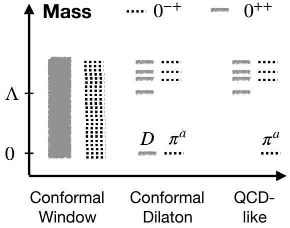
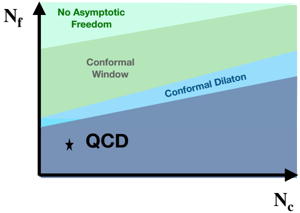
Dilaton & Scale Invariance





L Del Debbio

Higgs Centre for Theoretical Physics
University of Edinburgh

in collaboration with R Zwicky

[ldd & R Zwicky, 2010 - now]

Goldstone theorem as an introduction

- a symmetry of the action is *spontaneously* broken if

$$Q^a|0\rangle \neq 0$$

- **Goldstone theorem**: for every broken generator, there is a massless particle in the spectrum of the theory

- chiral symmetry in massless QCD

$$SU(3)_L \times SU(3)_R \to SU(3)_V \quad \Longrightarrow \quad \text{massless octet}$$

- Ward identities: GMOR

$$m_\pi^2 = 2mB + O(m^2)\,, \quad B = \left|\frac{\langle\bar{\psi}\psi\rangle}{F_\pi^2}\right|_{m=0}$$

anomalous breaking

- an *anomalous* symmetry is a symmetry of the action that is broken by quantum corrections

- $\Longrightarrow$ no massless Goldstone bosons

  example: $U(1)_A$ and Witten-Veneziano formula for the η'

- Witten-Veneziano

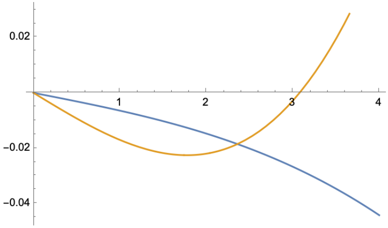
$$
\begin{aligned}
&\partial_\mu J_5^\mu(x) = 2n_f q(x) \\
&J_5^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)\,, \quad q(x) = \frac{g^2}{16\pi^2}\mathrm{Tr}\, G_{\mu\nu}(x)\tilde{G}^{\mu\nu}(x) \\
&\Longrightarrow m_{\eta'}^2 = \frac{2n_f}{F_\pi^2}\,\chi|_{\mathrm{YM}} \sim 1/N_c
\end{aligned}
$$

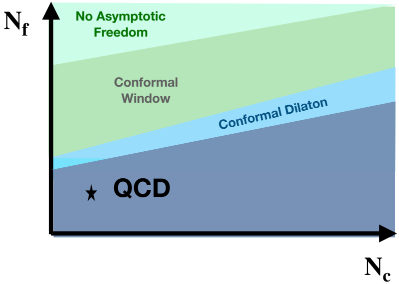
the problem with scale invariance

- like the $U(1)_A$ symmetry of QCD, scale invariance is broken, in this case by the trace anomaly
- **what is a dilaton?**
- in a QCD-like theory, no light degree of freedom associated to spontaneously broken dilatations
- **however** the effect of the anomaly “vanishes” in a theory where there is an IR fixed point

phase diagram of gauge theories

$$\beta(g) = -b_0 g^3 - b_1 g^5 + O(g^5)$$





IR conformal scaling

two-point correlator

$$C_{HH}(t;\delta g,\hat{m},\mu) = \int d\mathbf{x}\, \langle H(t,\mathbf{x}) H(0)^\dagger \rangle$$
$$= \frac{|\langle 0|H(0)|H\rangle|^2}{2m_H}\, e^{-m_H t} + \dots$$

RG transformation

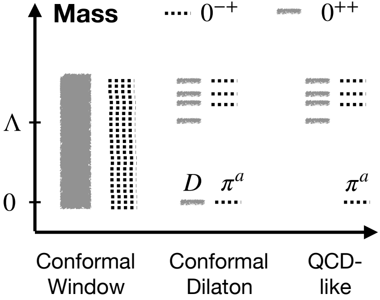
$$\mu = b\mu', \quad C_{HH}(t;\delta g,\hat{m},\mu) = b^{-2\gamma_H} C_{HH}(t; b^{y_g}\delta g, b^{y_m}\hat{m},\mu')$$

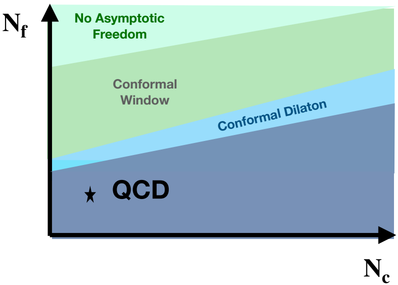
scale transformation

$$C_{HH}(t;\delta g,\hat{m},\mu) = b^{-2(d_H+\gamma_H)} C_{HH}(b^{-1}t; b^{y_g}\delta g, b^{y_m}\hat{m},\mu)$$
$$= \hat{m}^{2\Delta_H/y_m}\, F(t\hat{m}^{1/y_m})$$

$$\boxed{m_H \propto \hat{m}^{1/y_m}}$$

conformal spectrum





axial and scale invariance

- axial transformations (broken generators of chiral symmetry)

$$\delta_{a5} q(x) = i\gamma_5 \tau^a q(x)$$
$$A_\mu^a = \bar{q}(x)\gamma_5\gamma_\mu \tau^a q(x)$$

$$\partial^\mu A_\mu^a(x) = 2mP^a(x) = 2m\,\bar{q}(x) i\gamma_5 \tau^a q(x)$$

- scale transformations (broken generator of scale invariance)

$$\delta_s \phi(x) = -\left[d_\phi + x\cdot\partial\right]\phi(x)$$
$$D_\mu(x) = x^\nu T_{\mu\nu}$$

$$\partial^\mu D_\mu(x) = \mathcal{A}(x) = \frac{\beta(g)}{2g} G(x)^2 + m\left[1+\gamma(g)\right]\bar{q}(x)q(x)$$

matrix elements in the IR

on-shell contribution of the anomaly

$$\begin{aligned}
\langle 0| \frac{\beta(g)}{2g} G^2(0) |D(q)\rangle &\propto \lim_{b\to\infty} e^{E_q bt} \, \langle 0| \frac{\beta(g)}{2g} G^2(bt,\mathbf{q}) \Phi_D(0)|0\rangle \\
&\propto \lim_{b\to\infty} e^{E_q bt} \, C_{T\Phi}\left(bt,\mathbf{q};\delta g,\hat{m},\mu\right) \\
&\propto \lim_{b\to\infty} e^{E_q bt} \, C_{T\Phi}\left(t,b\mathbf{q};b^{y_g}\delta g,b^{y_m}\hat{m},\mu\right)
\end{aligned}$$

if the theory has an IR fixed point, $y_g < 0$

$$\begin{aligned}
&\lim_{b\to\infty} \frac{\beta\left(g^* + b^{y_g}\delta g\right)}{g^* + b^{y_g}\delta g} = \\
&\quad = \lim_{b\to\infty} \frac{\beta'\left(g^*\right) b^{y_g}\delta g}{g^*} \left(1 - \frac{b^{y_g}\delta g}{g^*}\right) = 0
\end{aligned}$$

conformal dilaton phase

matrix elements & decay constants

$$\Gamma^{(ab)}_{5\mu}(q) = \langle 0|A^a_\mu(0)|\pi^b(q)\rangle = iF_\pi q_\mu \delta^{ab} \ ,$$
$$\Gamma_\mu(q) \quad = \langle 0|D_\mu(0)|D(q)\rangle = iF_D q_\mu \ ,$$

ward identities in the symmetric limit

$$i^{-1} q^\mu \Gamma^{(ab)}_{5\mu}(0) = 2m \, \langle 0|P^a(0)|\pi^b(q)\rangle \ = F_\pi m_\pi^2 \delta^{ab} \stackrel{\text{sym}}{\to} 0 \ ,$$
$$i^{-1} q^\mu \Gamma_\mu(0) \quad = \langle 0|\mathcal{A}(0)|D(q)\rangle + \ldots = F_D m_D^2 \quad \stackrel{\text{sym}}{\to} 0 \ .$$

SSB: F_π, F_D finite, and $m_\pi, m_D \to 0$

conformal dilaton phase - scaling

$$C_{HH}\left(t,\mathbf{p};\hat{F}_\pi,\delta g,\hat{m},\mu\right) = \int d\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}}\,\langle 0|H(t,\mathbf{x})H(0)^\dagger|0\rangle$$

$$\begin{aligned} C_{HH}\left(t,\mathbf{0};\hat{F}_\pi,\delta g,\hat{m},\mu\right) &= \\ &= \hat{m}^{2\Delta_H/y_m}\, C_{HH}\left(\hat{m}^{1/y_m}t,\mathbf{0};\hat{m}^{-1/y_m}\hat{F}_\pi,0,1,\mu\right) \end{aligned}$$

$$\Longrightarrow \quad \boxed{m_H \propto \hat{m}^{1/y_m}\,\mathcal{F}_H\left(\hat{m}^{-1/y_m}\hat{F}_\pi\right)}$$

conjecture:

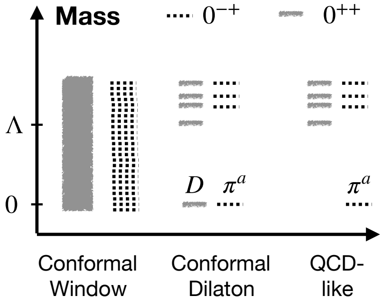
$$\lim_{x\to\infty}\mathcal{F}_H(x) = \begin{cases} \kappa & \Rightarrow \quad m_H = \mathcal{O}\left(\hat{m}^{1/y_m}\right) \\ \kappa x & \Rightarrow \quad m_H = \mathcal{O}\left(\hat{F}_\pi\right) \end{cases},$$

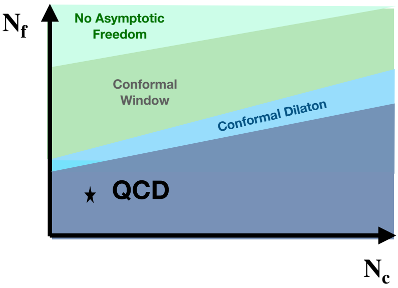
spectrum in the conformal dilaton phase

J^{PC}	Goldstone	
0^{-+}	$m_\pi = \mathcal{O}(m_q^{\frac{1}{1+\gamma_*}})$	$F_\pi = \mathcal{O}(\Lambda)$
0^{++}	$m_D = \mathcal{O}(m_q^{\frac{1}{1+\gamma_*}})$	$F_D = \mathcal{O}(\Lambda)$

J^{PC}	non-Goldstone	$(\frac{\eta_{F_{D',\pi'}}}{1+\gamma_*} \geq 1)$
0^{-+}	$m_{\pi'} = \mathcal{O}(\Lambda)$	$F_{\pi'} = \mathcal{O}(m_q^{\frac{\eta_{F_{\pi'}}}{1+\gamma_*}})$
0^{++}	$m_{D'} = \mathcal{O}(\Lambda)$	$F_{D'} = \mathcal{O}(m_q^{\frac{\eta_{F_{D'}}}{1+\gamma_*}})$

conformal dilaton spectrum





gravitational form factors

$$T_{\mu\nu}^{(\phi)}(p,p') = \langle \phi(p') | T_{\mu\nu}(0) | \phi(p) \rangle$$
$$= 2\mathcal{P}_\mu \mathcal{P}_\nu G_1(q^2) + (q_\mu q_\nu - q^2 \eta_{\mu\nu}) \frac{m_\phi^2}{q^2} G_2(q^2) \,,$$

in the presence of a fixed point

$$T^{(\phi)\mu}{}_{\mu}(p,p') = \langle \phi(p') | \mathcal{A}(0) | \phi(p) \rangle = 0$$

hence

$$2m_\phi^2 (1 - \frac{q^2}{4m_\phi^2}) G_1(q^2) - (d-1) m_\phi^2 G_2(q^2) = 0$$

conformal dilaton

$$G_2(q^2) = \frac{2}{d-1}(1 - \frac{q^2}{4m_\phi^2}) G_1(q^2) \,,\ m_\phi^2 \neq 0$$

$$\boxed{G_2(0) = \frac{2}{d-1}}$$

possible tests by lattice simulations

- compute the matrix element of $T_{\mu\nu}$ and look for the pole

$$G_2(0)\frac{m_\phi^2}{q^2}$$

- scaling of the spectrum

$$m_D \sim \hat{m}^{1/y_m}$$
$$m_\pi \sim \hat{m}^{1/y_m}$$
$$M = O(\Lambda)$$

- finite volume effects need to be under control

conclusions

- characterized a conformal dilaton phase, where scale invariance is spontaneously broken and a dilaton can exist
- importance of having an IR fixed point so that the anomalous breaking vanishes
- implementation of Ward identities consistent with a massless pole
- conformal dilaton phase is already being used to build dilatonic Higgs models