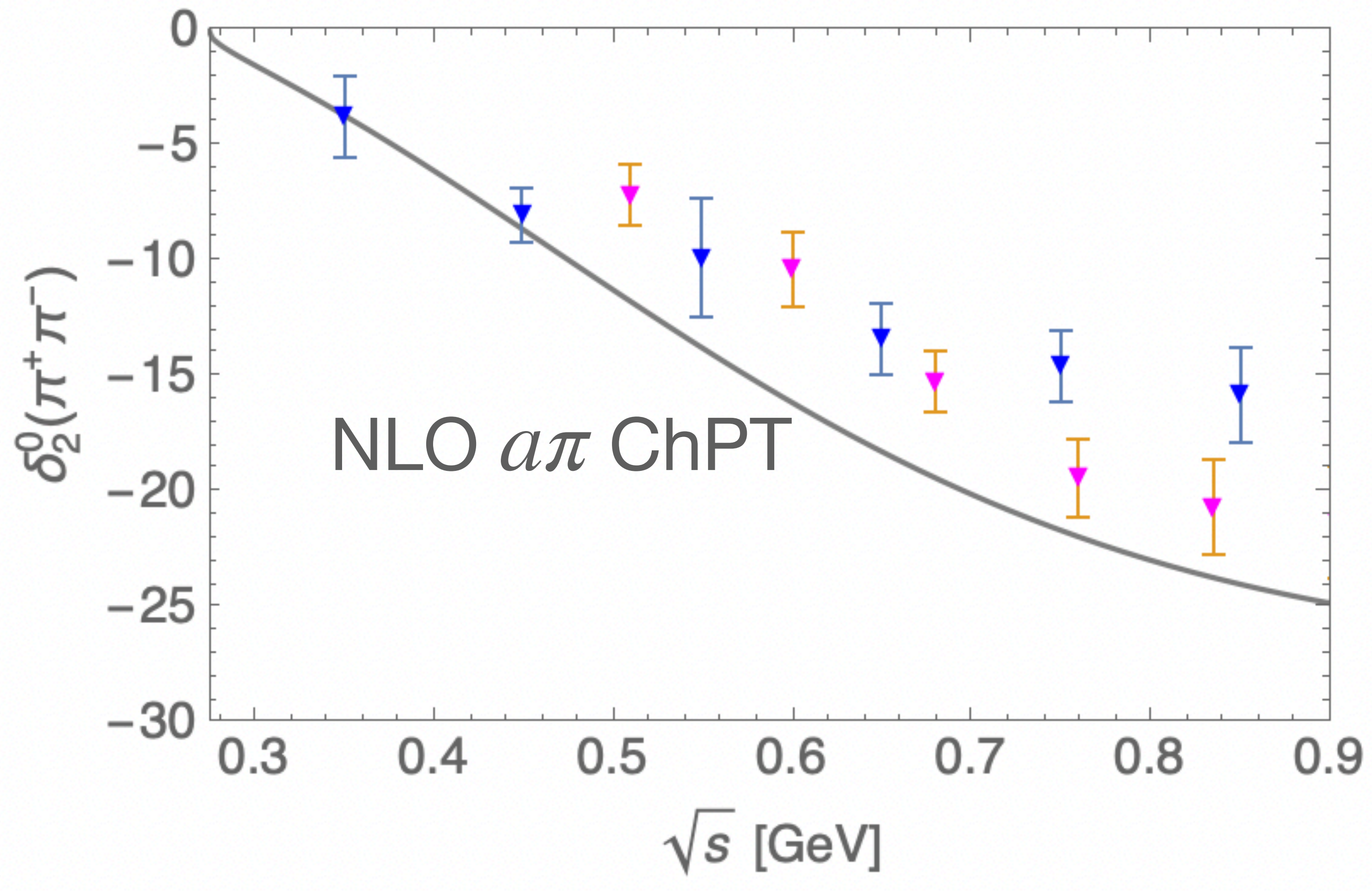
<u>ChPT cannot produce resonances</u>

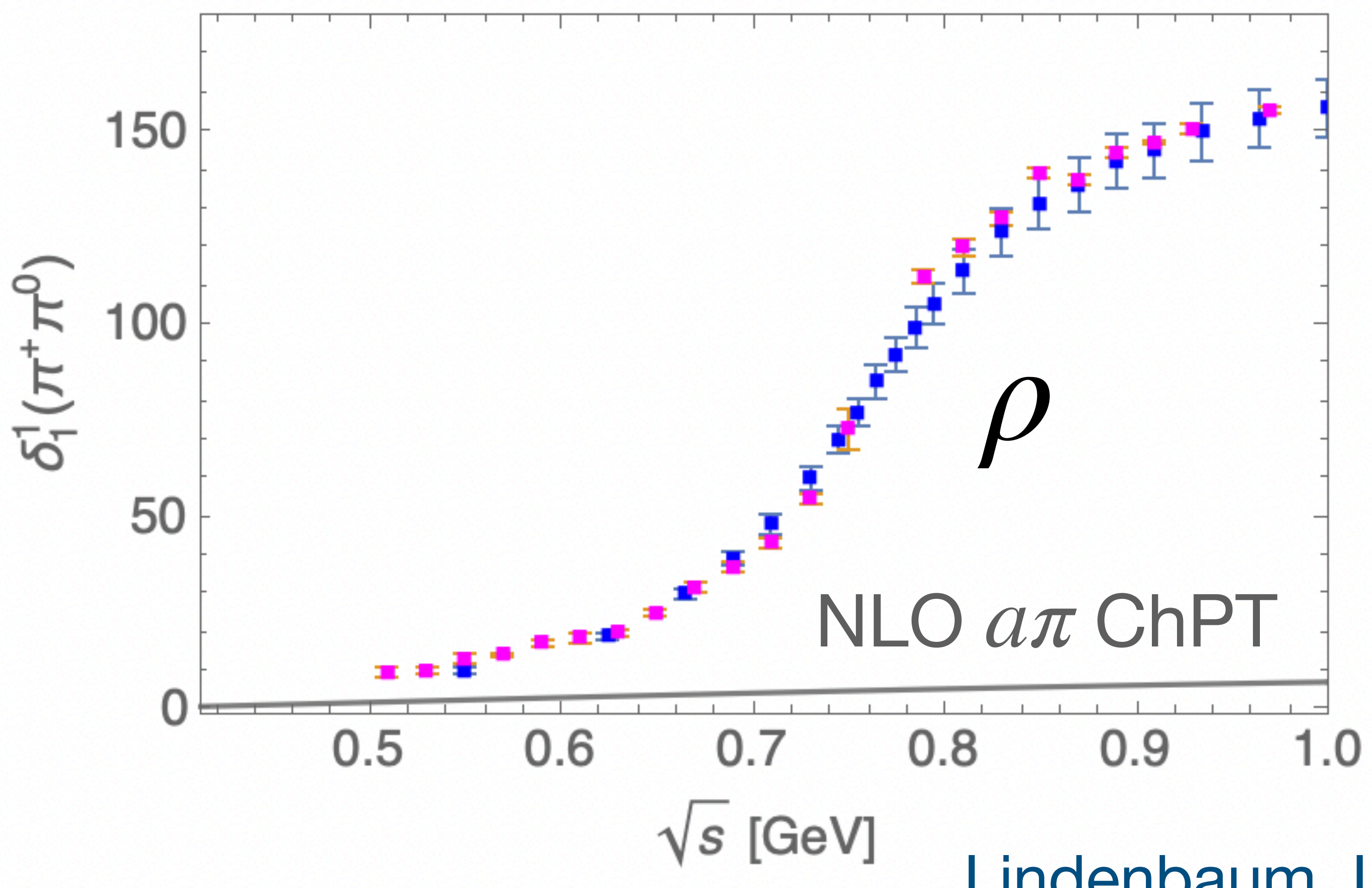


<u>Unitarity</u> $\Rightarrow (\delta_a)_I^\ell = (\delta_{\pi-\text{scatt}})_I^\ell$

Comparing NLO $a\pi$ ChPT to $\pi\pi$ data: $(\delta_a)_I^\ell \neq (\delta_{\pi-\text{scatt}})_I^\ell$







☹

Lindenbaum, Longacre ’92
Estabrooks, Martin ’74
…….

Unitarization to extend the validity of ChPT

✤Inverse Amplitude Method (IAM): [Truong, PRL **61**, 2526]

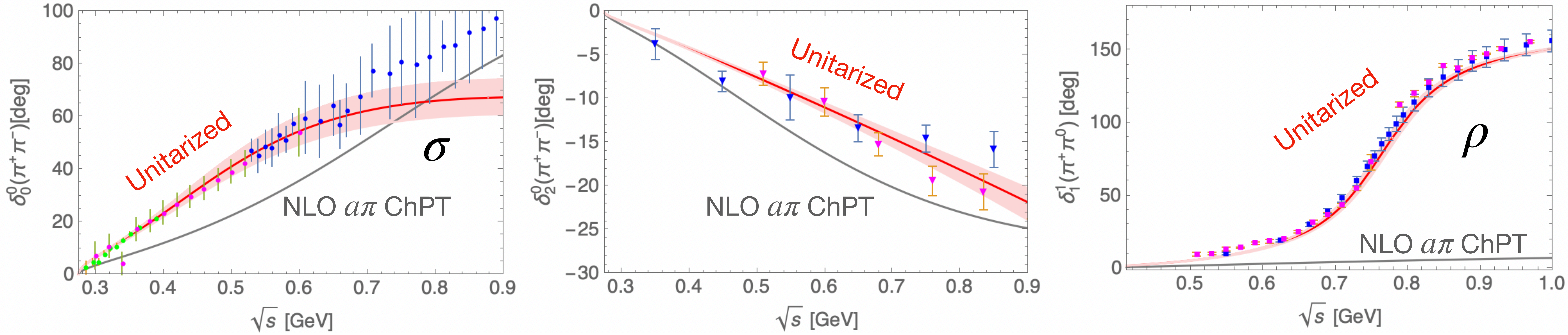
Definite I, J amplitudes

$$A_{IJ}(s) = \frac{A^{(2)}_{IJ}(s)}{1 - A^{(4)}_{IJ}(s)/A^{(2)}_{IJ}(s)}$$

The IAM amplitude satisfies unitarity and has the correct low-energy expansion of ChPT up to $\mathcal{O}(p^4)$

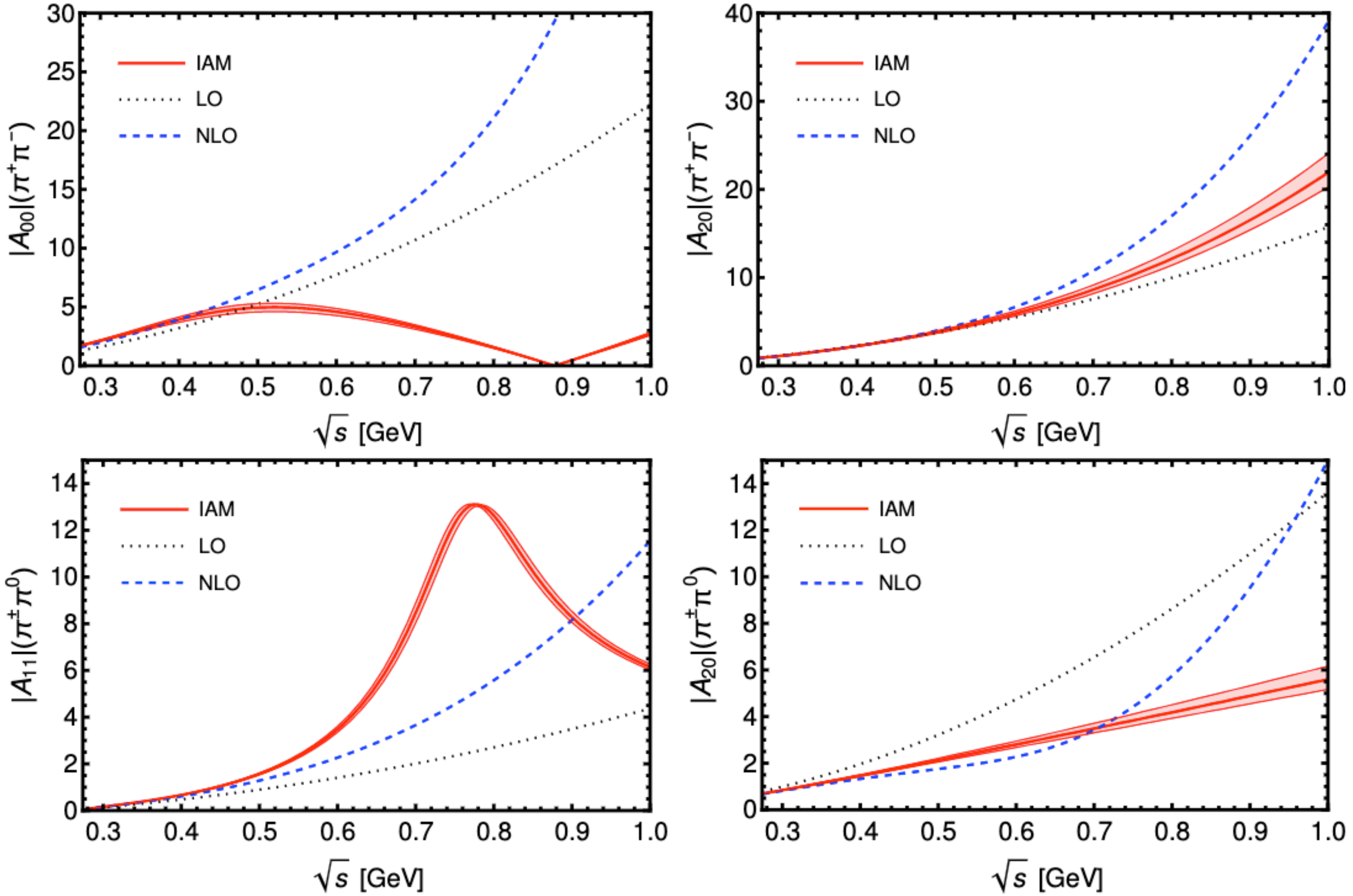
IAM LECs from fit to $\pi\pi$ scatt. [Dobado, Pelaez 1997]

✓ Phases obtained in IAM correspond to phases of $\pi\pi$ scattering: Watson th.!

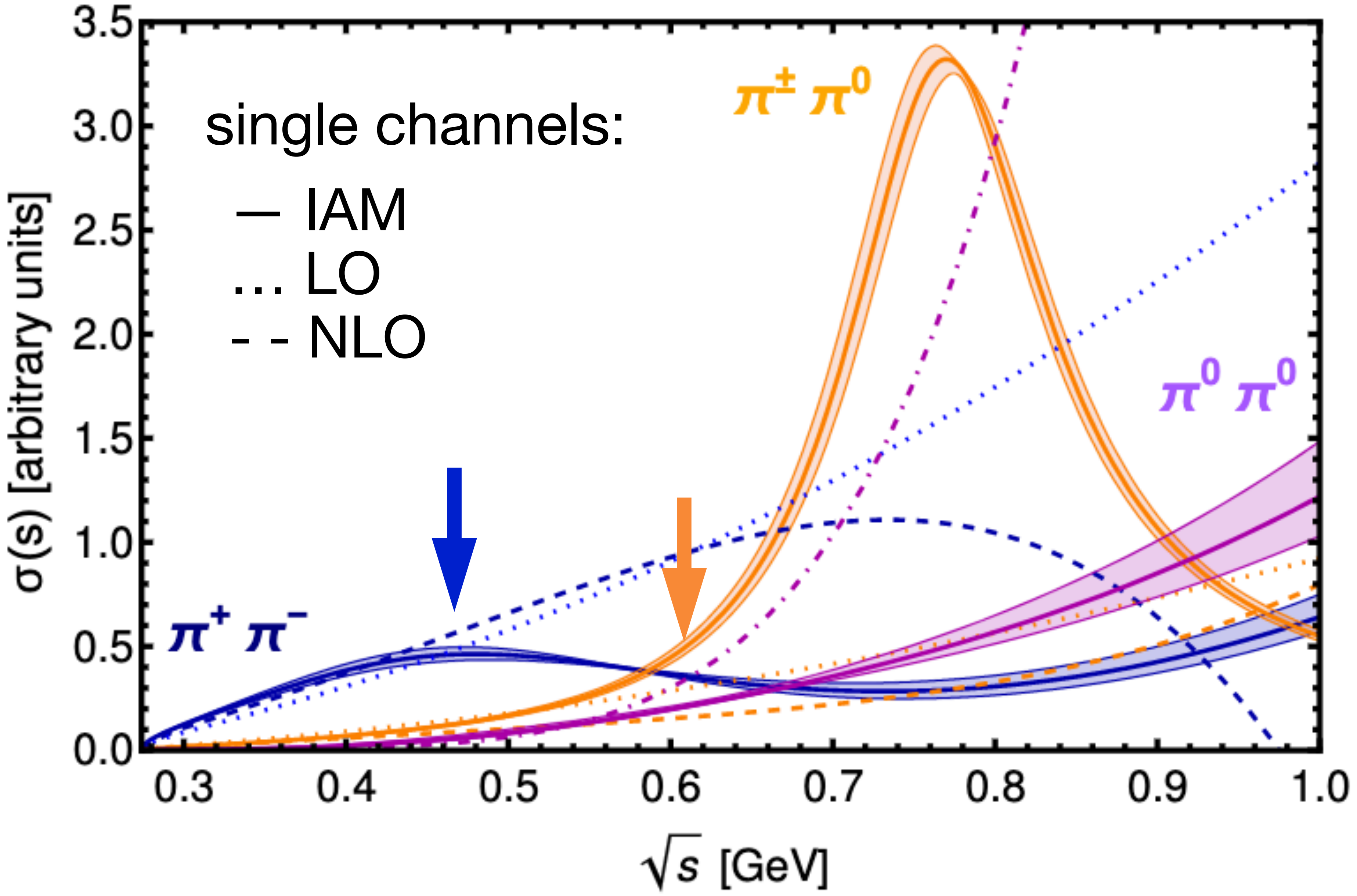


Partial wave amplitudes

Growth with energy of ChPT amplitudes is tamed by unitarization!

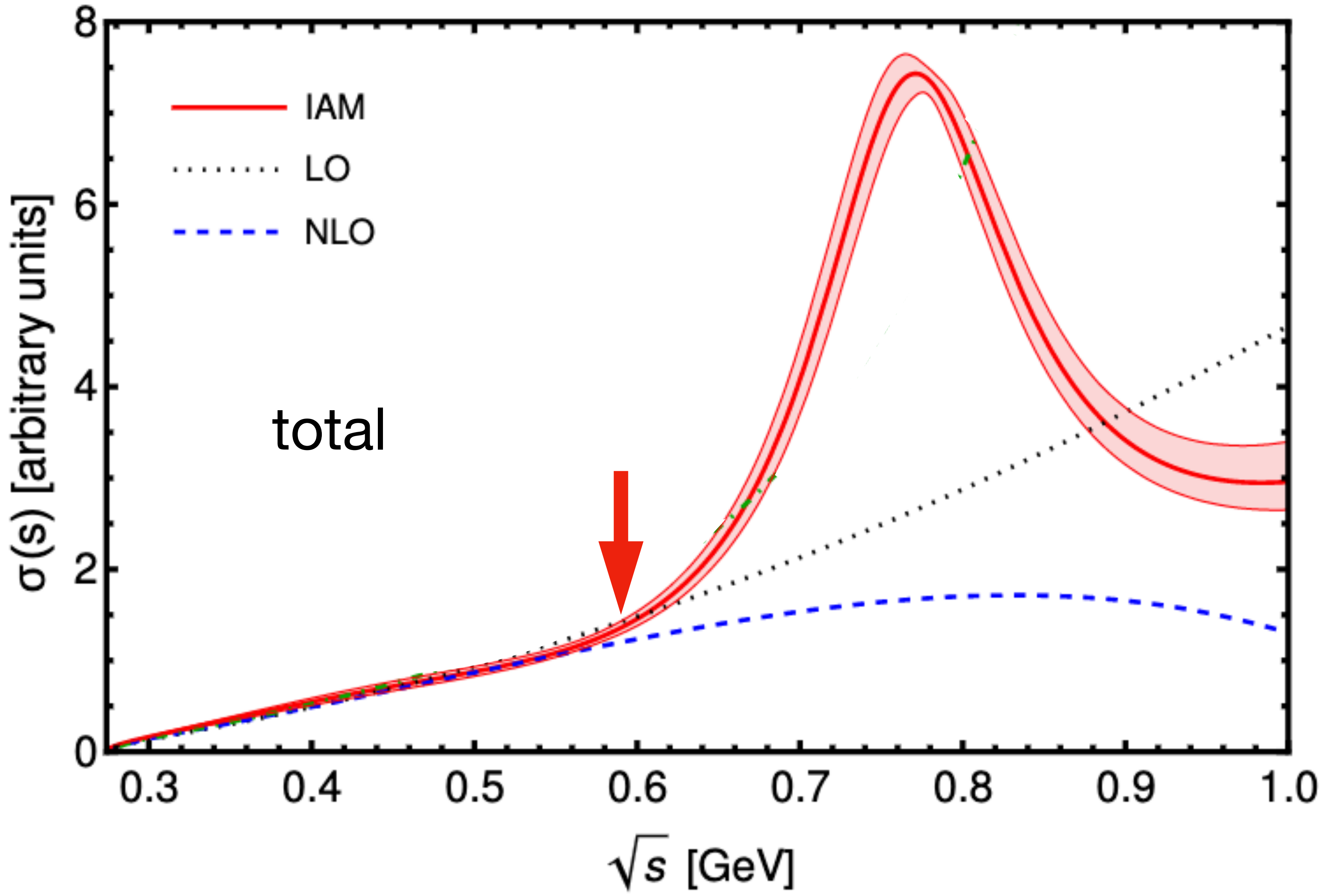


Cross Sections



- ✦ $\pi^+\pi^-$ ChPT departs from IAM at $\sqrt{s} \simeq 450$ MeV, $\sigma(550)$ resonance in $I = J = 0$ channel

- ✦ $\pi^\pm\pi^0$ ChPT departs from IAM at $\sqrt{s} \simeq 600$ MeV, $\rho(770)$ resonance in $I = J = 1$ channel



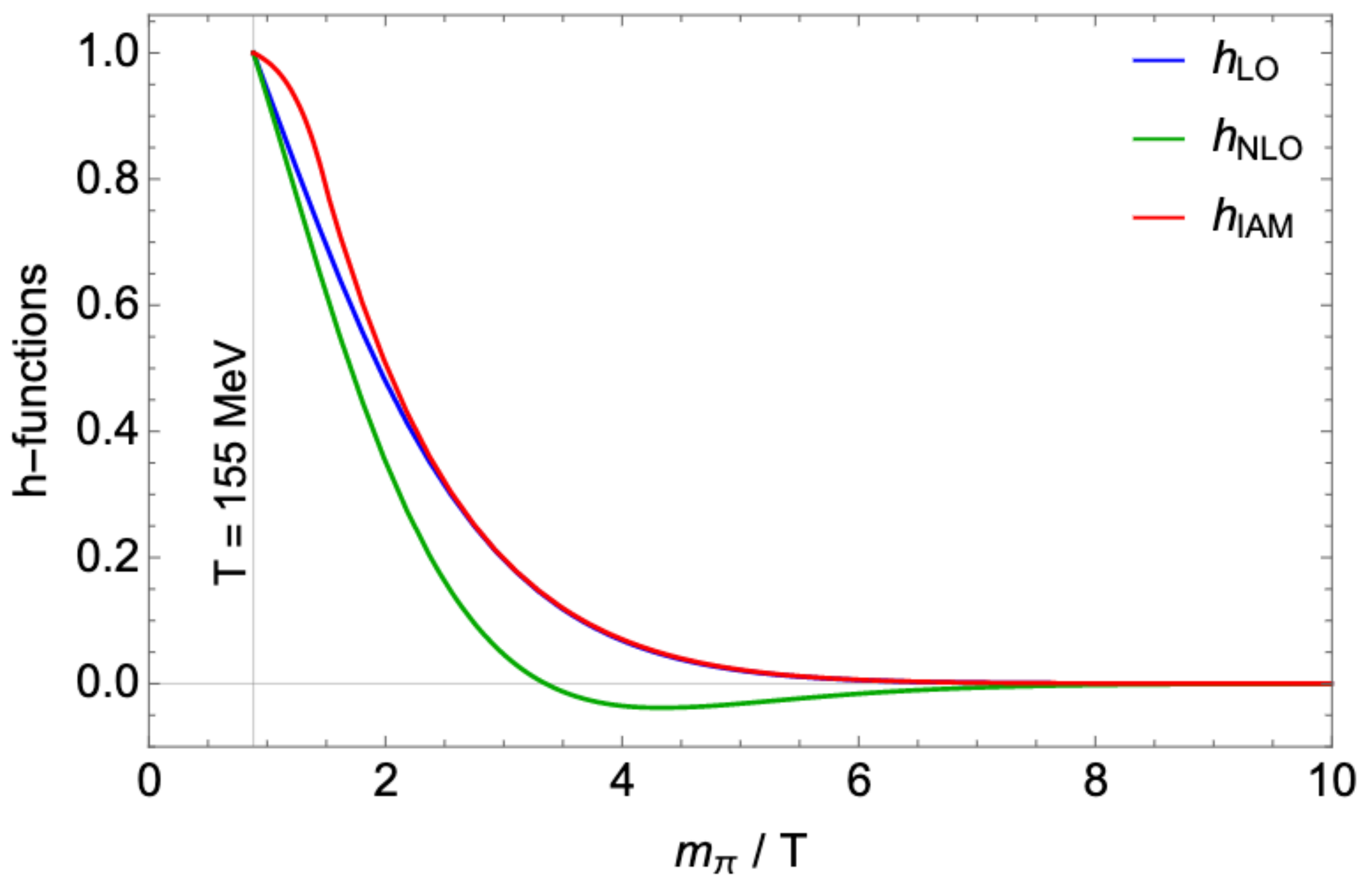
$$\sigma_{\text{tot}} = \sigma_{a\pi^0\to\pi^+\pi^-} + \sigma_{a\pi^+\to\pi^+\pi^0} + \sigma_{a\pi^-\to\pi^-\pi^0} + \underbrace{\sigma_{a\pi^0\to\pi^0\pi^0}}_{\text{not present at LO}}$$

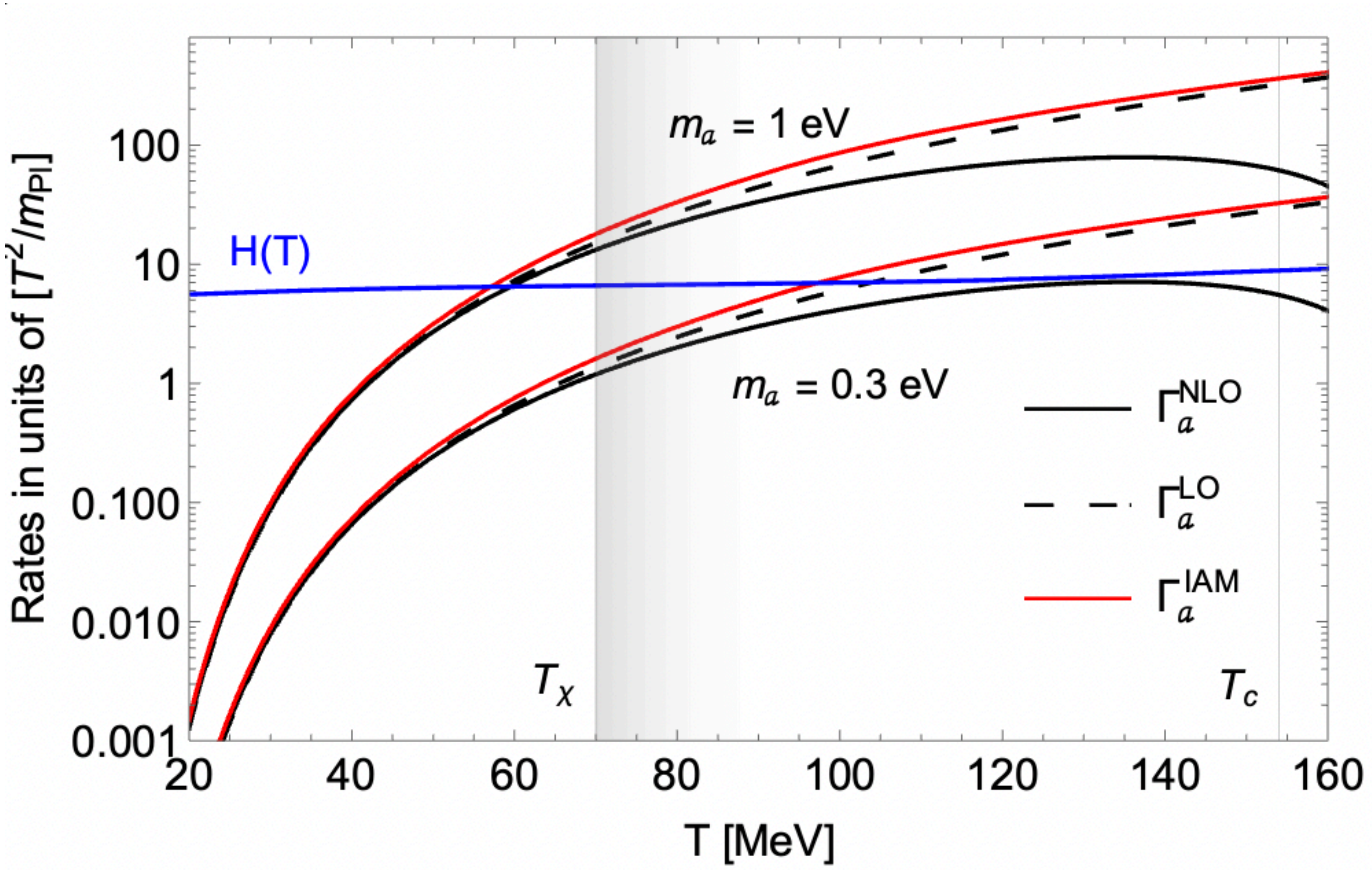
Accidental feature:

- ✦ Compensation between $\pi^+\pi^-$ and $\pi^\pm\pi^0$ channels makes $\sigma^{\text{tot}}_{\text{IAM}} \simeq \sigma^{\text{tot}}_{\text{LO}}$ up to $\sqrt{s} \simeq 600$ MeV

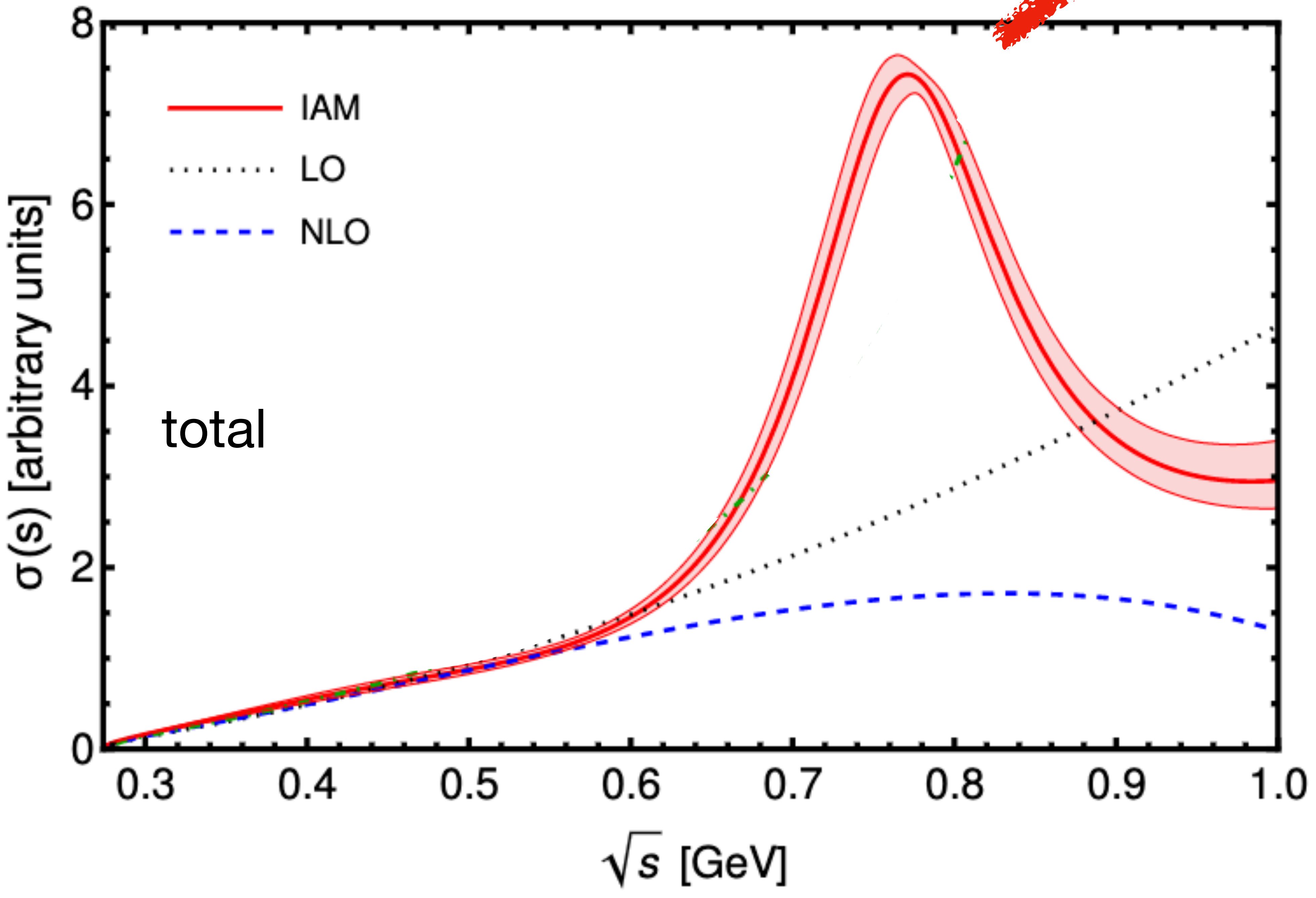
Thermal rates

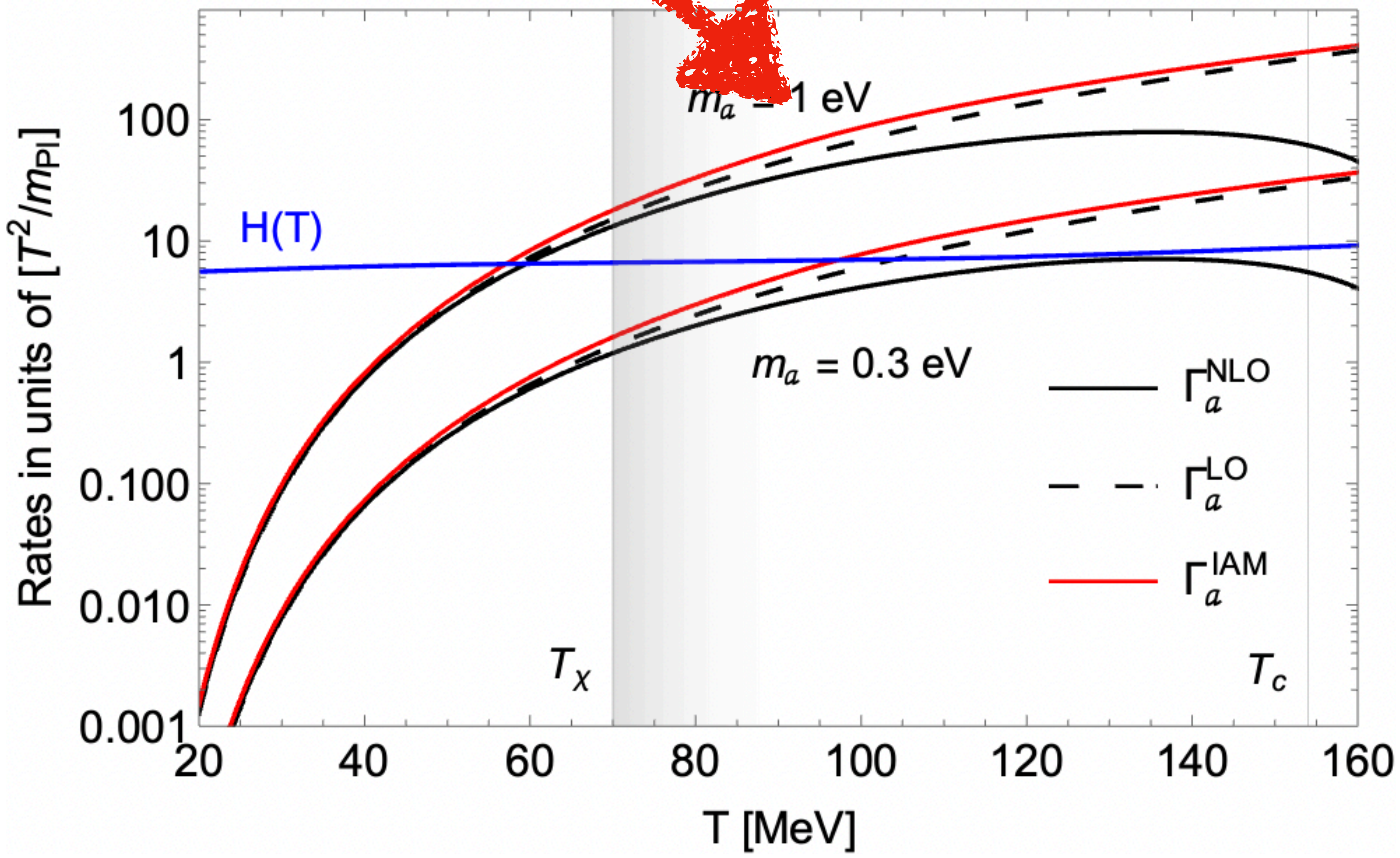
$$\Gamma_a^{\mathrm{IAM}}(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.181\ T^5 h_{\mathrm{IAM}}(m_\pi/T)$$

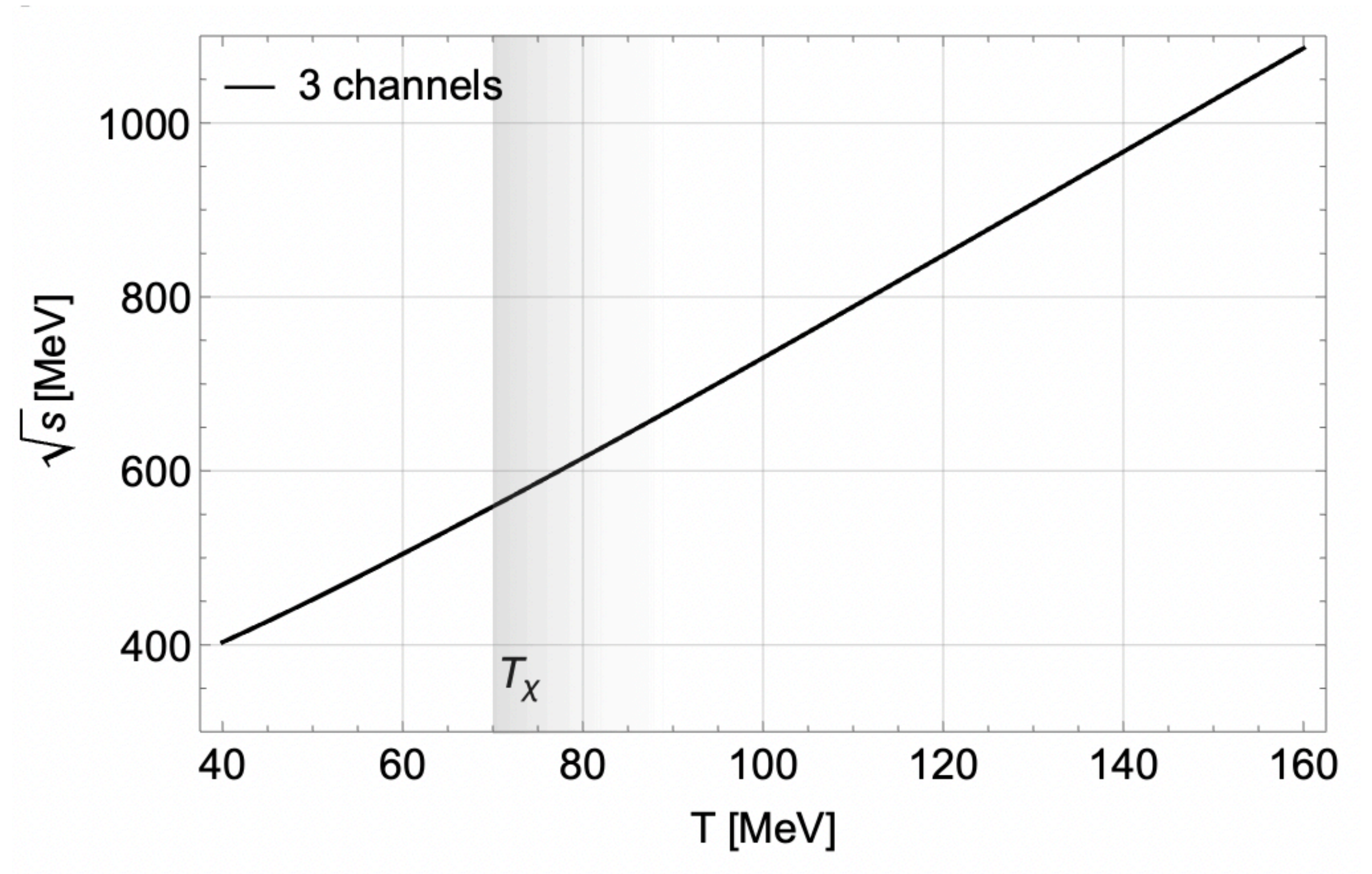




Thermal rates



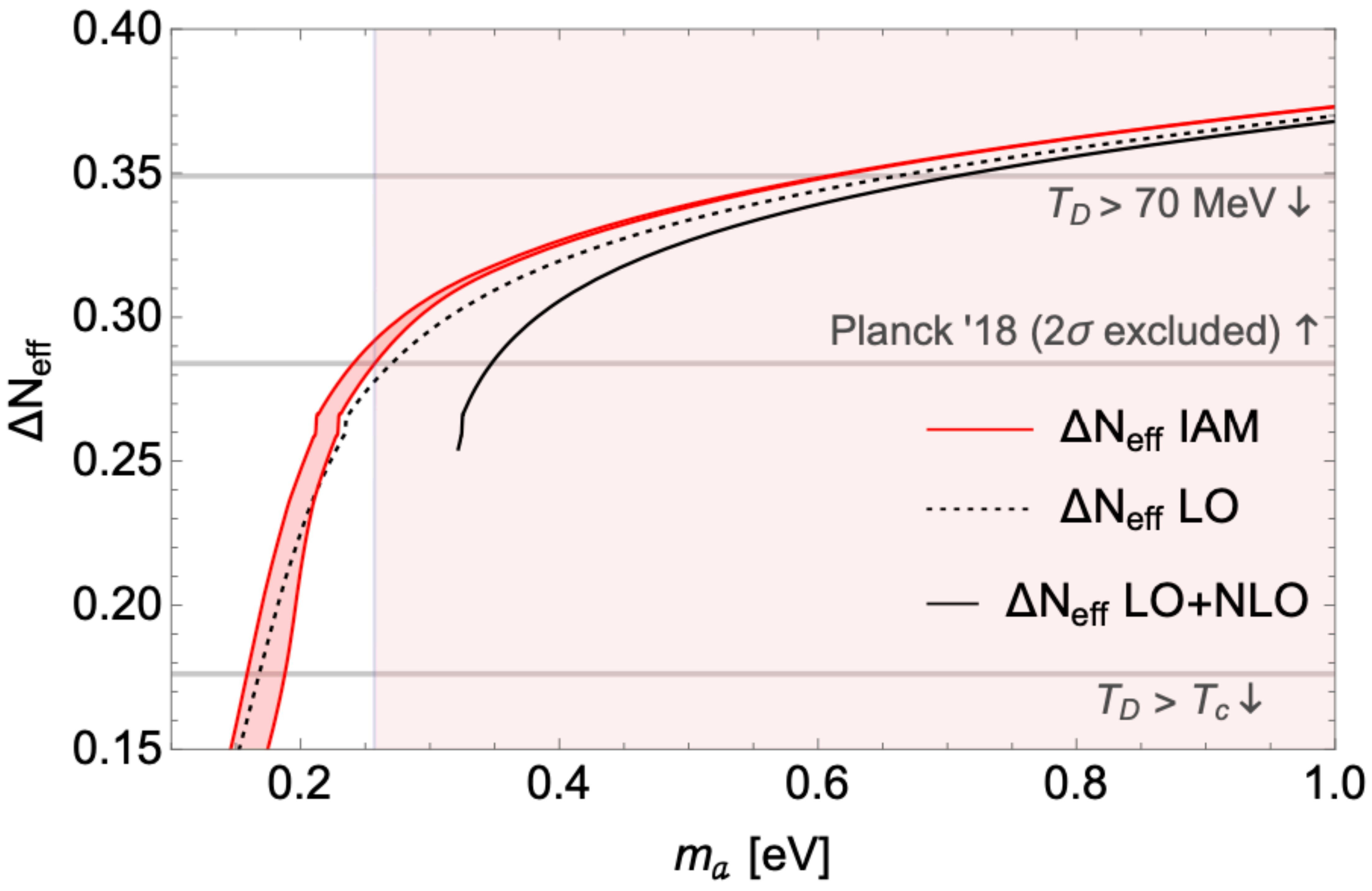




✦ $\Gamma_{\rm IAM} \simeq \Gamma_{LO}$ for $\sqrt{s} \lesssim 600$ MeV$\rightarrow$ $T \lesssim 70$ MeV

✦ ρ resonance appears at $\sqrt{s} \sim 750$ MeV $\rightarrow T \sim 100$ MeV

$\Delta N_{\rm eff}$: IAM vs LO



✦ LO and NLO ChPT reliable up to $T_D = 70$ MeV

✦ IAM valid up to $T_c \simeq 155$ MeV, gives a bound $m_a \leq 0.26$ eV

Conclusions

- Using ChPT at $T \sim 100 \text{ MeV}$ corresponds to $\sqrt{s} \sim 750 \text{ MeV}$, way above its validity!

- Unitarization provides a way to extend ChPT up to $T_c \simeq 155 \text{ MeV}$, including resonances and satisfying unitarity;

- Accidentally, <u>for the total rates,</u> IAM is very close to LO

- IAM Hot Dark Matter bound $m_a \leq 0.26 \text{ eV}$

✤ To Do: Kaons relevant at $\sqrt{s} \sim 800 \text{ MeV}, \ f_0(880)$, + including thermal effects

✤ Describe axion thermal production in the intermediate region between 155 MeV and 1 GeV *

*Especially relevant for future sensitivities

Thanks for the attention!

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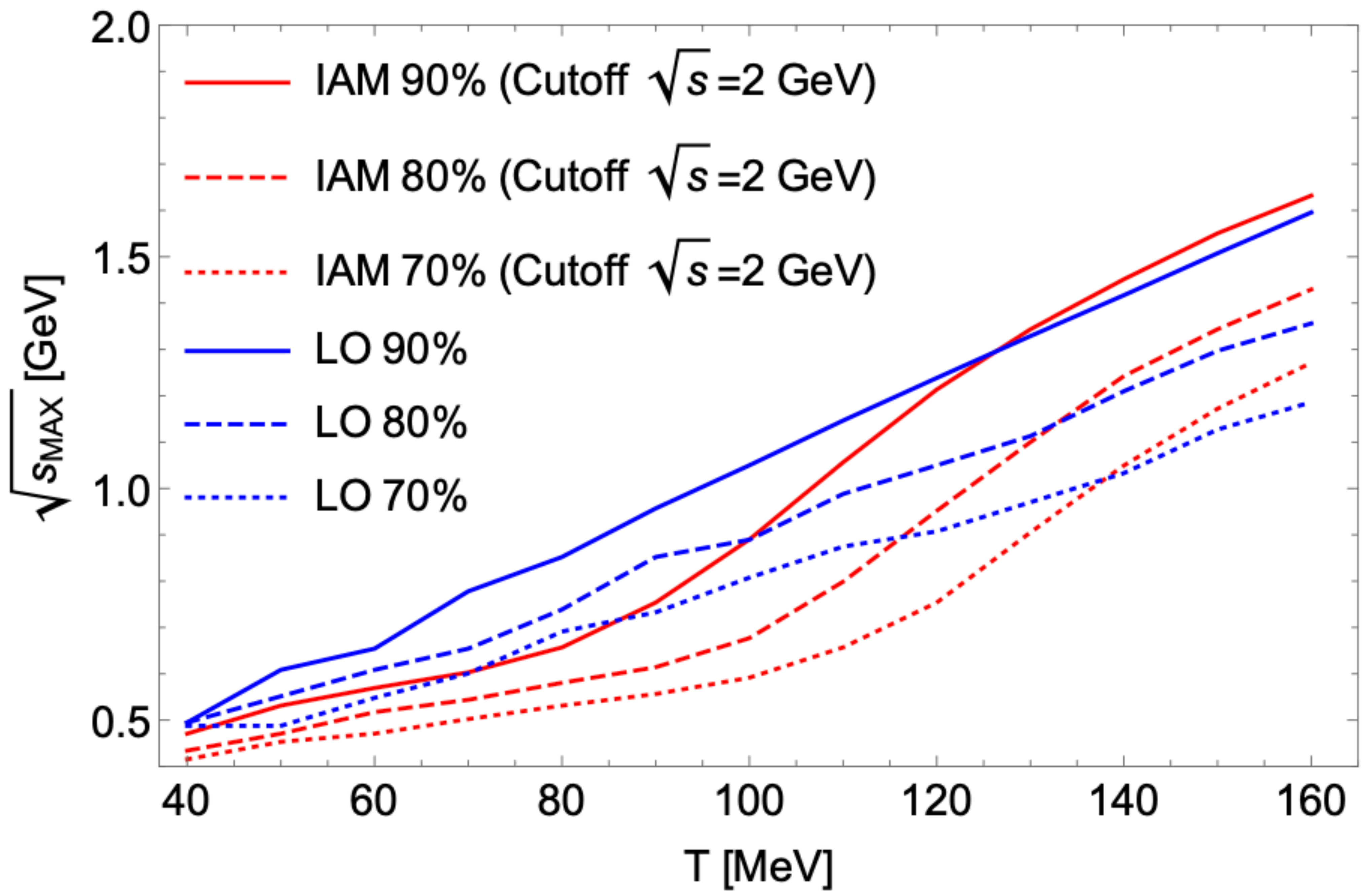
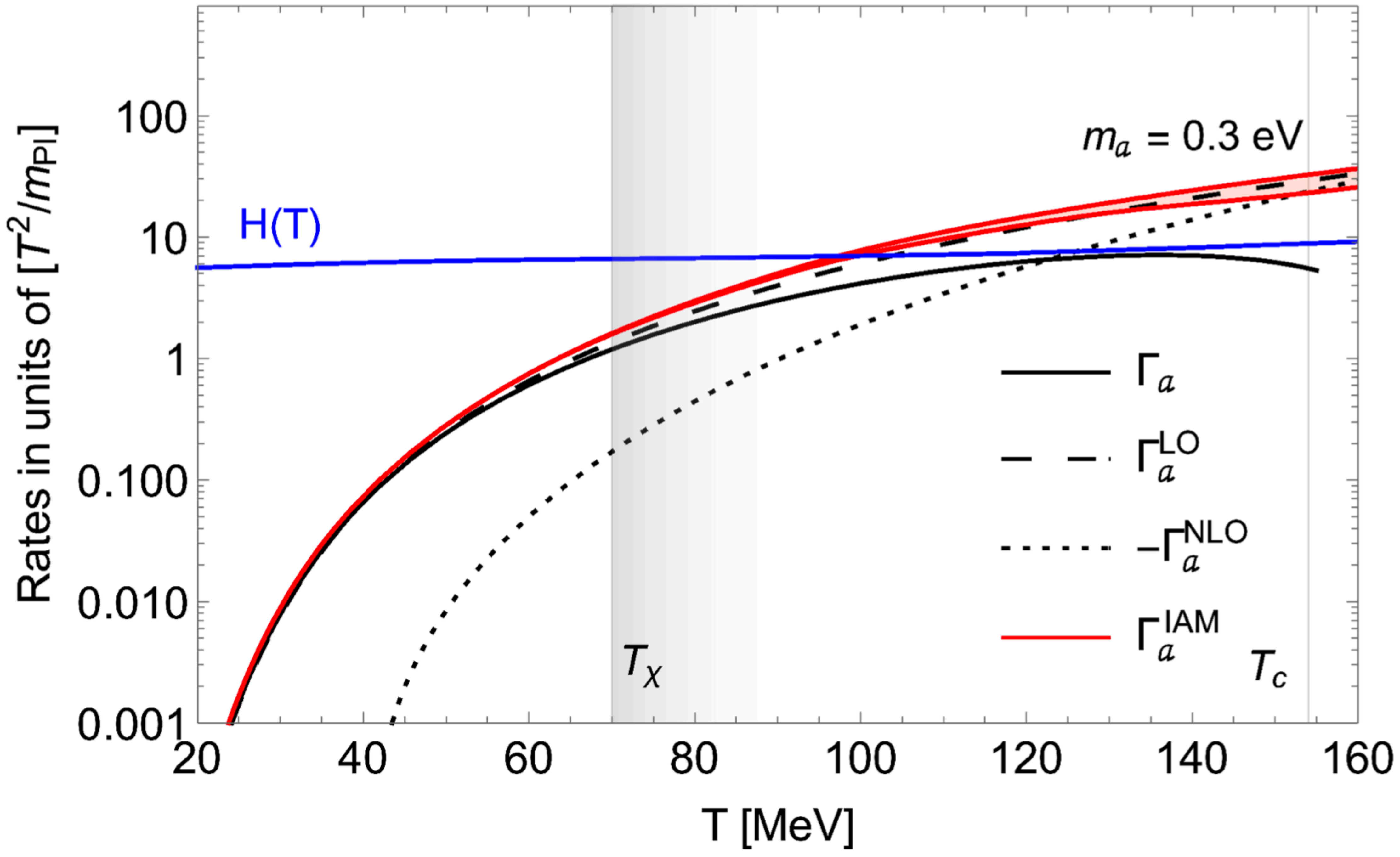
Backup

Energy contributions to thermal rate



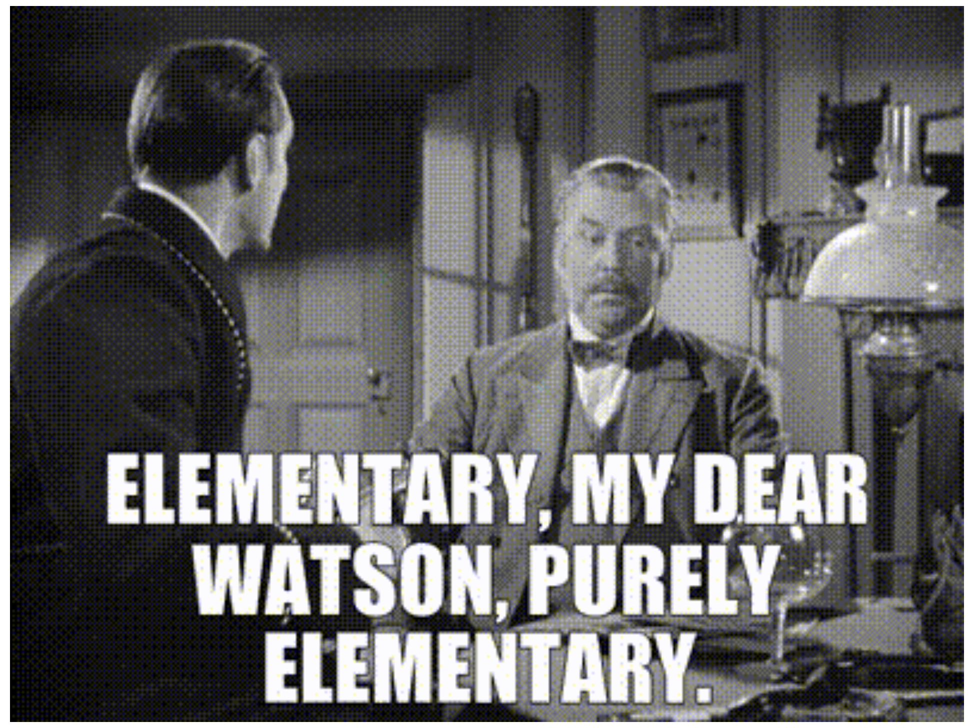
- At least 70% of the contribution to the thermal rates in IAM stems from $\sqrt{s} < 1\ \mathrm{GeV}$, where IAM is under control

- Error: difference between the total thermal rate in IAM and the one cut at $\sqrt{s_{\mathrm{MAX}}} < 1\ \mathrm{GeV}$



Watson Theorem

[K. M. Watson, Phys. Rev. 88, 1163 (1952)]



The axion interacts weakly, but $\pi\pi$ final-state interactions are strong and resonant

$$\rightarrow T_I^\ell = |T_I^\ell| e^{i(\delta_\pi)_I^\ell}$$

$$A_I^\ell = |A_I^\ell| e^{i(\delta_a)_I^\ell}$$

$$T^\dagger T = i(T^\dagger - T)$$

$$\boxed{\text{Im}\, A_I^\ell(s) = \frac{\sigma(s)}{32\pi} A_I^\ell(s) T_I^\ell(s)^* \theta(s - 4m_\pi^2)}$$

$$\sigma = \sqrt{1 - 4m^2/s}$$

Unitarity $\Rightarrow (\delta_a)_I^\ell = (\delta_\pi)_I^\ell$

IAM “derivation” [Truong, PRL **61**, 2526]

$$\text{Im}\, t(s) = \sigma(s)|t(s)|^2 \Rightarrow \text{Im}\frac{1}{t(s)} = -\sigma(s)$$

$$t(s) = \frac{1}{\text{Re}t^{-1}(s) - i\sigma(s)}$$

$$\sigma = \sqrt{1 - 4m^2/s}$$

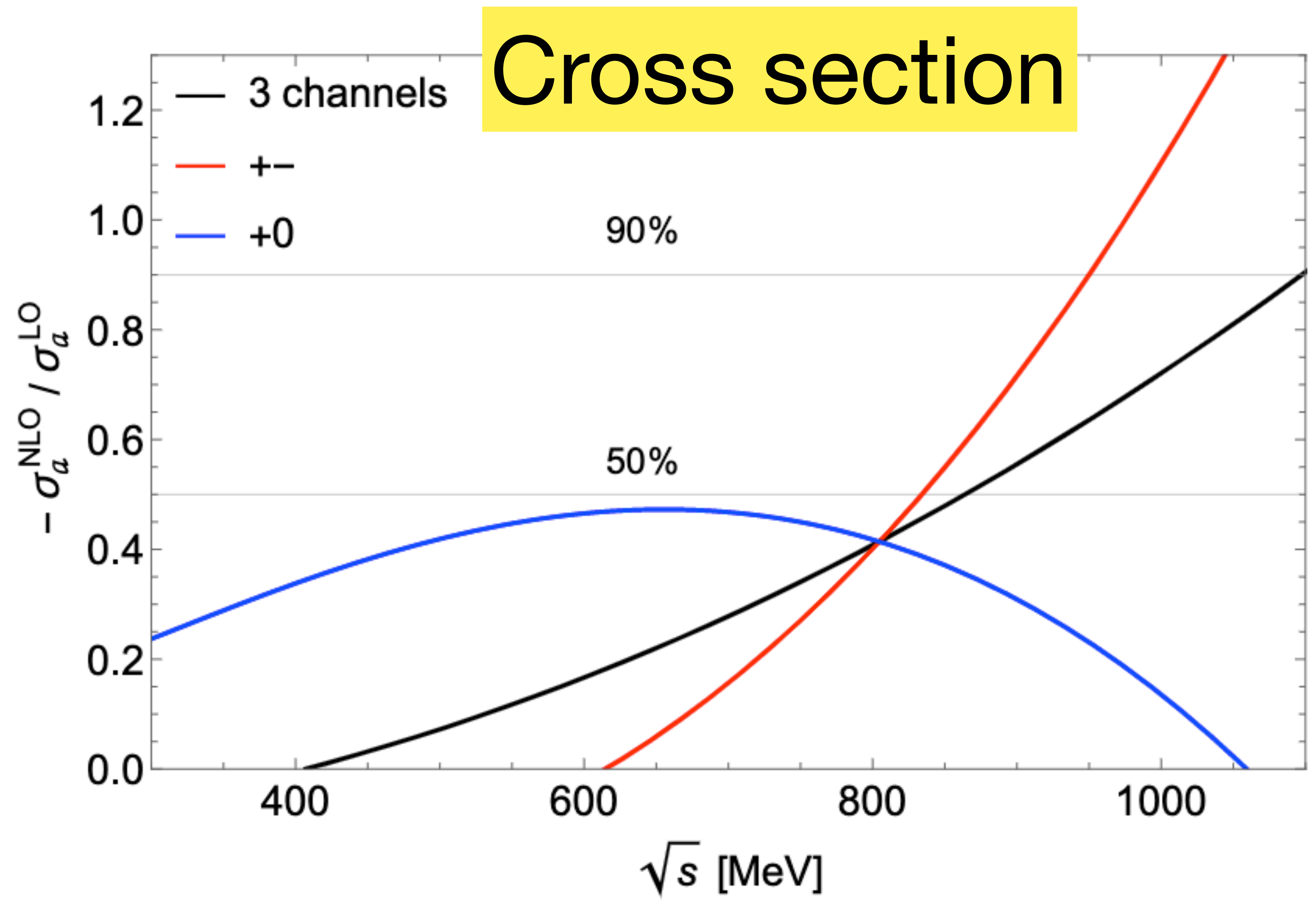
Replace Re t^{-1} by $\mathcal{O}(p^4)$ ChPT expansion

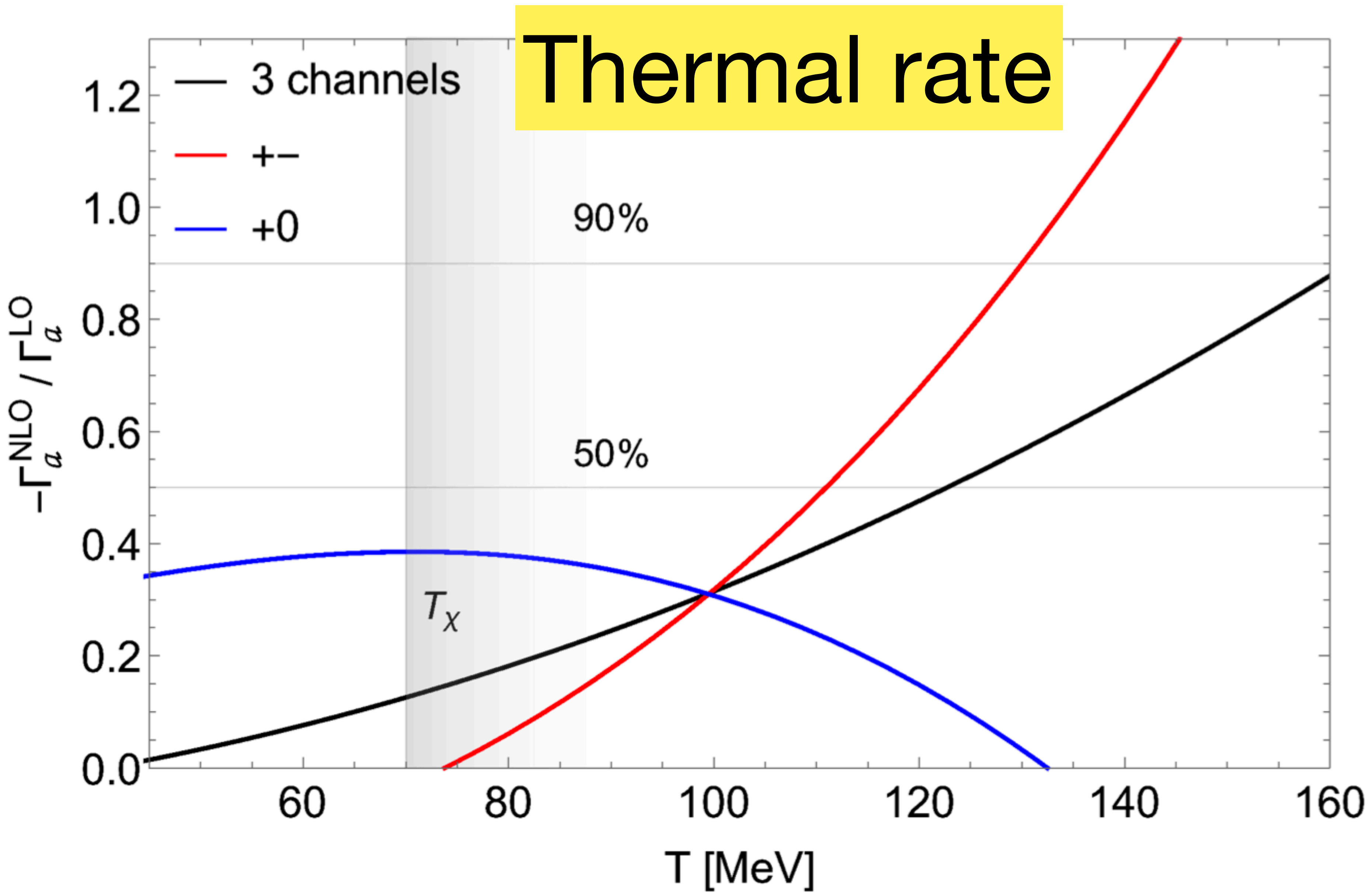
$$t^{\text{IAM}}(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

satisfies unitarity

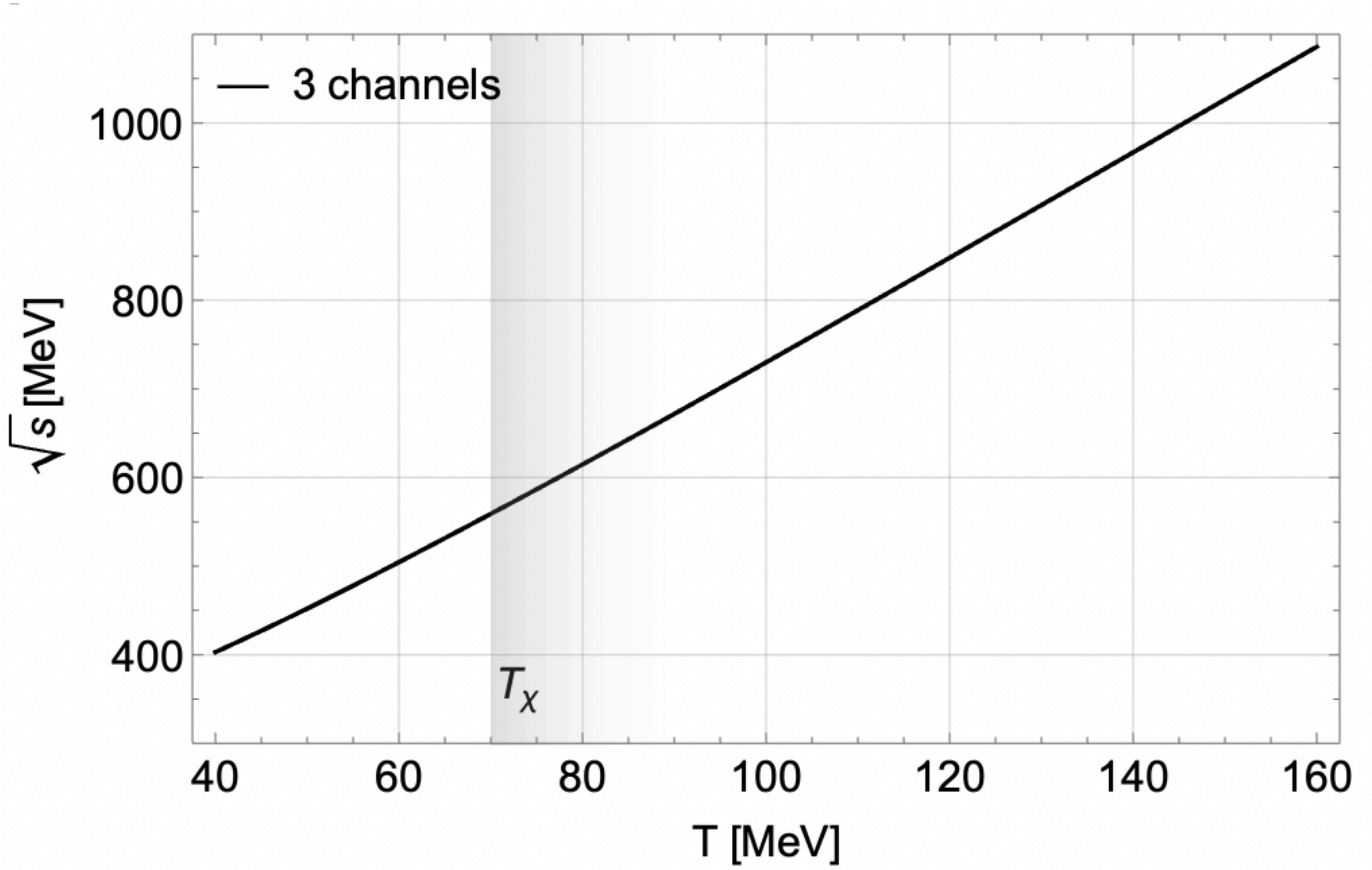
Reproduces simultaneously the low-energy expansion (Padé approx.) and the lightest resonances without including them explicitly in the Lagrangian

Breakdown of ChPT: σ and Γ





- ✦ Matching the % we get correspondence between $\sqrt{s}$ and T:



$\Delta N_{\rm eff}$, the origins

$$
\begin{aligned}
\rho =&\rho_\gamma + \rho_\nu + \rho_a \\
=&\rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}^{\rm SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4\right] \\
=&\rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]
\end{aligned}
$$

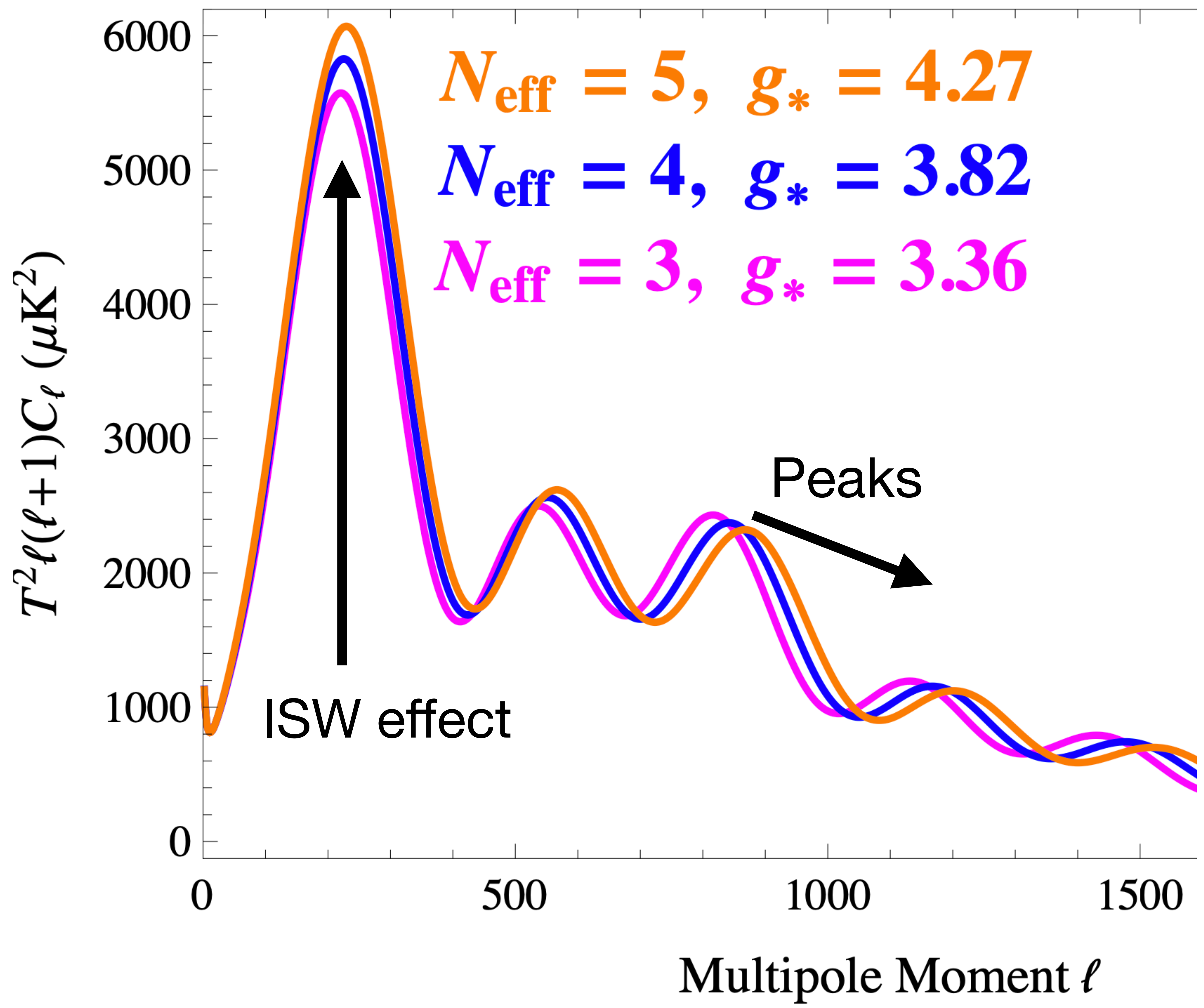
$$
\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4
$$

$$
\frac{T_a}{T_\nu} = \left(\frac{43}{4 g_S(T_D)}\right)^{1/3}
$$

$$
\Delta N_{\rm eff} = 0.027 \left(\frac{106.75}{g_S(T_D)}\right)^{4/3}
$$

Effects of $N_{\rm eff}$ on the CMB

- $N_{\rm eff} \uparrow \Rightarrow H \uparrow$, time for photons diffusion in the plasma decreases, reducing Silk damping and restricting it to higher ℓ. $\ell_{\rm dump} \uparrow$
- $H \uparrow$ Acoustic oscillation length scale decreases, increasing the sound horizon. $\ell_{\rm sound} \uparrow$
- Overall less dumping but more peaks dumped. $H \uparrow \Rightarrow \ell_s/\ell_d \uparrow$
- Also, gravitational red/blue shift increased on 1st peak scales (ISW)

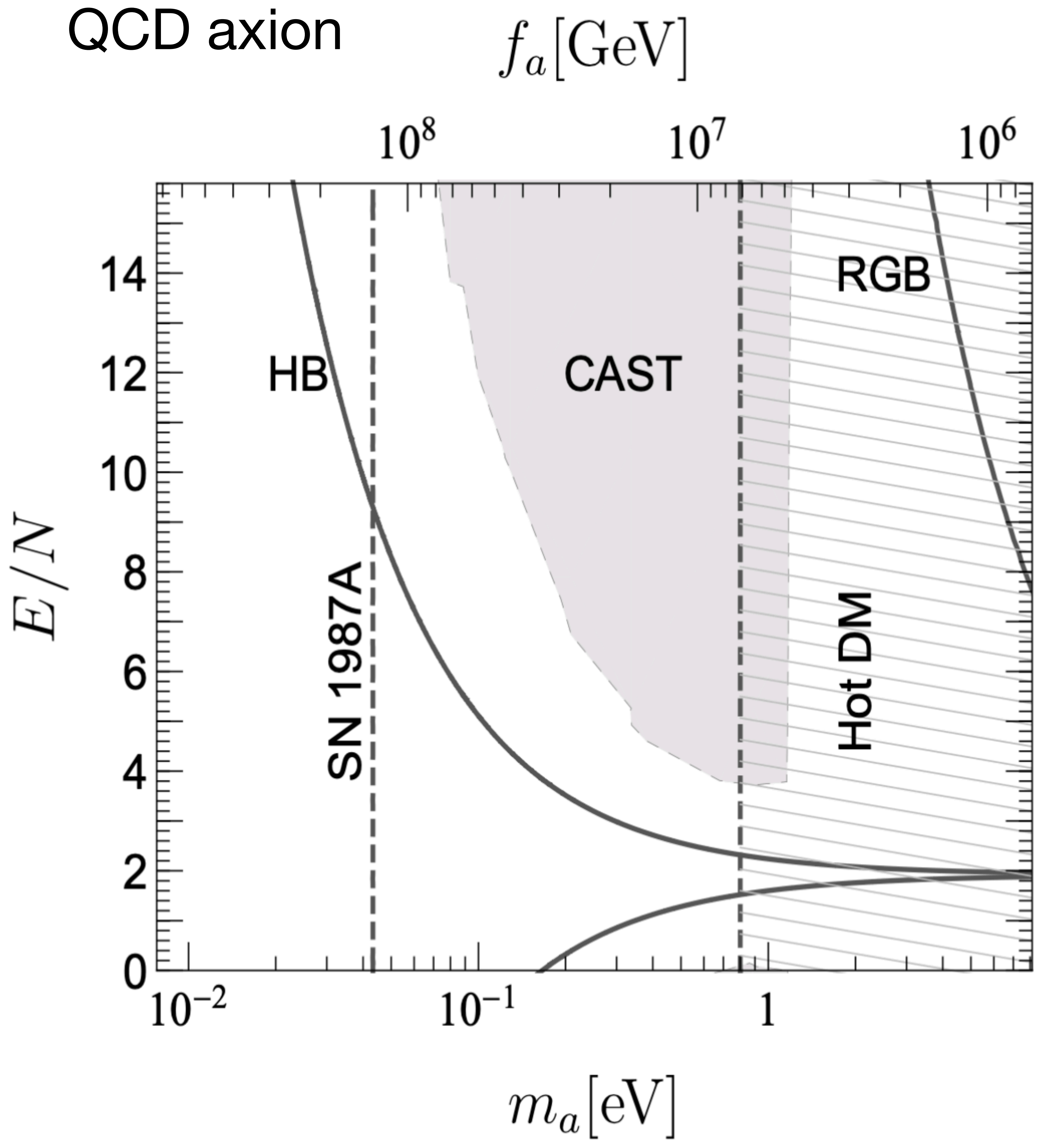


[Brust, Kaplan, Walters, arXiv:1303.5379]

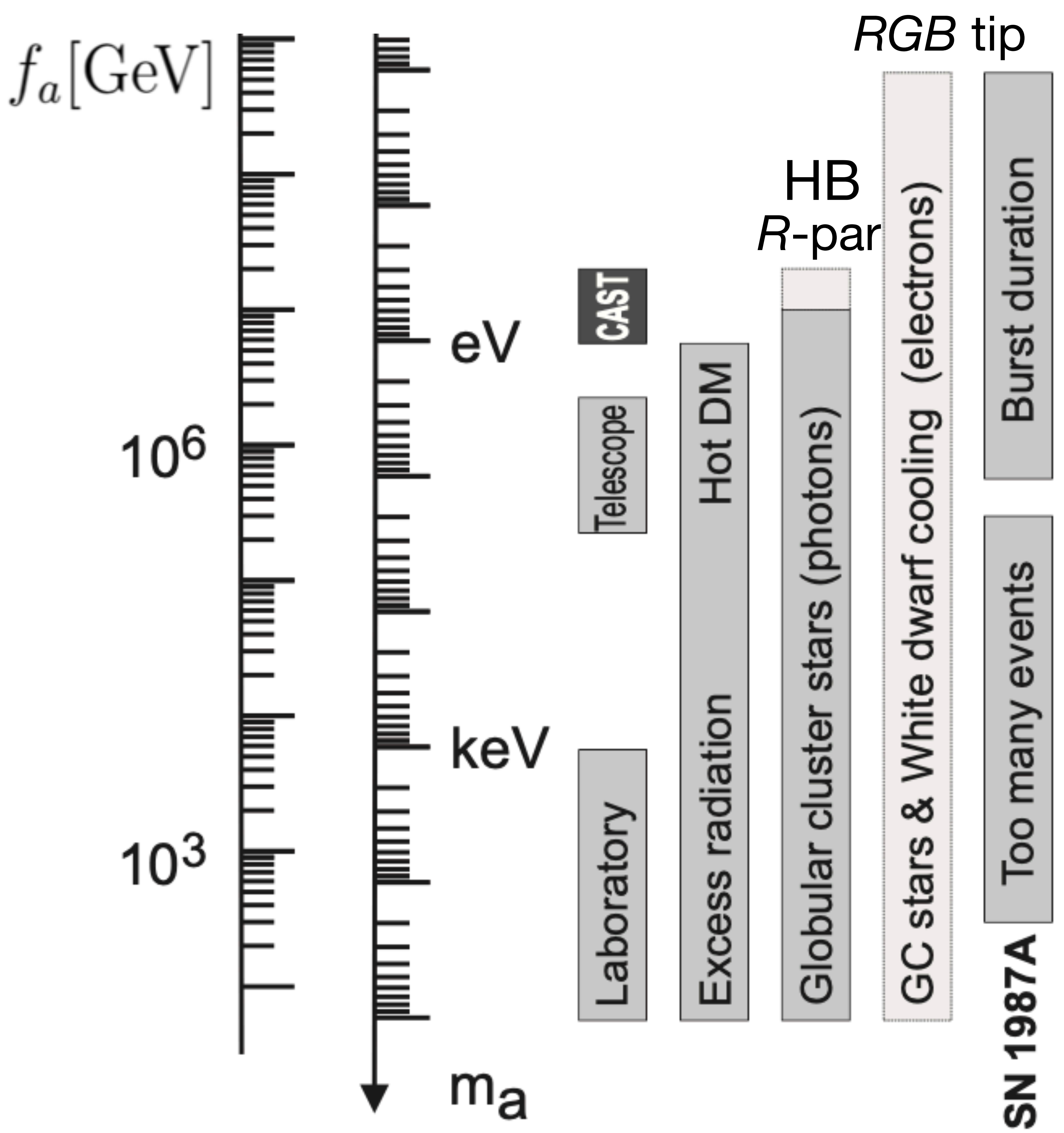
[Silk, *Astrophys.J.* 151 (1968)]
[Sachs, Wolfe, *Astrophys. J.* 147 (1967)]
[Bowen, Hansen, Melchiorri, Silk, Trotta, arXiv: astro-ph/0110636]
[Brust, Kaplan, Walters, arXiv:1303.5379]

ASTRO Bounds

QCD axion



[Di Luzio et al., Phys. Rept. **870** (2020)]



- $g_{ae}^0 = 0$ in KSVZ models
- SN bound - large astrophysical uncertainties

  see e.g. [Bar, Blum, D'Amico, 1907.05020]
- $g_{a\gamma}$ can be accidentally suppressed

  [Di Luzio, Mescia, Nardi, 1705.05370]