

The MUonE experiment

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NePSi 23, Pisa, 15-17 Feb 2023

<https://agenda.infn.it/event/32931/>

The MUonE project

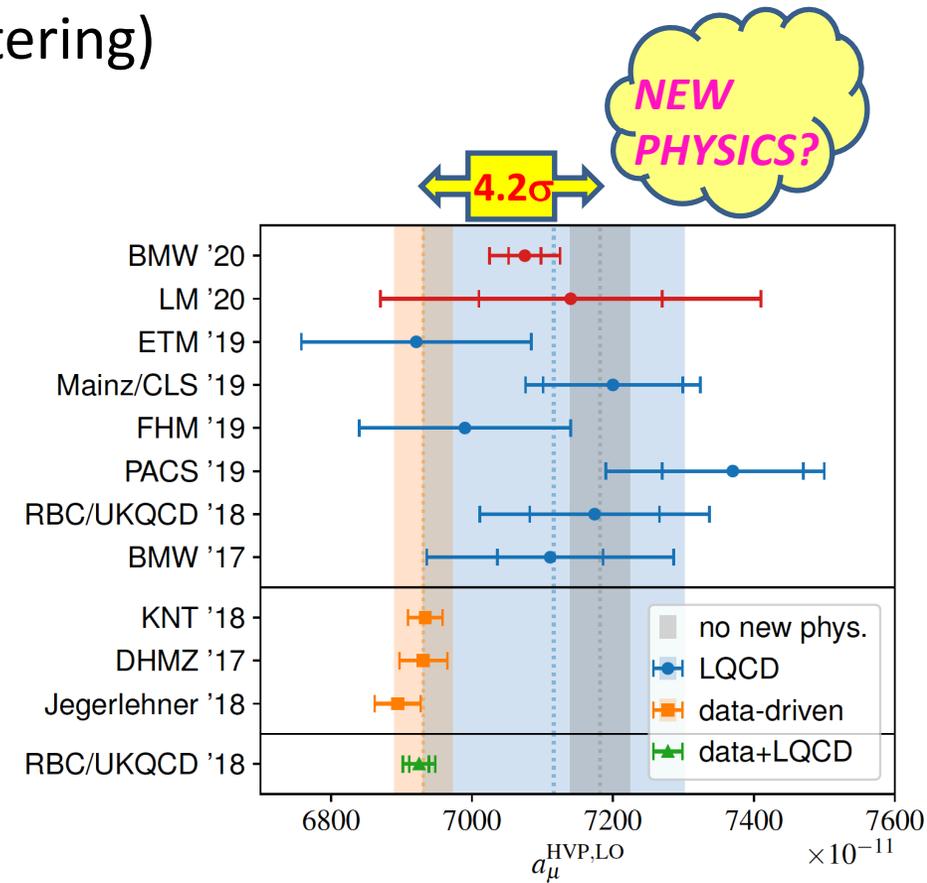
(MUon ON Electron elastic scattering)

Three possible methods to determine a_μ^{HLO} (leading hadronic contribution to the muon anomalous magnetic moment):

- Pure theory (Lattice QCD)
- Data-driven, from R-ratio measurements (timelike)
- Data-driven in spacelike domain (MUonE)



The MUonE experiment aims at an independent and precise determination of the leading hadronic contribution to the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$, based on an alternative method, complementary to the existing ones.



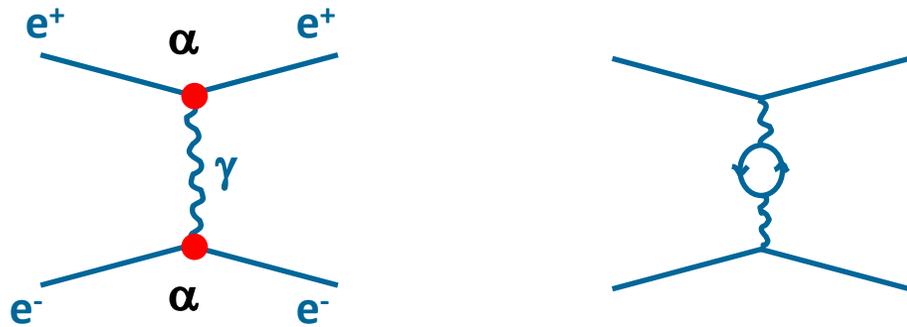
from M.Ce' [talk at Mainz \(2022\)](#)

Measurement of $\Delta\alpha_{\text{had}}(t)$ spacelike at LEP

OPAL measurement: [Eur.Phys.J.C45\(2006\)1](#)

Bhabha scattering at small angle, with $1.8 < -t < 6.1 \text{ GeV}^2$

about 10^7 events, precision at the per mille level



$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt} \left[\frac{\alpha(t)}{\alpha_0} \right]^2 (1 + \varepsilon)(1 + \delta_\gamma) + \delta_Z$$

Born term for t-channel single γ exchange

$$\left(\frac{1}{1 - \Delta\alpha(t)} \right)^2$$

Effective coupling factorized

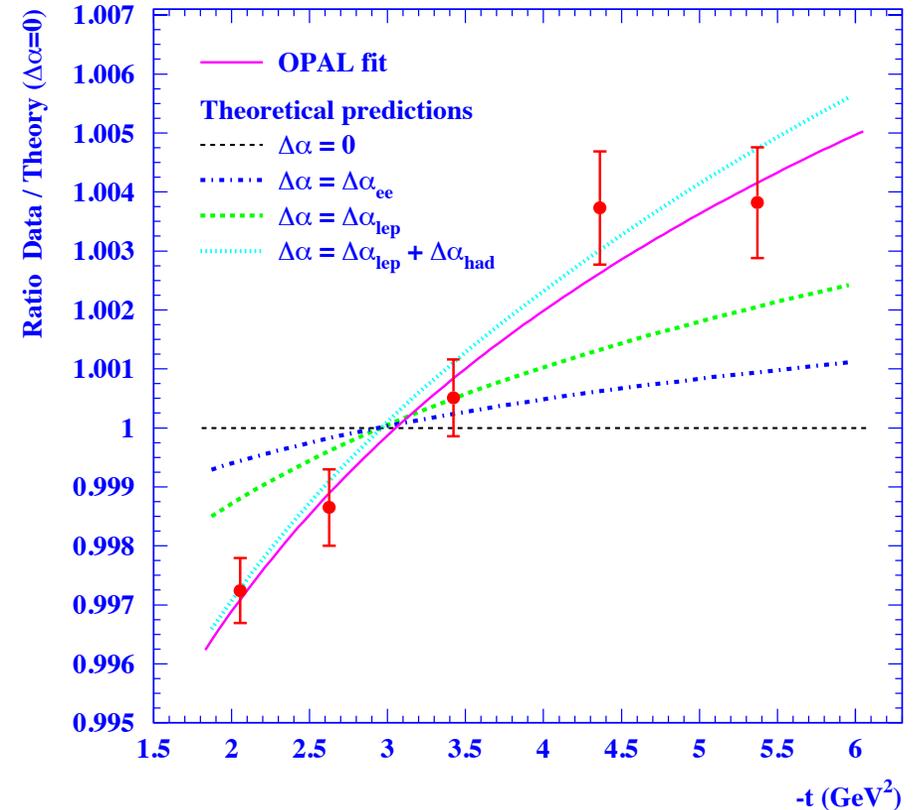
Photonic radiative corrections

Z interference correction

s-channel γ exchange correction

$$e^+e^- \rightarrow e^+e^- \quad \sqrt{s} \approx 91.2 \text{ GeV}$$

OPAL



Other measurements in the space-like region by L3, VENUS

a_μ^{HLO} : the MUonE approach (space-like data)

B. E. Lautrup, A. Peterman and E. de Rafael, Phys. Rept. 3 (1972) 193

[C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, Phys.Lett.B746\(2015\)325](#) :

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

propose to determine a_μ^{HLO} from the measurement of the running of α in Bhabha scattering

$\Delta\alpha_{\text{had}}$ is the hadronic contribution to the running of α in the space-like region ($t < 0$):

$$\alpha(t) = \frac{\alpha}{1 - \Delta\alpha(t)}$$

$$\Delta\alpha = \Delta\alpha_{\text{lep}} + \Delta\alpha_{\text{had}}$$

$$0 \leq -t < \infty$$

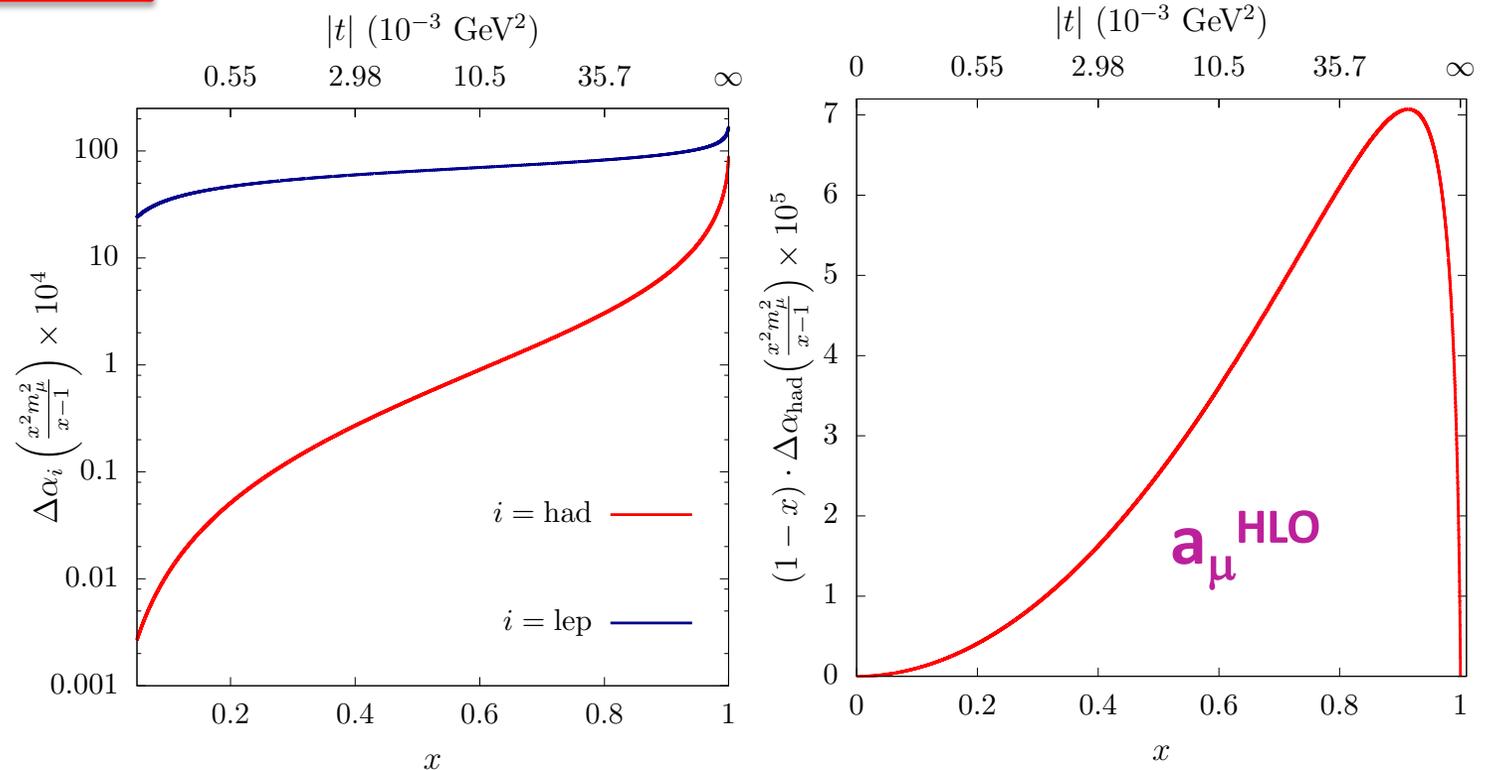
$$0 \leq x < 1$$

$$t(x) = -\frac{x^2 m_\mu^2}{1-x}$$

Integrand function smooth: no resonances

Low-energy enhancement:

peak of the integrand at $x \cong 0.9 \rightarrow t = -0.11 \text{ GeV}^2 \rightarrow \Delta\alpha_{\text{had}} \sim 10^{-3}$



MUonE experiment idea

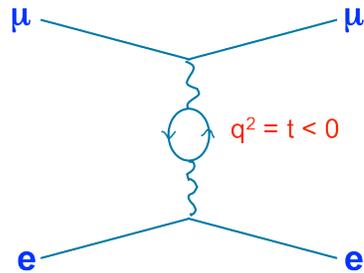
Eur. Phys. J. C (2017) 77:139
DOI 10.1140/epjc/s10052-017-4633-z

Regular Article - Experimental Physics

Measuring the leading hadronic contribution to the muon $g-2$ via μe scattering

G. Abbiendi^{1,a}, C. M. Carloni Calame^{2,b}, U. Marconi^{3,c} , C. Matteuzzi^{4,d}, G. Montagna^{2,5,e}, O. Nicrosini^{2,f}, M. Passera^{6,g}, F. Piccinini^{2,h}, R. Tenchini^{7,i}, L. Trentadue^{8,4,j}, G. Venanzoni^{9,k}

[Eur.Phys.J.C77\(2017\)139](#)



$$\frac{d\sigma}{dt} \approx \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha(0)} \right|^2 \approx \frac{d\sigma_0}{dt} \left| \frac{1}{1 - \Delta\alpha(t)} \right|^2$$

running of α

$$\Delta\alpha(t) = \underbrace{\Delta\alpha_{lep}(t)}_{\text{known from QED}} + \underbrace{\Delta\alpha_{had}(t)}_{\text{to be measured}}$$

From $\Delta\alpha_{had}(t)$ determine a_μ^{HLO} by the space-like approach: [Phys.Lett.B746\(2015\)325](#)

$$a_\mu^{HLO} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{had}[t(x)]$$

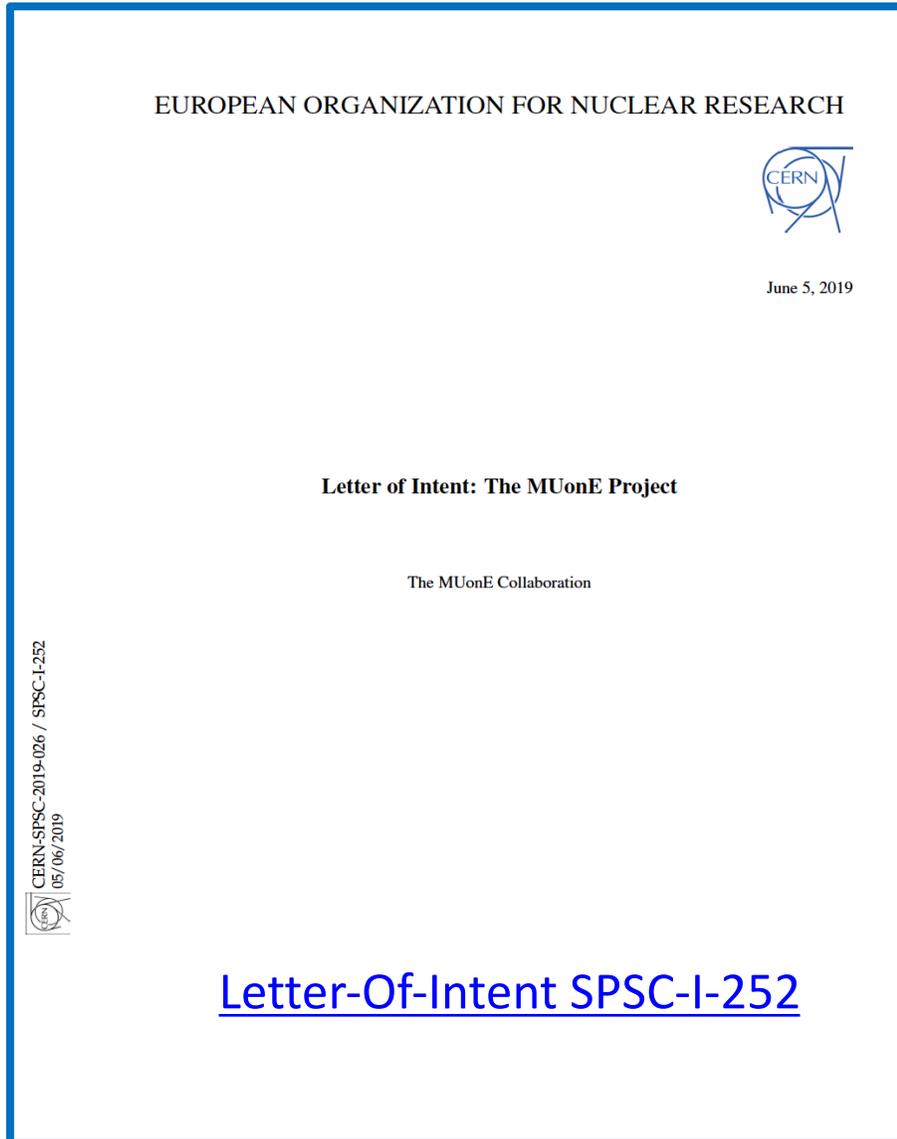
Very precise measurement of the running of α_{QED} from the shape of the differential cross section of elastic scattering of $\mu(150-160\text{GeV})$ on atomic electrons of a fixed target with low Z (Be or C)
→ CERN SPS

MUonE experiment proposal

5 June 2019: LoI submitted to SPSC

22 January 2020: SPSC acknowledges the fundamental interest of the proposal and approves a Test Run, which should verify the detector design and the potential outcomes

Test Run to be completed in 2023



Being formed
still growing up



INFN +Univ. (Bologna, Milano-Bicocca, Padova, Pavia, Perugia, Pisa, Trieste)
Exp-Th



CERN
Exp-Th



Imperial College (London), Liverpool U., Durham U.
Exp-Th



Krakow IFJ Pan
Exp



The MUonE
Collaboration



Cornell U., Northwestern U., Regis U., Virginia U.
Exp



Budker Inst. (Novosibirsk)
Exp



Demokritos INPP (Athens)
Exp-Th



Shanghai Jiao Tong U.
Exp



PSI (Villigen), U.Zürich, ETH Zürich
Th



Mainz U., Max-Planck Inst.
Exp-Th

+ other involved theorists from: New York City Tech (USA), Vienna U. (A)

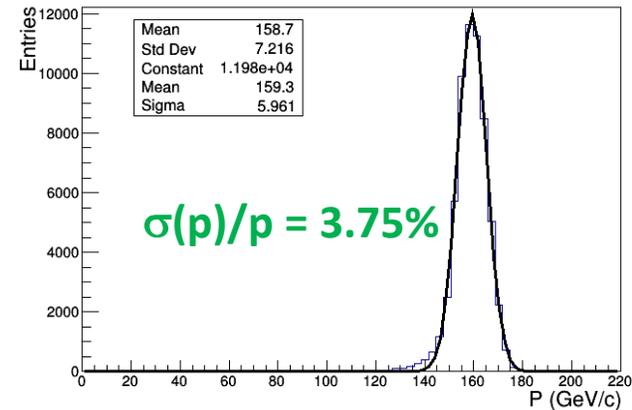
Location @ CERN & M2 beam parameters

[MUonE Letter-Of-Intent SPSC-I-252](#)

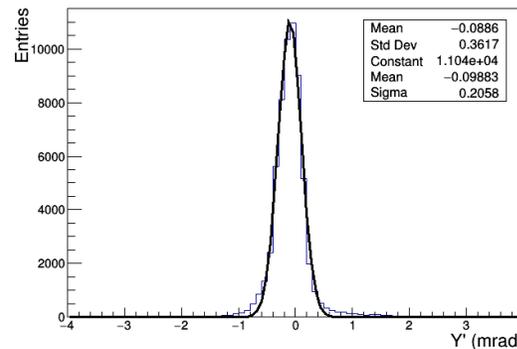
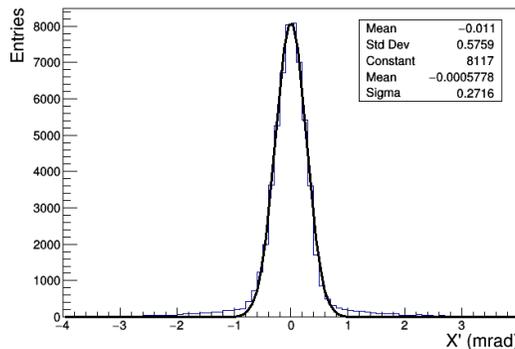


Upstream of the COMPASS detector, after its Beam Momentum Station (BMS), on the M2 beam line : available ~ 40 m

Beam Momentum

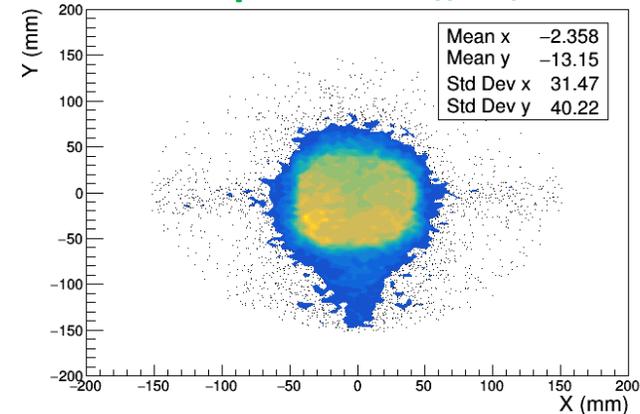


Very small divergence ~0.2-0.3 mrad

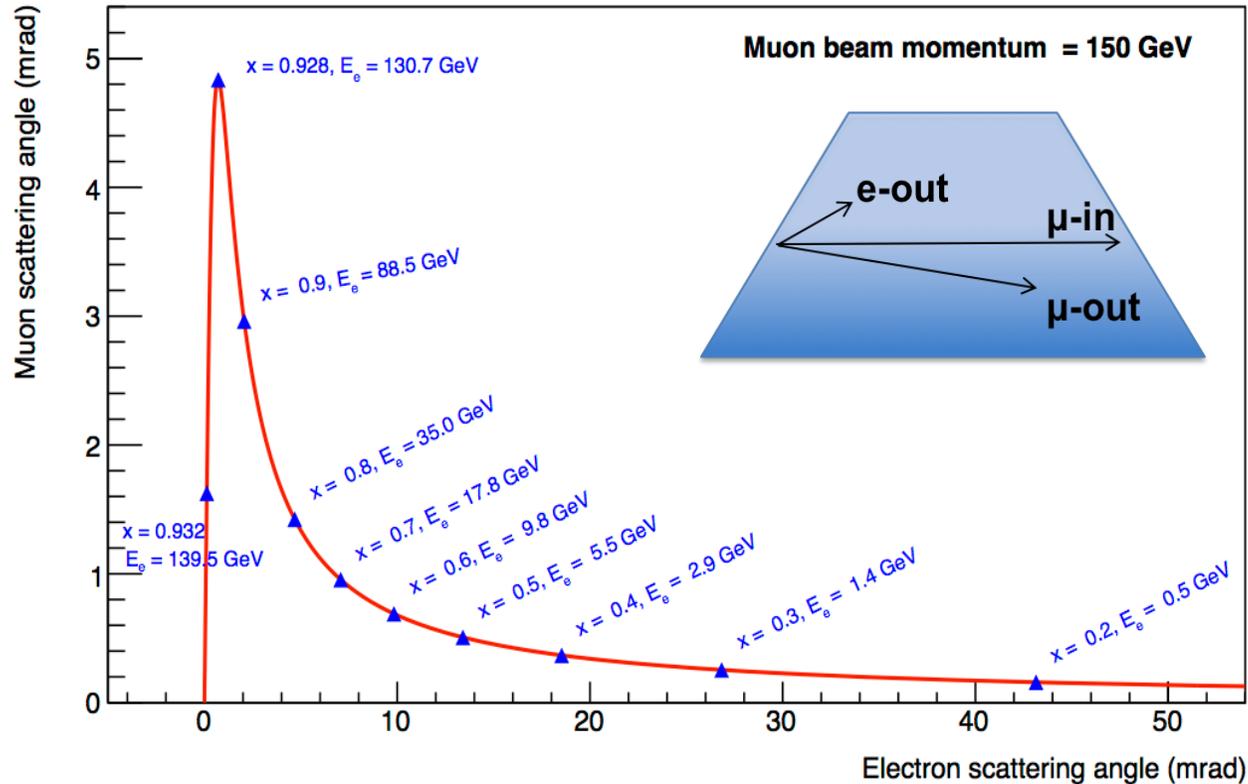


M2 beam typical max intensity: $5 \times 10^7 \mu/s$
SPS Fixed Target cycle ~15-20 s / Spill duration ~ 5s

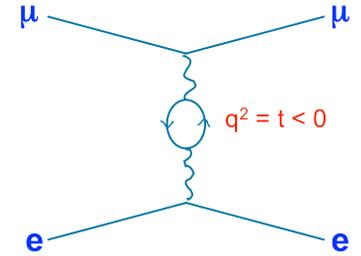
Beam spot size $\sigma_x \sim \sigma_y \sim 3\text{cm}$



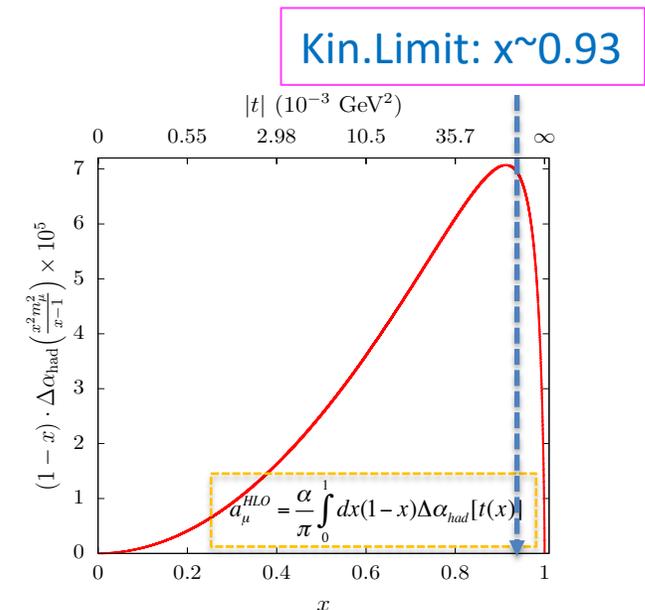
μ -e Elastic scattering: pros



- **Simple kinematics:** $t \approx -2 m_e E_e$
 E_e can be determined from the scattering angle θ_e and the beam energy
- Scattering angles θ_e and θ_μ are correlated
- Events are planar



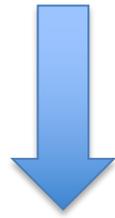
- For $E(\text{beam})=160$ GeV the phase space covers **88%** of the a_μ^{HLO} integral
- ❖ Smooth extrapolation to the full integral with a proper fit model



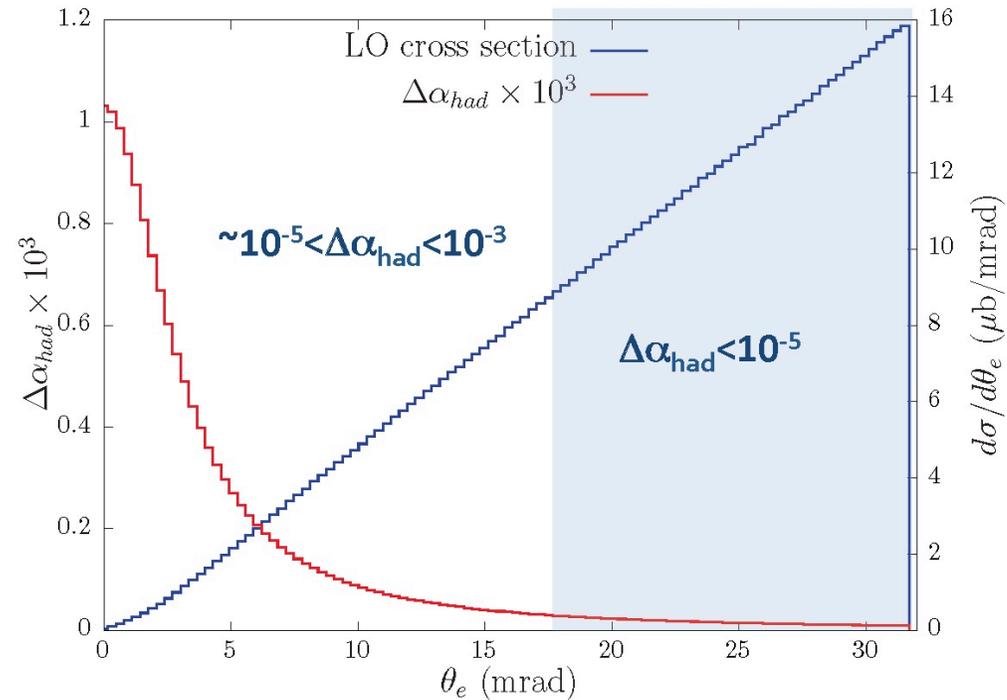
μ -e elastic scattering: challenges

Observable effect $\sim 10^{-3}$

wanted accuracy $\sim 10^{-2}$



Required precision $\sim 10^{-5}$
on the shape of $d\sigma/dt$



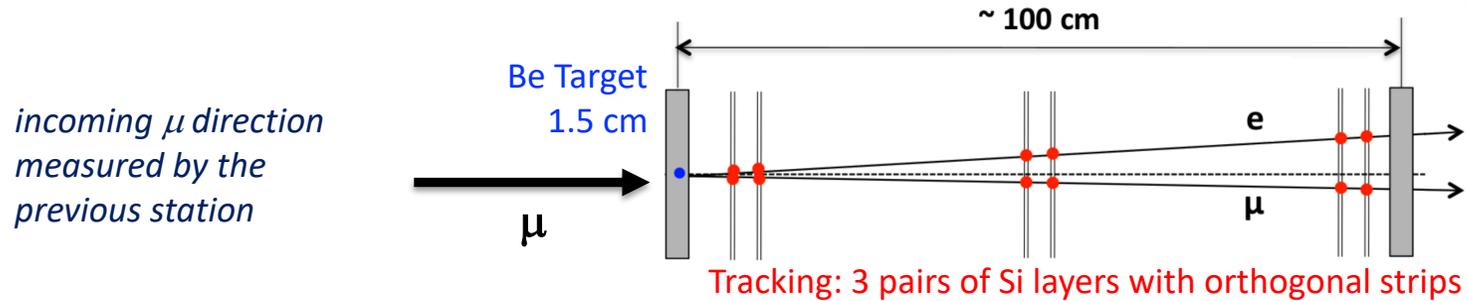
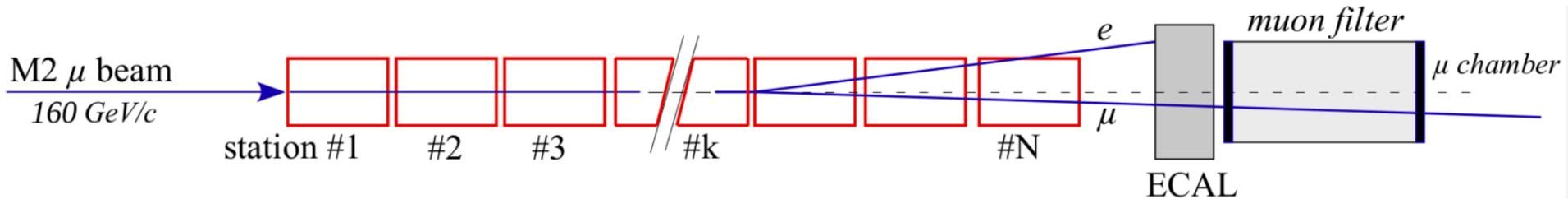
- Large statistics to reach the necessary sensitivity
- Minimal distortions of the outgoing e/ μ trajectories within the target material and small rate of radiative events

MUonE Detector Layout

[Letter-Of-Intent SPSC-I-252](#)

The detector concept is simple, the challenge is to keep the systematics at the same level as the statistical error .

- Modular structure of 40 independent and precise tracking stations, with split light targets equivalent to 60cm Be
- ECAL and Muon filter after the last station, to help the ID and background rejection

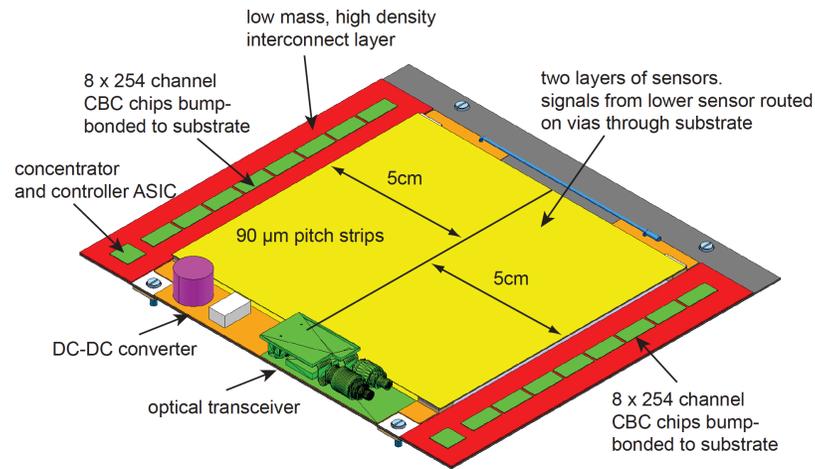


- Boosted kinematics: $\theta_e < 32 \text{ mrad}$ (for $E_e > 1 \text{ GeV}$), $\theta_\mu < 5 \text{ mrad}$:
 - ❖ the whole acceptance can be covered with a $10 \times 10 \text{ cm}^2$ silicon sensor at 1m distance from the target, reducing many systematic errors

Detector choice: CMS-upgrade Outer Tracker 2S

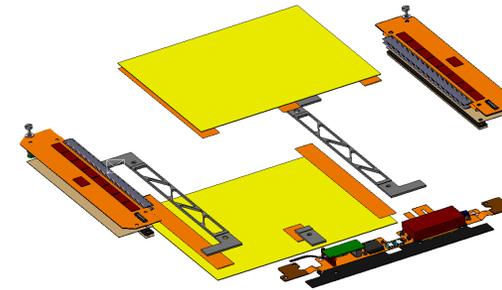
[MUonE Letter-Of-Intent SPSC-I-252](#)

Details: see [CMS Tracker Upgrade TDR](#)



Two close-by planes of strips reading the same coordinate, providing track elements (**stubs**)

suppression of background from single-layer hits or large-angle tracks

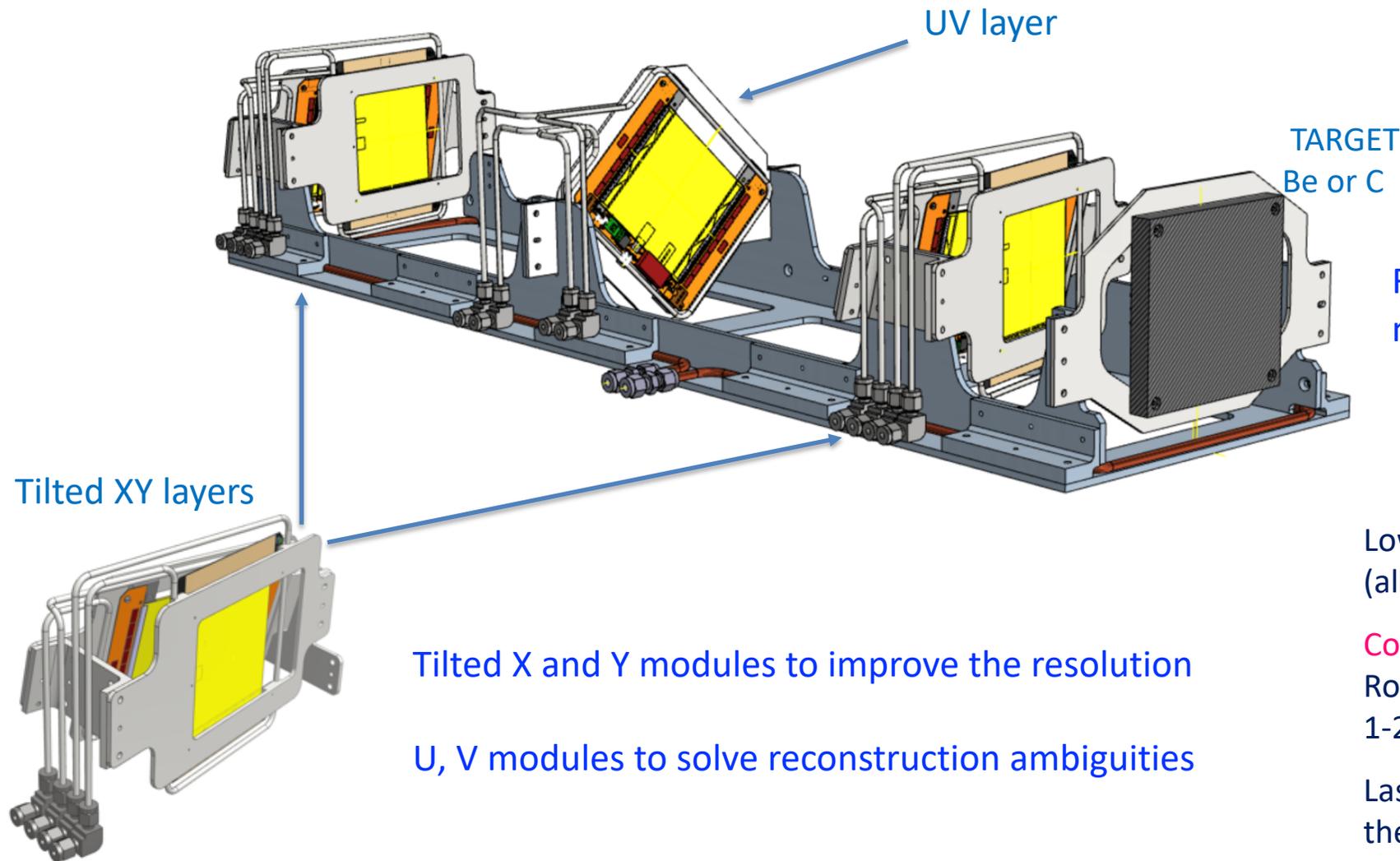


➤ Large active area $10 \times 10 \text{ cm}^2$
-> complete/uniform angular coverage with a single sensor

➤ Position resolution $\sim 20 \mu\text{m}$
-> improvable to $\sim 10 \mu\text{m}$ with a 15° - 20° tilt around the strip axis and/or with effective staggering of the planes (with a microrotation)

MAIN Difference w.r.t. LHC operation: signal is asynchronous while sampling has fixed clock at 40MHz -> can be overcome with a specific configuration of the FE

MUonE tracking station



Length 1m
Transverse size 10cm

TARGET
Be or C

Relative positions of modules
must be stable within $10\mu\text{m}$

Low CTE support structure: INVAR
(alloy of 65%Fe, 35%Ni)

Cooling system, tracker enclosure,
Room temperature stabilized within
1-2 °C

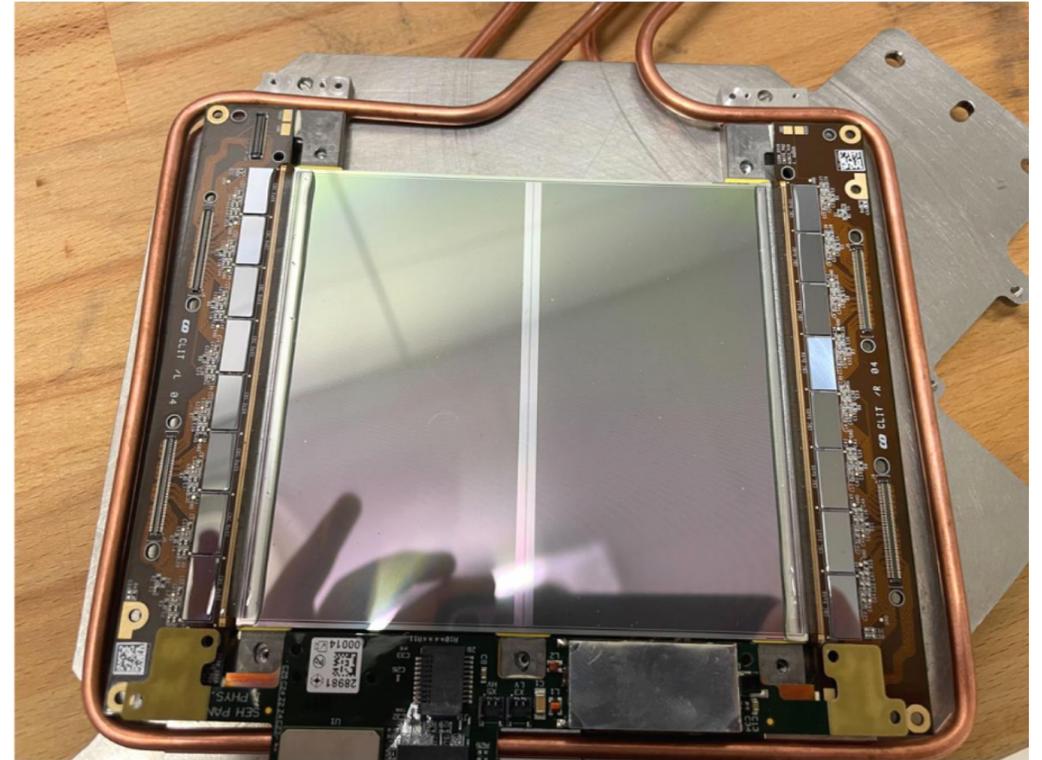
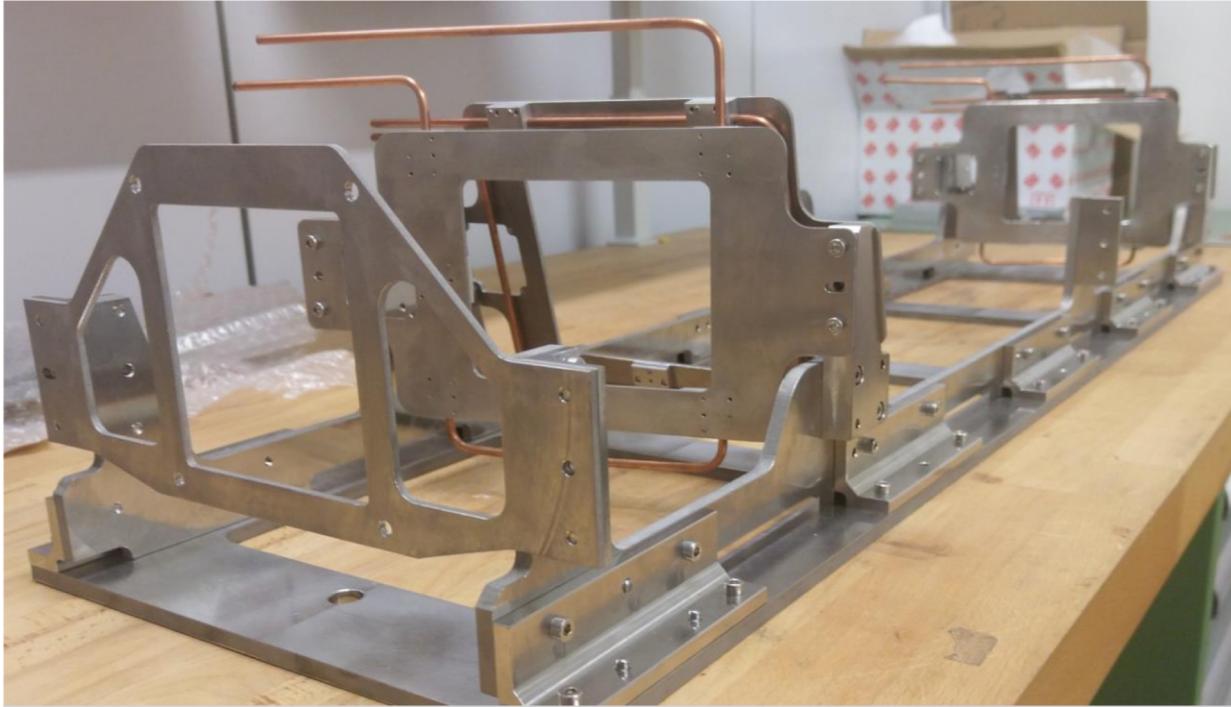
Laser holographic system to monitor
the stability

Tilted X and Y modules to improve the resolution

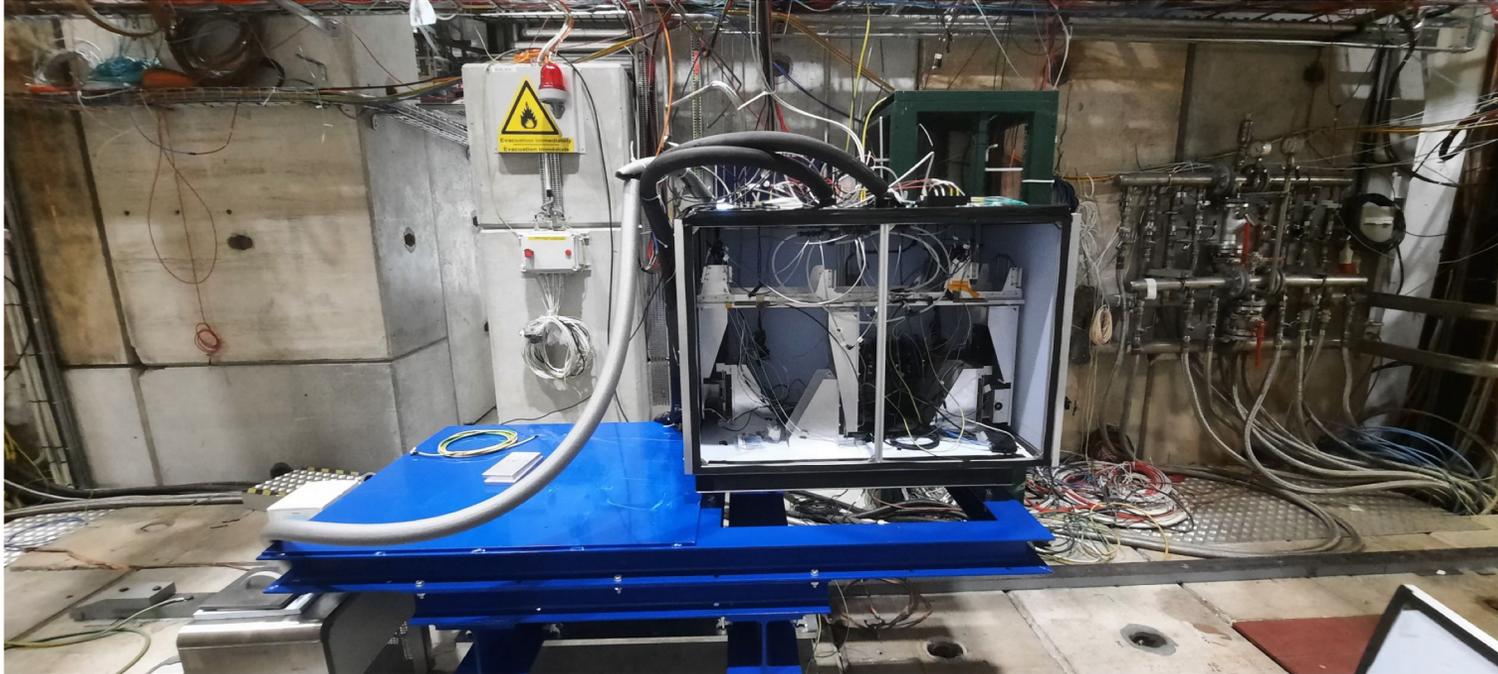
U, V modules to solve reconstruction ambiguities

Support structure and 2S module on its frame

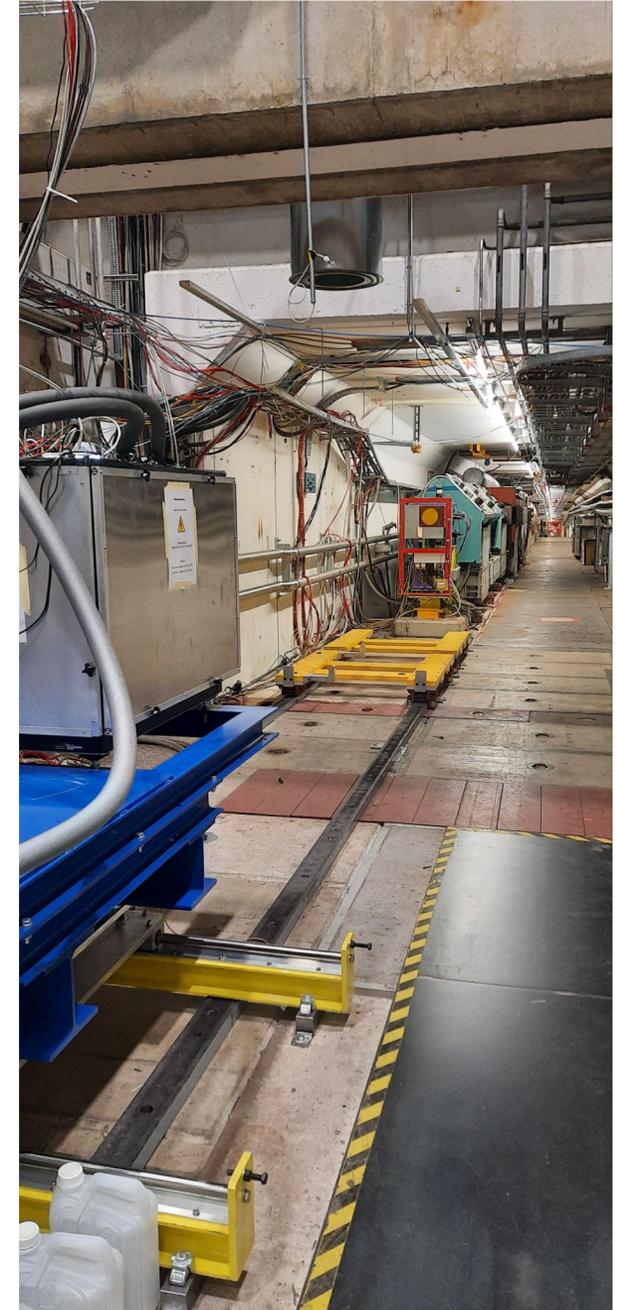
The support structure in INVAR



Tracker beam test setup



- Joint tests with CMS-Tracker in 2021-2022
- In 2022 the MUonE setup was placed on rails to allow easy movements in and out the M2 beam



Readout & DAQ

- Station is read out optically (IpGBT)
 - ~60m optical fibre from detector to barracks
 - Serenity implements IpGBT serial link and FE decoding, stub data processing at 40MHz and optionally event filtering
 - Packaged events transmitted over 10GbE links to servers
- Servers receive 10GbE data
 - Ryzen 9 5900X server PCs
 - 128GB RAM, 1TB NVMe SSD, 40TB RAID
 - Buffering, packaging, DQM, ship to EOS
- 10/100GbE switch
 - Direct 100GbE connection to B513/EOS
- FC7s for aux functions

IpGBT MFC

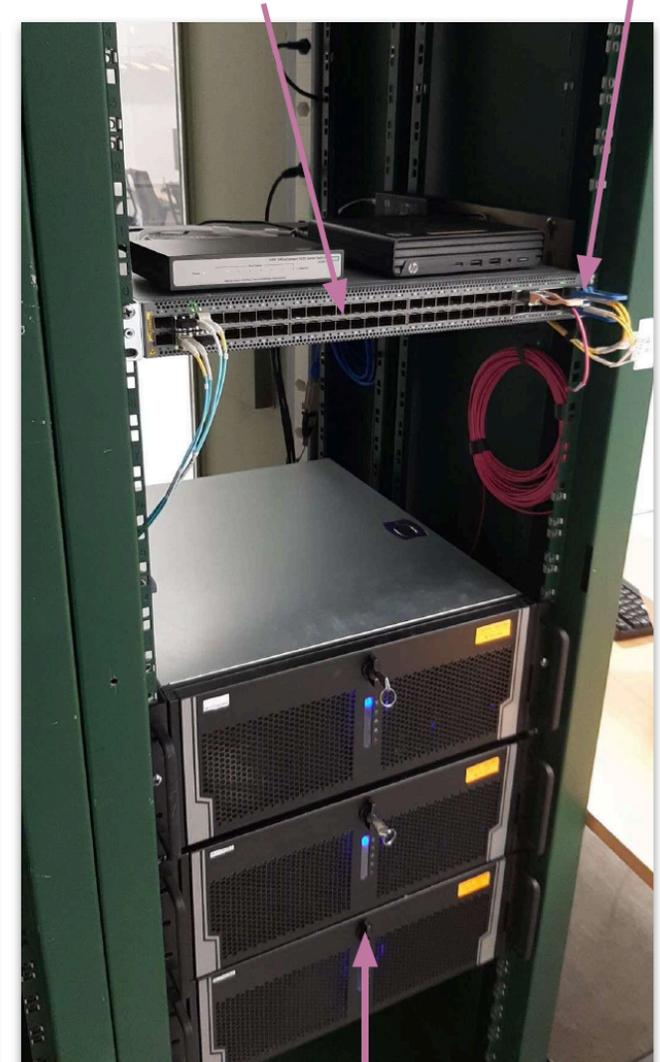
aux FC7s

10/100GbE switch

to EOS



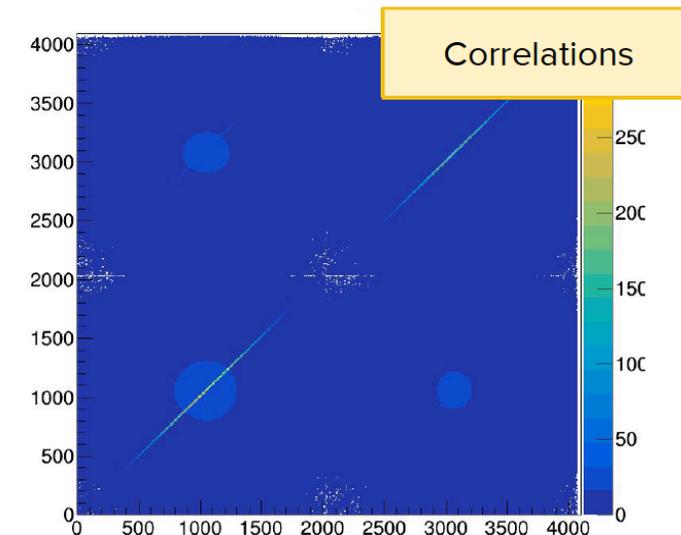
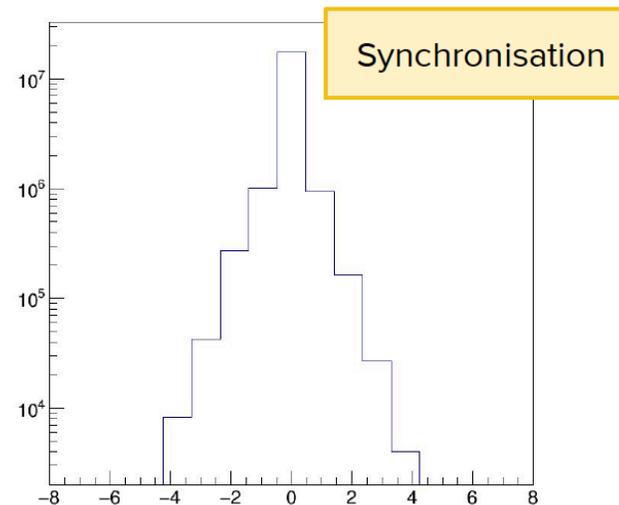
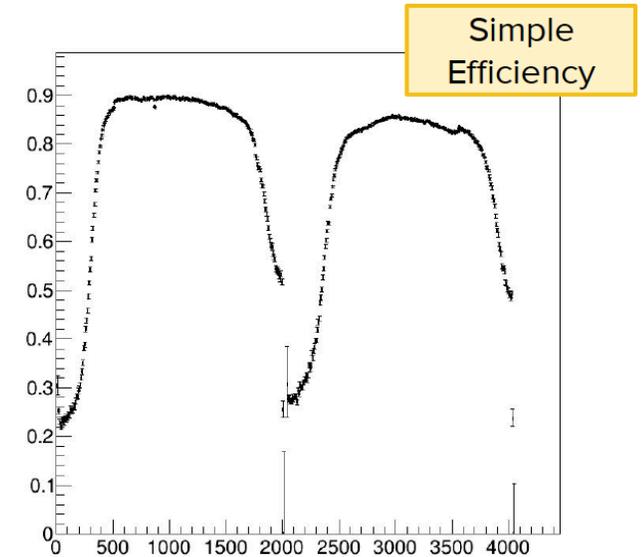
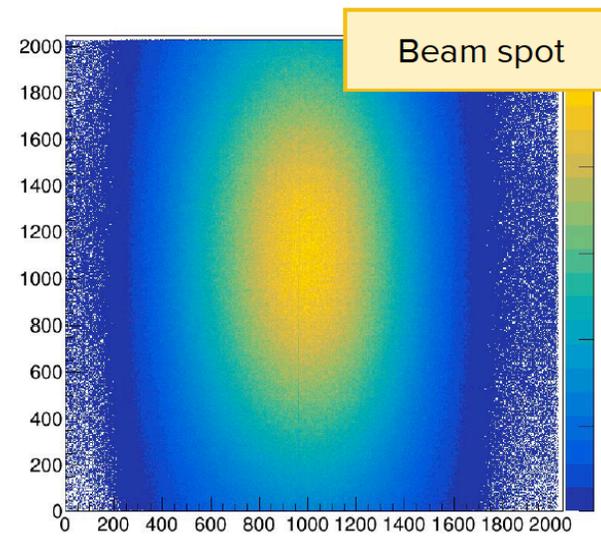
Serenity



Server PCs

Online DQM

- Data stored on EOS for current data-taking run is sampled
 - Plots displayed on webUI
- Performed more complex calculations on data:
 - Synchronisation between modules
 - System alignment
 - Beam spot
 - Stub efficiency
- Many previous analyses have now become part of the DQM!

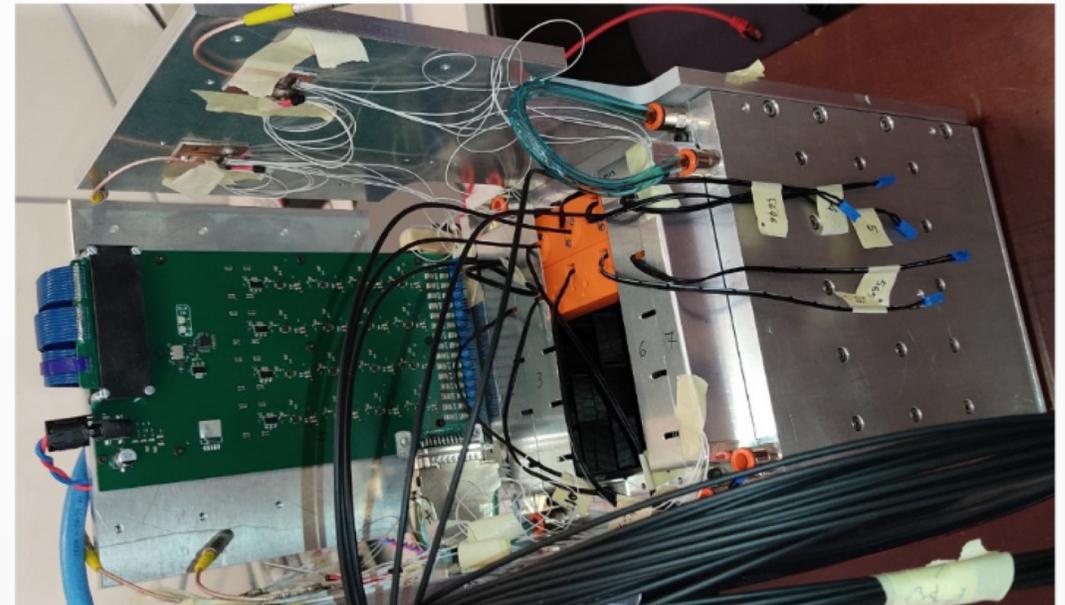


ECAL

- 5x5 PbWO₄ crystals:
 - area: 2.85×2.85 cm², length: 22cm (~25 X₀).
- Total area: ~14×14 cm².
- Readout: APD sensors.

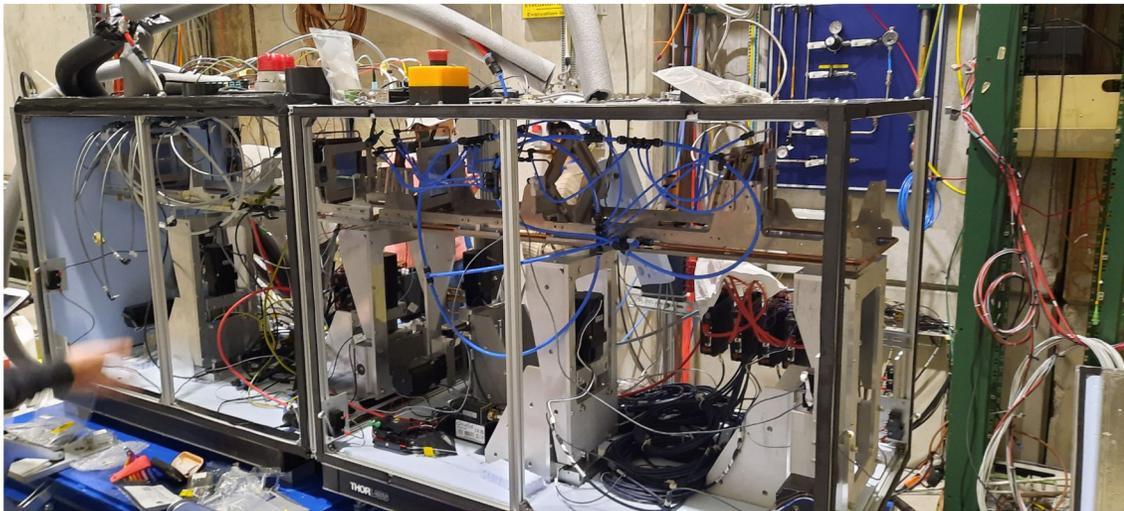
Beam Test: 20-27 July 2022,
CERN East Area.

- Electrons in range 1-4 GeV.
- Overall debug of detector, DAQ.
- Absolute energy calibration, energy resolution.
- Calorimeter being installed downstream of the tracking station at the M2 beam line.



Test with a full station and a calorimeter (October '22)

- A fully equipped tracking station and the calorimeter
- One week of test in M2 as main users performed the last October
- High intensity, asynchronous muon beam (160 GeV), up to 2×10^8 muon/spill, for the first time
 - or low intensity 40 GeV electrons
- Tracker system always on, DAQ reliable throughout
- Some 100TB of stub data collected



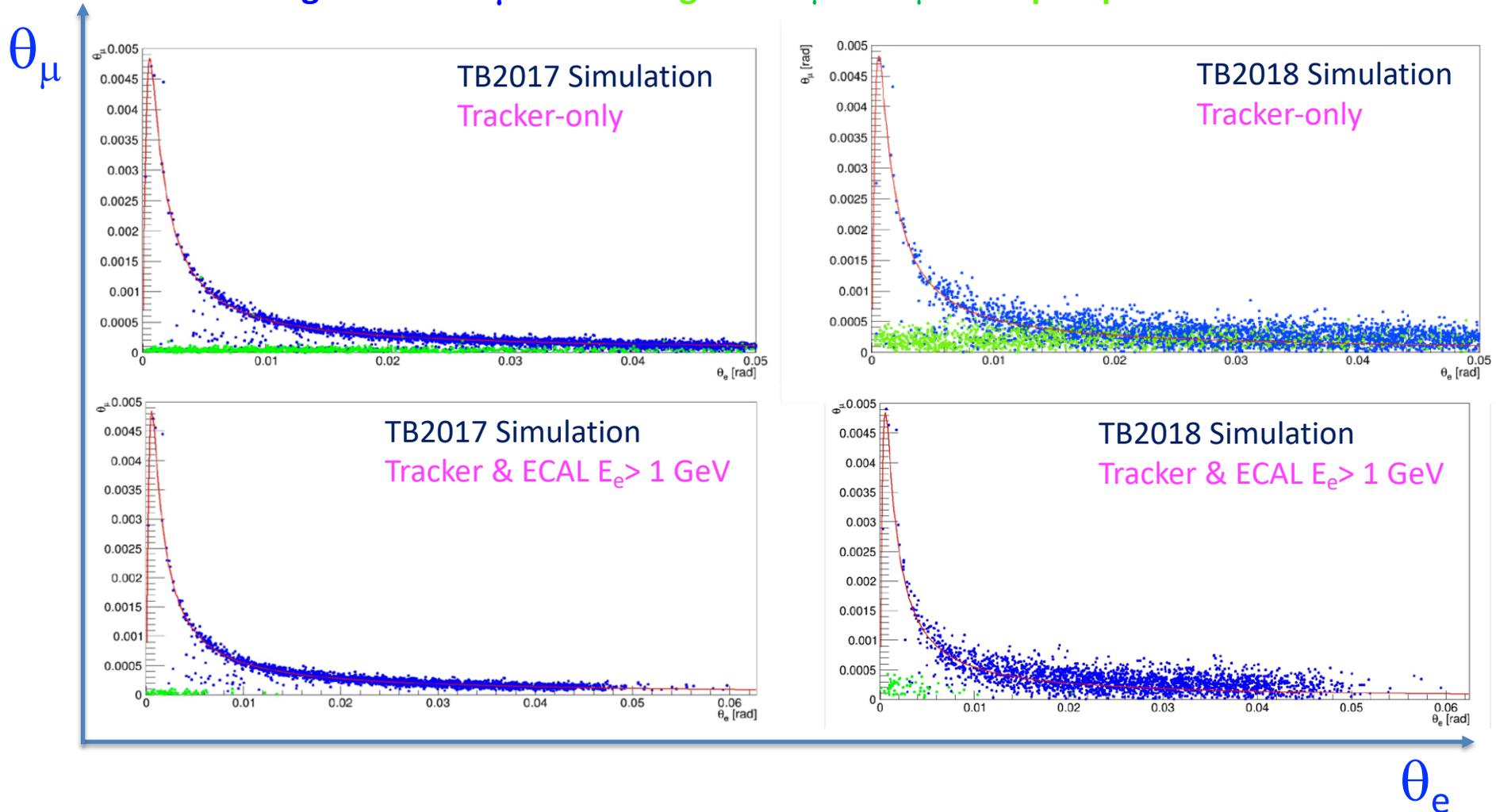
GEANT4 simulations

Effect of the position resolution on θ_μ vs θ_e distribution:

(Left) TB2017: UA9 resolution $7\mu\text{m}$; (Right) TB2018: resolution $\sim 35\text{-}40\mu\text{m}$

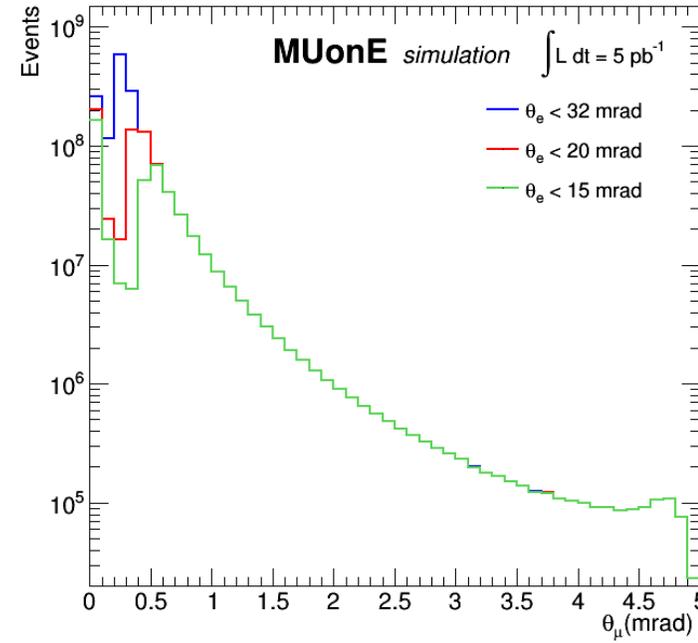
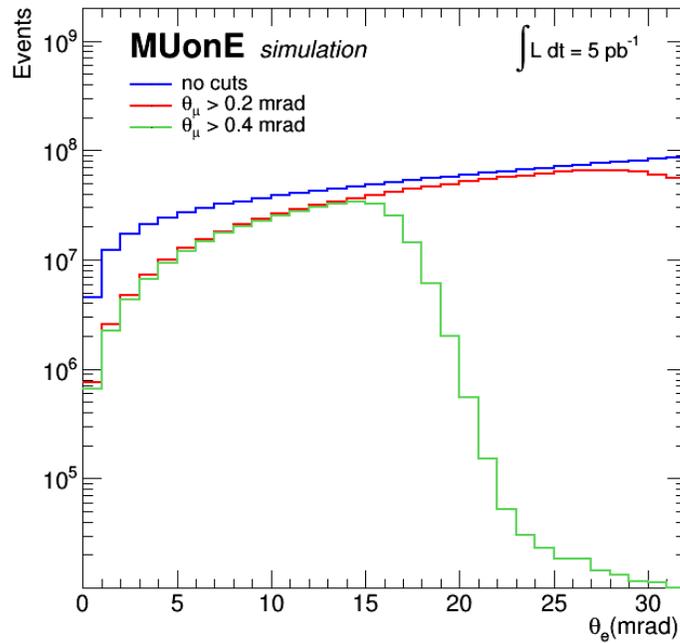
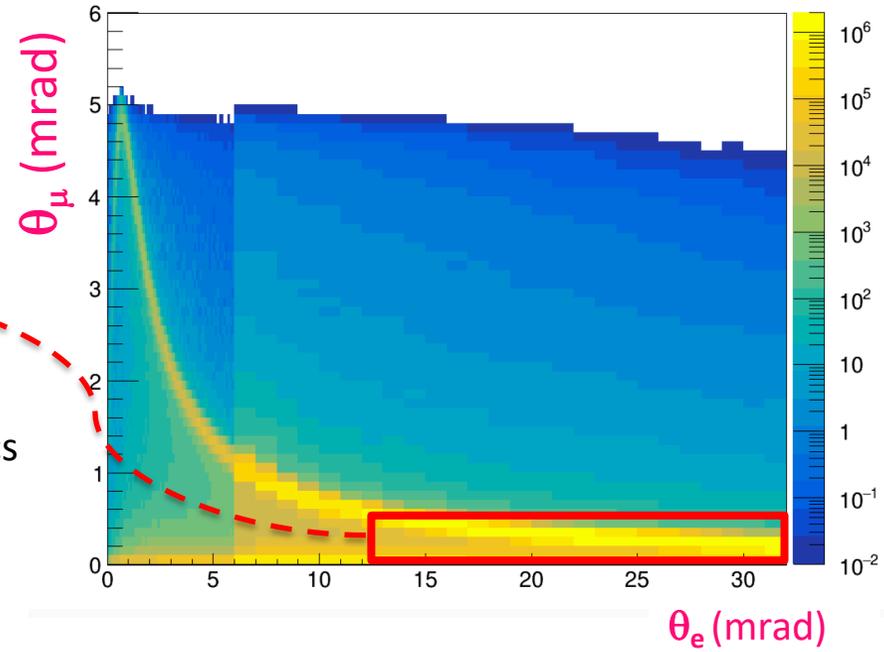
Signal: elastic μ

Background: $\mu\text{N} \rightarrow \mu\text{Ne}^+e^-$ pair production



Event kinematics

Normalisation region
 huge statistics
 vanishing signal ($\Delta\alpha_{\text{had}}$)
 convenient for studies of systematics



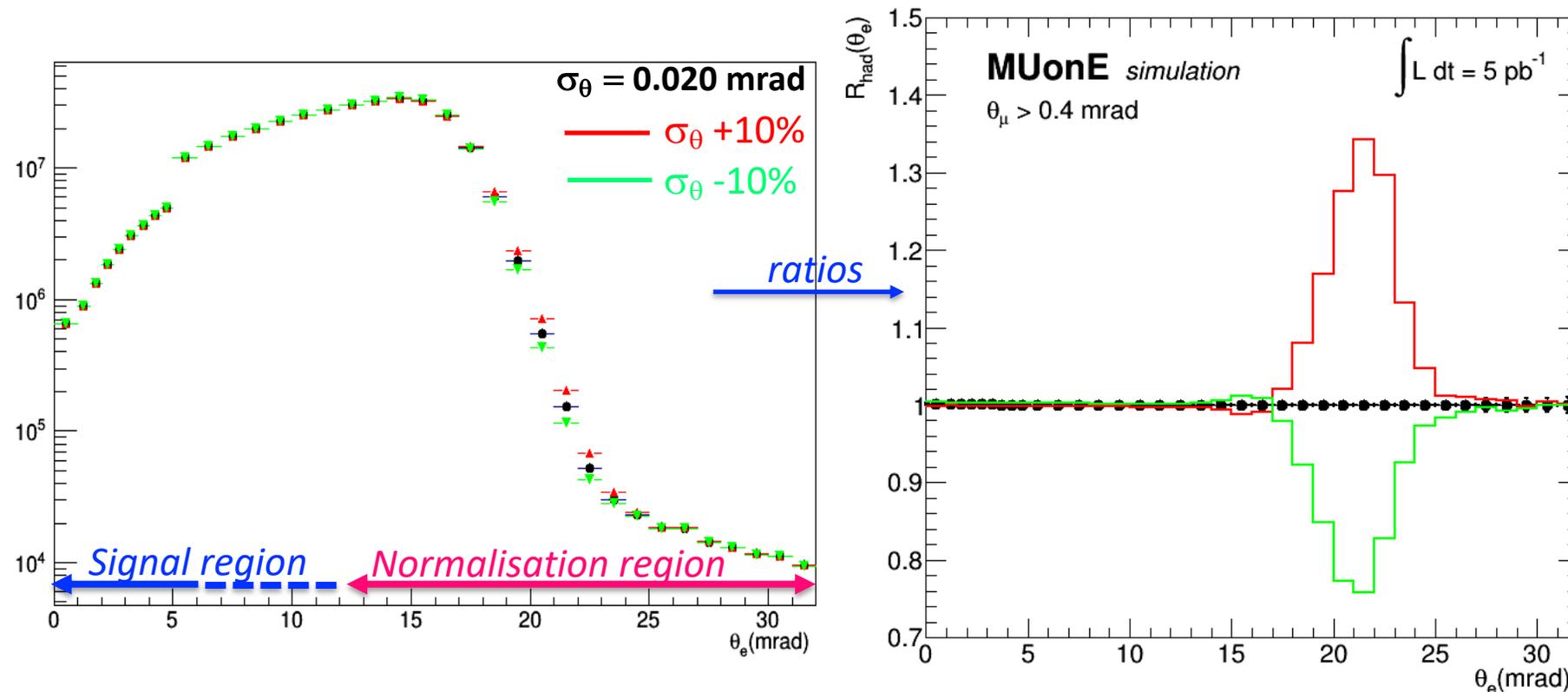
Probing systematics in the normalisation region

The **intrinsic angular resolution** can be probed by looking at the θ_e distribution after a cut on θ_μ distribution, e.g. cutting at $\theta_\mu > 0.4$ mrad

→ Effect of a $\pm 10\%$ error w.r.t. the nominal $\sigma_\theta = \mathbf{0.020}$ mrad

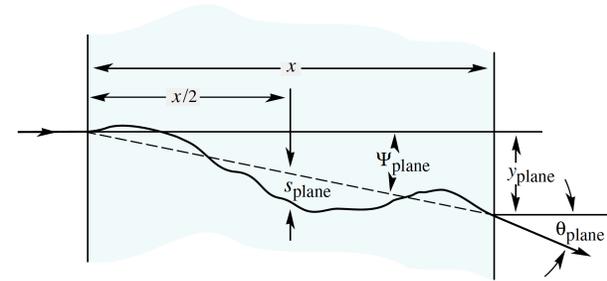
Huge distortion of 20-30% around electron angles of 20 mrad

No effect in the signal region

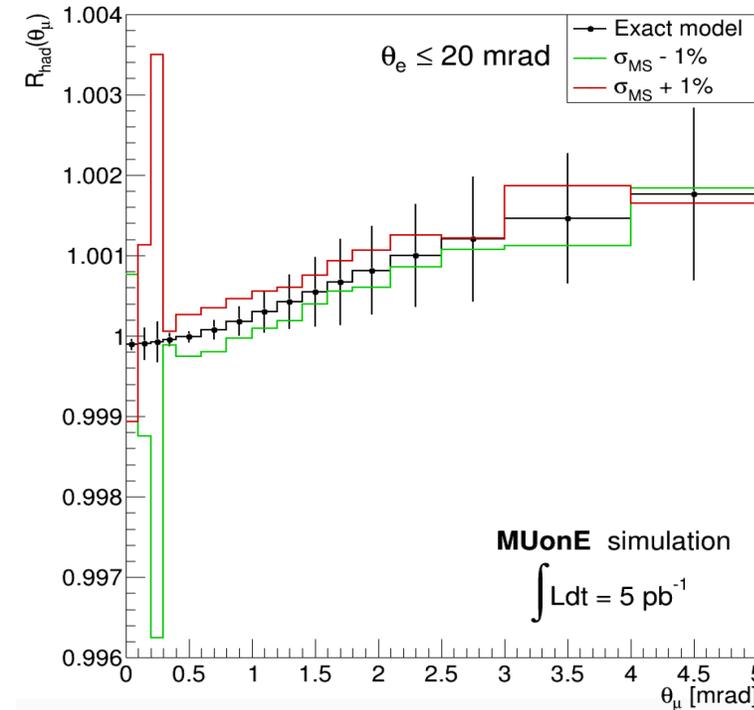
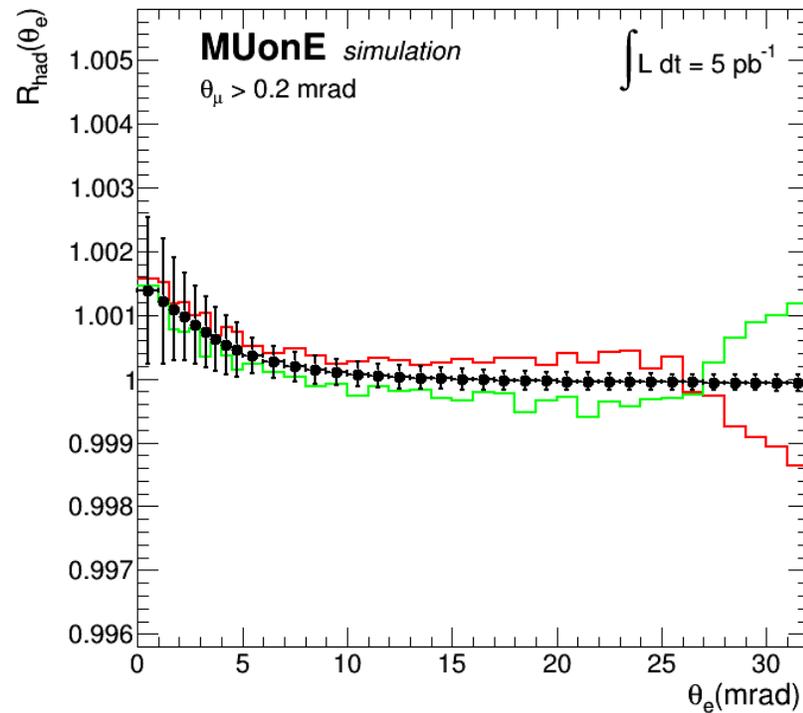


Systematics: Multiple Coulomb Scattering

Effect of a flat error of $\pm 1\%$ on the core width of multiple scattering



— +1%
— -1%



Multiple scattering previously studied in a Beam Test in 2017: [JINST 15 \(2020\) P01017](#)
with 12–20 GeV electrons on 8-20 mm C targets

$\Delta\alpha_{had}$ parameterisation

Physics-inspired from the calculable contribution of lepton-pairs and top quarks at $t < 0$



$$q^2 = t < 0 \quad \Delta\alpha_{had}(t) = k \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \log \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\}$$

M with dimension of mass squared, related to the mass of the fermion in the vacuum polarization loop
 k depending on the coupling $\alpha(0)$, the electric charge and the colour charge of the fermion

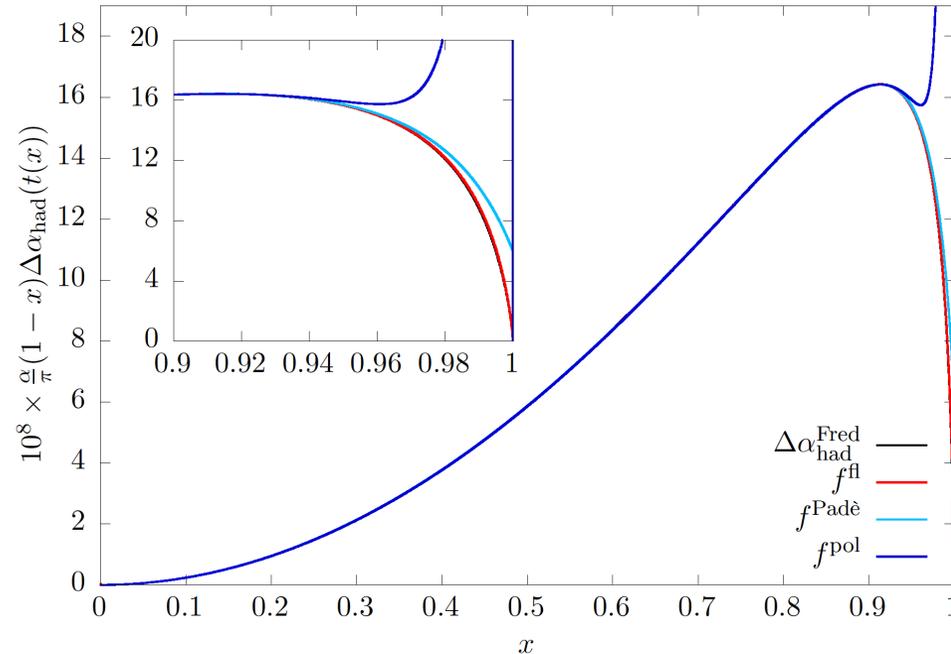
Low- $|t|$ behavior dominant in the MUonE kinematical range:

$$\Delta\alpha_{had}(t) = -\frac{1}{15} \frac{k}{M} t$$

for $t \rightarrow -0$

$$\Delta\alpha_{had}(t) = \frac{k}{3} \left(\log \frac{|t|}{M} - \frac{5}{3} \right)$$

for $t \rightarrow -\infty$

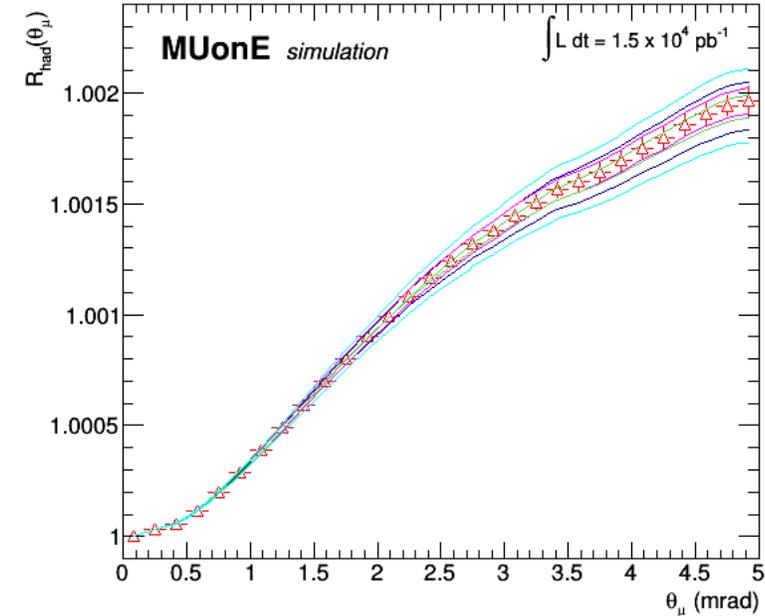


Template fit

Full description in:

[Phys. Scripta 97 \(2022\) 054007](#) [[arXiv:2201.13177](#)]

Define a grid of points (K,M) in the parameter space covering a region of $\pm 5\sigma$ around the expected values (with σ the expected uncertainty). Step size taken to be 0.5σ . This defines $21 \times 21 = 441$ templates for the relevant distributions.



For every template in the grid calculate the χ^2 obtained with the pseudodata distribution:

$$\chi^2(K, M) = \sum_i^{\text{bins}} \frac{R_i^{\text{data}} - R_i^{(K, M)}}{\sigma_i^{\text{data}}}$$

- Neglect the statistical errors of the templates as in the ratios they are vanishingly small.

Minimise the χ^2 interpolating across the grid by parabolic approximation.

Final errors correspond to $\Delta\chi^2=1$.

Hadronic running of α

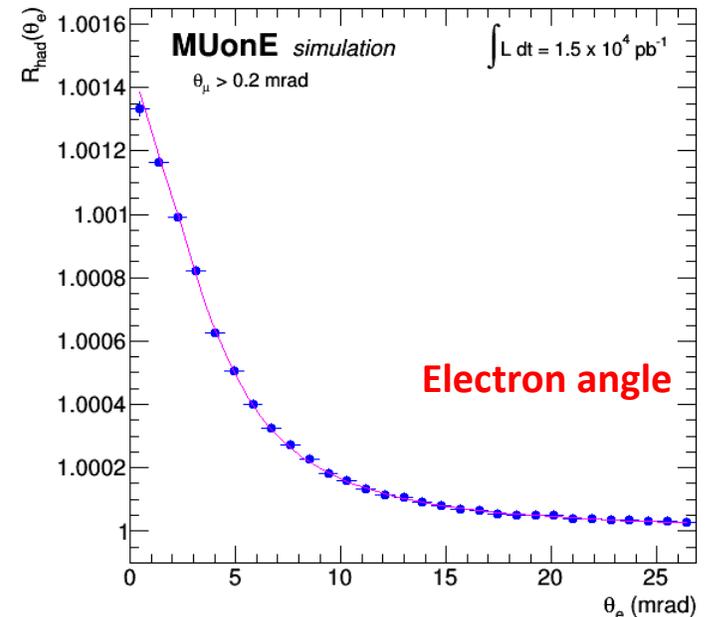
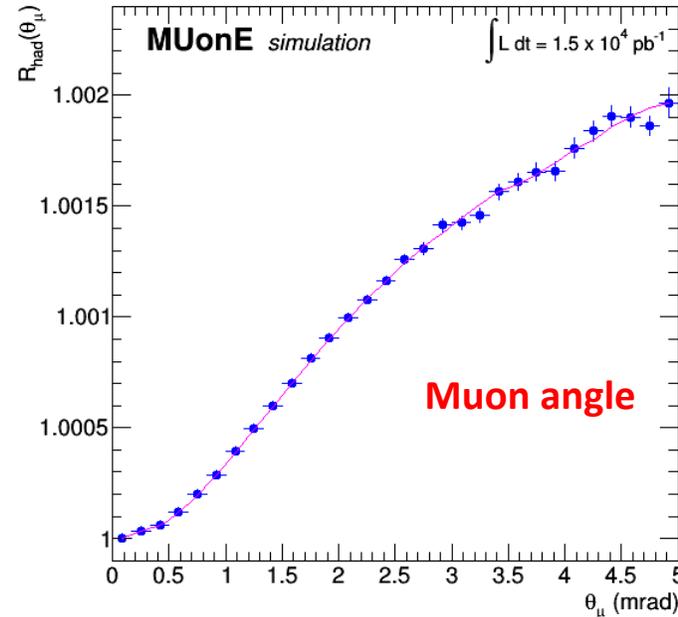
Most easily displayed by taking ratios of the MC predicted angular distributions (pseudodata) and the predictions obtained from the same MC sample reweighting $\alpha(t)$ to correspond to only the leptonic running:

$$R_{had}(\theta) = \frac{d\sigma(\theta, \Delta\alpha_{had})}{d\sigma(\theta, \Delta\alpha_{had}=0)}$$

Observable effect $\sim 10^{-3}$
 wanted precision $\sim 10^{-2}$

→ required precision $\sim 10^{-5}$

Example toy experiment: the expected distributions are obtained from the nominal integrated luminosity, corresponding to 3-year run



Expected Fit result

$$a_{\mu}^{HLO} = (688.8 \pm 2.4) \times 10^{-10}$$

Stat.err. 0.35%

Closure test ok:

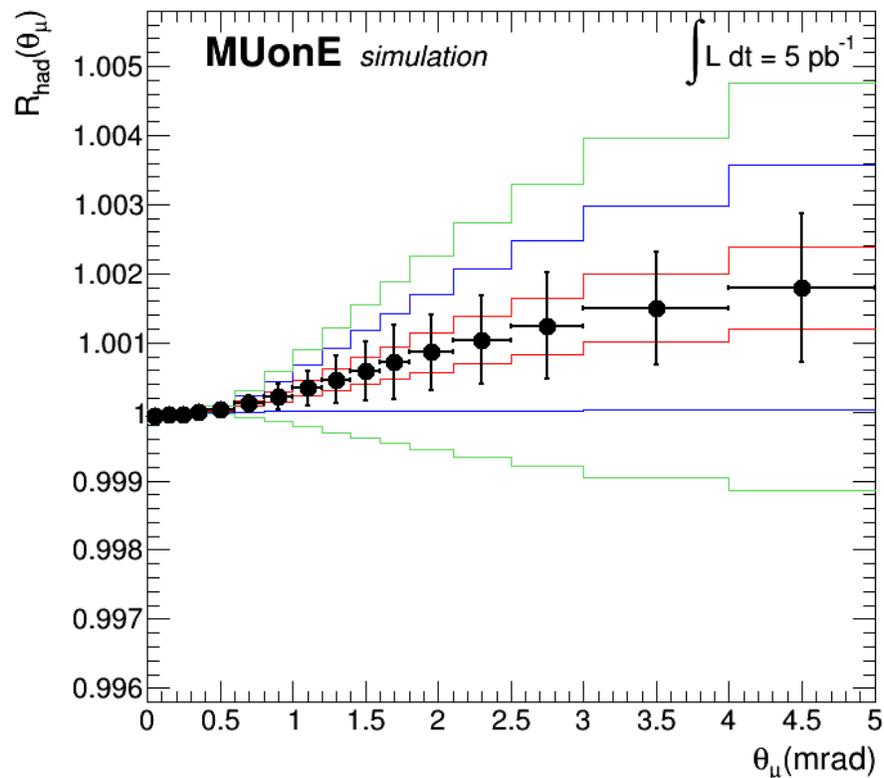
expected from the used input parameterization (Jegerlehner's) $a_{\mu}^{HLO} = 688.6 \times 10^{-10}$
 → Negligible error from the fit method: 0.2×10^{-10}

Expected sensitivity of a First Physics Run

Expected integrated Luminosity with the Test Run setup:

assuming full beam intensity and full detector efficiency $\sim 1\text{pb}^{-1}/\text{day}$,

in one week $\sim 5\text{pb}^{-1} \rightarrow \sim 10^9 \mu\text{e}$ scattering events with $E_e > 1 \text{ GeV}$ ($\theta_e < 30 \text{ mrad}$)



Template fit with just one fit parameter $K = k/M$ in the $\Delta\alpha_{\text{had}}$ parameterization.
The other parameter fixed at its expected value: $M = 0.0525 \text{ GeV}^2$

Initial sensitivity to the hadronic running of α .

Pure statistical level: 5.2σ

2D (θ_μ, θ_e) $K = 0.136 \pm 0.026$

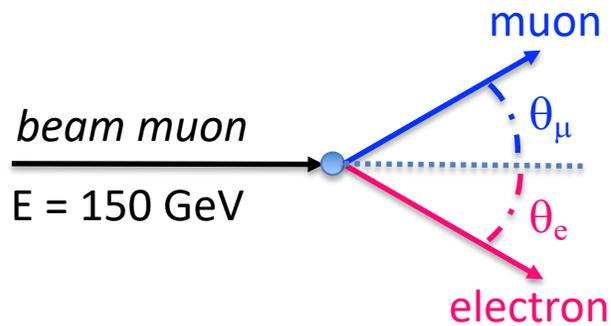
Definitely we will have sensitivity to the leptonic running (ten times larger)

Systematics: Beam Energy scale

Time dependency of the beam energy profile has to be continuously monitored during the run:

- SPS monitor
 - COMPASS BMS
- } needed external infos

However, the absolute beam energy scale has to be calibrated by a physics process:
kinematical method on elastic μe events

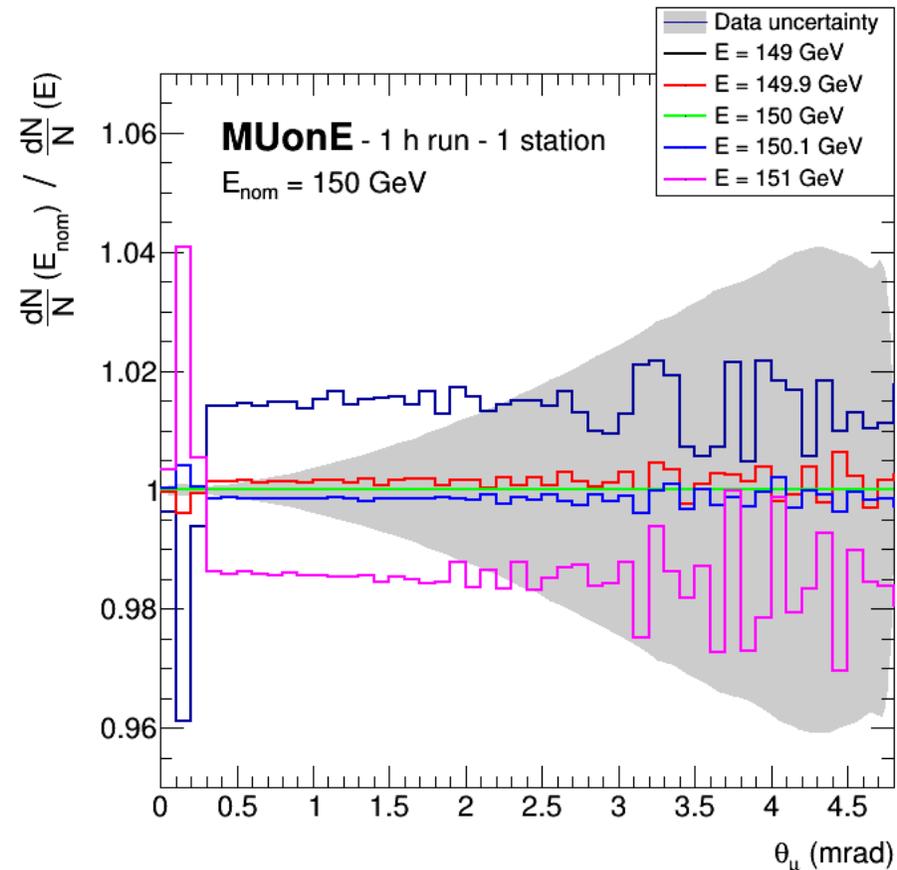


For equal angles:

$$\theta_\mu = \theta_e \equiv \theta \approx \sqrt{\frac{2m_e}{E}}$$

Can reach <3 MeV uncertainty in a single station in less than one week
From SPS E scale $\sim 1\%$: 1.5 GeV

Effect of a syst shift of the average beam energy on the θ_μ distribution: 1h run / 1 station



Simultaneous fit of signal and nuisances

Full description in Riccardo Pilato's PhD thesis

- Template fit using CMS Combine tool
 - Likelihood fit with systematics included as nuisance parameters, simultaneously extracted along with the signal parameters
<https://cms-analysis.github.io/HiggsAnalysis-CombinedLimit/>
- Currently including 4 nuisance parameters, related to:
 - Normalisation (uncertainty in the integrated luminosity)
 - Average beam energy
 - Intrinsic angular resolution
 - Multiple Coulomb Scattering (core width)
- Recent improvement: two-step workflow suited to the Test Run luminosity
 1. fit the main systematic effects (nuisance parameters) in the normalisation region (where signal is ~ 0). Use a short run of ~ 1 h time station-by-station ($\sim 35/\text{nb}$) assuming SM for the hadronic running.
 2. fit signal + nuisance parameters using as starting values and prior uncertainties for the nuisances the values determined in (1). Use the full Test Run statistics ($\sim 5/\text{pb}$).

Example Fit results

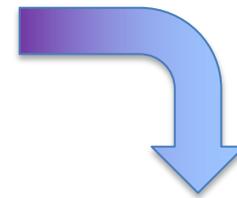
From Riccardo Pilato's PhD thesis

Pseudodata with statistics equivalent to the Test Run (5/pb). Input Signal parameter: $K=0.136$.
 Normalisation nuisance $\nu=0$. Distorted by the following 3 simultaneous shape systematics:

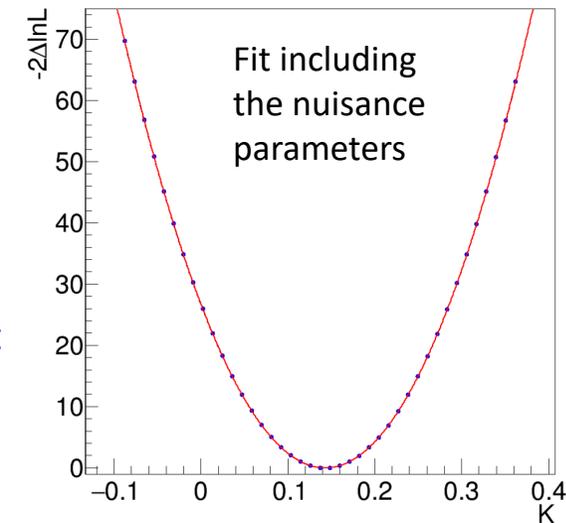
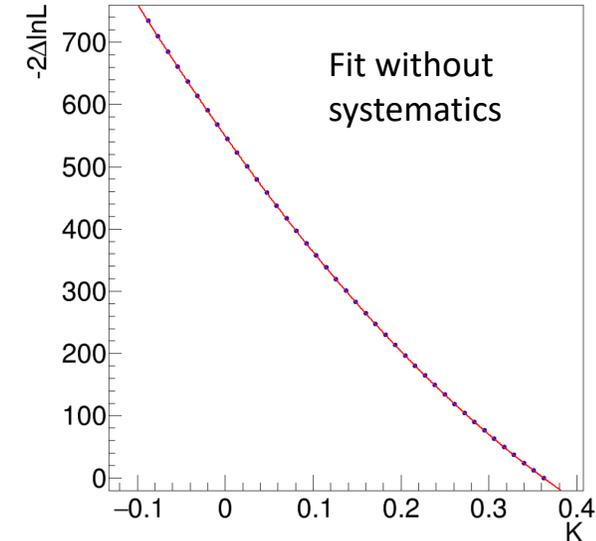
	Nominal configuration	Shift in the pseudo-data
Beam energy scale	$E_{beam} = (150 \pm 1)$ GeV	+ 6 MeV
Multiple scattering	$\sqrt{x/X_0} = 0.1458 \pm 1\%$	+ 0.5%
Angular intrinsic resolution	$\sigma_{Intr} = 0.02$ mrad $\pm 10\%$	+ 5%

Results:

Selection cuts	Fit results	
$\theta_e \leq 32$ mrad $\theta_\mu \geq 0.2$ mrad	$K = 0.133 \pm 0.028$	$\mu_{MS} = (0.47 \pm 0.03)\%$ $\mu_{Intr} = (5.02 \pm 0.02)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.003$
$\theta_e \leq 20$ mrad $\theta_\mu \geq 0.4$ mrad	$K = 0.133 \pm 0.033$	$\mu_{MS} = (0.46 \pm 0.04)\%$ $\mu_{Intr} = (5.02 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.6)$ MeV $\nu = -0.008 \pm 0.007$
$\theta_e \leq 32$ mrad $\theta_\mu \geq 0.4$ mrad	$K = 0.133 \pm 0.033$	$\mu_{MS} = (0.46 \pm 0.04)\%$ $\mu_{Intr} = (5.03 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.6)$ MeV $\nu = -0.009 \pm 0.007$
$\theta_e \leq 20$ mrad $\theta_\mu \geq 0.2$ mrad	$K = 0.133 \pm 0.031$	$\mu_{MS} = (0.47 \pm 0.03)\%$ $\mu_{Intr} = (5.02 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.006$
$\theta_{L,R} \in [0.2, 32]$ mrad	$K = 0.132 \pm 0.029$	$\mu_{MS} = (0.45 \pm 0.02)\%$ $\mu_{Intr} = (5.04 \pm 0.02)\%$ $\mu_{E_{Beam}} = (6.9 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.003$
$\theta_{L,R} \in [0.4, 20]$ mrad	$K = 0.133 \pm 0.034$	$\mu_{MS} = (0.43 \pm 0.03)\%$ $\mu_{Intr} = (5.05 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.8 \pm 0.6)$ MeV $\nu = -0.008 \pm 0.007$



Fit Results are in excellent agreement with the input values for all the selections, both with and without particle identification



Conclusions & Plans

- **MUonE** experiment proposal: measuring the running of α_{QED} from the shape of the differential cross section for elastic scattering of $\mu(160\text{GeV})$ on atomic electrons at the CERN SPS
 - Getting a_{μ}^{HLO} with a novel method integrating over the space-like region
 - Independent and complementary to the standard method integrating over the time-like region and to lattice QCD calculations
 - Competitive precision $\sim 0.35\text{-}0.5\%$ on a_{μ}^{HLO} allowing to better constrain the theory prediction, will help to solve the muon $g-2$ puzzle
- Impressive progress on the needed theoretical calculations and tools
 - (see talks by E.Budassi and C.L.Del Pio)
- Successful beam tests in 2021-22 with one tracking station
 - Stable operation under very high intensity
 - Good quality data collected (analysis ongoing)
- Test Run 2023: prove the feasibility of the method
 - 2 or 3 tracking stations (minimum: one before / one after the target)
 - Integration of the ECAL readout with the full DAQ
 - Implementation of online (in-FPGA) event selection
 - Simple track and vertex reconstruction
 - Selection of two-track events
 - Study backgrounds
 - Study the alignment in test beams
 - Study the beam energy calibration and validate our method
- Write the experimental proposal
- Move to the experiment with 10 stations to get a first measurement before LS3



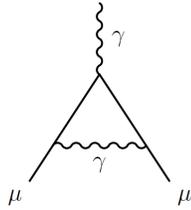
More infos:
papers, conferences, theses see:
<https://web.infn.it/MUonE/>

BACKUP

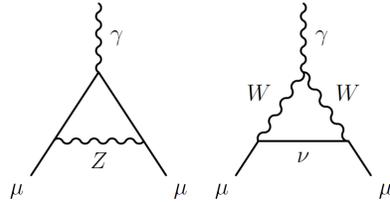
Muon g-2 Theory prediction

Currently accepted Standard Model prediction:
 White Paper of the Muon g-2 Theory Initiative
[Aoyama et al., Phys.Rept.887\(2020\)1](#)

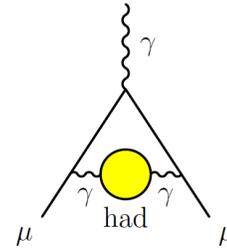
$$a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{EWK} + a_{\mu}^{had}$$



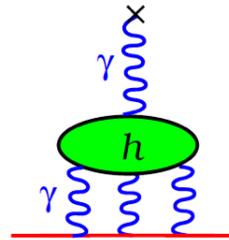
QED



EWK



HADRONIC: VP



HADRONIC: Light-By-Light

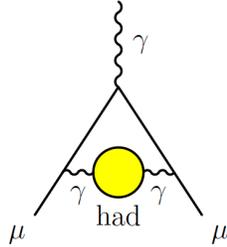
Order	$\mathcal{O}(\alpha)$	$\mathcal{O}(G_F m_{\mu}^2) = \mathcal{O}\left(\frac{\alpha}{s_W^2} \frac{m_{\mu}^2}{M_W^2}\right)$	$\mathcal{O}(\alpha^2)$	$\mathcal{O}(\alpha^3)$
SIZE	10^{-3}	10^{-9}	7×10^{-8}	10^{-9}
Uncertainty	10^{-12}	10^{-11}	4×10^{-10}	2×10^{-10}

QED LO term (Schwinger) = $\alpha/2\pi \sim 0.00116$
 QED corrections known up to 5 loops,
 uncertainty related to missing 6 loops!

Hadronic contributions
-not calculable by pQCD-

→ Dominant Theoretical uncertainty
LO Hadronic Vacuum Polarization
 Relative uncertainty: 0.6%

$a_\mu^{\text{HVP,LO}}$: standard data-driven approach (time-like)



Dispersion relations, optical theorem:

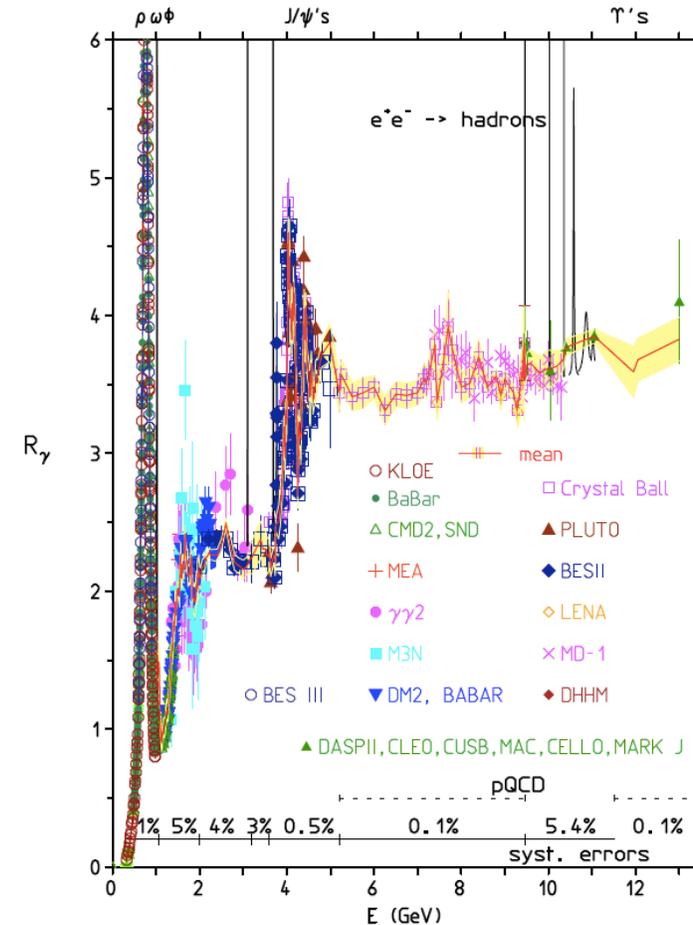
$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{\widehat{K}(s)R(s)}{s^2} ds$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad \widehat{K} \text{ smooth}$$

Traditionally the integral is calculated by using the experimental measurements up to an energy cutoff, beyond which perturbative QCD can be applied.

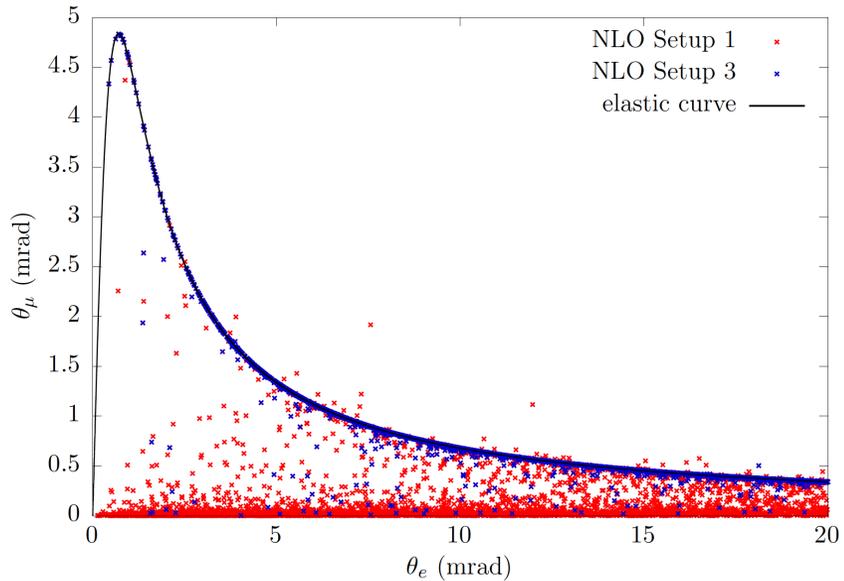
Main contribution: low-energy region ($1/s^2$ enhancement), highly fluctuating due to hadron resonances and thresholds effects

Radiative corrections to $R(s)$ crucial



F.Jegerlehner, EPJ Web Conf. 1c18 (2016) 01016

Radiative events and elastic selection



[M.Alacevich et al, JHEP02\(2019\)155](#)

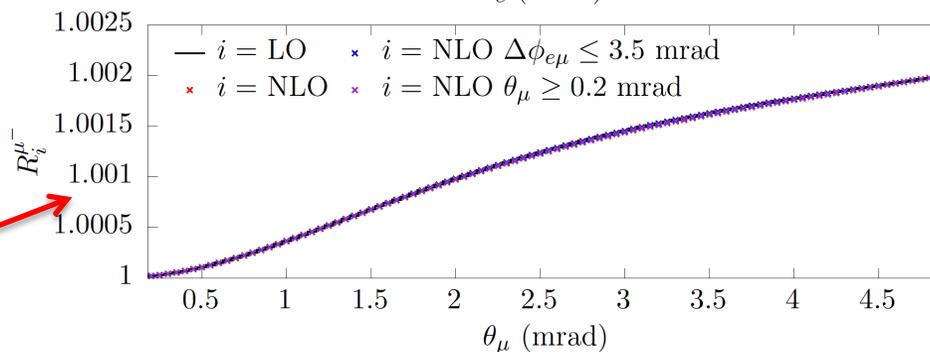
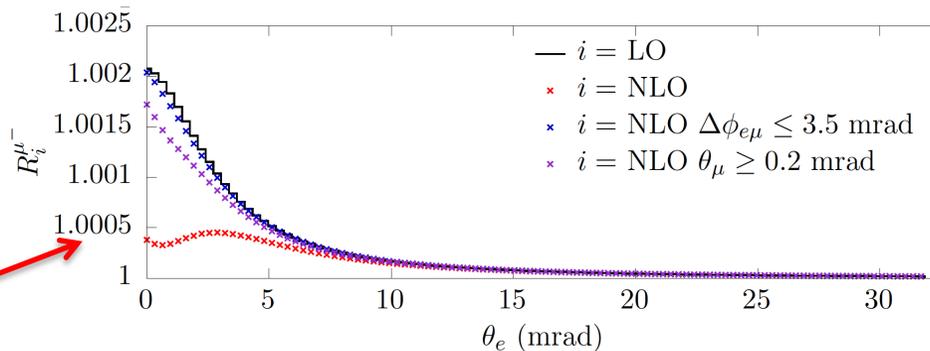
NLO:

Setup 1 is the inclusive selection (no cuts)

Setup 3 has an acoplanarity cut $|\pi - (\phi_e - \phi_\mu)| < 3.5$ mrad

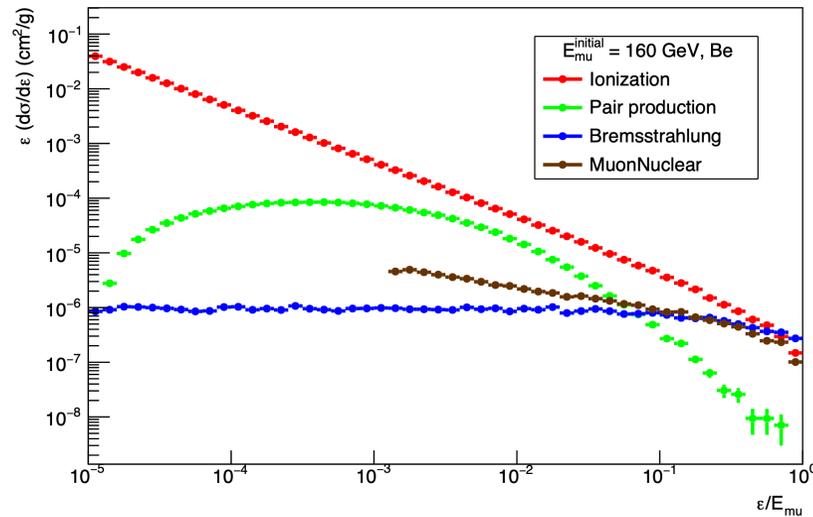
Without any selection the signal sensitivity of the electron angle is destroyed -> necessary to implement an “elastic” selection

Instead the muon angle is a robust observable, stable w.r.t. radiative corrections -> it can be used with an inclusive selection (theoretically advantageous)

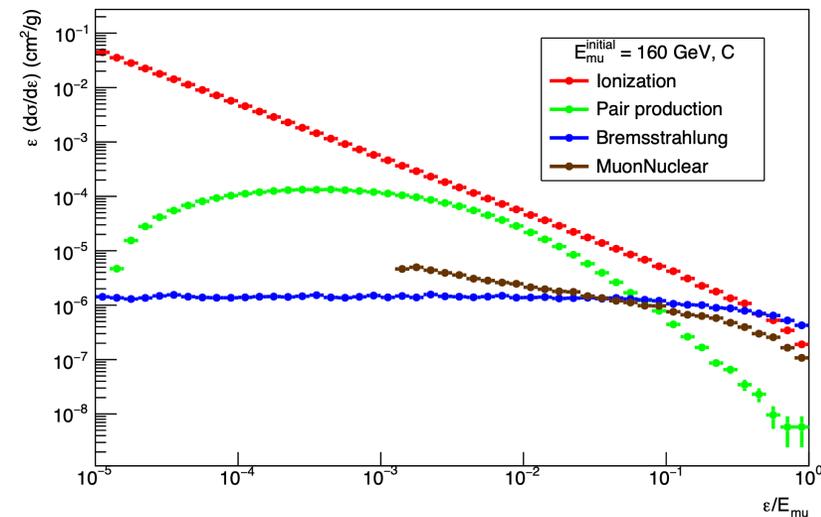


GEANT4: μ interaction cross sections

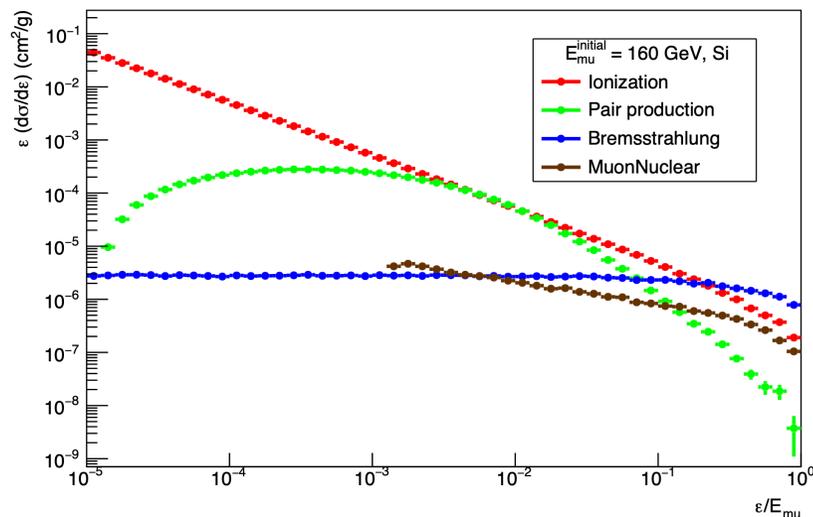
Differential macroscopic cross section: beryllium



Differential macroscopic cross section: carbon



Differential macroscopic cross section: silicon



GEANT4 simulation

ϵ Muon Energy loss fraction

σ Macroscopic cross section

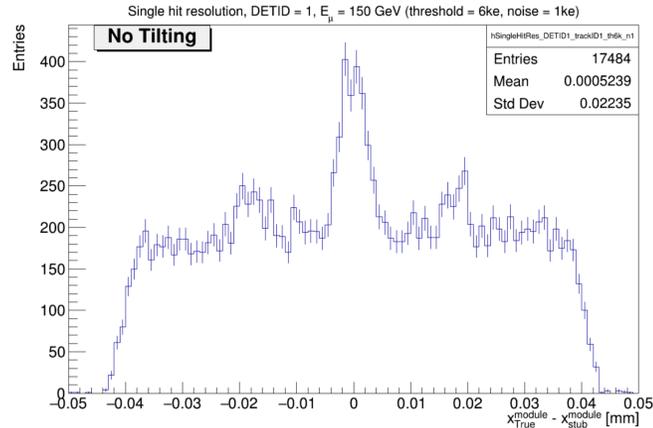
$$\sigma = \sigma_A n_A / \rho_A$$

σ_A Atomic cross section

n_A density of atoms per unit volume

ρ_A material density in g/cm³

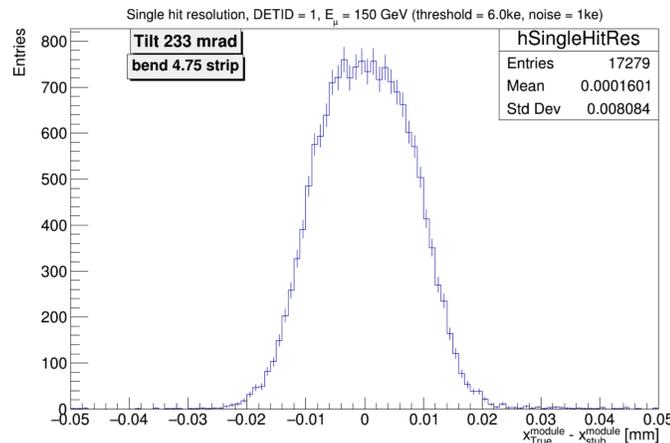
Simulation: Intrinsic Resolution – Tilted geometry



Strip digital readout: with $90\mu\text{m}$ pitch the expected resolution is $90/\sqrt{12} \cong 26\mu\text{m}$ on a single sensor layer for single-strip clusters

Tilting a sensor around an axis parallel to the strips → Charge sharing between adjacent strips, improving the resolution

The best is obtained when $\langle \text{cluster width} \rangle \sim 1.5$ (same number of clusters made of 1 or 2 strips) for a tilt angle ~ 15 degrees

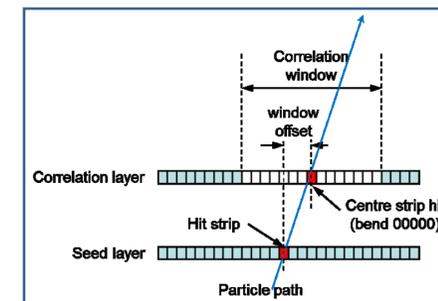


Further improvement: a small tilt of 25mrad is equivalent to an half-strip staggering of the two sensor layers of a 2S module

Final resolution:
22 μm → 8-11 μm

measured coordinate (x) determined by hit position on one layer and direction of the track stub

Tilt angle [mrad]	$\langle \text{bend} \rangle$ [strips]	threshold [σ]	resolution [μm]	$\langle \text{cluster width} \rangle$ [strips]
210	4.25	5	7.8	1.51
221	4.5	5.5	11.5	1.51
233	4.75	6	8.0	1.50
245	5	6.5	11.2	1.51
257	5.25	7	8.7	1.50
268	5.5	7.5	11.0	1.49



FastSim analysis strategy

- NLO MESMER MC
- $\Delta\alpha_{\text{had}}(t)$ from F.Jegerlehner's code(hadr5n12.f) $\rightarrow a_{\mu}^{\text{HLO}} = 688.6 \times 10^{-10}$
- Detector resolution effects parametrized in a simplified way (including only: multiple scattering on 1.5cm Be target and intrinsic resolution $\sigma_{\theta}=0.02$ mrad)
 - Neglecting: scattering on the Si planes, non-Gaussian tails, residual backgrounds
 - Neglecting: detailed track simulation and reconstruction
- Fit is done directly on the angular distributions of scattered μ and e
 - No attempt to estimate t (or x) event by event
 - $\theta_e < 32$ mrad (geometric acceptance)
 - $\theta_{\mu} > 0.2$ mrad (remove most of the background)
 - Both 1D and 2D distributions fitted. 2D is the most robust.
 - Ideally there is no need to identify the outgoing muon and electron, provided the event is a signal one. In this case we simply label the two angles as θ_L, θ_R ("Left" and "Right" w.r.t. an arbitrary axis)
- Shape-only fit: the absolute normalization shall not count.

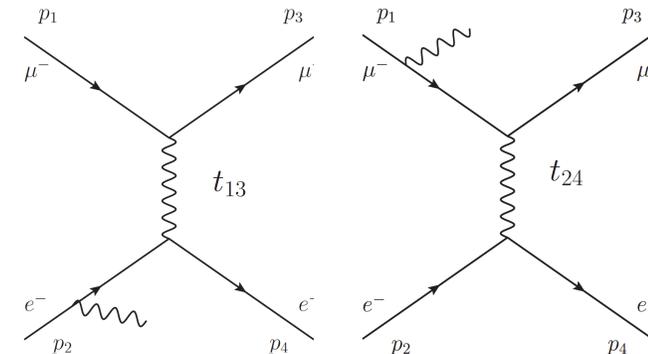
Template Fit technique

- MC templates for any useful distribution are built by reweighting the events to correspond to a given functional form of $\Delta\alpha_{had}(t)$
- $\Delta\alpha_{had}(t)$ is conveniently parameterised with the “Lepton-Like” form, one-loop QED calculation.

The 2->3 matrix element for one-photon emission at NLO can be split in 3 parts (radiation from mu or e leg and their interference), each one with a different running coupling factor → **3 coefficients**

NOTE: at NNLO one needs 11 coefficients

By saving the relevant coefficients at generation time we can easily reweight the events according to the chosen parameters in the $\Delta\alpha_{had}(t)$



Determination of a_{μ}^{HLO} by the Master Integral

- From the fitted (K,M) values the hadronic contribution to $\Delta\alpha_{\text{had}}(t)$ is determined from the Lepton-Like parameterisation:

$$\Delta\alpha_{\text{had}}(t) = k \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \log \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\}$$

- Then, by using the master integral, we have the result in the full phase space:

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

- The result for the nominal luminosity is $a_{\mu}^{\text{HLO}} = (688.8 \pm 2.4) \times 10^{-10}$
 - statistical uncertainty of 0.35%
- The expectation from the used Jegerlehner's parameterization is:
 $a_{\mu}^{\text{HLO}} = 688.6 \times 10^{-10}$
 - difference from our fit is 0.2×10^{-10} , negligible w.r.t. the statistical uncertainty