The unitarity fit and lepton flavour violation or Much Ado About Nothing

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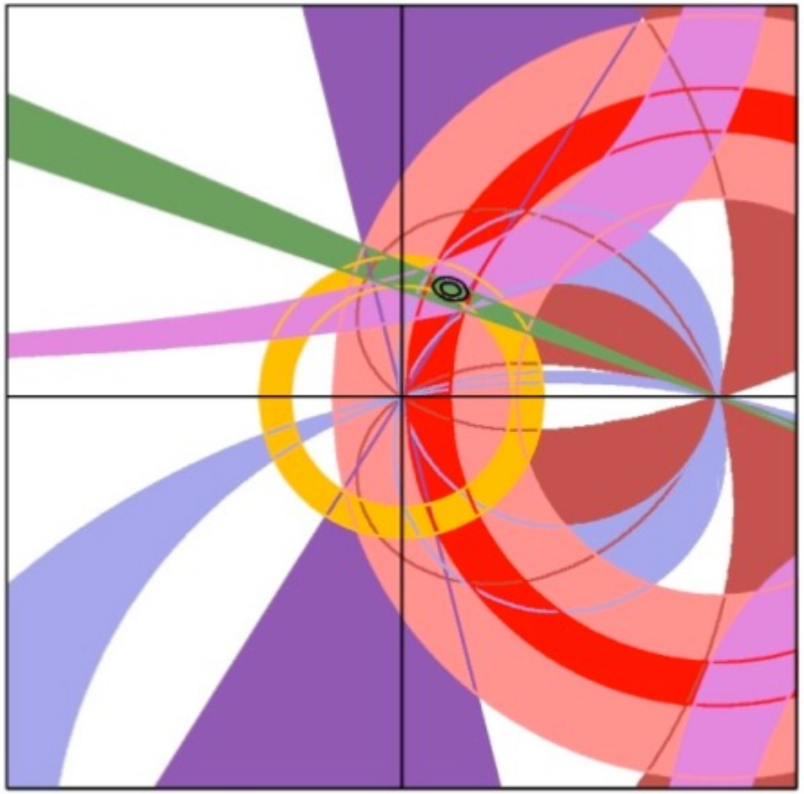
PLAN OF THE TALK



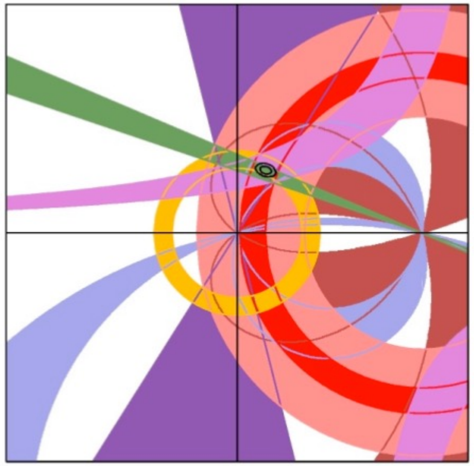
- *General introduction to the Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *Some news in lattice calculations;*
- *Future directions, new/old ideas*
- *Conclusion*

New UTfit Analysis of the Unitarity Triangle in the Cabibbo-Kobayashi-Maskawa scheme

arXiv:2212.03894v1

Thanks to M. Bona, A. Di Domenico, C. Kelly, V. Lubicz, C. Sachrajda, L. Silvestrini, S. Simula, L. Vittorio,

STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (Flavor Physics)



- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM (``direct" vs ``indirect" determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example sin 2β and Δm_s)*
- *It could lead to new discoveries (CP violation, Charm, !?)*
- *The discovery potential of precision flavor physics should not be underestimated*

Flavour Physics

1963: *Cabibbo Angle*
1964: ***CP violation in K decays*** *
1970 ***GIM Mechanism***
1973: ***CP Violation needs at least three quark families (CKM)*** *
1975: ***discovery of the tau lepton – 3rd lepton family*** *
1977: ***discovery of the b quark - 3rd quark family*** *
2003/4: ***CP violation in B meson decays***

*** Nobel Prize***

The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavor physics requires the control of hadronic effects for which lattice QCD simulations are essential

$$Q^{EXP} = V_{CKM}\langle F|\hat{O}|I\rangle$$

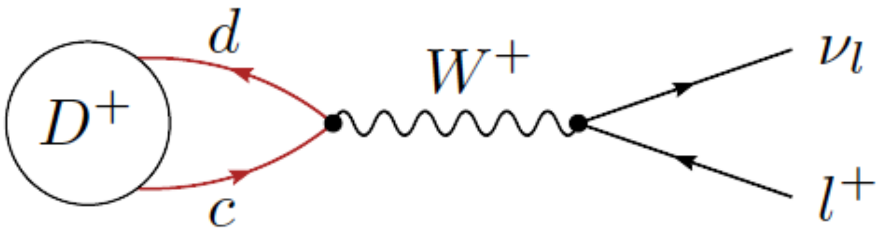
SM

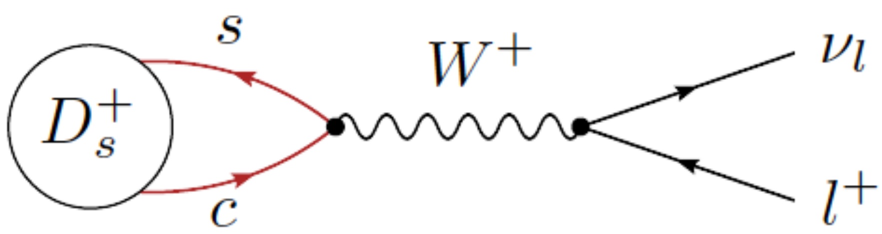
$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s)\langle F|\hat{O}_i|I\rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s)\langle F|\hat{O}_{i'}|I\rangle$$

BSM

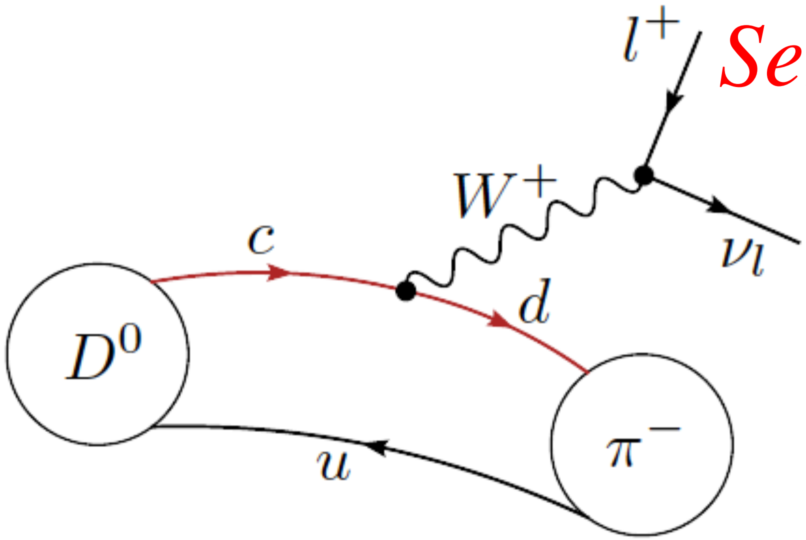
What can be computed and what cannot be computed

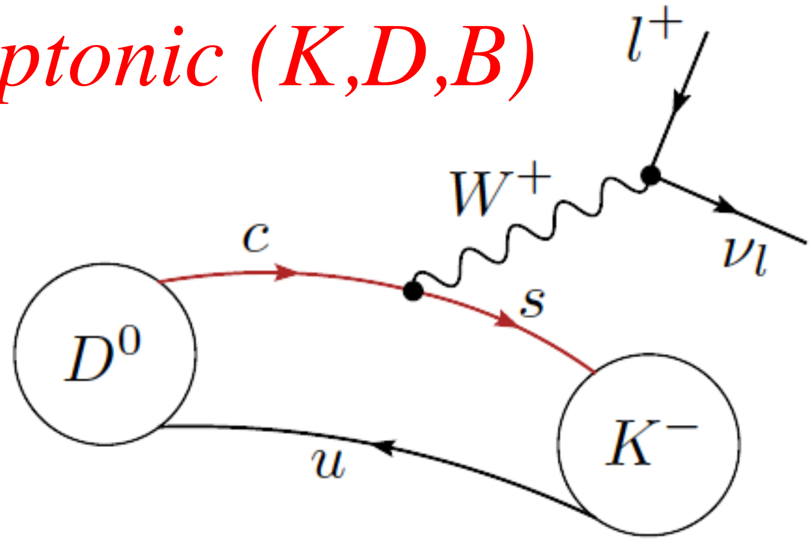
Leptonic (π,K,D,B)





Semileptonic (K,D,B)

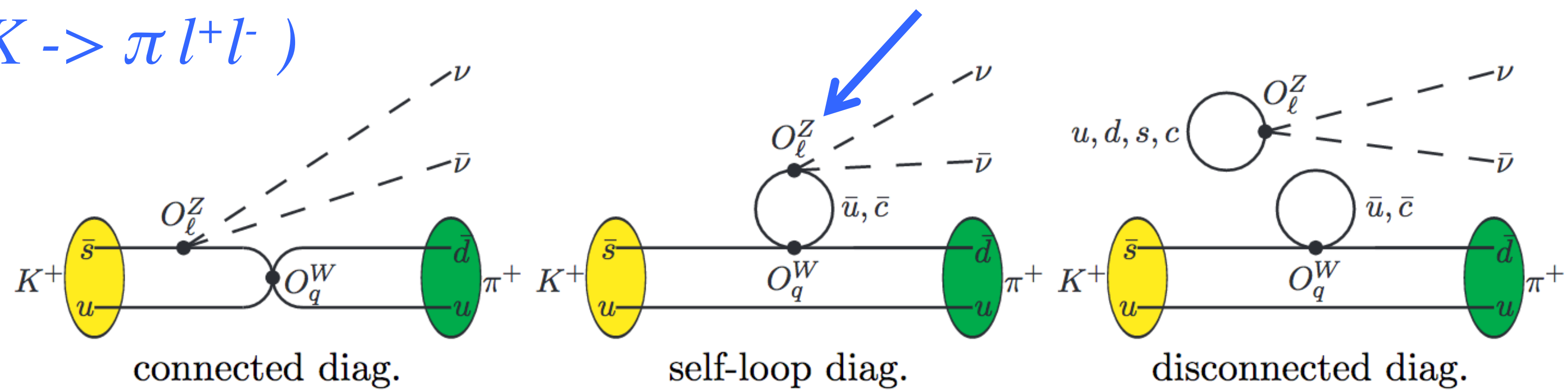




(some) Radiative and Rare

long distance effects

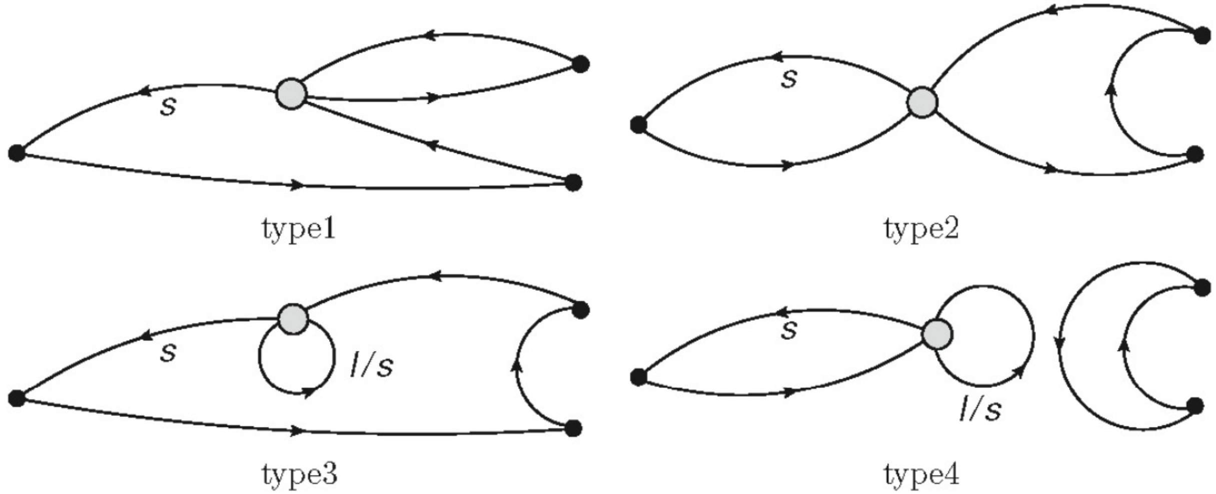
(also $K \to \pi\, l^+ l^-$)



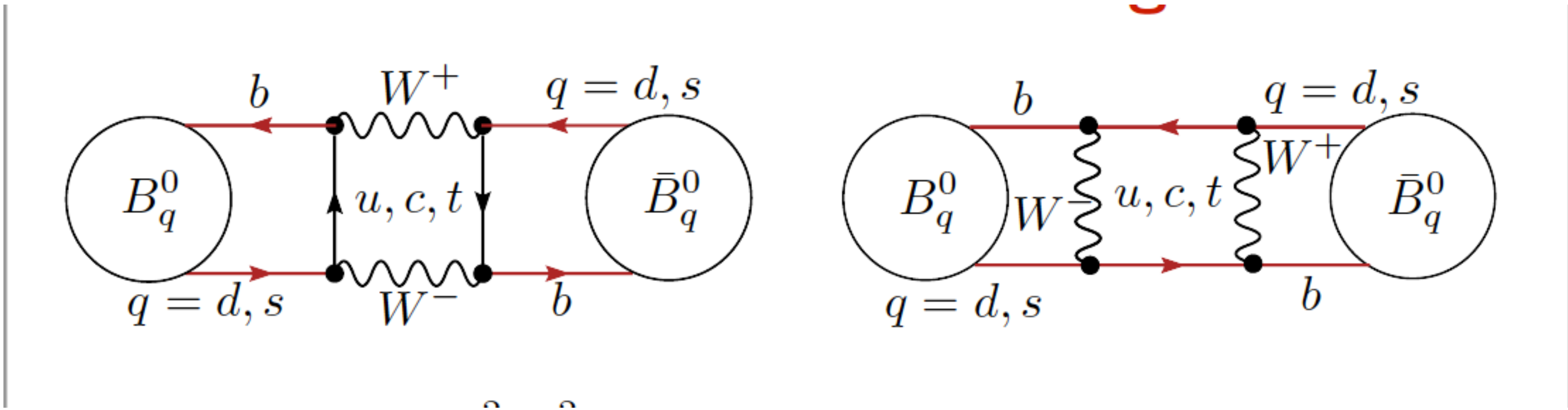
Non-leptonic

but only below the inelastic threshold (may be also 3 body decays)

$B \to \pi\pi, K\pi$, etc. No !



Neutral meson mixing (local)



+ some long distance contributions to K and D neutral meson mixing + short distance contributions to $B \to K^{(*)} l^+ l^-$

Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and ~~CP~~ violation originate, is determined by the coupling of the Higgs boson to fermions.

$$\mathcal{L}^{\text{quarks}} = \mathcal{L}^{\text{kinetic}} + \mathcal{L}^{\text{gauge}} + \mathcal{L}^{Yukawa}$$

CP invariant

~~CP~~ and symmetry breaking are striclty correlated

$$\mathcal{L}(\Lambda_{Fermi}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{kin} + \mathcal{L}^{gauge}_{SM} + \mathcal{L}^{Yukawa}_{SM} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \ldots$$

EWSB

has many accidental simmetries

may violate accidental simmetries

Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)

Almost no CP violation at tree level

Flavour Physics is extremely sensitive to New Physics (NP)

In competition with Electroweak Precision Measurements

WHY RARE DECAYS ?

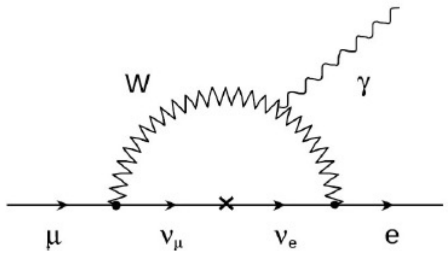
Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

Proton decay — **baryon and lepton number conservation**

$\mu \rightarrow e + \gamma$

lepton flavor number

$\nu_i \rightarrow \nu_k$ **found !**



$$\mathcal{B}(\mu \to e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$q_i \rightarrow q_k + \nu \bar{\nu}$

$q_i \rightarrow q_k + l^+ l^-$

$q_i \rightarrow q_k + \gamma$

these decays occur only via loops because of GIM and are suppressed by CKM

THUS THEY ARE SENSITIVE TO NEW PHYSICS

CP Violation in the Standard Model

After the diagonalisation of the quark mass matrix

$$L_{CC}^{weak\,int} = \frac{g_W}{\sqrt{2}}\left(J_\mu^- W_\mu^+ + h.c.\right)$$

$$\rightarrow \quad \frac{g_W}{\sqrt{2}}\left(\bar{u}_L \mathbf{V}^{CKM} \gamma_\mu d_L W_\mu^+ + \ldots\right)$$

N(N-1)/2 angles and (N-1)(N-2) /2 phases

N=3 3 angles + 1 phase KM

the phase generates complex couplings i.e. CP violation;

6 masses +3 angles +1 phase = 10 parameters

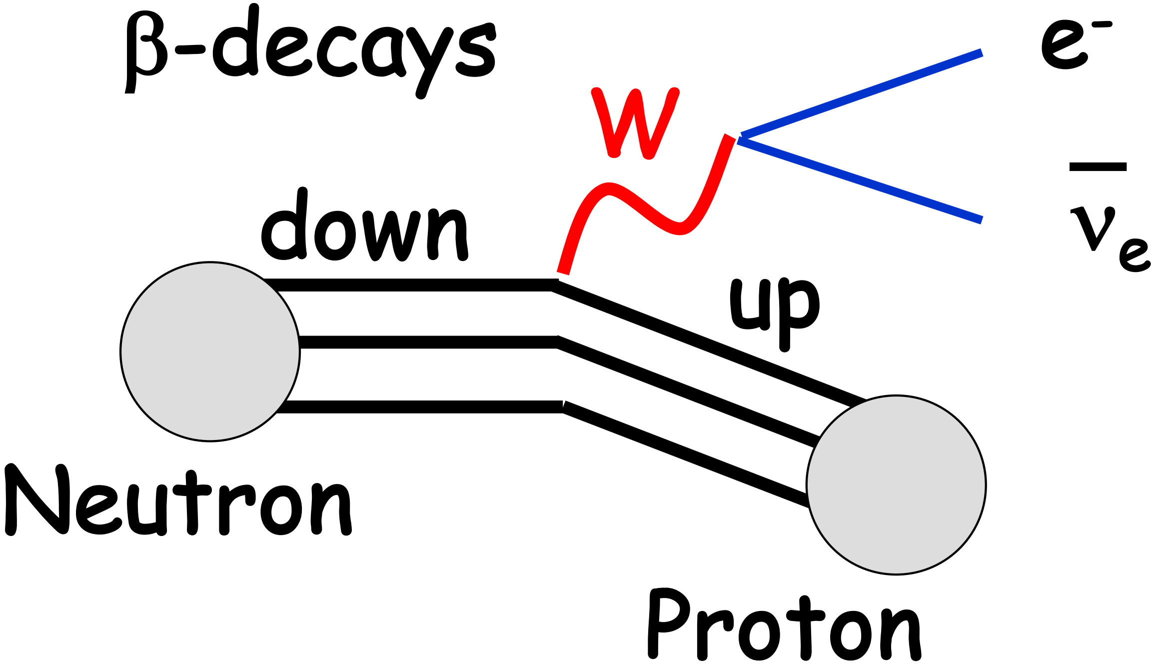
V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{tb}	V_{ts}	V_{tb}

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

Quark masses & Generation Mixing

β-decays



$|V_{ud}|$

$|V_{ud}| = 0.9735(8)$

$|V_{us}| = 0.2196(23)$

$|V_{cd}| = 0.224(16)$

$|V_{cs}| = 0.970(9)(70)$

$|V_{cb}| = 0.0406(8)$

$|V_{ub}| = 0.00409(25)$

$|V_{tb}| = 0.99(29)$

(0.999)

updated values later

The Wolfenstein Parametrization

$1 - 1/2\,\lambda^2$	λ	$A\,\lambda^3(\rho - i\,\eta)$
$-\lambda$	$1 - 1/2\,\lambda^2$	$A\,\lambda^2$
$A\,\lambda^3 \times (1 - \rho - i\,\eta)$	$-A\,\lambda^2$	1

V_{ub}

$+\ O(\lambda^4)$

V_{td}

$\lambda \sim 0.2 \quad A \sim 0.8$

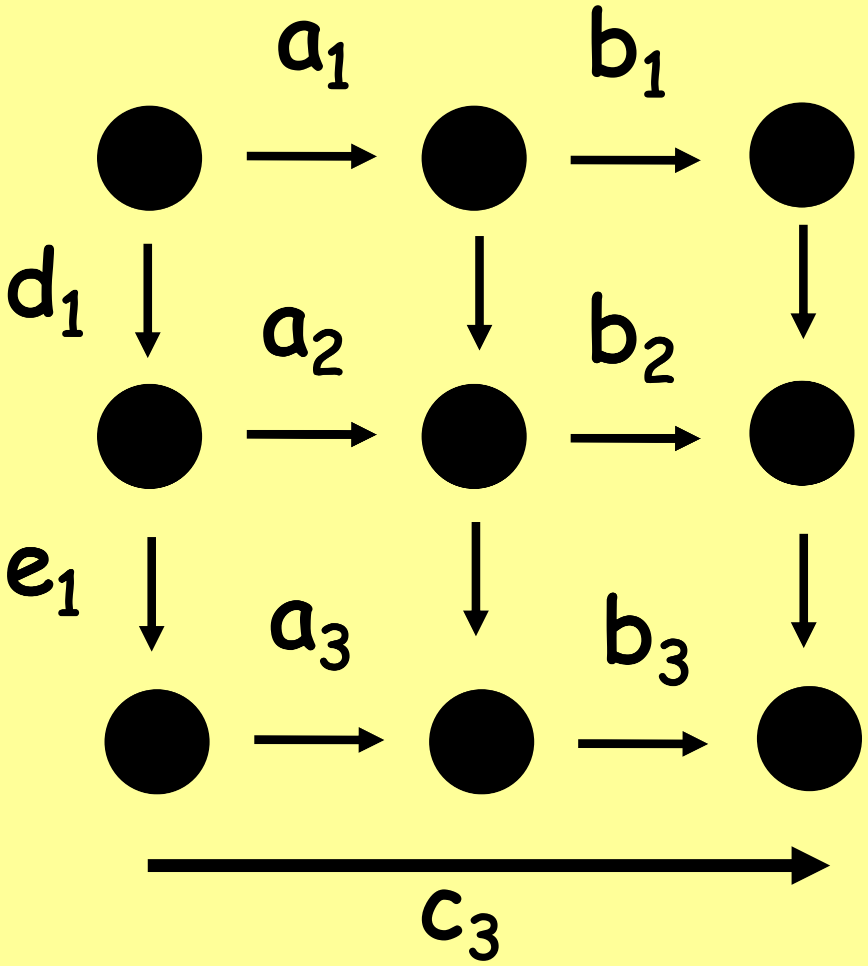
$\eta \sim 0.2 \quad \rho \sim 0.3$

$\sin\theta_{12} = \lambda$

$\sin\theta_{23} = A\,\lambda^2$

$\sin\theta_{13} = A\,\lambda^3(\rho - i\,\eta)$

The Bjorken-Jarlskog Unitarity Triangle



| V_{ij} | is invariant under phase rotations

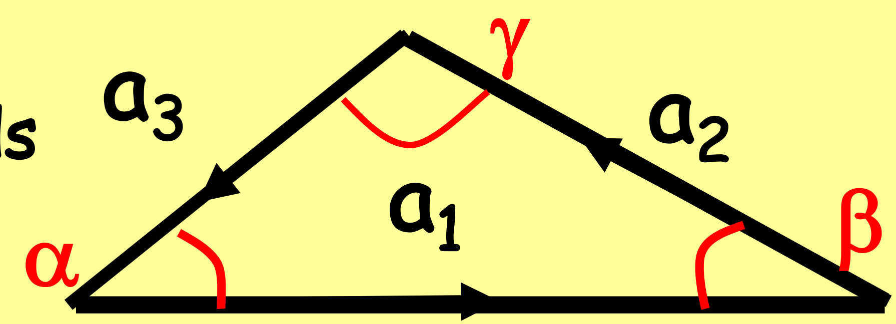
$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$

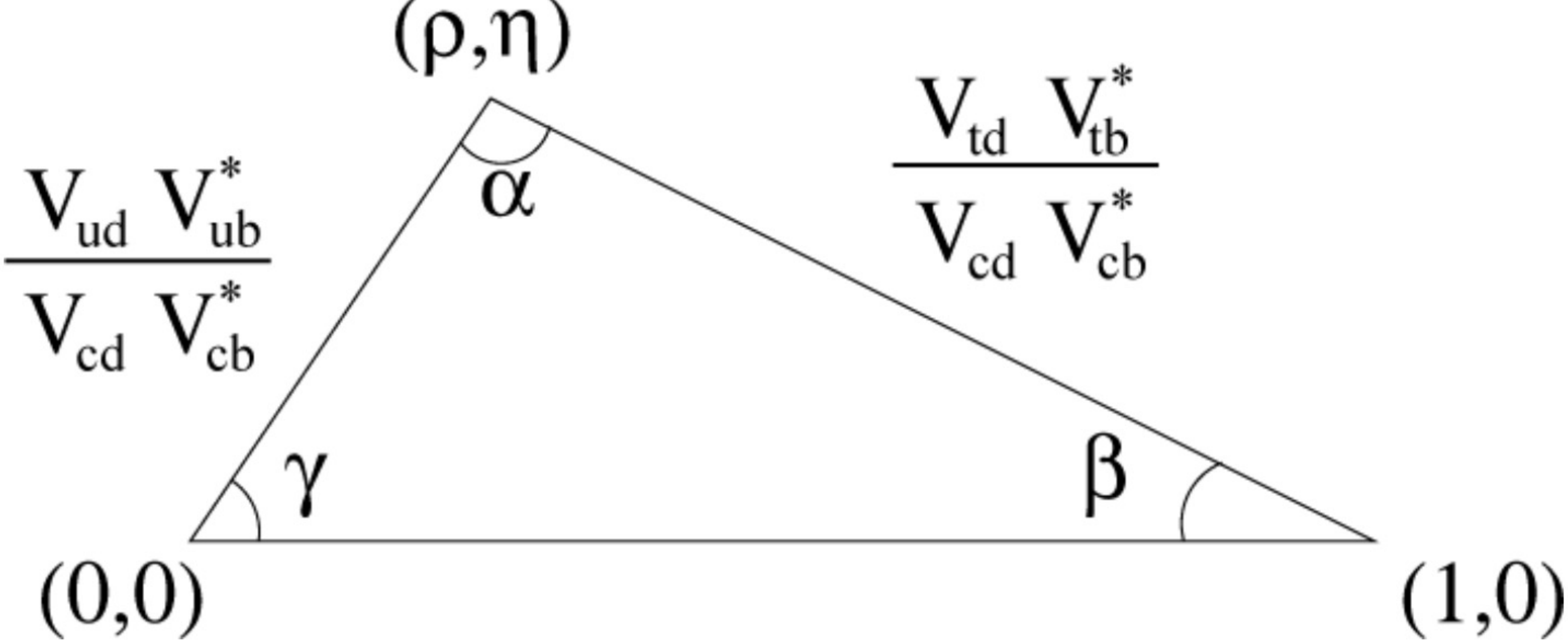
$a_2 = V_{21} V_{22}^*$ $a_3 = V_{31} V_{32}^*$

$a_1 + a_2 + a_3 = 0$

$(b_1 + b_2 + b_3 = 0$ etc.)

Only the orientation depends on the phase convention





The Standard Triangle of the Standard Model

STRONG CP VIOLATION

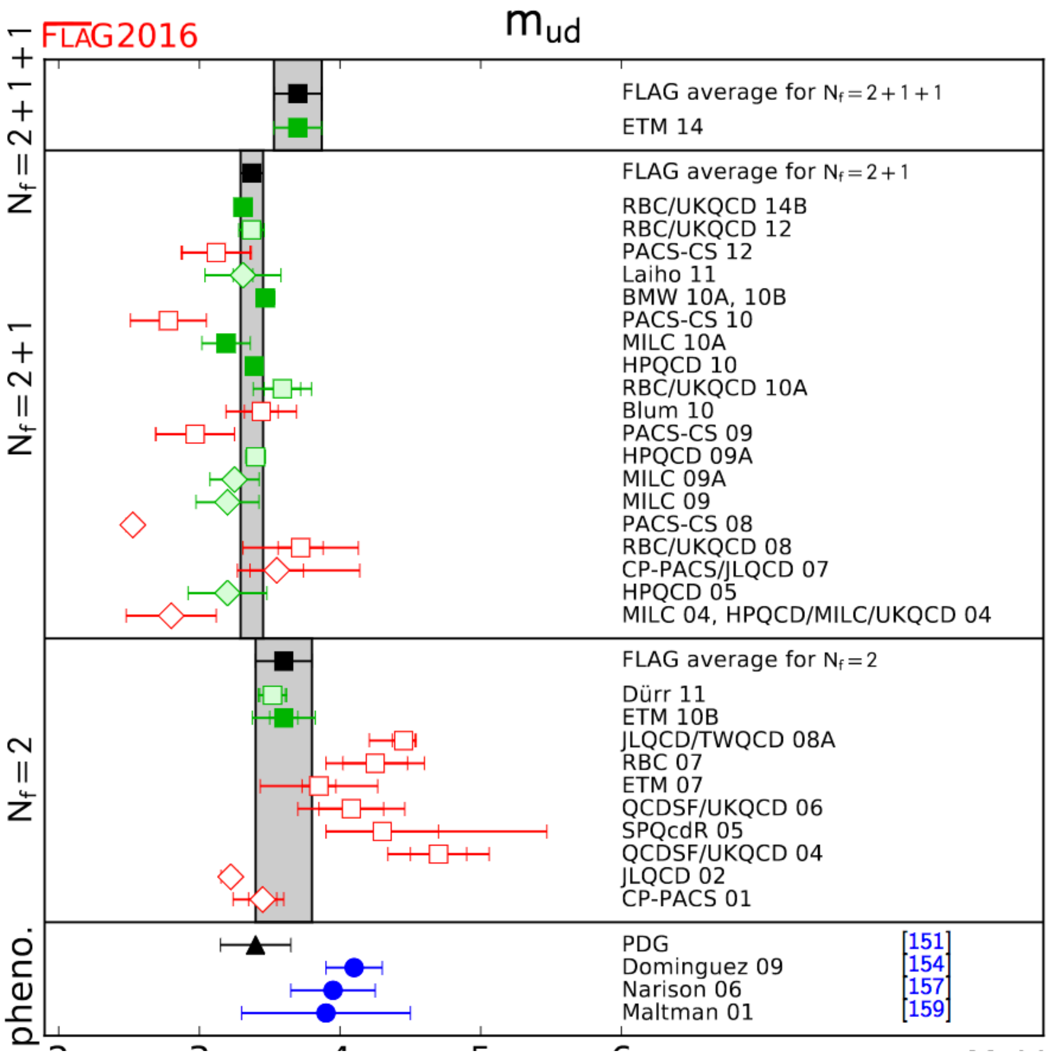
$$\mathcal{L}_\theta = \theta \, \tilde{G}^{\mu\nu a} G^a{}_{\mu\nu} \qquad \tilde{G}^a{}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^a{}_{\rho\sigma}$$

$$\mathcal{L}_\theta \sim \theta \, \vec{E}^a \cdot \vec{B}^a$$

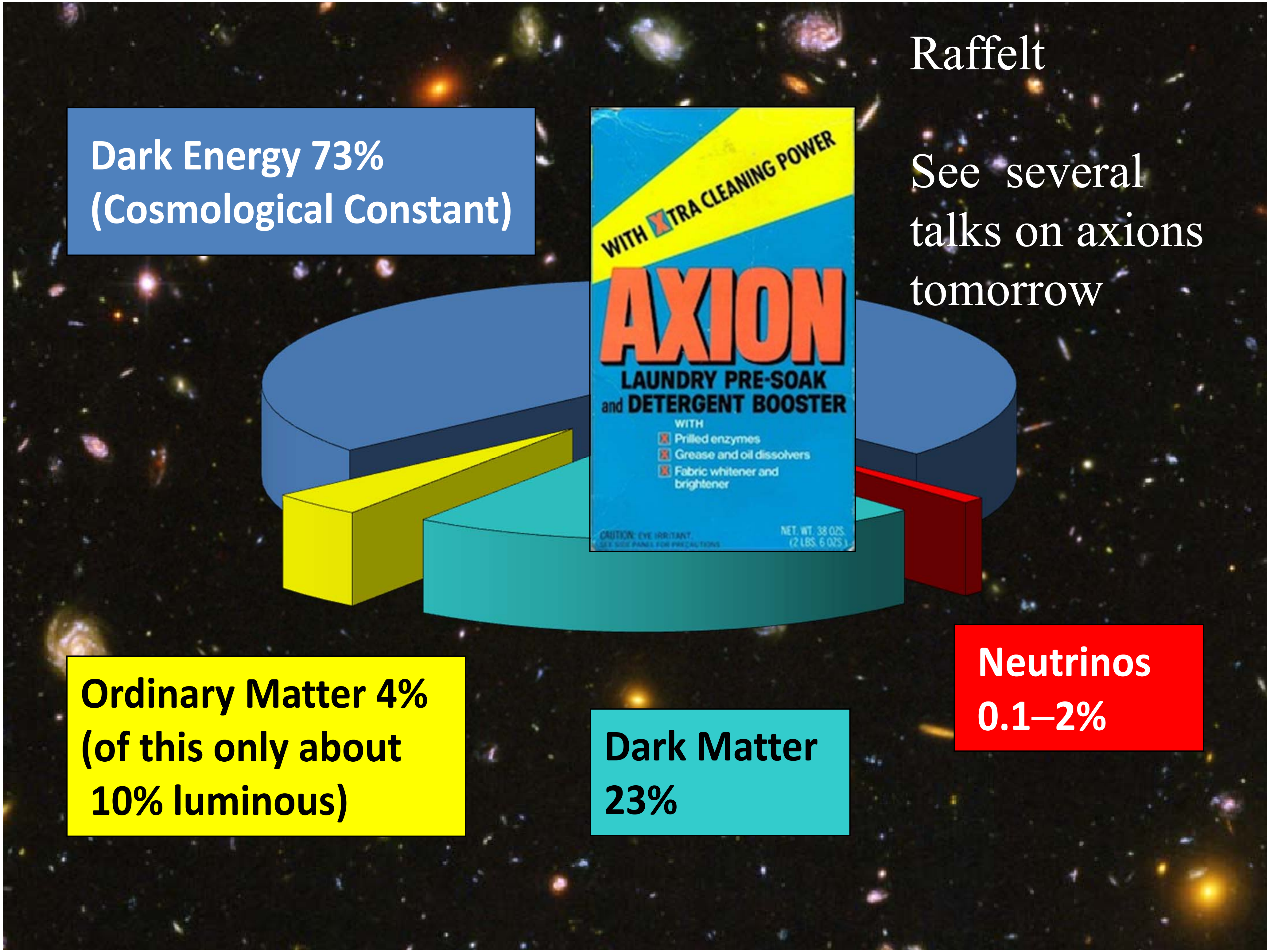
This term violates CP and gives a contribution to the electric dipole moment of the neutron

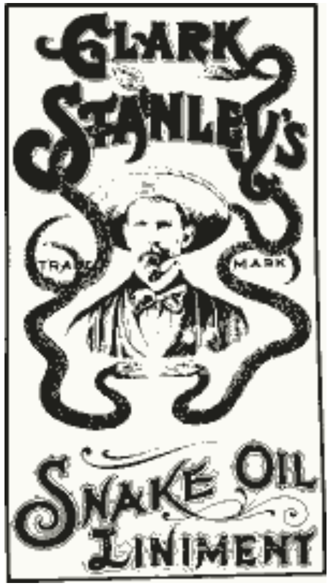
$$e_n < 3 \;\; 10^{-26} \text{ e cm}$$

$\theta < 10^{-10}$ which is quite unnatural !!



N_f	m_u	m_d	m_u/m_d	R	Q
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)


Raffelt
See several talks on axions tomorrow
Dark Energy 73%
(Cosmological Constant)
WITH XTRA CLEANING POWER
AXION
LAUNDRY PRE-SOAK
and DETERGENT BOOSTER
WITH
Prilled enzymes
Grease and oil dissolvers
Fabric whitener and brightener
NET. WT. 38 OZS.
(2 LBS. 6 OZS.)
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%



UTfit

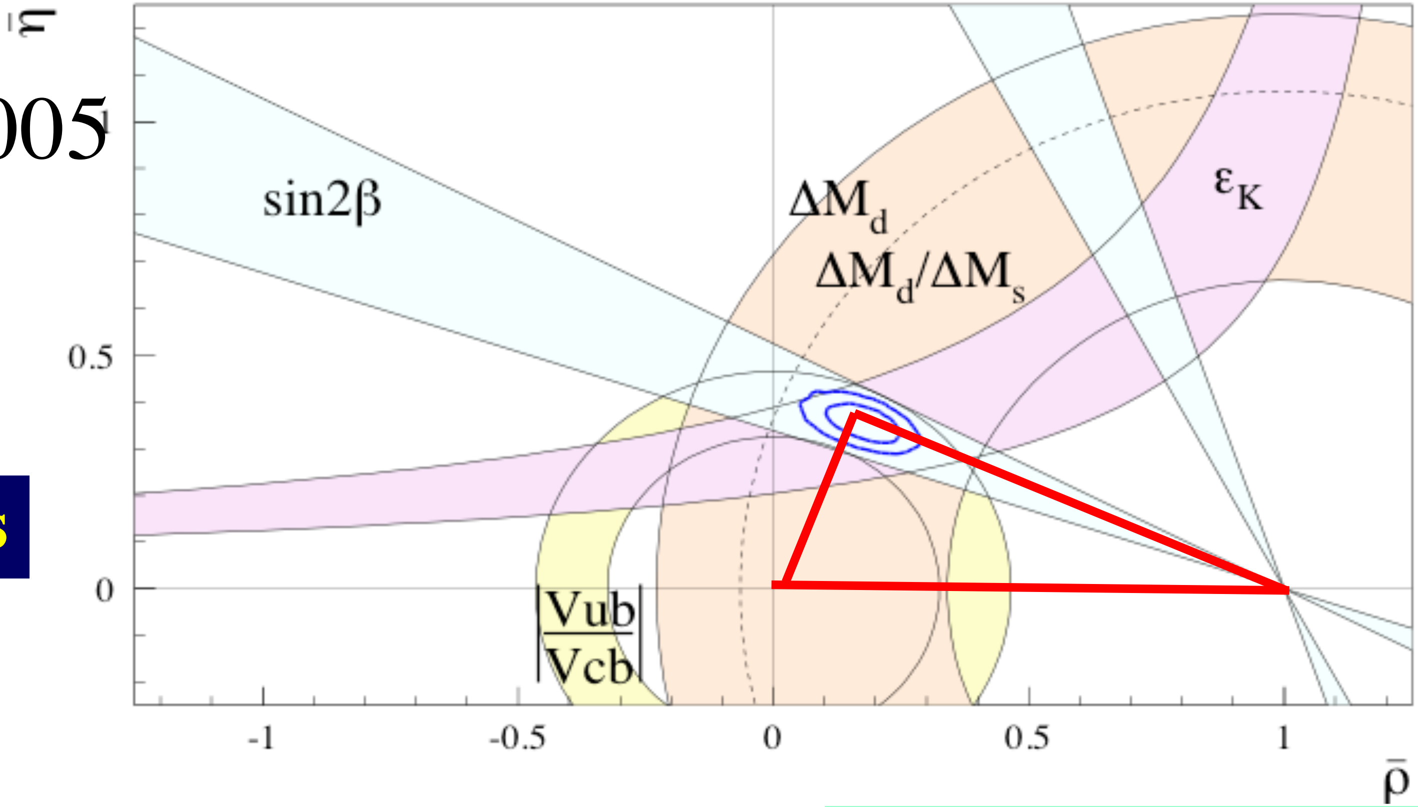
www.utfit.org

RELOADED

M.Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco, V. Lubicz, G. Martinelli, D. Morgante, M. Pierini, L. Silvestrini, S. Simula, C. Tarantino, V. Vagnoni, M. Valli, and L.Vittorio

Unitary Triangle SM

2005



semileptonic decays

Experimental co[…]

Meas.	V_{CKM} ×other	$(\bar{\rho}, \bar{\eta})$
$\frac{b\to u}{b\to c}$	$\|V_{ub}/V_{cb}\|^2$	$\bar{\rho}^2+\bar{\eta}^2$
Δm_d	$\|V_{td}\|^2 f_{B_d}^2 B_{B_d}$	$(1-\bar{\rho})^2+\bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\|\frac{V_{td}}{V_{ts}}\right\|^2 \xi^2$	$(1-\bar{\rho})^2+\bar{\eta}^2$
ϵ_K	$f(A,\bar{\eta},\bar{\rho},B_K)$	$\propto \bar{\eta}(1-\bar{\rho})$
$A(J/\psi K^0)$	$sin2\beta$	$\frac{2\bar{\eta}(1-\bar{\rho})}{\sqrt{\bar{\eta}^2+(1-\bar{\rho})^2}}$

K^0 - $\overline{K}^0$ mixing

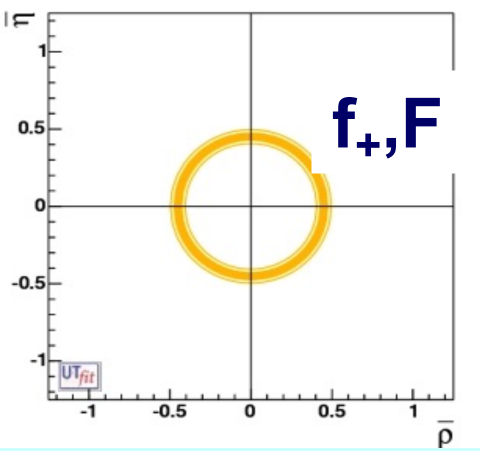
B_d

$B^0_{d,s}$ - $\overline{B}^0_{d,s}$ mixing

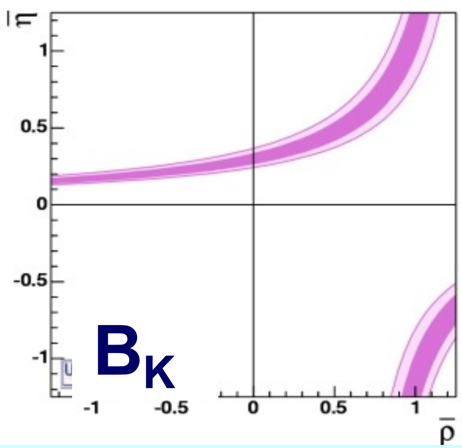
Quantities used in the Standard UT Analysis

levels @
68% (95%) CL

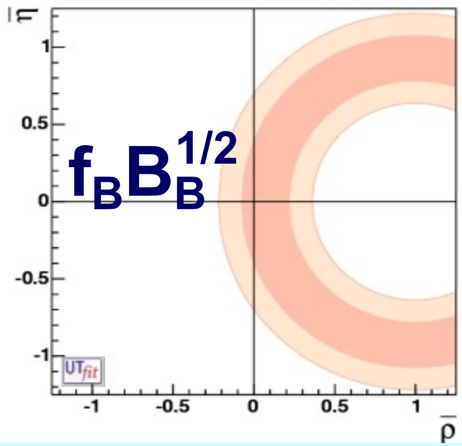
V_{ub}/V_{cb}



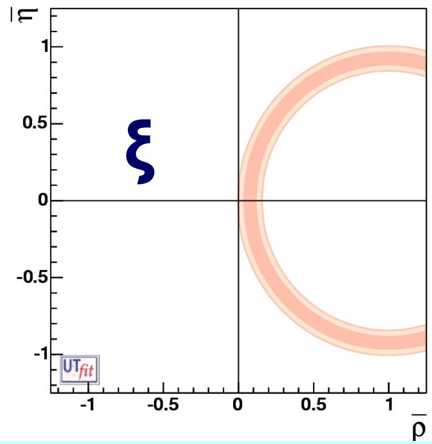
ε_K



Δm_d



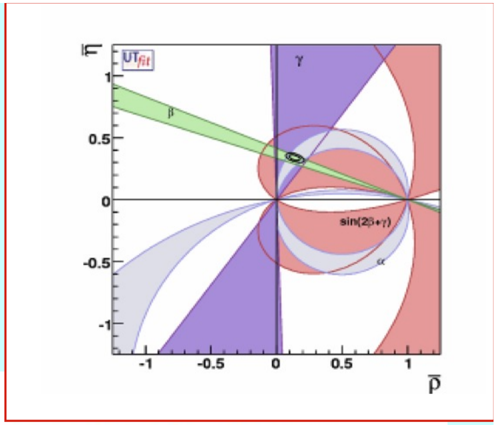
$\Delta m_d/\Delta m_s$



Inclusive vs Exclusive
Opportunity for lattice QCD

UT-LATTICE

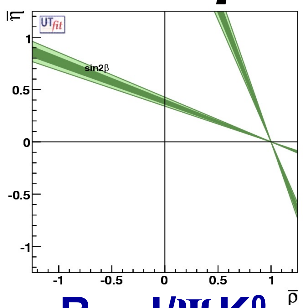
Other Quantities used in the UT Analysis



UT-ANGLES

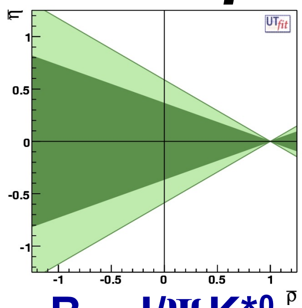
Several new determinations of UT angles are now available, thanks to the results coming from the B-Factory experiments

$\sin 2\beta$



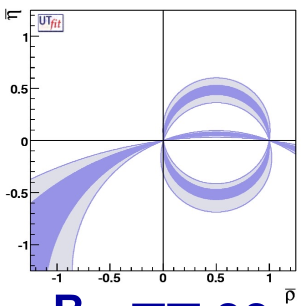
$B \to J/\Psi K^0$

$\cos 2\beta$



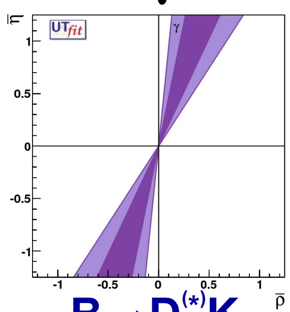
$B \to J/\Psi K^{*0}$

α



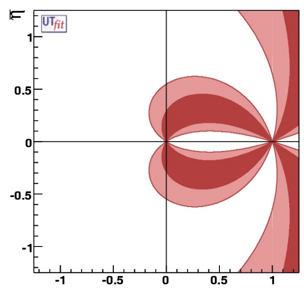
$B \to \pi\pi, \rho\rho$

γ



$B \to D^{(*)}K$

$\sin(2\beta+\gamma)$

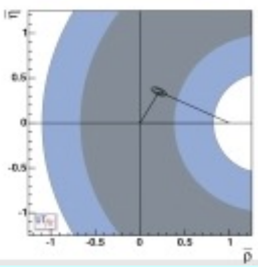


$B \to D^{(*)}\pi, D\rho$

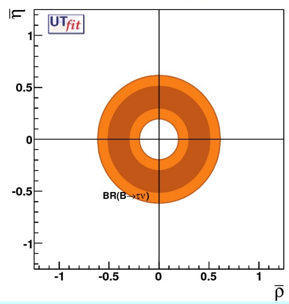
New Constraints from B and K rare decays (not used yet)

New bounds are available from rare B and K decays. They do not still have a strong impact on the global fit and they are not used at present.

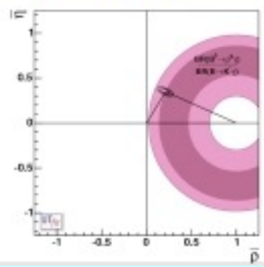
$K \to \pi \nu \bar{\nu}$

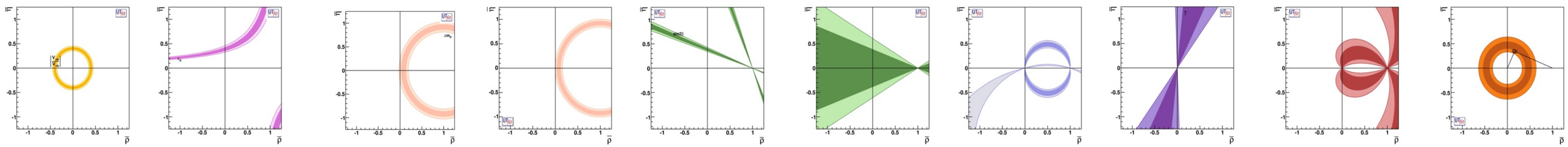


$B \to \tau \nu$



$(B \to \rho/\omega\, \gamma)/(B \to K^*\gamma)$

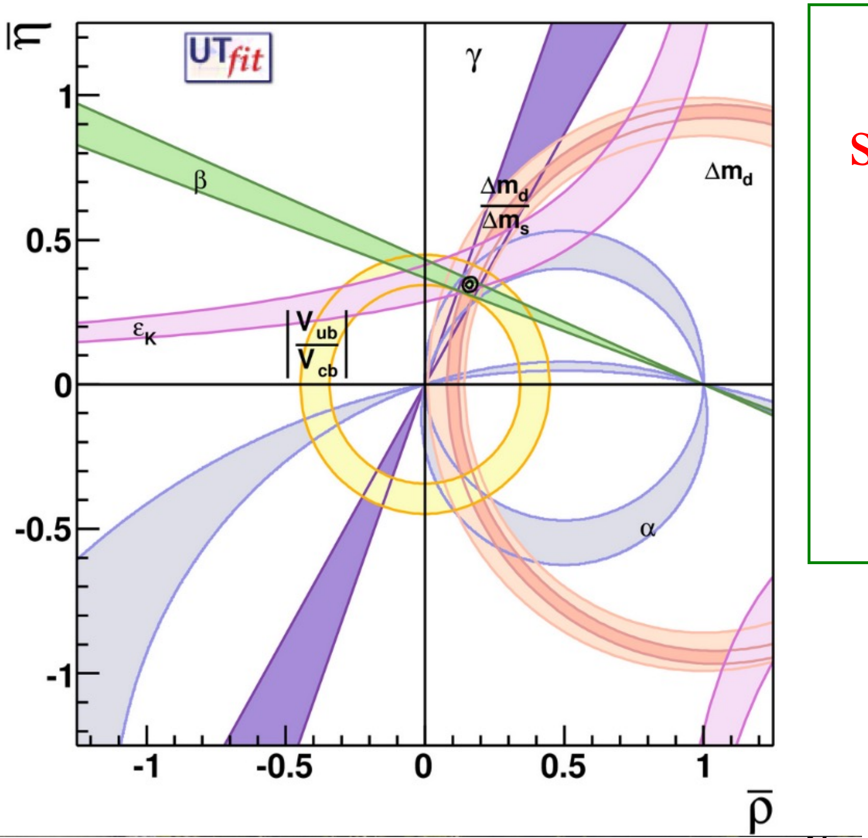




2023 results

$\overline{\rho} = 0.1609 \pm 0.0095$ $\overline{\eta} = 0.347 \pm 0.010$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$\alpha = (94.9 \pm 4.7)^0$

$\sin 2\beta = 0.688 \pm 0.0206$

$\beta = (22.46 \pm 0.68)^0$

$\gamma = (66.1 \pm 3.5)^0$

$A = 0.828 \pm 0.011$

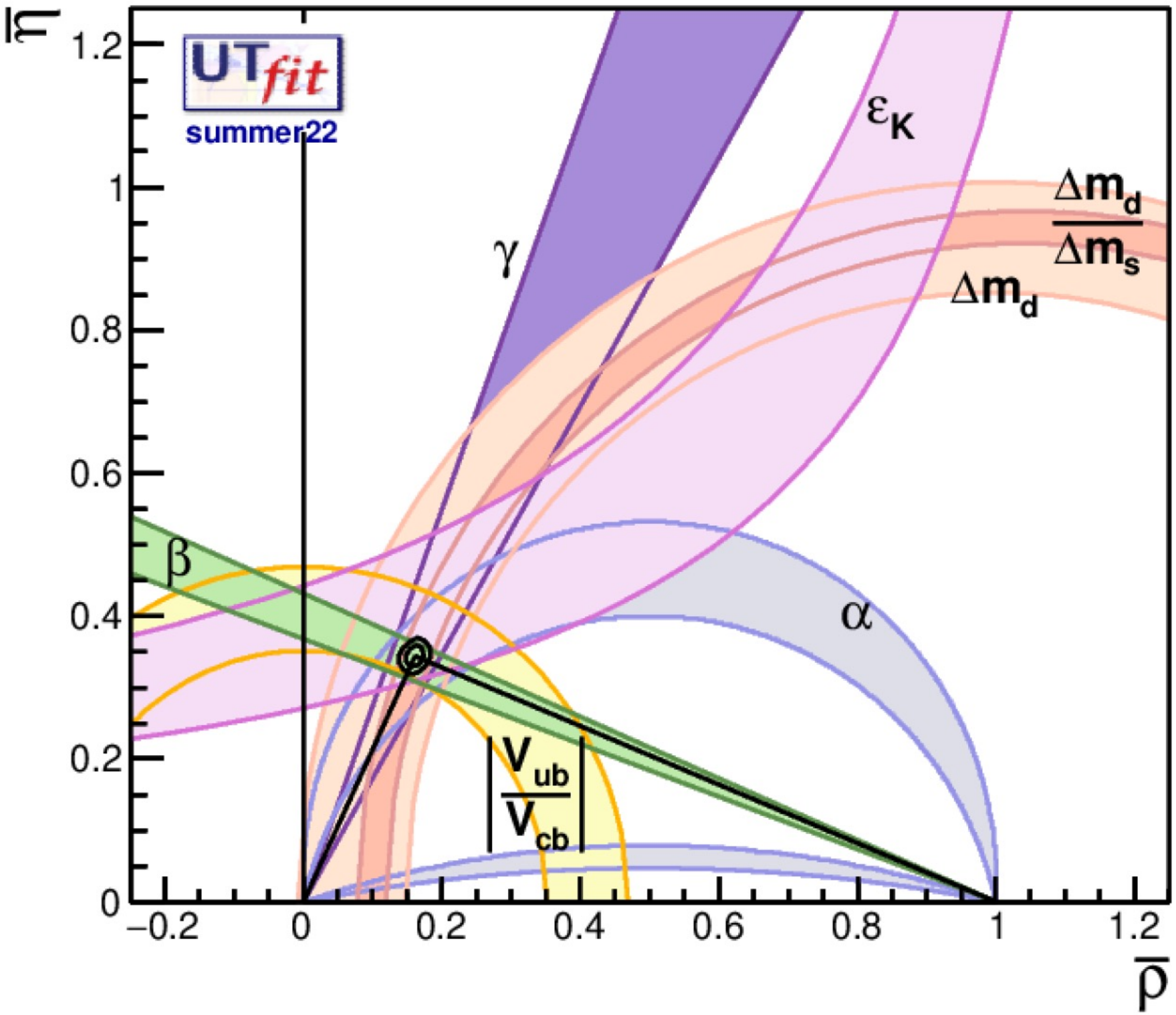
$\lambda = 0.22519 \pm 0.00083$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

Unitarity Triangle analysis in the SM:

zoomed in..



levels @
95% Prob

~6%

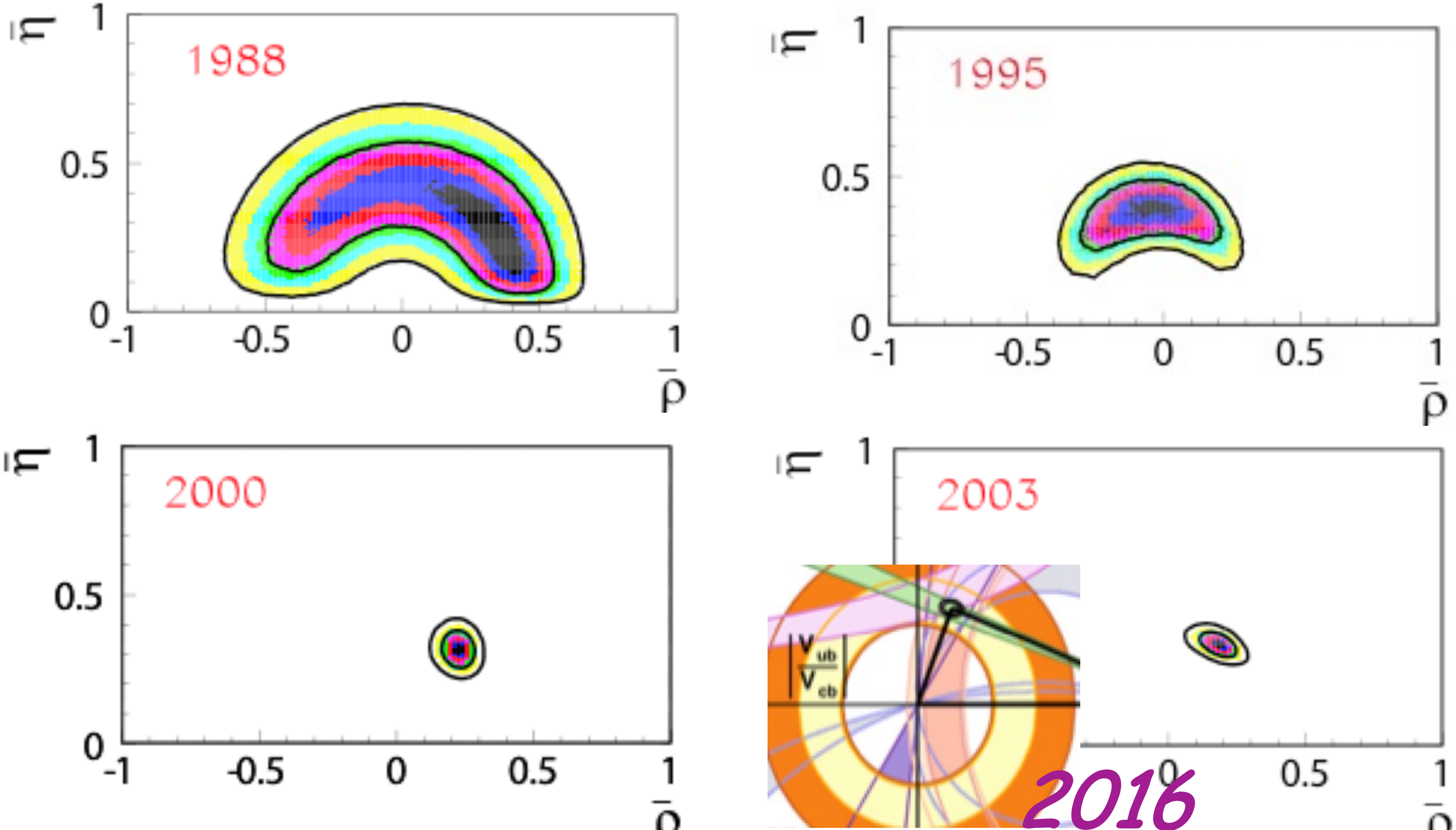
$\rho = 0.1609 \pm 0.0095$

$\eta = 0.347 \pm 0.010$

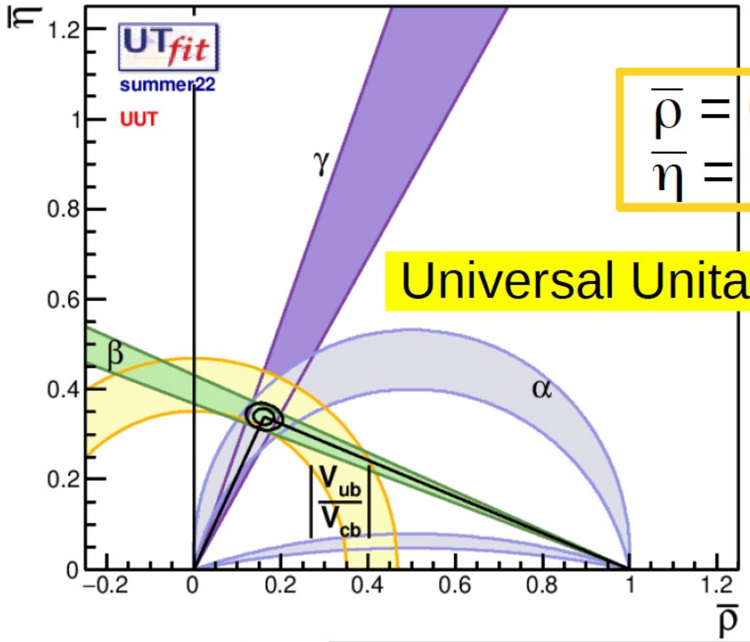
~3%

PROGRESS SINCE 1988

Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)



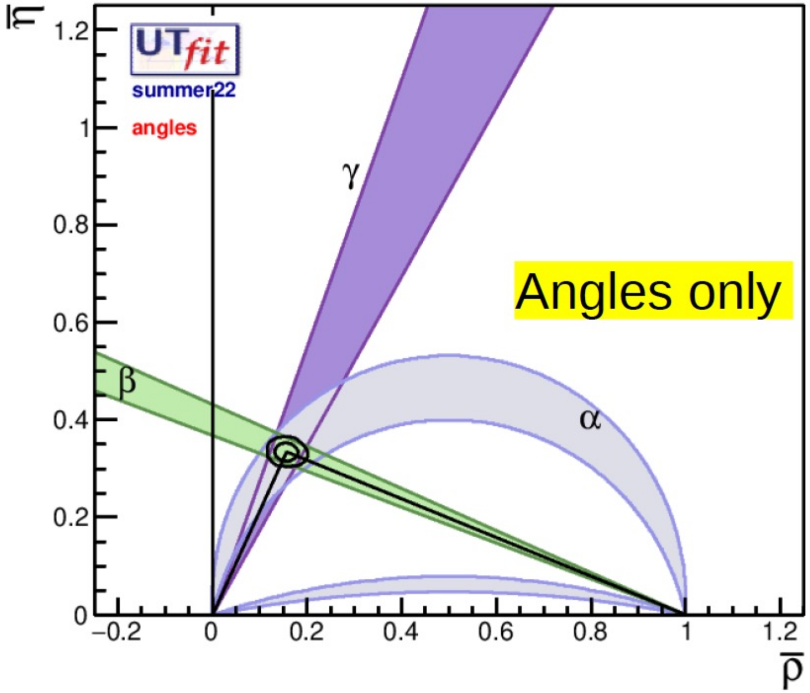
Some interesting configurations



Universal Unitary Triangle

$\bar{\rho} = 0.161 \pm 0.017$ ~10%

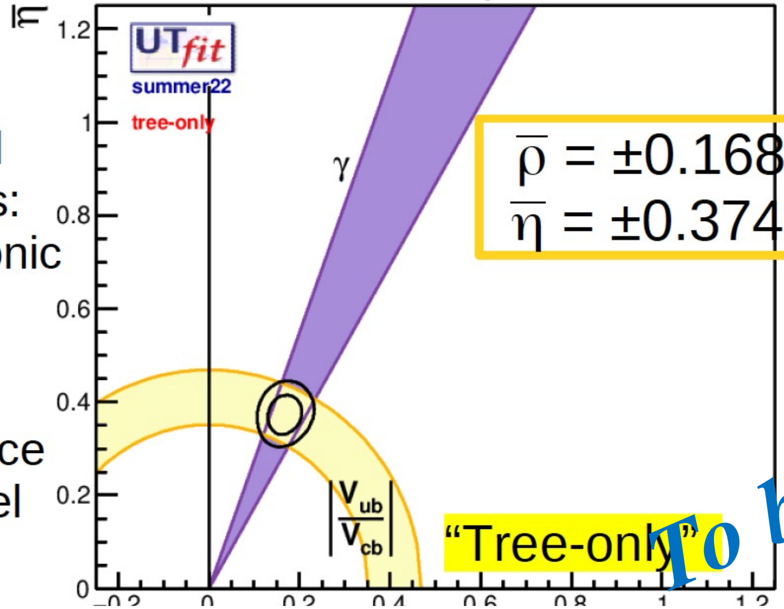
$\bar{\eta} = 0.339 \pm 0.011$ ~3%



Angles only

Tree-level processes: Semileptonic and DK B decays

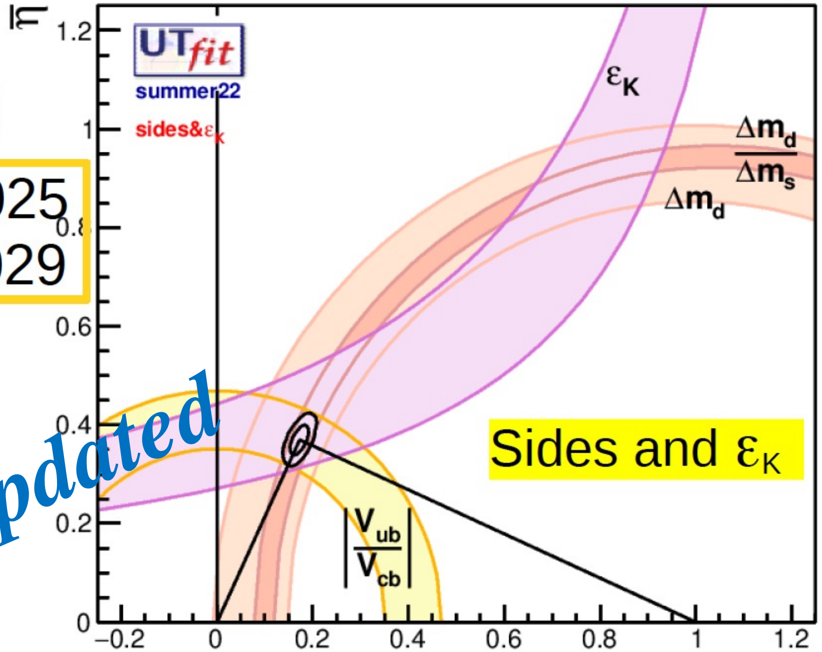
→ reference for model building



"Tree-only"

$\bar{\rho} = \pm 0.168 \pm 0.025$ ~15%

$\bar{\eta} = \pm 0.374 \pm 0.029$ ~8%



Sides and ε_K

To be updated

compatibility plots

A way to “measure” the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

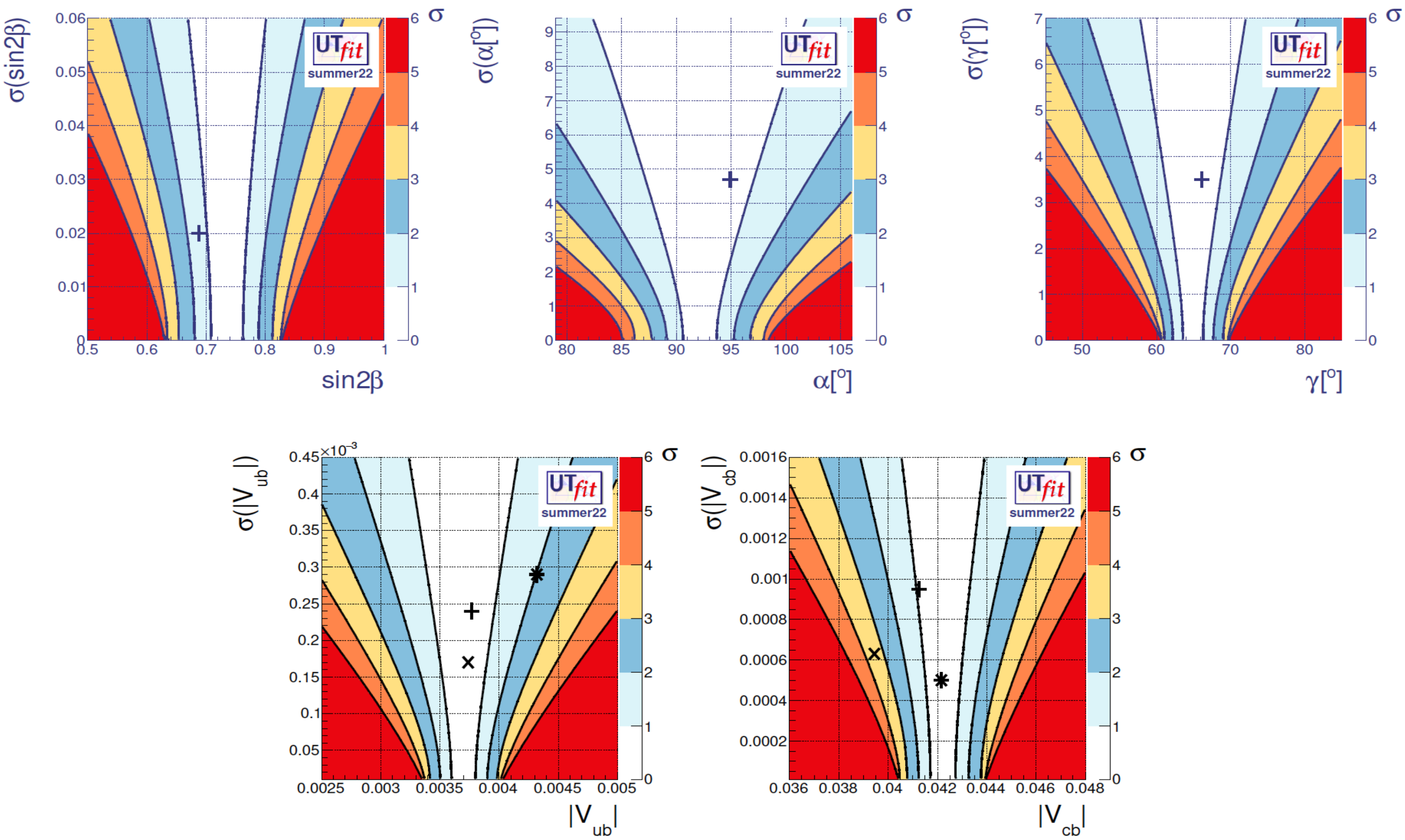


FIG. 5. *Pull plots (see text) for* $\sin 2\beta$ *(top-left),* α *(top-centre),* γ *(top-right),* $|V_{ub}|$ *(bottom-left) and* $|V_{cb}|$ *(bottom-right) inputs. The crosses represent the input values reported in Table I. In the case of* $|V_{ub}|$ *and* $|V_{cb}|$ *the* x *and the* * *represent the values extracted from exclusive and inclusive semileptonic decays respectively.*

V_{cb} and V_{ub}

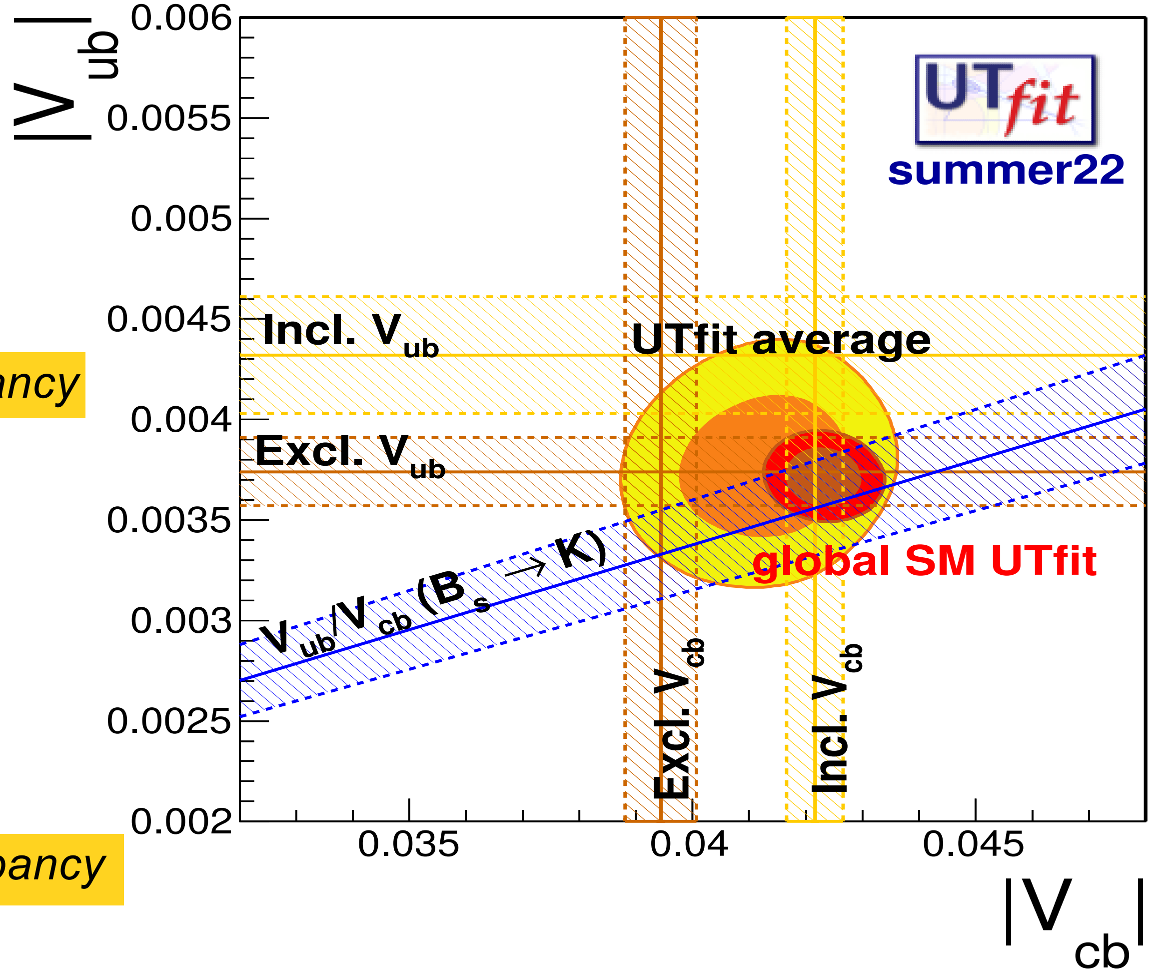
from FLAG 2021

$|V_{cb}|$ *(excl)* = (39.44 ± 0.63) 10^{-3}

$|V_{cb}|$ *(incl)* = (42.16 ± 0.50) 10^{-3}

from Bordone et al.
arXiv:2107.00604

~ 3.2σ discrepancy



$|V_{ub}|$ *(excl)* = (3.74 ± 0.17) 10^{-3}

$|V_{ub}|$ *(incl)* = (4.32 ± 0.29) 10^{-3}

from GGOU HFLAV 2021
adding a flat uncertainty
covering the spread
of central values

~1.6σ discrepancy

$|V_{ub} / V_{cb}|$ *(LHCb)* = (9.46 ± 0.79) 10^{-2} — From B_s to K at high q^2

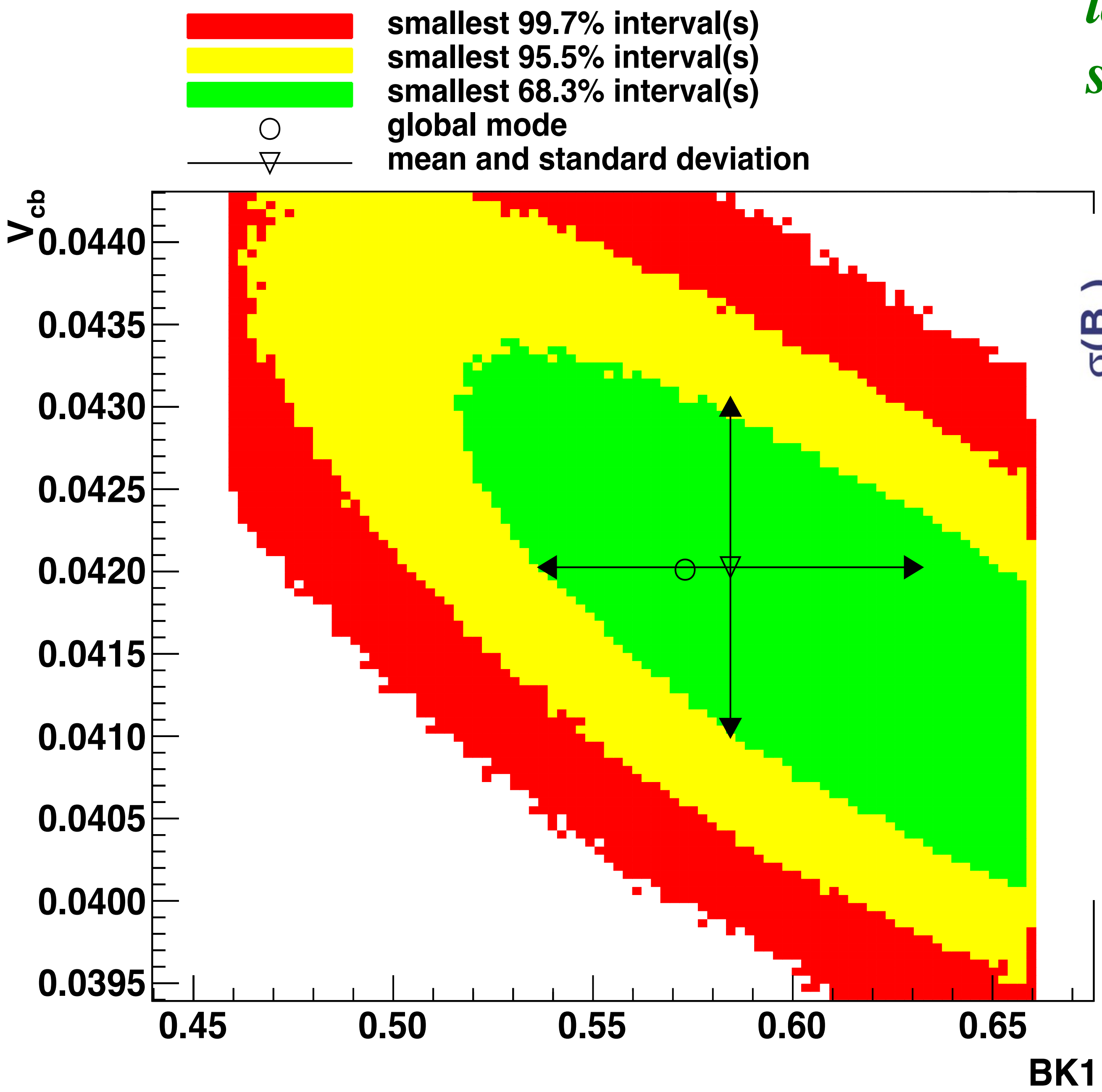
$|V_{ub} / V_{cb}|$ *(LHCb)* = (7.9 ± 0.6) 10^{-2} — From Λ_b, excluded following FLAG guidelines

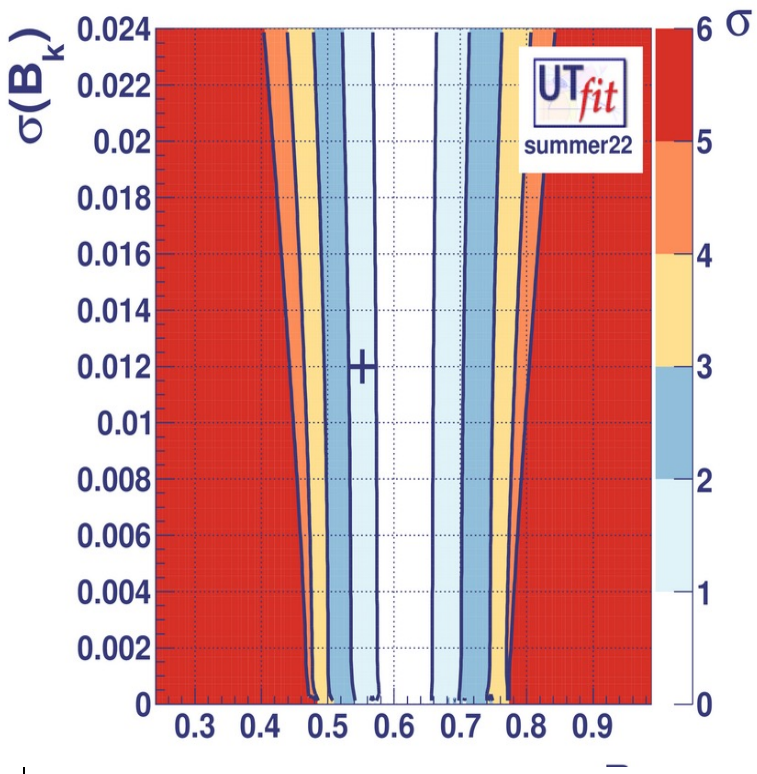
From global SM fit $|V_{cb}|$ = (42.00 ± 0.47) 10^{-3} $|V_{ub}|$ = (3.715 ± 0.093) 10^{-3}

Utfit Prediction Vcb= (42.22 ± 0.51) 10^{-3} *Vub*= (3.70 ± 0.11) 10^{-3}

UT-fit Preliminary

- ε_K large Vcb
- B mixing with large lattice matrix elements smaller Vcb





Power corrections to the CP-violation parameter ε_K

M. Ciuchini[(a)], E. Franco[(b)], V. Lubicz[(c,a)],
G. Martinelli[(d,b)], L. Silvestrini[(b)], C. Tarantino[(c,a)]

$$\varepsilon_K^{exp} = 2.228 \pm 0.011) \cdot 10^{-3}$$

2021: an estimate from the 1/mc expansion of the effective Hamiltonian + UTfit

$$\varepsilon_K = 2.00\,(15) \times 10^{-3}$$

Computing the long-distance contributions to ε_K

Ziyuan Bai
Columbia University, USA
bzyhty@gmail.com

Norman Christ* †
Columbia University, USA
E-mail: nhc@phys.columbia.edu

RBC and UKQCD Collaborations

2015: a real exploratory calculation

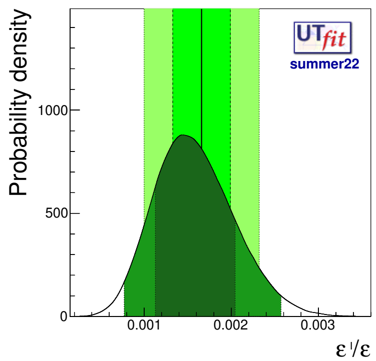
no physical masses, no extrapolation to the continuum

$$|\varepsilon| = (1.806(41) + 0.891(11) + 0.209(6) + 0.112(13)) \times 10^{-3} = 3.019(45) \times 10^{-3}$$

tt ut_{SD} ut_{LD} $\mathrm{Im}(A_0)$,

e'/e from RBC now in Utfit:

e'/ e= 15.2(4.7) x10^-4

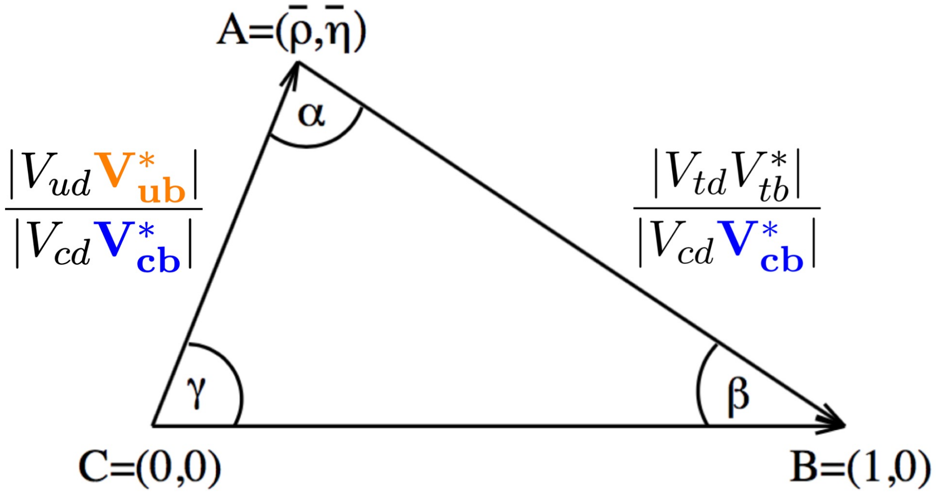


Exclusive semileptonic B → {D(*),π} decays through unitarity

Work in collaboration with M. Naviglio. S. Simula and L. Vittorio

(PRD '21 (2105.02497), PRD '21 (2105.07851), PRD '22 (2105.08674), 2109.15248, 2204.05925, 2202.10285)

See talk by A. Vaquero



Mr. Nosferatu
from Transylvania

Main Results from the Dispersive Matrix Method

to show the relevant, attractive features of the Dispersion Matrix (DM) approach [arXiv:2105.02497], which is a rigorously model-independent tool for describing the hadronic form factors (FFs) in their whole kinematical range

- entirely based on first principles (i.e. lattice QCD simulations of 2- and 3-point Euclidean correlators)
- independent on any assumption about the momentum dependence of the FFs
- unitarization of the input theoretical data (including also kinematical constraints)
- **no mixing among theoretical calculations and experimental data to describe the shape of the FFs**

* results for $B_{(s)} \to D^{(*)}_{(s)} \ell \nu_\ell$ decays: extraction of $|V_{cb}|$ and theoretical determination of $R(D^{(*)}_{(s)})$ using LQCD results for the FFs (from FNAL/MILC and HPQCD) [2105.08674, 2109.15248, 2204.05925]

decay	$\lvert V_{cb}\rvert^{DM} \cdot 10^3$	inclusive [2107.00604]	exclusive [FLAG 21]
$B \to D$	41.0 ± 1.2		
$B \to D^*$	41.3 ± 1.7		
$B_s \to D_s$	42.4 ± 2.0		
$B_s \to D_s^*$	41.4 ± 2.6		
average	41.4 ± 0.8	42.16 ± 0.50	39.36 ± 0.68
difference		$\simeq 0.8\sigma$	$\simeq 1.9\sigma$

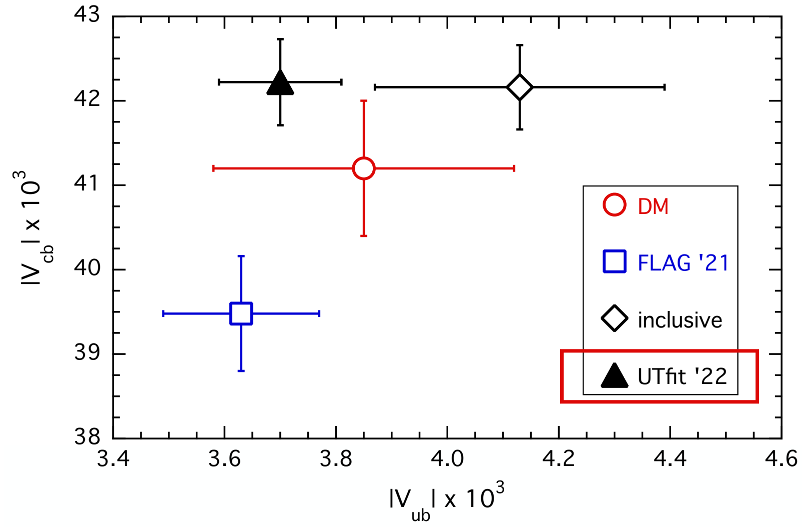
observable	DM	experiment	difference
$R(D)$	0.296 (8)	0.340 (27) (13)	$\simeq 1.4\sigma$
$R(D^*)$	0.275 (8)	0.295 (11) (8)	$\simeq 1.3\sigma$
$R(D_s)$	0.298 (5)		
$R(D_s^*)$	0.250 (6)		

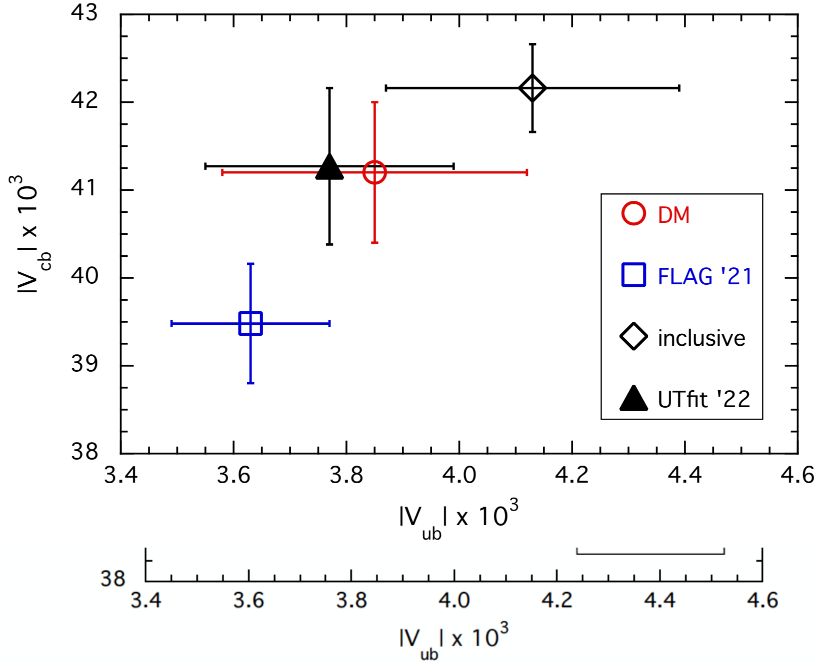
Utfit 42.22 (0.51)

*** reduced tensions in both $|V_{cb}|$ and $R(D^{(*)})$ ***

From S. Simula

- *universal: it can be applied to any exclusive semileptonic decays of mesons and baryons*

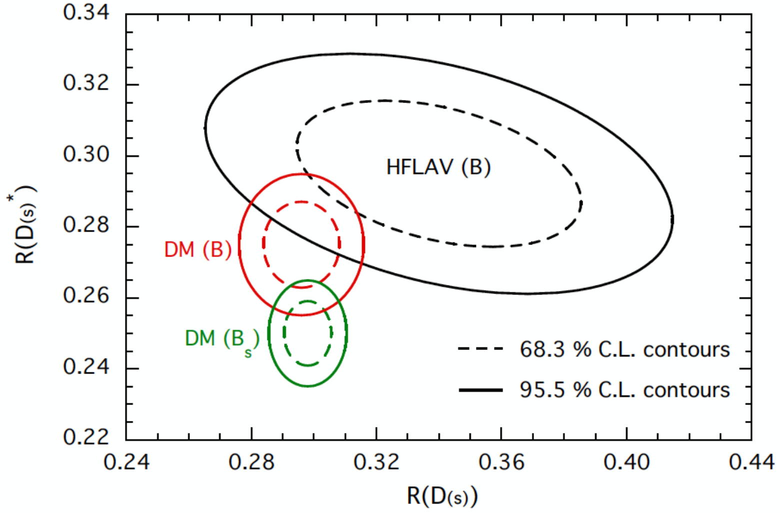




	decays	DM	FLAG '21	inclusive
$\lvert V_{cb}\rvert \cdot 10^3$	$B_{(s)} \to D_{(s)}^{(*)}$	41.4 (8)	39.48 (68)	42.16 (50)
$\lvert V_{ub}\rvert \cdot 10^3$	$B_{(s)} \to \pi, K$	3.85 (27)	3.63 (14)	4.13 (26)

SU(3) breaking effects need further investigation

see Ludovico's slides in discussion session



	DM	HFLAV '19
R(D)	0.296 (8)	0.340 (27) (13)
R(D*)	0.275 (8)	0.295 (11) (8)
R(D_s)	0.298 (5)	
R(D_s*)	0.250 (6)	

reduced tensions in both $|V_{cb}|$, $|V_{ub}|$ and $R(D^{(*)})$
when theory and experiments are not fitted simultaneously

RADIATIVE CORRECTIONS see talk by M. Di Carlo

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking and em effects cannot be neglected anymore:

f_π = 130.2(0.8) MeV ε =0.6% f_K = 155.7(0.3) MeV ε =0.2%

f_K/f_π = 1.1932(19) ε =0.16% $F^{K\pi}(0)$ =0.9698(17) ε =0.18%

A remark on useful and useless precision of lattice calculations:

1) ε_K and long distance charm contributions

2) isospin breaking and electromagnetic corrections to f_K and f_π

Radiative corrections to neutron decay, the Sacred Graal

Real & virtual photon emission

e-Print: 2302.01298
Phys.Rev.D 105 (2022) 11, 11450,
Phys.Rev.D 103 (2021) 5, 053005

see also talk by G. Gagliardi

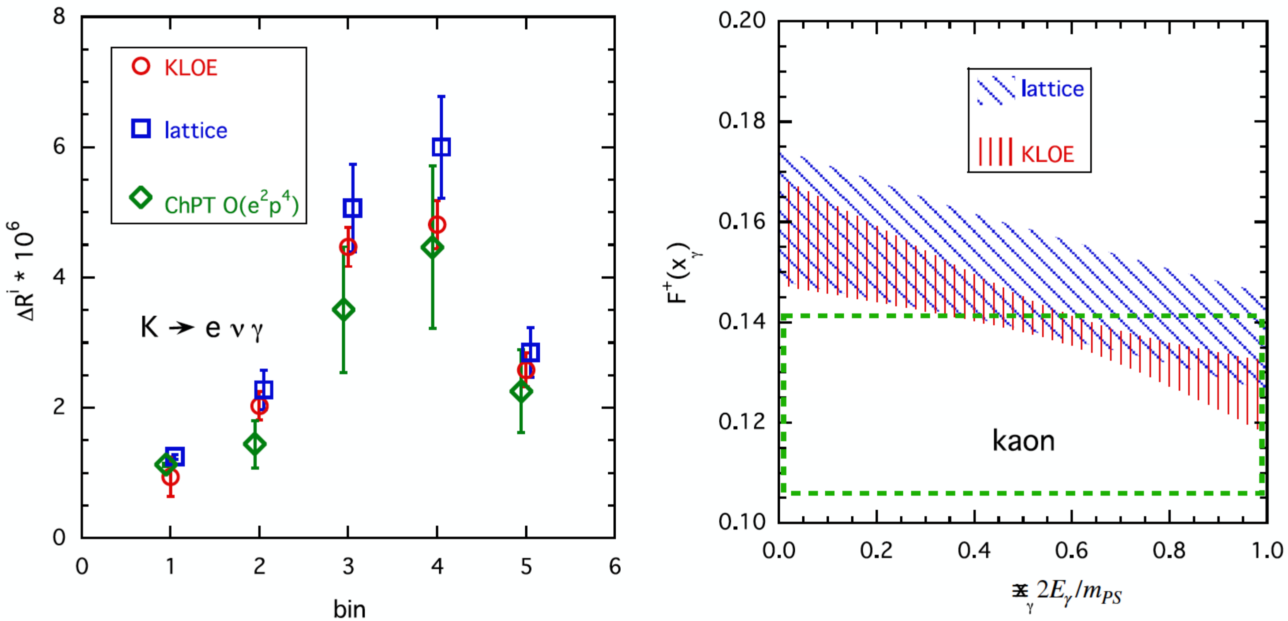
KLOE experiment $K \to e\nu_e\gamma$

[EPJC '09]

$$\Delta R^{exp,i} = \int_{E_\gamma^i}^{E_\gamma^{i+1}} dE_\gamma \frac{1}{\Gamma_{K\mu2[\gamma]}} \left[\frac{d\Gamma(K_{e2\gamma})}{dE_\gamma}\right]_{p_e>200\text{ MeV}} \to \Delta R^{pt,i} + \Delta R^{SD,i} + \Delta R^{INT,i}$$

(kinematical cut due to K_{e3} decays)

five bins : $E_\gamma^i = \{10,\ 50,\ 100,\ 150,\ 200,\ 250\}$ MeV
$E_\gamma^{max} \simeq 250$ MeV



$\Delta R^{pt,i}$: relevant in the first bin only

$\Delta R^{INT,i}$: negligible

$\Delta R^{SD,i} \propto \left[F_V(x_\gamma) + F_A(x_\gamma)\right]^2$

ChPT $O(e^2p^4)$: $F_V(x_\gamma) = \dfrac{m_{PS}}{4\pi^2 f_{PS}}$

$F_A(x_\gamma) = \dfrac{8m_{PS}}{f_{PS}}(L_9^r + L_{10}^r)$

$F^+(x_\gamma) = F_V(x_\gamma) + F_A(x_\gamma) \simeq 0.123 \pm 0.018$

FIG. 1. Left panel: comparison of the KLOE experimental data $\Delta R^{\text{exp},i}$ [9] (red circles) with the theoretical predictions $\Delta R^{\text{th},i}$, (blue squares) evaluated with the vector and axial form factors of Ref. [8] given in Eqs. (13)–(17), for the 5 bins (see Table IV). The green diamonds correspond to the prediction of ChPT at order $\mathcal{O}(e^2p^4)$, based on the vector and axial form factors given in Eq. (53). Right panel: comparison of the form-factor $F^+(x_\gamma)$ extracted by the KLOE collaboration in Ref. [9] and the theoretical prediction from Eqs. (13)–(17). The shaded areas represent uncertainties at the level of 1 standard deviation.

******* good consistency *******

B meson real photon emissions

Factorization at leading power in an expansion of the decay amplitude in Λ_{QCD}/E_γ and Λ_{QCD}/mb has been established to all orders in the strong coupling α_s. In this approximation, the branching fraction depends only on the leading-twist B-meson light-cone distribution amplitude (LCDA) $\phi_+(\omega, \mu)$

More precisely, it is proportional to $1/\lambda_B$, the most important LCDA parameter in exclusive decays, is uncertain by a large factor ranging from 200 MeV favoured by non-leptonic decays to 460 MeV from QCD sum rules.

The radiative leptonic decay has therefore been suggested as a measurement of λ_B

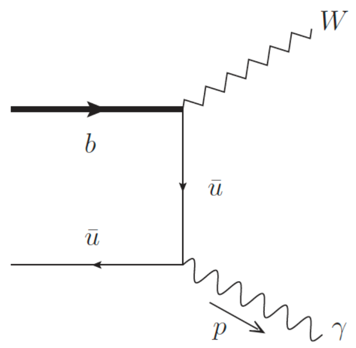


Figure 1. Leading contribution to $B \to \gamma \ell \nu_\ell$.

For large photon energies the form factors can be written as [9]

$$F_V(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \xi(E_\gamma) + \Delta\xi(E_\gamma)\,,$$
$$F_A(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \xi(E_\gamma) - \Delta\xi(E_\gamma)\,. \tag{2.7}$$

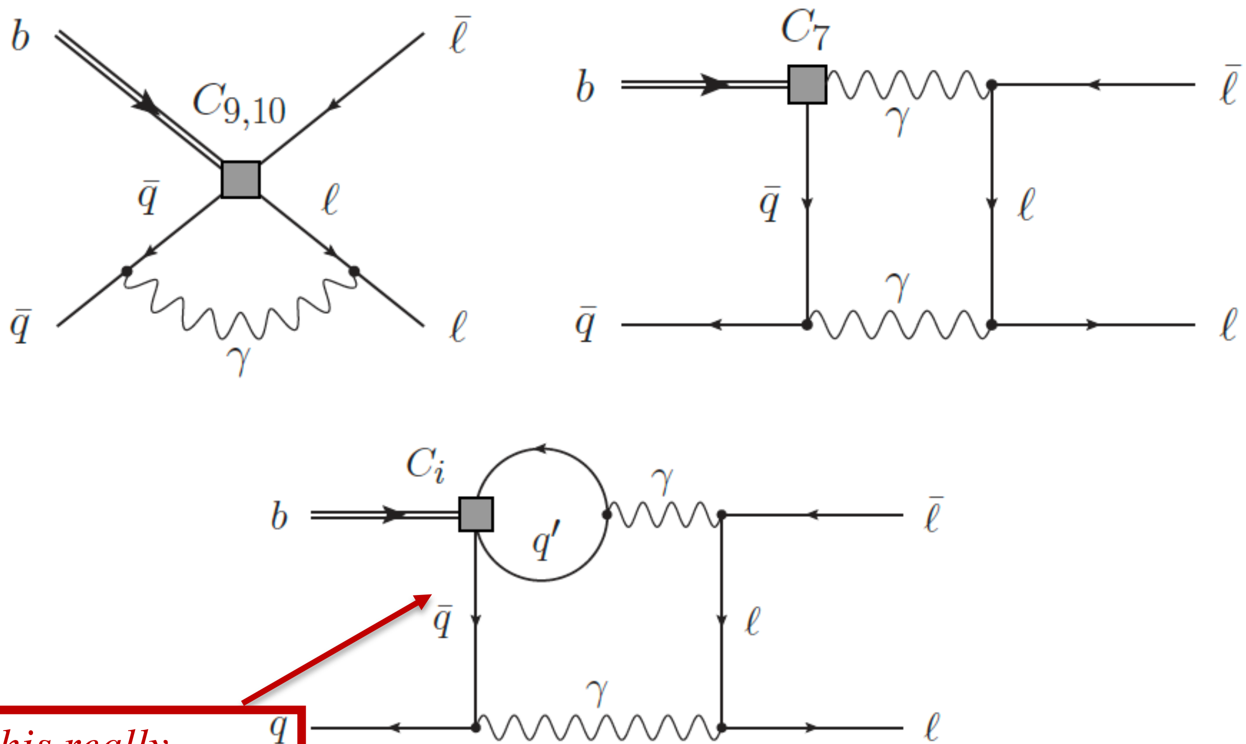
The first term is equal in both expressions and represents the leading-power contribution in the heavy-quark expansion (HQE). It originates only from photon emission from the light spectator quark in B meson (Fig. 1). In the above, f_B is the decay constant of B meson, and the quantity λ_B is the first inverse moment of the B-meson LCDA,

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu)\,. \tag{2.8}$$

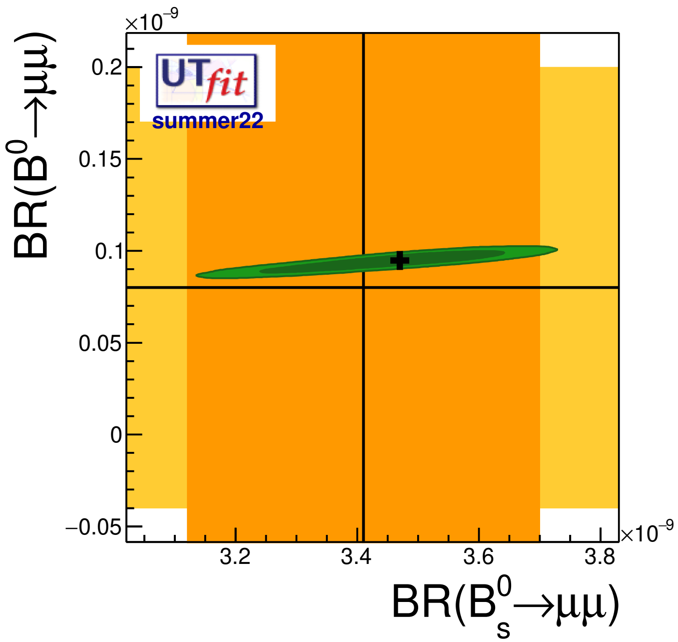
Further applications in decays of heavy neutral B mesons: Virtual corrections (some questions still open)

Enhanced electromagnetic correction to the rare B-meson decay $B_{s,d} \to \mu^+\mu^-$

Martin Beneke,[1] Christoph Bobeth,[1,2] and Robert Szafron[1]



is this really reabsorbed in the coefficient of O_9?



Further applications in decays of heavy neutral B mesons: real corrections (some questions still open)
see the talk by L. Vittorio

$B_s^0 \to \mu^+\mu^-\gamma$ from $B_s^0 \to \mu^+\mu^-$

Francesco Dettori[a], Diego Guadagnoli[b] and Méril Reboud[b,c]

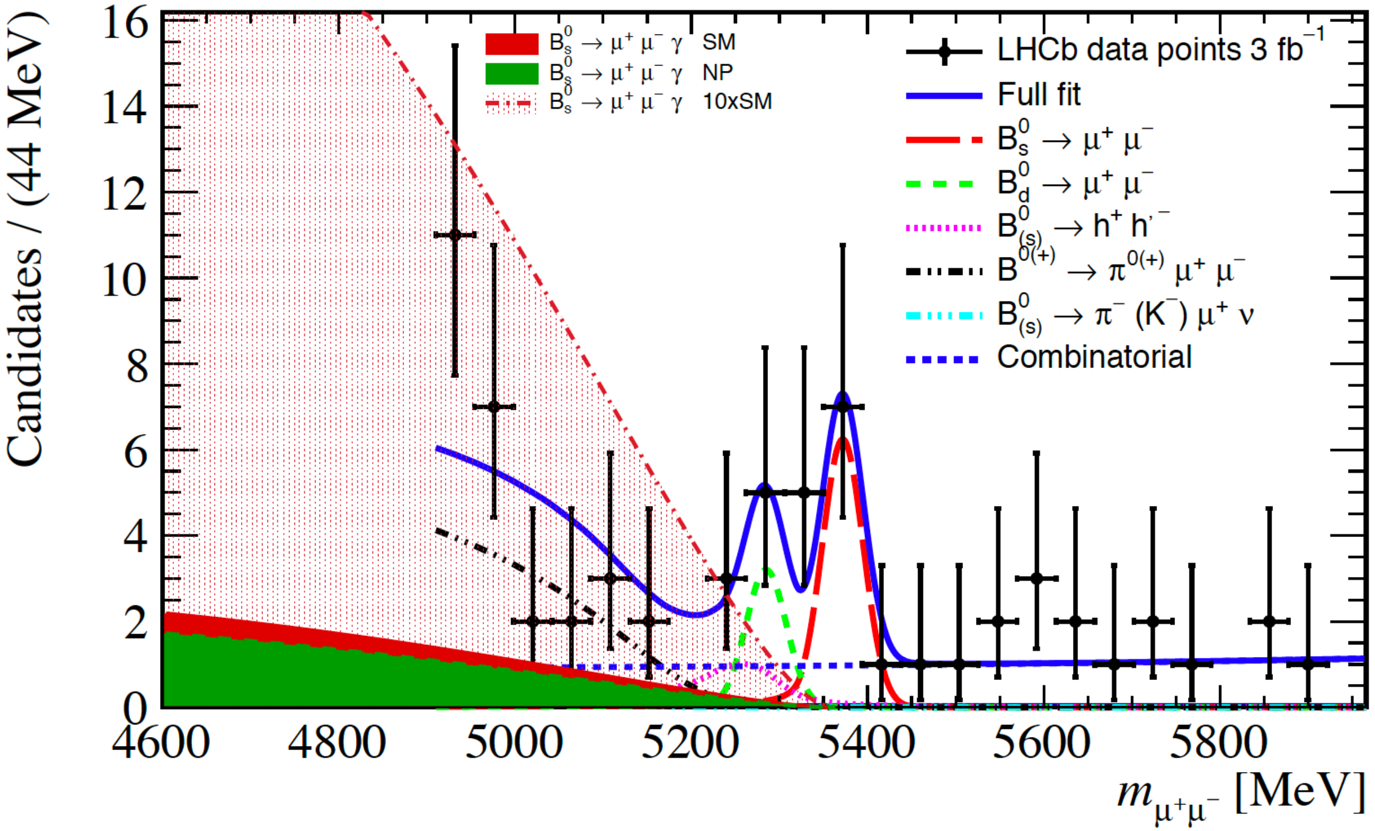
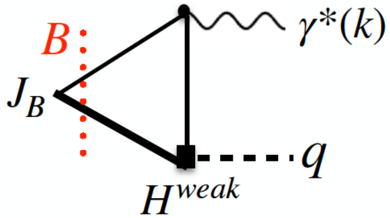
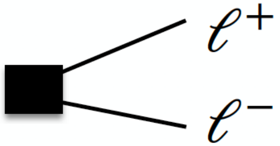


Figure 3: Dimuon invariant mass distribution from LHCb's measurement of $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ [52] overlayed with the contribution expected from $B_s^0 \to \mu^+\mu^-\gamma$ decays (ISR only). Assumes flat efficiency versus $m_{\mu^+\mu^-}$. The line denoted as '$B_s^0 \to \mu^+\mu^-\gamma$ NP' refers to the $V-A$ case with $\delta C_9 = -12\% \, C_9^{\rm SM}$ (see also Fig. 2). The two filled curves are not stacked onto each other.

Particle(s) from weak vertex with momenta q



- **FCNC** Qb= Qq (need long distance in addition) : $F(q^2, k^2)$



$H^{weak} \sim O_{9,10} : B_{d,s} \to \ell^+\ell^-\gamma$ $\quad F(q^2) = F(q^2,0)$

Bobeth's talk

$H^{weak} \sim O_7 : B_{d,s} \to \ell^+\ell^-\gamma$ $\quad F^*(k^2) = F(0,k^2)$

flavoured

axion

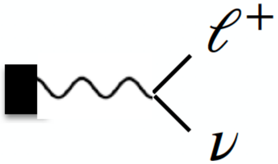
$H^{weak} \sim \bar{q}\gamma_\mu b_L \partial^\mu a : B_{d,s} \to \ell^+\ell^- a$ $\quad F(m_a^2, k^2) \to F^*(k^2)$

Ziegler's talk

or dark photon, scalar DM, …

Xin-Yu Tuo et al. arXiv:2103.11331
G. Gagliardi et al. arXiv:2202.03833 [hep-lat]

- **FCCC** Qb ≠ Qq :



$H^{weak} \sim V_{ub}\, \bar{u}\gamma_\mu b_L \ell \gamma^\mu \nu_L : B_u \to \ell^+ \nu \gamma$

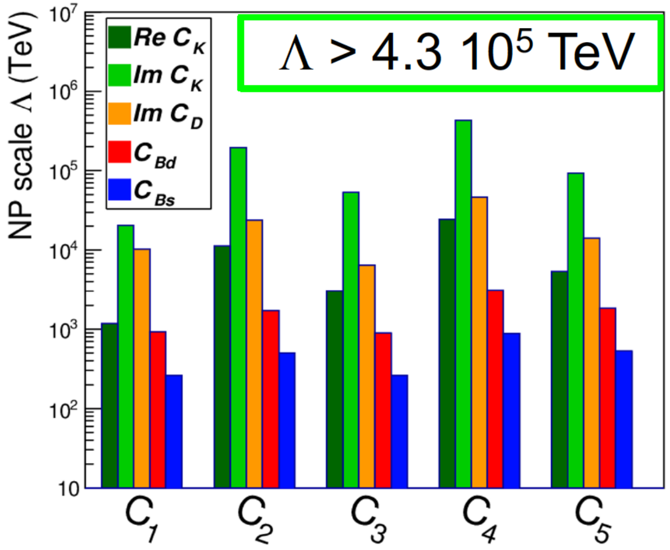
- Physics: helicity suppression of $B \to f_i \bar{f}_j$ relieved in radiative decay!

Beyond the SM

*for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_W (~ 0.03).

Wilson Coefficients results

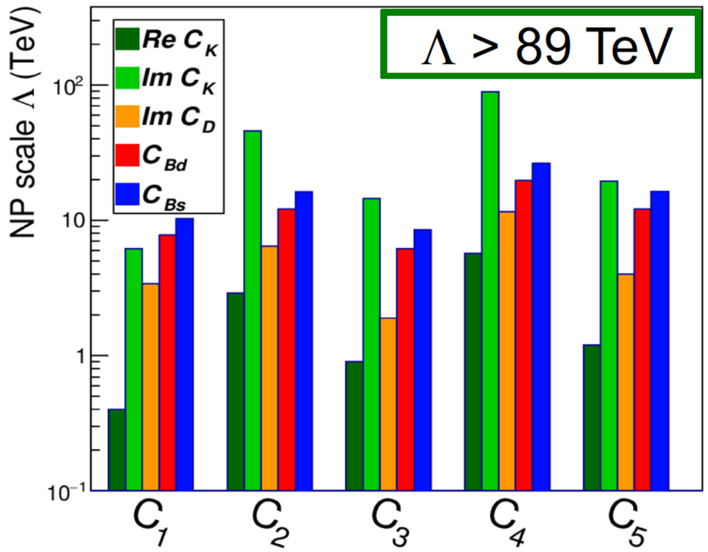
Generic: $C(\Lambda) = \alpha/\Lambda^2$, F_i~1, arbitrary phase, α ~ 1 for strongly coupled NP



- α ~ α_W in case of loop coupling
- through weak interactions*

$\Lambda > 1.3\ 10^4$ TeV

NMFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$, F_i~$|F_{SM}|$, arbitrary phase

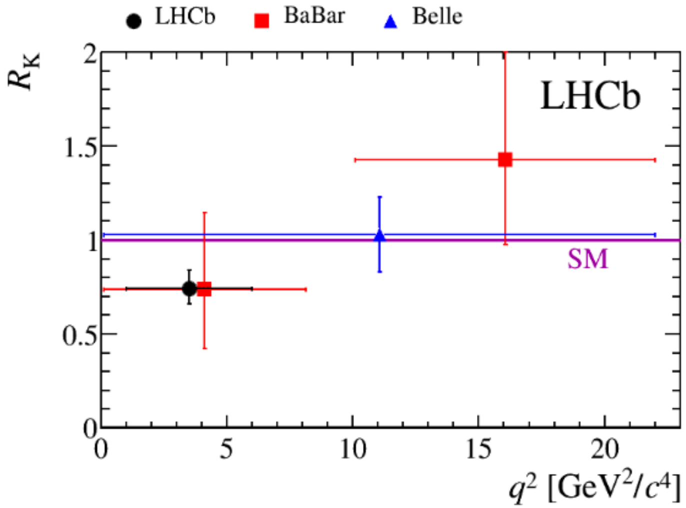


- α ~ α_W in case of loop coupling
- through weak interactions*

$\Lambda > 2.7$ TeV

Reminder: $R_K=B(B^+\to K^+\mu^+\mu^-)/B(B^+\to K^+e^+e^-)$

- Test of lepton universality : $R_K \sim 1$ in SM, with negligible theoretical uncertainties



LHCb, PRL 113 151601
Belle, PRL 103 171801
BaBar, PRD 86 032012

$$R_K(1 < q^2 < 6\ \mathrm{GeV}^2) = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst})$$

Old slide

- Compatible with SM at 2.6σ
- Experimentally challenging
  - lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test: $B^0\to K^{*0}\, l^+ l^-$, $B_s\to\phi\, l^+ l^-$, $\Lambda_B\to\Lambda\, l^+ l^-$

Excitement

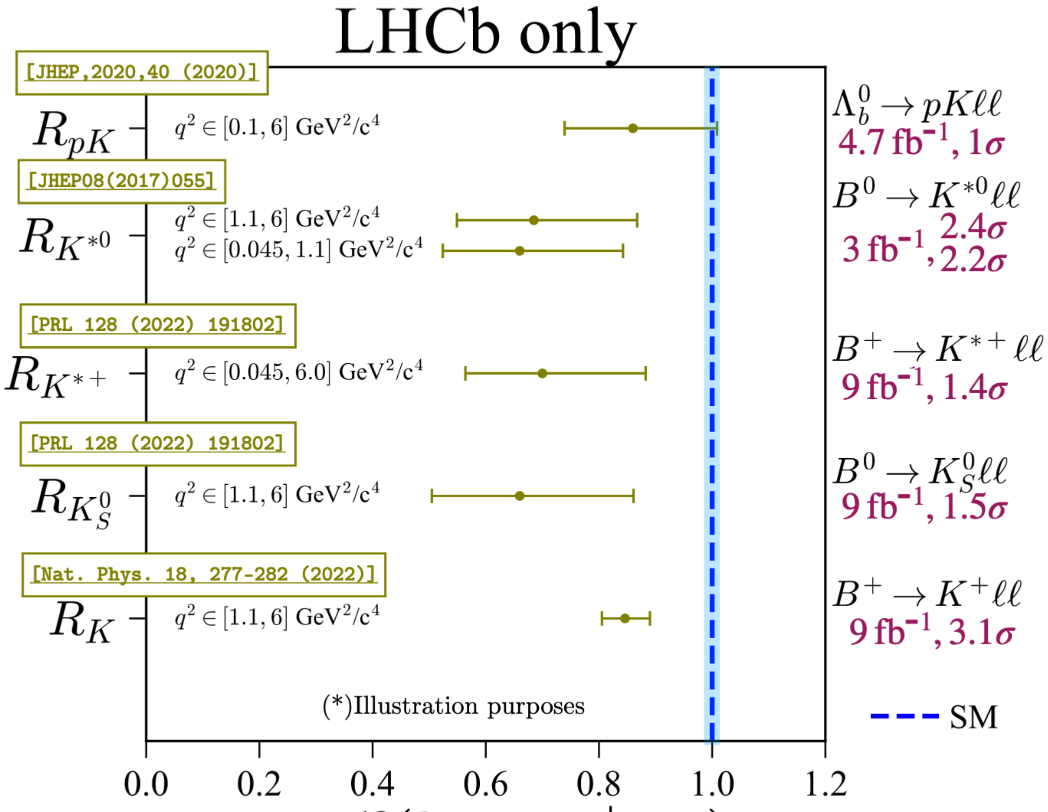
Analysis

Lepton Flavour Universality (LFU) tests in $b \to s\ell^+\ell^-$

- Coherent pattern of tension to SM in LFU test with $b \to s\ell^+\ell^-$ transition:

- R_X ratio extremely well predicted in SM
  - Cancellation of hadronic uncertainties at 10^{-4}
  - $\mathcal{O}(1\%)$ QED correction [Eur.Phys.J.C 76 (2016) 8]
  - Statistically limited

- **Any departure from unity is a clear sign of New Physics**



$$R_X = \frac{\mathcal{B}(b \to s\mu^+\mu^-)}{\mathcal{B}(b \to se^+e^-)}$$

(*) Measurements from Belle not shown (larger statistical uncertainties)

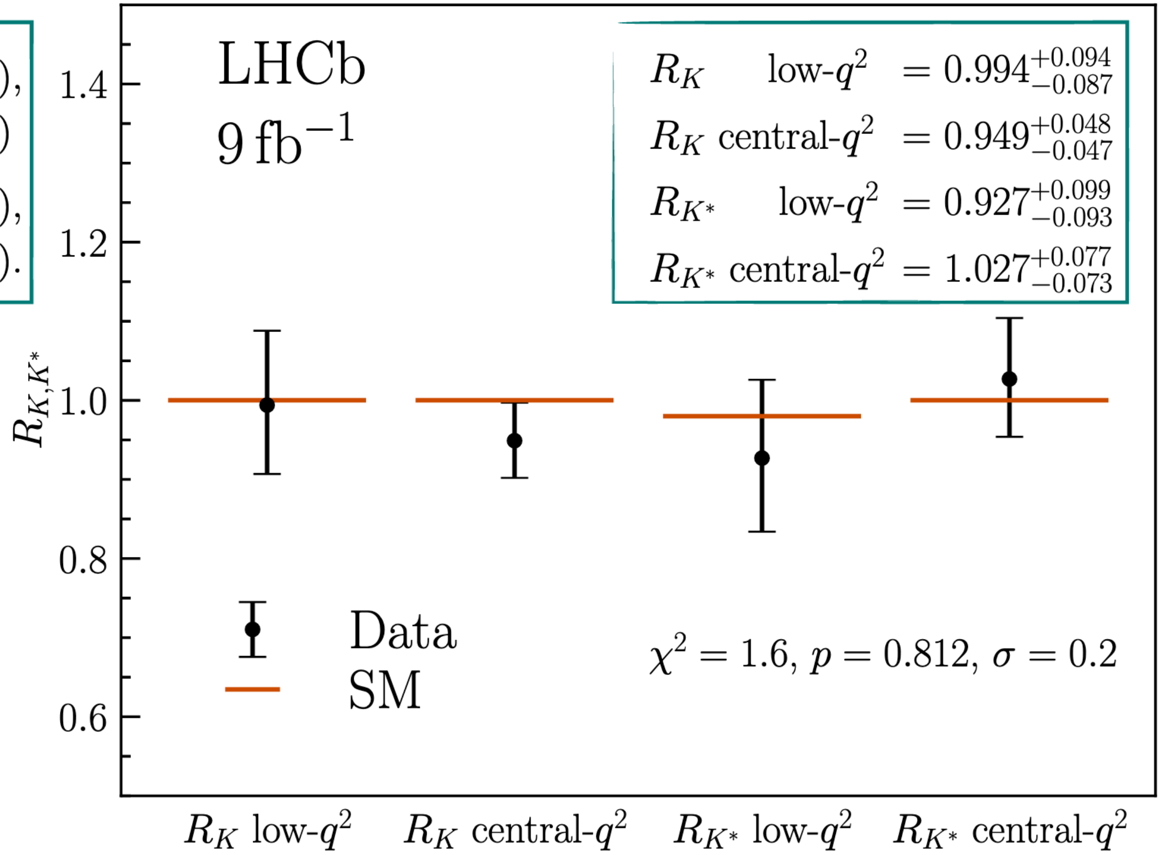
Harakiri!

Results

$$\text{low-}q^2 \begin{cases} R_K = 0.994\ ^{+0.090}_{-0.082}\ (\text{stat})\ ^{+0.027}_{-0.029}\ (\text{syst}), \\ R_{K^*} = 0.927\ ^{+0.093}_{-0.087}\ (\text{stat})\ ^{+0.034}_{-0.033}\ (\text{syst}) \end{cases}$$

$$\text{central-}q^2 \begin{cases} R_K = 0.949\ ^{+0.042}_{-0.041}\ (\text{stat})\ ^{+0.023}_{-0.023}\ (\text{syst}), \\ R_{K^*} = 1.027\ ^{+0.072}_{-0.068}\ (\text{stat})\ ^{+0.027}_{-0.027}\ (\text{syst}). \end{cases}$$

- Most precise and accurate LFU test in $b \to s\ell\ell$ transition
- Compatible with SM with a simple χ^2 test on 4 measurement at 0.2 σ



absence says more than presence

FRANK HERBERT
(Dune)

THANKS FOR YOUR ATTENTION

Back up Slides

The Dispersive Matrix (DM) method $B \to D$

$t \equiv q^2$

$$\mathbf{M} = \begin{pmatrix} \langle \phi f|\phi f\rangle & \langle \phi f|g_t\rangle & \langle \phi f|g_{t_1}\rangle & \cdots & \langle \phi f|g_{t_n}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle \end{pmatrix}$$

$$\langle h_1|h_2\rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}(t) z}$$

The conformal variable z is related to the momentum transfer as:

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}$$

[0, t_{max}=t_-] ⇨ [z_{max}, 0]

$$t_\pm \equiv (m_B \pm m_D)^2$$

$$\det \mathbf{M} \geq 0$$

The DM method

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

We also have to define the kinematical functions

$$\phi_0(z, Q^2) = \sqrt{\frac{2n_I}{3}} \sqrt{\frac{3t_+t_-}{4\pi}} \frac{1}{t_+ - t_-} \frac{1+z}{(1-z)^{5/2}} \left(\beta(0) + \frac{1+z}{1-z}\right)^{-2} \left(\beta(-Q^2) + \frac{1+z}{1-z}\right)^{-2},$$

$$\phi_+(z, Q^2) = \sqrt{\frac{2n_I}{3}} \sqrt{\frac{1}{\pi(t_+ - t_-)}} \frac{(1+z)^2}{(1-z)^{9/2}} \left(\beta(0) + \frac{1+z}{1-z}\right)^{-2} \left(\beta(-Q^2) + \frac{1+z}{1-z}\right)^{-3}, \quad \beta(t) \equiv \sqrt{\frac{t_+ - t}{t_+ - t_-}}$$

Thus, we need **these external inputs** to implement our method:

- estimates of the FFs, computed on the lattice, @ $\{t_1,\ldots,t_n\}$: from Cauchy's theorem (for generic m)

$$\langle g_{t_m} | \phi f \rangle = \phi(t_m, Q^2) f(t_m)$$

LQCD data!

$$\langle g_{t_m} | g_{t_l} \rangle = \frac{1}{1 - \bar{z}(t_l) z(t_m)}$$

Q^2

- non-perturbative values of the susceptibilities, since from the dispersion relations (calling the Euclidean quadratic momentum)

$$\chi(Q^2) \geq \langle \phi f | \phi f \rangle$$

Since the susceptibilities are computed on the lattice, we can in principle use whatever value of $Q^2 \equiv -q^2$

The DM method

The positivity of the original inner products guarantees that $\det \mathbf{M} \geq 0$ namely

LOWER bound

$$\boxed{\beta - \sqrt{\gamma}} \leq f(z) \leq \boxed{\beta + \sqrt{\gamma}}$$

UPPER bound

$$\beta = \frac{1}{d(z)\phi(z)} \sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_j} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

UNITARITY FILTER: unitarity is satisfied if γ is semipositive definite, namely if

$$\chi \geq \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$$

This is a parametrization-independent unitarity test of the LQCD input data

A detailed discussion of the treatment of statistical errors and constraints was also presented (simplified with respect to L. Lellouch NPB, 479 (1996))

Non-perturbative computation of the susceptibilities

The possibility to compute the χs on the lattice allows us to choose *whatever value of Q^2* (i.e. near the region of production of the resonances)

→

NOT POSSIBLE IN PERTURBATION THEORY!!!

$$(m_b + m_c)\Lambda_{QCD} << (m_b + m_c)^2 - q^2$$

POSSIBLE IMPROVEMENT IN THE STUDY OF THE FFs through our method

Work in progress...

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^+}(t)\ , \xrightarrow{W.\ I.} \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t')\right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^-}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^-}(t)\ , \xrightarrow{W.\ I.} \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t')\right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^+}(t)$$

Non-perturbative computation of the susceptibilities

Let us choose for the moment zero Q^2:

$$\boxed{\chi_{0^+}(Q^2=0)} = \int_0^\infty dt\ t^2\ C_{0^+}(t)\ ,$$

$$\boxed{\chi_{1^-}(Q^2=0)} = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^+}(t)\ .$$

$$\chi_{0^+}(Q^2=0) = \frac{1}{12}(m_b-m_c)^2\int_0^\infty dt\ t^4\ C_S(t)$$

$$\chi_{0^-}(Q^2=0) = \frac{1}{12}(m_b+m_c)^2\int_0^\infty dt\ t^4\ C_P(t)$$

$$C_{0^+}(t) = \boxed{\widetilde{Z}_V^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \boxed{\widetilde{Z}_V^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \boxed{\widetilde{Z}_A^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \boxed{\widetilde{Z}_A^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_S(t) = \boxed{\widetilde{Z}_S^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)q_2(x)\ \bar{q}_2(0)q_1(0)\right]|0\rangle\ ,$$

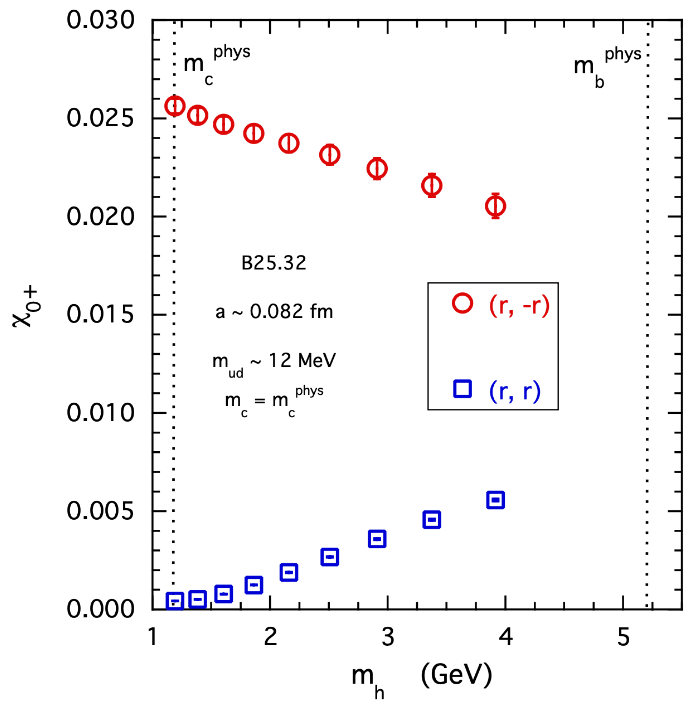
$$C_P(t) = \boxed{\widetilde{Z}_P^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_5 q_1(0)\right]|0\rangle\ ,$$

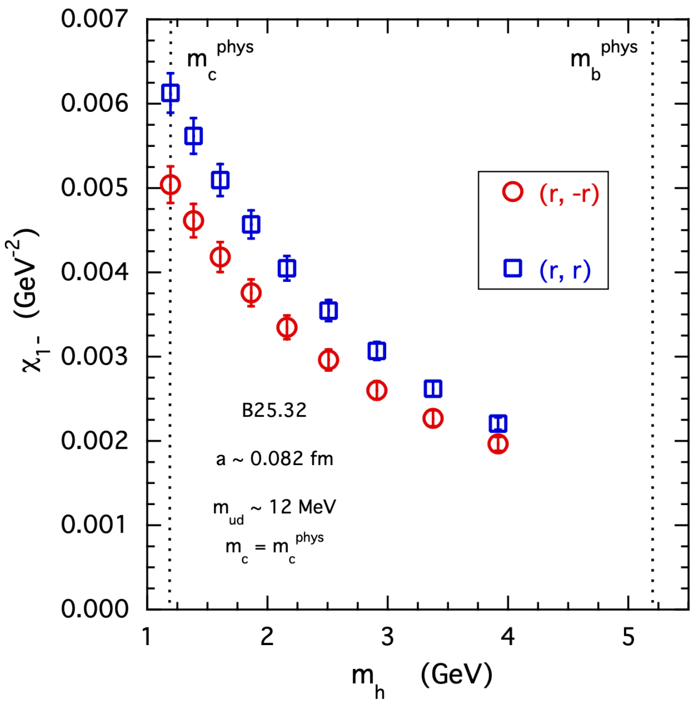
Z: appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

Non-perturbative computation of the susceptibilities

***r*: (unphysical) Wilson parameter**





Following set of nine quark masses:

$$m_h(n) = \lambda^{n-1}\ m_c^{phys} \qquad \text{for } n = 1, 2, \ldots$$

$$m_h(1) = m_c^{phys}$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602$$

$$m_h(9) \simeq 3.9\ \text{GeV} \simeq 0.75\ m_b^{phys}$$

$$m_h = a\mu_h/(Z_P a)$$

Contact terms &

Large discretisation effects

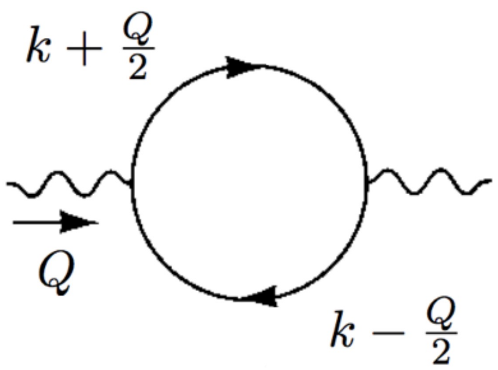
Contact terms & perturbative subtraction

In **twisted mass LQCD** (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\Big],$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i\mu_{q,i}\gamma_5\tau^3}{\mathring{p}^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}, \qquad i = 1,2$$

$$\mathring{p}_\mu \equiv \frac{1}{a}\sin(ap_\mu), \qquad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2}a\hat{p}_\mu^2, \qquad \hat{p} \equiv \frac{2}{a}\sin\Big(\frac{ap_\mu}{2}\Big).$$



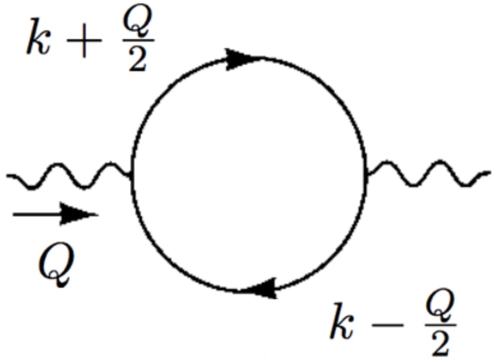
$$\Pi_V^{\alpha\beta} = a^{-2}(Z_1^I + (r_1^2 - r_2^2)Z_2^I + (r_1^2 - r_2^2)(r_1^2 + r_2^2)Z_3^I)g^{\alpha\beta}$$
$$+ (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1\mu_2 Z^{\mu_1\mu_2})g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2 - r_2^2)Z_2^{Q^2})Q\cdot Q g^{\alpha\beta}$$
$$+ (Z_1^{Q^\alpha Q^\beta} + (r_1^2 - r_2^2)Z_2^{Q^\alpha Q^\beta})Q^\alpha Q^\beta + r_1 r_2(a^{-2}Z_1^{r_1r_2}g^{\alpha\beta} + (Z_2^{r_1r_2} + (r_1^2 + r_2^2)Z_3^{r_1r_2}$$
$$+ (r_1^4 + r_2^4)Z_4^{r_1r_2})Q\cdot Q g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1r_2} + \mu_2^2 Z_6^{r_1r_2})g^{\alpha\beta}) + O(a^2),$$

CONTACT TERMS!!!

Contact terms & perturbative subtraction

In **twisted mass LQCD** (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

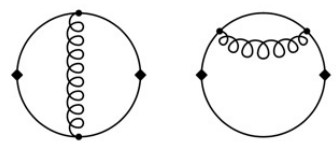
$$\chi_j^{free} = \boxed{\chi_j^{LO}} + \boxed{\chi_j^{discr}}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

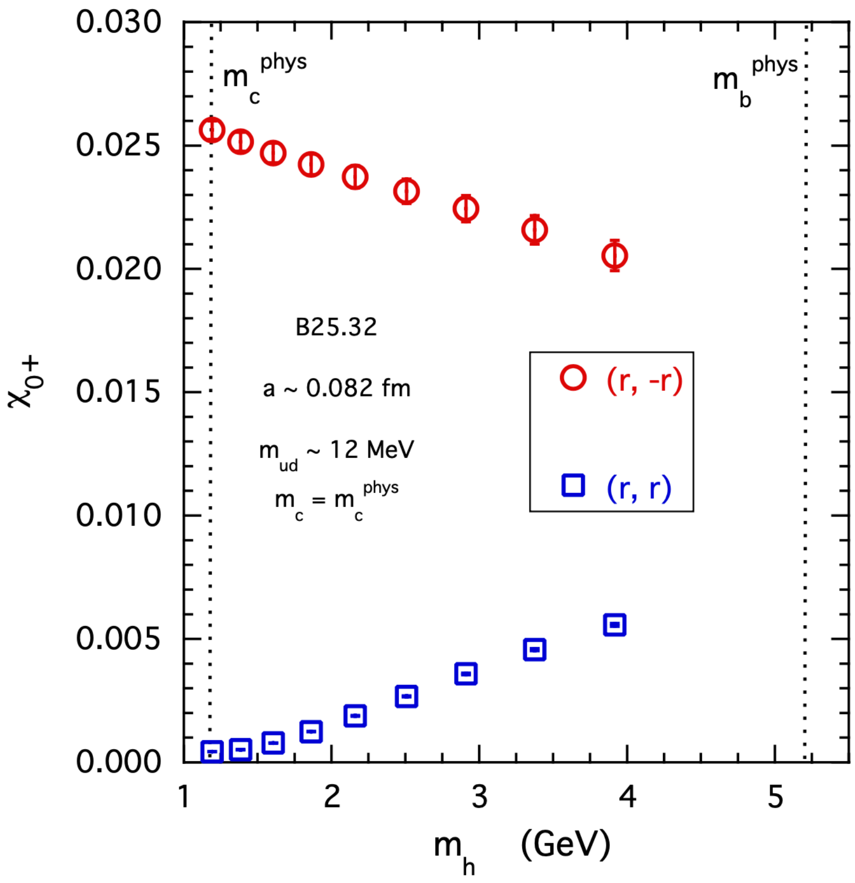
$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

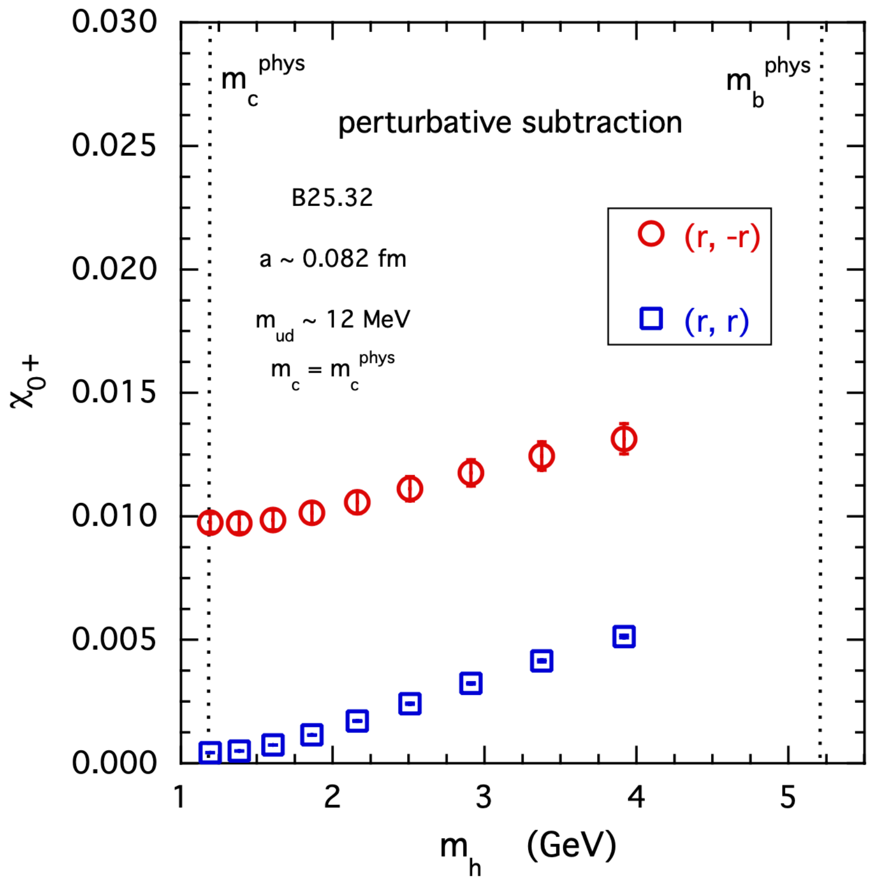
Higher order corrections?



Work in progress…

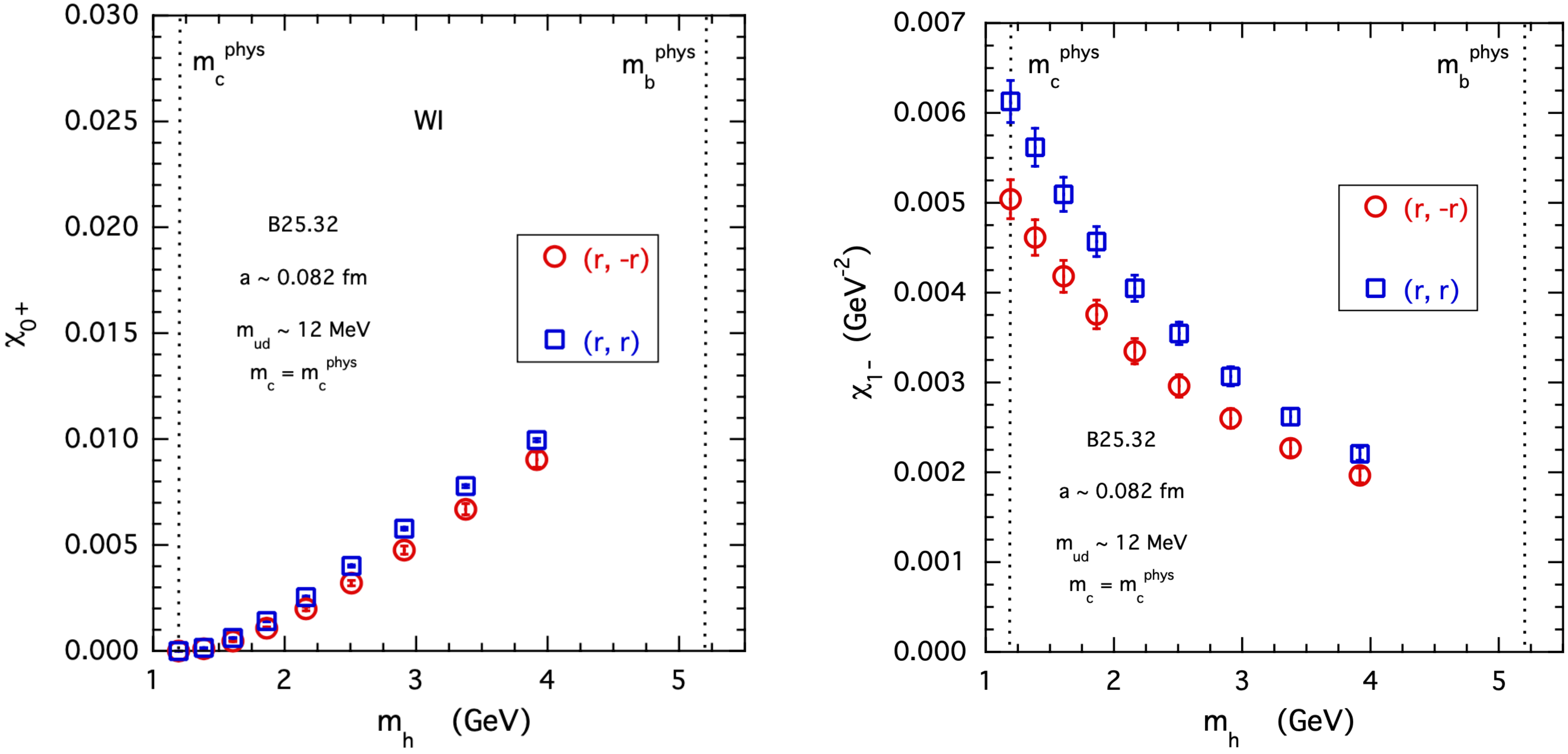
Contact terms & perturbative subtraction





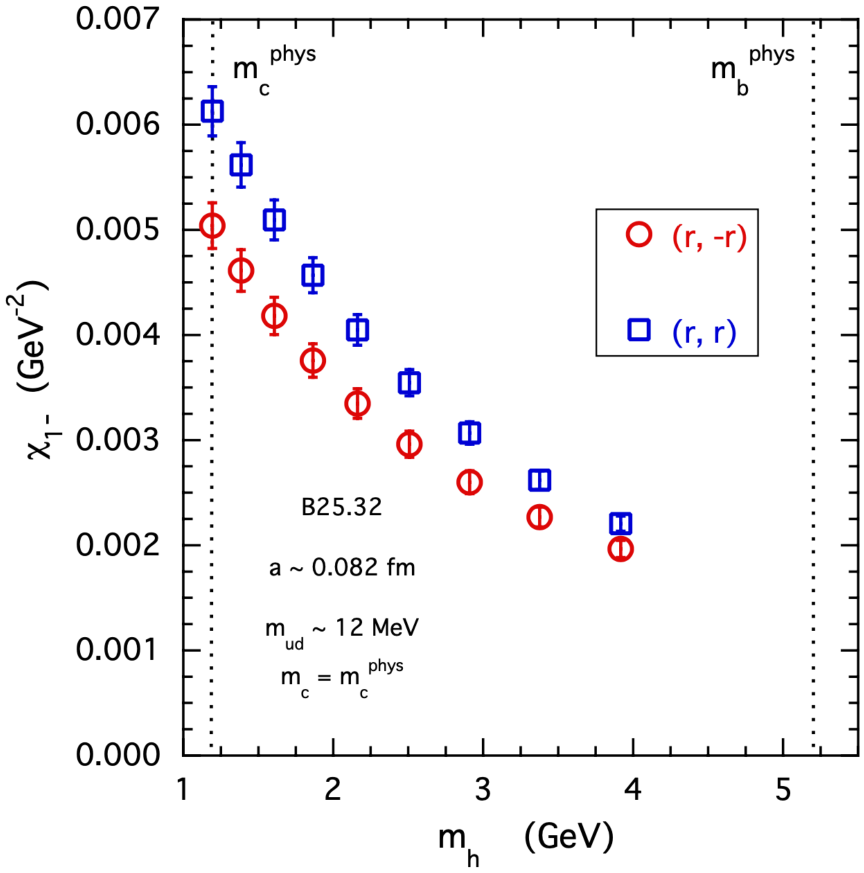
NOT ENOUGH…

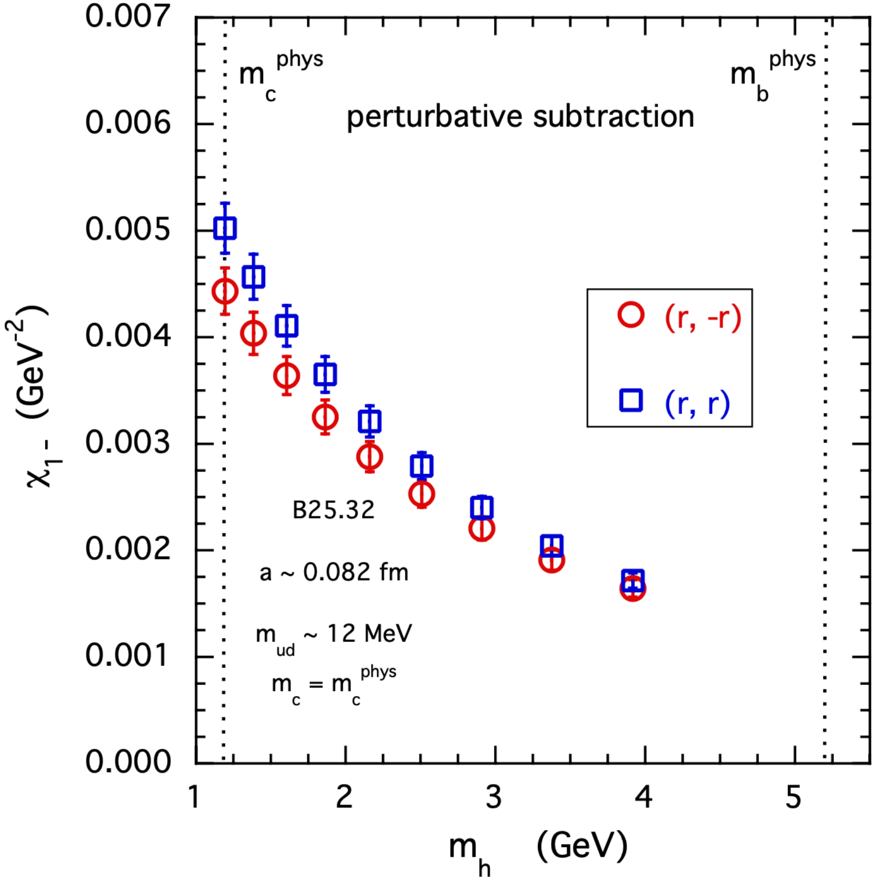
Non-perturbative computation of the susceptibilities



Much better using the Ward Identity

Contact terms & perturbative subtraction





OK

An extrapolation to the continuum limit was implemented

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}}$$

to ensure that

$$\lim_{n\to\infty} R_j(n) = 1$$

$$\rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1 \;,$$

$$\rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2$$

All the details are deeply discussed in *arXiv:2105.07851*. In this way, we have obtained **the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, *in prep.*) transition current densities:**

b → c

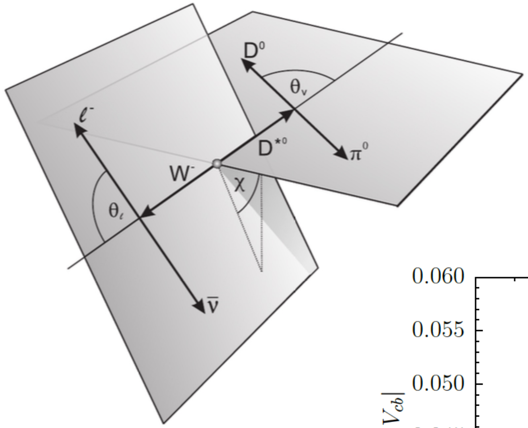
	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}[10^{-4}\ \mathrm{GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}[10^{-4}\ \mathrm{GeV}^{-2}]$	3.894	—	4.69(30)	—

b → u

Non-perturbative	With subtraction
2.04(20)	—
2.34(13)	—
4.88(1.16)	4.45(1.16)
4.65(1.02)	—

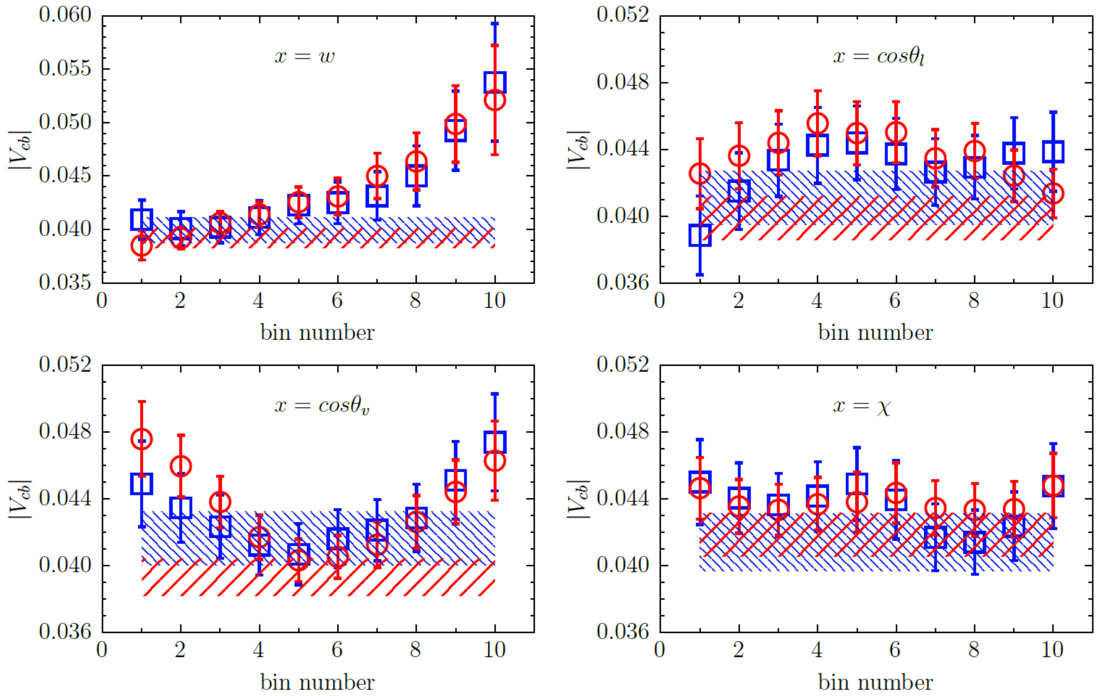
Differences with PT? **~4% for 1⁻, ~7% for 0⁻, ~20 % for 0⁺ and 1⁺**

Bigi, Gambino PRD '16
Bigi, Gambino, Schacht PLB '17
Bigi, Gambino, Schacht JHEP '17



Exclusive Vcb determination from B -> D*

$$d\Gamma/dx \,, \qquad x = w, \cos\theta_l, \cos\theta_v, \chi$$



Blue squares: arXiv:1702.01521

Red points: arXiv:1809.03290

In the w differential decay rate data systematically above the result of the fit

This problem is known and has been studied, for example, in Nucl. Instrum. Meth. A346 (1994) 306-311

Our interpretation is that there is a problem related to the experimental calibration and to the covariance matrix

experimental data for $B \to D^* \ell \nu_\ell$ decays

- two sets of data from Belle collaboration arXiv:1702.01521 and arXiv:1809.03290
- four different differential decay rates $d\Gamma/dx$ where $x = \{w, \cos\theta_v, \cos\theta_\ell, \chi\}$: 10 bins for each variable

total of 80 data points

*** we do not mix theoretical calculations with experimental data to describe the shape of the FFs ***

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}} \qquad i = 1, \ldots, N_{bins}$$

* issue with the covariance matrix $C_{ij}^{exp.}$ of the Belle data: $\Gamma^{exp.} \equiv \sum_{i=1}^{10} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$ should be the same for all the variables x

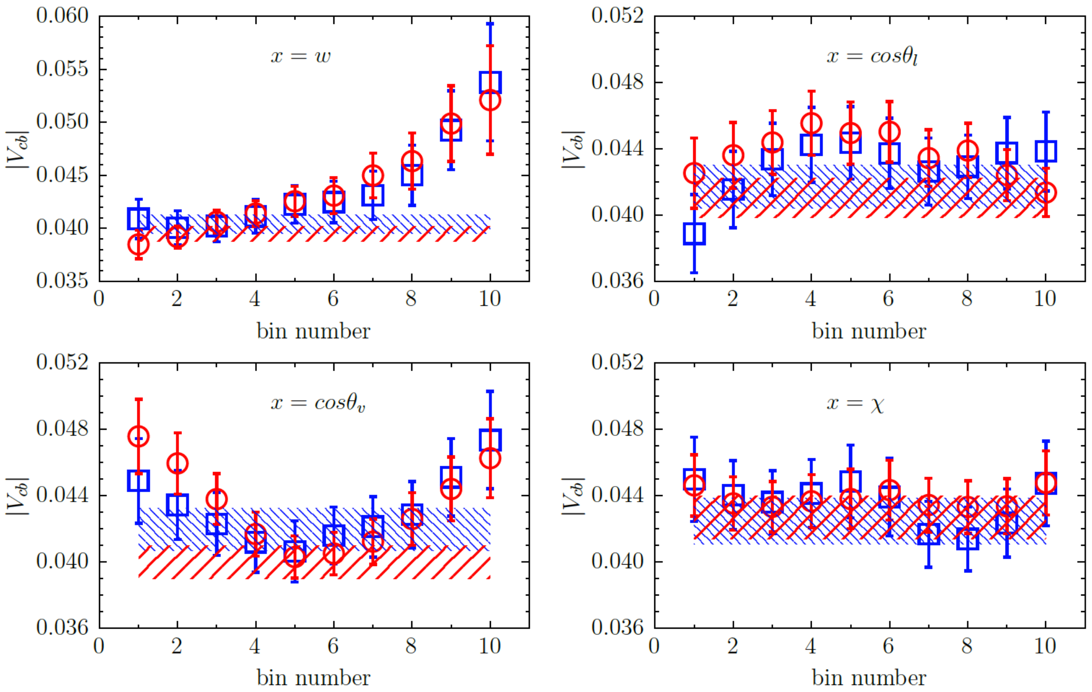
(see D'Agostini, arXiv: 2001.07562)

- we recover the above property by evaluating the correlation matrix of the experimental ratios

$$\frac{1}{\Gamma^{exp.}} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$$

and by considering the new covariance matrix of the experimental data given by (see arXiv:2105.08674)

$$\widetilde{C}_{ij}^{exp.} \to \rho_{ij}^{ratios} \sqrt{C_{ii}^{exp.} C_{jj}^{exp.}}$$



blue data: Belle 1702.01521

red data: Belle 1809.03290

significative improvement of the $\chi^2/(\text{d.o.f.})$

bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} ,$$

$$\sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} ,$$

Exclusive Vcb determination from B → D*

Belle 1702.01521

Belle 1809.03290

experiment	$\|V_{cb}\|(x = w)$	$\|V_{cb}\|(x = \cos\theta_l)$	$\|V_{cb}\|(x = \cos\theta_v)$	$\|V_{cb}\|(x = \chi)$
Ref. [11]	0.0404 (9)	0.0417 (13)	0.0420 (13)	0.0425 (14)
$\chi^2/(\text{d.o.f.})$	1.05	0.89	0.69	0.73
Ref. [12]	0.0395 (7)	0.0410 (12)	0.0400 (10)	0.0427 (13)
$\chi^2/(\text{d.o.f.})$	1.22	1.36	2.02	0.41

<u>averaging procedure</u>

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k \ ,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N} \sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu_x)^2,$$

→

$$|V_{cb}| = (41.2 \pm 1.6) \cdot 10^{-3}$$

with the original covariance of Belle data

$$|V_{cb}| = (40.5 \pm 1.7) \cdot 10^{-3}$$

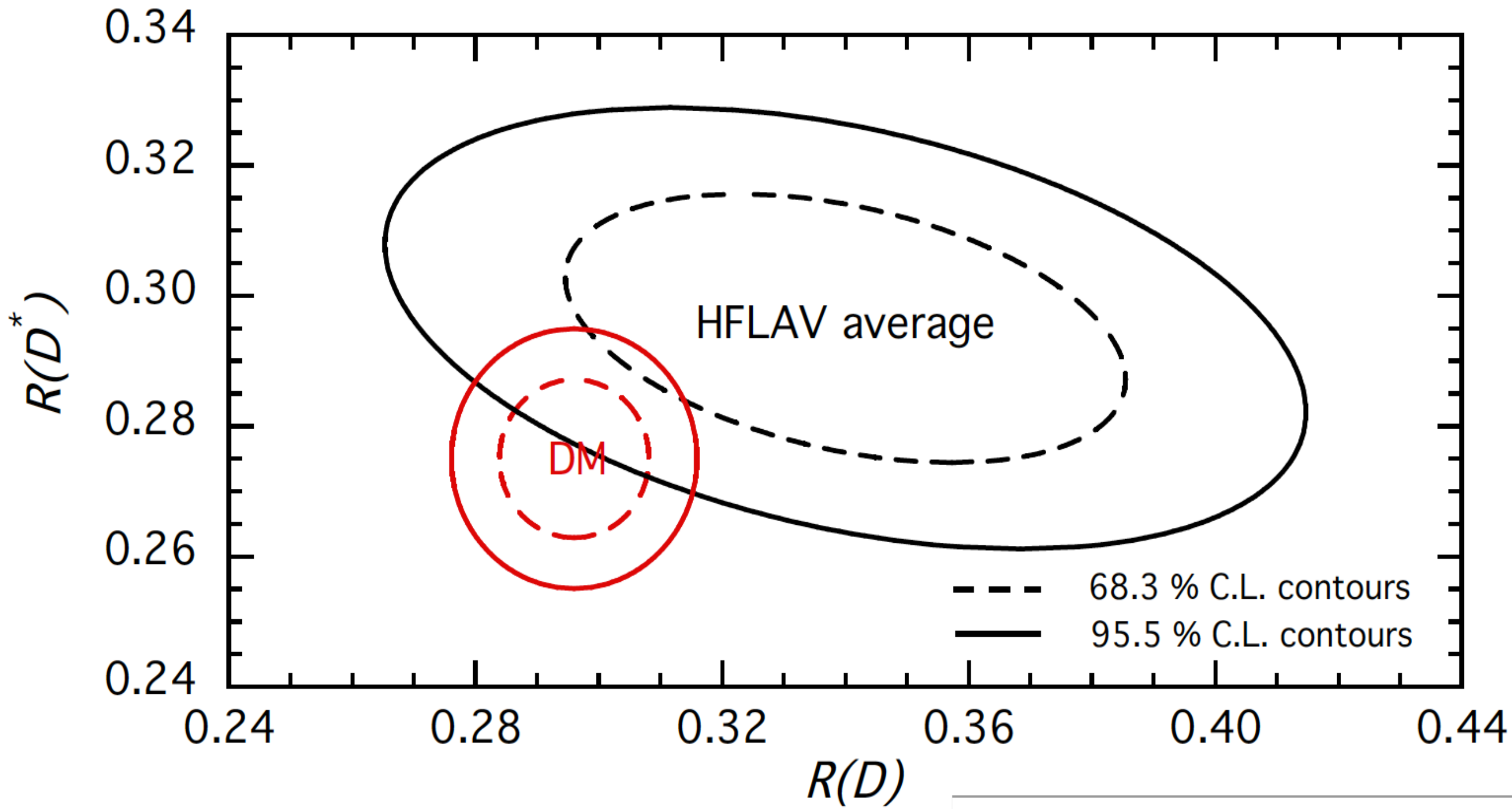
$|V_{cb}|_{excl.} \cdot 10^3 = 39.6^{+1.1}_{-1.0}$ Gambino et al., arXiv:1905.08209

$|V_{cb}|_{excl.} \cdot 10^3 = 39.56^{+1.04}_{-1.06}$ Jaiswal et al., arXiv:2002.05726

$|V_{cb}|_{excl.} \cdot 10^3 = 38.86 \pm 0.88$ FLAG '21, arXiv:2111.09849

the use of exp. data to describe the shape of the FFs leads to smaller errors, but the use of truncated BGL fits does not guarantee that the final error is not underestimated

$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50$ (Bordone et al: arXiv:2107.00604)



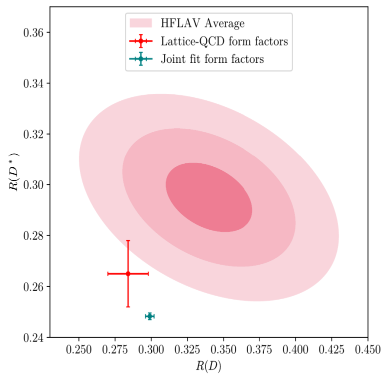
Is there really a problem with Lepton Flavor Universality in $B \to D^{(*)}$ decays ?

or

Much ado about nothing

S. Meinel CKM21

$R(D^{(*)})$



Lattice-QCD-only SM predictions:

$$R(D^*)_{\text{FNAL/MILC 2021}} = 0.265 \pm 0.013$$
$$R(D)_{\text{FNAL/MILC 2015}} = 0.284 \pm 0.014$$
$$R(D)_{\text{FLAG 2021}} = 0.2934 \pm 0.0053$$

[FNAL/MILC, arXiv:2105.14019; HFLAV, arXiv:1909.12524/EPJC 2021]

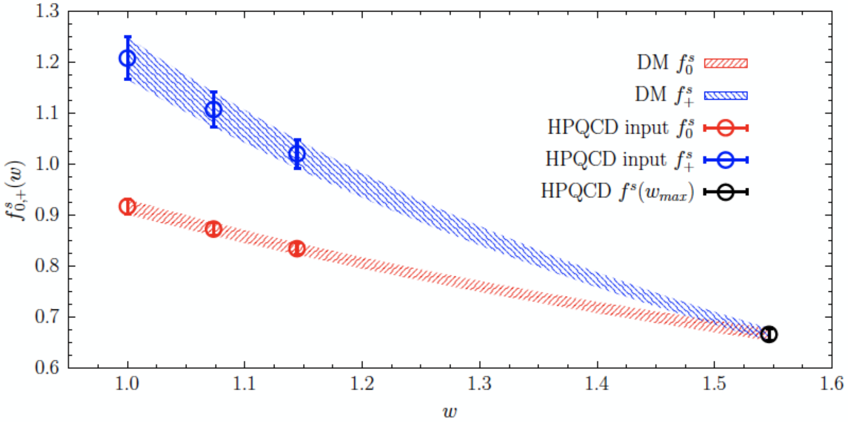
See the talks by Judd Harrison, Shoji Hashimoto, Takashi Kaneko, Gumaro Rendon, and Silvano Simula for more lattice results on heavy→heavy decays!

form factors for $B_s \to D_s^{(*)} \ell \nu_\ell$ decays

[arXiv:2204.05925]

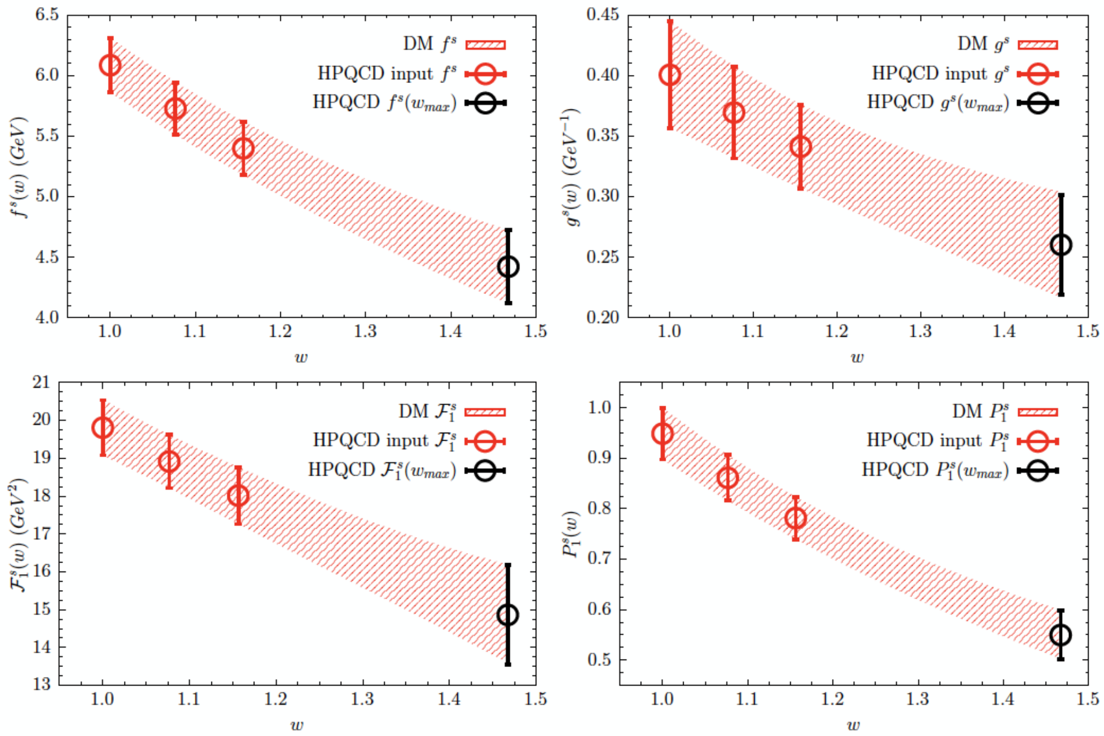
* lattice QCD form factors from **HPQCD arXiv:1906.00701**($B_s \to D_s$) and **arXiv:2105.11433** ($B_s \to D_s^*$) in the form of BCL fits in the whole kinematical range
* we extract 3 data points for the FFs at small values of the recoil and we apply the DM approach

$B_s \to D_s \ell \nu_\ell$



* nice agreement in the whole kinematical range

$B_s \to D_s^* \ell \nu_\ell$



DM confronts BGL

two important differences in the DM method with respect to BGL parametrization

- No series expansion to describe the FFs → ***NO TRUNCATION ERRORS***

particularly relevant for semileptonic decays characterized by a very large q^2 range

$B \to \pi \ell \nu$ Maximum q^2 = 26.46 GeV^2

$\Lambda_b \to p \ell \nu$ Maximum q^2 = 21.9 GeV^2

- Unitarity check of FFs data completely independent of the parameterization

The DM approach

i) reproduces exactly the known data

ii) allows to extrapolate the form factor in the whole kinematical range

iii) in a parameterization-independent way

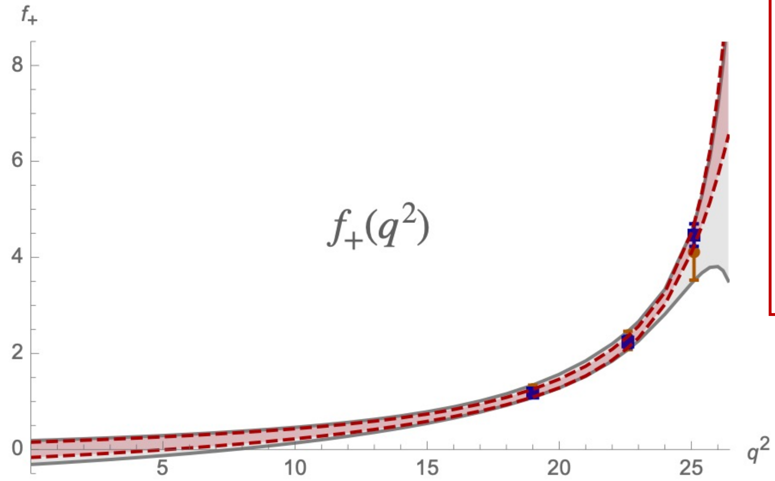
iv) providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points (important for estimating uncertainties)

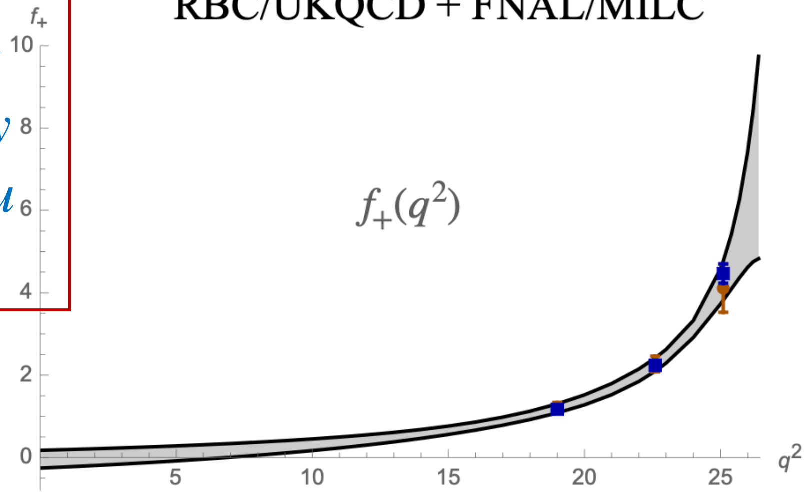
Semileptonic B → π decays *(in prep.)*

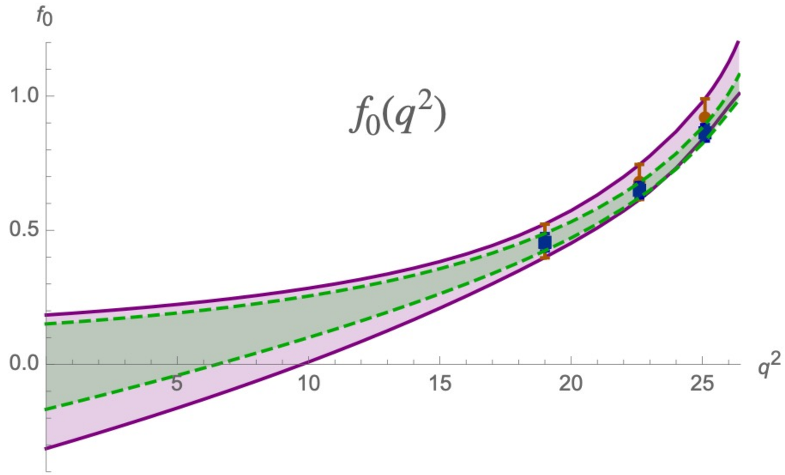
Solid: RBC/UKQCD
Dashed: FNAL/MILC

Non-perturbative susceptibility for the $b \to u$ current

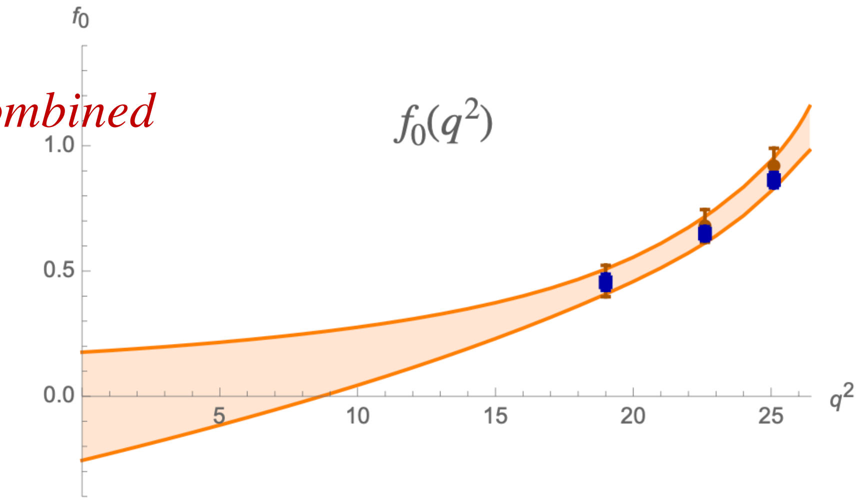
RBC/UKQCD + FNAL/MILC







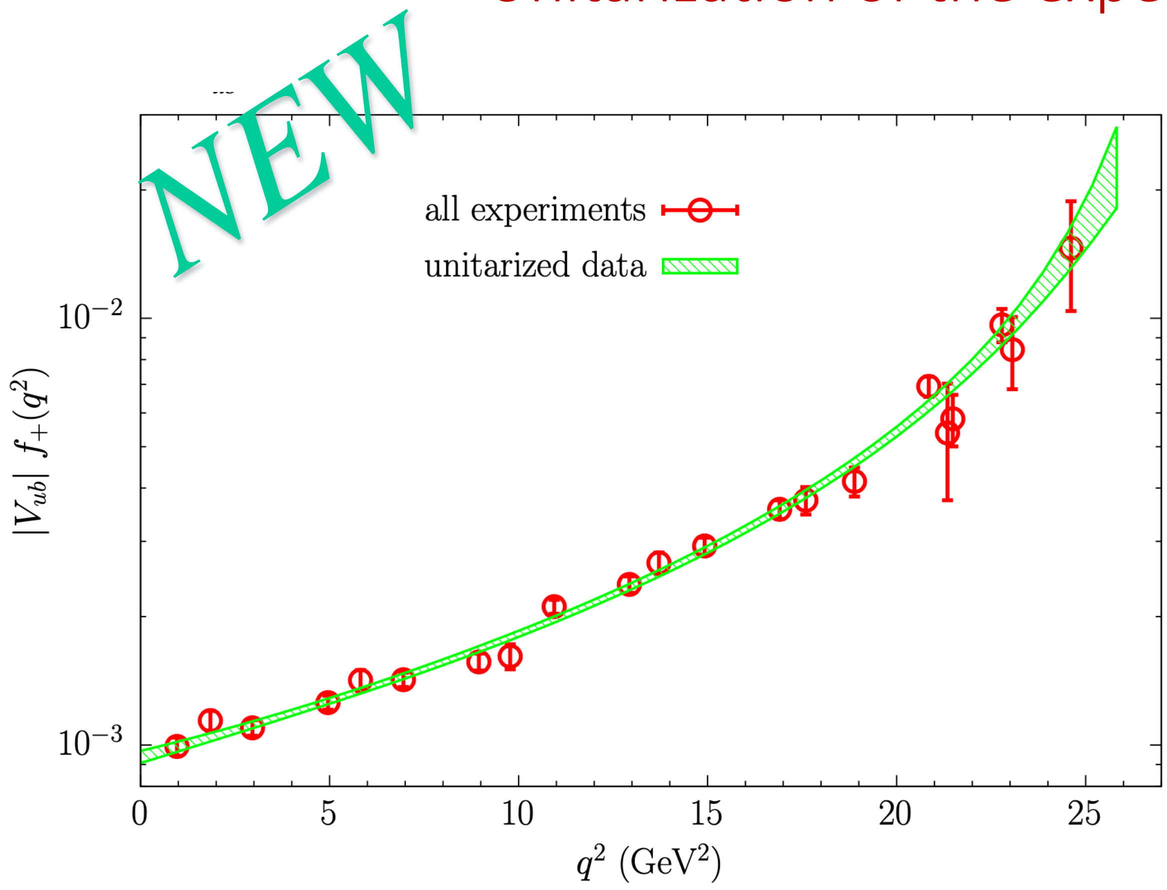
combined



	$f_+(0) = f_0(0)$
RBC/UKQCD	-0.06 ± 0.25
FNAL/MILC	-0.01 ± 0.16
Combined	-0.04 ± 0.22
LCSR	0.28 ± 0.03

- **3 RBC/UKQCD data (points) for each FF [arXiv:1501.05363]**
- **3 FNAL/MILC data (squares) for each FF [arXiv:1503.07839]**

Unitarization of the experimental data



$$|\mathbf{V_{ub}}|_{\mathbf{DM}} \times \mathbf{10^3} = \mathbf{3.88 \pm 0.32}$$

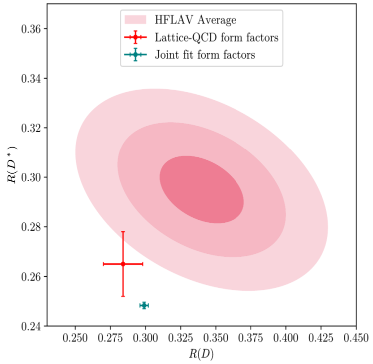
Reference	$\lvert V_{ub}\rvert \times 10^3$
FLAG '21	3.74 ± 0.17
HFLAV '18 & PDG '20	4.32 ± 0.29
Belle Coll. '21	4.10 ± 0.28

Exclusive and the inclusive values are compatible at the 1σ level

* construct the experimental values of $|V_{ub} f_+(q_i^2)| = \sqrt{\Delta\Gamma_i / z_i}$ (z_i = kinematical coefficient in the i-th bin)
* apply the DM method on the data points $|V_{ub} f_+(q_i^2)|$ using the unitarity bound $|V_{ub}|^2 \chi_{1^-}(0)$ with an initial guess for $|V_{ub}|$
* determine $|V_{ub}|$ using the theoretical DM bands and iterate the procedure until consistency for $|V_{ub}|$ is reached

S. Meinel CKM21

$R(D^{(*)})$



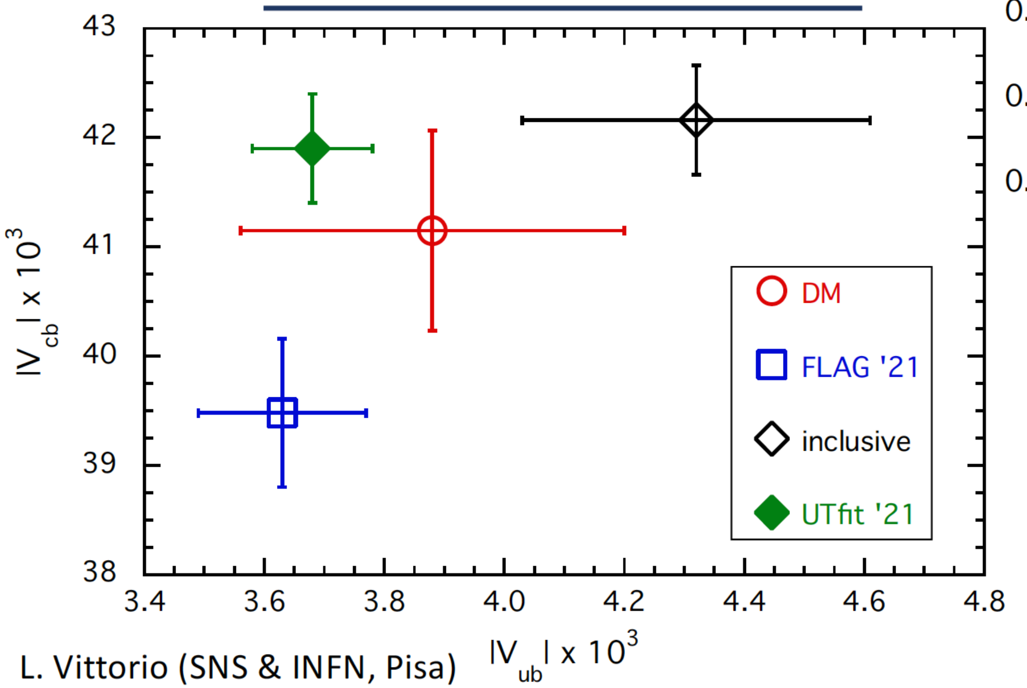
Lattice-QCD-only SM predictions:

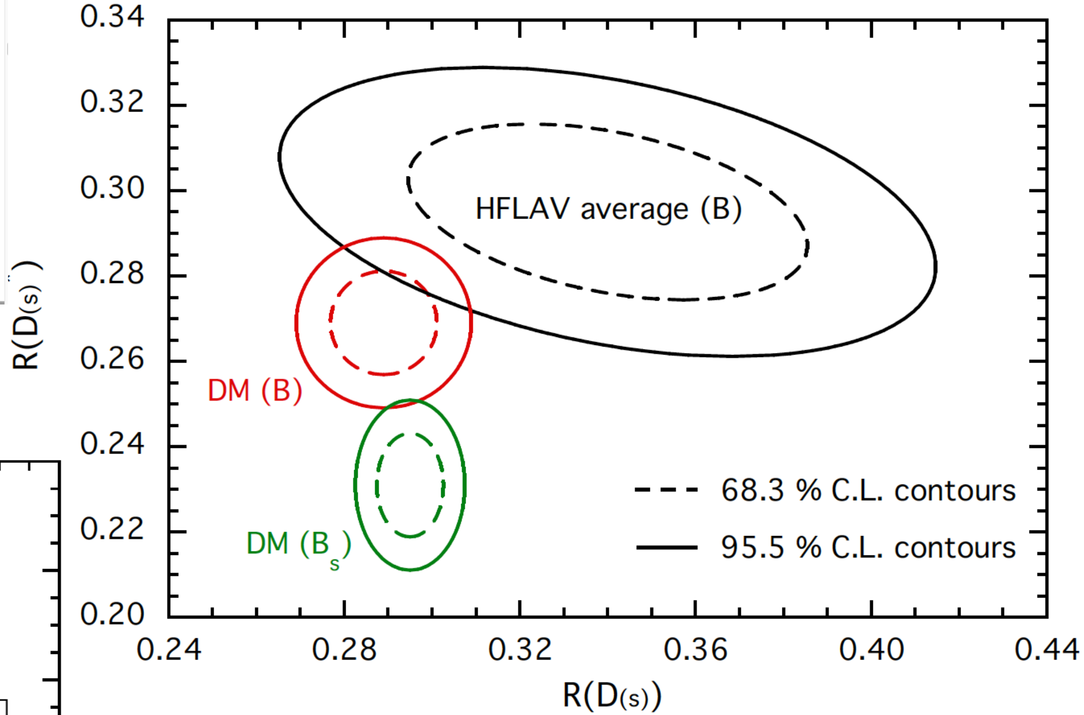
$$R(D^*)_{\text{FNAL/MILC 2021}} = 0.265 \pm 0.013$$
$$R(D)_{\text{FNAL/MILC 2015}} = 0.284 \pm 0.014$$
$$R(D)_{\text{FLAG 2021}} = 0.2934 \pm 0.0053$$

[FNAL/MILC, arXiv:2105.14019; HFLAV, arXiv:1909.12524/EPJC 2021]

See the talks by Judd Harrison, Shoji Hashimoto, Takashi Kaneko, Gumaro Rendon, and Silvano Simula for more lattice results on heavy→heavy decays!

CKM matrix elements





LFU observables

IMPORTANT: *the difference between* the red and the green area *comes from the difference in the LQCD computations by FNAL/MILC and HPQCD Collaborations*

Conclusions

*The **Dispersion Matrix approach** is a powerful tool to implement unitarity in the analysis of exclusive semileptonic decays of mesons and baryons*

- it does not rely on any assumption about the momentum dependence of the hadronic form factors

- it can be based entirely on first principles (*i.e*. unitarity and analiticity) using non-perturbative lattice determinations of both the relevant form factors and the dispersive bounds (the susceptibilities) from appropriate 2-point and 3-point (Euclidean) correlation functions

- it predicts band of values that are equivalent to all possible BGL fits satisfying unitarity and reproducing exactly a given set of data points. Larger but more reliable uncertainties

- It is not biased by the fit of the experimental data

- it is universal, namely it can be applied to any exclusive semileptonic decay e.g. baryon decays

Conclusions 2

New insight on both:

- *the |Vcb|, |Vub| puzzles (exclusive and inclusive determinations compatible @ the 1σ level)*

We found problems with the Belle covariance matrix

- *the R(D(*)) anomalies (theoretical values and measurements compatible @ the 1.6σ level)*

- *No apparent deviation in the down sector, what about the up one?*