A novel phenomenological approach to radiative leptonic Bs-meson decays

Work in collaboration with Diego Guadagnoli, Camille Normand and Silvano Simula

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

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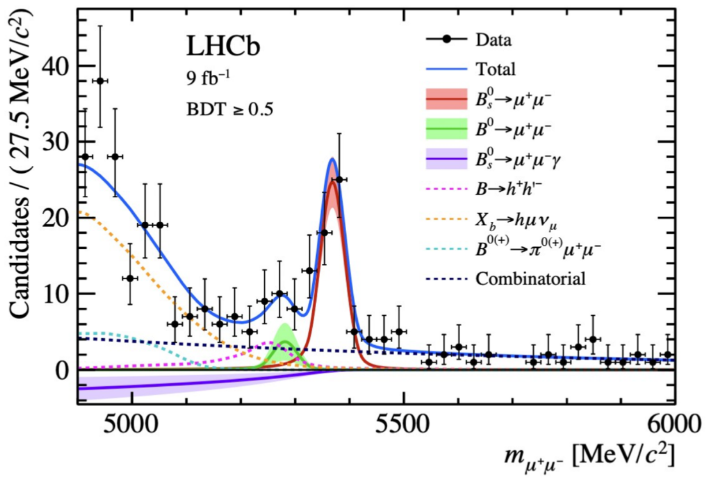

(from LHCb-PAPER-2021-007)

Radiative-and-leptonic Bs decays

A novel possibility to analyze b → s quark transitions is the study of rare radiative-and-leptonic Bs decays. This is experimentally challenging, and yet LHCb has recently set a limit (very close to the SM signal):

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Description of these decays within the SM??

Radiative-and-leptonic decays in the SM

Standard WET approach:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}}\lambda_{\rm CKM}\left[\frac{\alpha_{em}}{4\pi}\left(\sum_i C_i \mathcal{O}_i + \sum_i C_i^{'} \mathcal{O}_i^{'}\right)\right]$$

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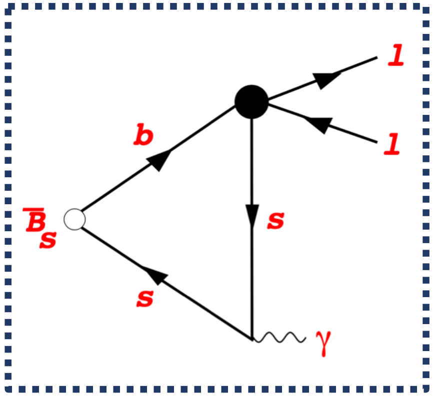

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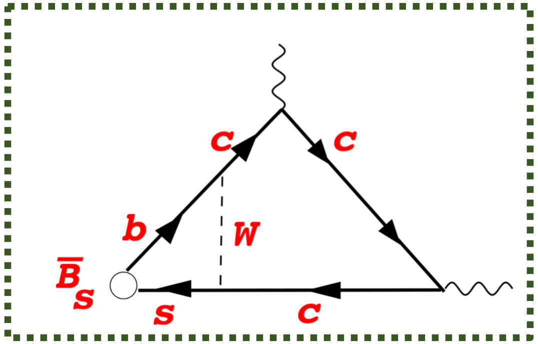

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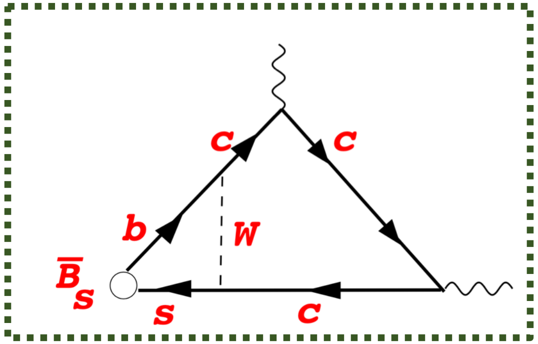
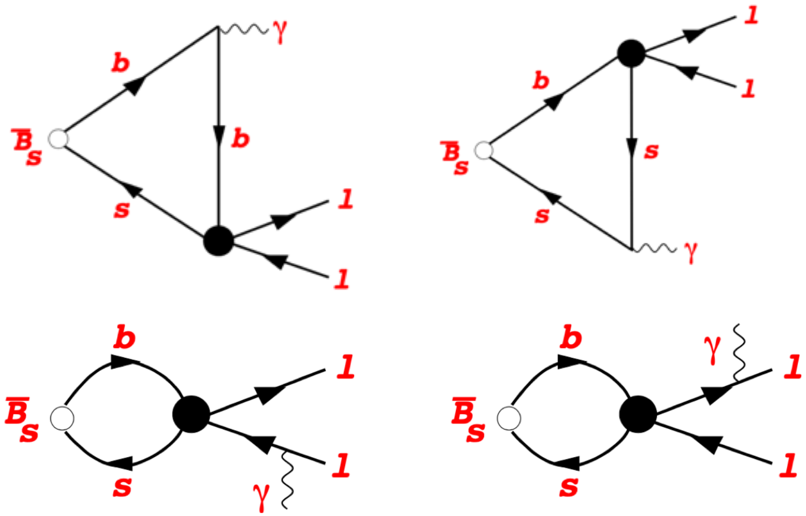
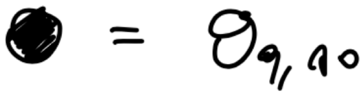


Important issue: <u>only 4 diagrams</u> give a relevant contribution at high-q2, *i.e.*





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Janowski, Pullin and Zwicky, JHEP 12 (2021) 008

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No direct computation on the lattice in the Bs-sector at present …

Which is the state-of-the-art?

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Our work wants to offer a new SM prediction of the **BR at high-q2** in addition to:

- Kozachuk, Melikhov and Nikitin **(KMN)**

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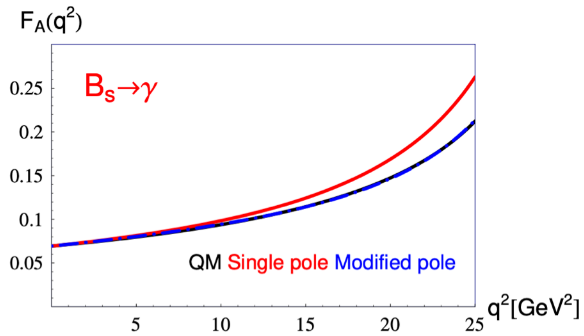

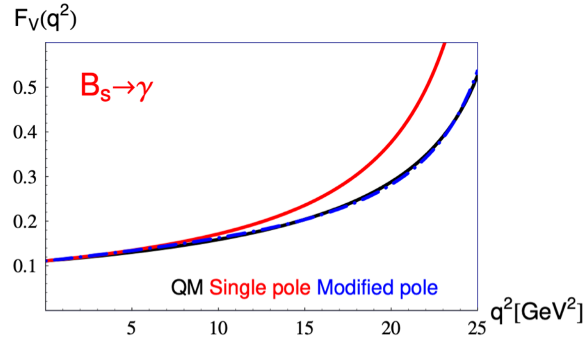

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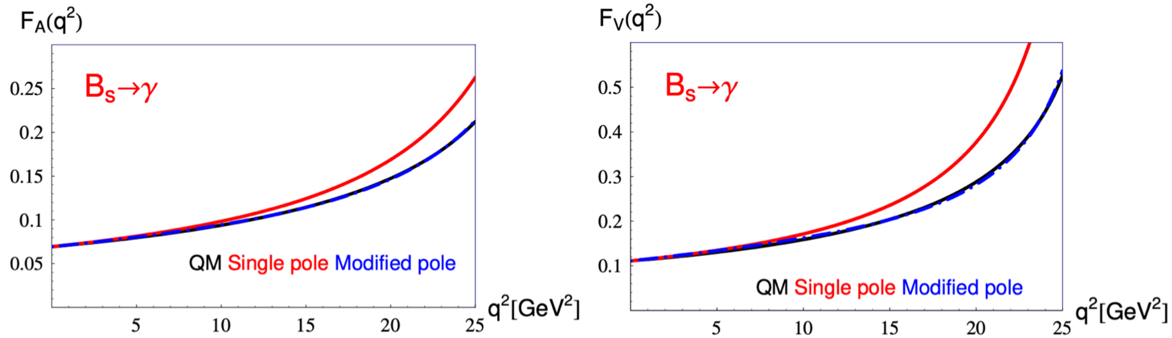

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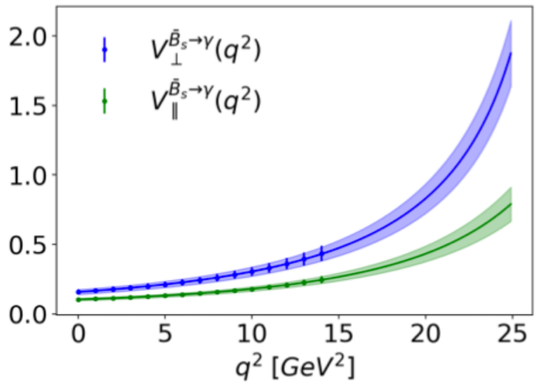
LIGHT-CONE SUM RULES



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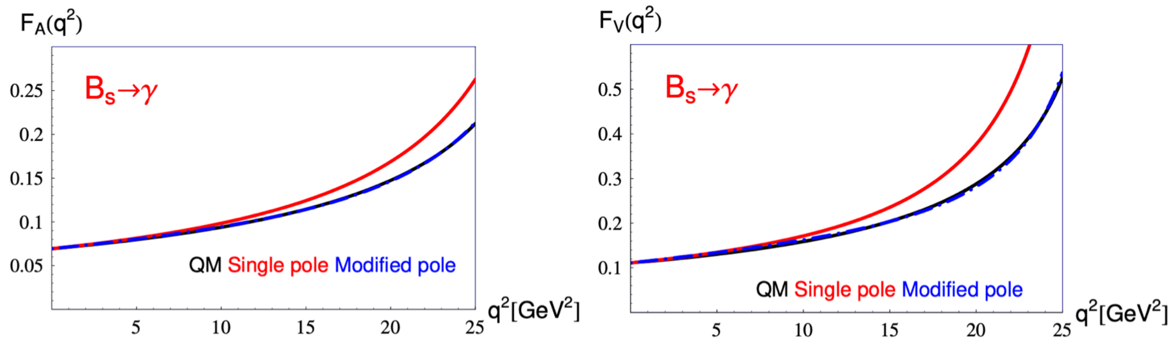

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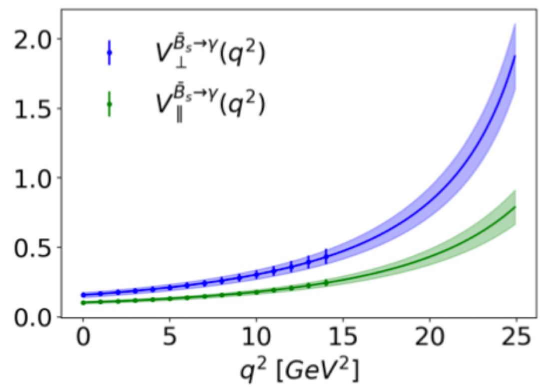

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SOFT-COLLINEAR EFFECTIVE THEORY (SCET): valid only at low-q2 !

Beneke, Bobeth and Wang , JHEP 12 (2020) 148

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Bottom line: **you are always forced to perform an *extrapolation to the high-q2 region* …**

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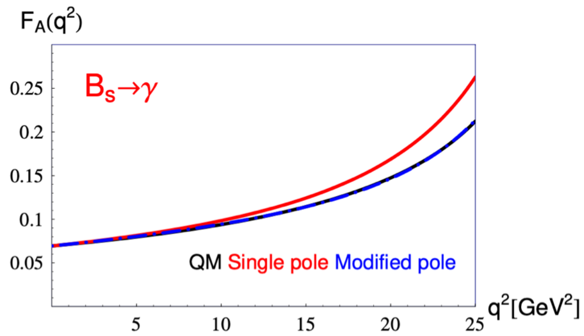

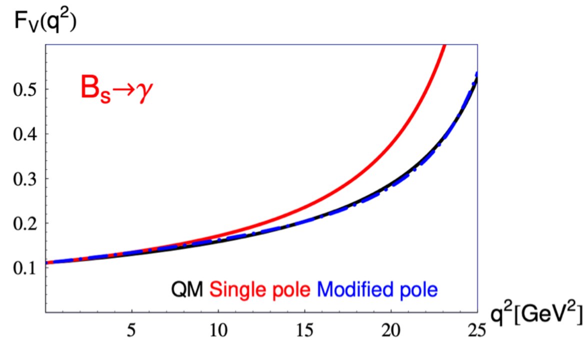

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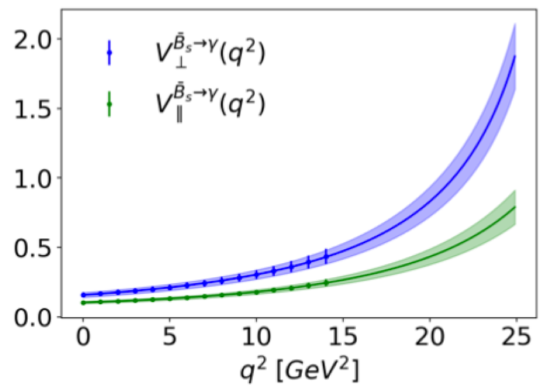

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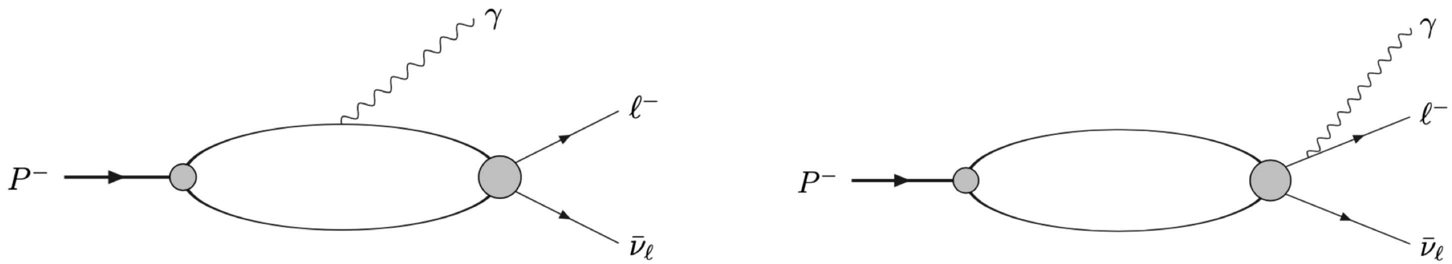

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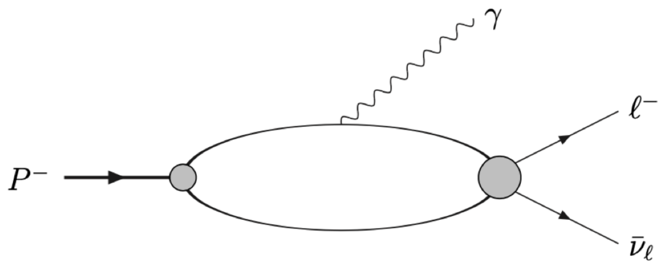

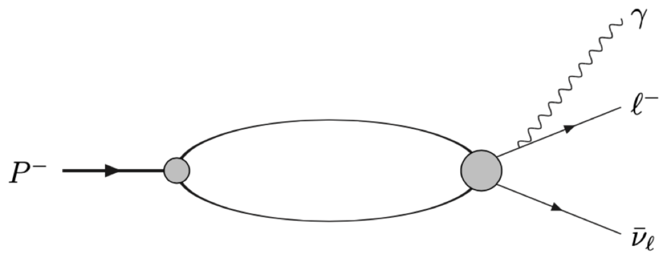

These are exactly the diagrams relevant for us at high-q2 in the Bs-sector!!

A new approach to the hadronic FFs

$$\langle\gamma(k,\epsilon)|O^V_\mu|\bar{B}_q(p_B)\rangle = s_e(P^\perp_\mu V_\perp(q^2) - P^\parallel_\mu (V_\parallel(q^2) + Q_{\bar{B}_q} f^{(pt)}_{B_q}) - P^{\rm Low}_\mu Q_{\bar{B}_q} f^{(pt)}_{B_q})$$

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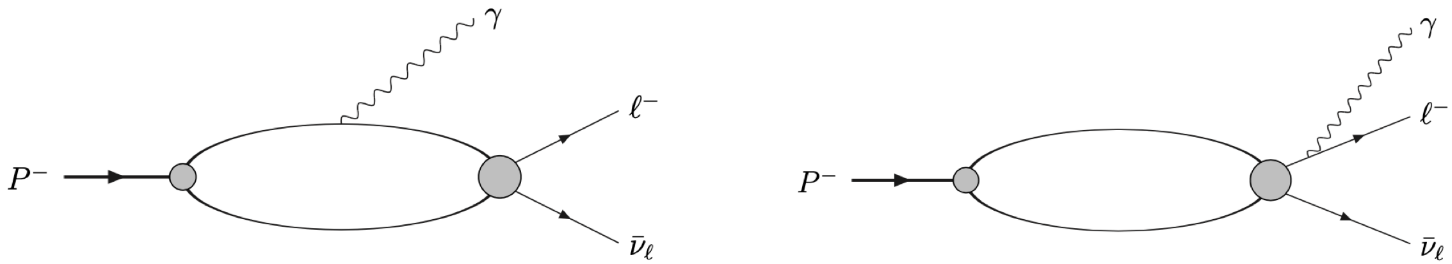

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GNSV PLAN: new analysis of the data for P = Ds and, then, extrapolation to the Bs sector!

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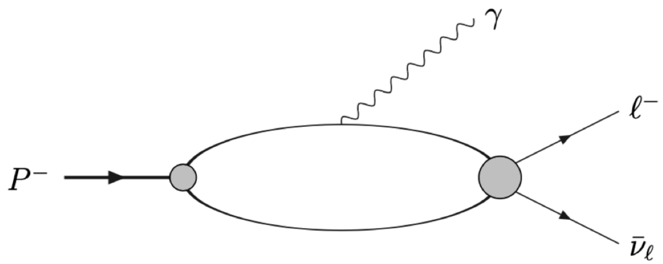

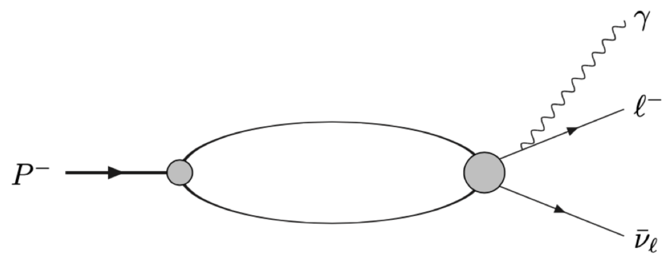

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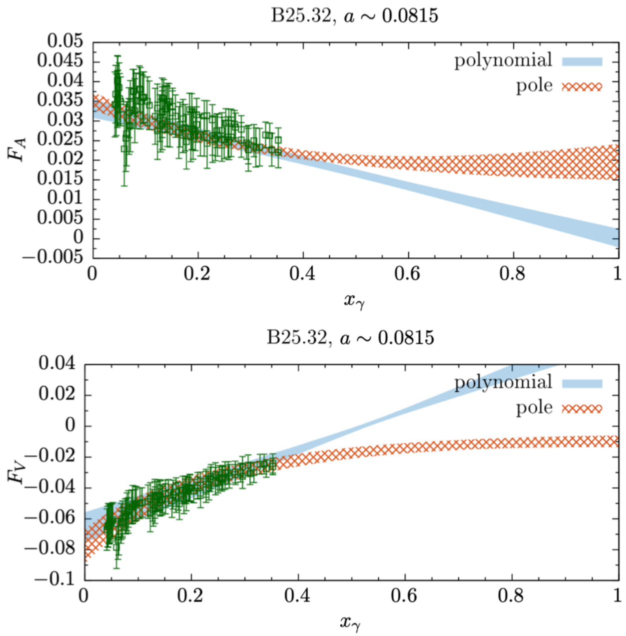
First lattice computation of lattice data for P = K, \pi (whole kinatical range) and P = D, Ds (only high-q2)!



To fix the conventions:

i)

$$x_\gamma = \frac{2p \cdot k}{m_P^2} \qquad \text{with} \qquad 0 \leq x_\gamma \leq 1 - \frac{m_\ell^2}{m_P^2}$$

ii)

$$F_V = -V_\perp \text{ and } F_A = -V_\parallel$$

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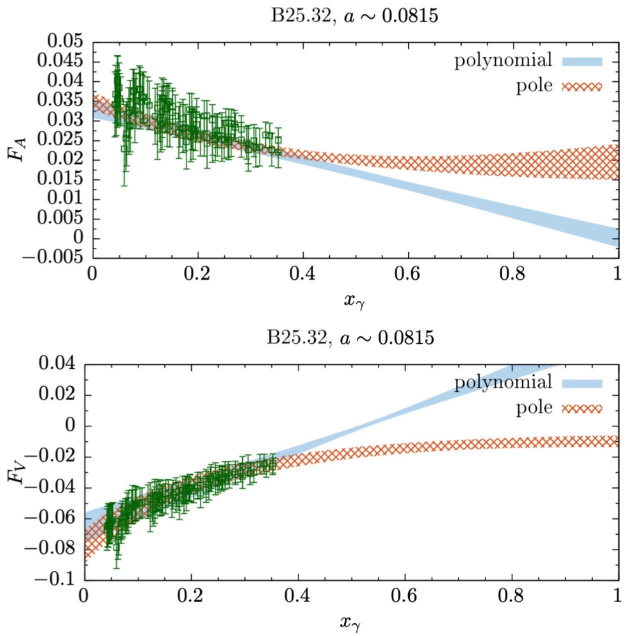

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To describe the hadronic Form Factors in the whole kinematical region (after the continuum and the chiral extrapolations):

$$F^P_{A,V}(x_\gamma) = C^P_{A,V} + D^P_{A,V}\, x_\gamma$$

LINEAR fit

$$F^{D_{(s)}}_{A,V}(x_\gamma) = \frac{\tilde{C}^{D_{(s)}}_{A,V}}{1 + \tilde{D}^{D_{(s)}}_{A,V}\, x_\gamma}$$

POLE-LIKE fit

A new approach to the hadronic FFs

Here we can introduce the first of our ideas. Each form factor has to respect a dispersive formula like

$$V_{\|[\perp]}^{\bar{B}\to\gamma}(q^2) = \frac{1}{\pi}\int_{\text{cut}}^{\infty} dt \frac{\text{Im}[V_{\|[\perp]}^{\bar{B}\to\gamma}(t)]}{t - q^2 - i0} = \frac{r_{\|[\perp]}^{V}}{1 - q^2/m_{B^*[B_1]}^2} + \dots$$

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The **residues** **are related to** **tricouplings!** **For instance, in the B-sector**

$$r_{\perp}^{(V,T)} = \left(\frac{m_B f_{B^*}}{m_{B^*}}, f_{B^*}^T\right) g_{BB^*\gamma}\,, \quad r_{\|}^{(V,T)} = \left(\frac{m_B f_{B_1}}{m_{B_1}}, f_{B_1}^T\right) g_{BB_1\gamma}$$

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The tricouplings describe a completely different kind of processes, i.e. **radiative non-leptonic decays of mesons as**

$$\Gamma(B^* \to B\gamma) = \frac{\alpha}{24}\left(1-\frac{m_B^2}{m_{B^*}^2}\right)^3 m_{B^*}^3 g_{BB^*\gamma}^2$$

$$\Gamma(B_1 \to B\gamma) = \frac{\alpha}{24}\left(1-\frac{m_B^2}{m_{B_1}^2}\right)^3 m_{B_1}^3 g_{BB_1\gamma}^2$$

Amundson et al., PLB 296 (1992) 415-419 [arXiv:hep-ph/9209241]

The extrapolation to the Bs-sector

BASIC PLAN: we want to infer the tricouplings in the Bs-sector by analyzing the LQCD data in the **region $x\gamma \in [0.1, 0.4]$** in **A. Desiderio et al., PRD 103 (2021) 014502** and by adopting an approach similar to **Becirevic, Haas and Kou, PLB 681 (2009) 257-263.** In other words, the idea is **to scale our required parameters to the Bs by using properly constrained heavy quark symmetry**.

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Take it as a *preview:* from a first computation the Branching Ratio at high-q2 will be $O(10^{-10})$

• the **integration range** can be **as large as q = [4.2, 5] GeV**,

• since the «indirect» method does not suffer the problem of low efficiencies on the reconstruction of photons ($\varepsilon \sim 0.02$), **it is well possible that the next LHCb analysis will be a measure** (and this would not be the case with other exp. methods …)

Conclusions

Rare radiative-and-leptonic Bs decays are a promising channel to investigate/put new bounds on possible NP effects affecting low-energy data in flavour physics in the next future *(after the disappearance of R(K(*)) anomalies …)*

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I leave with a **final wondering**, which is a methodological question: is there a solid and systematic way to implement dispersion relations, unitarity and analiticity to describe the FFs (in the whole kinematical region) in radiative-and-leptonic decays of mesons? Efforts already done for semileptonic transitions:

- Dispersion Matrix method in b \to c decays

Di Carlo et al, PRD 104 (2021) 054502;
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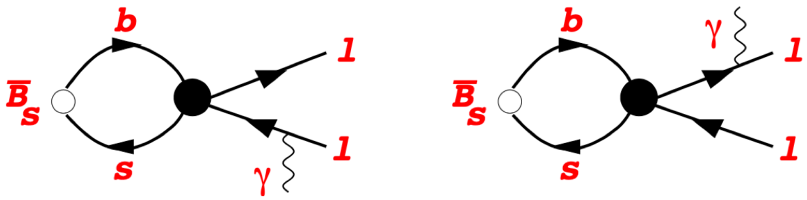
→ **Potential issue for future precision studies (also on the lattice)!**

THANKS FOR

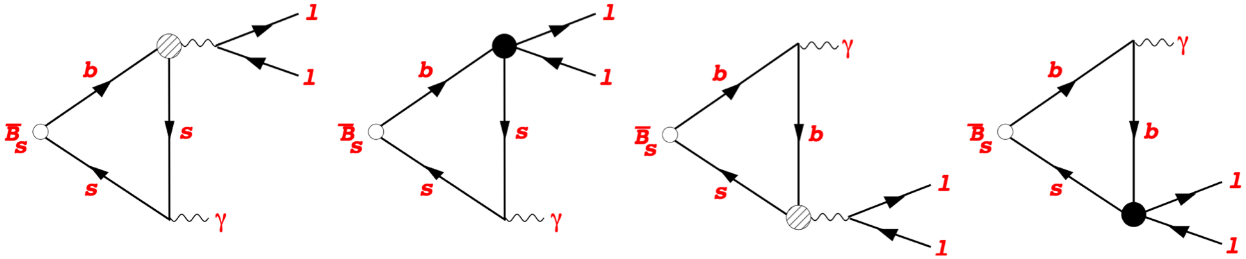
YOUR ATTENTION!

Radiative-and-leptonic decays in the SM

3. Bremsstrahlung



1. Direct emission of the photon from valence quarks



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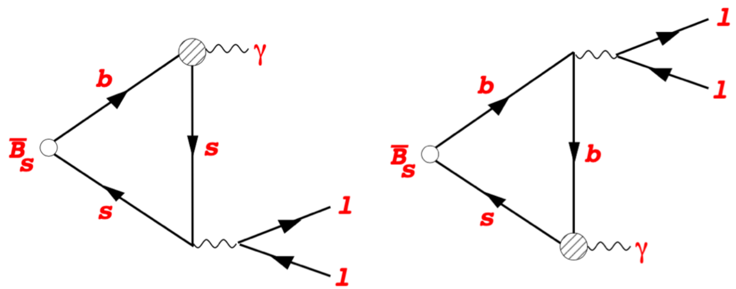

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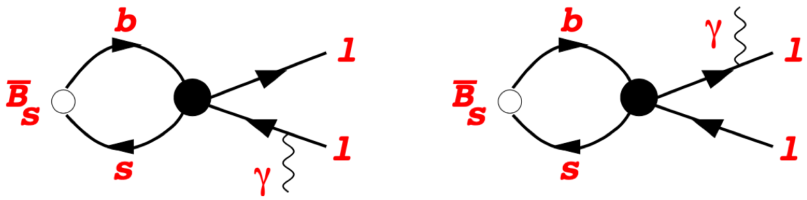

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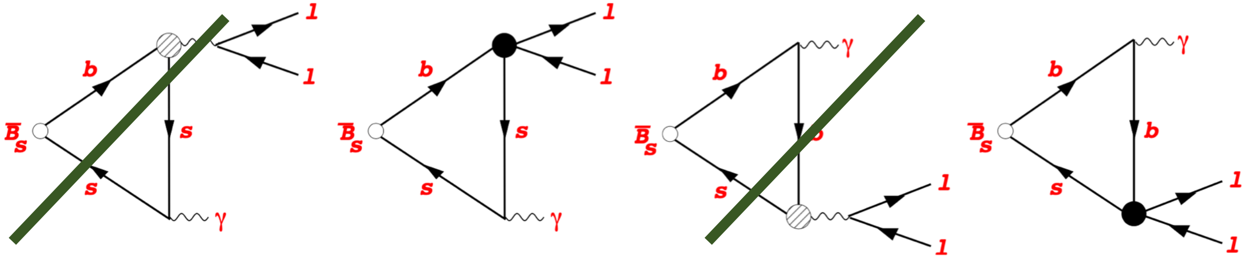

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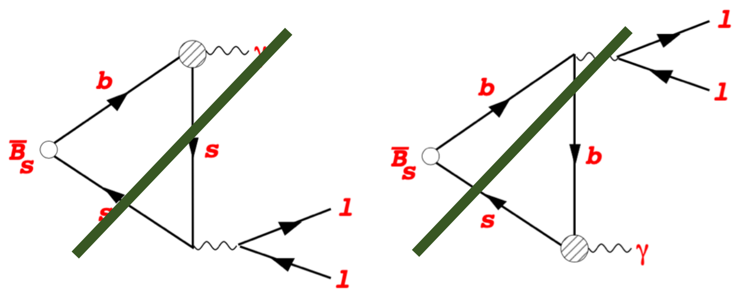

Some diagrams are suppressed at high-q2!

Radiative-and-leptonic decays in the SM

But that's not all!

$$H_{\rm eff}^{b\to s\bar{c}c} = -\frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^* \, \{C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\}$$

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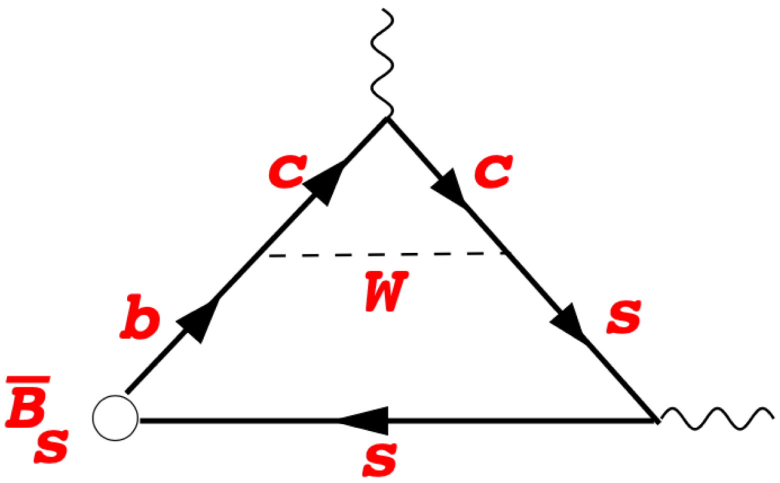
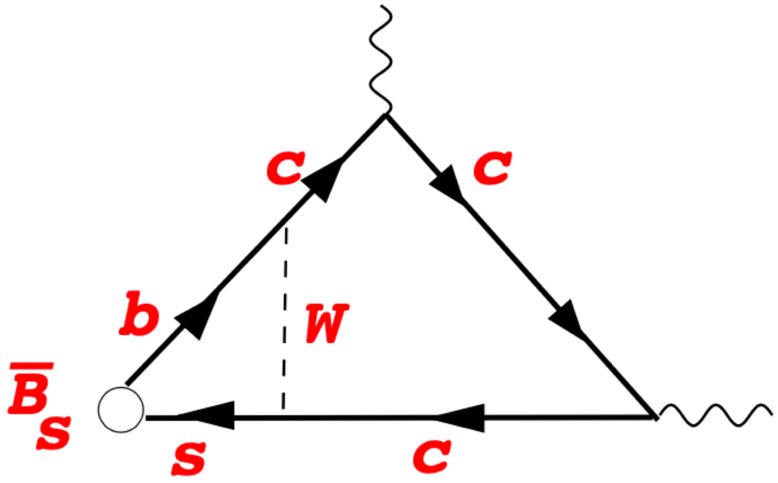


Charm loops



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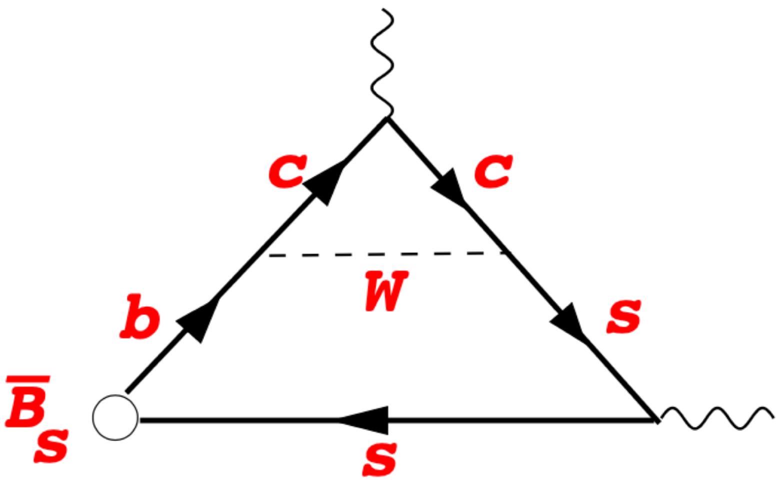

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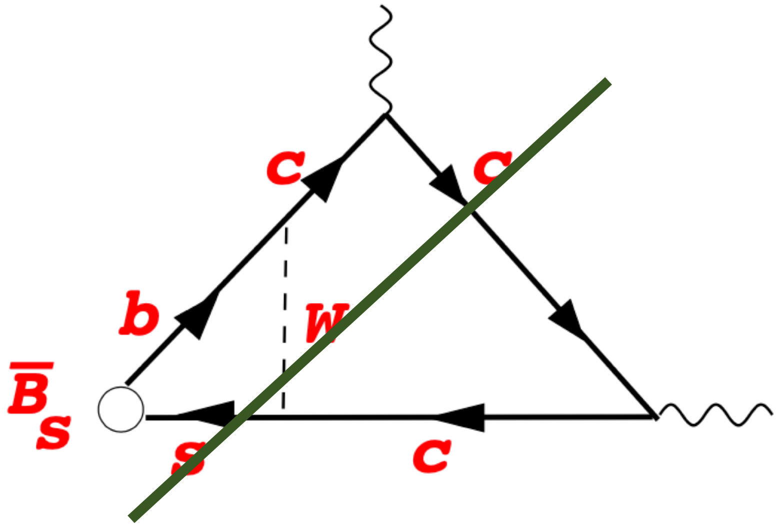

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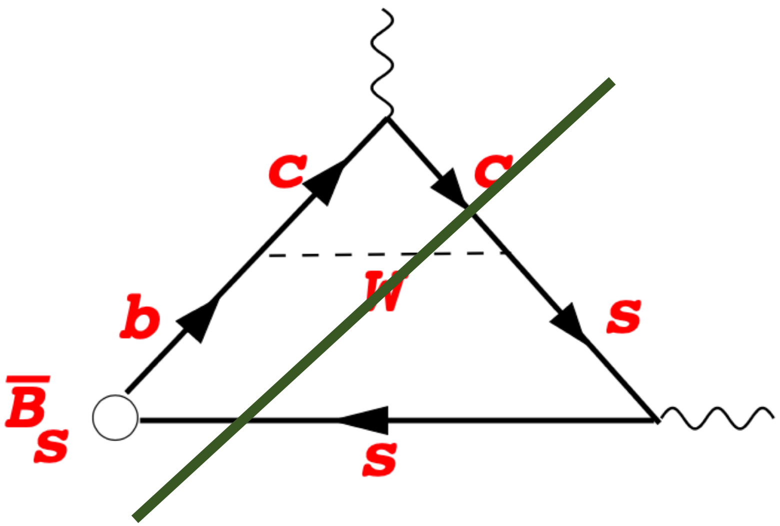
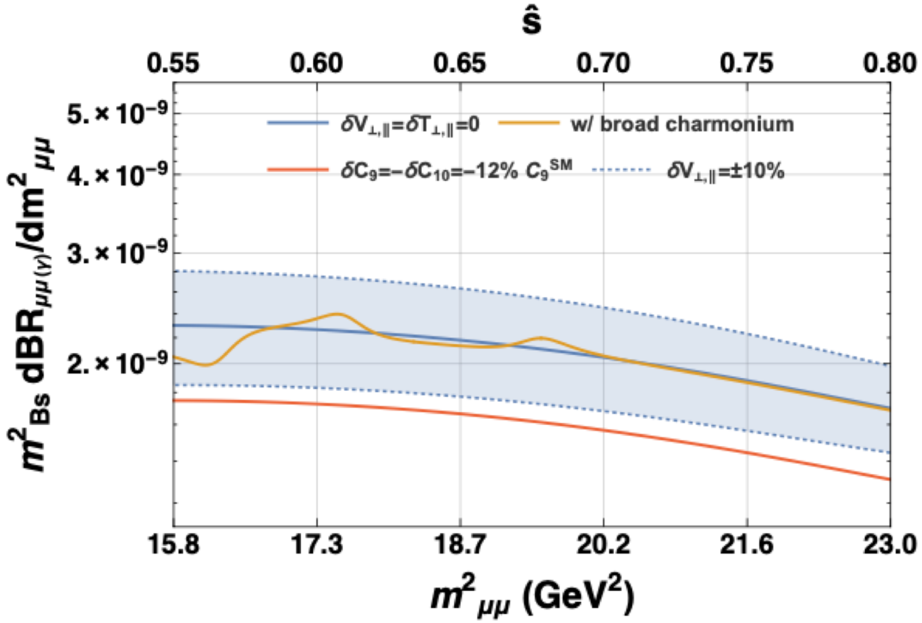


Charm loops

Negligible impact w.r.t. hadronic uncertainties!



Guadagnoli, Reboud and Zwicky, JHEP 11(2017) 184

Desiderio et al. (PRD 103 (2021) 014502) LQCD data

This generates the data in the Appendix D.4:

x_γ	F_A	Δ_{F_A}
0	0.09307	0.00598514
0.1	0.0849608	0.00486963
0.2	0.0776688	0.00422644
0.3	0.0709899	0.00389515
0.4	0.0647833	0.00400167
0.5	0.0589483	0.00469896
0.6	0.0534115	0.00595597
0.7	0.0481175	0.00763605
0.8	0.0430239	0.00962076
0.9	0.0380979	0.0118319
1	0.0333133	0.0142191

F_A Correlation Matrix

1.000	0.959	0.862	0.725	0.537	0.338	0.186	0.091	0.034	0.001	-0.020
0.959	1.000	0.966	0.861	0.664	0.423	0.224	0.091	0.006	-0.048	-0.084
0.862	0.966	1.000	0.957	0.802	0.569	0.357	0.206	0.105	0.037	-0.010
0.725	0.861	0.957	1.000	0.937	0.768	0.583	0.437	0.334	0.261	0.208
0.537	0.664	0.802	0.937	1.000	0.942	0.824	0.713	0.626	0.561	0.512
0.338	0.423	0.569	0.768	0.942	1.000	0.966	0.906	0.848	0.802	0.764
0.186	0.224	0.357	0.583	0.824	0.966	1.000	0.984	0.956	0.928	0.903
0.091	0.091	0.206	0.437	0.713	0.906	0.984	1.000	0.993	0.979	0.965
0.034	0.006	0.105	0.334	0.626	0.848	0.956	0.993	1.000	0.996	0.989
0.001	-0.048	0.037	0.261	0.561	0.802	0.928	0.979	0.996	1.000	0.998
-0.020	-0.084	-0.010	0.208	0.512	0.764	0.903	0.965	0.989	0.998	1.000

x_γ	F_V	Δ_{F_V}
0	-0.120018	0.0155225
0.1	-0.0989568	0.0117214
0.2	-0.0824261	0.00951697
0.3	-0.0684115	0.00783577
0.4	-0.05594	0.00806476
0.5	-0.0444834	0.0110619
0.6	-0.03373	0.0158477
0.7	-0.0234841	0.0215682
0.8	-0.0136163	0.0278383
0.9	-0.00403801	0.0344741
1	0.00531392	0.0413737

F_V Correlation Matrix

1.000	0.933	0.886	0.898	0.782	0.543	0.379	0.288	0.236	0.205	0.184
0.933	1.000	0.989	0.935	0.667	0.317	0.107	-0.004	-0.066	-0.103	-0.127
0.886	0.989	1.000	0.955	0.679	0.314	0.093	-0.024	-0.090	-0.130	-0.157
0.898	0.935	0.955	1.000	0.862	0.571	0.369	0.255	0.188	0.147	0.119
0.782	0.667	0.679	0.862	1.000	0.908	0.787	0.707	0.656	0.623	0.600
0.543	0.317	0.314	0.571	0.908	1.000	0.973	0.938	0.912	0.893	0.879
0.379	0.107	0.093	0.369	0.787	0.973	1.000	0.993	0.982	0.972	0.965
0.288	-0.004	-0.024	0.255	0.707	0.938	0.993	1.000	0.998	0.993	0.990
0.236	-0.066	-0.090	0.188	0.656	0.912	0.982	0.998	1.000	0.999	0.997
0.205	-0.103	-0.130	0.147	0.623	0.893	0.972	0.993	0.999	1.000	1.000
0.184	-0.127	-0.157	0.119	0.600	0.879	0.965	0.990	0.997	1.000	1.000

Desiderio et al. (PRD 103 (2021) 014502) LQCD data

This generates the data in the Appendix D.4:

This is the region of the **direct** computations on the lattice!

x_γ	F_A	Δ_{F_A}
0	0.09307	0.00598514
0.1	0.0849608	0.00486963
0.2	0.0776688	0.00422644
0.3	0.0709899	0.00389515
0.4	0.0647833	0.00400167
0.5	0.0589483	0.00469896
0.6	0.0534115	0.00595597
0.7	0.0481175	0.00763605
0.8	0.0430239	0.00962076
0.9	0.0380979	0.0118319
1	0.0333133	0.0142191

F_A Correlation Matrix

1.000	0.959	0.862	0.725	0.537	0.338	0.186	0.091	0.034	0.001	-0.020
0.959	1.000	0.966	0.861	0.664	0.423	0.224	0.091	0.006	-0.048	-0.084
0.862	0.966	1.000	0.957	0.802	0.569	0.357	0.206	0.105	0.037	-0.010
0.725	0.861	0.957	1.000	0.937	0.768	0.583	0.437	0.334	0.261	0.208
0.537	0.664	0.802	0.937	1.000	0.942	0.824	0.713	0.626	0.561	0.512
0.338	0.423	0.569	0.768	0.942	1.000	0.966	0.906	0.848	0.802	0.764
0.186	0.224	0.357	0.583	0.824	0.966	1.000	0.984	0.956	0.928	0.903
0.091	0.091	0.206	0.437	0.713	0.906	0.984	1.000	0.993	0.979	0.965
0.034	0.006	0.105	0.334	0.626	0.848	0.956	0.993	1.000	0.996	0.989
0.001	-0.048	0.037	0.261	0.561	0.802	0.928	0.979	0.996	1.000	0.998
-0.020	-0.084	-0.010	0.208	0.512	0.764	0.903	0.965	0.989	0.998	1.000

x_γ	F_V	Δ_{F_V}
0	-0.120018	0.0155225
0.1	-0.0989568	0.0117214
0.2	-0.0824261	0.00951697
0.3	-0.0684115	0.00783577
0.4	-0.05594	0.00806476
0.5	-0.0444834	0.0110619
0.6	-0.03373	0.0158477
0.7	-0.0234841	0.0215682
0.8	-0.0136163	0.0278383
0.9	-0.00403801	0.0344741
1	0.00531392	0.0413737

F_V Correlation Matrix

1.000	0.933	0.886	0.898	0.782	0.543	0.379	0.288	0.236	0.205	0.184
0.933	1.000	0.989	0.935	0.667	0.317	0.107	-0.004	-0.066	-0.103	-0.127
0.886	0.989	1.000	0.955	0.679	0.314	0.093	-0.024	-0.090	-0.130	-0.157
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0.782	0.667	0.679	0.862	1.000	0.908	0.787	0.707	0.656	0.623	0.600
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0.379	0.107	0.093	0.369	0.787	0.973	1.000	0.993	0.982	0.972	0.965
0.288	-0.004	-0.024	0.255	0.707	0.938	0.993	1.000	0.998	0.993	0.990
0.236	-0.066	-0.090	0.188	0.656	0.912	0.982	0.998	1.000	0.999	0.997
0.205	-0.103	-0.130	0.147	0.623	0.893	0.972	0.993	0.999	1.000	1.000
0.184	-0.127	-0.157	0.119	0.600	0.879	0.965	0.990	0.997	1.000	1.000

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0.5	0.0589483	0.00469896
0.6	0.0534115	0.00595597
0.7	0.0481175	0.00763605
0.8	0.0430239	0.00962076
0.9	0.0380979	0.0118319
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0.959	1.000	0.966	0.861	0.664	0.423	0.224	0.091	0.006	-0.048	-0.084
0.862	0.966	1.000	0.957	0.802	0.569	0.357	0.206	0.105	0.037	-0.010
0.725	0.861	0.957	1.000	0.937	0.768	0.583	0.437	0.334	0.261	0.208
0.537	0.664	0.802	0.937	1.000	0.942	0.824	0.713	0.626	0.561	0.512
0.338	0.423	0.569	0.768	0.942	1.000	0.966	0.906	0.848	0.802	0.764
0.186	0.224	0.357	0.583	0.824	0.966	1.000	0.984	0.956	0.928	0.903
0.091	0.091	0.206	0.437	0.713	0.906	0.984	1.000	0.993	0.979	0.965
0.034	0.006	0.105	0.334	0.626	0.848	0.956	0.993	1.000	0.996	0.989
0.001	-0.048	0.037	0.261	0.561	0.802	0.928	0.979	0.996	1.000	0.998
-0.020	-0.084	-0.010	0.208	0.512	0.764	0.903	0.965	0.989	0.998	1.000

This is the region of the **direct** computations on the lattice!

FA has NO Structure Independent (SI) contributions !!
The extrapolation to the Bs-sector can thus be done without taking care of the subtraction of the SI term …

$$\langle \gamma(k,\epsilon)|O_\mu^V|\bar{B}_q(p_B)\rangle = s_e(P_\mu^\perp V_\perp(q^2) - P_\mu^\parallel (V_\parallel(q^2) + Q_{\bar{B}_q} f_{B_q}^{(pt)}) - \ldots$$