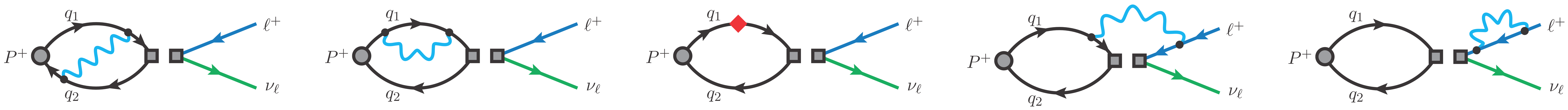

Light-meson leptonic decay rates from lattice QCD+QED calculations

Matteo Di Carlo

16th February 2023

THE UNIVERSITY of EDINBURGH

New Physics Signals
(NePSi — NeΨ)

UNIVERSITÀ DI PISA

Testing the Standard Model with flavour physics

Indirect searches of new physics using CKM matrix unitarity constraints

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

in the Standard Model:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

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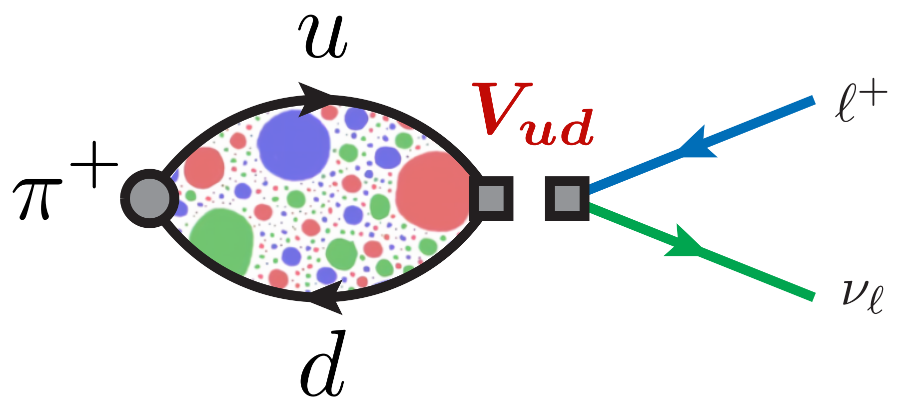
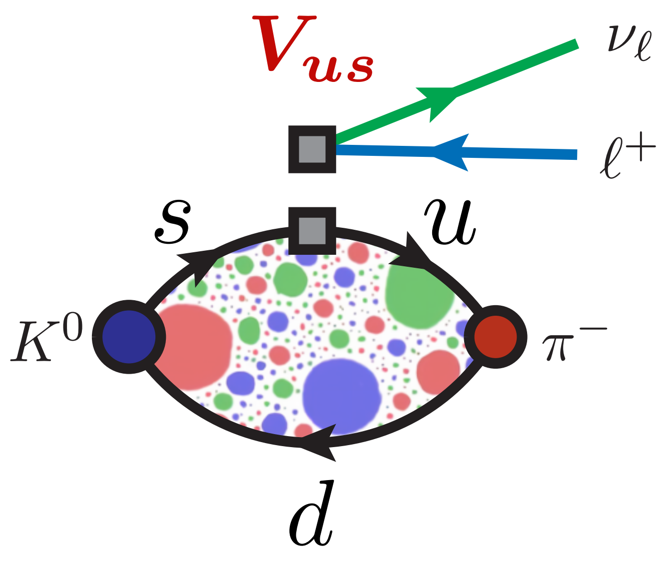
Matrix elements can be extracted e.g. from leptonic and semileptonic decays of mesons

$$\underbrace{\frac{\Gamma\left[K \to \ell\nu_\ell(\gamma)\right]}{\Gamma\left[\pi \to \ell\nu_\ell(\gamma)\right]}}_{\text{experiments}} \propto \boxed{\left|\frac{V_{us}}{V_{ud}}\right|^2} \underbrace{\left(\frac{f_K}{f_\pi}\right)^2}_{\text{QCD}}$$

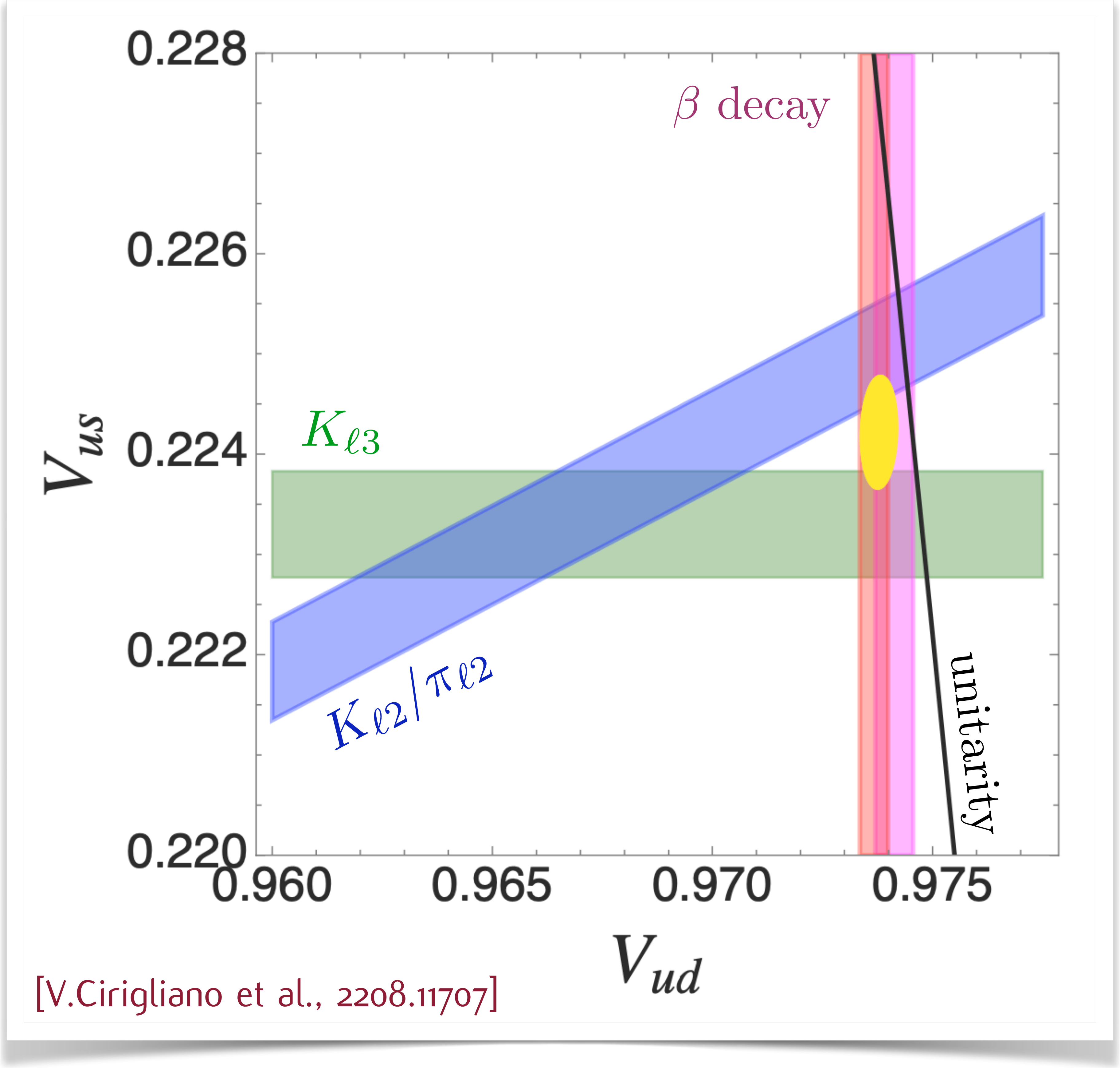
$$\underbrace{\Gamma\left[K \to \pi\ell\nu_\ell(\gamma)\right]}_{\text{experiments}} \propto \boxed{|V_{us}|^2} \underbrace{|f_+^{K\pi}(0)|^2}_{\text{QCD}}$$





Testing the Standard Model with flavour physics

possible tensions?



[V.Cirigliano et al., 2208.11707]

Possible $\sim 3\sigma$ **tensions** in the V_{us}-V_{ud} plane
(best fit vs CKM unitarity, leptonic vs semileptonic, ...)

Experimental and theoretical control of these quantities is of crucial importance to solve the issue

Testing the Standard Model with flavour physics

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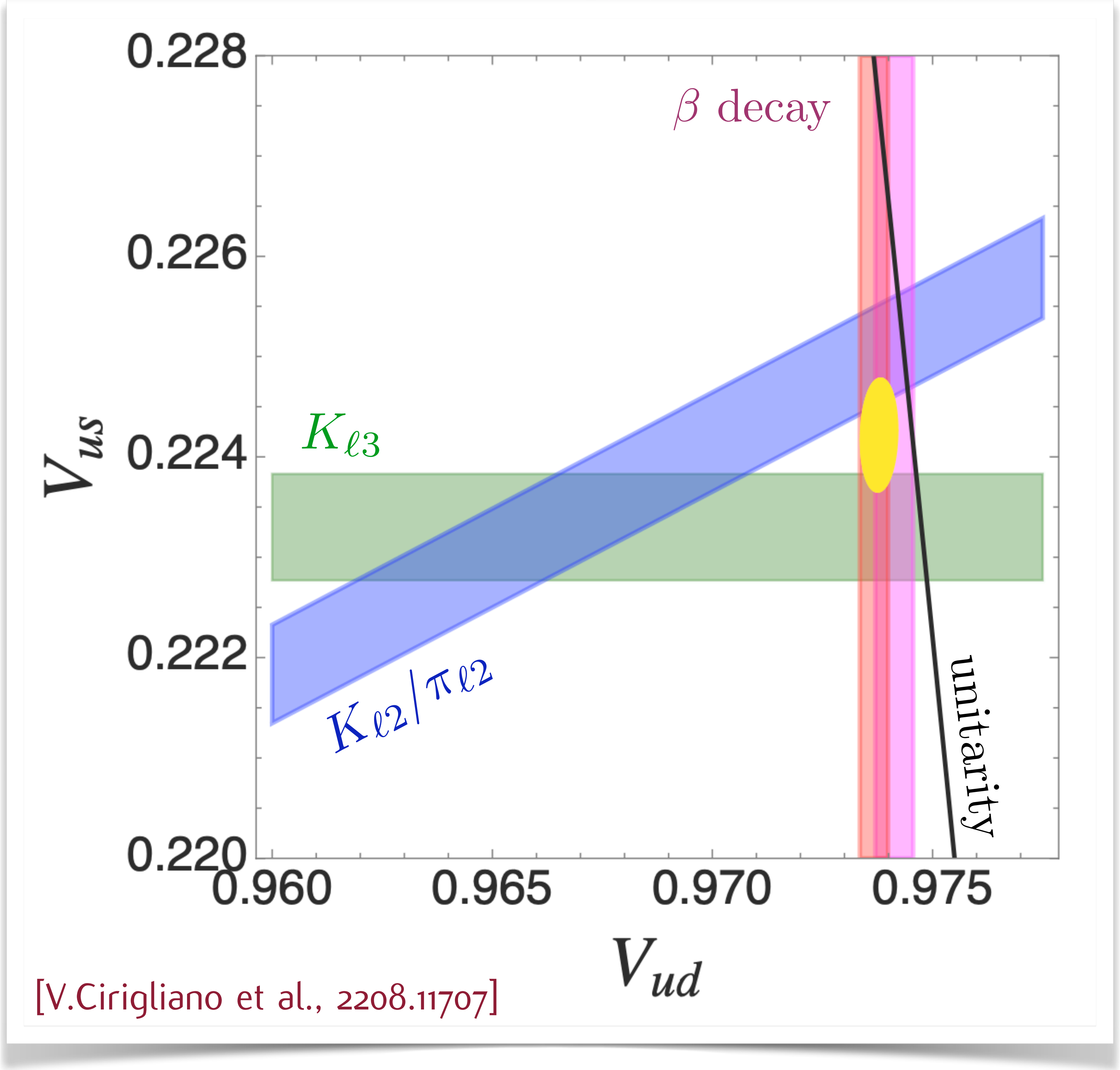

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- clarify situation with **new measurements** of leptonic and semileptonic decay rates (e.g. at NA62)

Testing the Standard Model with flavour physics

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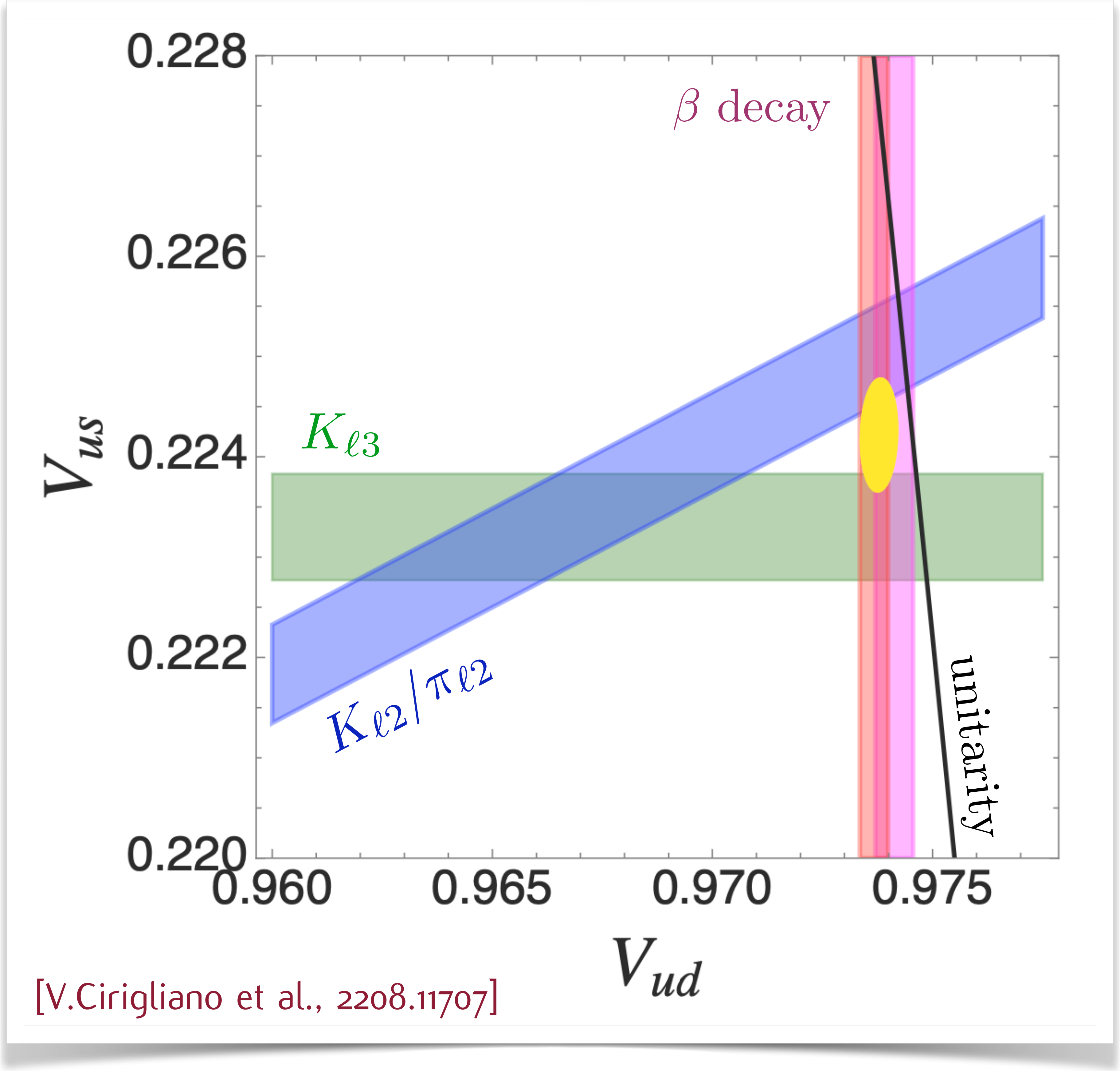

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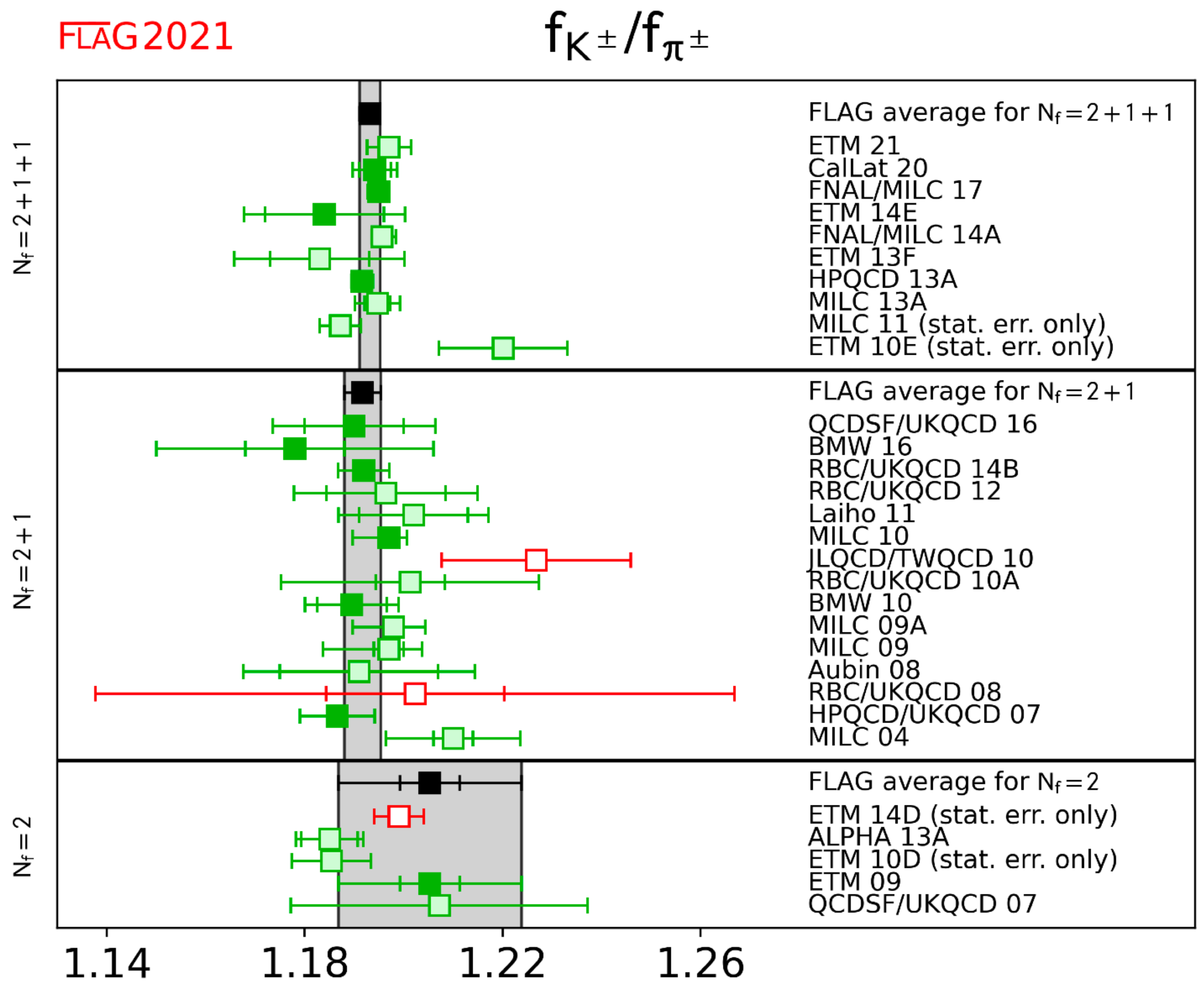
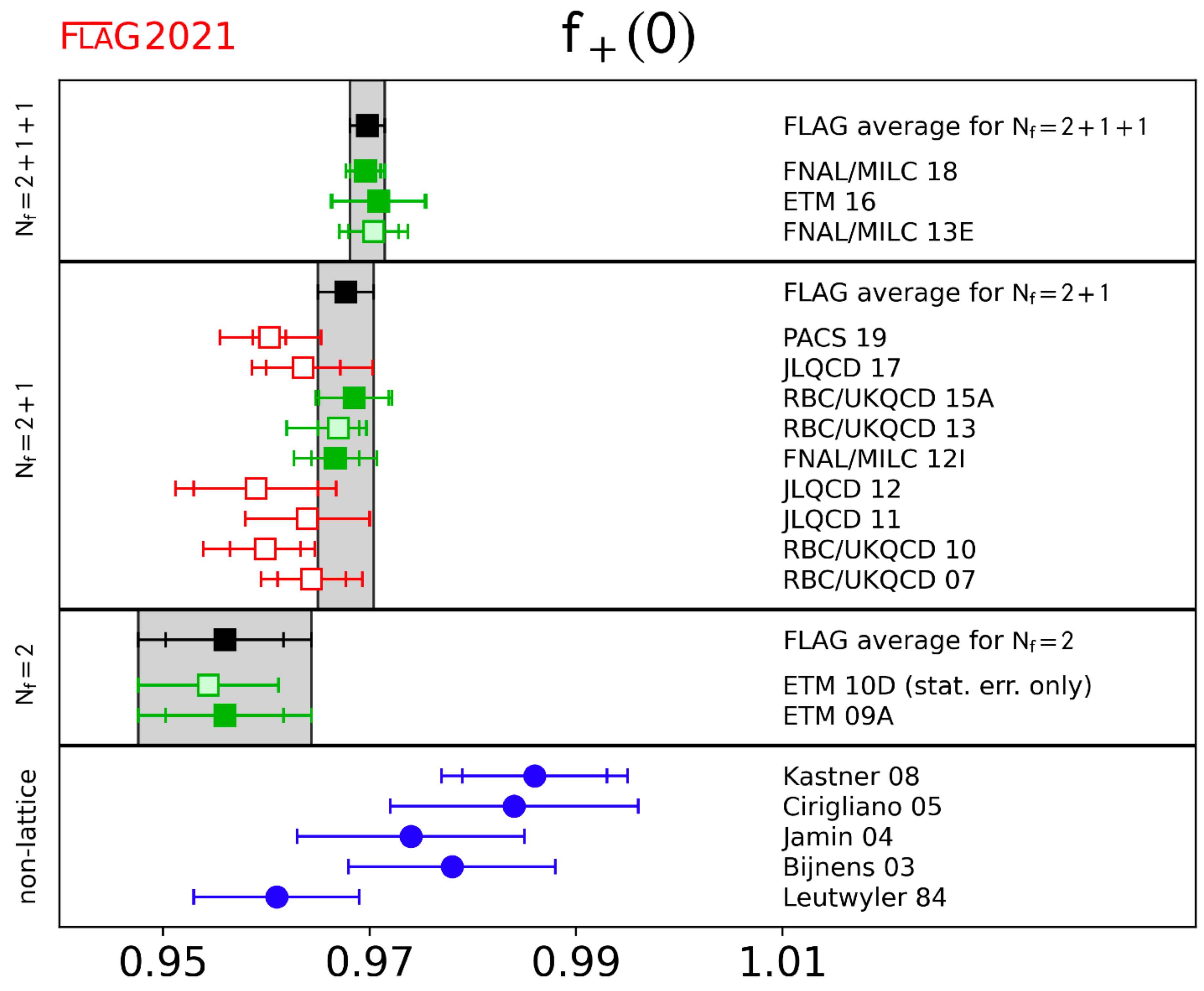
Possible $\sim 3\sigma$ **tensions** in the V_{us}-V_{ud} plane (best fit vs CKM unitarity, leptonic vs semileptonic, ...)

Experimental and theoretical control of these quantities is of crucial importance to solve the issue

- clarify situation with **new measurements** of leptonic and semileptonic decay rates (e.g. at NA62)
- improve predictions including **radiative corrections and isospin-breaking effects**

Some lattice QCD results



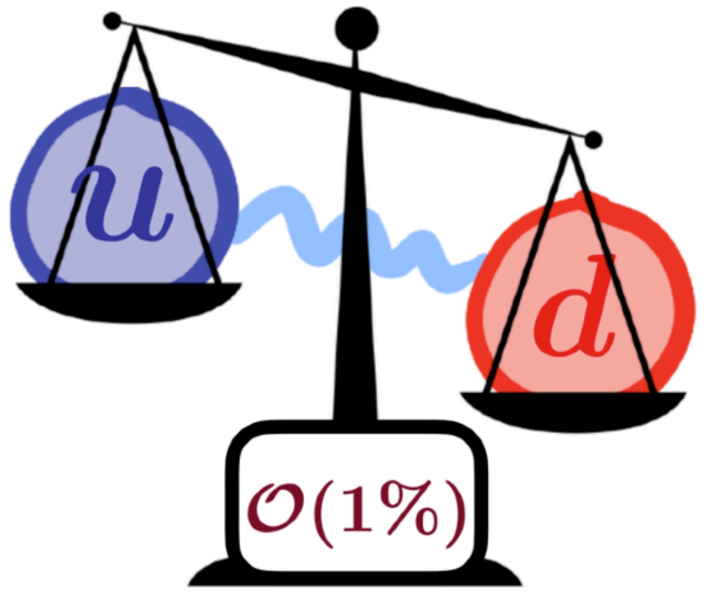


FLAG 2021
Flavour Lattice Averaging Group

f_K/f_π and $f_+^{K\pi}(0)$ determined from lattice QCD with **sub percent precision!**

Isospin-breaking effects on the lattice

Current level of precision requires the inclusion of isospin breaking (IB) corrections



- **strong** effects $[m_u - m_d]_{\text{QCD}} \neq 0$
- **electromagnetic** effects $\alpha \neq 0$

Different ways to include them on the lattice...

Isospin-breaking effects on the lattice

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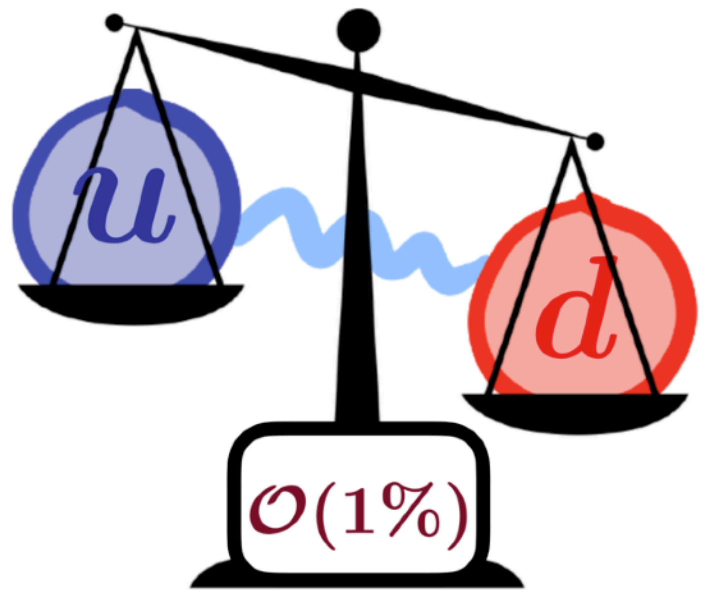

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Different ways to include them on the lattice...

In this calculation:

- RM123 perturbative approach

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S_{\mathrm{iso}} - \Delta S}$$
$$= \langle \mathcal{O} \rangle_{\mathrm{iso}} + \langle \Delta S \, \mathcal{O} \rangle_{\mathrm{iso}} + \dots$$

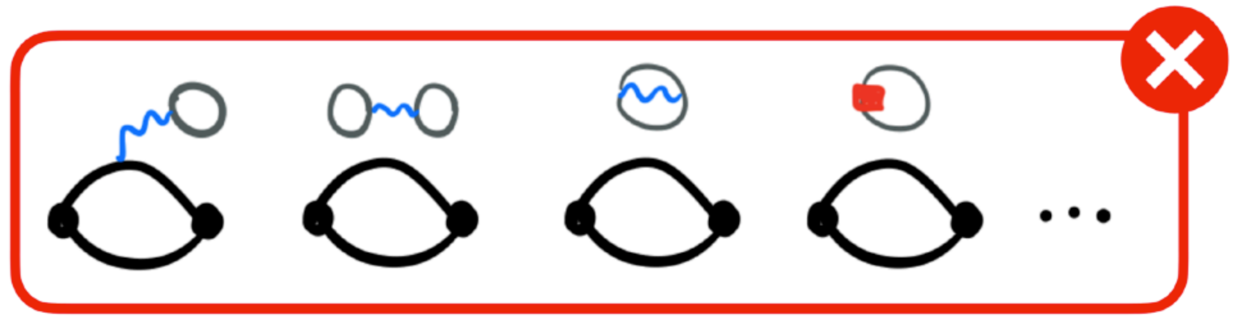
- QED$_{\mathrm{L}}$ photon prescription

$$\Delta^{\gamma}_{\mu\nu}(x) = \frac{1}{V} \sum_{k_0} \sum_{\mathbf{k} \neq 0} \Delta^{\gamma}_{\mu\nu}(k) \, \mathrm{e}^{\mathrm{i}k \cdot x}$$

& power-like finite volume effects

- "electro-quenched" QED

neutral sea quarks

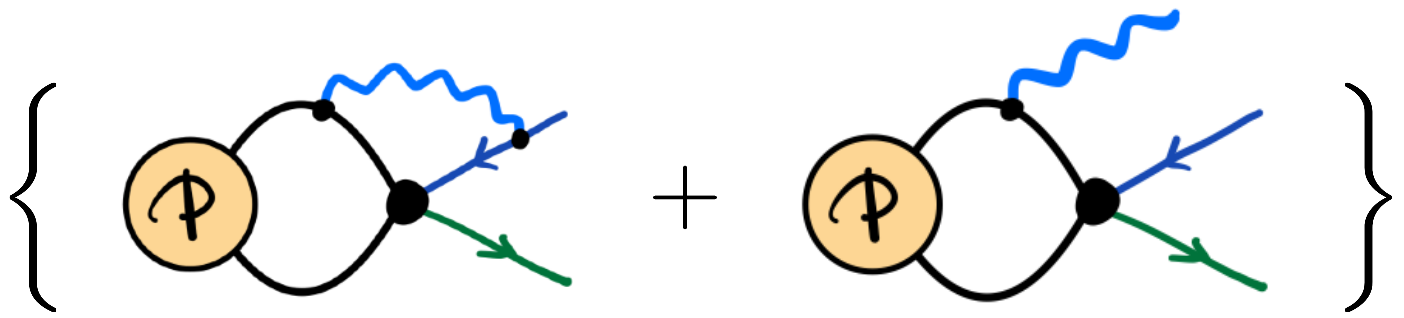


Decay rate at $\mathcal{O}(\alpha)$

F. Bloch & A. Nordsieck, PR 52 (1937) 54

The RM123+Soton recipe

$$\Gamma(P_{\ell 2}) = \lim_{\Lambda_{\mathrm{IR}}\to 0}$$



IR finite

IR divergent

IR divergent

Decay rate at $\mathcal{O}(\alpha)$

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N. Carrasco et al., PRD 91 (2015)
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MDC et al., PRD 100 (2019)

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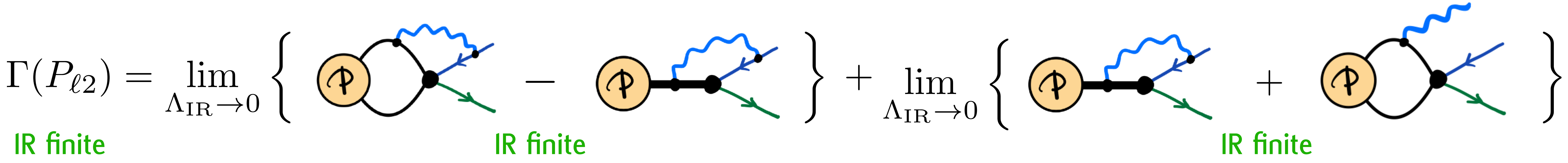

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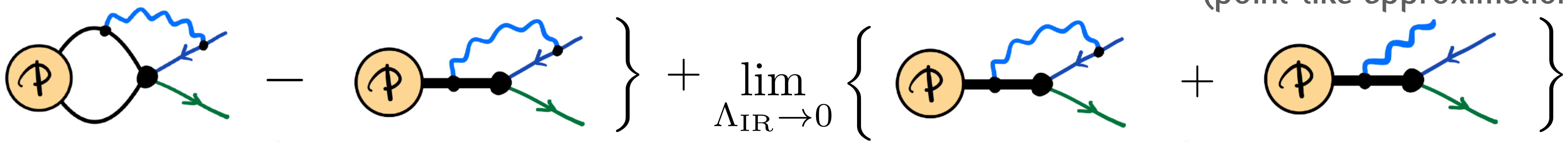

$$\Gamma(P_{\ell 2}) = \lim_{\Lambda_{\mathrm{IR}}\to 0}\Bigg\{ \qquad - \qquad \Bigg\} + \lim_{\Lambda_{\mathrm{IR}}\to 0}\Bigg\{ \qquad + \qquad \Bigg\}$$

(point-like approximation)

IR finite

IR finite

IR finite

Decay rate at $\mathcal{O}(\alpha)$

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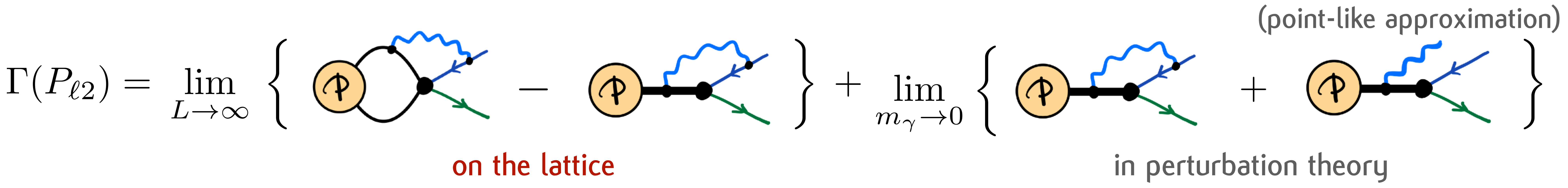

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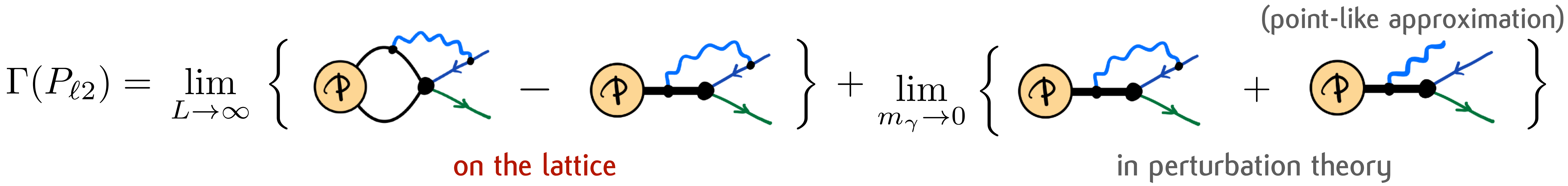


Possible extensions:

- compute **structure-dependent real photon emission** on the lattice

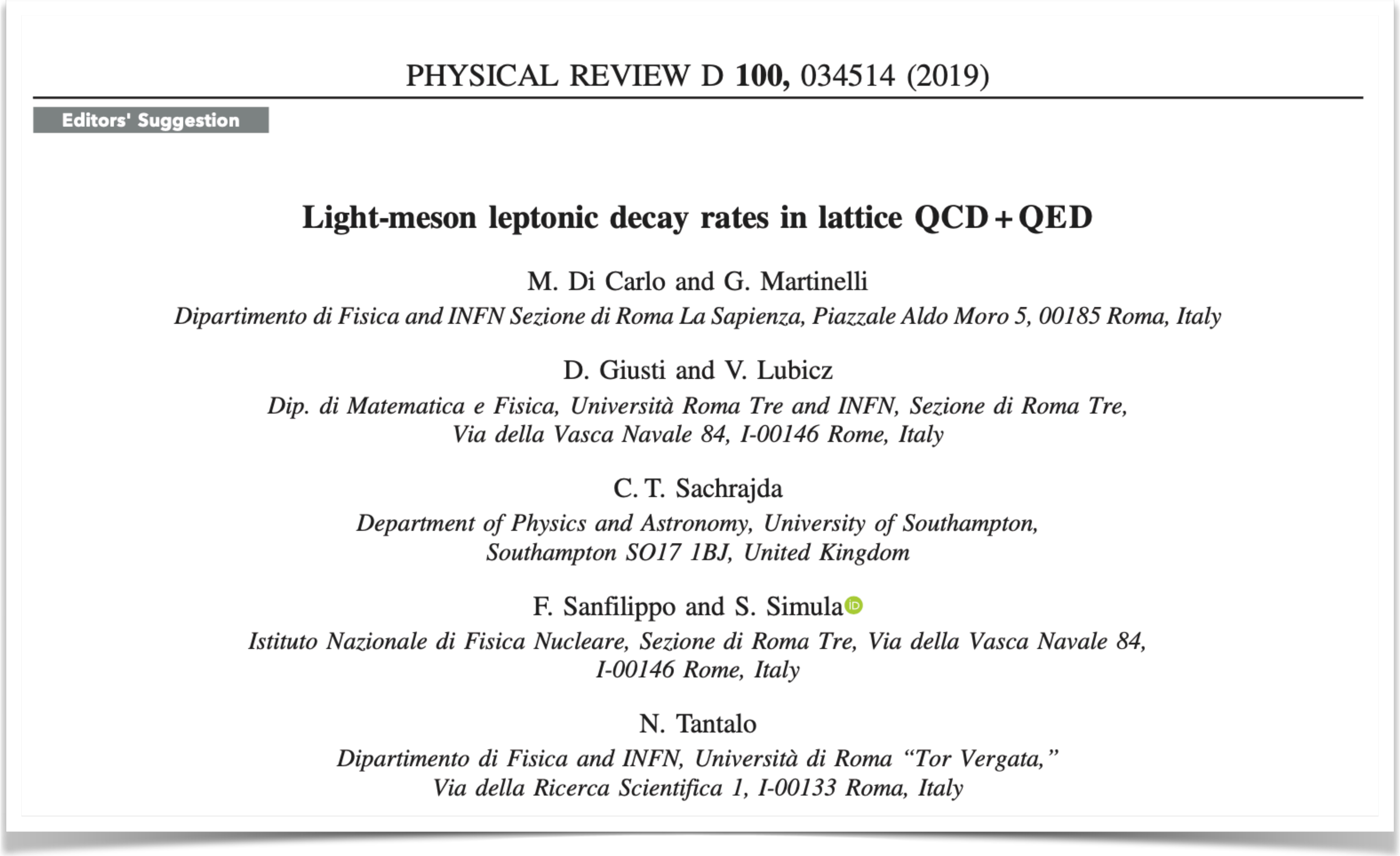
G.M. de Divitiis et al., [1908.10160]
R. Frezzotti et al., PRD 103 (2021)
A. Desiderio et al., PRD 102 (2021)
C. Kane et al., [1907.00279 & 2110.13196]
D. Giusti et al., [2302.01298]

- nice progress also on **virtual photon emission**: see **G.Gagliardi's talk @11.50**

G. Gagliardi et al., PRD 105 (2022)

- $\Gamma(K_{\mu 2})$ and $\Gamma(\pi_{\mu 2})$ separately
- Twisted Mass fermions
- multiple volumes and 3 lattice spacings
- unphysical pion masses ($\gtrsim 230$ MeV)

1904.08731



PHYSICAL REVIEW D **100,** 034514 (2019)

Editors' Suggestion

Light-meson leptonic decay rates in lattice QCD + QED

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Via della Ricerca Scientifica 1, I-00133 Roma, Italy

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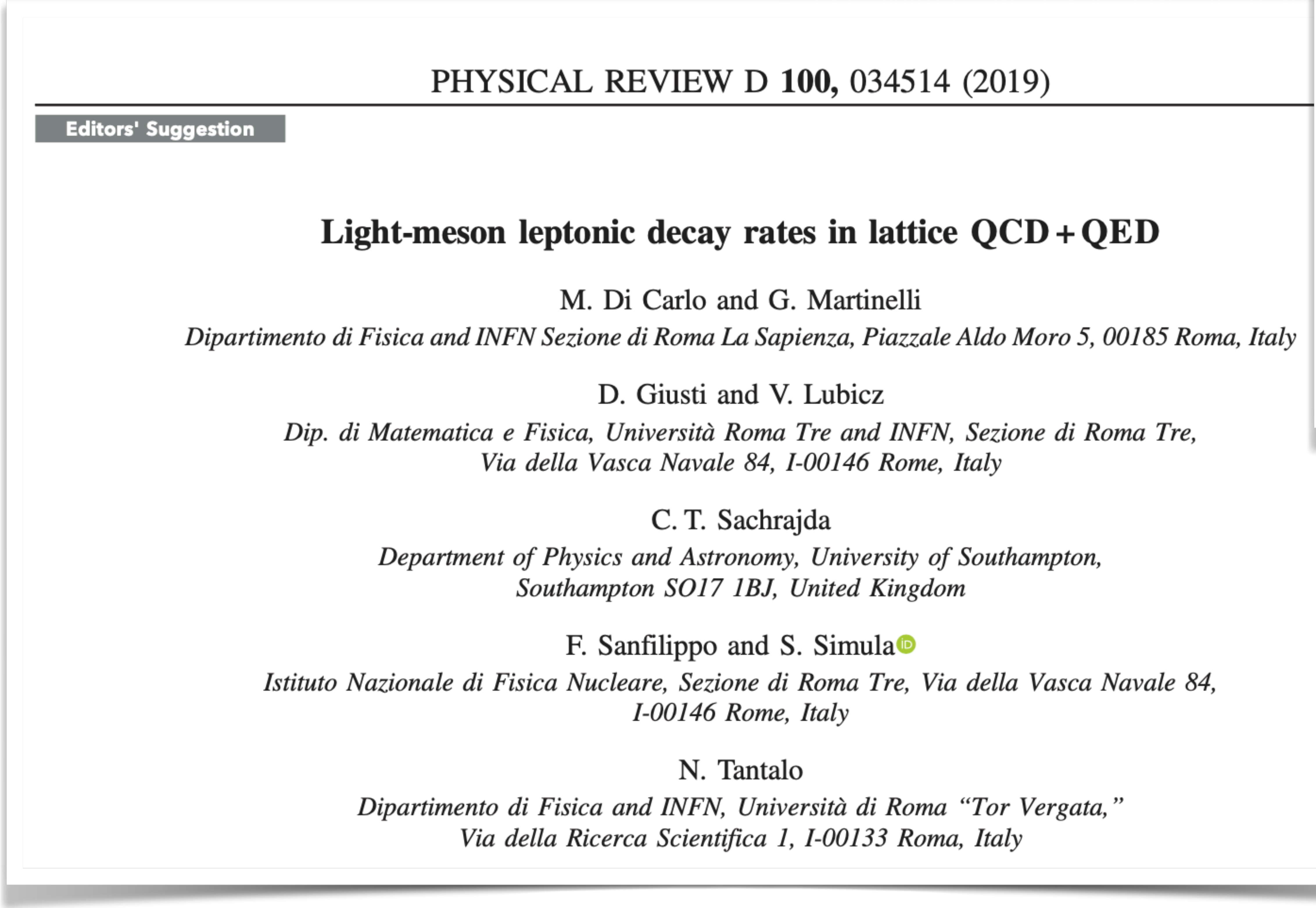

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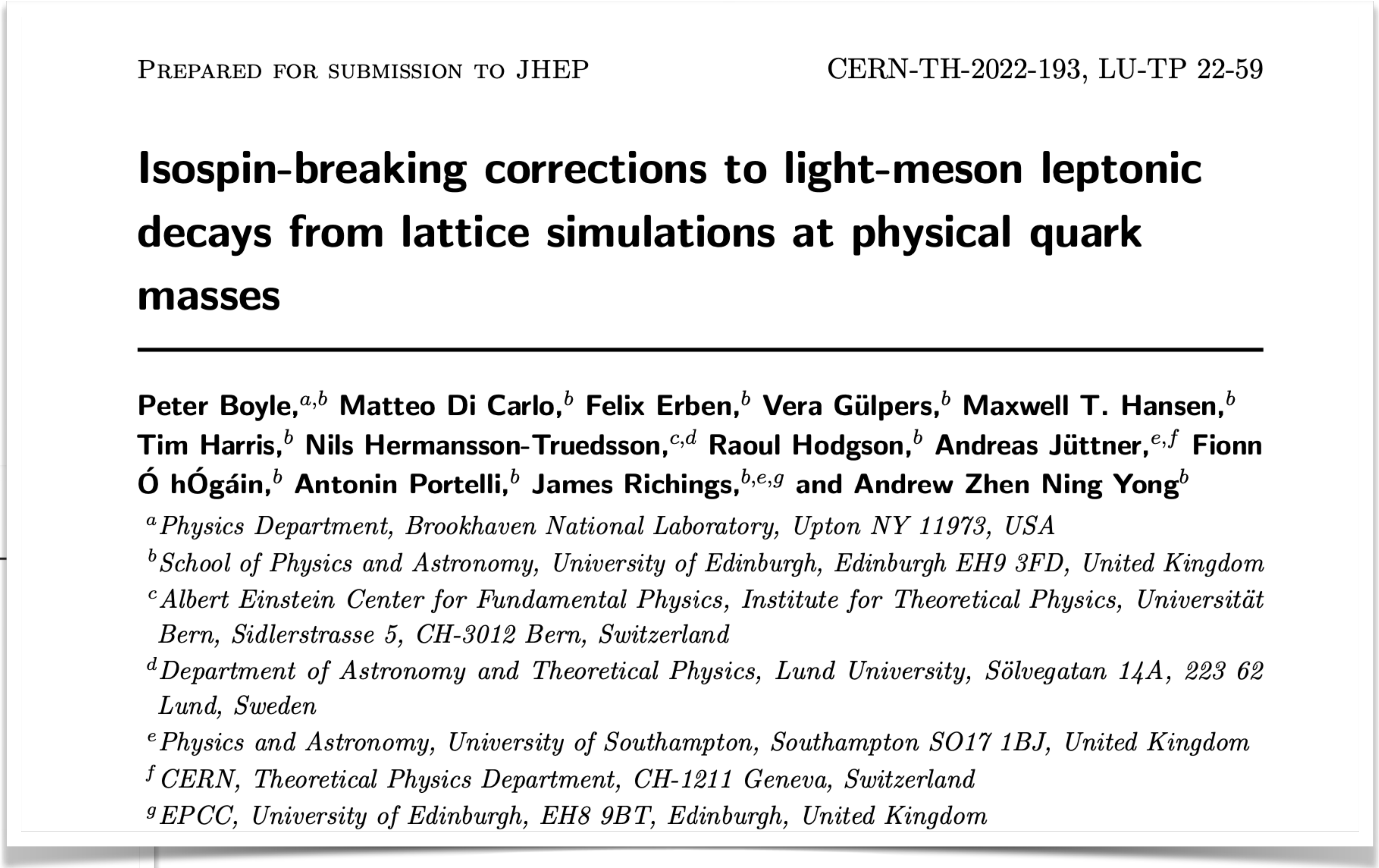

Prepared for submission to JHEP CERN-TH-2022-193, LU-TP 22-59

Isospin-breaking corrections to light-meson leptonic decays from lattice simulations at physical quark masses

Peter Boyle,[a,b] **Matteo Di Carlo,**[b] **Felix Erben,**[b] **Vera Gülpers,**[b] **Maxwell T. Hansen,**[b] **Tim Harris,**[b] **Nils Hermansson-Truedsson,**[c,d] **Raoul Hodgson,**[b] **Andreas Jüttner,**[e,f] **Fionn Ó hÓgáin,**[b] **Antonin Portelli,**[b] **James Richings,**[b,e,g] **and Andrew Zhen Ning Yong**[b]

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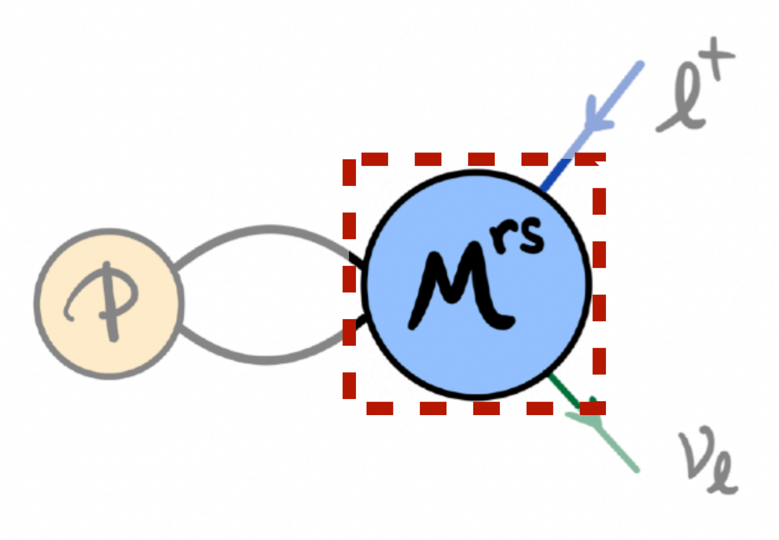
[b] *School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3FD, United Kingdom*
[c] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, Universität Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland*
[d] *Department of Astronomy and Theoretical Physics, Lund University, Sölvegatan 14A, 223 62 Lund, Sweden*
[e] *Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[f] *CERN, Theoretical Physics Department, CH-1211 Geneva, Switzerland*
[g] *EPCC, University of Edinburgh, EH8 9BT, Edinburgh, United Kingdom*

2211.12865

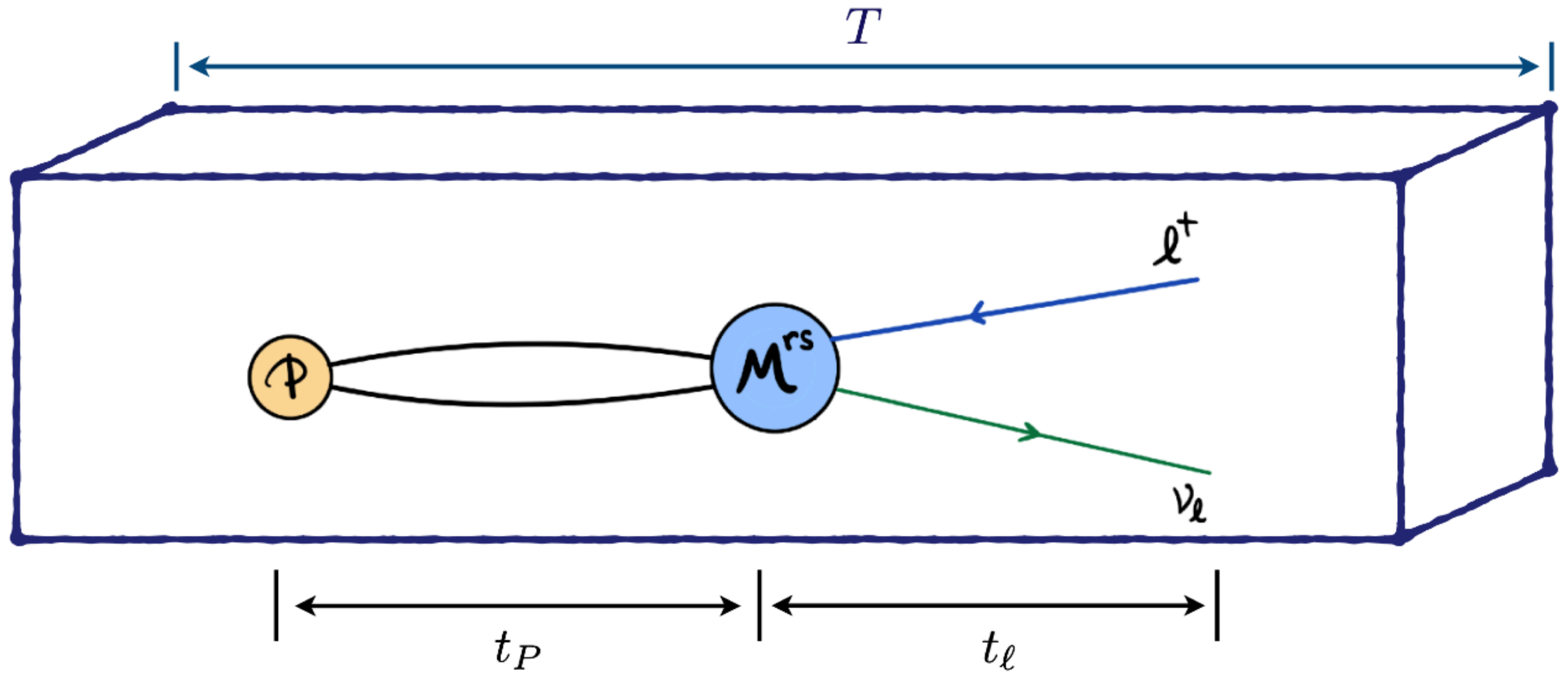
- ratio $\Gamma(K_{\mu 2})$ / $\Gamma(\pi_{\mu 2})$
- Domain Wall fermions
- single volume and lattice spacing
- physical quark masses

From correlators to matrix elements

Our goal:



How we realise it:



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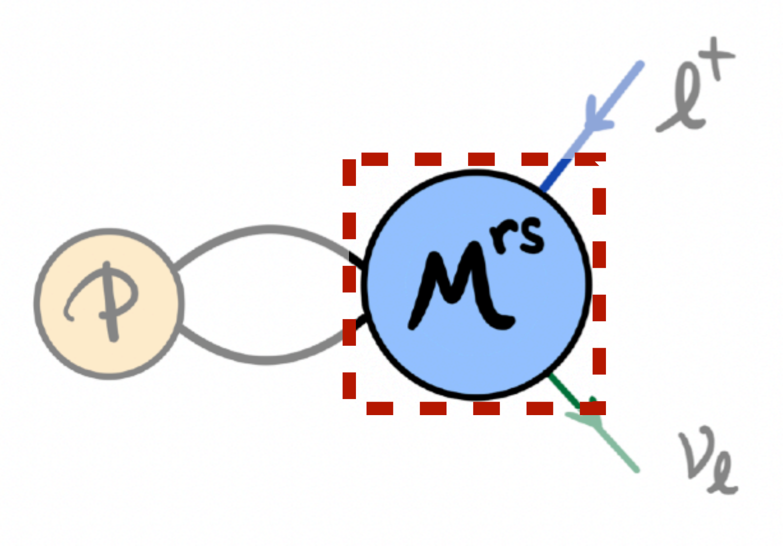

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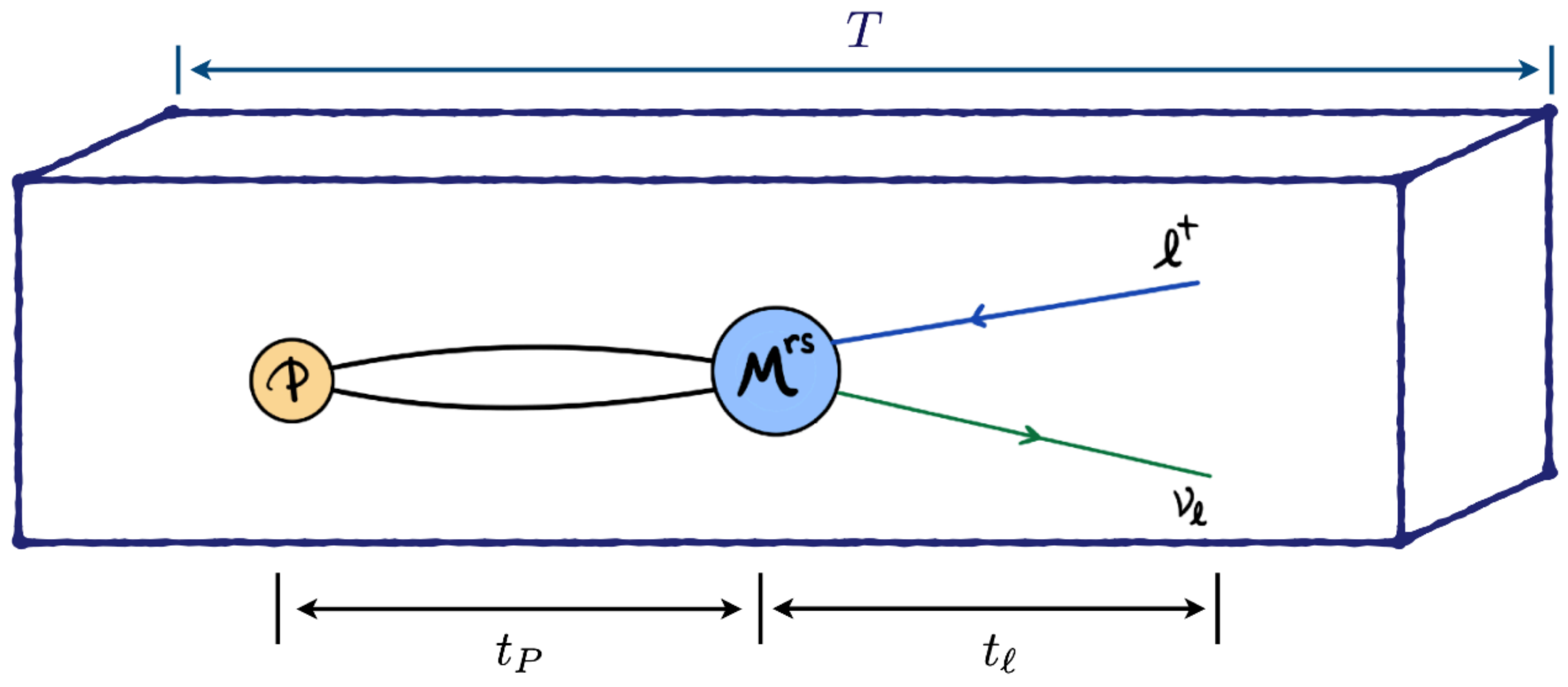
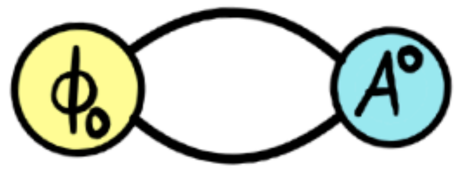


Tree-level decay amplitude: $|\mathcal{M}_0(\mathbf{p}_\ell)|^2 = |\mathcal{A}_{P,0}|^2|\mathcal{L}_0(\mathbf{p}_\ell)|^2 \quad \mathcal{A}_{P,0} = \langle 0|A^0|P\rangle_0 = im_{P,0}\,[f_{P,0}]$



$$= \langle 0|A^0(0)\phi^\dagger(-t)|0\rangle \quad \rightarrow \quad \frac{Z_{P,0}\mathcal{A}_{P,0}}{2m_{P,0}}\ \mathrm{e}^{-m_{P,0}t}$$

$$Z_{P,0} = \langle P, \mathbf{p} = \mathbf{0}|\phi^\dagger|0\rangle_0$$

$$= \langle 0|\phi(0)\phi^\dagger(-t)|0\rangle \quad \rightarrow \quad \frac{Z_{P,0}^2}{2m_{P,0}}\ \mathrm{e}^{-m_{P,0}t}$$

From correlators to matrix elements

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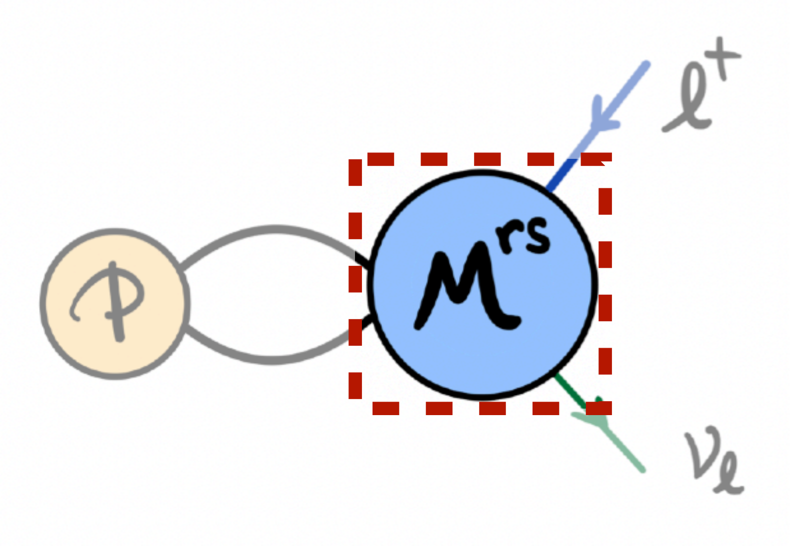

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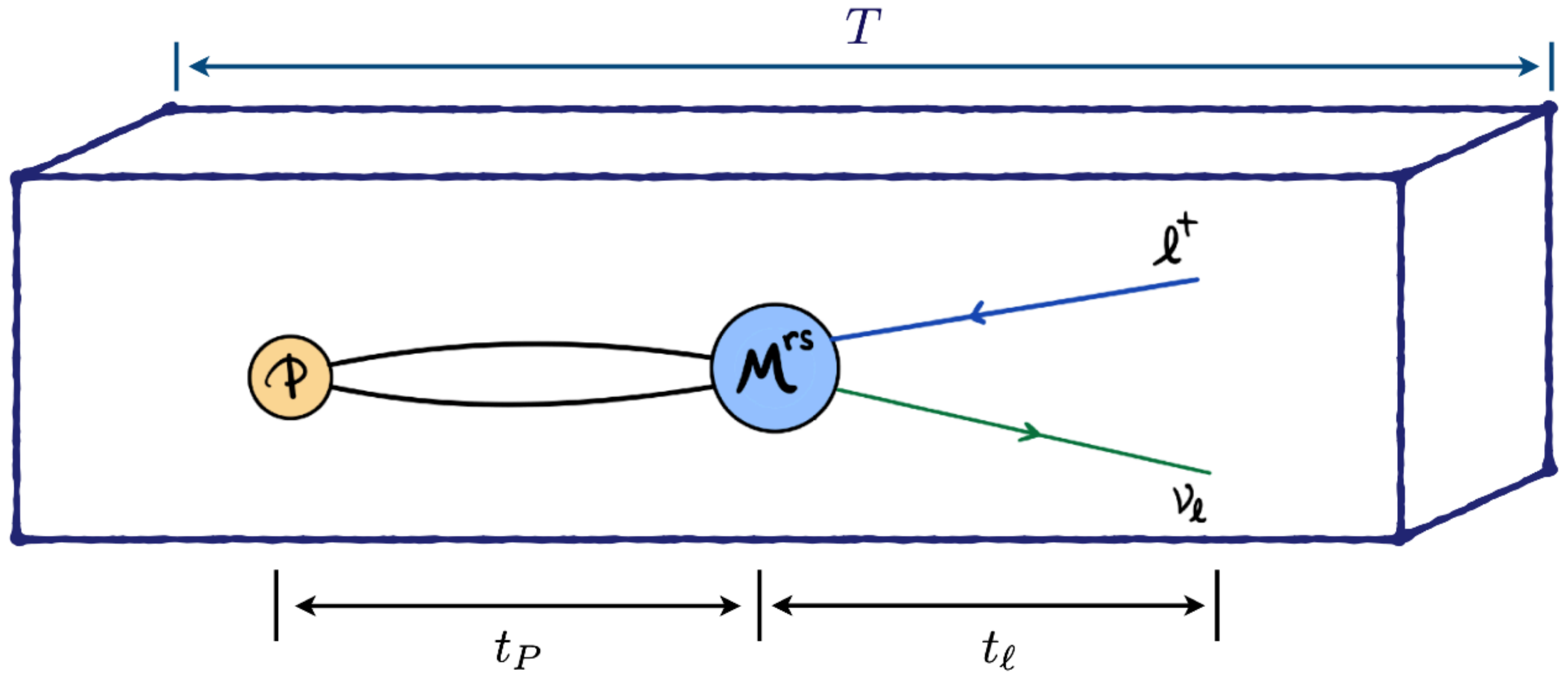

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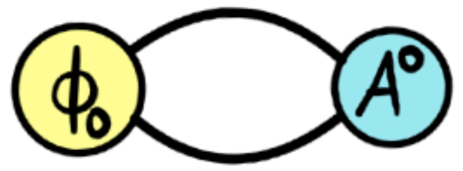
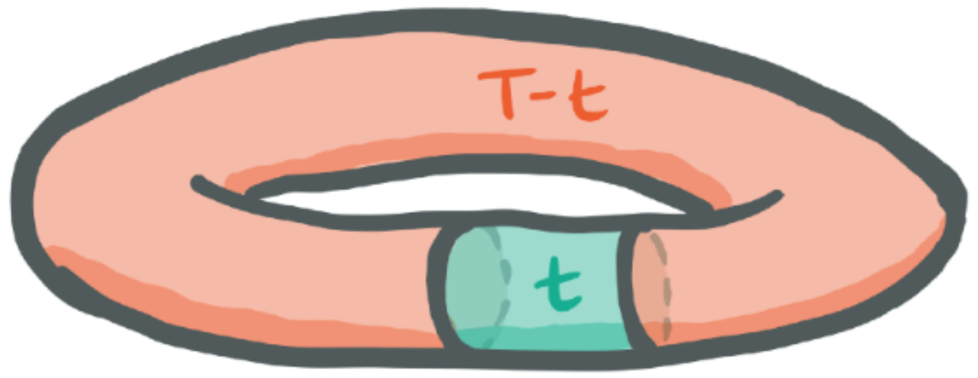


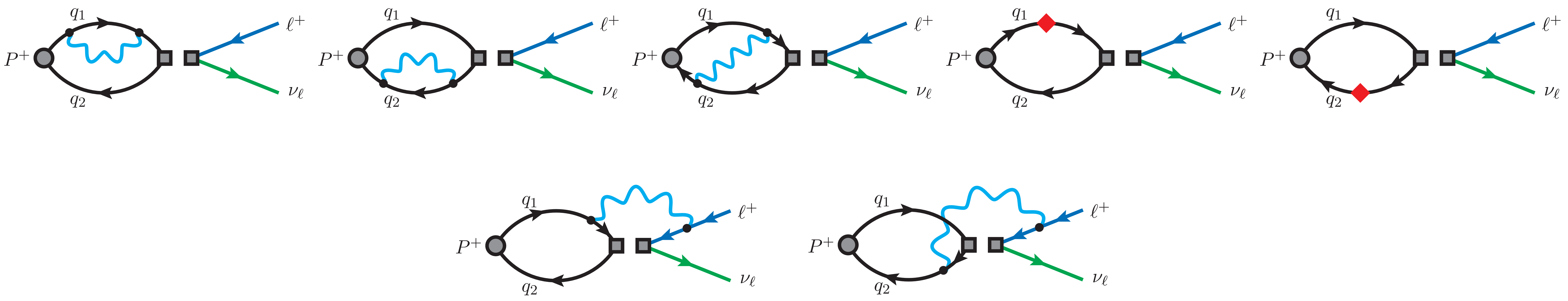
$$= \langle 0|A^0(0)\phi^\dagger(-t)|0\rangle_T \rightarrow \frac{Z_{P,0}\mathcal{A}_{P,0}}{2m_{P,0}}\left\{\mathrm{e}^{-m_{P,0}t} - \mathrm{e}^{-m_{P,0}(T-t)}\right\}$$

$$Z_{P,0} = \langle P, \mathbf{p} = \mathbf{0}|\phi^\dagger|0\rangle_0$$

$$= \langle 0|\phi(0)\phi^\dagger(-t)|0\rangle_T \rightarrow \frac{Z_{P,0}^2}{2m_{P,0}}\left\{\mathrm{e}^{-m_{P,0}t} + \mathrm{e}^{-m_{P,0}(T-t)}\right\}$$



IB corrections to the decay amplitude



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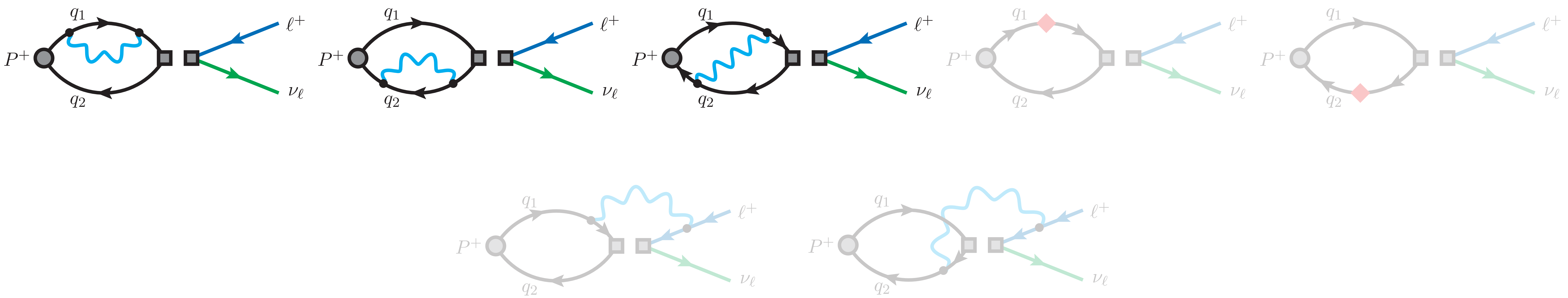

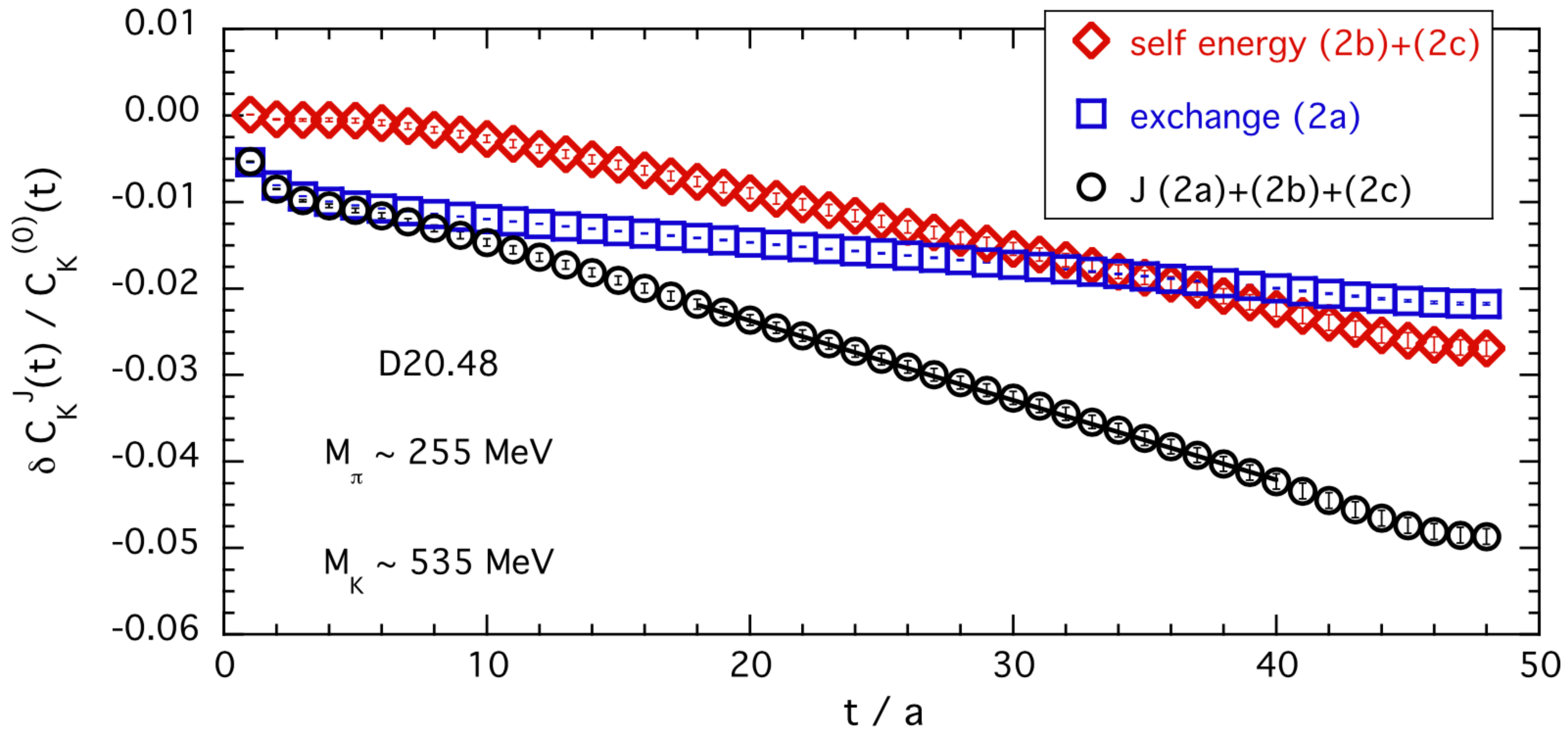

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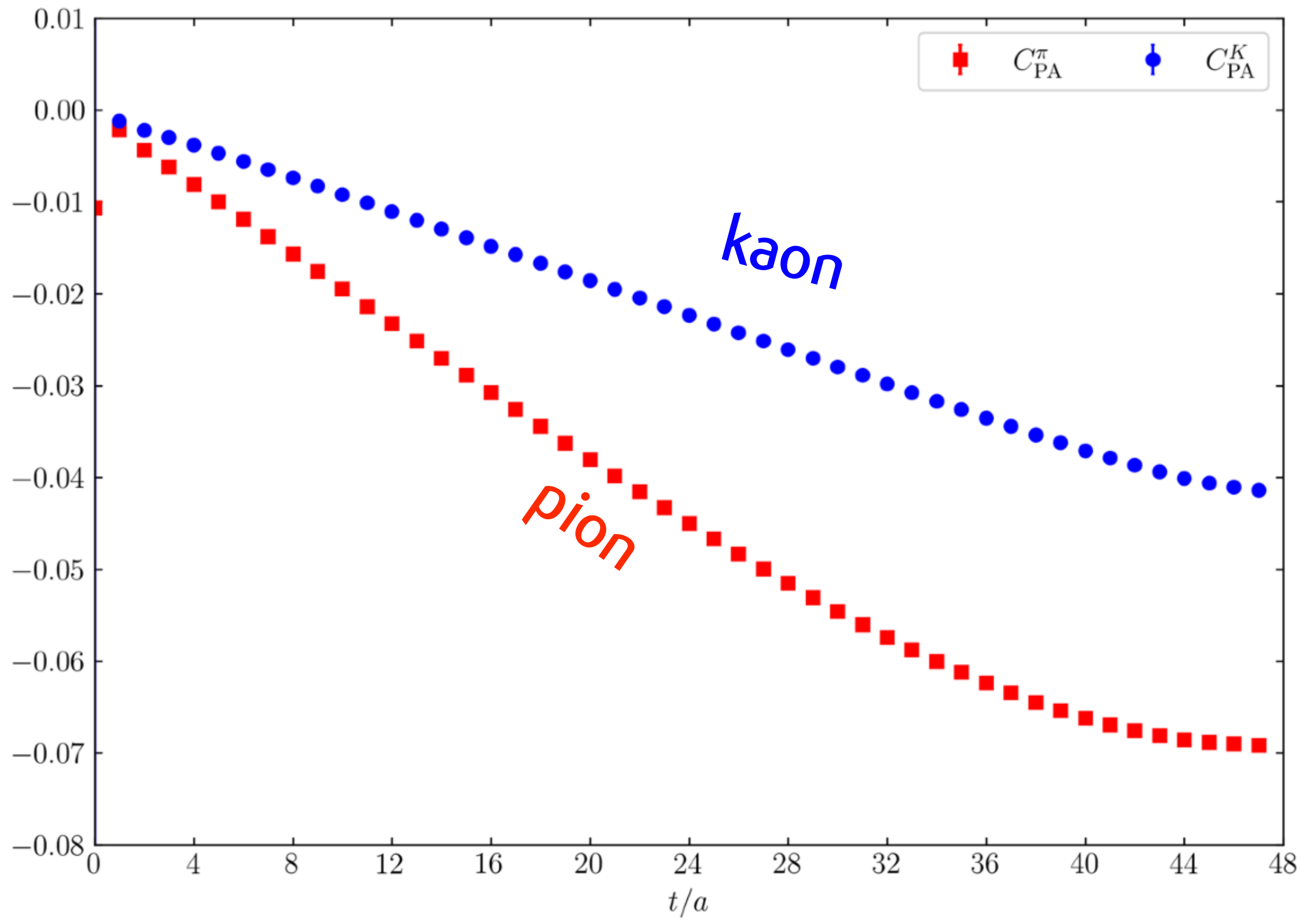

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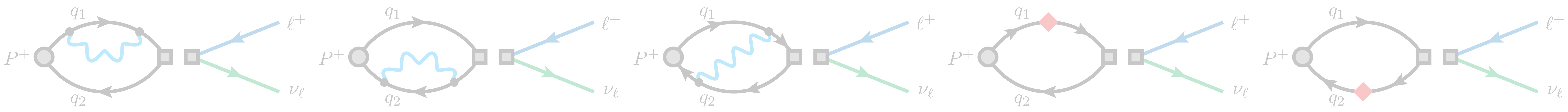
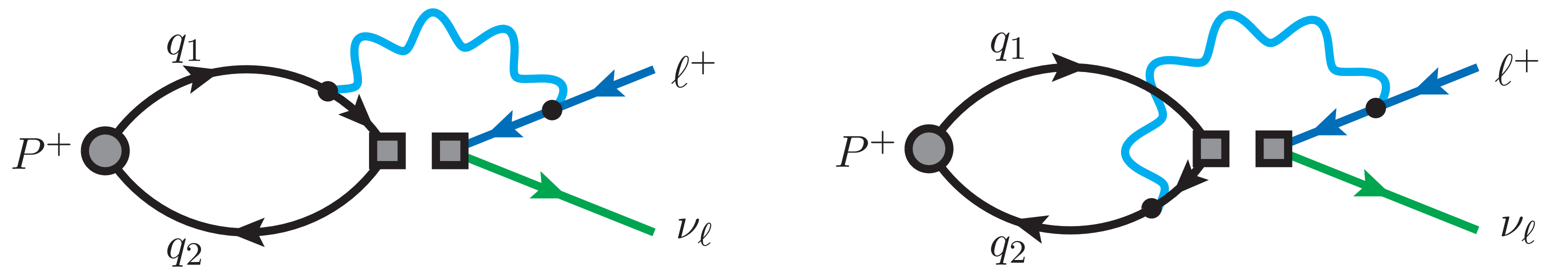
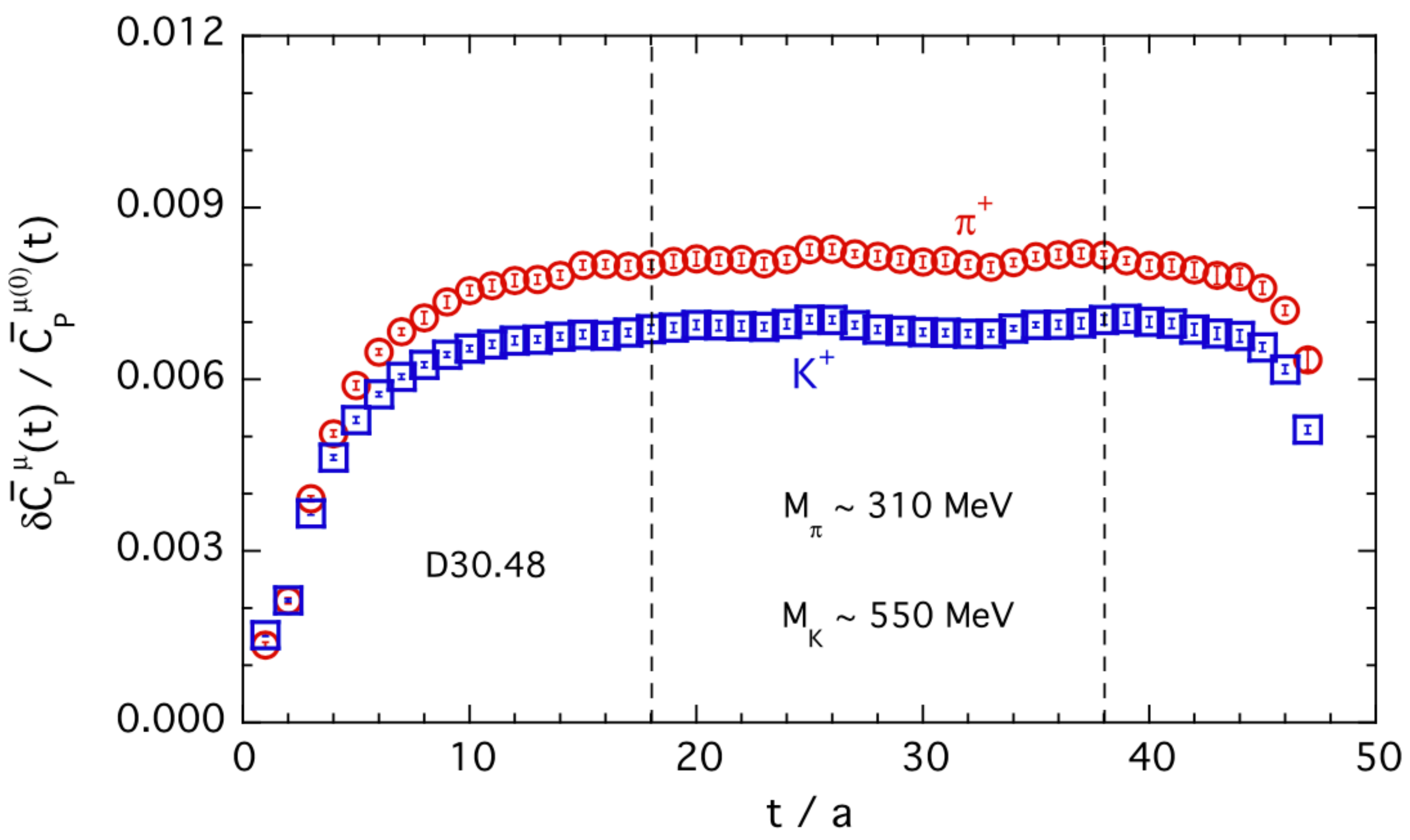
IB corrections to the decay amplitude







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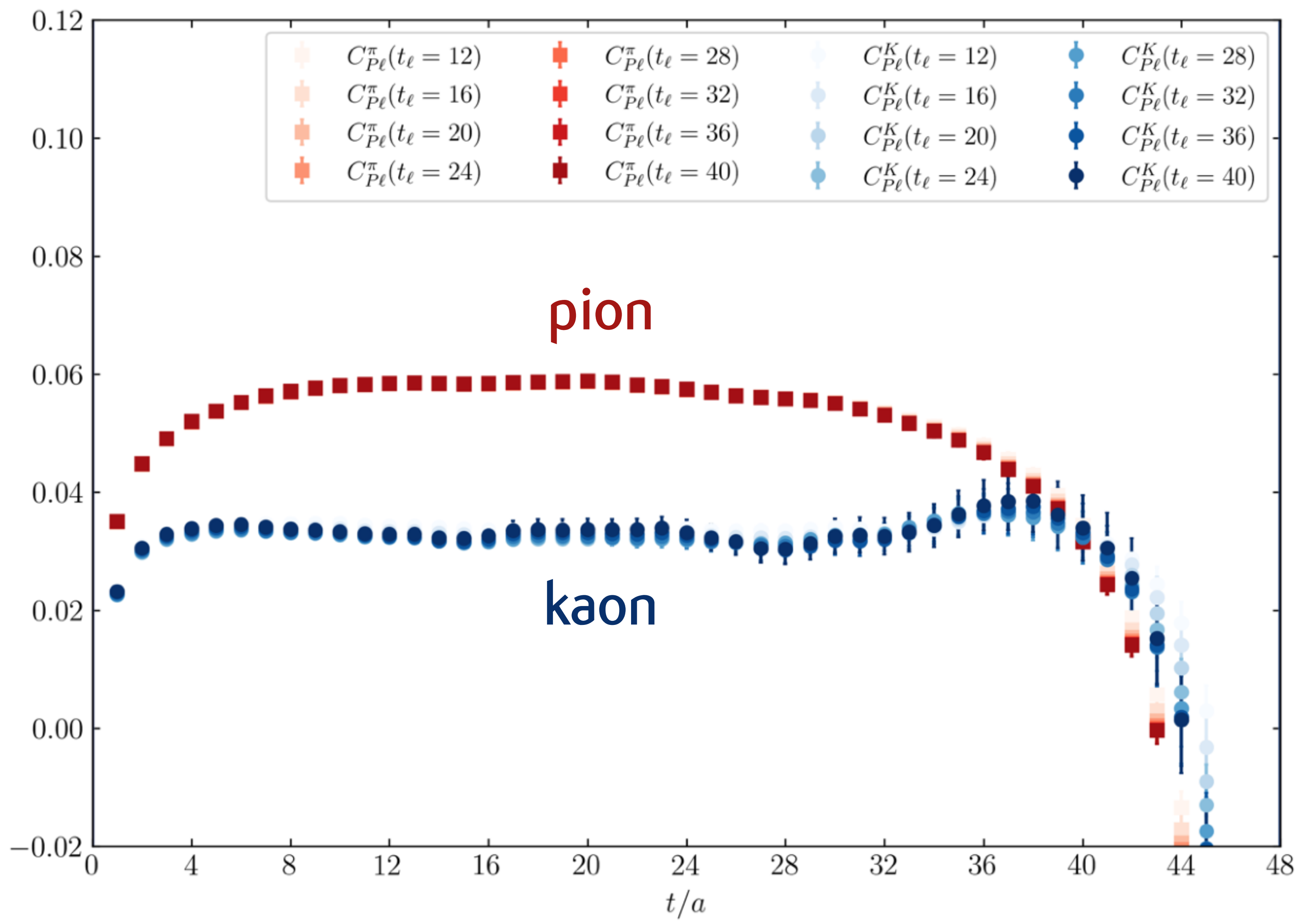

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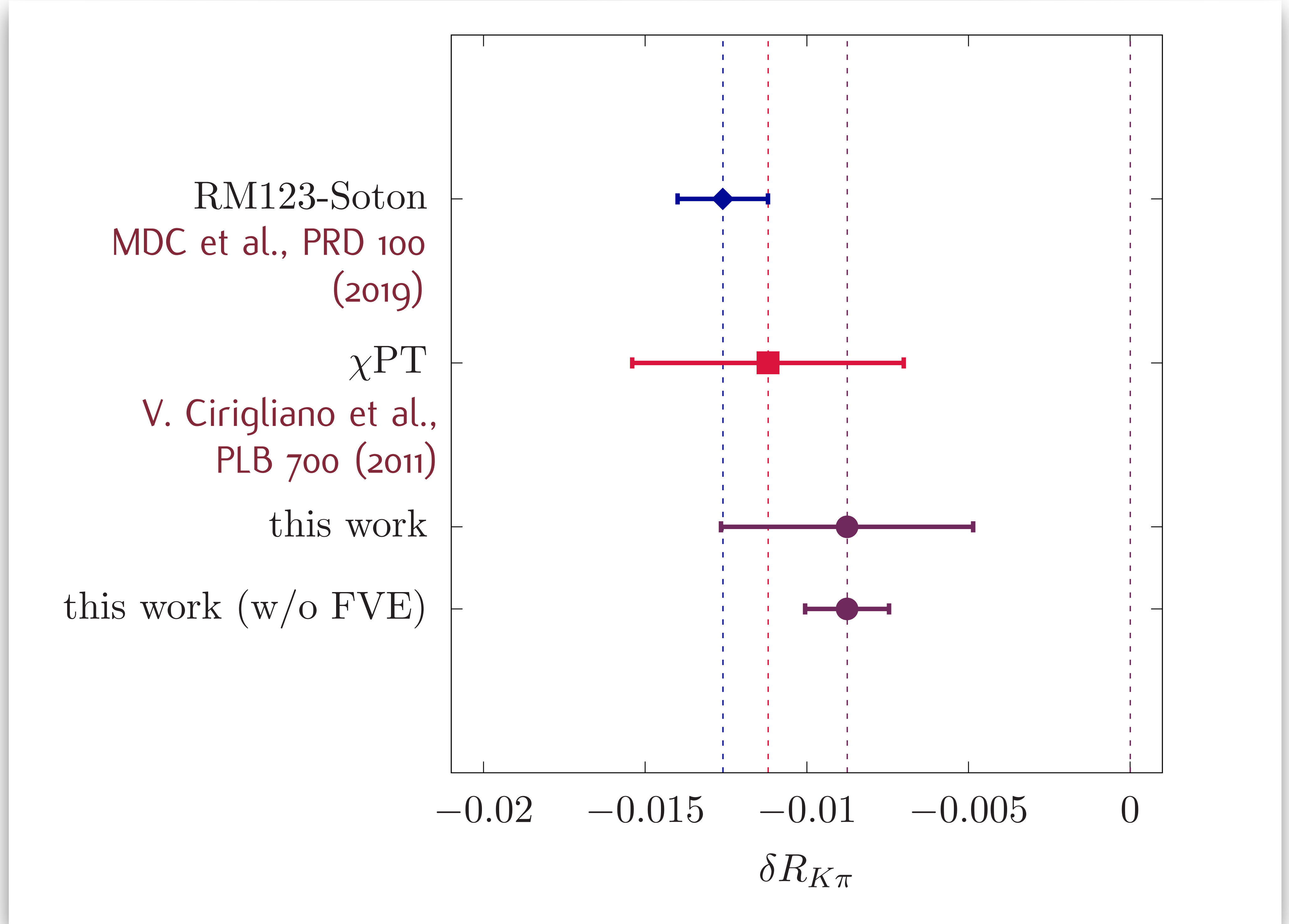
Results for $\delta R_{K\pi}$



$$\boxed{\delta R_{K\pi} = -0.0086\,(3)_{\text{stat.}}\,(^{+11}_{-4})_{\text{fit}}\,(5)_{\text{disc.}}\,(5)_{\text{quench.}}\,(39)_{\text{vol.}}}$$

RM123S: $\delta R_{K\pi} = -0.0126\,(14)$ **χPT:** $\delta R_{K\pi} = -0.0112\,(21)$

- Our new result is **compatible** with previous lattice calculation (RM123S) and with χPT
- The error is dominated by a large systematic uncertainty related to **finite-volume effects**

Solid evidence that $\delta R_{K\pi}$ can be computed from first principles non-perturbatively on the lattice!

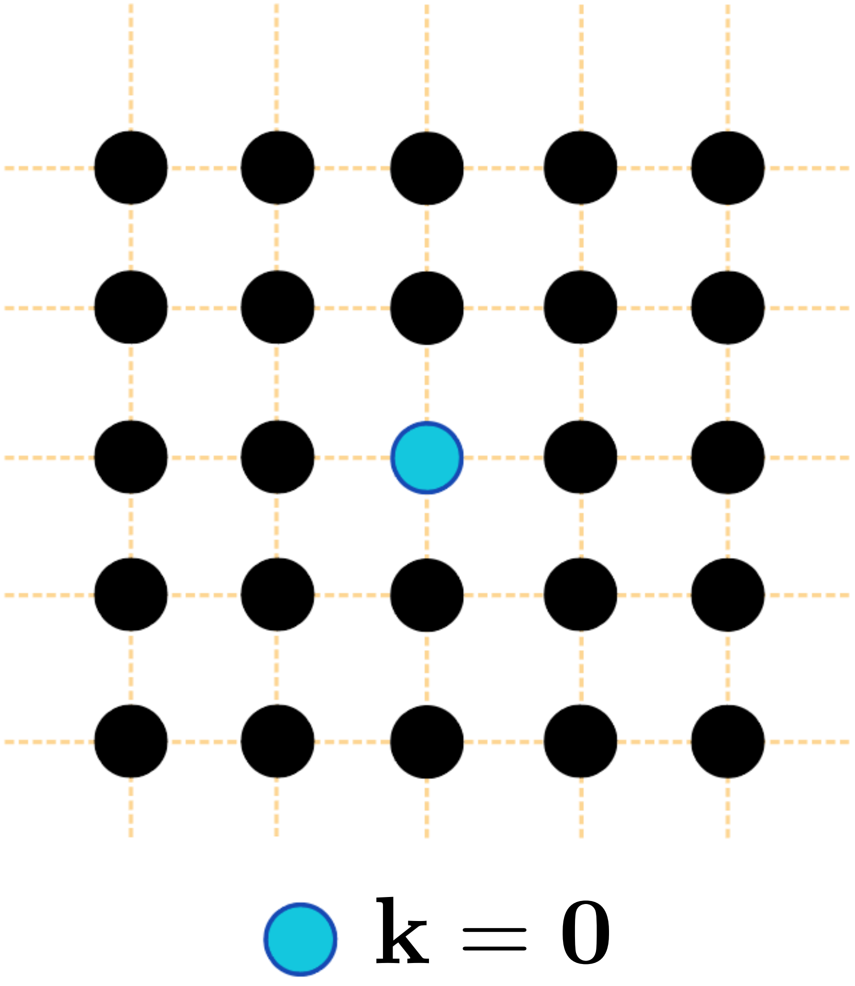
Finite-volume QED$_L$ effects

$$\Delta f(L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \right) \int \frac{\mathrm{d}k_0}{2\pi} \mathcal{F}(k_0, \mathbf{k}) = f(L) - f(\infty)$$

Including QED: photon zero modes require a regularisation

$$\Delta g(L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \right) \int \frac{\mathrm{d}k_0}{2\pi} \frac{\mathcal{G}(k_0, \mathbf{k})}{k_0^2 + |\mathbf{k}|^2}$$

$$D^{\mu\nu}(k_0, \mathbf{k}) = \delta^{\mu\nu} \frac{1}{k_0^2 + |\mathbf{k}|^2}$$



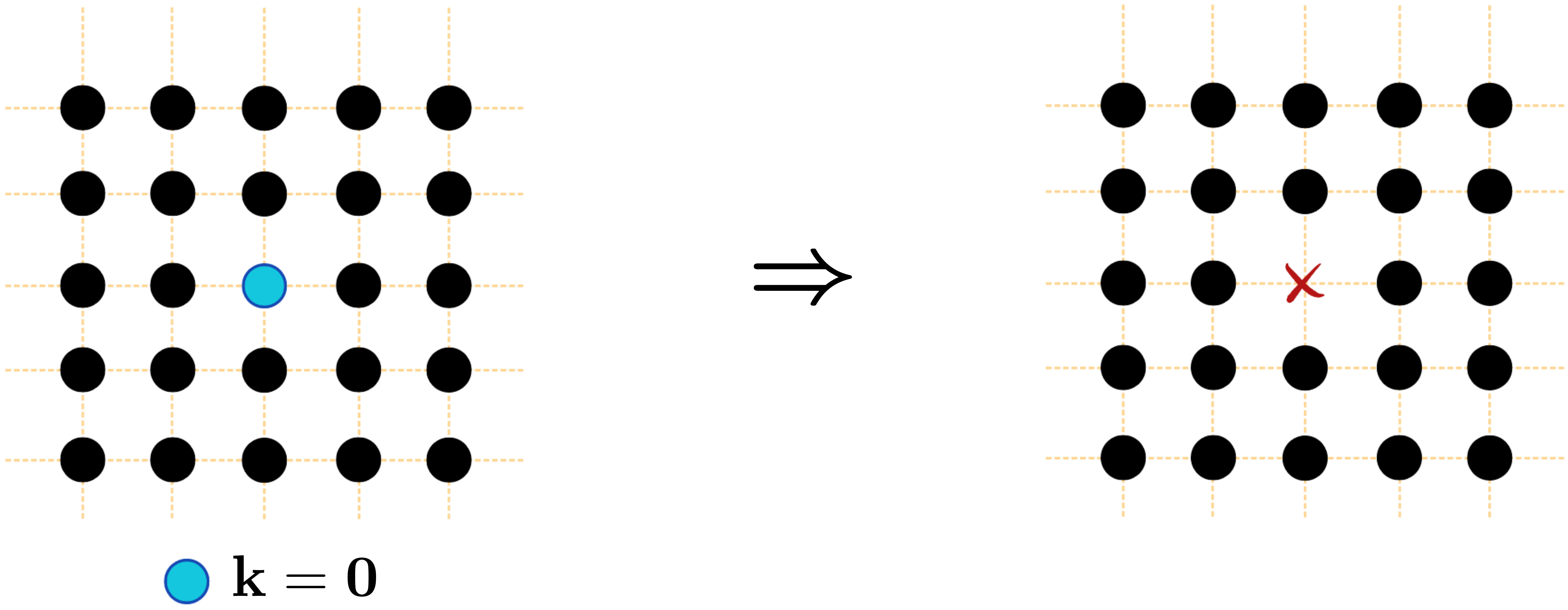
Finite-volume QED$_\mathrm{L}$ effects

$$\Delta f(L) = \left(\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)\int \frac{\mathrm{d}k_0}{2\pi}\mathcal{F}(k_0,\mathbf{k}) = f(L) - f(\infty)$$

Including QED: photon zero modes require a regularisation ⟶ QED$_\mathrm{L}$ M. Hayakawa & S. Uno, PTP 120 (2008)

$$\Delta' g(L) = \left(\frac{1}{L^3}\sum_{\mathbf{k}\neq\mathbf{0}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)\int \frac{\mathrm{d}k_0}{2\pi}\,\frac{\mathcal{G}(k_0,\mathbf{k})}{k_0^2+|\mathbf{k}|^2}$$

$$D_L^{\mu\nu}(k_0,\mathbf{k}) = \delta^{\mu\nu}\,\frac{1-\delta_{\mathbf{k},\mathbf{0}}}{k_0^2+|\mathbf{k}|^2}$$



Finite-volume QED effects

Leptonic decay rate

V. Lubicz et al., PRD 95 (2017)
N. Tantalo et al., [1612.00199v2]
MDC et al., PRD 105 (2022)

$$\Gamma_0(L) = \Gamma_0^{\rm tree}\left\{1 + 2\,\frac{\alpha}{4\pi}\,Y(L)\right\}$$

$$Y(L) = Y_{\log}(L) + Y_0 + \frac{1}{m_P L}\,Y_1 + \frac{1}{(m_P L)^2}\,Y_2 + \frac{1}{(m_P L)^3}\,Y_3^{\rm pt} + \frac{1}{(m_P L)^3}Y_3^{\rm SD} + \mathrm{O}(1/L^4) + \mathrm{O}(\mathrm{e}^{-\alpha L})$$

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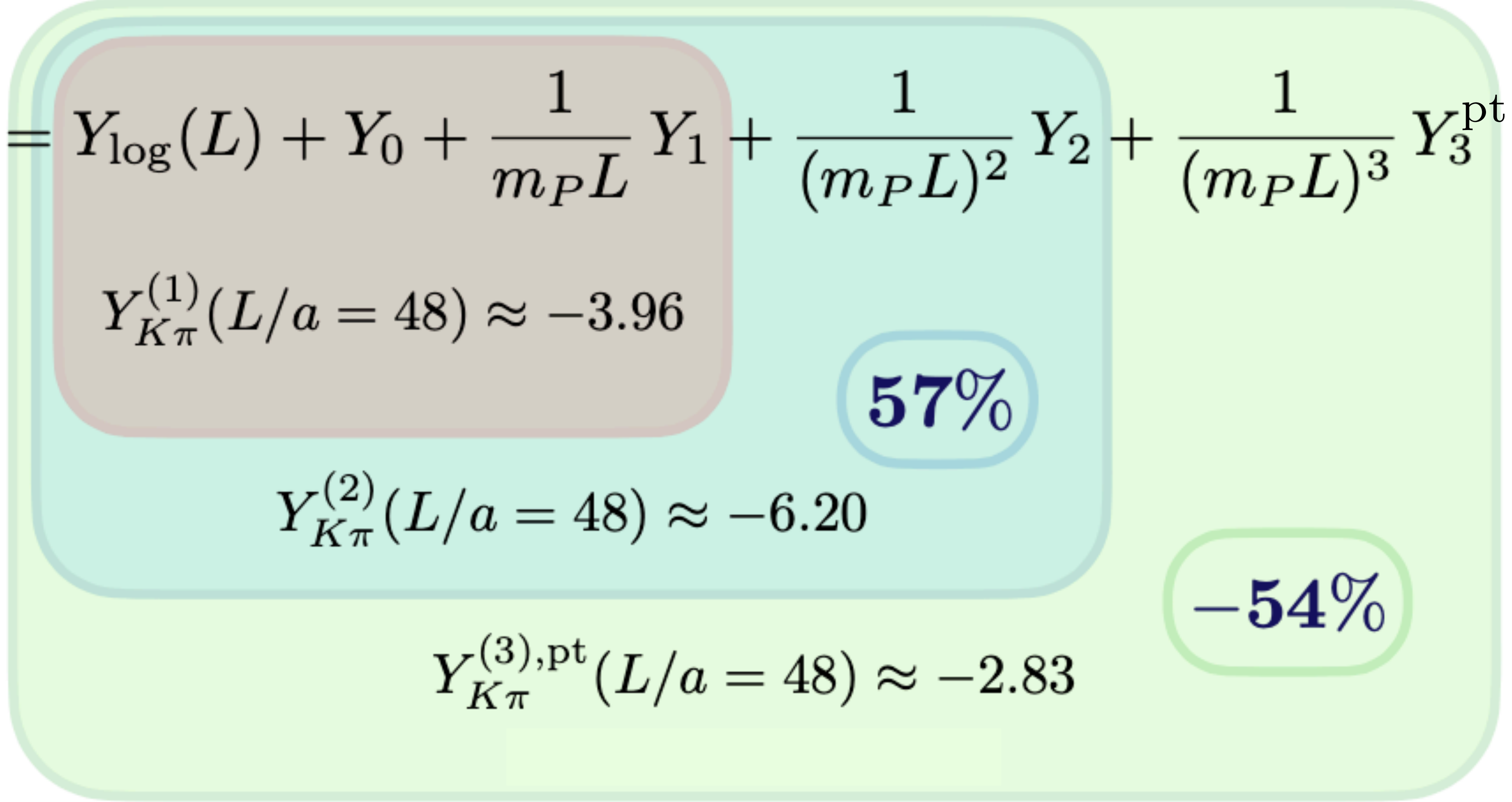

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−54%

$$Y_{K\pi}^{(3),\text{pt}}(L/a = 48) \approx -2.83$$

Finite volume scaling should be carefully studied!

Where we are and where to go?

Finite volume effects produce large **systematic uncertainty**

$$\delta R_{K\pi} = -0.0086\,(13)(39)_{\text{vol.}}$$

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$$\left(\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)$$

adopt or **develop QED formulations**
with **reduced** finite volume effects

Prospects for $|V_{us}/V_{ud}|$

An exercise on the error budget

$$\left|\frac{V_{us}}{V_{ud}}\right|^2 = \left[\frac{\Gamma(K_{\ell 2})}{\Gamma(\pi_{\ell 2})}\,\frac{M^3_{K^+}}{M^3_{\pi^+}}\,\frac{(M^2_{K^+}-M^2_{\mu^+})^2}{(M^2_{\pi^+}-M^2_{\mu^+})^2}\right]_{\rm exp}\cdot\left[\frac{f_{K,0}}{f_{\pi,0}}\right]^2(1+\delta R_{K\pi})$$

- Using our new result $\boxed{\delta R_{K\pi} = -0.0086\,(13)(39)_{\rm vol.}}$

$[f_{K,0}/f_{\pi,0}]$		$\lvert V_{us}/V_{ud}\rvert$
FLAG21 2+1 average	1.1930 (33)	$0.23154\ (28)_{\rm exp}\,(15)_{\delta R}\,(45)_{\delta R,{\rm vol.}}\,(65)_{f_P}$

- Using RM123S result $\boxed{\delta R_{K\pi} = -0.0126\,(14)}$

$[f_{K,0}/f_{\pi,0}]$		$\lvert V_{us}/V_{ud}\rvert$
FLAG19 2+1+1 average	1.1966 (18)	$0.23131\ (28)_{\rm exp}\,(17)_{\delta R}\,(35)_{f_P}$

Conclusions

- Current **tensions** in the first row CKM unitarity can be only solved (or confirmed) by a combined effort of **theory** and **experiments**

- **New results** for radiative virtual correction $\delta R_{K\pi}$ from lattice calculation with Domain Wall fermions at the physical point

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(looking forward to seeing results with different QED prescriptions: $\mathrm{QED_C}$, $\mathrm{QED_m}$, $\mathrm{QED_\infty}$)

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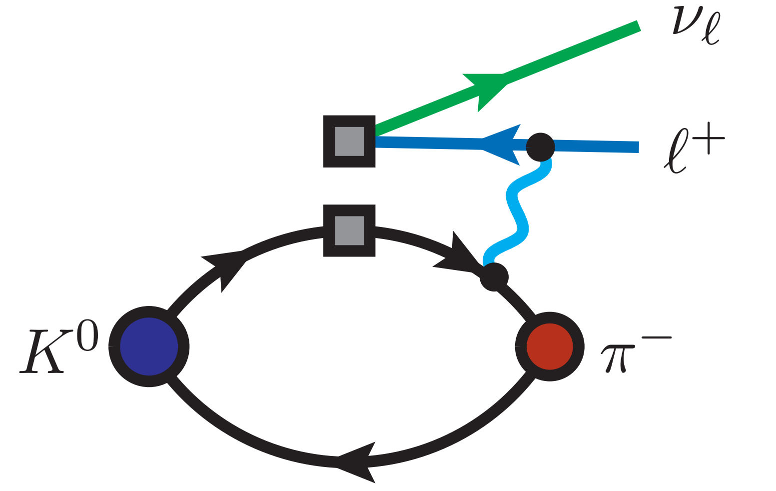
… and future prospects

$$\left(\frac{1}{L^3}\sum_{\mathbf{k}\neq\mathbf{0}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)$$

investigate & tame effects due to non-locality of $\mathrm{QED_L}$

$$\lim_{a\to 0}\ \lim_{L\to\infty}$$

repeat calculation on different ensembles



study semi-leptonic kaon decays

Thank you