Connected HVP contribution to the muon's g-2 from lattice QCD (+QED)

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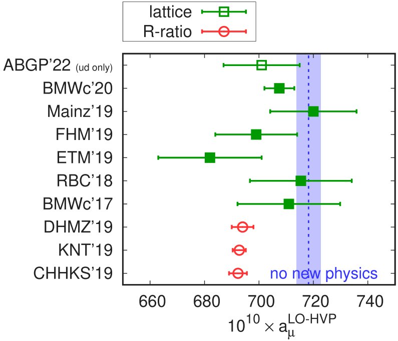
Outline

1. Motivation
2. C* boundary conditions
3. Connected HVP from QCD and QCD+QED with C*
4. Conclusions & outlook

Motivation

Contribution	value $\times 10^{11}$
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP	6845(40)
HLbL	92(18)
Total SM value	116 591 810(43)
Exp. (E821 + E989)	116 592 061(41)
Difference: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)

[arXiv: 2006.04822, 2104.03281, 2203.15810]



[L. Lellouch, g-2 Workshop '22]

- HVP is the main contribution to the SM prediction's uncertainty
- great efforts in the lattice community to reduce the error

Motivation

To reach the subpercent precision for the HVP on lattice
$\implies$ isospin breaking corrections $\sim O(\alpha) \sim O(\frac{m_d - m_u}{\Lambda_{QCD}}) \sim O(10^{-2})$

To address the problem of the QED effects:

- non-local constraints to remove the zero-modes of the photon [Hayakawa & Uno, 0804.2044]
  e.g QED_L, where $\sum_{\vec{x}} A_\mu(x_0, \vec{x}) = 0$
- massive photon (QED_M) [Endres et al., 1507.08916]
- QED_∞ [Blum et al., 1705.01067; Feng & Jin, 1812.09817]
- $\text{C}^\star$ boundary conditions $\rightarrow$ local and gauge-invariant prescription [Kronfeld & Wiese, 1991, 1992; Polley, 1993; Lucini et al., 1509.01636]

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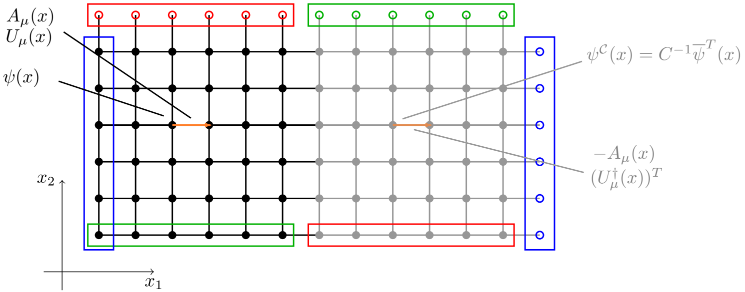
C* boundary conditions

Idea: include the charge conjugate of the fields in the simulations



- physical ($0 < x_1 < L_1$) and mirror lattice ($L_1 < x_1 < 2L_1$)
- C* boundary conditions in the spatial directions
- periodic conditions on the extended lattice e.g $\psi(x + 2L_i\hat{i}) = \psi(x)$

C⋆ boundary conditions[1]

- $\psi(x)$ defined on the whole lattice
- $\bar{\psi}(x)$ given by $-\psi^T(x + L_1\hat{1})C$

	Periodic boundaries		C⋆ boundaries
Action:	$\sum_{x\in\Lambda_{\text{phys}}} \bar{\psi}(x)D\psi(x)$	$\longrightarrow$	$\sum_{x\in\Lambda_{\text{phys+mir}}} -\frac{1}{2}\psi^T(x)CTD\psi(x)$
Int. meas.:	$\left[\mathcal{D}\psi\right]_{\Lambda_{\text{phys}}} \left[\mathcal{D}\bar{\psi}\right]_{\Lambda_{\text{phys}}}$	$\longrightarrow$	$\left[\mathcal{D}\psi\right]_{\Lambda_{\text{phys+mirror}}}$
Determinant:	$\det(D)$	$\longrightarrow$	$\text{Pf}(CTD)$
Wick-contr.:	$\overbrace{\psi(x)\bar{\psi}}(y) = D^{-1}(x\vert y)$	$\longrightarrow$	$\overbrace{\psi(x)\psi}^T(y) = -D^{-1}(x\vert y)TC^{-1}$

T: translation operator by L_i
C: charge conjugation operator

[1] For details see 1509.01636, 1908.11673

C* boundary conditions: A&D

[+] $A_\mu(x) = -A_\mu(x + L_i\hat{i}) \implies p_i \neq 0$

[+] charged particles propagation is allowed

[+] suppressed finite-volume effects for QED

[-] double volume

[-] flavour mixing (exponentially suppressed with the volume)

[-] charge conservation partially violated (exponentially suppressed with the volume)

RC* collaboration's ensembles

Ensembles generated through the openQ*D code [2]
https://gitlab.com/rcstar/openQxD

ensemble	V	flavor	α	a [fm][3]	$m_{\pi^\pm}[MeV]$
A400a00b324	64×32^3	$3+1$	0	0.05393(24)	398.5(4.7)
B400a00b324	80×48^3	$3+1$	0	0.05400(14)	401.9(1.4)
A450a07b324	64×32^3	$1+2+1$	0.007299	0.05469(32)	451.2(4.3)
A380a07b324	64×32^3	$1+2+1$	0.007299	0.05323(28)	383.6(4.4)
A500a50b324	64×32^3	$1+2+1$	0.05	0.05257(14)	495.0(2.8)
A360a50b324	64×32^3	$1+2+1$	0.05	0.05054(27)	358.6(3.7)
C380a50b324	96×48^3	$1+2+1$	0.05	0.050625(79)	386.5(2.4)

- Lüscher-Weisz SU(3) gauge action ($\beta = 3.24$)
- Wilson action for compact U(1) field
- four flavours of $O(a)$-improved Wilson fermions

[2] For details see Lucke et al., 2108.11989; Bushnaq et al., 2209.13183

[3] a is determined using the $N_f = 2+1$ value of $\sqrt{8t_0} = 0.415$ fm from Bruno et al., 1608.08900

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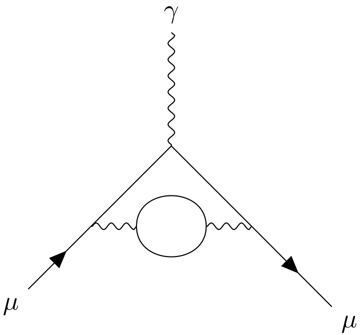
Strategy for the calculation

- Time-momentum representation

$$G(t) = -\frac{1}{3} \sum_{k=1,2,3} \sum_{\vec{x}} \langle V_k(x) V_k(0) \rangle$$

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t) \tilde{K}(t; m_\mu)$$



- two discretizations for the vector current

$$V_\mu^l(x) = \sum_f q_f \overline{\psi}_f(x) \gamma_\mu \psi_f(x)$$

$$V_\mu^c(x) = \sum_f \frac{1}{2} q_f \left[\bar{\psi}_f(x+\hat{\mu}) \left(1+\gamma_\mu\right) U_\mu^\dagger(x) \psi_f(x) - \bar{\psi}_f(x) \left(1-\gamma_\mu\right) U_\mu(x) \psi_f(x+\hat{\mu}) \right]$$

- extrapolation of the signal at large t (single-exponential for now)

Hadronic Vacuum Polarization

By considering the different Wick contractions:

$$\langle V_k^l(x) V_k^l(0)\rangle = \sum_{f,f'} q_f q_{f'}\,\mathrm{tr}\left[\gamma_k D_f^{-1}(x|x)\right]\cdot \mathrm{tr}\left[\gamma_k D_{f'}^{-1}(0|0)\right] +$$

$$-\sum_f q_f^2\,\mathrm{tr}\left[\gamma_k D_f^{-1}(x|0)\gamma_k D_f^{-1}(0|x)\right]$$

- $D_f^{-1}(x|y)$ quark propagator from y to x
- γ_k Dirac matrices ($k = 1, 2, 3$)

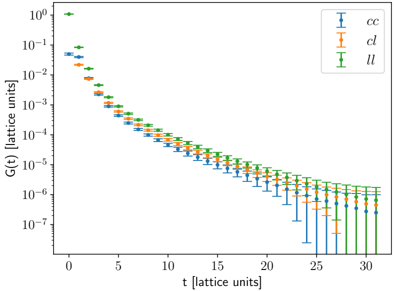
QCD configurations → leading HVP (w/o IBE effects)

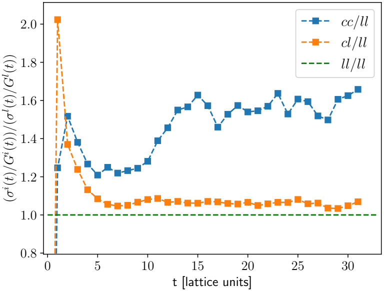
QCD+QED configurations → full HVP

QCD with C⋆ b.c.

Ensemble	V	β	$\kappa_{u,d,s}$	κ_c	$c_{\mathrm{sw,SU(3)}}$
A400a00b324	64×32^3	3.24	0.1344073	0.12784	2.18859
B400a00b324	80×48^3	3.24	0.1344073	0.12784	2.18859

- study of the signal-to-noise ratio for different discretizations of the correlator: $G^{ll}(t)$, $G^{cl}(t)$, $G^{cc}(t)$





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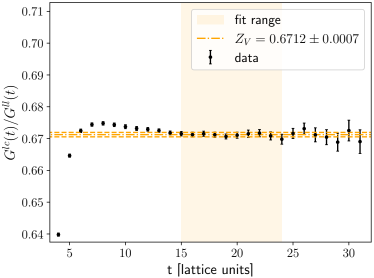
- V^l_μ requires a renormalization constant and $O(a)$-improvement

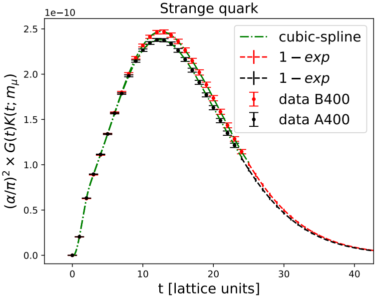
$$V^R_{\mu,f} = Z^{m_f}_V (V^l_{\mu,f} + a c_V \partial_\nu T_{\mu\nu,f})$$ [Bhattacharya et al., 0511014]

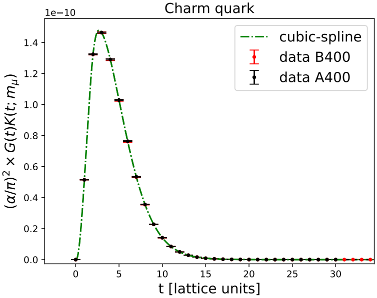
$$R(t) = \frac{\sum_{\vec{x},k} \langle V^c_f(x) V^l_f(0) \rangle}{\sum_{\vec{x},k} \langle V^l_f(x) V^l_f(0) \rangle}$$

Ensemble	$Z^{m_s}_V$	$Z^{m_c}_V$
A400a00b324	0.6712(7)	0.6066(2)
B400a00b324	0.6707(5)	0.6066(4)



- comparison of the results for lattice volumes 64×32^3 and 80×48^3





ensemble	type	am_V^s	$a_\mu^s \times 10^{-10}$	am_V^c	$a_\mu^c \times 10^{-10}$
A400a00b324	*ll*	0.2808(22)	46.7(7)	0.8463(5)	7.83(8)
	cl	0.2796(29)	46.2(7)	0.8462(5)	6.18(7)
B400a00b324	*ll*	0.2794(19)	48.5(7)	0.8458(9)	7.81(9)
	cl	0.2791(20)	48.0(7)	0.8454(8)	6.16(7)

QCD+QED with C* b.c.

Simulation details:

ensemble	V	β	α	κ_u	$\kappa_{d,s}$	κ_c	$c_{SW}^{SU(3)}$	$c_{SW}^{U(1)}$
A380a07b324	64×32^3	3.24	0.007299	0.13459164	0.13444333	0.12806355	2.18859	1
A360a50b324	64×32^3	3.24	0.05	0.135560	0.134617	0.129583	2.18859	1

- Wilson action with compact U(1) field

$$S_{g,U(1)} = \frac{1}{8\pi q_{el}^2 \alpha} \sum_x \sum_{\mu \neq \nu} [1 - P_{\mu\nu}^{\mathrm{U}(1)}(x)]$$

  q_{el} elementary charge, $P_{\mu\nu}^{\mathrm{U}(1)}$ plaquette

- $c_{SW}^{\mathrm{SU}(3)}$ correct up to $O(\alpha)$ terms, tree level improvement for U(1)
- $N_f = 1 + 2 + 1 \rightarrow$ non-physical degenerate d and s quarks

QCD+QED with C* b.c.

Evaluation of the connected HVP:

- signal-to-noise ratio of $G^{ll}(t), G^{cl}(t), G^{cc}(t)$ similar to the QCD case $\rightarrow$ we evaluate $G^{cl}(t)$
- the conserved current requires renormalization (not included yet)

$$V_\mu^{\mathrm{R}}(x) = V_\mu^{\mathrm{c}}(x) + \mathcal{O}\left(\partial_\nu F^{\nu\mu}\right)$$ [Collins et al., 0512187]

- isospin breaking effects automatically included

QCD+QED with C* b.c.

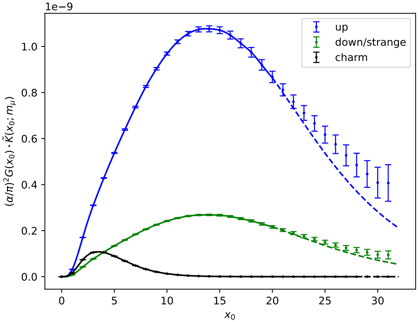


Figure: Integrand of the connected HVP contribution for the A380a07b324 ensemble

ensemble	α_R	flavor	$a_\mu^{\mathrm{HVP}} \times 10^{10}$
A360a50b324	0.007081(19)	up	309(11)
		down/strange	77(2)
		charm	10.62(11)
A380a07b324	0.040633(80)	up	331(7)
		down/strange	83(2)
		charm	9.78(10)

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Conclusions & Outlook

- We computed the connected HVP on two QCD and two QCD+QED ensembles with $C^\star$ b.c.
- Signal-to-noise ratio good for QCD and at the physical α_{EM}
- The results are not extrapolated: $a \sim 0.05$, $360 < m_\pi < 400$ MeV
- On-going:
  - Variance reduction: low mode averaging [De Grand & Schaefer, 0401011]
  - Disconnected contributions
  - Isospin breaking effects in QCD through a perturbative expansion (RM123) [De Divitiis et al., 1303.4896]
- Long-term:
  - Chiral extrapolation
  - Continuum extrapolation

Backup slides

- renormalization scheme [4]: $(8t_0)^{1/2}$, $\alpha_R(t_0)$, ϕ_0, ϕ_1, ϕ_2, ϕ_3

$$\begin{aligned}
\phi_0 &= 8t_0(m^2_{K^\pm} - m^2_{\pi^\pm}) && \to 0 && (\text{fixes } m_s - m_d)\\
\phi_1 &= 8t_0(m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0}) && \to \phi_1^{\text{phys}} && (\text{fixes } m_s + m_d + m_u)\\
\phi_2 &= 8t_0(m^2_{K^0} - m^2_{K^\pm})/\alpha_R && \to \phi_2^{\text{phys}} && (\text{fixes } \delta m_{strong}/\delta_{EM})\\
\phi_3 &= \sqrt{8t_0}(m^2_{D_S^\pm} + m^2_{D^\pm} + m^2_{D^0}) && \to \phi_3^{\text{phys}} && (\text{fixes } m_c)
\end{aligned}$$

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A500a50b324	64×32^3	1 + 2 + 1	3.24	0.05
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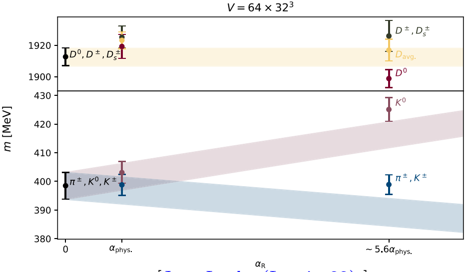


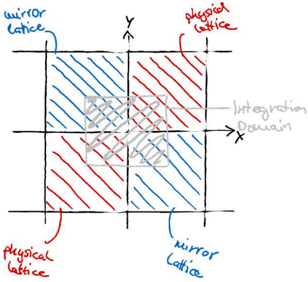
[Jens Lucke (Lattice22)]

[4] we used the CLS $N_f = 2+1$ value of $\sqrt{8t_0} = 0.415$ fm [Bruno et al., 1608.08900

C⋆ b.c.

- C⋆ boundaries in $\hat{i} \implies p_i = \frac{\pi}{L}(2\mathbb{Z}+1)$ for A_μ
- the vector current is a C-odd operator
- zero-momentum projection is not possible
- the spatial integration domain in TMR should be set to

$$\left(-\frac{V}{2},\frac{V}{2}\right)^3$$

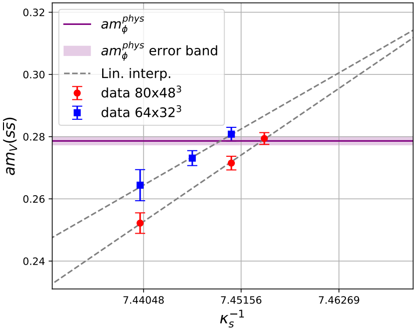


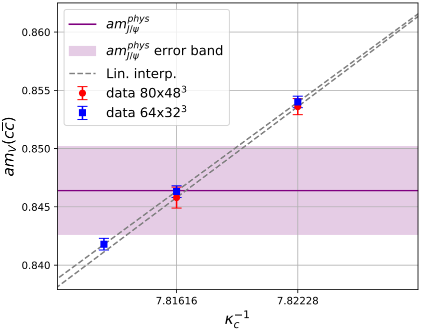
[Sofie Martins(Lattice22)]

QCD with C* b.c.

- tuning of $\kappa_{s,c}^{\text{val}}$: change the valence hopping parameters such that $m_V^{s\bar{s}}$, $m_V^{c\bar{c}}$ match

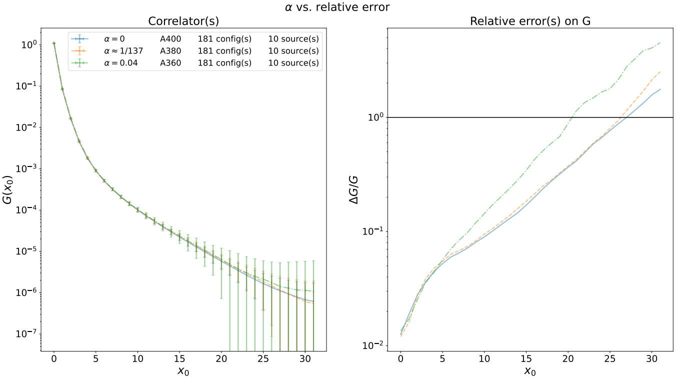
$$m_\phi^{phys} = 1019.461(20)\text{MeV} \quad m_{J/\psi}^{phys} = 3096.900(6)\text{MeV}$$





QCD+QED with C* b.c.

- comparison of the relative error for three values of α



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