NeΨ23 | NePSi 23

Pisa 17 February 2023

The cosmic evolution of the QCD axion, and lattice simulations

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A.Kotov, MpL, A.Trunin, in progress
PoS Lattice2021
Phys.Lett.B23(2021) 136749
Strong-2020 NA6 white paper, **MpL et al**:Topology and Axion Section, **C.Bonanno** editor, ePrint: 2301.04382

$$m_A^2 f_A^2 = \chi_{top}$$

A vast parameter space for the QCD axion mass & coupling

$$m_A^2 f_A^2 = \chi_{top}$$

— valid at all temperatures —

A vast parameter space for the QCD axion mass & coupling

constrained once the cosmological evolution is taken into account

Background —

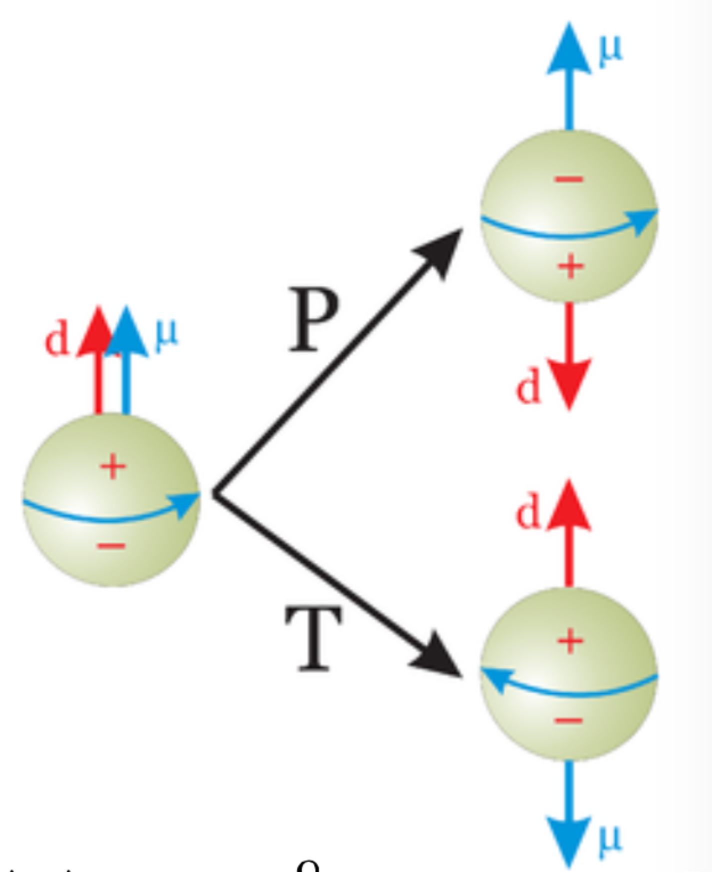
Strong CP problem, topology, axions

Topology and the Strong CP problem

CP violation in QCD?

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = q(x)$$



Diagnostic of CP violation: Neutron electric dipole moment d_n

- QCD sum rules: $d_n = 2.4 \times 10^{-16} \theta$ e cm **Pospelov(1999)**
- Chiral perturbation theory: $d_n = 3.6 \times 10^{-16} \theta$ e cm **Pich(1991)**
- Experiments: $d_n = (0.0 \pm 1.1 \text{ (stat)} \pm 0.2 \text{ (sys)}) \times 10^{-26}$ e cm
  $|d_n| < 1.8 \times 10^{-26}$ e cm 90% C.L, **nEDM, Abel et al. (2020)**

$$\theta < 0.5 \times 10^{-10}$$

SOLUTION OF THE STRONG CP PROBLEM: THE AXION

Peccei-Quinn(1977),Weinberg(1977),Wilczek(1977)

Introducing a new U(1) global symmetry which spontaneously break at a scale f_A

Promoting θ to a dynamical variable: the axion–QCD Lagrangian:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \partial^2_\mu a^2 + \frac{a}{f_A}\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \\ &= \mathcal{L}_{QCD} + \partial^2_\mu a^2 + \frac{g^2}{32\pi^2}\left(\frac{a}{f_A} + \theta\right) F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\end{aligned}$$

The minimum of the effective potential is now dynamically set to

$$\theta = 0$$

THE GRAND CANONICAL PARTITION FUNCTION

$$\mathcal{Z}(\theta, T) = \int \mathcal{D}[\Phi]\, e^{-T \sum_t \int d^3x \mathcal{L}(\theta)} \; = e^{-VF(\theta,T)}.$$

$$P_Q = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} e^{-VF(\theta)}, \; Q = \int d^4x\, q(x)$$

Taylor expansion..

$$F(\theta, T) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{2n!}\, C_n$$

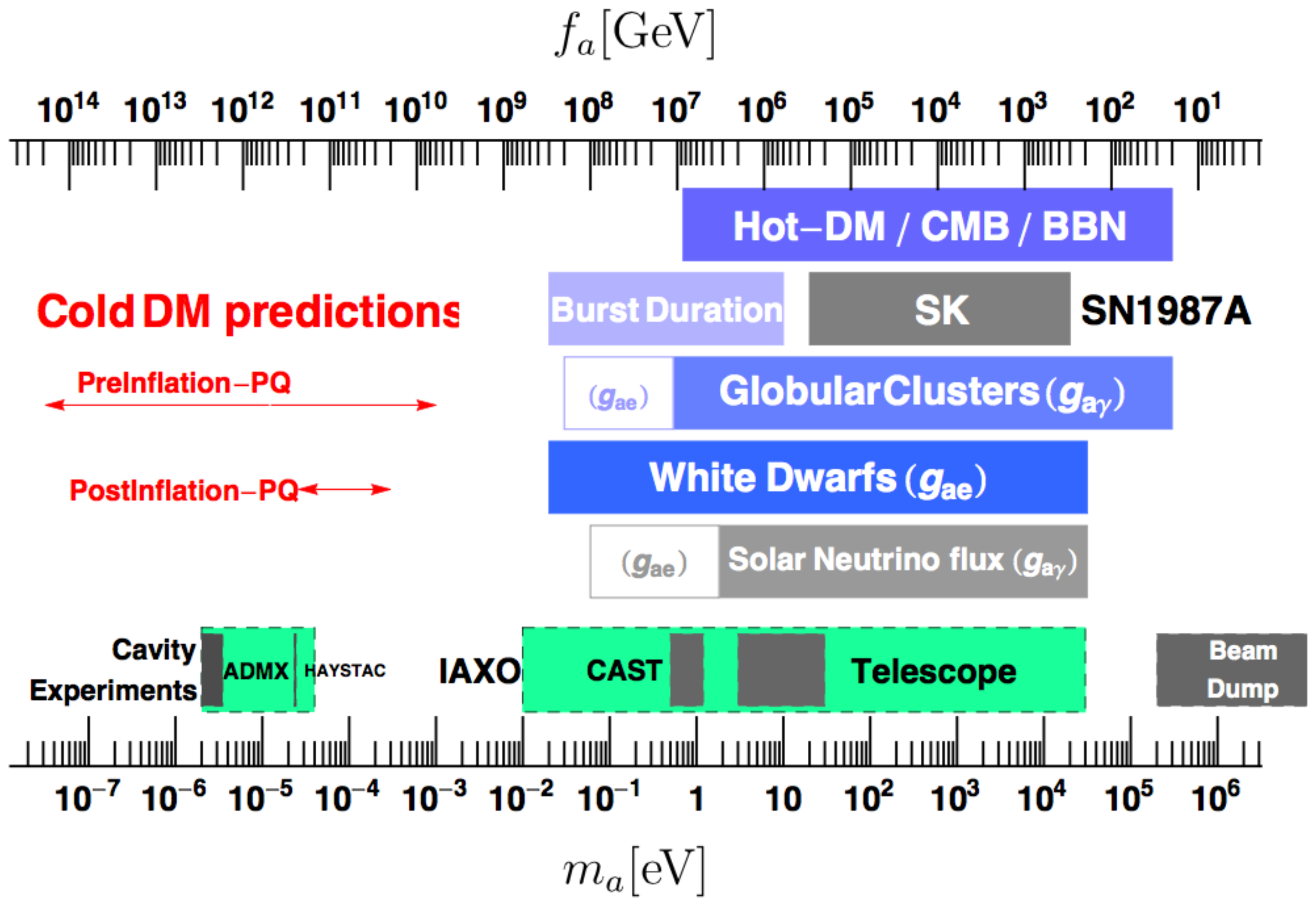
..and cumulants of the topological charge:

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

THE AXION MASS

At leading order in $1/f_A$ – well justified as $f_A \gtrsim 4 \times 10^8$ GeV

$$m_A^2(T) f_A^2 = \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0} \equiv \chi_{top}(T)$$



At low temperature ChPT gives:

$$m_A^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_A^2},$$

In general

$$\boxed{m_A^2 f_A^2 = \chi_{top},}$$

valid for any temperature.

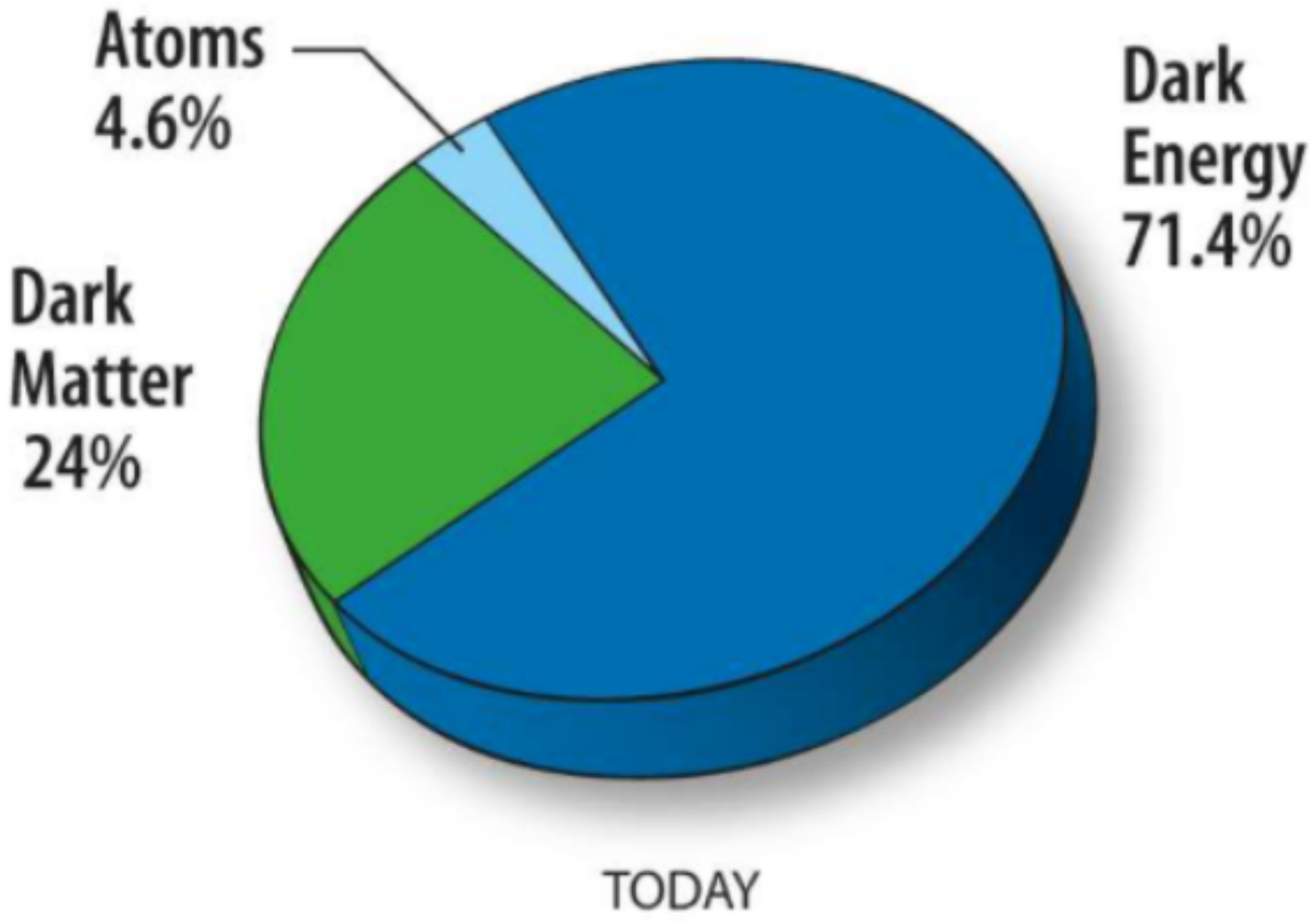
$$m_A = 5.70(6)(4)\,\mu\text{eV} \left(\frac{10^{12}\,\text{GeV}}{f_A} \right);$$

In brief:

the strong CP problem in QCD

can be solved by introducing the AXION

a new particle which is a viable **dark matter candidate**



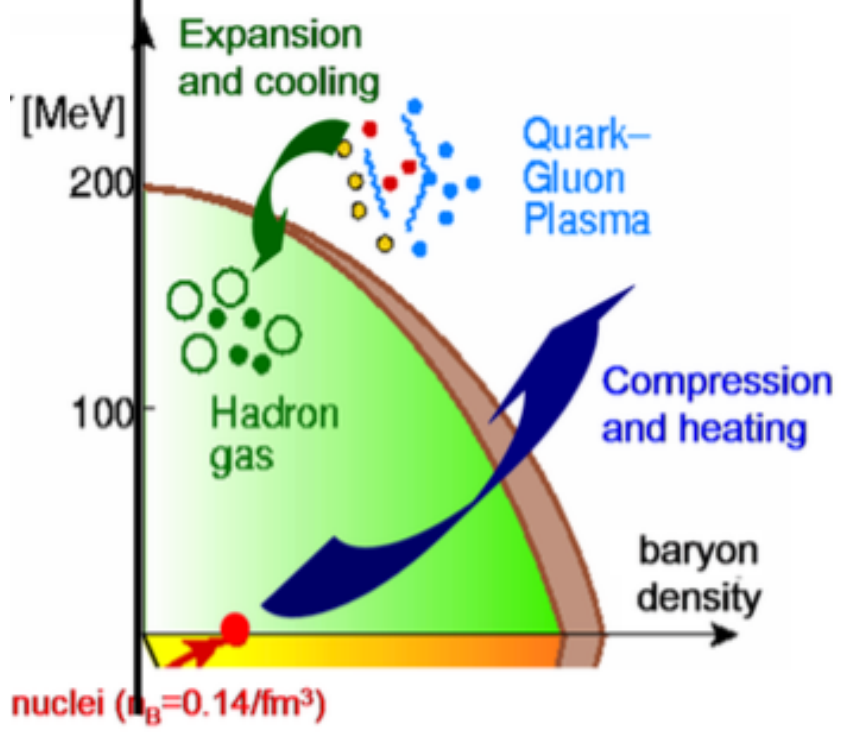
$$m_A^2 f_A^2 = \chi_{top},$$

Axion cosmology and the freeze-out

Time from Big Bang

Temperature

Quark Gluon Plasma: Topology



Time from Big Bang

Temperature

Hubble parameter

$$H(T) \simeq T^2/M_P$$

Axions's freezout

$$3H(T) = m_a(T)$$

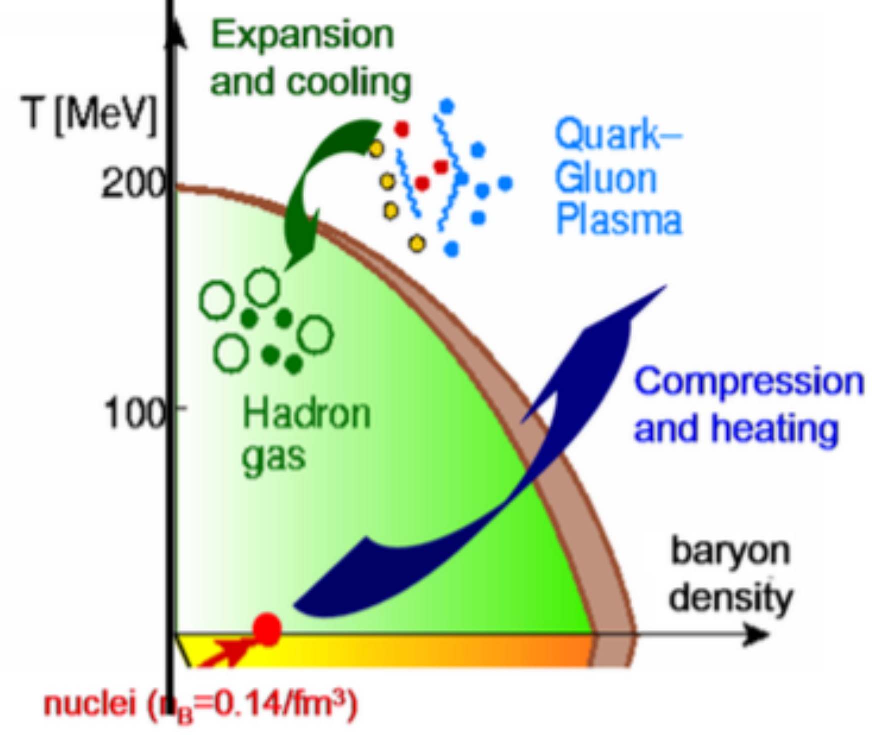
$$m_a(T) = \sqrt{\chi(T)}/f_a$$

Quark Gluon Plasma: Topology

Axions' mass and density today

After freezout $\dfrac{n_a}{s}$ constant

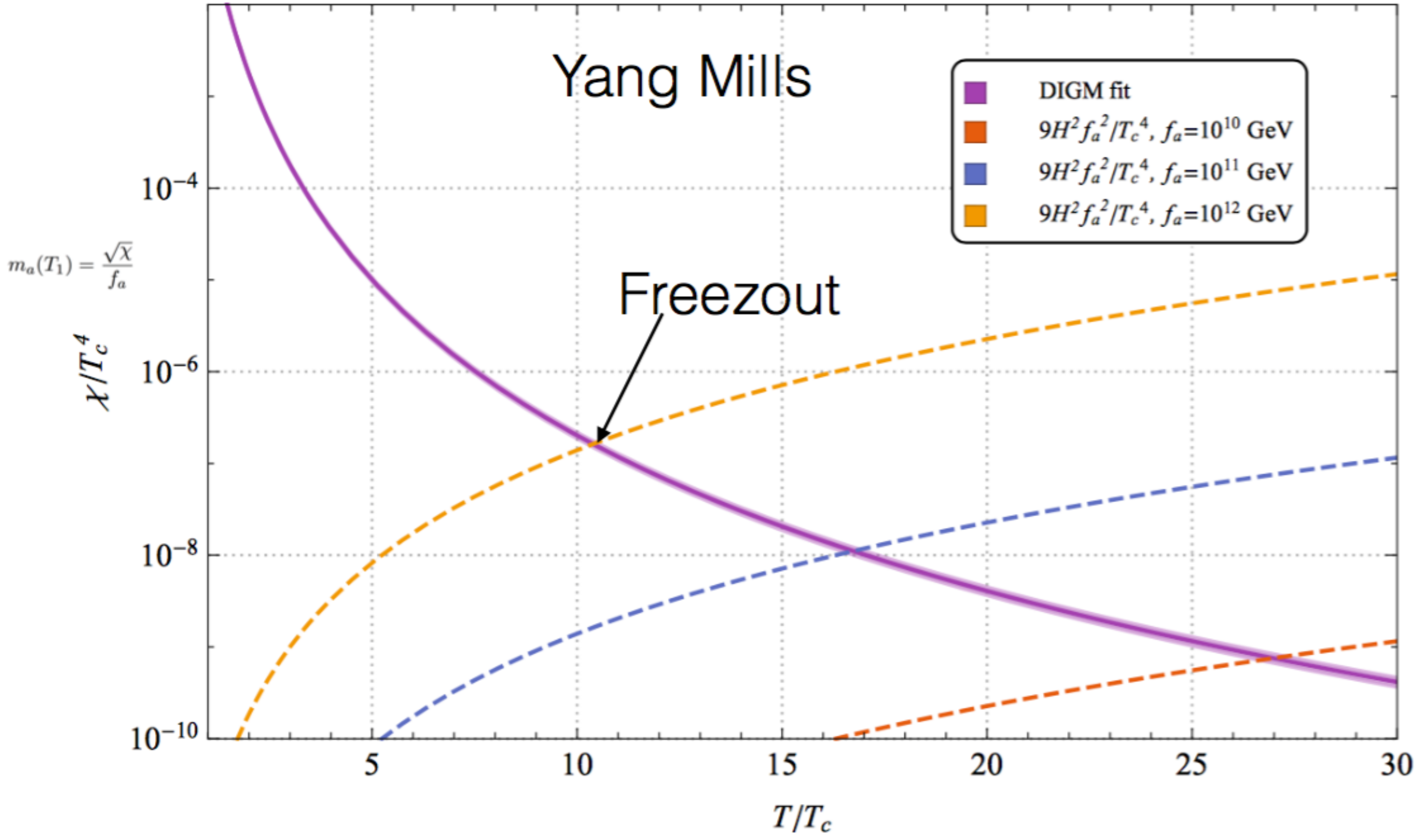
$$\rho_{a,0} = \frac{n_a}{s} m_a s_0$$



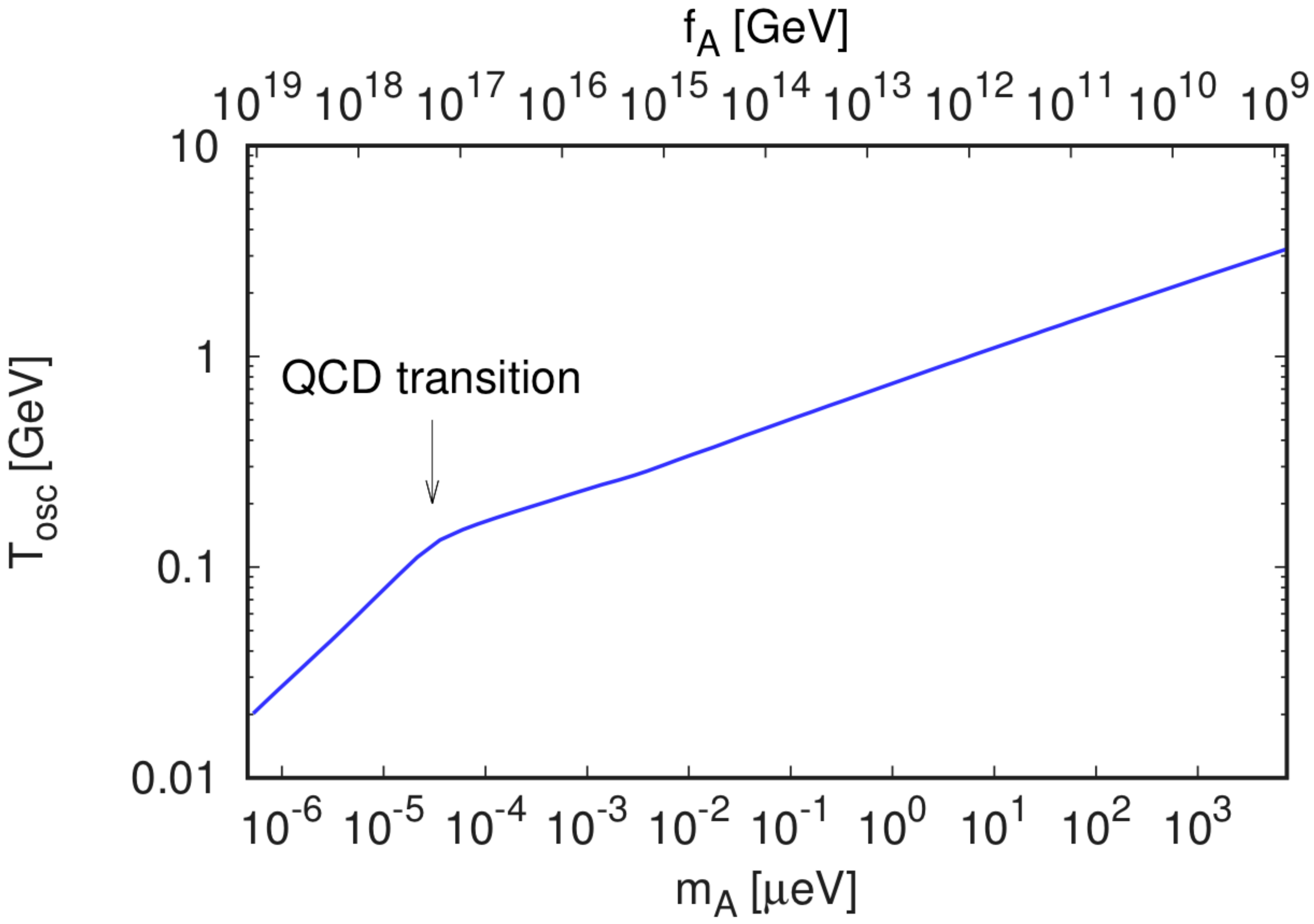
Wantz, Shellard 2010

Axion freezout : $3H(T) = m_a(T) = \sqrt{\chi(T)}/f_a$

First numerical study: Berkowitz Buchoff Rinaldi 2015 Yang-Mills



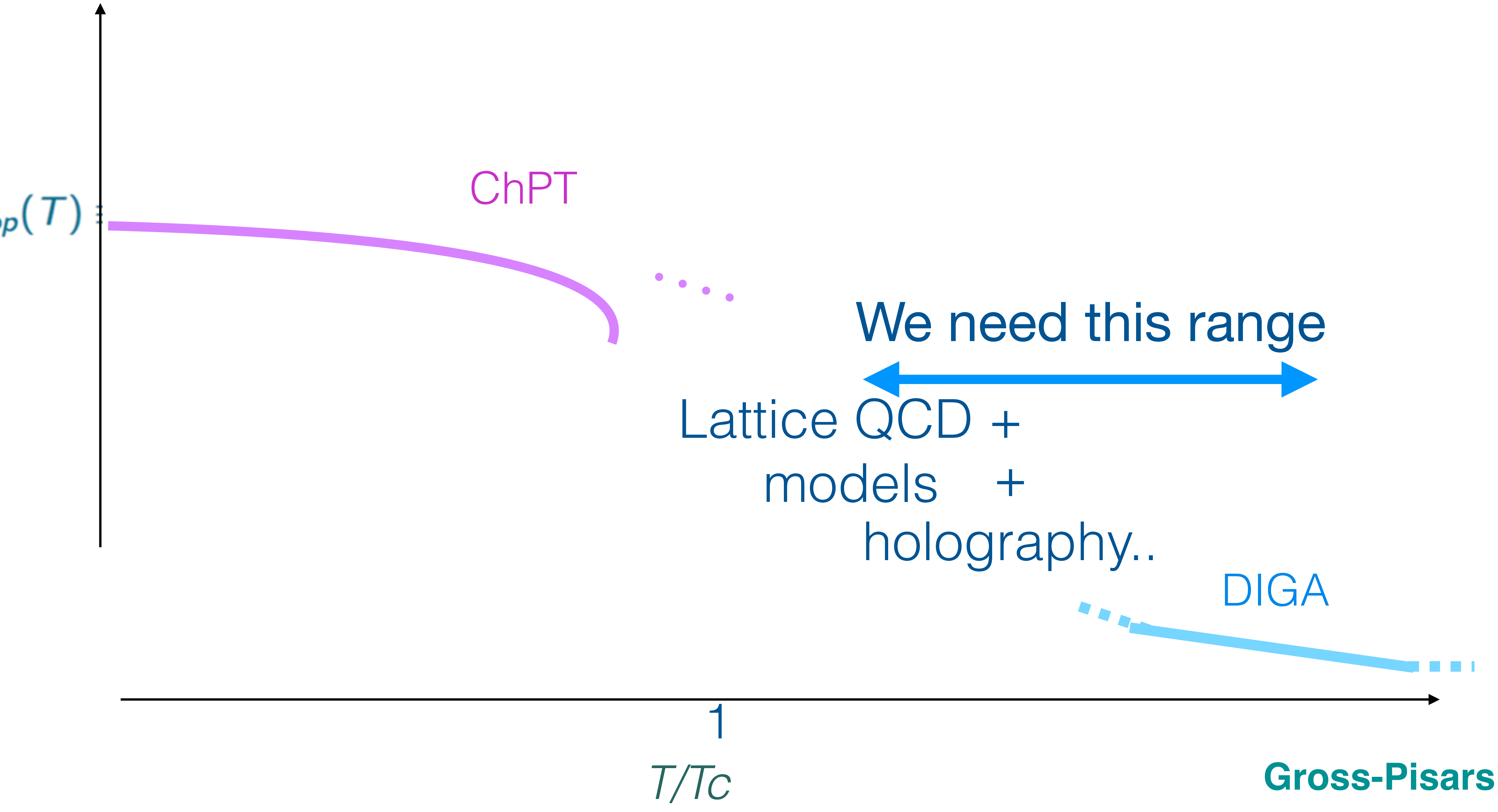
Axion density at freezout controls axion density today



Borsanyi et al, Nature 2016

Figure S30: The oscillation temperature as a function of the axion mass. For this figure we assume that all the observed dark matter comes from the axions. Within this pre-inflation scenario the roll-down comes from a single θ_0 angle. The bend on the figure represents the QCD transition temperature. It reflects the very different behaviour of $\chi(T)$. Above the QCD transition χ rapidly drops, whereas below the QCD transition it has a much milder –almost constant– behaviour.

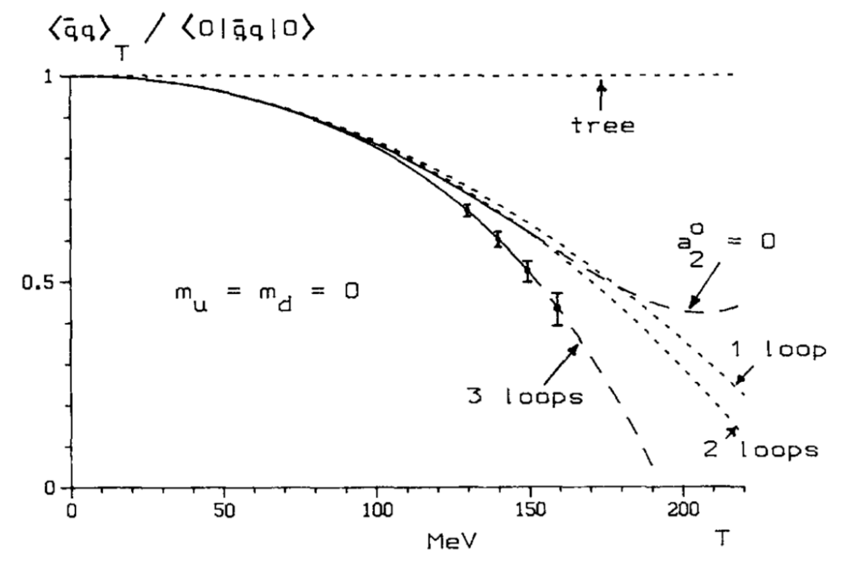
What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$



We need this range

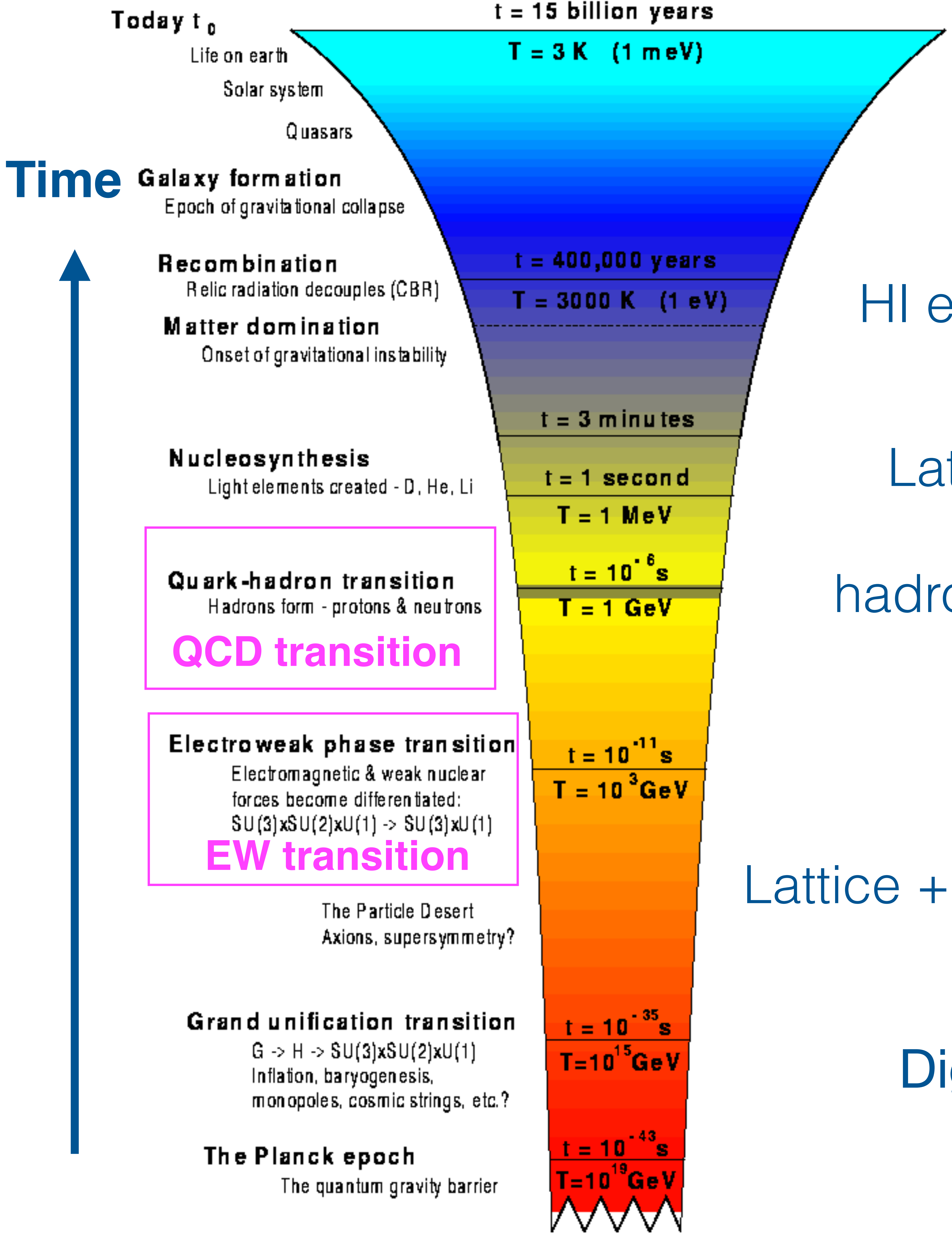
Lattice QCD +
models +
holography..

$$\langle \bar{q}q \rangle \overset{m\to 0}{=} \langle 0|\bar{q}q|0\rangle \left(1 - \frac{T^2}{8F^2} - \frac{T^4}{384F^4} - \frac{T^6}{288F^6}\ln\frac{\Lambda_q}{T} + O(T^8)\right)$$



Gross-Pisarski 1988

Gasser-Leutwyler 1987-1989



HI experiments: T < 500 MeV

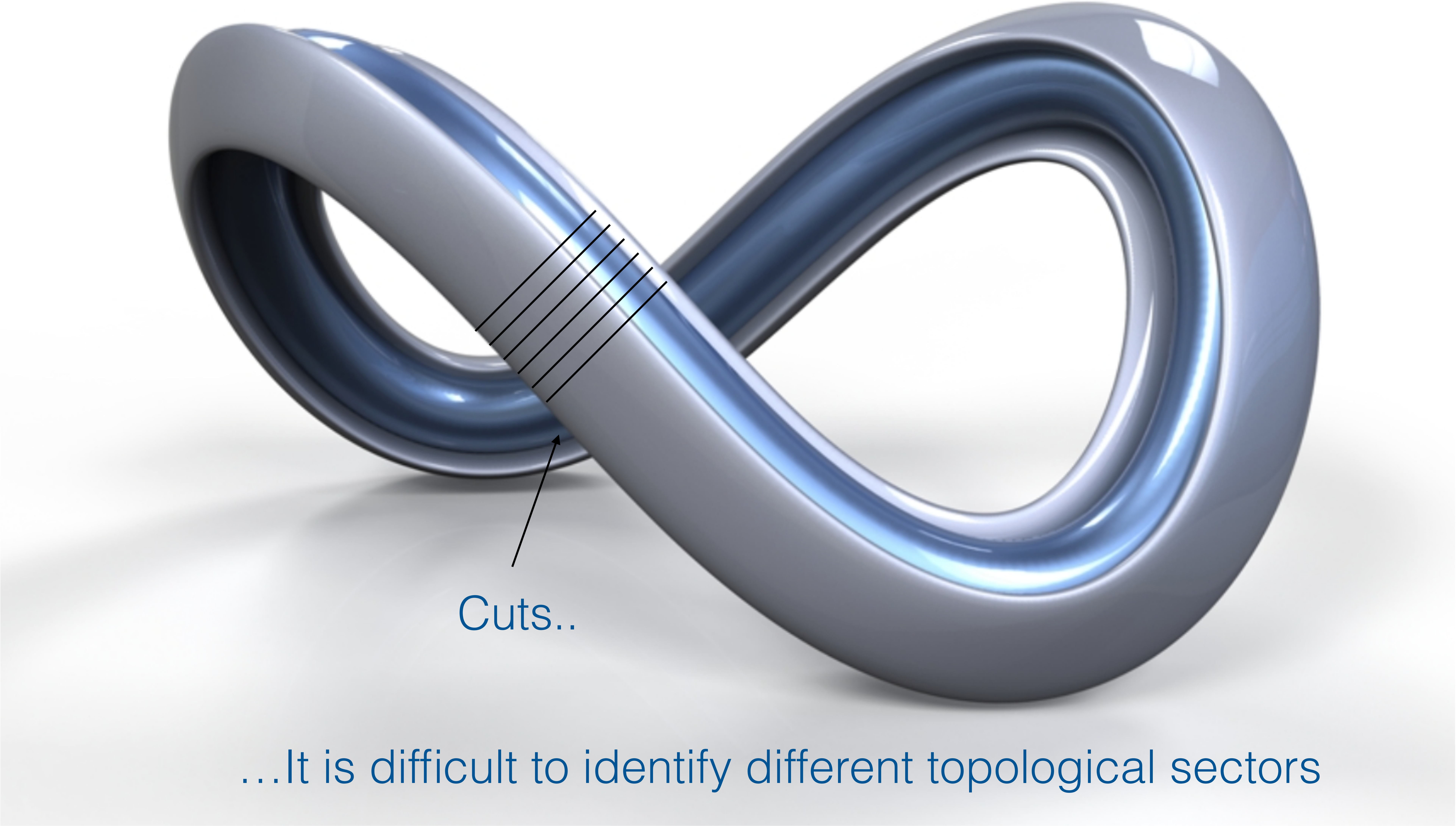
Lattice: T < 600-700 MeV -
sufficient for Tc,
hadron spectrum in the plasma
and
QGP dynamics

Lattice + **extrapolations:** very high T O(10 GeV)
needed to study axions

Diga provides analytic guidance

Results

Topology on a lattice



…It is difficult to identify different topological sectors

…and large temperatures require huge statistics..

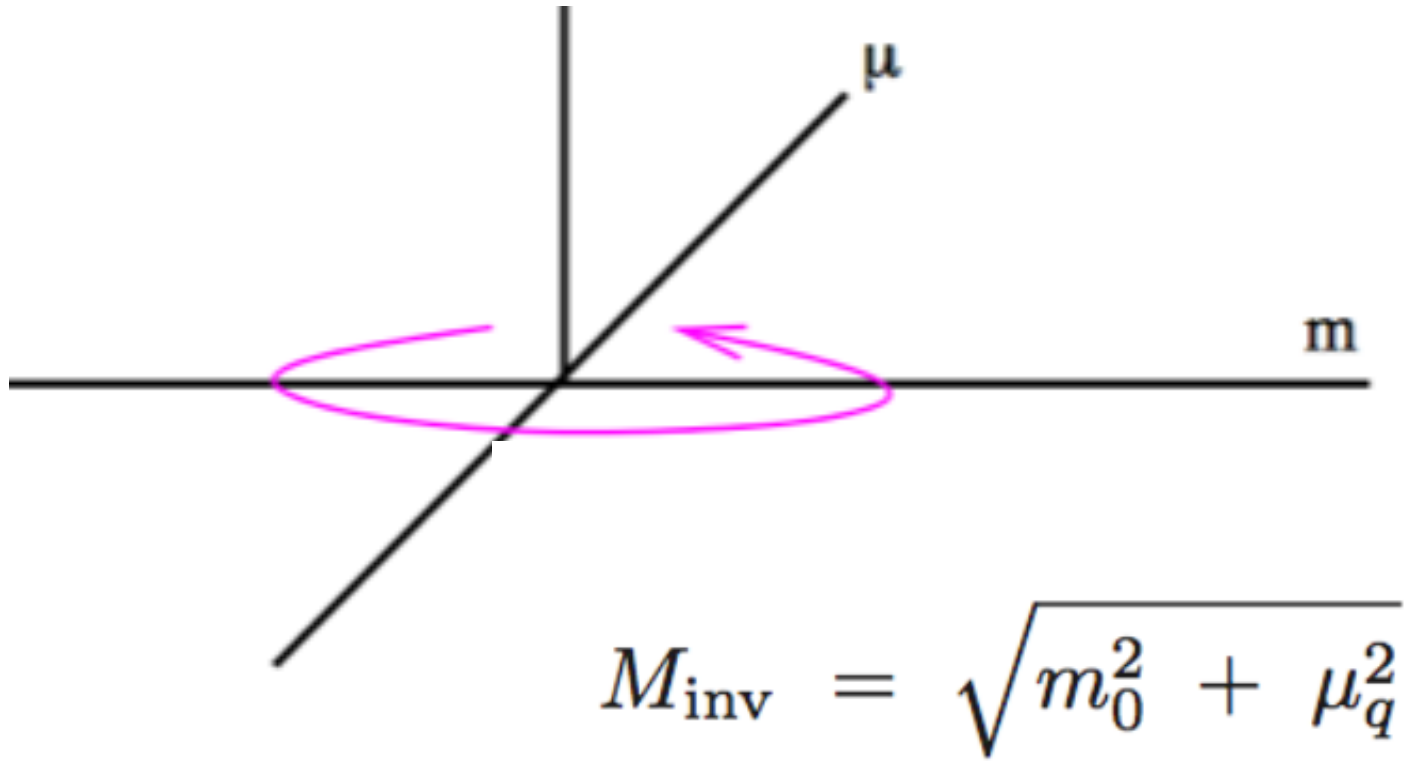
Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

A twisted mass term in flavor space:

$i\mu\tau_3\gamma_5$ for two degenerate light flavors

is added to the standard mass term in the Wilson Lagrangian



$$M_{\rm inv} = \sqrt{m_0^2 + \mu_q^2}$$

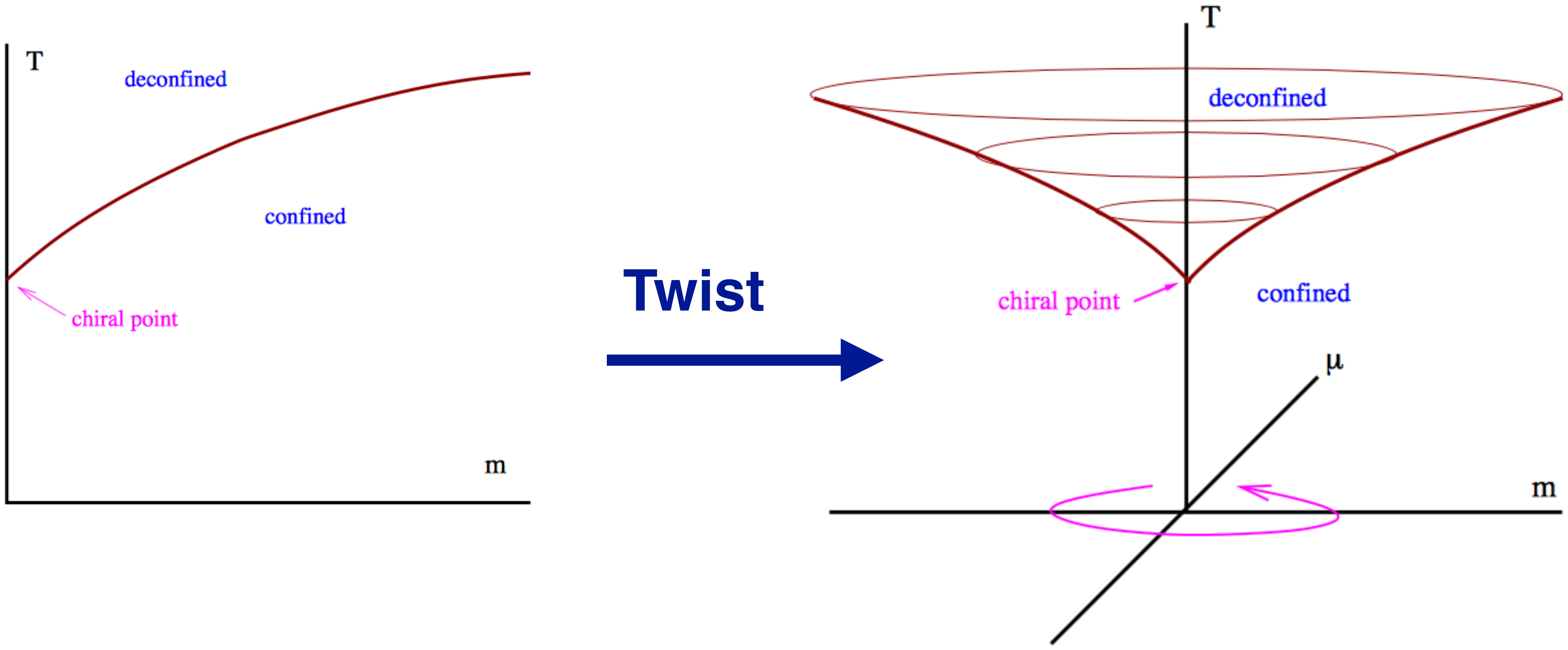
Consequences:

-simplified renormalization prope
-automatic O(a) improvement
-control on unphysical zero modes

Successful phenomenology at T=0

ETMC collaboration 2003—

Continuum scenario at finite temperature



$$M_{\rm inv} = \sqrt{m_0^2 + \mu_q^2} \quad ; \quad \tan(\omega) = \frac{\mu_q}{m_0}$$

Nf = 2+1+1 Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

'twisted' mass terms in flavor space:

$i\mu\tau_3\gamma_5$ for two degenerate light flavors

$i\mu_\sigma\tau_1\gamma_5 + \tau_3\mu_\delta$ for two heavy flavors

are added to the standard Wilson Lagrangian

Consequences:

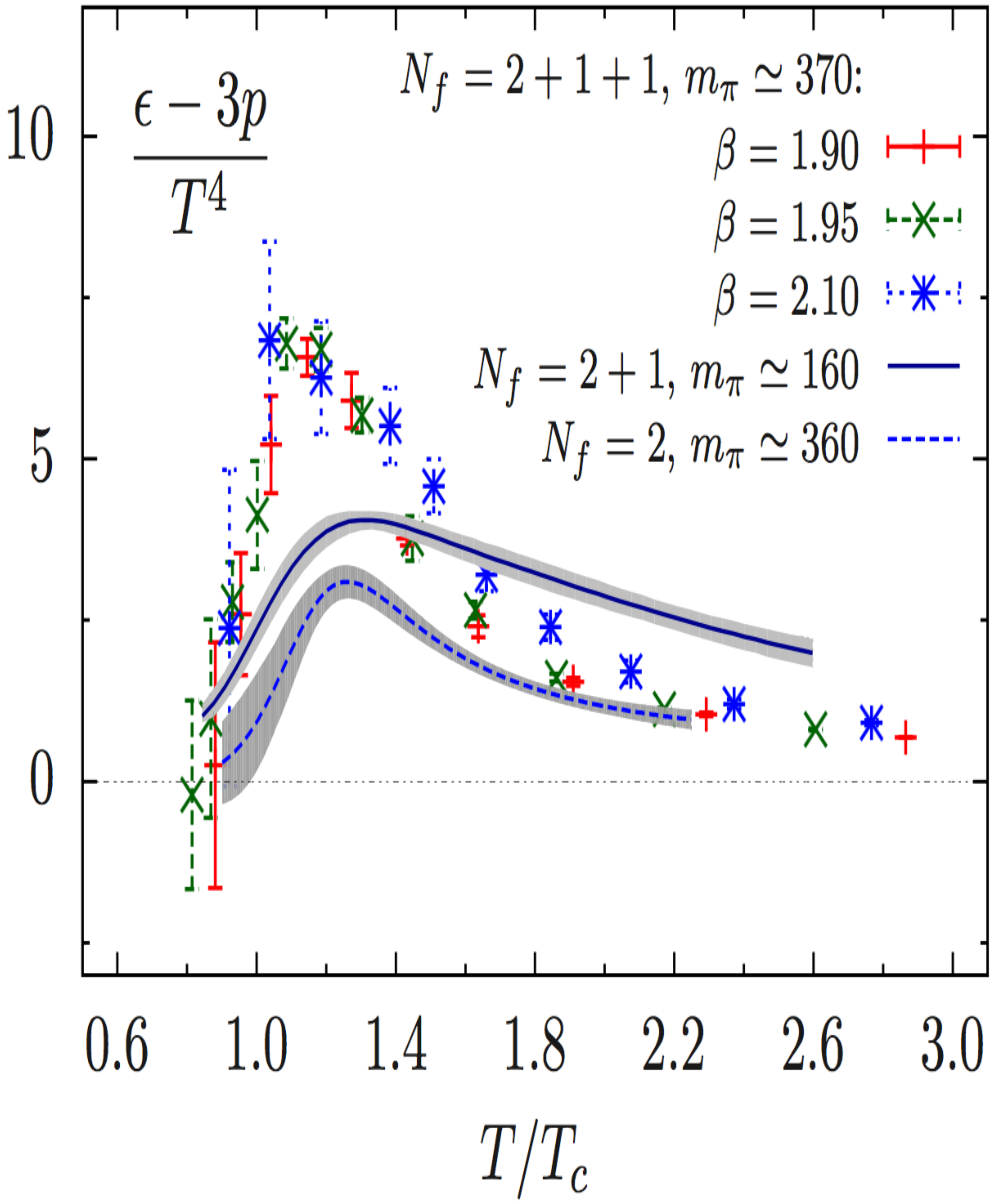
-simplified renormalization properties
-automatic O(a) improvement
-control on unphysical zero modes

Successful phenomenology at T=0

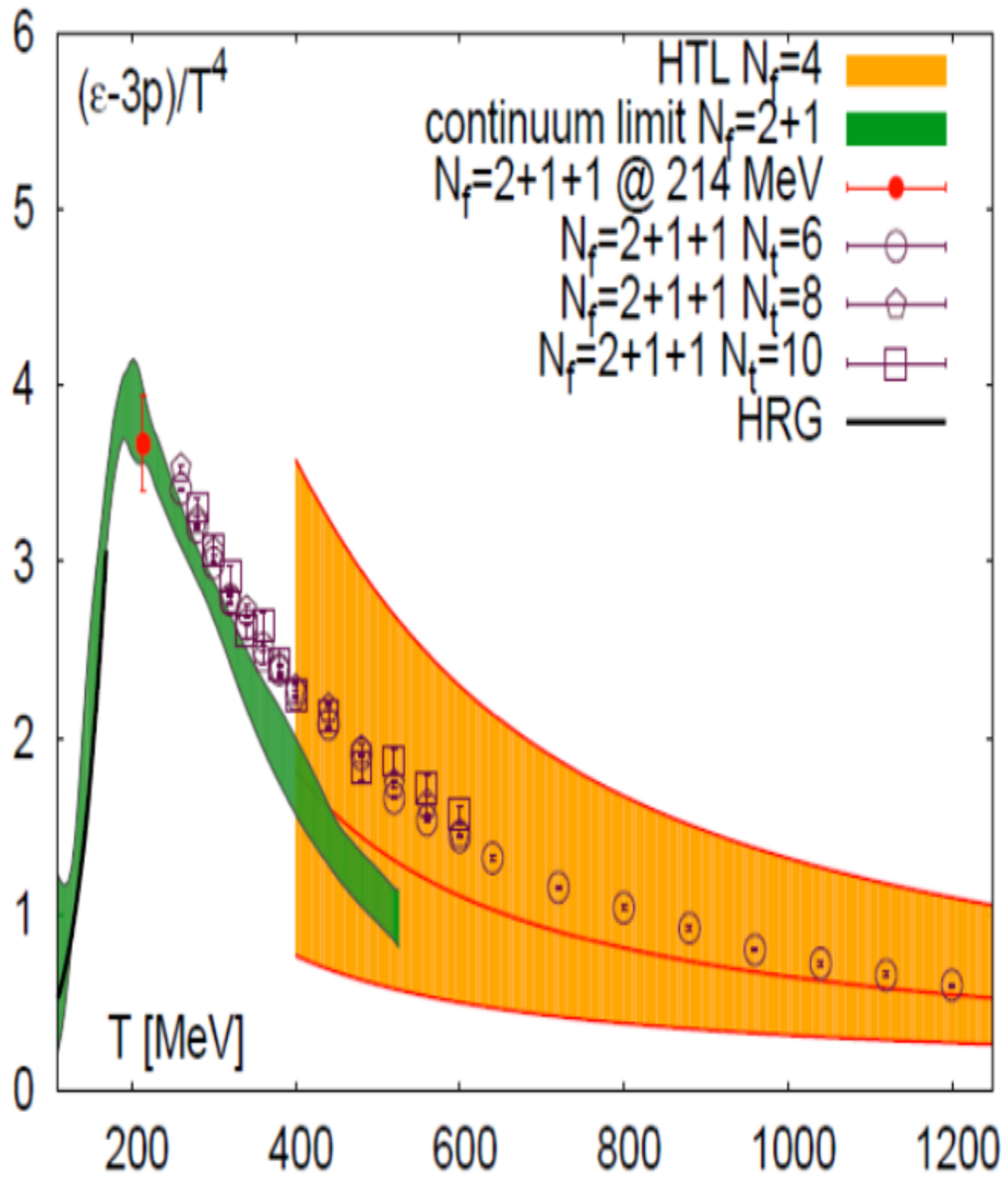
ETMC collaboration 2003—

Trace anomaly: effects of a dynamical charm

Tmft



Wuppertal-Budapest



Staggered

Setup

Twisted mass - Maximal twist Physical strange and charm quarks

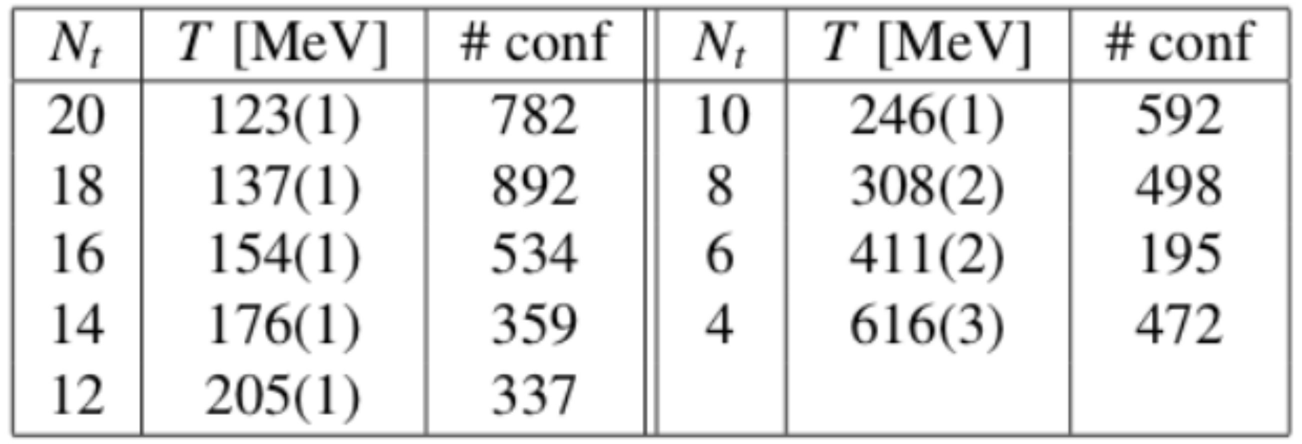
Nf = 2 +1 + 1, $m_\pi^{phys} < m_\pi < 470 \text{ MeV}$ $a = 0.06 - 0.09 \text{ fm}$

Fixed scale approach - Temperature range $130 \text{ MeV} < T < 500 \text{ MeV}$

Observables: Chiral condensate and Susceptibility,

[light mesons' screening masses,,η']

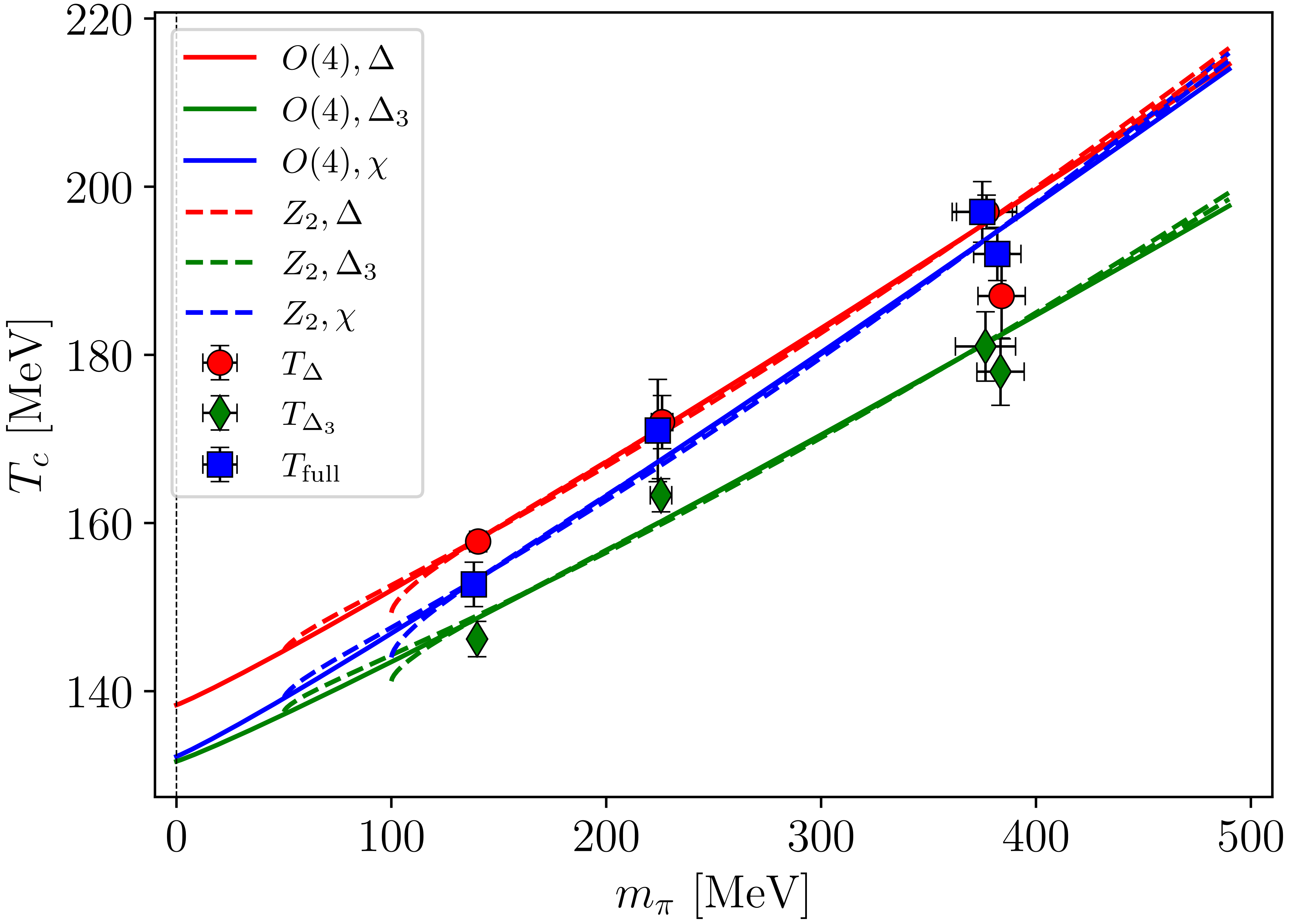
Statistics for physical pion mass



N_t	T [MeV]	# conf	N_t	T [MeV]	# conf
20	123(1)	782	10	246(1)	592
18	137(1)	892	8	308(2)	498
16	154(1)	534	6	411(2)	195
14	176(1)	359	4	616(3)	472
12	205(1)	337			

Heavier masses: Burger, Ilgenfritz, MpL, Trunin *Phys.Rev.D* 98 (2018) 9, 094501

Warmup: Scaling of the pseudo critical temperatures



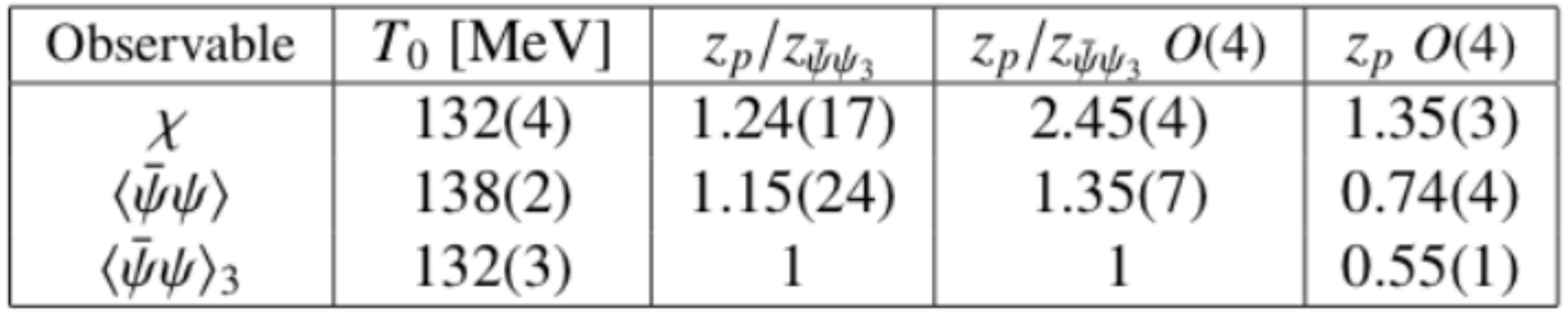
Consistent (not a proof) with O4

Robust extrapolation:

$$T_0 \equiv T_c(m_\pi \to 0) = 134^{+6}_{-4}\ \text{MeV}$$

Check O4:

$$T_c(m_\pi) = T_0 + A z_p m_\pi^{2/\beta\delta}$$



Observable	T_0 [MeV]	$z_p/z_{\bar\psi\psi_3}$	$z_p/z_{\bar\psi\psi_3}$ $O(4)$	z_p $O(4)$
χ	132(4)	1.24(17)	2.45(4)	1.35(3)
$\langle\bar\psi\psi\rangle$	138(2)	1.15(24)	1.35(7)	0.74(4)
$\langle\bar\psi\psi\rangle_3$	132(3)	1	1	0.55(1)

O4 vs Z2

$$T_c(m_\pi) = T_0 + B(m_\pi^2 - m_c^2)^{1/\beta\delta}$$

m_c = 100 MeV still OK

m_c = 0 still OK, indistinguishable from O4

Topological and chiral susceptibility

Kogut, Lagae, Sinclair 1999

HotQCD, 2012

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{5,disc}$$

From:

$$m \int d^4x \bar{\psi}\gamma_5\psi = Q_{top}$$

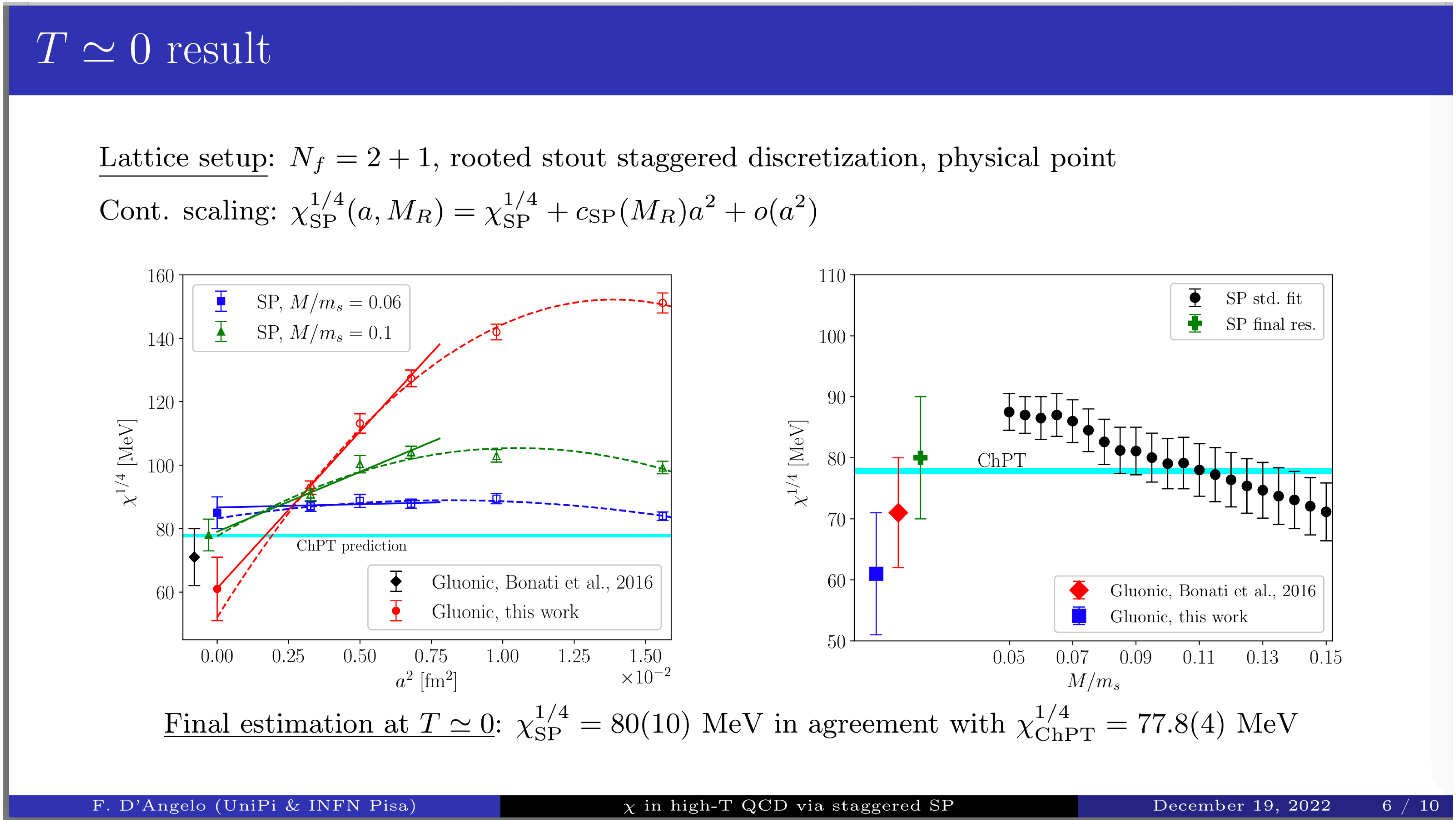
$$\begin{tikzcd}
\chi_{5,con} & \pi : \bar{q}\gamma_5 \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \mathrm{x}\, SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q}q \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{con} + \chi_{disc} \\
\chi_{con} & \delta : \bar{q}\frac{\tau}{2} q \arrow[r, leftrightarrow] & \eta : \bar{q}\gamma_5 q & \chi_{5,con} - \chi_{5,disc}
\end{tikzcd}$$

$$\chi_\pi - \chi_\delta = \chi_{\rm disc} = \chi_{5,{\rm disc}} \ , \qquad \text{for} \quad T \geq T_c \ , \ m_l \rightarrow 0$$

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{disc}$$

[Slide from Francesco D’Angelo @SM&FT22]

Systematics from staggered fermions - Spectral projector



$T \simeq 0$ result

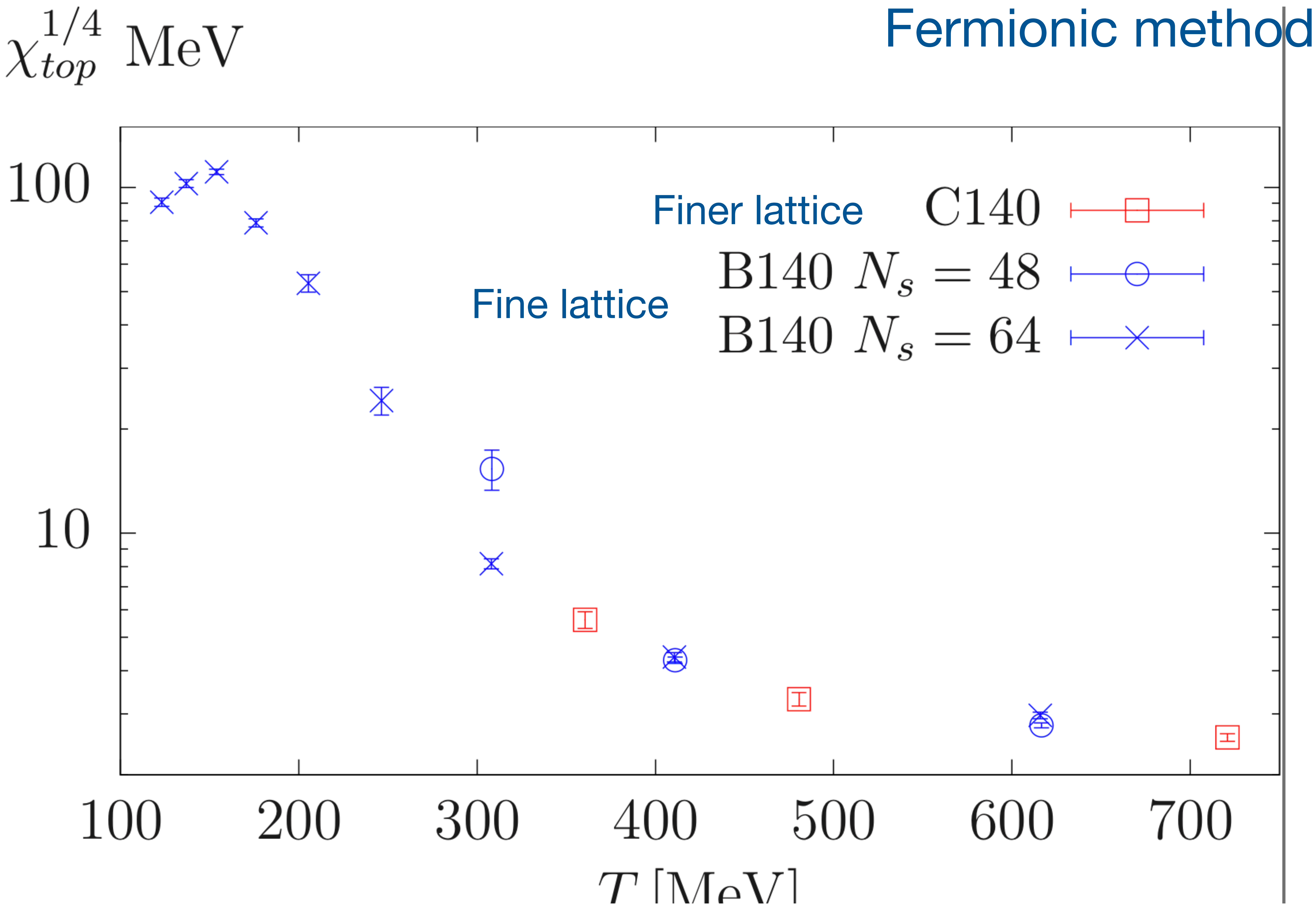
Lattice setup: $N_f = 2+1$, rooted stout staggered discretization, physical point

Cont. scaling: $\chi_{\mathrm{SP}}^{1/4}(a, M_R) = \chi_{\mathrm{SP}}^{1/4} + c_{\mathrm{SP}}(M_R)a^2 + o(a^2)$

Final estimation at $T \simeq 0$: $\chi_{\mathrm{SP}}^{1/4} = 80(10)$ MeV in agreement with $\chi_{\mathrm{ChPT}}^{1/4} = 77.8(4)$ MeV

Systematics from twisted mass Wilson fermions 2+1+1 flavours

Fermionic method



Finer lattice

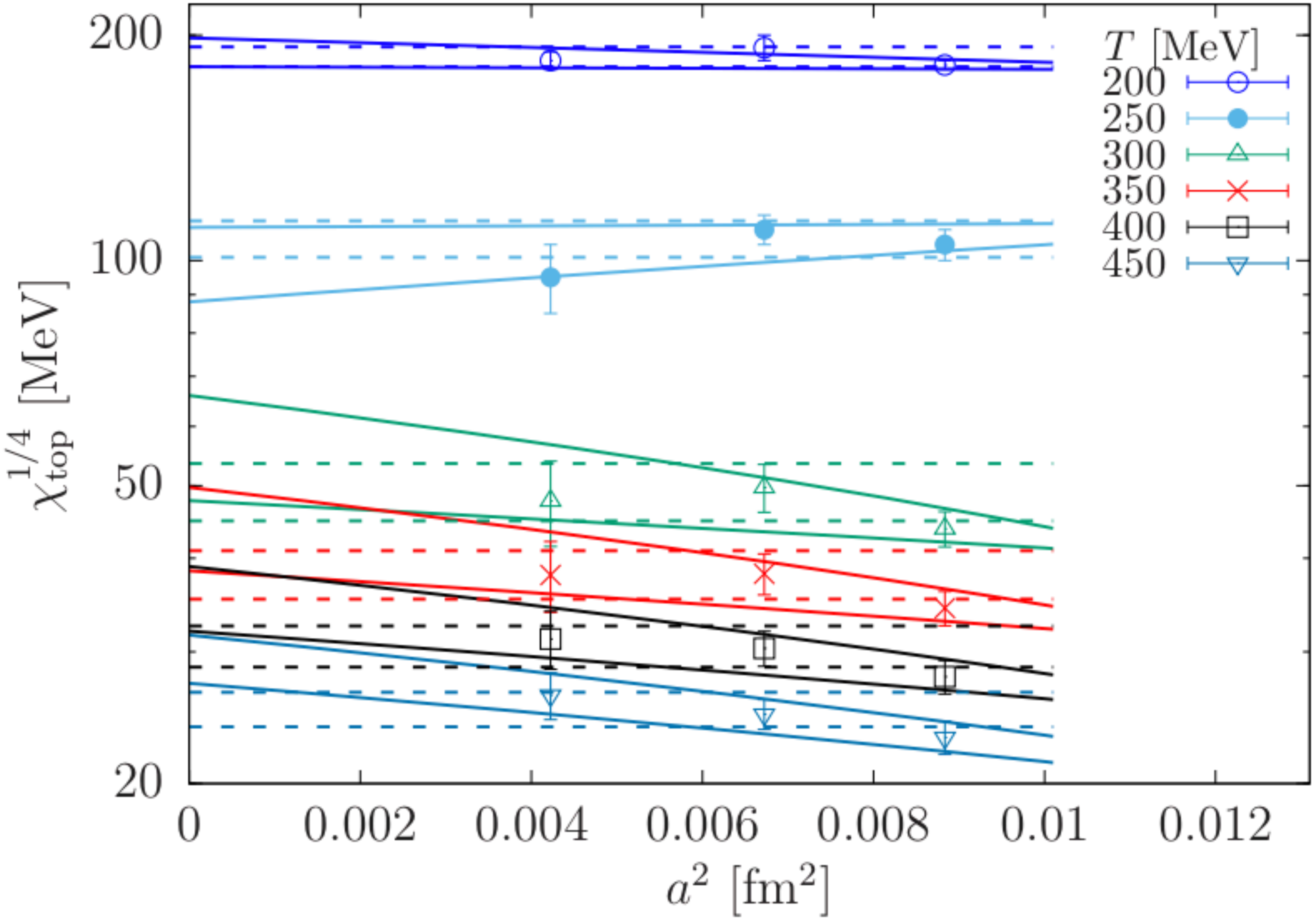
Fine lattice

Kotov MpL Trunin (2021 + in progress)

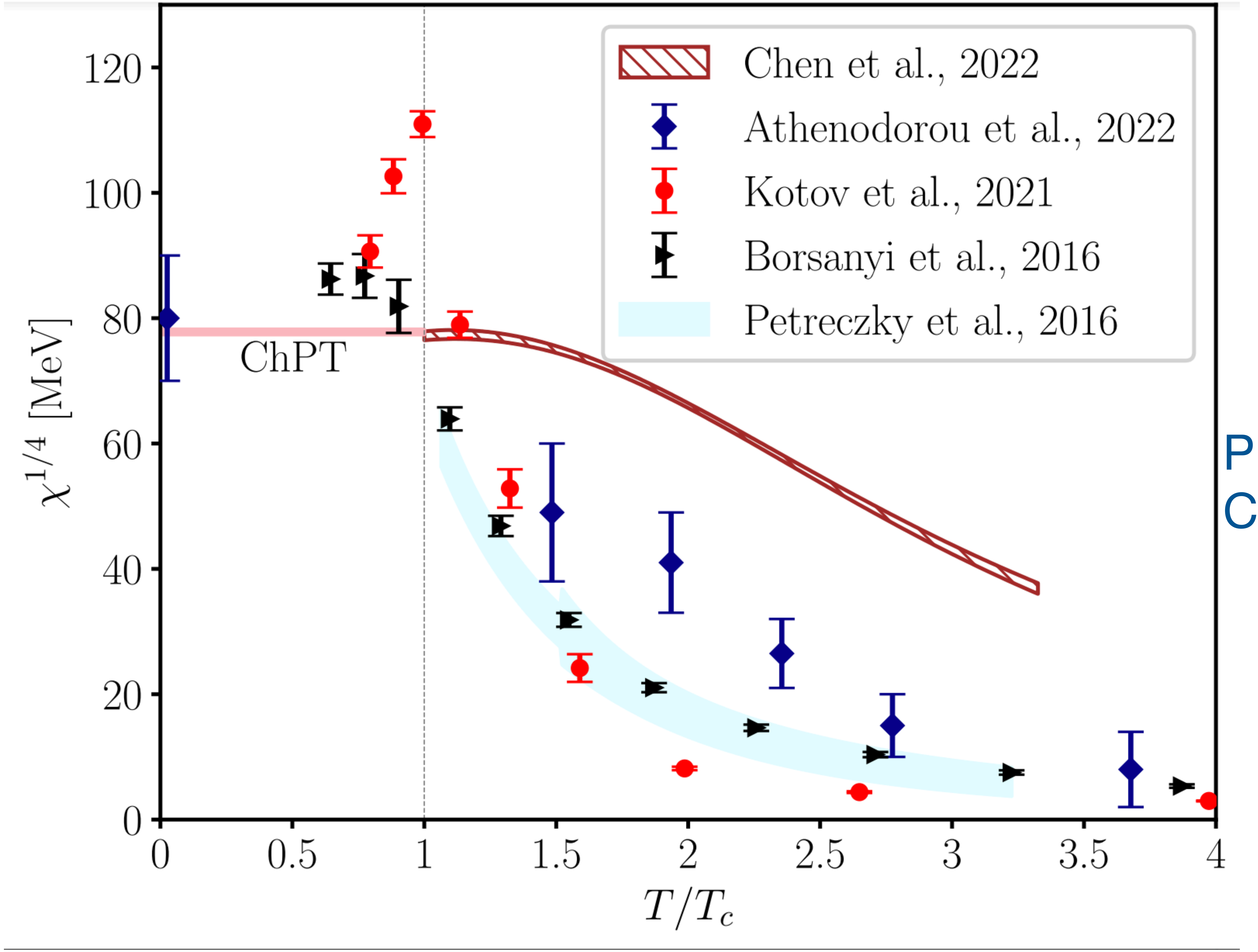
Systematics from twisted mass Wilson fermions 2+1+1 flavours

Heavier pion masses

Fermionic method



Burger, Ilgenfritz, Mueller -Preussker, MpL Trunin 2018



Plot by
Claudio Bonanno

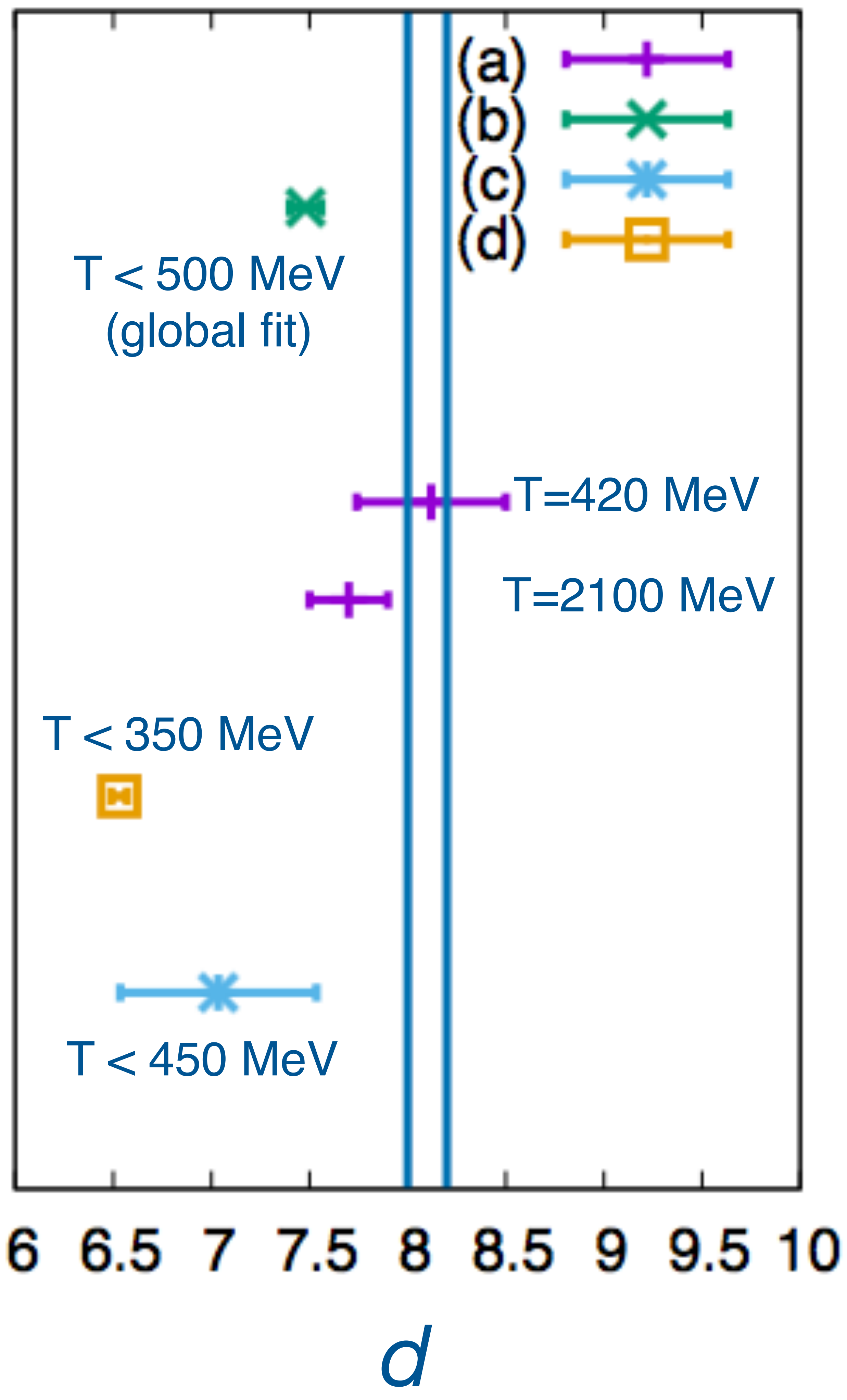
Na6-Strong2020, MpL et al, 2023

For axion freeze-out extrapolation needed $\chi(T) = A\,T^{d}$

Summary of the *d* parameters

Y. Taniguchi, K. Kanaya, H. Suzuki and T. Umeda (2017) (d),

Borsanyi et al. (2016) (a)
Petreczky, Schlaeder, Scharma (2016) (b)
☐ Burger et al. (2018) ☐(c)
DIGA, Nf = 3
DIGA, Nf=4

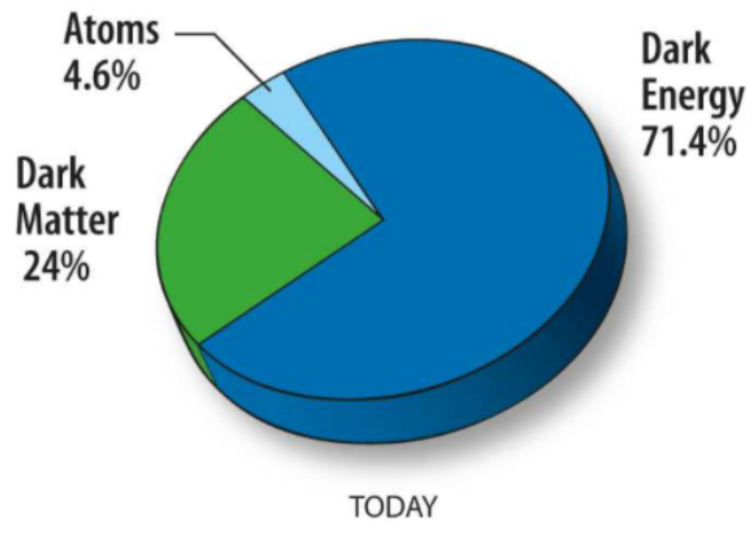


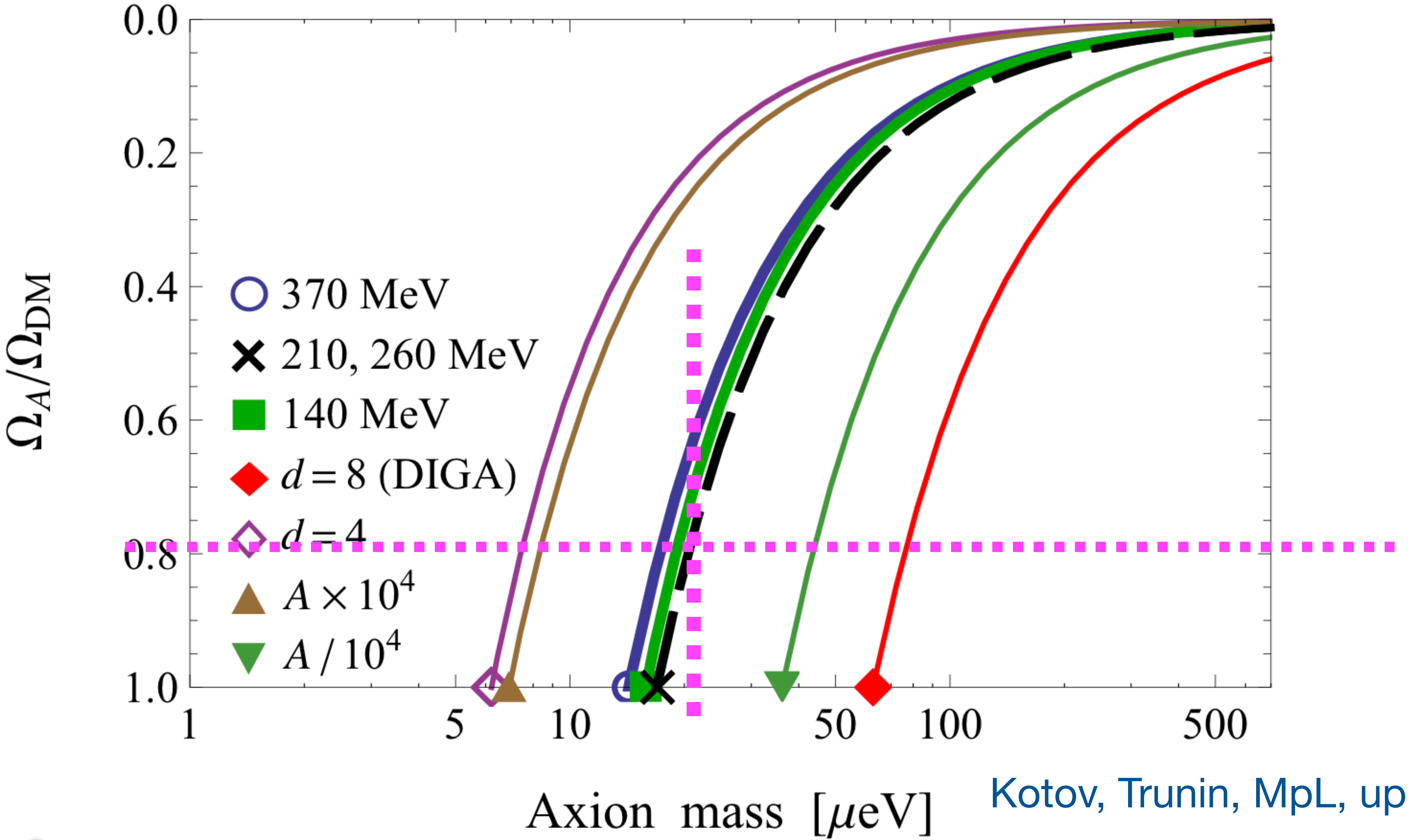
For T > 300 MeV the DIGA exp
is approached from below

Tc < T < 250 — 300 MeV ??

Limits on the (post-inflactionary) axion mass

$$\Omega_A = F(A, d, \ldots)\, m_A^{-\frac{3.053+d/2}{2.027+d/2}}$$





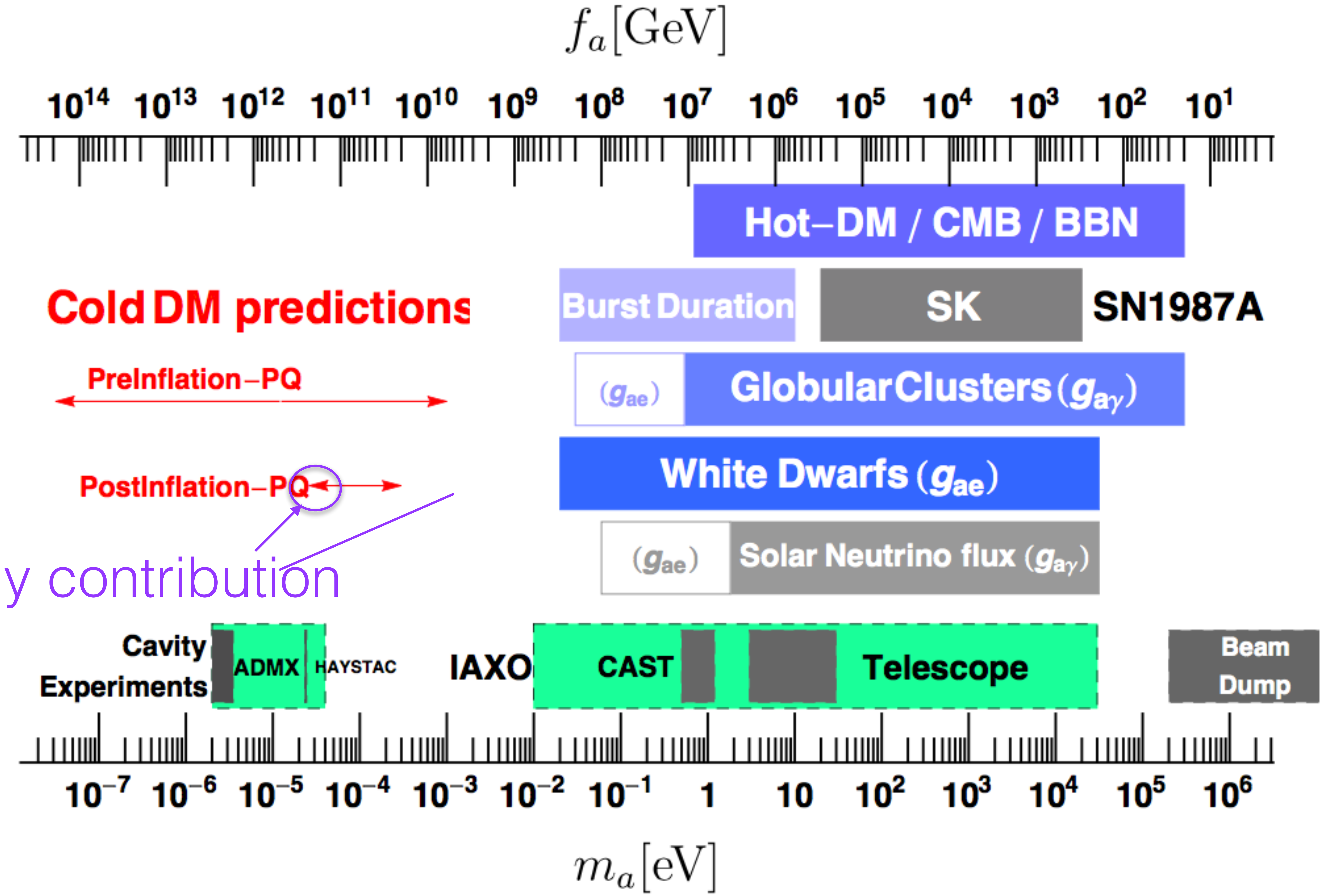
$$\Omega_a = \frac{\rho_{a,0}}{\rho_c};$$

Kotov, Trunin, MpL, updated 2023

Burger,Trunin, Ilgenfritz,Mueller-Preussker,MpL 2019

Example: if axions constitute 80% DM, our results give a lower bound for the axion mass of $\simeq 30\mu\mathrm{eV}$

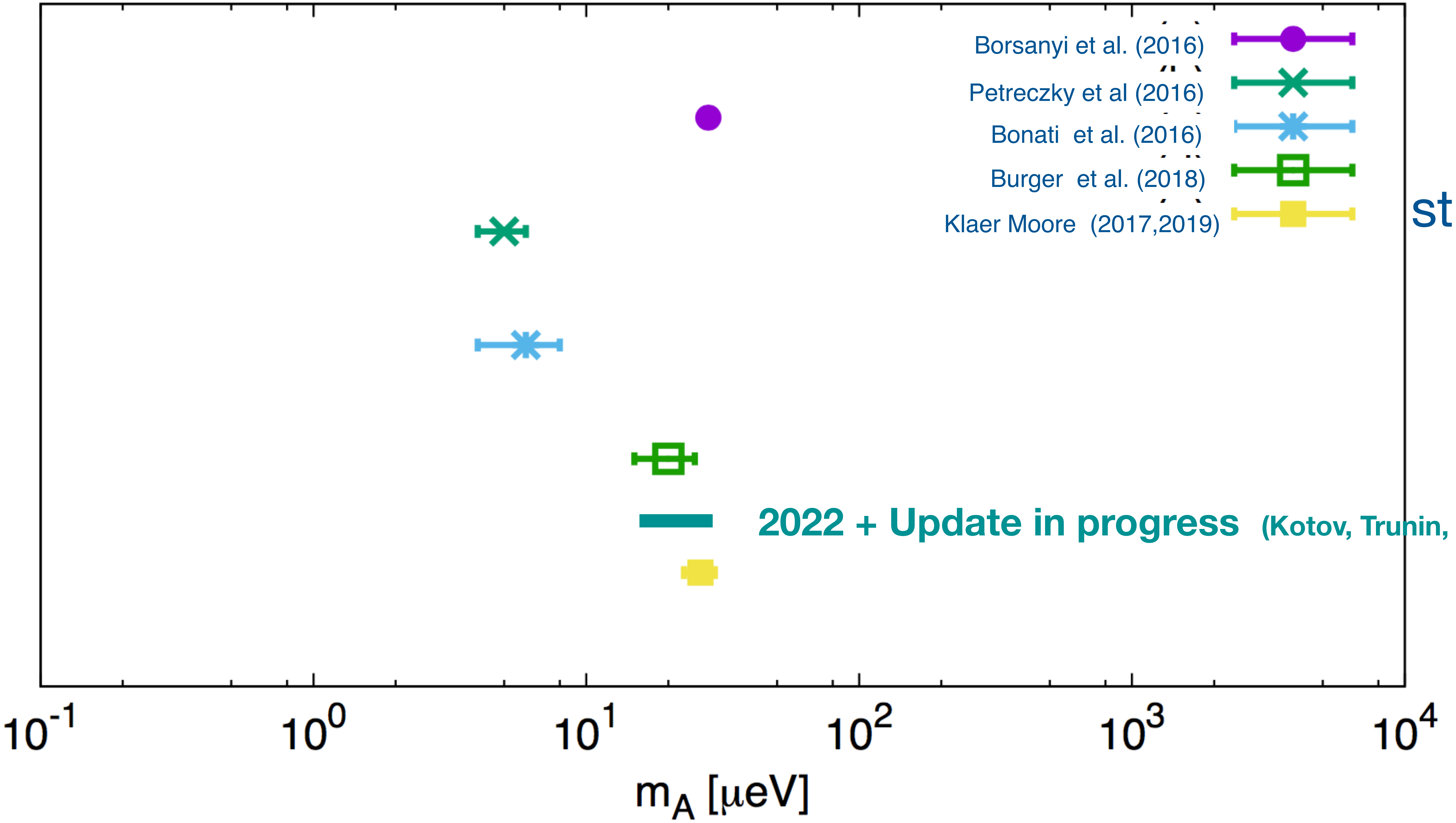
Limits on the axion mass



Lattice topology contribution

Issue: string contribution ??

Lower limits on post-infl. axion mass from lattice QCD



2022 + Update in progress (Kotov, Trunin, MpL)

strings contribution

Slide from Maurizio Giannotti

Axion Dark matter

Predictable initial angle.

Axion abundance depends also on production from topological defects

Estimating the axion string contribution from topological defects is very difficult. Numerical simulations still make very different predictions.

Important numerical advances thanks to Adaptive Mesh Refinement

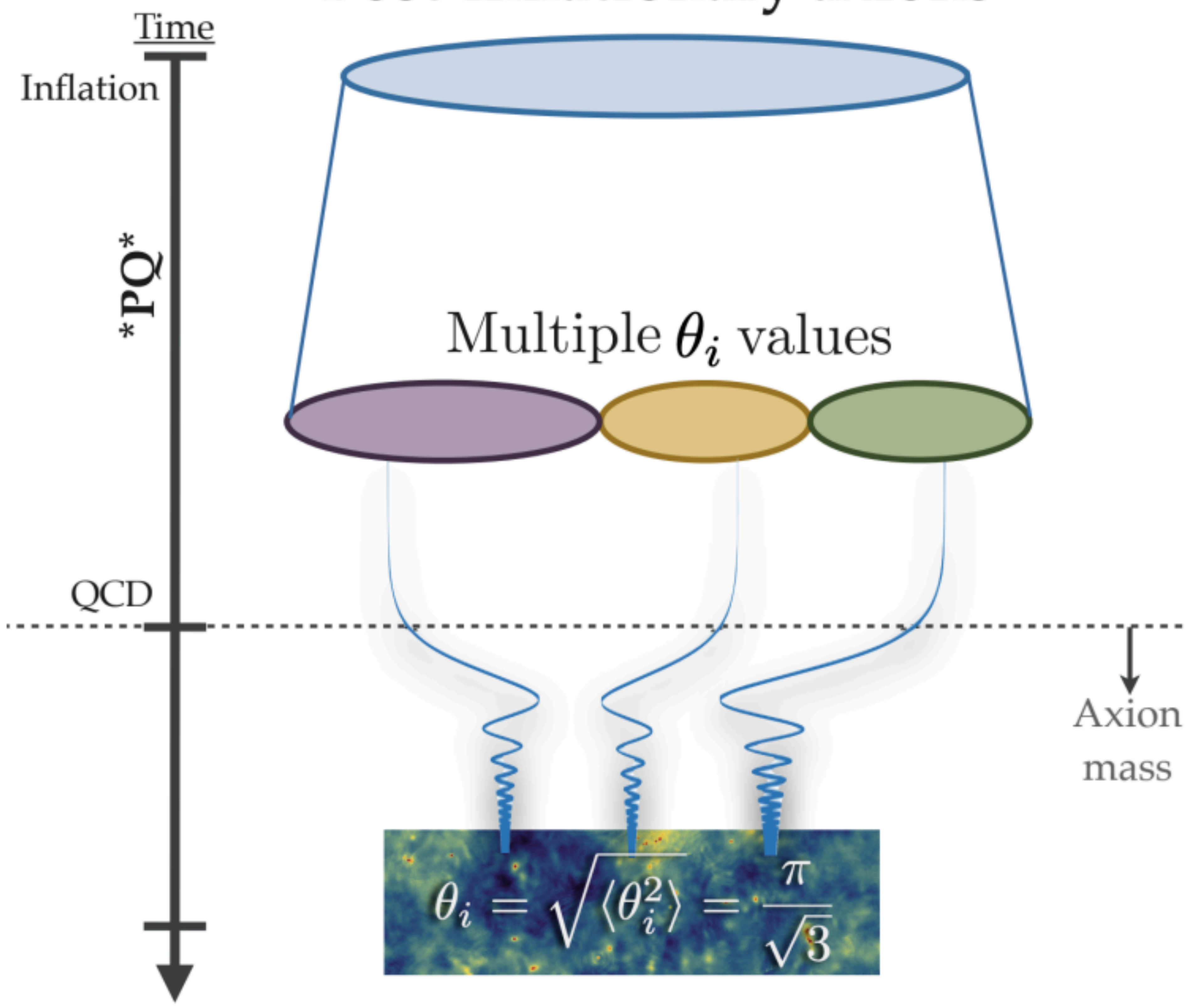
M. Buschmann et al., *Nature Commun.* 13 (2022) 1, 1049.

Still controversial. More work required.

- M. Gorghetto, E. Hardy, arXiv:2212.13263
- O'Hare, Pierobon, Redondo, Wong, Phys.Rev.D 105 (2022)
- M. Gorghetto, E. Hardy, G. Villadoro, SciPost Phys. 10, 050 (2021)

Scenario 2:

Post-inflationary axions



$$\theta_i = \sqrt{\langle \theta_i^2 \rangle} = \frac{\pi}{\sqrt{3}}$$

Ensemble of initial misalignment angles
→ Density set by single stochastic average

Figure Credits: C. O'Hare (2021)
https://cajohare.files.wordpress.com/2021/10/axions.pdf

The two faces of QCD topology

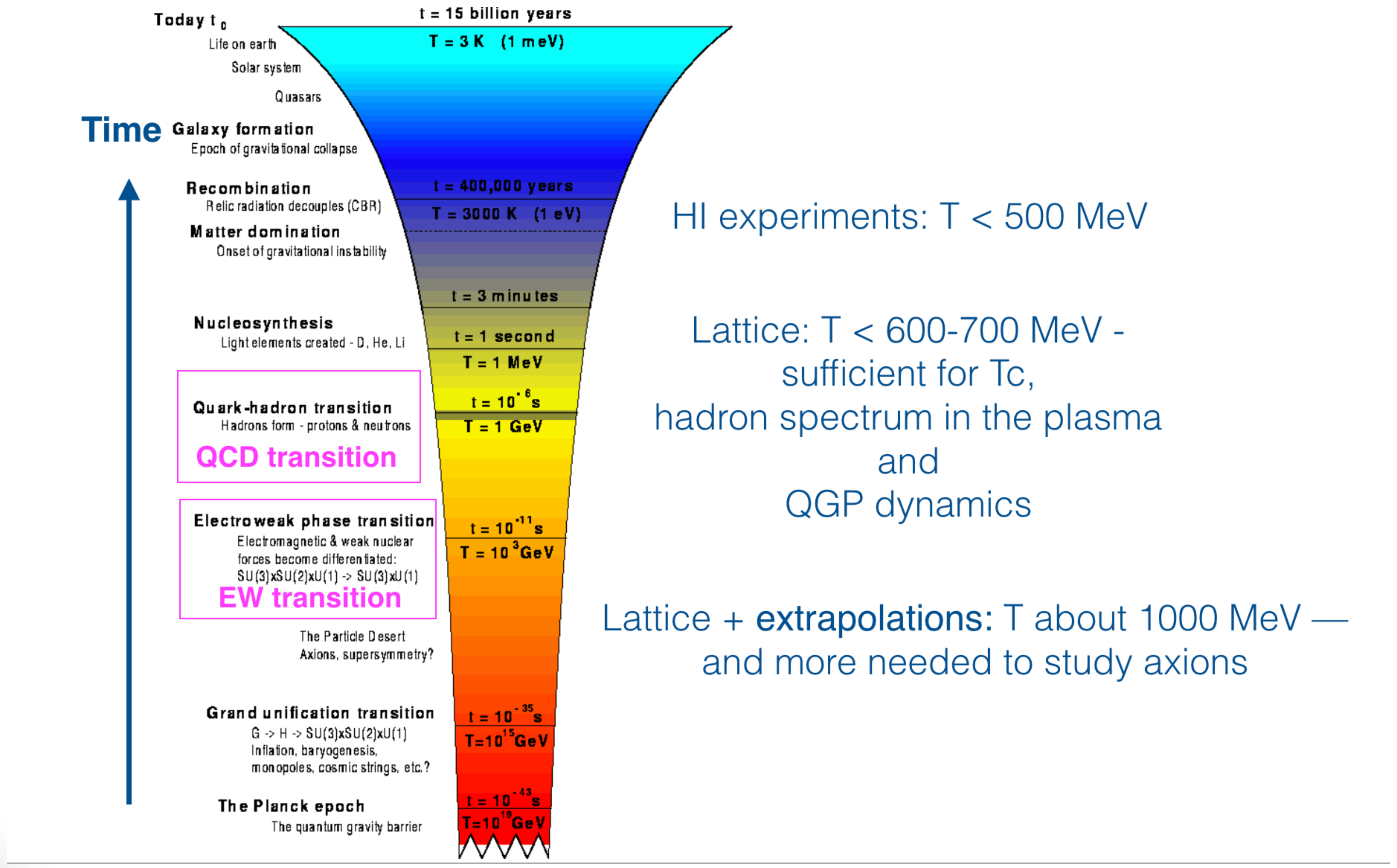
Window to Dark Matter

Strong interactions dynamics

The other face of topology:

Role in the strongly coupled Quark Gluon Plasma

Topology from low to high but not so high Temperature



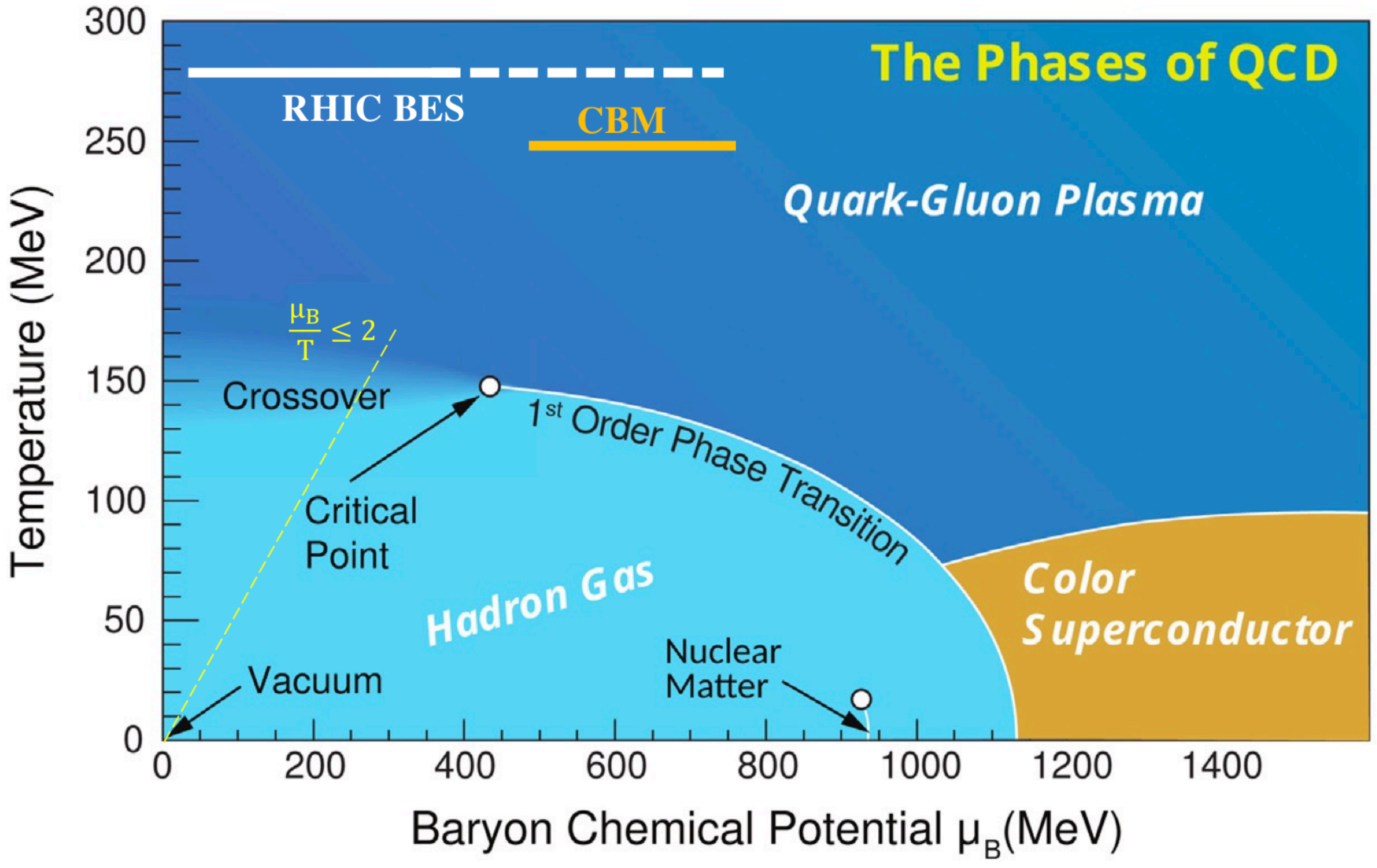
HI experiments: T < 500 MeV

Lattice: T < 600-700 MeV -
sufficient for Tc,
hadron spectrum in the plasma
and
QGP dynamics

Lattice + **extrapolations:** T about 1000 MeV —
and more needed to study axions

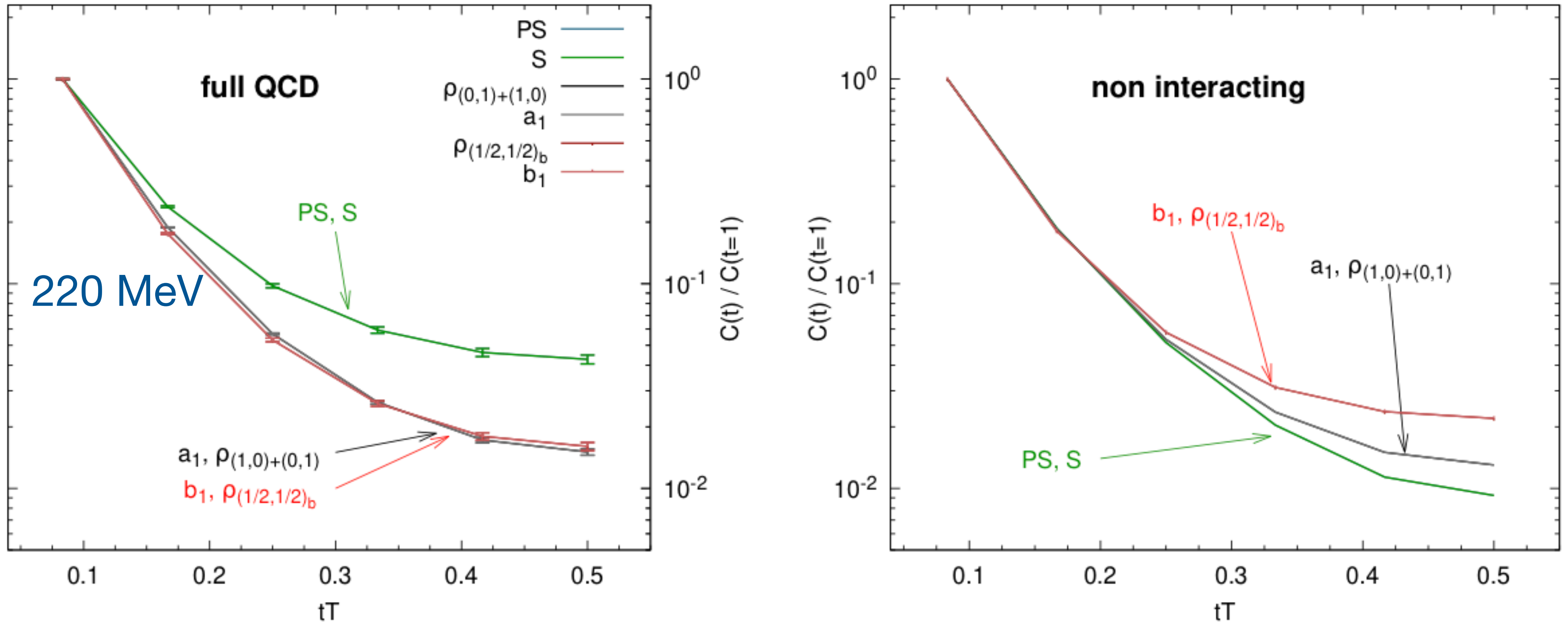
A threshold in the plasma ?

Alexandru and Horvath (2019-2021);
Glozman et al; Glozman, Philipsen, Pisarski (2016-2022);
Burger, Kotov, MpL, Trunin (2018-2022..)



Almanool et al., 2022

Spectrum and the anomalous threshold



C. Rohrhofer, Y. Aoki, L.Y. Glozman and S. Hashimoto, 2021

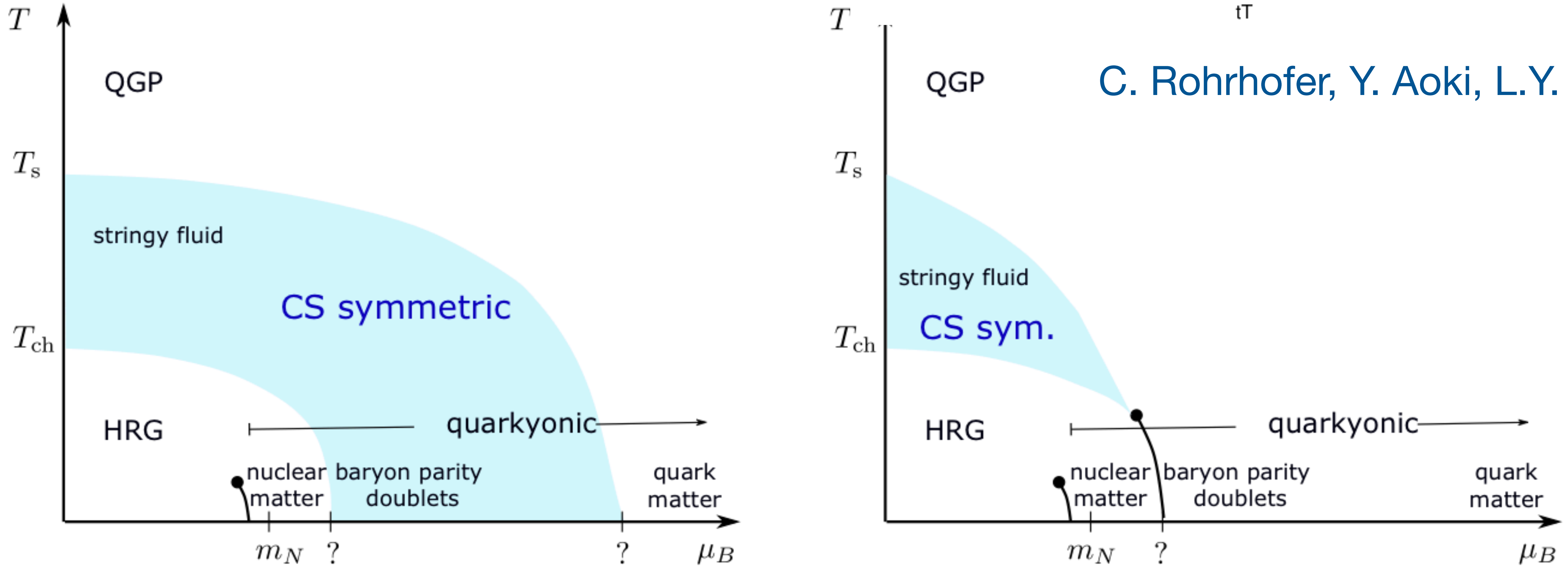


Figure 8: Possibilities for the QCD phase diagram with a chiral spin and $SU(4)$-symmetric band.

Philipsen et al. 2022

Generic features of a critical region

Mass

Tc

Mean field

Ginzburg region
Non-trivial
criticality

Mean field

. Is the ‘threshold’ related with the crossover from the Ginzburg region to mean field ?
. Is the threshold related with the onset of power-law of topological susceptibility?

Analysis tool : scaling of the singular part of the Free Energy

Assumption: Free Energy = Singular + Regular

Mean field

Ginzburg region
Non-trivial
criticality

Mean field

Playing with the order parameter

also mentioned in the PhD thesis by Wolfgang Unger

'Beating' the regular terms/additive renormalization for more stringent universality checks

$$\Delta_3 \equiv (\bar\psi\psi - m\chi_L) \equiv (\bar\psi\psi - m\frac{\partial\bar\psi\psi}{\partial m}) \qquad \equiv m(\chi_T - \chi_L)$$

Advantage wrt standard subtracted condensate: admits EoS

Note: Transverse and longitudinal susceptibilities

$$\chi_T = \frac{\bar\psi\psi}{m}$$

$$\chi_L = \frac{\partial\bar\psi\psi}{\partial m}.$$

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)},$$

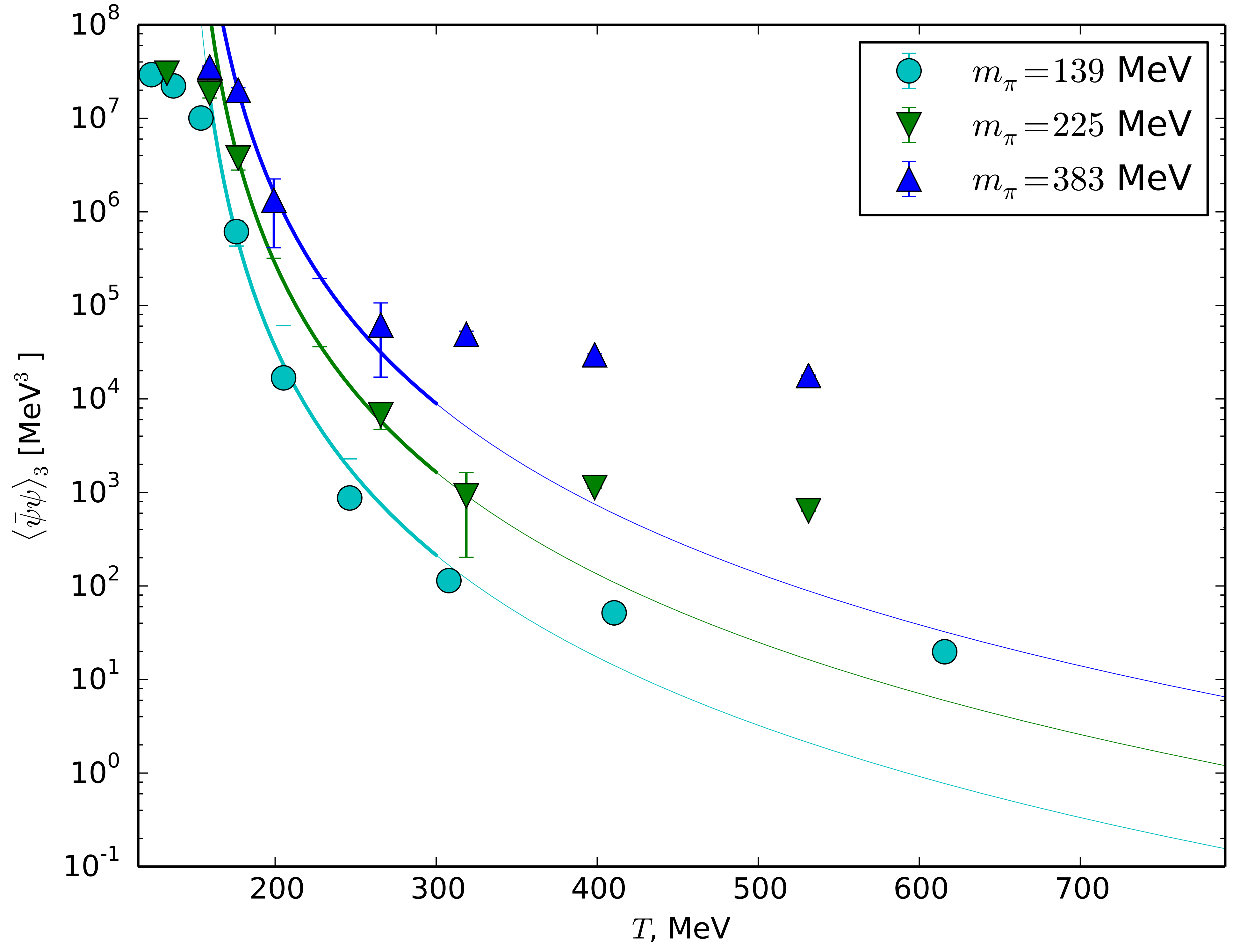
$$R_\pi(0,m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

Searching for the scaling window in temperature

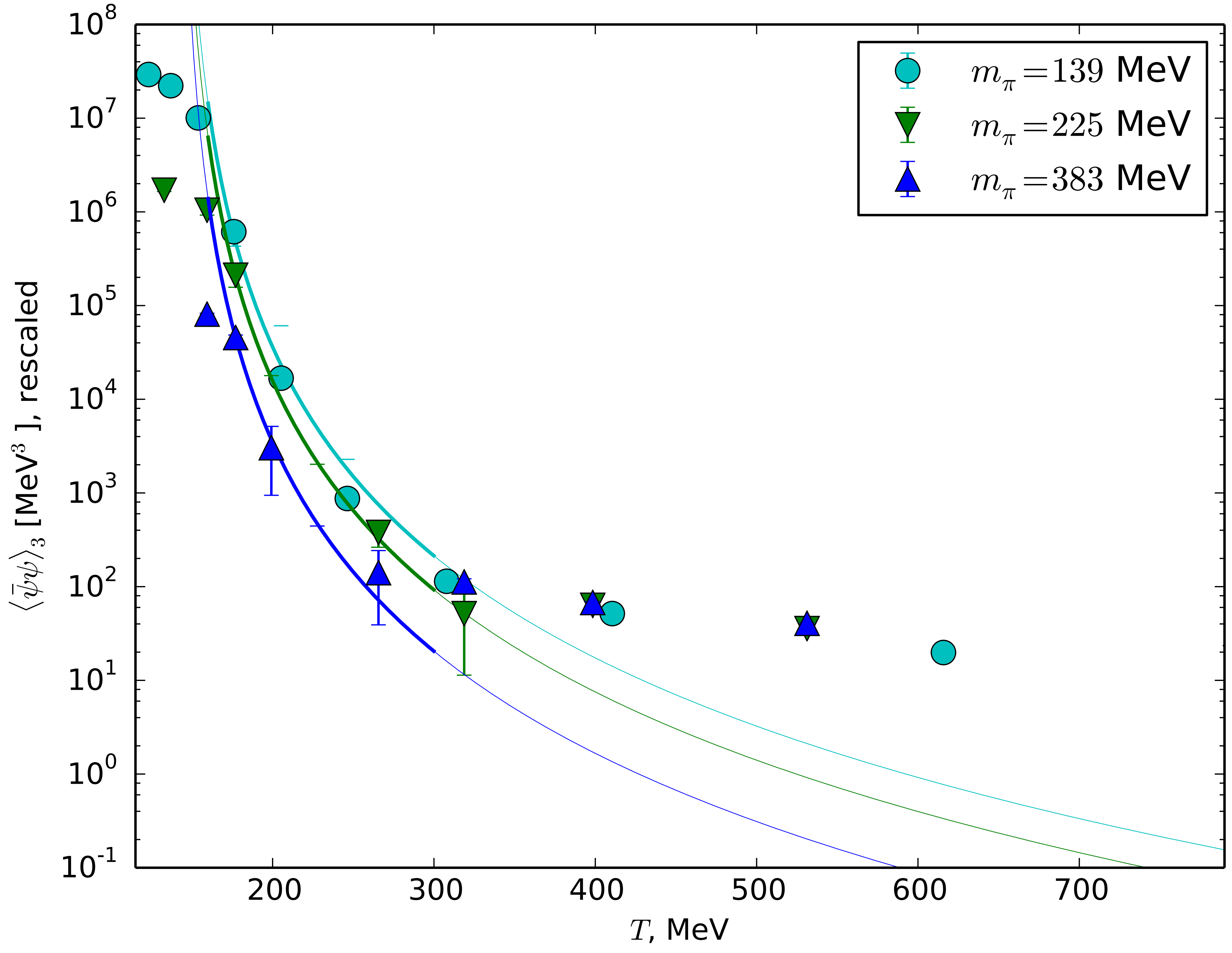
Kotov, MpL, Trunin 2021

Ginzburg region T < 300 MeV



$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$

Simple analytic behaviour T > 300 MeV



$$\Delta_3 \propto m_\pi^6$$

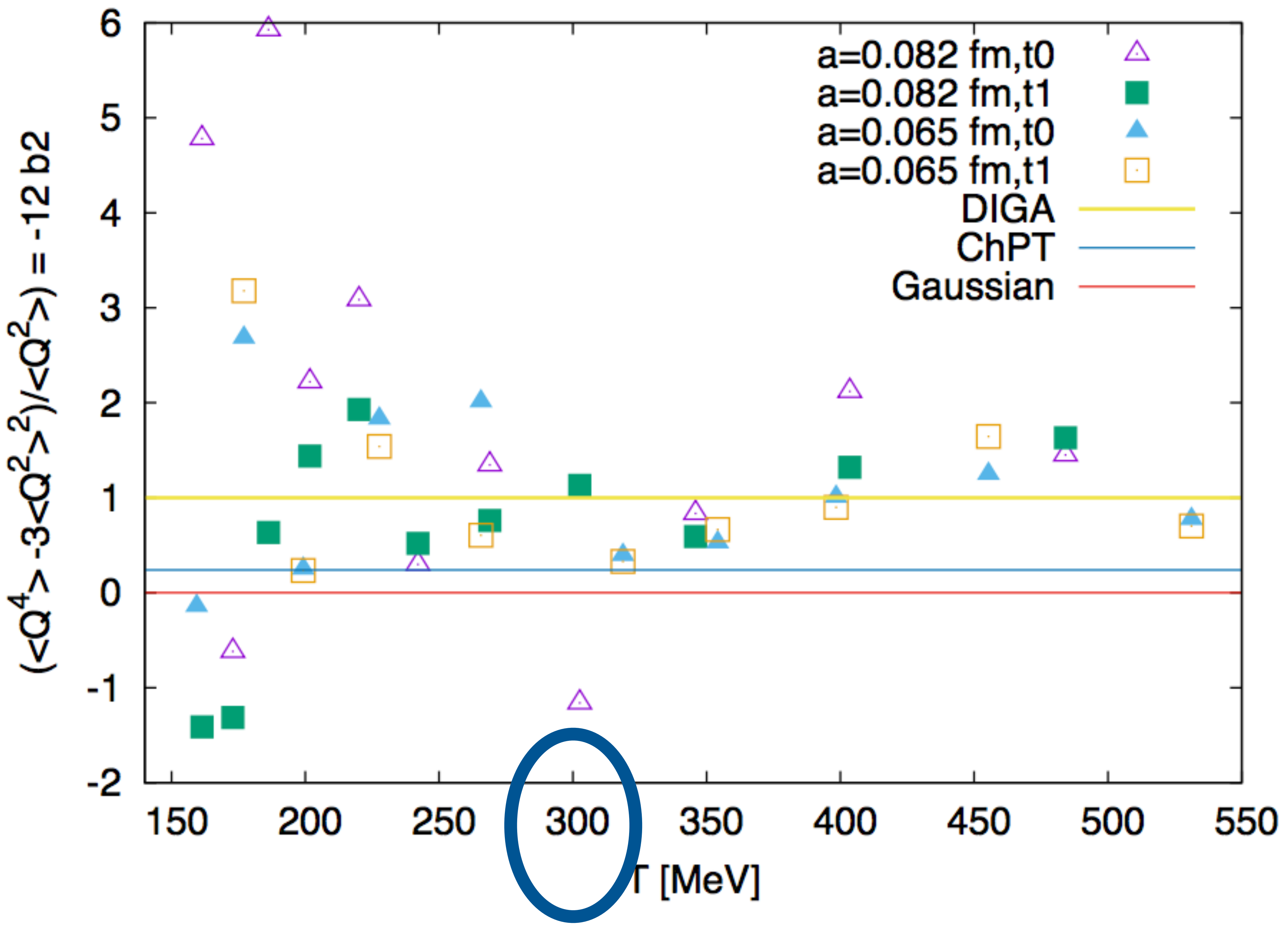
Evidence of a related crossover in topology

Beyond topological susceptibility, higher order cumulants

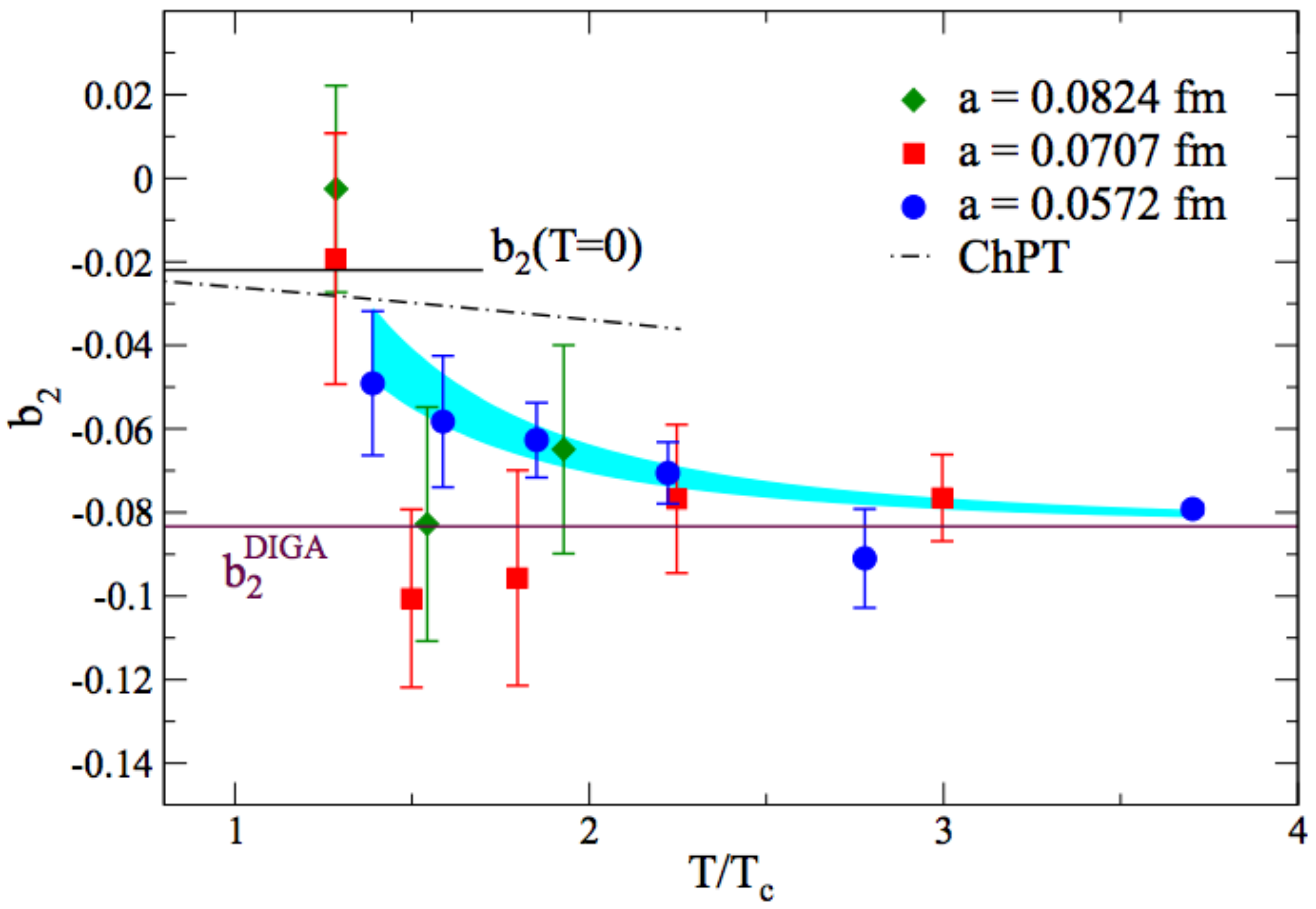
$T > 250\text{-}300$ MeV

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

d'Elia, Vicari (2013)



Trunin et al (2018)



Bonati et al. (2016)

Summary

Axion and Topology in QCD

1. Axions: Limits on the post-inflationary axion mass

   Lattice issues: Results not entirely settled; extrapolation to high temperature regime may be subtle and needs further studies.
   Phenomenological issues: Requirements on mass accuracy?
   Interest for axion potential? Input from string crucial

2. QGP: Interesting threshold at the border of the strongly-coupled plasma

   Interplay of topology, chiral and axial symmetry still to be explored