High precision calculations for the MUonE experiment

NePsi 23, Pisa

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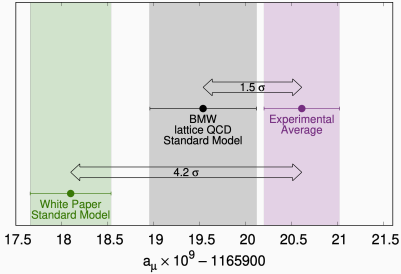
μONe

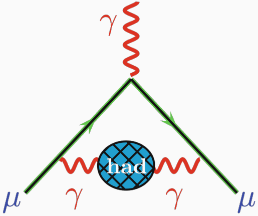
The muon $g-2$ & Spacelike approach

Starting point: $g_\mu - 2$ & Theoretical Approaches

$$a_\mu^{\text{SM}} \times 10^{11} = 116591810(43)$$
$$a_\mu^{\text{EXP}} \times 10^{11} = 116592061(41)$$

$$a_\mu^{\text{QED}} \times 10^{11} = 116584718.931(104)$$
$$a_\mu^{\text{EW}} \times 10^{11} = 153.6(1.0)$$
$$a_\mu^{\text{HLbL}} \times 10^{11} = 92(18)$$
$$a_\mu^{\text{HVP}} \times 10^{11} = 6845(40)$$
$$a_\mu^{\text{SM}} \times 10^{11} = 116591810(43)$$





T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166

B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801.

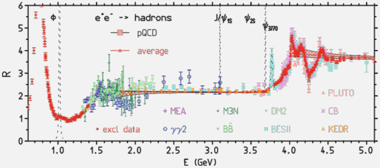
Borsanyi, S. *et al.* *Nature* **593**, 51–55 (2021).

Timelike & Spacelike approaches

Timelike approach:

$$a_\mu^{\text{HLO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{m_\pi^2}^{\infty} ds \frac{K(s) R^{\text{had}}(s)}{s^2}\right];$$

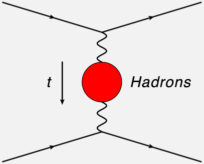
$$R^{\text{had}}(s) = \frac{\sigma(e^+e^- \to \text{had}, s)}{\frac{4}{3}\frac{\pi\alpha^2}{s}}$$



Spacelike approach:

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\text{had}}[t(x)];$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

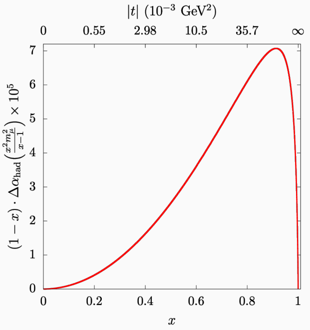


Lautrup, B. E., et al., Phys. Rept. 3 (1972) 193-259

C. M. Carloni Calame et al. Phys. Lett. B 746 (2015), 325-329

Going spacelike

- $\Delta\alpha_{\text{had}}$: measured in a single experiment with a spacelike process.
- A high-precision experiment is needed: 10 ppm.



μe scattering on a low Z target is an ideal process:

- pure t-channel process
- M2 muon beam ($E_\mu \approx 160$ GeV) is available at CERN
- $\sqrt{s} \approx 0.4$ GeV and $-0.143 < t < 0$ GeV^2. We can cover 87% of the integral with data. We can then extrapolate up to $x \to 1$.

G. Abbiendi's talk

C. M. Carloni Calame et al. Phys. Lett. B 746 (2015), 325-329
G. Abbiendi et al., Eur. Phys. J.C77(2017), no. 3 139.

State of the art of $\mu e \to \mu e$ scattering calculations

Theory work

- C. M. Carloni Calame et al. Phys. Lett. B 746 (2015), 325-329;
- P. Mastrolia, M. Passera, *et al.*, JHEP11 (2017) 198;
- S. Di Vita, S. Laporta, P. Mastrolia, *et al.*, JHEP09 (2018) 016;
- M. Alacevich, C. M. Carloni Calame *et al.*, JHEP 02 (2019) 155;
- M. Fael and M. Passera, Phys. Rev. Lett.122(2019), no. 19 192001;
- M. Fael, JHEP02 (2019) 027;
- C.M. Carloni Calame *et al.*, Towards muon-electron scattering at NNLO, JHEP 11 (2020) 028;
- P. Banerjee *et al.*, SciPost Phys. 9 (2020), 027;
- P. Banerjee *et al.*, Eur.Phys.J.C 80 (2020) 6, 591;
- E. Budassi *et al.*, JHEP 11 (2021) 098;
- E. Balzani, S. Laporta, M. Passera, arXiv: 2112.05704 [hep-ph];
- A.V. Nesterenko, arXiv:2112.05009 [hep-ph];
- R. Bonciani, *et al.*, Phys.Rev.Lett. 128 (2022) 2, 022002;
- M. Fael, F. Lange, K. Schönwald, M. Steinhauser, arXiv:2202.05276 [hep-ph];
- A. Broggio, *et al.*, JHEP 01 (2023), 112.

Numerical implementations for μe scattering NLO and NNLO: State of the Art

MESMER (Monte Carlo Event Generator)
MCMULE (Monte Carlo Integrator)

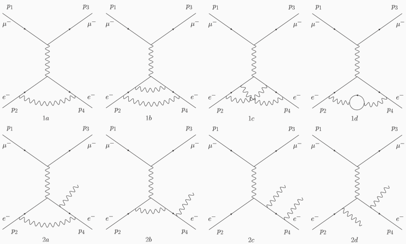
Possible "Contamination" from New Physics

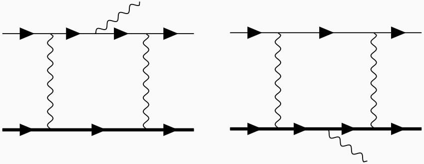
- A. Masiero, P. Paradisi and M. Passera, Phys.Rev.D 102 (2020) 7, 075013.
- P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053.

Towards muon-electron scattering at NNLO in QED

Photonic NNLO corrections: exact contributions

- Virtual NNLO photonic contributions are included exactly for electron or muon leg emission. 2-loop QED vertex from factors taken from Mastrolia and Remiddi.





- 1-loop corrections to real photon emission exactly included: *e.g.* pentagon diagrams.
- Double real emission included exactly.

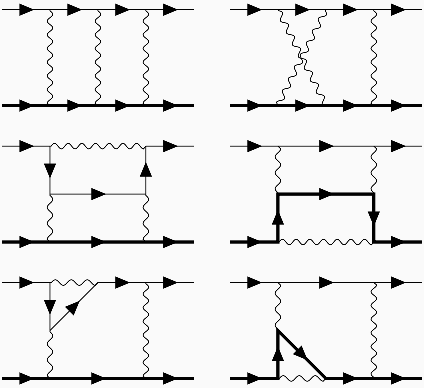
M. Alacevich, C. M. Carloni Calame, M. Chiesa, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 02 (2019) 155.

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

P. Mastrolia and E. Remiddi, Nucl.Phys.B 664 (2003), 341-356.

P. Banerjee, T. Engel, A. Signer, Y. Ulrich. SciPost Phys. 9 (2020), 027; P. Banerjee *et al.*, Eur.Phys.J.C 80 (2020) 6, 591.

Photonic NNLO corrections: approximated contributions



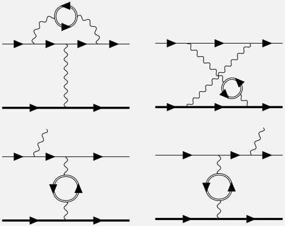
- Of the two-loop virtual diagrams with a virtual photon insertion on top of NLO boxes, only the IR part is included exactly (YFS).
- The non-IR remnants are approximate.
- All photonic NNLO effects weigh at most some % at the Phase Space boundaries.
- Full $\mu e \to \mu e$ at NNLO (Padova&PSI).

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

A. Broggio, T. Engel, A. Ferroglia, M.K. Mandal, P. Mastrolia *et al.*, JHEP 01 (2023), 112.

NNLO Lepton Pair Contributions: Virtual

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$



$$\frac{-ig_{\mu\nu}}{q^2 + i\varepsilon} \to -ig_{\mu\nu} \left(\frac{\alpha}{3\pi}\right) \int_{4m_\ell^2}^{\infty} \frac{dz}{z}$$

$$\times \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}} .$$

- Integration over z is performed numerically with MC techniques.
- Master Integral techniques for a subset of such diagrams to cross-check results.
- Interplay between real photon radiation and leptonic loop insertions.
- IR divergences are cancelled by a sub-set of the virtual contributions.

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

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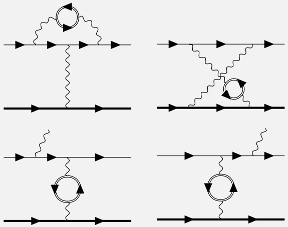

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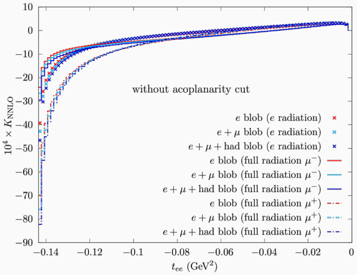
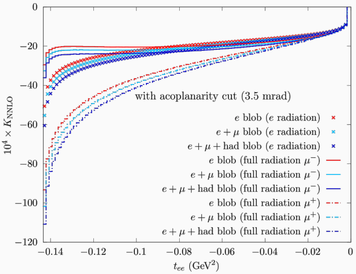


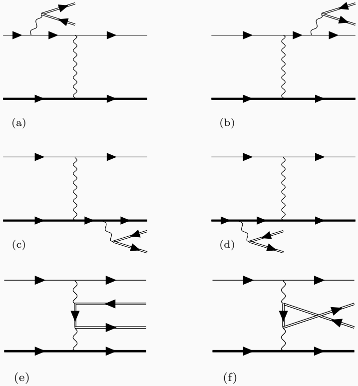


E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

NNLO Lepton Pair Contributions: Real

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

- $2 \rightarrow 4$ LIPS.
- The QED matrix elements have been calculated with FORM and cross-checked with RECOLA.
- Cuts: a set of elasticity cuts must be imposed to reduce a potentially large background



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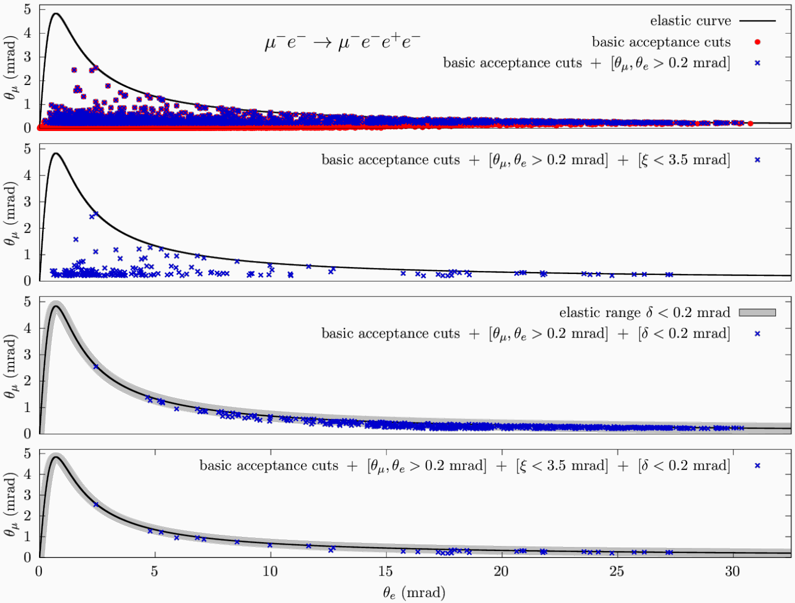
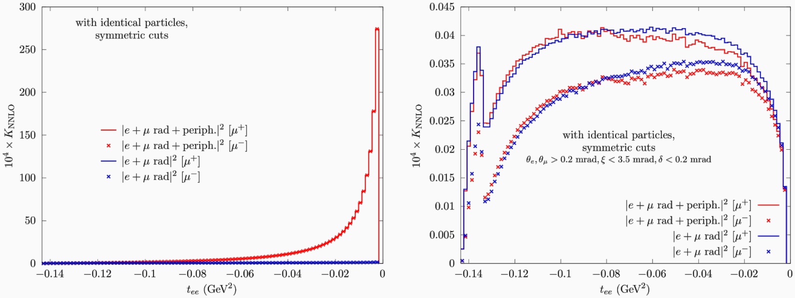
B. Ruijl, T. Ueda and J. Vermaseren, FORM version 4.2.

A. Denner, *et al.* Recola2: REcursive Computation of One-Loop Amplitudes 2, Comput. Phys. Commun. 224 (2018) 346.

Elasticity cut



Real NNLO Lepton Pair Contributions: Results



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Take-Home Message and Outlook

- Important efforts to develop NNLO fixed-order Monte Carlo event generators for μe scattering.

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- Higher-order QED corrections must be included to reach the required precision, e.g. by matching a QED Parton Shower with exact NNLO matrix elements.
- $\mu N \to \mu N + e^+e^-$ is a relevant background that needs to be addressed.

Backup slides

Backup: elasticity cut

Elasticity curve can be parametrised as follows:

$$\theta_\mu(\theta_e) = \arctan\left[\frac{2m_e r\cos\theta_e\sin\theta_e}{E^i_\mu - r\left(rE^i_\mu + 2m_e\right)\cos^2\theta_e}\right],$$

where r is defined as:

$$r = \frac{\sqrt{(E^i_\mu)^2 - m^2_\mu}}{E^i_\mu + m_e}$$

and E^i_μ is the incident muon energy in the laboratory reference frame.

Backup: Event selection criteria

- **Basic acceptance cuts**
- When we have 4 particles in the final state we require that only 2 are detected ($E_i > 200$ MeV and $\vartheta_i < 100$ mrad).

On top of it, we added 3 selection cuts to select elastic events:

- **cut 1**: $\vartheta_e > 0.2$ mrad and $\vartheta_\mu > 0.2$ mrad
- **cut 2**: $\xi = |\pi - |\phi_e - \phi_\mu|| < \xi_c = 3.5$ mrad
- **cut 3**: Elasticity distance $\delta < \delta_c = 0.2$ mrad. δ is defined as the distance from the elastic curve:

$$\delta = \min_{\theta_e} \sqrt{(\theta_e - \theta_e^0)^2 + (\theta_\mu(\theta_e) - \theta_\mu^0)^2}.$$

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

Backup: Sketch of the NLO calculation

NLO contributions:

$$\sigma_{\text{NLO}} = \sigma_{2\to2} + \sigma_{2\to3}$$

- Leading Order and NLO virtual contributions::

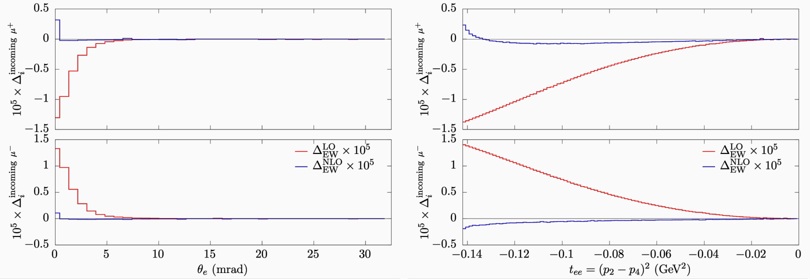
$$\sigma_{2\to2} = \sigma_{\text{LO}} + \sigma^{\text{v}}_{\text{NLO}} = \frac{1}{F}\int d\Phi_2 \left\{ \left|\mathcal{M}_{\text{LO}}\right|^2 + 2\,\text{Re}\left[\mathcal{M}^{\dagger}_{\text{LO}}\mathcal{M}^{\text{v}}_{\text{NLO}}(\lambda)\right]\right\}$$

- NLO Real contributions:

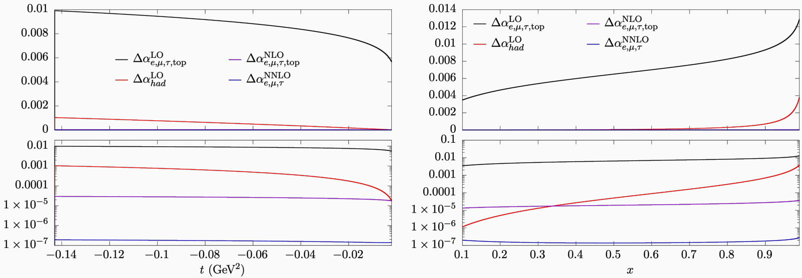
$$\sigma_{2\to3} = \frac{1}{F}\left(\int_{\lambda<E_\gamma<\Delta E} d\Phi_3 \left|\mathcal{M}^{\gamma}_{\text{NLO}}\right|^2 + \int_{E_\gamma>\Delta E} d\Phi_3 \left|\mathcal{M}^{\gamma}_{\text{NLO}}\right|^2\right)$$

- Same strategy used at NNLO

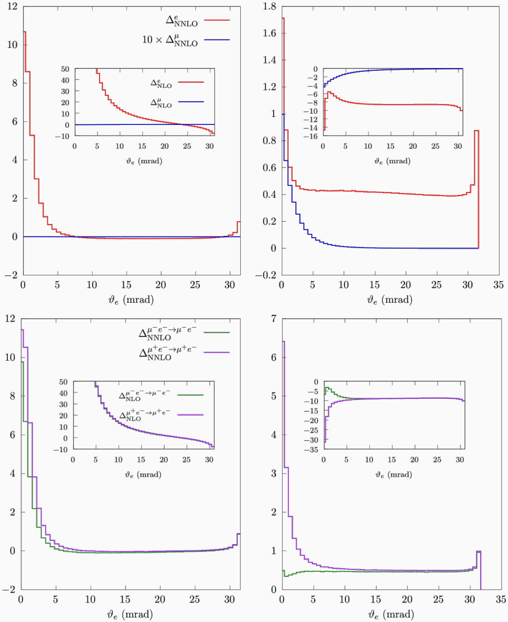
Backup: NLO EW corrections



Backup: $\Delta\alpha$



Backup: Photonic NNLO corrections: Results



Backup: YFS approximation

At NLO for virtual box diagrams, YFS misses terms of order:

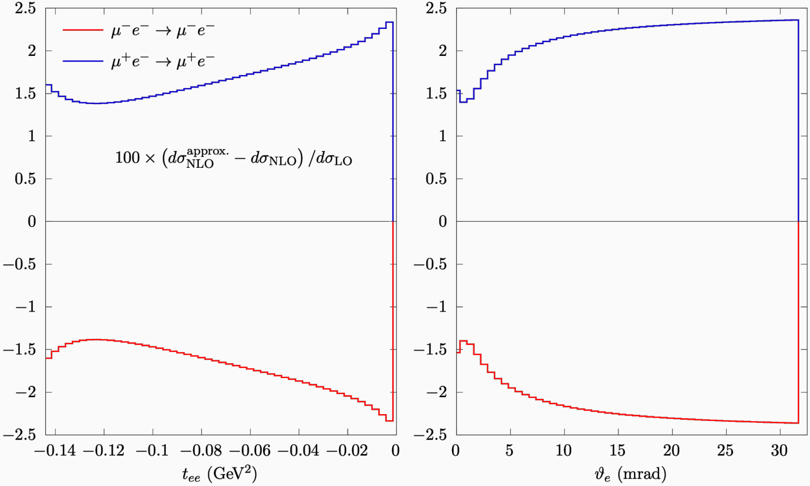
$$\frac{\alpha}{\pi} \ln \frac{m_\mu^2}{m_e^2} \simeq 0.025.$$

Therefore, for NNLO boxes YFS is expected to be accurate up to terms of order:

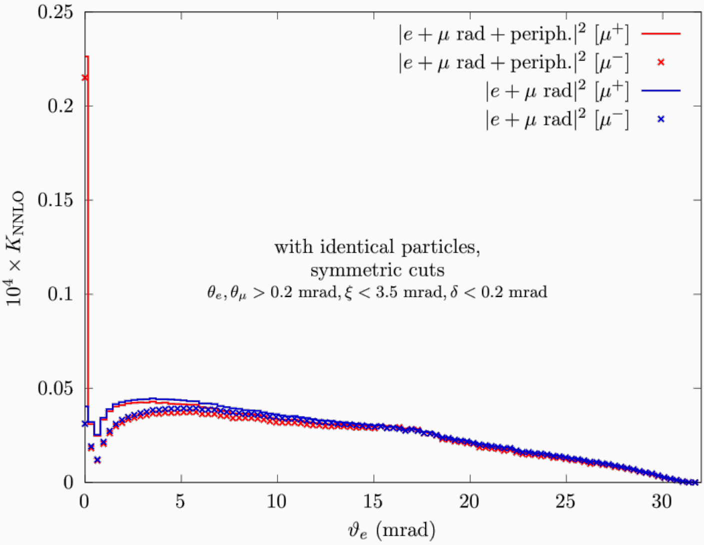
$$\left(\frac{\alpha}{\pi}\right)^2 \ln^2 \frac{m_\mu^2}{m_e^2} \simeq 6 \times 10^{-4}.$$

Improving the accuracy requires the inclusion of exact NNLO boxes, at least their leading terms in m_e.

Backup: YFS @ NLO



Backup: Real NNLO Lepton Pair Contributions: More Results



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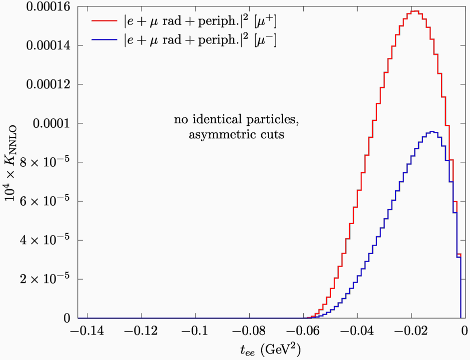
Backup: Muon pair production



- $\mu e \to \mu e + \mu^+\mu^-$ contributions are well below 10 ppm without cuts.
- By imposing standard (symmetrical) cuts, the process is kinematically forbidden.

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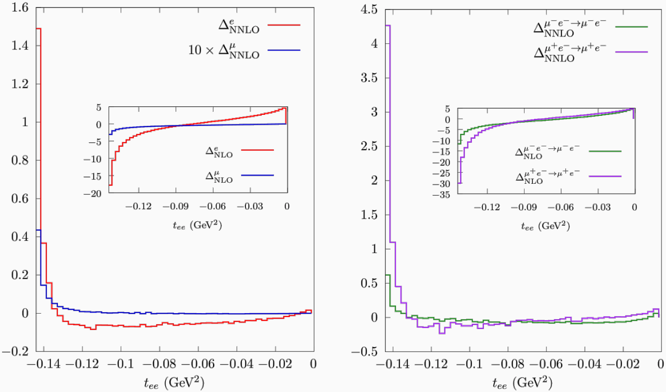
Backup: Photonic NNLO corrections: Results

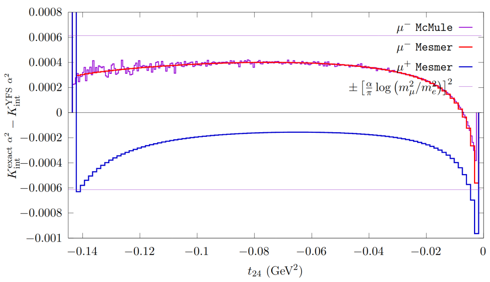


$$\Delta_i^{\rm NNLO} = \frac{d\sigma_i^{\rm NNLO}}{d\sigma_i^{\rm LO}} \times 100$$

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

Exact NNLO photonic corrections

- The complete two-loop corrections to $f\bar{f} \to F\bar{F}$ have been calculated by Bonciani *et al.* for $m_f = 0$. Using **crossing symmetry** it can be used for $\mu e \to \mu e$;
- The amplitudes with $m_e = 0$ can undergo the **massification** procedure, to get the collinear divergences in terms of $\ln\left(Q^2/m_e\right)$;
- NNLO double boxes are CPU expensive (>1 s/event on a single core).



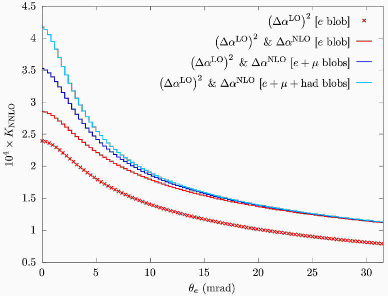
Difference between YFS-approximated and exact NNLO photonic K factor.
Preliminary

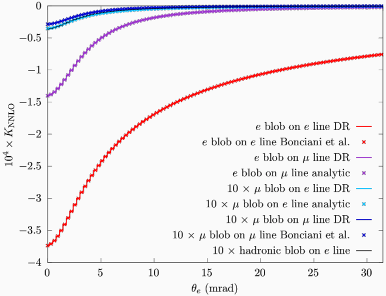
R. Bonciani *et al.*, PRL 128 (2022) 2.

T. Engel *et al.*, JHEP 02 (2019) 118. T. Engel, 2209.11110 [hep-ph].

A Broggio *et al.*, JHEP 01 (2023), 112.

Backup: Virtual NNLO Lepton Pair Contributions: Results





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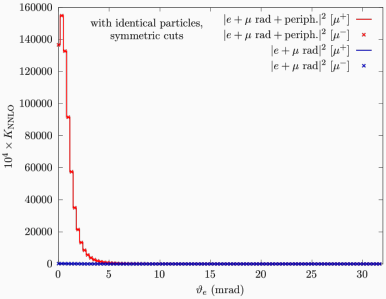

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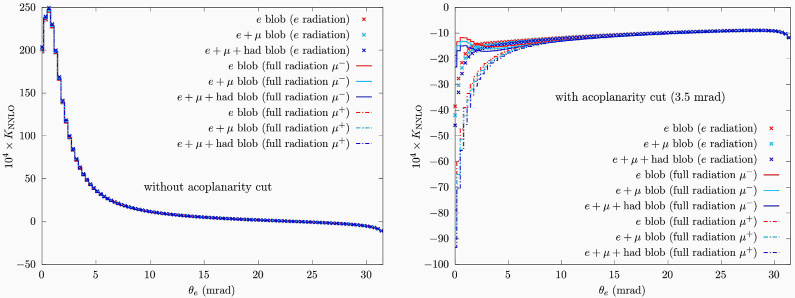
Backup: Virtual NNLO Lepton Pair Contributions: More Results



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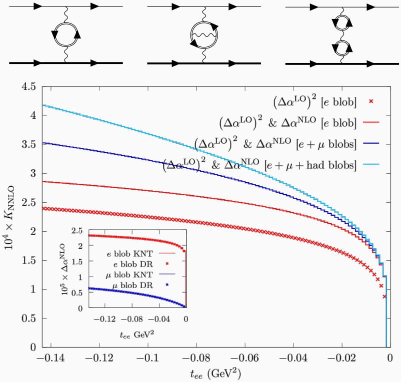
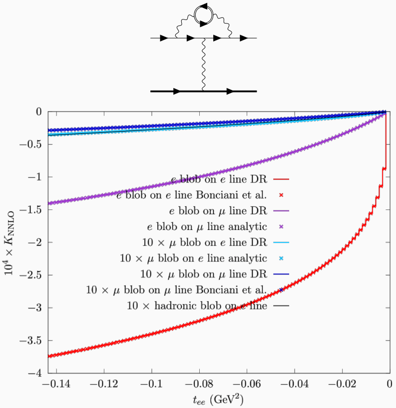
Backup: Virtual NNLO Lepton Pair Contributions: Results





$$K_{\rm NNLO} = \frac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{\rm LO}}$$

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