QCD Axion vs Lattice QCD:

Past successes and future challenges

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Definition of QCD axion:

pseudo-Goldstone boson whose shift symmetry is only/mostly broken by the QCD anomaly

Peccei Quinn '77
Weinberg, Wilczek '78
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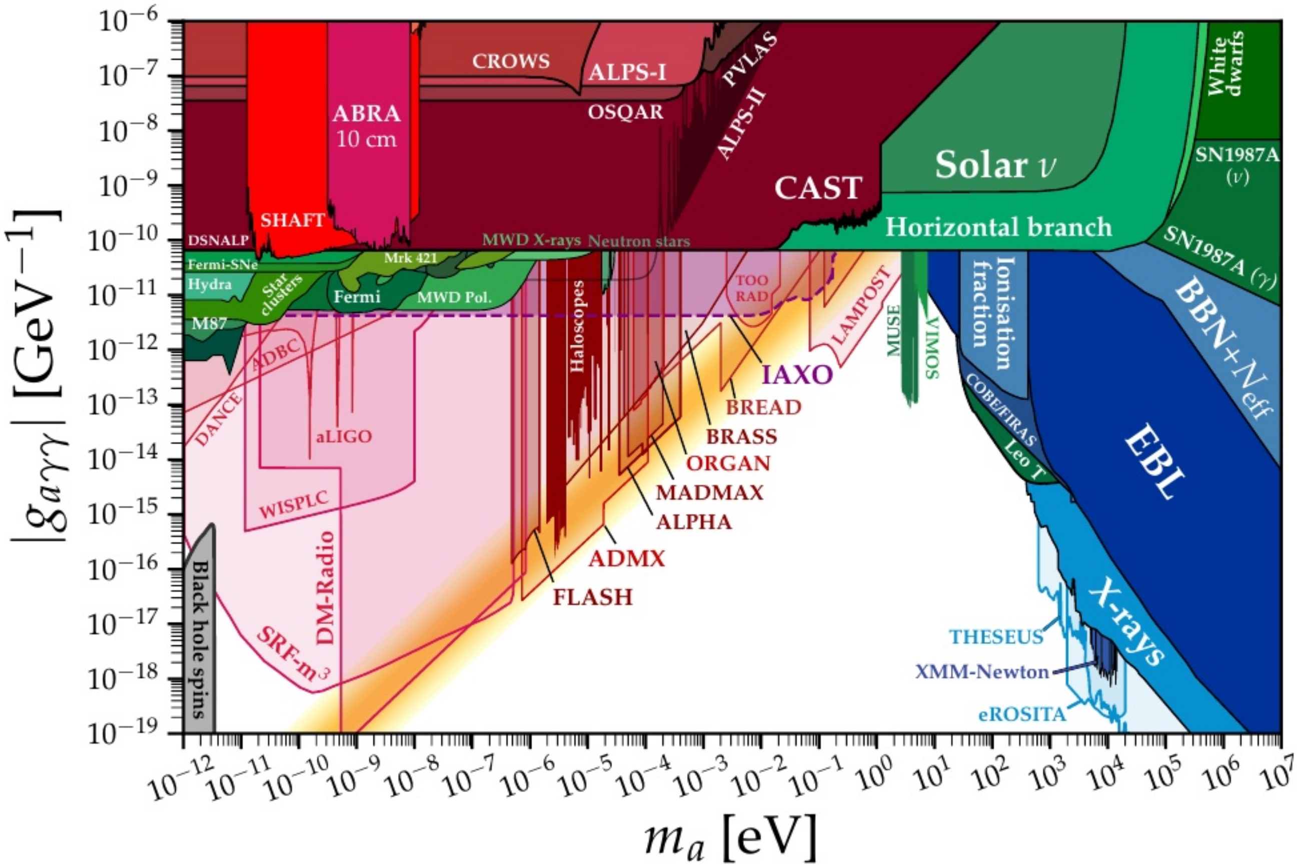
- QCD dynamics relaxes θ_{QCD} to zero: no neutron EDM (up to CKM)
- QCD axion can naturally explain observed DM abundance
- Predictive/Testable Theory: $m_a \leftrightarrow g_{a-\text{SM}} \propto 1/f_a$

Axion Present and Future Searches



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Axions Vs Lattice QCD:

- The Mass
- The Couplings to Nucleons
- The Couplings to Photons
- The Thermal Mass
- The Thermal Width

The Mass:

$$m_a^2 = \frac{\chi_{\rm top}}{f_a^2}$$

$$\chi_{\rm top} = \int d^4x \left\langle \frac{G\tilde{G}(x)}{32\pi^2} \frac{G\tilde{G}(0)}{32\pi^2} \right\rangle$$

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From Chiral Perturbation Theory

Weinberg '78

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

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Grilli, Hardy, Pardo, GV - 1511.02867

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lattice average:

$$z \equiv \frac{m_u^{\overline{\rm MS}}(2\ \text{GeV})}{m_d^{\overline{\rm MS}}(2\ \text{GeV})} = 0.48(3)$$

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$$7(4)\cdot 10^{-3} \qquad \Delta m_\pi^2|_{\rm (strong)}$$

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$4.8 \pm 1.4 \cdot 10^{-3}$

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$$m_a = 5.70(6)(4)\ \mu{\rm eV} \left(\frac{10^{12}{\rm GeV}}{f_a} \right)$$

The Mass:

Gorghetto, GV - 1812.01008

$$m_a = \Big[\underbrace{5.815(22)_z(04)_{f_\pi}}_{\text{LO}} \underbrace{-0.121(38)_{\ell_i^r}}_{\text{NLO}} \underbrace{-0.022(07)_{\ell_i^r}(05)_{c_i^r}}_{\text{NNLO}} \underbrace{+0.019(06)_{k_i^r}}_{\text{EM}}\Big] \,\mu\text{eV}\, \frac{10^{12}\ \text{GeV}}{f_a}$$

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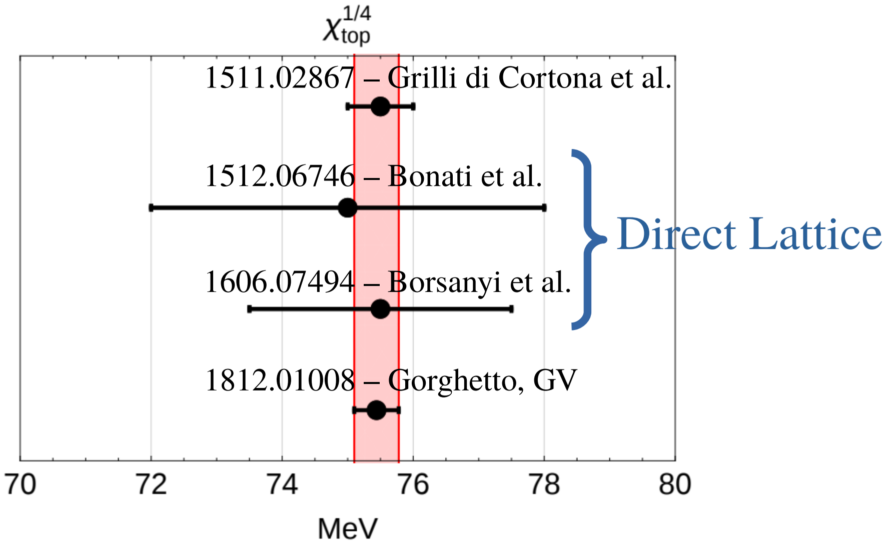

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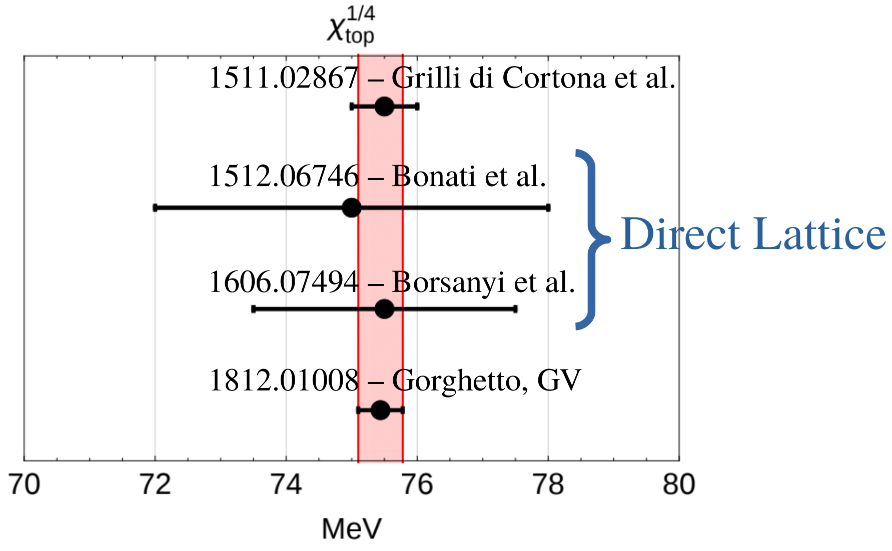
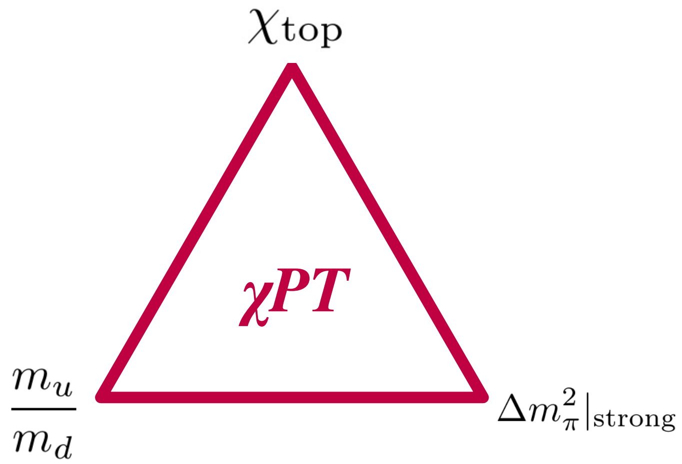
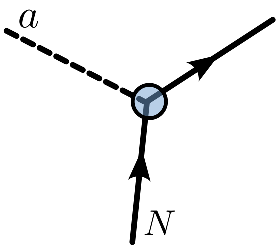




The Coupling to Nucleons:

Grilli, Hardy, Pardo, GV - 1511.02867



$$\frac{\partial_\mu a}{2f_a} \sum_q c_q \, \bar{q}\gamma^\mu \gamma_5 q$$

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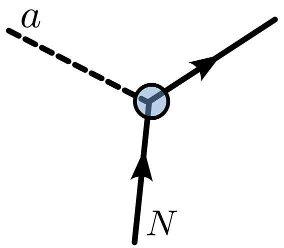

$$\frac{\partial_\mu a}{2 f_a} \sum_q c_q \, \bar{q} \gamma^\mu \gamma_5 q \quad \Rightarrow \quad \frac{\partial_\mu a}{2 f_a} c_N \bar{N} \gamma^\mu \gamma_5 N$$

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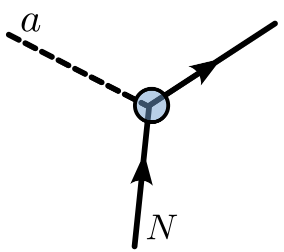

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$$c_N = \sum_q c_q \Delta q$$

$$\langle N | \bar{q} \gamma^\mu \gamma_5 q | N \rangle = s^\mu \Delta q$$

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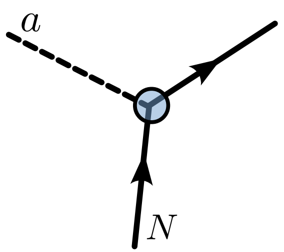

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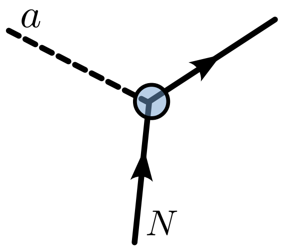

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$$g_0^{ud} = \Delta u + \Delta d = 0.521(53)$$

$$\Delta s = -0.026(4)$$

$$\Delta c = \pm 0.004$$

from Lattice @ 2015…

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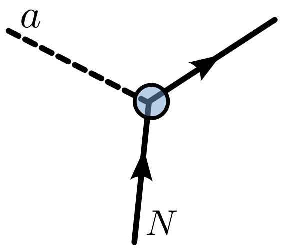

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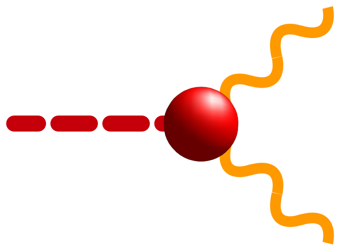
from Lattice @ 2015…

$$\begin{cases} c_p = -0.47(3) + 0.88(3)c_u^0 - 0.39(2)c_d^0 - 0.038(5)c_s^0 \\ \qquad - 0.012(5)c_c^0 - 0.009(2)c_b^0 - 0.0035(4)c_t^0\,, \\ c_n = -0.02(3) + 0.88(3)c_d^0 - 0.39(2)c_u^0 - 0.038(5)c_s^0 \\ \qquad - 0.012(5)c_c^0 - 0.009(2)c_b^0 - 0.0035(4)c_t^0\,, \end{cases}$$

The Coupling to Photons:

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u}\right\}$$



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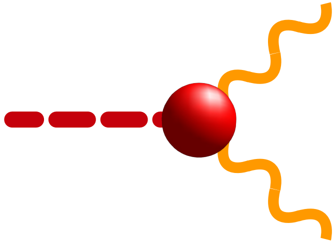

E/*N* =

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8/3	(DFSZ, GUT-KSVZ,...)
2	(Unificaxion,...)

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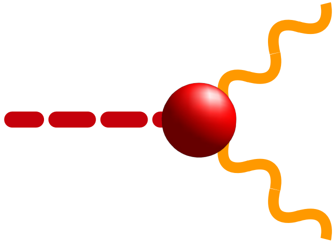

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tree ~ - 2

$$a \to \pi \to \gamma\gamma$$

The Coupling to Photons:

Grilli, Hardy, Pardo, GV - 1511.02867

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left\{ \frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u} + \frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\left(5\tilde{c}_3^W + \tilde{c}_7^W + 2\tilde{c}_8^W\right) - \frac{m_d - m_u}{m_d + m_u} l_7^r\right]\right\}$$

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$a \to \pi \to \gamma\gamma$

NLO $= 0.033(6)$

from $\pi \to \gamma\gamma \quad \eta \to \gamma\gamma$

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Grilli, Hardy, Pardo, GV - 1511.02867

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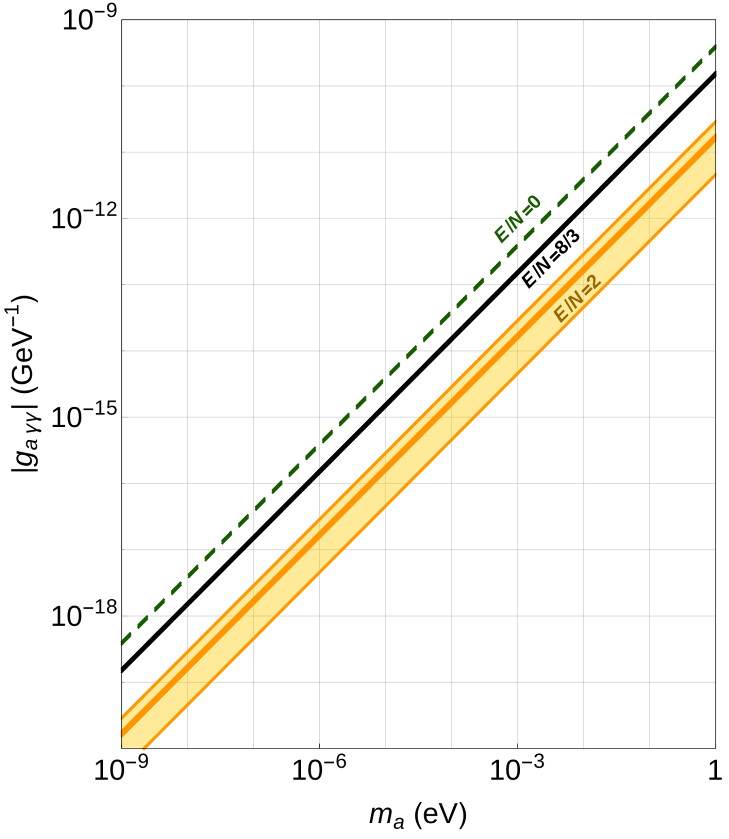
$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

$$g_{a\gamma\gamma} = \begin{cases} -2.227(44) \cdot 10^{-3}/f_a & E/N = 0 \\ 0.870(44) \cdot 10^{-3}/f_a & E/N = 8/3 \\ 0.095(44) \cdot 10^{-3}/f_a & E/N = 2 \end{cases}$$



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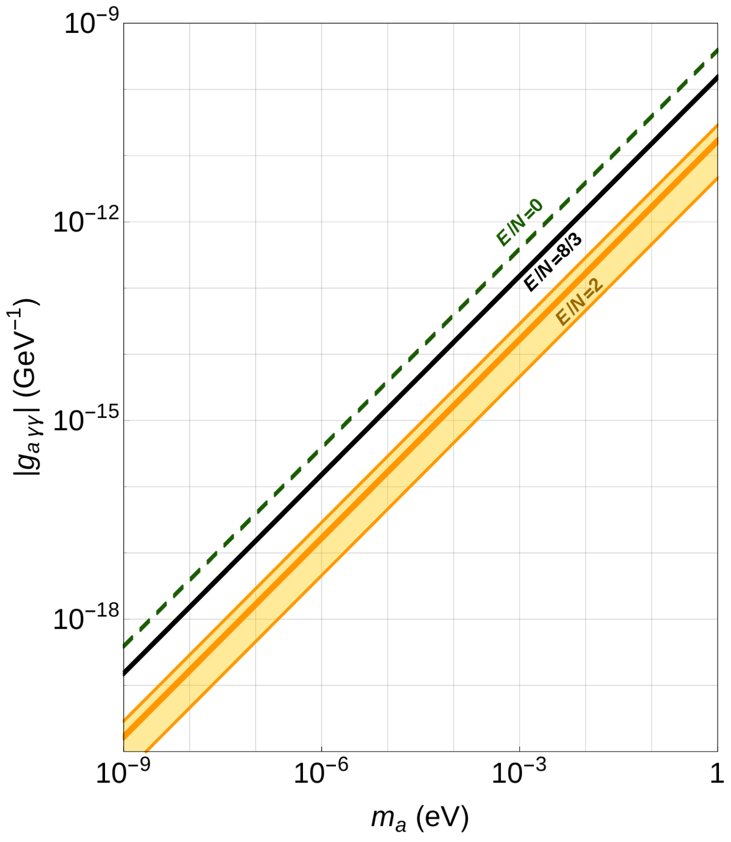

$a\gamma\gamma$ from Lattice?

Any hope for $\pi\gamma\gamma$ from Lattice?

$$\langle j_A^\mu \, j_{em}^\nu \, j_{em}^\rho \rangle$$

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$T < T_c \sim m_\pi$	$\chi_{\rm top}(T) \simeq \chi_{\rm top}(0) \left[1 - \frac{3}{2(2\pi)^{3/2}} \frac{m_\pi^2}{f_\pi^2} \left(\frac{m_\pi}{T}\right)^{\frac{3}{2}} e^{-m_\pi/T}\right]$ Grilli, Hardy, Pardo, GV - 1511.02867

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$T \gg \Lambda_{QCD}$	$\chi_{\rm top}(T) \sim \kappa \left(\frac{T_c}{T}\right)^{\alpha}$ $\quad \alpha(N_f = 3) \simeq 8$ dilute instanton gas approx. (NLO corrections?)	Gross, Pisarski, Yaffe - '81

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$T \gtrsim T_c$?

see M.P. Lombardo's talk

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The Thermal Mass:

$$m_a^2(T) = \frac{\chi_{\rm top}(T)}{f_a^2} \qquad \chi_{\rm top}(T) = \int d^4x \left\langle \frac{G\tilde{G}(x)}{32\pi^2} \frac{G\tilde{G}(0)}{32\pi^2} \right\rangle_T$$

$T < T_c \sim m_\pi$	$\chi_{\rm top}(T) \simeq \chi_{\rm top}(0) \left[1 - \frac{3}{2(2\pi)^{3/2}} \frac{m_\pi^2}{f_\pi^2} \left(\frac{m_\pi}{T}\right)^{\frac{3}{2}} e^{-m_\pi/T}\right]$	Grilli, Hardy, Pardo, GV - 1511.02867

$T \gtrsim T_c$? → Controls Axion Abundance $\Omega_a = 0.1\ k_\alpha \left[\frac{\theta_0}{2.15}\right]^2 \left[\frac{f_a}{10^{12}\ \mathrm{GeV}}\right]^{1+\frac{1}{2+\alpha/2}} \left[\frac{60}{g_\star}\right]^{\frac{1}{2}-\frac{1}{4+\alpha}}$

see M.P. Lombardo's talk

$T \gg \Lambda_{QCD}$	$\chi_{\rm top}(T) \sim \kappa \left(\frac{T_c}{T}\right)^\alpha \qquad \alpha(N_f = 3) \simeq 8$	Gross, Pisarski, Yaffe - '81

dilute instanton gas approx. (NLO corrections?)

The Thermal Width:

$$\Gamma^{>}_{\text{top}} \equiv \int d^4x \, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu) \, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

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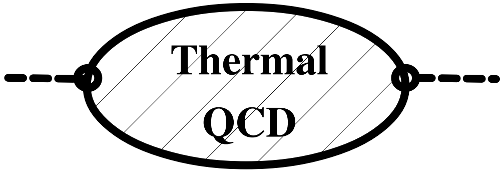

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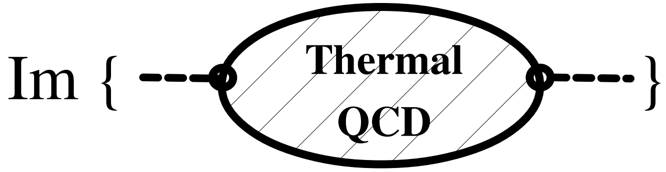

Thermal Width → Relativistic Axions from SM bath:

Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

Abundance depends on decoupling temperature
(which depends on coupling f_a)

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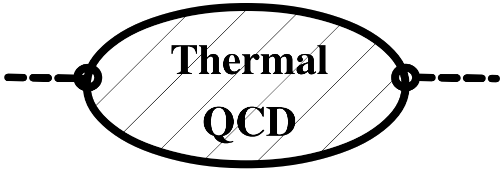
Im {  }

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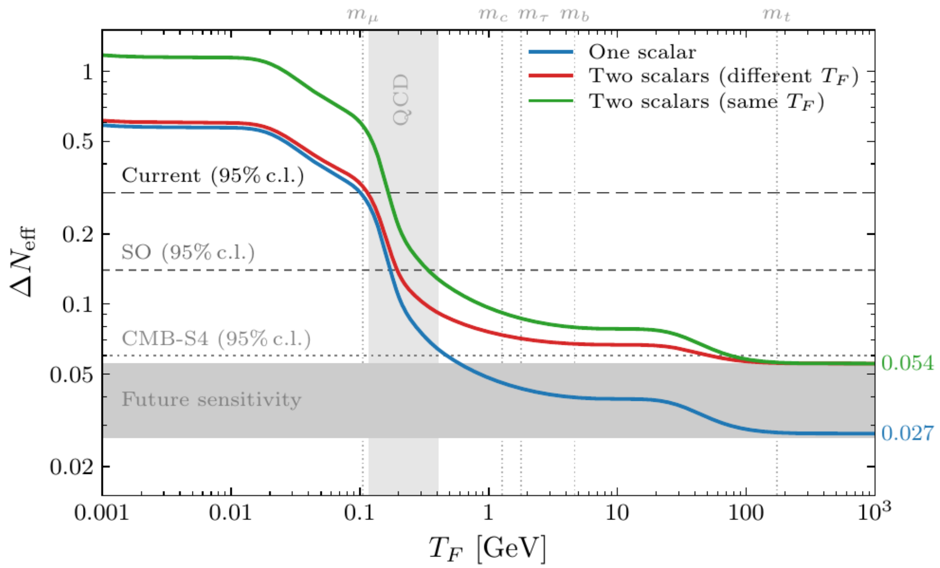

from 2109.12088

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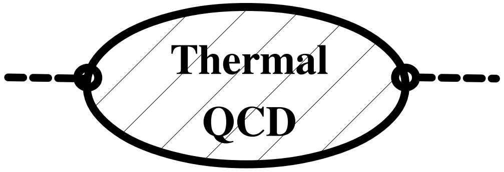

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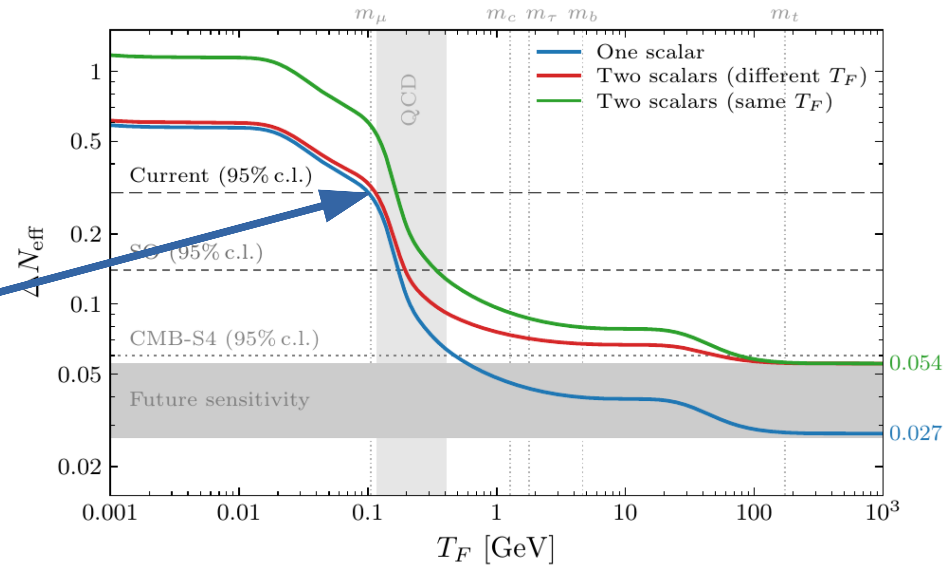

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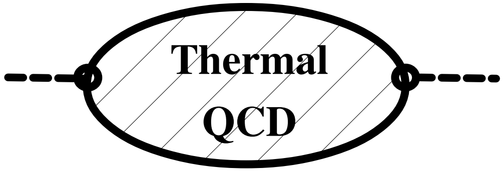

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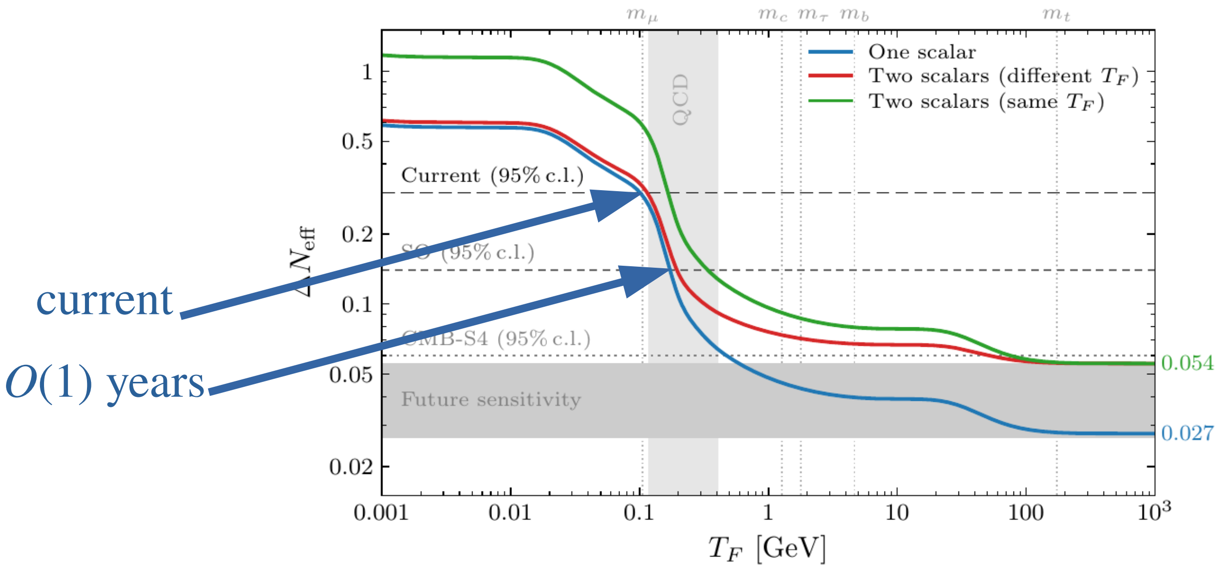

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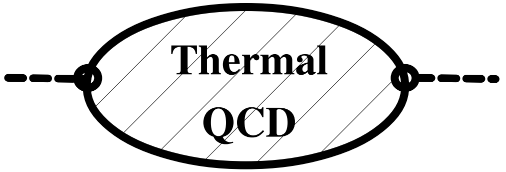

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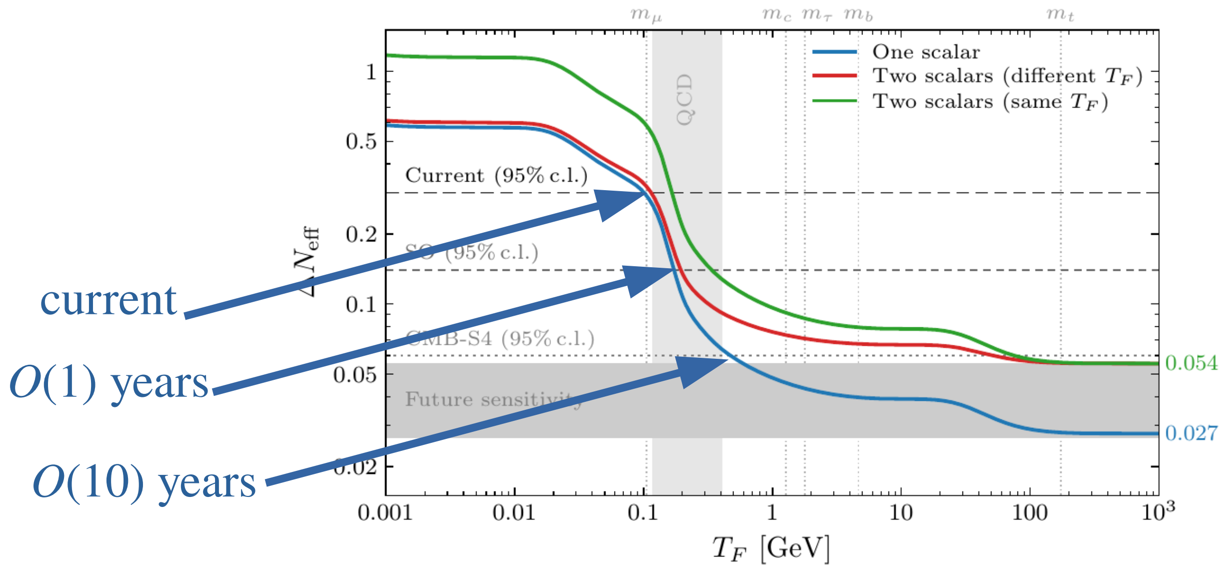

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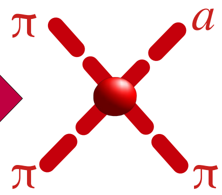
The Thermal Width:

$T < T_c$	$\Gamma^{<} = \frac{1}{2E} \int \left(\prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i} \right) f_1^{\rm eq} f_2^{\rm eq} (1 + f_3^{\rm eq}) (2\pi)^4 \delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu) \lvert \mathcal{M} \rvert^2$



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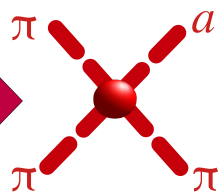

χPT +
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see G.Piazza's talk

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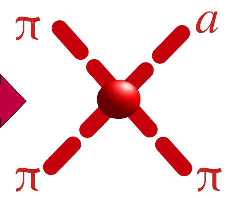

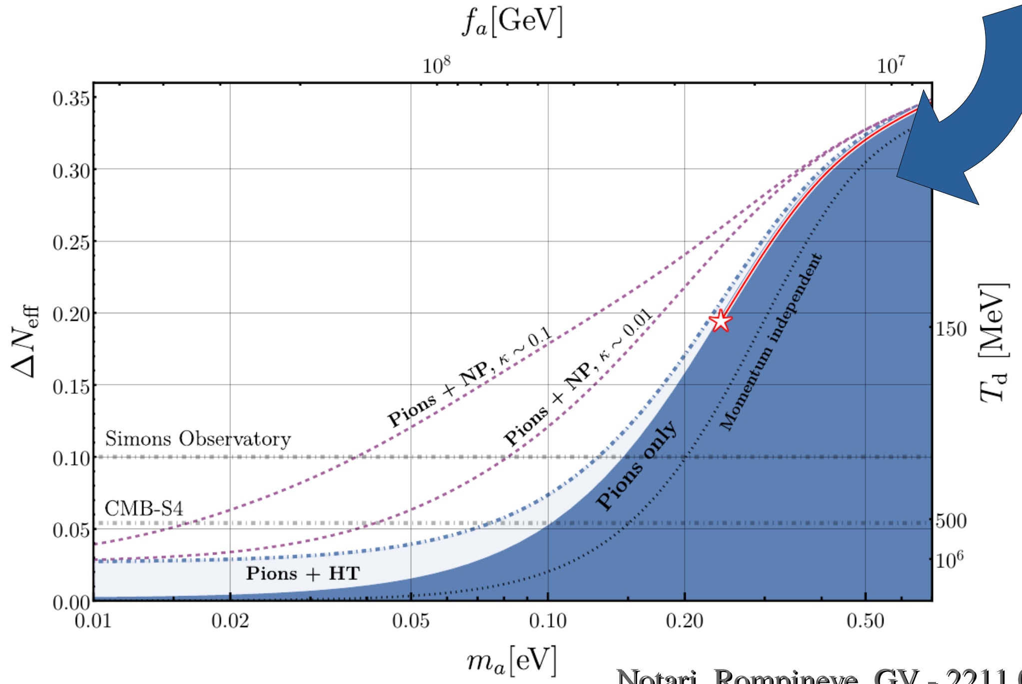

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Notari, Rompineve, GV - 2211.03799

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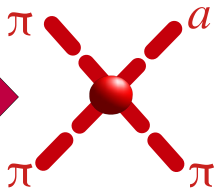

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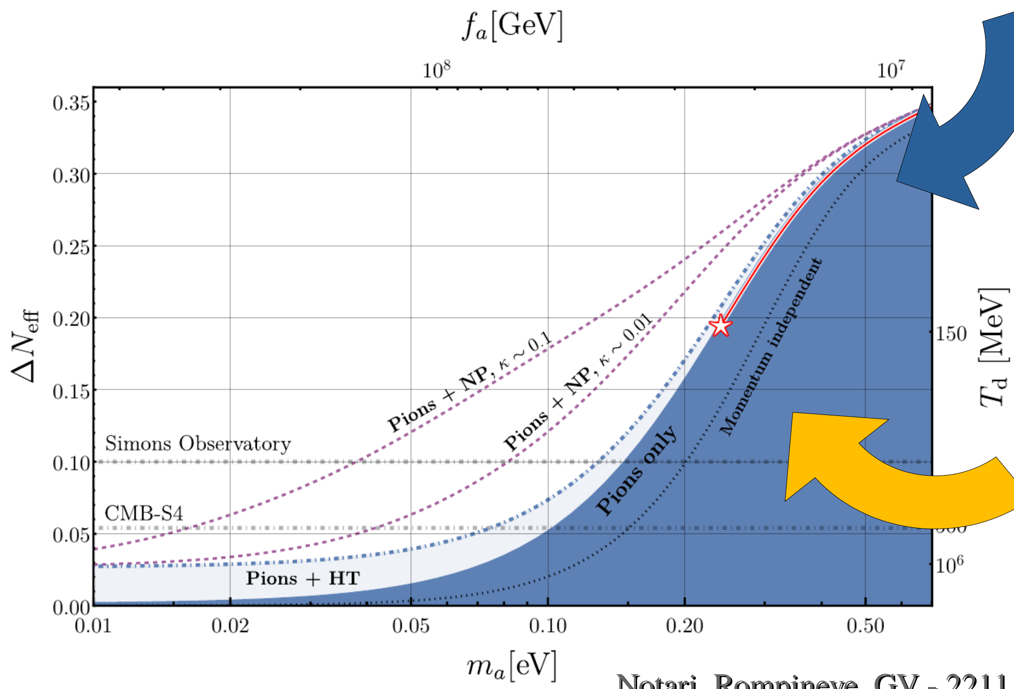

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should be considered a lower bound

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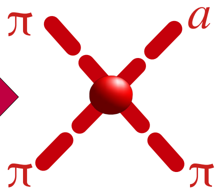

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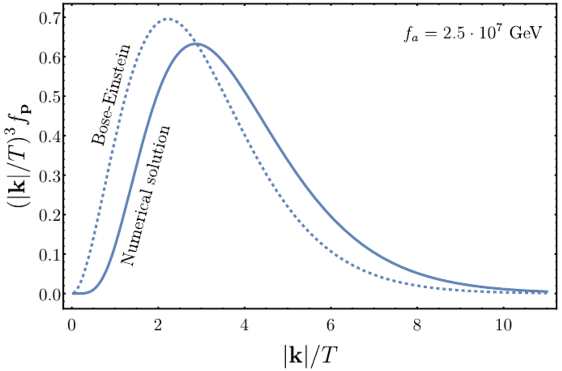
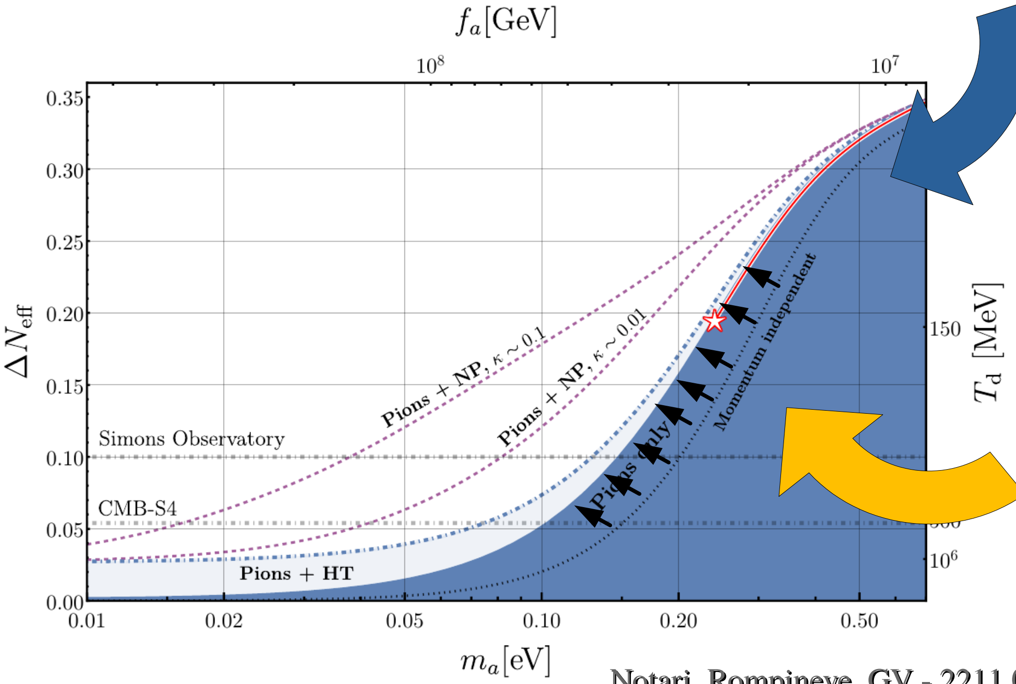
Important Effects:

1. momentum dependence
~ 40 %





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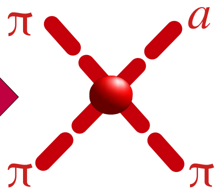

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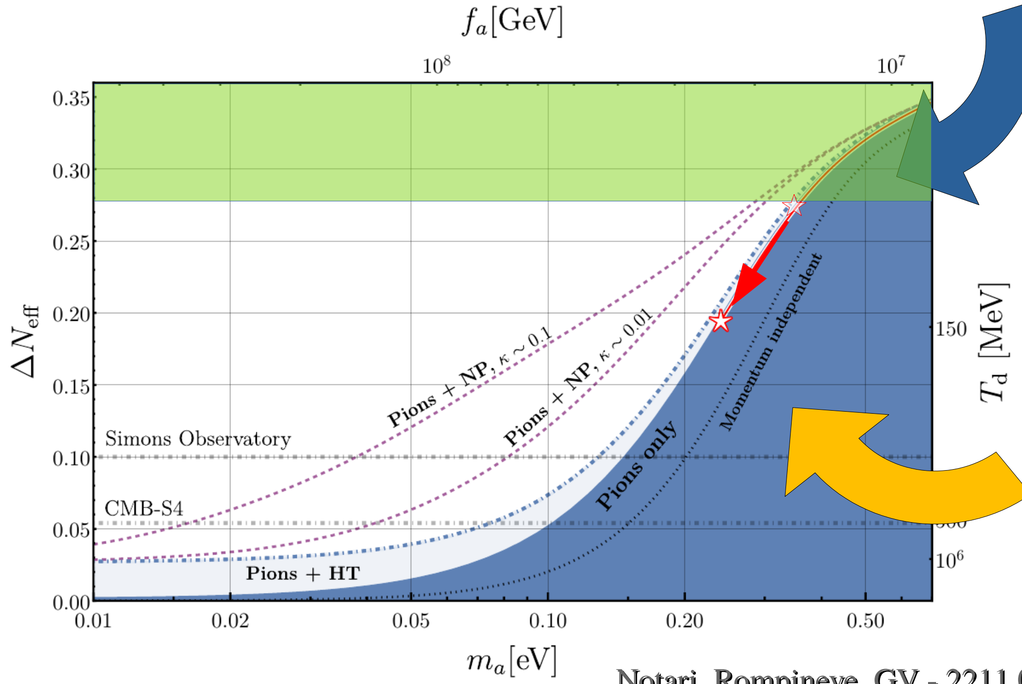
see G.Piazza's talk

Important Effects:

1. momentum dependence
~ 40 %

2. finite axion mass effects
~ 30 %



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The Thermal Width:

Upcoming data requires understanding Thermal Width @ $T > T_c$

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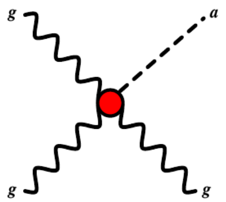

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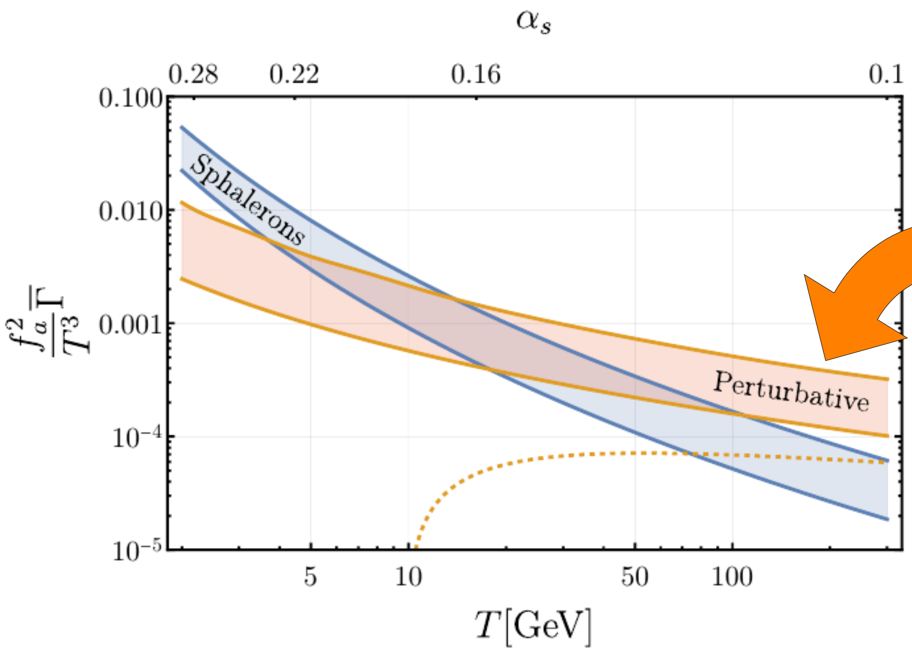

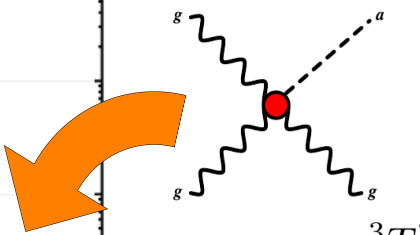

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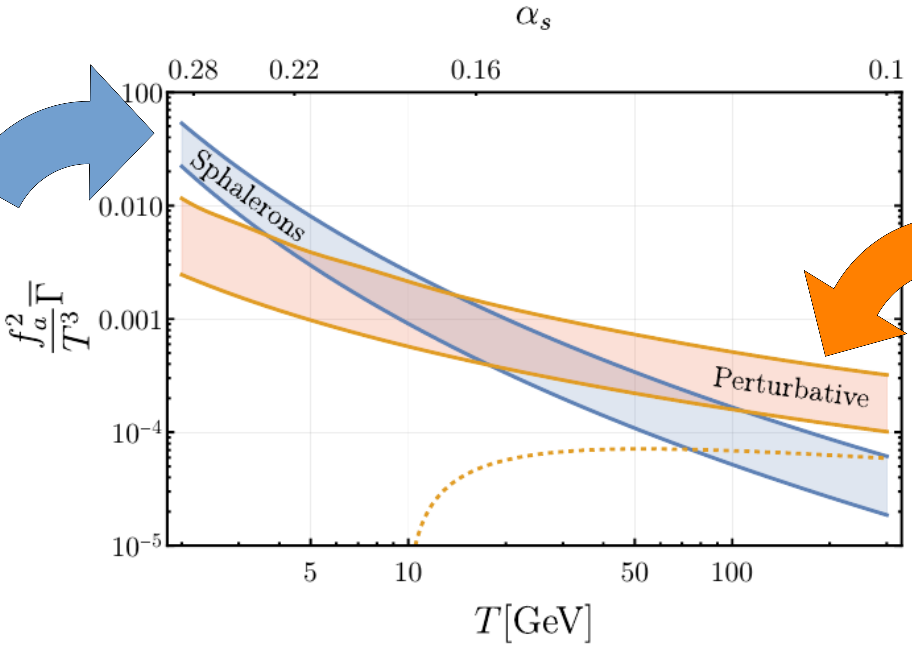

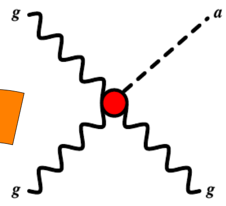

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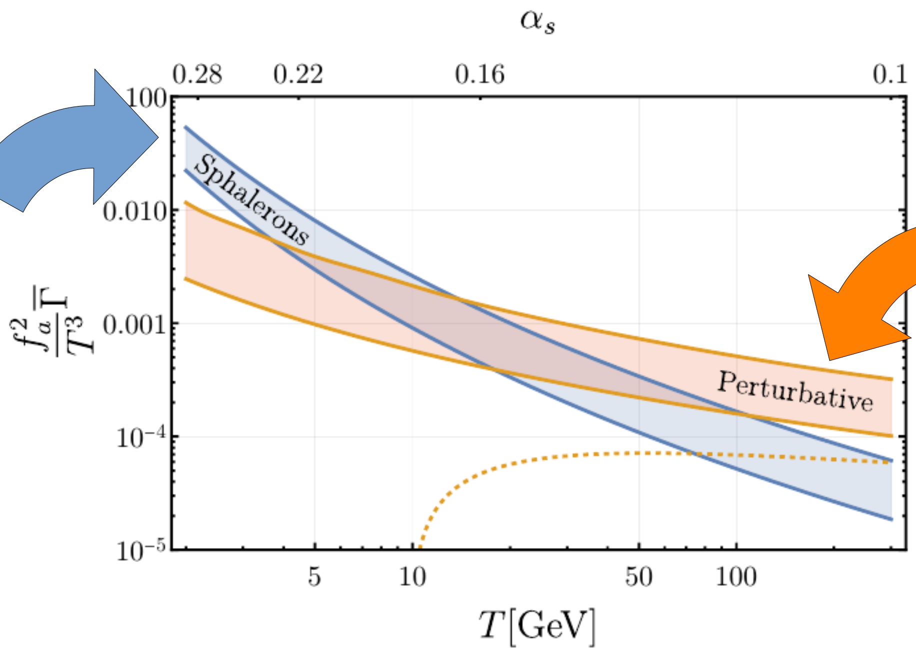

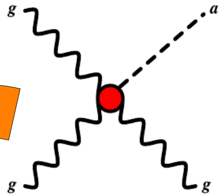

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Plausible that both estimates are unreliable (both rely on perturbativity/semiclassics)

definitely below 1 GeV

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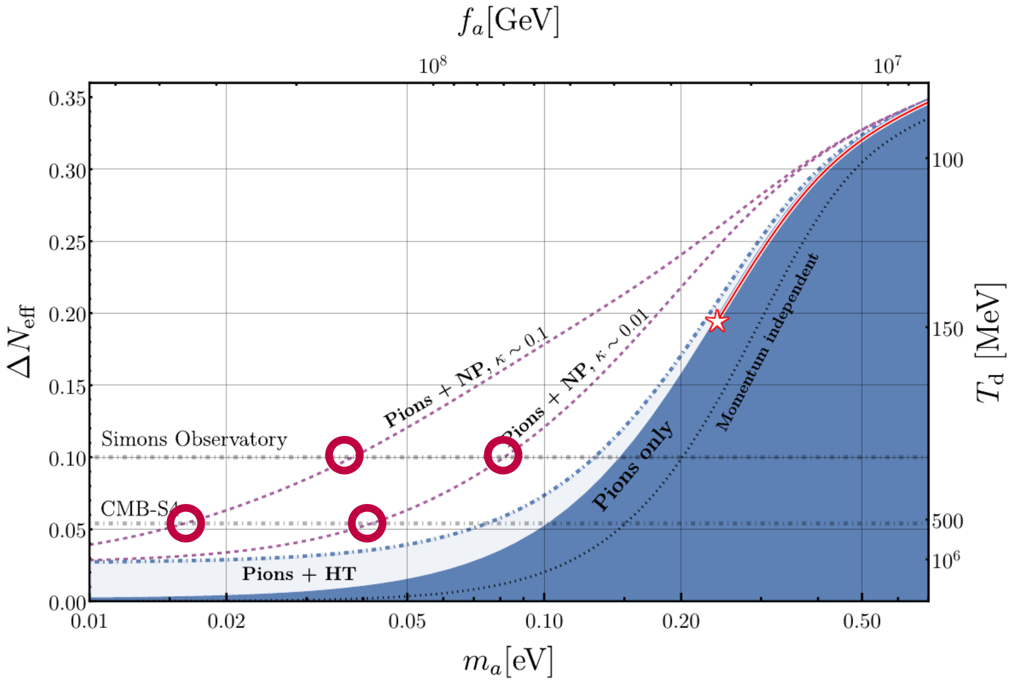

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Challenge for Lattice QCD: Compute Γ_k for $T > T_c$

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$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0}, 0) q(\vec{x}, \tau) \rangle$$

$$= -\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \, \rho(\omega) \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)}$$

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Important to exploit upcoming experiments!

Thank you!