Window contributions to the Muon HVP from twisted mass lattice QCD

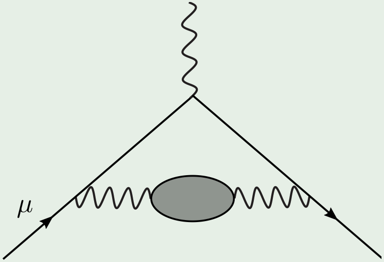
Francesco Sanfilippo

INFN Roma Tre

15-17th February 2023

Introduction on a_μ

Definition of a_μ



Magnetic moment of the muon: $\mu_\mu = -g_\mu \frac{e}{2m_\mu} S$

$$a_\mu \equiv \frac{g_\mu - 2}{2} = a_\mu^{QED} + a_\mu^{weak} + \boldsymbol{a_\mu^{HVP}} + a_\mu^{LBL}$$

$a_\mu^{HVP} \rightarrow$ non-perturbative hadronic contribution

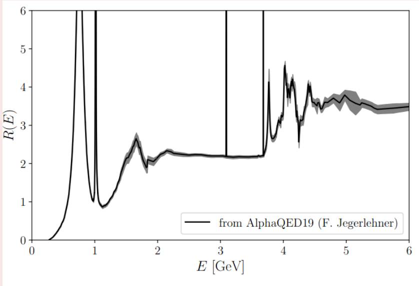
R-Ratio determination of a_μ^{had}

Historically "computed" via dispersive relation:

$$a_\mu^{HVP} = \frac{\alpha_{em}^2}{3\pi^2} \int_{M_\pi^2}^{\infty} dE^2 \frac{K(E)}{E^2} R(E)$$

from $R(E)$, the measured cross section:

$$e^+ e^- \rightarrow hadr$$



QCD determination: time momentum representation

Polarization-function based approach

From a pure theoretical point of view, a_μ can be computed from $\Pi\left(Q^2\right)$:

$$a_\mu^{\mathrm{LO-HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \cdot \Big(\Pi(Q^2) - \Pi(0)\Big).$$

Correlation function decomposition

Polarization function $\Pi\left(Q^2\right)$ can be extracted from the hadronic vector currents correlators:

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \left\langle J_\mu(x) J_\nu(0)\right\rangle = \Big(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\Big)\,\Pi(Q^2).$$

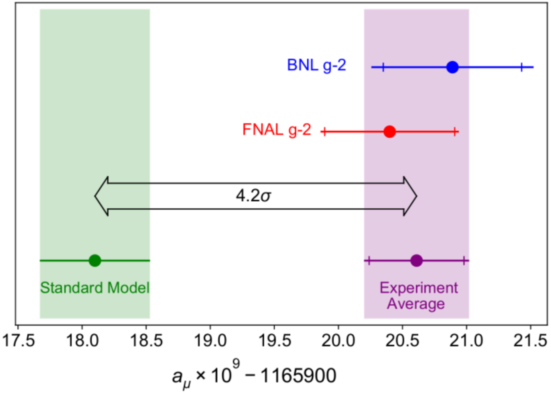
Time momentum approach

Customarily done with time momentum representation [Bernecker & Meyer, 2011]:

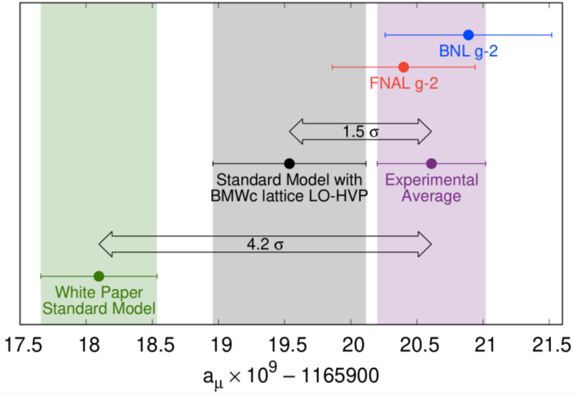
$$a_\mu^{\mathrm{LO-HVP}} = 2\alpha_{em}^2 \int_0^\infty dt\; t^2 K(m_\mu t)\, V(t), \quad V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \left\langle J_i(\vec{x},t) J_i(0)\right\rangle .$$

The new $g_\mu - 2$ puzzle

[Fermilab plot, from PRL 126, 141801 (2021) Muon $g-2$ collaboration]



[BMWc version, from L. Lellouch slides at SchwingerFest, LA (June 2022)]

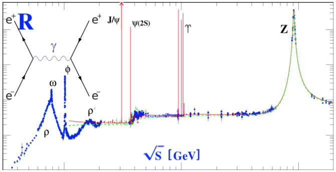


Calculation using dispersive approach in $\sim 4\sigma$ disagreement with experiment

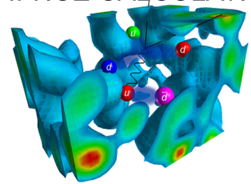
?? What's the deal ??

A triangular puzzle...

DISPERSIVE PREDICTION



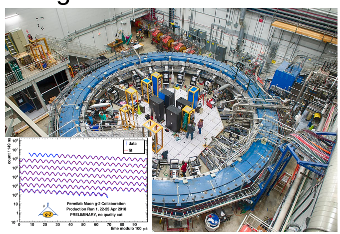
LATTICE CALCULATION



4.2σ DISAGREEMENT

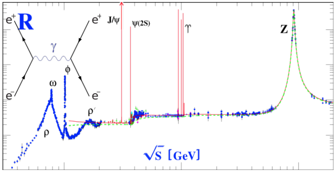
1.5σ “AGREEMENT”

g-2 EXPERIMENT



A triangular puzzle...

DISPERSIVE PREDICTION



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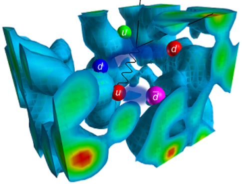

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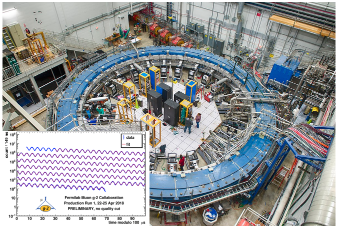


PROBLEMS IN R-RATIO?

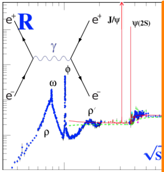
EXPERIMENTAL ISSUES?

NEW PHYSICS SPOILS?

PROBLEMS IN BMWc?

New physics explanation?

DISPERSIVE PREDICTION



LATTICE CALCULATION

New physics behind the new muon g-2 puzzle?

Luca Di Luzio,[1,2] Antonio Masiero,[1,2] Paride Paradisi,[1,2] and Massimo Passera[2]

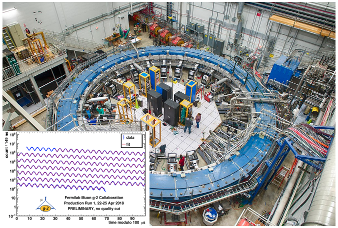
[1] *Dipartimento di Fisica e Astronomia 'G. Galilei', Università di Padova, Italy*
[2] *Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova, Italy*

The recent measurement of the muon g-2 at Fermilab confirms the previous Brookhaven result. The leading hadronic vacuum polarization (HVP) contribution to the muon g-2 represents a crucial ingredient to establish if the Standard Model prediction differs from the experimental value. A recent lattice QCD result by the BMW collaboration shows a tension with the low-energy $e^+e^- \to$ hadrons data which are currently used to determine the HVP contribution. We refer to this tension as the new muon g-2 puzzle. In this Letter we consider the possibility that new physics contributes to the $e^+e^- \to$ hadrons cross-section. This scenario could, in principle, solve the new muon g-2 puzzle. However, we show that this solution is excluded by a number of experimental constraints.

4.2σ DIS GREEMENT

1.5σ “AGR

g-2 EXPERIMENT



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New physics explanation?

DISPERSIVE PREDICTION

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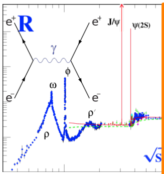

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The $(g\text{-}2)_\mu$ window discrepancy: a GeV-scale new physics explanation

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[b] Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Piazzale A. Moro 2, I-00185 Roma, Italy
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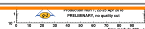
Abstract

Recent lattice determinations of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment $a_\mu^{\rm HVP}$ indicate an exacerbation of the discrepancy with the data driven dispersive method at the level of $\sim 4.5\sigma$. This disagreement could be due to some process beyond the standard model affecting the determination of a_μ from $e^+e^- \to$ hadrons data within a certain energy range. Recently, we built a new physics mechanism that could explain the lattice versus data-driven discrepancy together with other a_μ related anomalies. Here we study how our theoretical construction performs in the short, intermediate and long distance windows. We find that, in agreement with lattice indications, the dominant effects are confined to the low and intermediate energy windows, while the high energy window remains largely unaffected.

PROBLEMS IN R-RATI

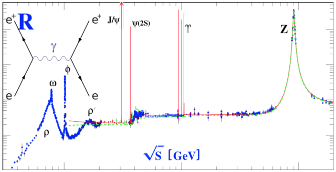
EXPERIMENTAL ISSU

NEW PHYSICS SPOIL



Clearing the discussion

DISPERSIVE PREDICTION



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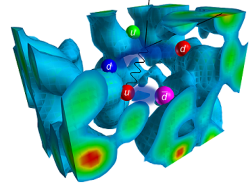

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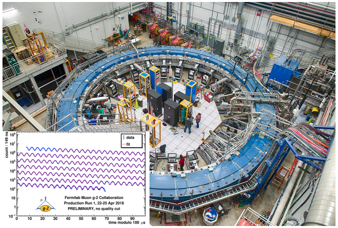
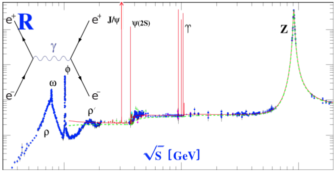


Direct theoretical comparison?

DISPERSIVE PREDICTION



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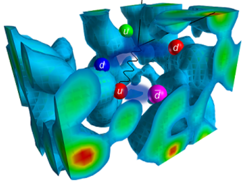

?

See A.De Santis talk @10am

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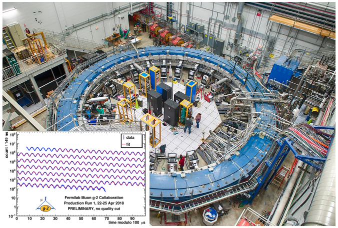
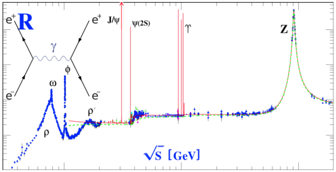
g-2 EXPERIMENT



Intermediate step: a_μ window

DISPERSIVE PREDICTION



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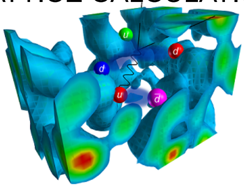
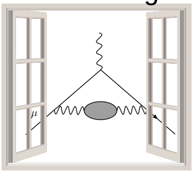


WINDOW g-2



Modified a_μ

a_μ: same observable, two approaches

$$\underbrace{\frac{\alpha_{em}^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{dE^2}{E^2}\tilde{K}\left(E\right)\boldsymbol{R}\left(\boldsymbol{E}\right)}_{disperisve,\,experimental} = a_\mu^{HVP} = \underbrace{2\alpha_{em}^2\int_0^{\infty}dt\ t^2 K(m_\mu t)\,\boldsymbol{V}\left(\boldsymbol{t}\right)}_{lattice,\,SM}$$

Direct theoretical comparison of $V\left(t\right)$ and $R\left(E\right)$?

- One could use lattice $V\left(t\right)$to compute $R\left(E\right)$
- Ambitious: $R\left(E\right)$ is the Inverse Laplace Transform of $V\left(t\right)$
- (again: see talk by **A.De Santis @10am** today for more info...)

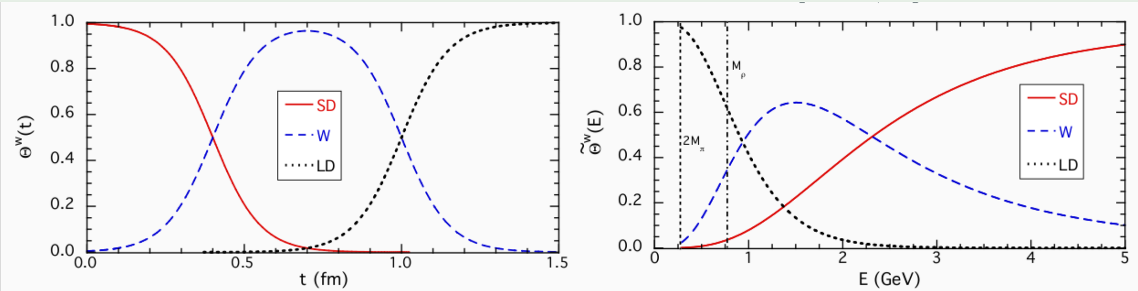
Intermediate step: a_μ window

Less ambitious but more effective: compare modified versions of a_μ more local in energy

$$2\alpha_{em}^2\int_0^{\infty}dt\ t^2 K(m_\mu t)\,\boldsymbol{V}\left(\boldsymbol{t}\right)\underline{\Theta\left(t\right)} = a_\mu^{\Theta} = \frac{\alpha_{em}^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{dE^2}{E^2}\tilde{K}\left(E\right)\boldsymbol{R}\left(\boldsymbol{E}\right)\underline{\tilde{\Theta}\left(E\right)}$$

Window observables for $g_\mu - 2$

Intermediate Window by RBC/UKQCD



Consider (mainly) the $[0.4 - 1]\ \mathrm{fm}$ contribution in the lattice computation.

$$\Theta^{SD}(t) + \Theta^{W}(t) + \Theta^{LD}(t) = 1$$

Historical motivation: restrict QCD to do what it is best at

Little cut off effects, little finite volume effects, good signal to noise ratio

Today: an important analysis tool

- Compare more in details lattice calculations of a_μ by different collaborations.
- Explore local portions of the R-ratio experimental measurement with predictions.

IN THIS WORK: a_μ^{SD} and a_μ^{W}

ETMC calculation of short and intermediate windows

Lattice calculation of the short and intermediate time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions

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[2] *Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Konstantinou Kavafi Street, 2121 Nicosia, Cyprus*
[3] *Dipartimento di Scienze Matematiche, Fisiche e Informatiche, Università di Parma and INFN, Gruppo Collegato di Parma, Parco Area delle Scienze 7/a (Campus), 43124 Parma, Italy*
[4] *Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma, Italy*
[5] *Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy*
[6] *HISKP (Theory), Rheinische Friedrich-Wilhelms-Universität Bonn, Nussallee 14-16, 53115 Bonn, Germany*
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[8] *NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany*
[9] *Dipartimento di Matematica e Fisica, Università Roma Tre and INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy*
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Extended Twisted Mass Collaboration

We present a lattice determination of the leading-order hadronic vacuum polarization (HVP) contribution to the muon anomalous magnetic moment, $a_\mu^{\rm HVP}$, in the so-called short

arXiv:2206.15084 - TO APPEAR ON PRD

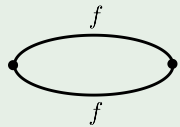
Outline of our twisted-mass lattice QCD calculation

What has been computed

u, d, s, c, quark-line connected and disconnected contributions to a_μ^{SD} and a_μ^{W} in the isospin symmetric limit $m_u{=}m_d$, neglecting α_{em}^3 QED effects.

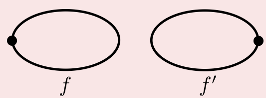
Connected contributions: $f = u, d, s, c$.

$$V^{f}_{conn}(t) \equiv -\frac{1}{3}\sum_{i=1,2,3}\int d^3x \langle J_i^f(\vec{x},t)\ J_i^f(0)\rangle = q_f^2 \times$$



Disconnected contributions: $f, f' = u, d, s, c$.

$$V^{ff'}_{disco}(t) \equiv -\frac{1}{3}\sum_{i=1,2,3}\int d^3x \langle J_i^f(\vec{x},t)\ J_i^{f'}(0)\rangle = -q_f q_{f'} \times$$



Twisted-mass (tm) and Osterwalder-Seiler (OS) currents

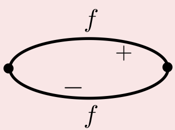
Twisted-mass regularization

Mass term: $\bar{\psi}^{\pm}\left(m \pm i\mu\gamma_5\right)\psi^{\pm}$ tuned to achieve $\mathcal{O}\left(a\right)$ improvement.

- For connected contributions, two different ways to regularize quarks within Twisted Mass
- The difference is a pure $O\left(a^2\right)$ effect $\rightarrow$ constrain continuum limit extrapolation.

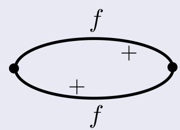
Twisted Mass choice

$$J_{\mu}^{f,tm} \propto \bar{\psi}_f^{+}\gamma_{\mu}\psi_f^{-}$$

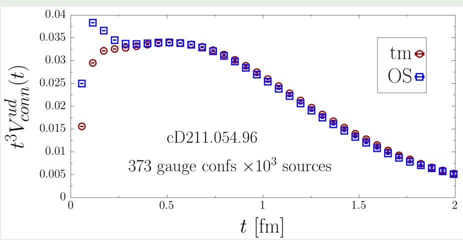


Osterwalder-Seiler choice

$$J_{\mu}^{f,OS} \propto \bar{\psi}_f^{+}\gamma_{\mu}\psi_f^{+}$$



The correlator

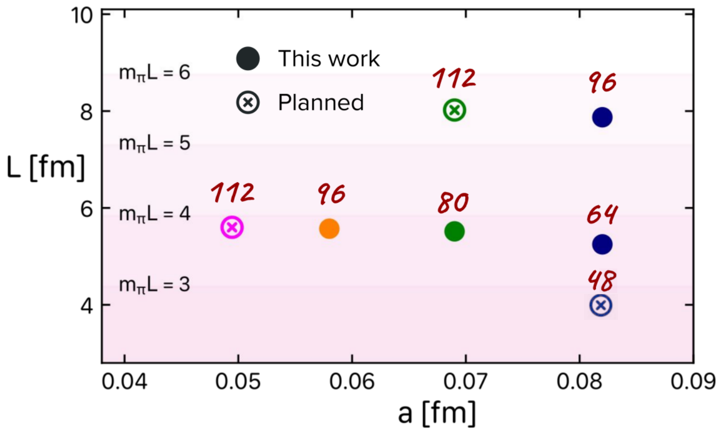


Comparison of the two regularizations:

- Large differences at small $\sim a$ distances.
- Subject to different finite volume effects.

Simulations at the $\simeq$ physical point

- Four ($\simeq$) physical point ensembles, with $a \in [0.057 \text{ fm} - 0.080 \text{ fm}]$.
- $L \sim 5.1$ fm and $L \sim 7.6$ fm to control Finite Size Effects (FSEs).
- $M_\pi \in [136, 141]$ MeV, $\quad M_\pi L > 3.5$, $\quad V = L^3 \times T$, $\quad T = 2L$.



Planned simulations

- at $a < 0.05$ fm,
- on larger volume at $a \sim 0.068$ fm,
- and smaller volume at $a \sim 0.0072$ fm

Details of the ETMC ensembles

Lattice parameters

ensemble	β	V/a^4	a (fm)	$a\mu_\ell$	M_π (MeV)	L (fm)
B.072.64	1.778	$64^3 \times 128$	0.0796 (1)	0.00072	140.2 (0.2)	5.10
B.072.96	1.778	$96^3 \times 192$	0.0796 (1)	0.00072	140.1 (0.2)	7.64
C.060.80	1.836	$80^3 \times 160$	0.0682 (1)	0.00060	136.6 (0.2)	5.46
D.054.96	1.900	$96^3 \times 192$	0.0569 (1)	0.00054	140.8 (0.3)	5.46

- Rernormalization Constants have $< 0.1\%$ uncertainties.
- Wilson-clover twisted mass fermions at maximal twist (automatic $\mathcal{O}(a)$ improvement).
- Small mistuning of M_π corrected both in valence and sea.
- B lattice spacing interpolated to $L = 5.46\,\mathrm{fm}$.

"Renormalization constants"

ensemble	Z_V	Z_A
B.072.64	0.706378 (16)	0.74284 (23)
B.072.96	0.706402 (15)	0.74274 (20)
C.060.80	0.725405 (13)	0.75841 (16)
D.054.96	0.744105 (11)	0.77394 (10)

Estimates per confs

ensemble	ℓ	s	c
B.072.64	10^3	16	4
B.072.96	10^3	16	4
C.060.80	10^3	16	4
D.054.96	10^3	64	24

Making contact with physical world

Continuum limit

Combined continuum fits employing both **tm** and **OS** lattice correlators.

Fit ansatz ($w = \{SD, W\}$):

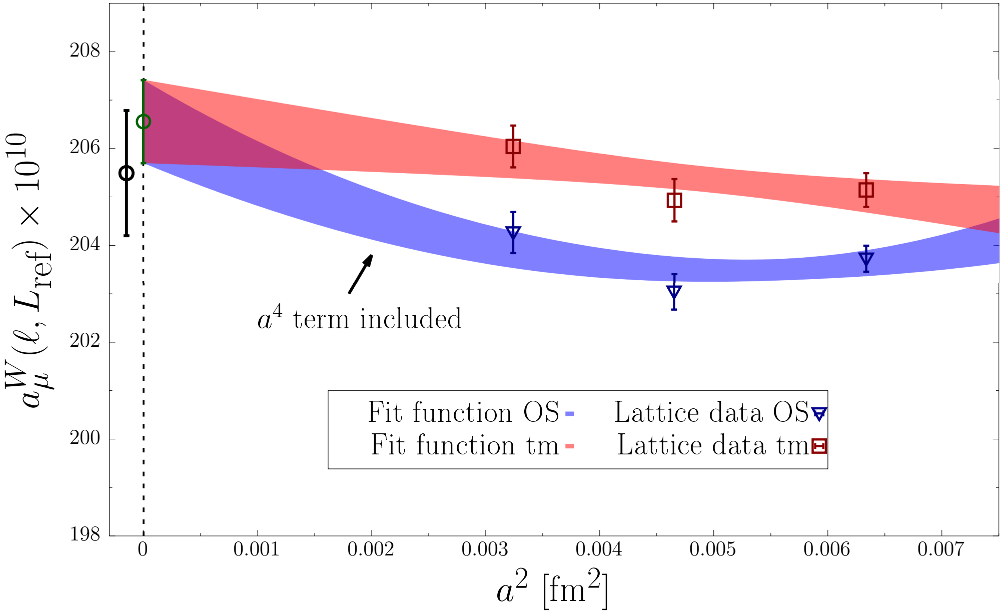
$$a_\mu^w(\ell) = \boldsymbol{a_\mu^{w,cont}}(\ell) \times \left[1 + \boldsymbol{D_1^r}\,\frac{a^2}{[\log(a^2/w_0^2)]^{n_r}} + \boldsymbol{D_2^r}\,a^4\right]$$

- $\boldsymbol{a_\mu^{w,cont}}$,$\boldsymbol{D_1^r}$ and $\boldsymbol{D_2^r}$ are free fitting parameters.
- $a_\mu^{w,cont}$ do not depend upon the regularization $r = \{tm, OS\}$.

Infinite volume limit within MLLGS method (for u, d quarks)

- Finite Volume Effects Mostly dominates the tail of the correlator: ρ state ($\pi\pi$ resonance)
- At finite volume: described in terms of discrete energy levels of two pions in a box as H.**M**ayer proposed:
  - **L**ellouch-**L**uscher framework to describe the interacting states
  - **G**ounaris **S**akurai model to parametrize phase shifts in the continuum
- Bring continuum result from $L = 5.46$ to infinite volume limit.

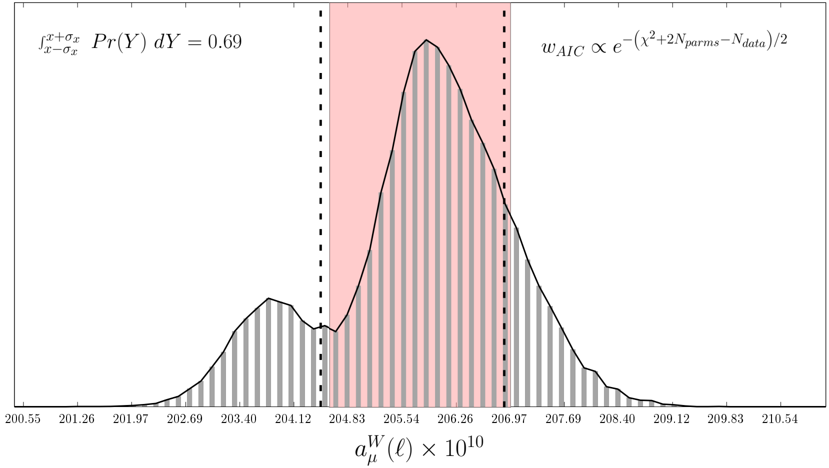
Light $(u+d)$ connected contribution to a_μ^W



Typical accuracy of $0.1 - 0.2\%$ for all ensembles and regularizations.

Analysis of the systematics

$$N_{\text{fits}} = 154\,, \qquad a_\mu^W(\ell) = 205.78 \pm 1.19 \times 10^{-10}$$



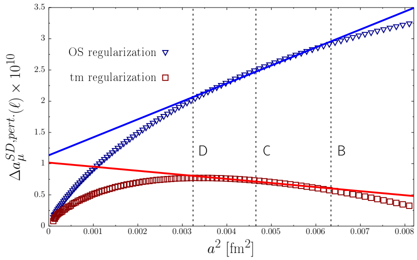
Akaike information criterion or maximum $\chi^2/d.o.f$ cut.

Light (u+d) connected contribution to a_μ^{SD}

Lattice evaluation of a_μ^{SD} suffers from dangerous $a^2 \log(a^2)$ artifacts generated by the short-times integration [**Cé, Harris, Meyer et al. (2021)**]

$$V(t \ll m^{-1}, a) \propto \frac{1}{t^3}\left[1 + \sum_{n=1}^{\infty} c_n \cdot \left(\frac{a}{t}\right)^{2n}\right], \qquad K(m_\mu t \ll 1) \propto t^2$$

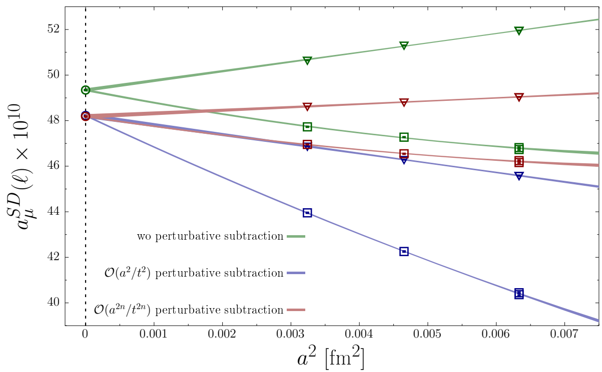
$$\Longrightarrow \quad a_\mu^{SD} \simeq \int_a^{t_0} dt\ V(t,a)\ t^2 K(m_\mu t) = A + D\ a^2 \log(a^2) + \mathcal{O}(a^2)$$



Naive continuum limit of free theory cut-offs

- $a^2 \log(a^2)$ cut-off effects already present in the free-theory correlator.
- $\Delta a_\mu^{SD,pert.}(\ell)$ are cut-off effects of the tm (OS) $\mathcal{O}(\alpha_s^0)$ massless correlator.

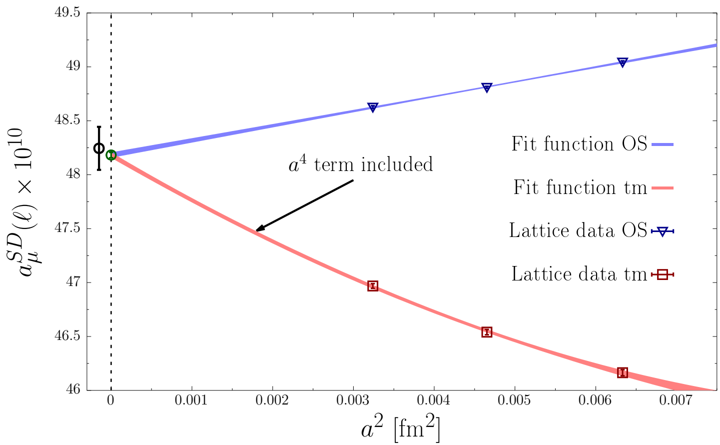
Perturbative $\mathcal{O}(\alpha_s^0)$ subtraction of cut-off effects



Three options

- No perturbative subtraction: continuum limit missed by $\simeq 1 \times 10^{-10}$ (effect larger than any other source of systematics).
- LO $\mathcal{O}(\frac{a^2}{t^2})$ subtraction: sufficient to get correct continuum limit.
- Full $\mathcal{O}(a^{2n}/t^{2n})$ subtraction: makes lattice data even flatter.

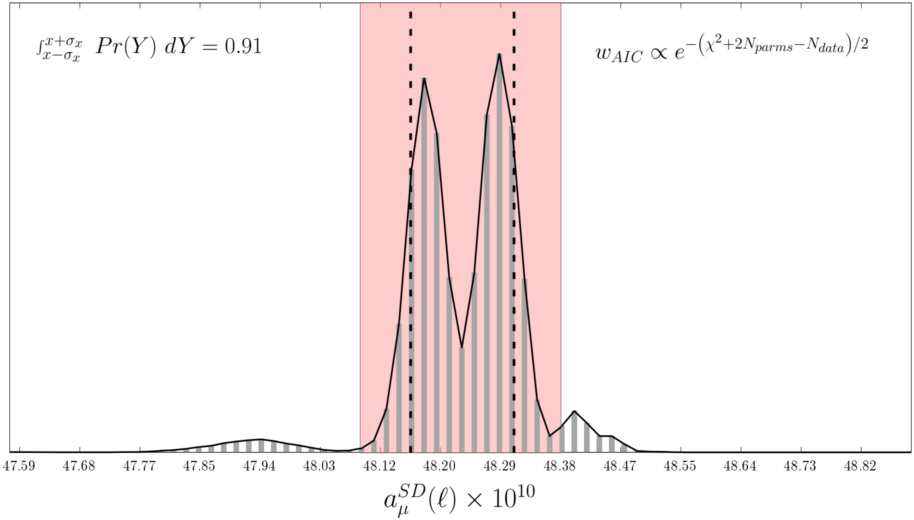
$a_\mu^{SD}(\ell)$



- Precision: better than 0.1%.
- $\mathcal{O}(a^{2n}/t^{2n})$ free-theory cut-off effects subtracted for both regularizations.
- Final error entirely due to systematics in continuum extrapolation.
- a^4 term on tm regularization necessary to have a good χ^2/dof.

Analysis of the systematics for $a_\mu^{SD}(\ell)$

$$N_{\text{fits}} = 155\,, \qquad a_\mu^{SD}(\ell) = 48.23 \pm 0.15 \times 10^{-10}$$



- Final error entirely due to systematics in continuum extrapolation.
- Alternative continuum limit extrapolation with ultra-short distance regulator.

Calculation details

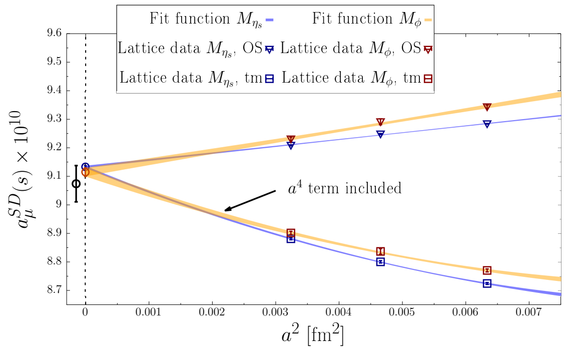
Strange contributions

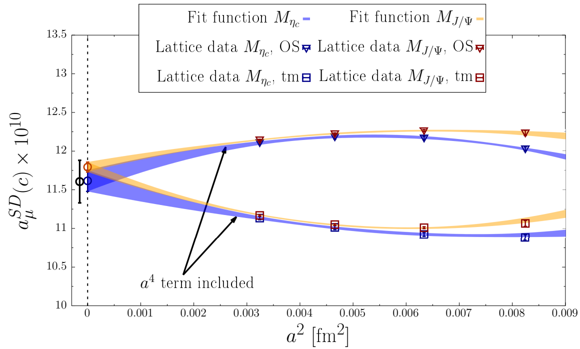
- Valence s quark mass tuned alternatively using M_{η_s} or M_ϕ as input.
- Both determinations included in final analysis of systematics.
- Subtraction of perturbative $\mathcal{O}(\alpha_s^0)$ cut-off effects in $a_\mu^{SD}(s)$.
- Finite size effects and M_π^{sea} mistuning effects not visible within accuracy.

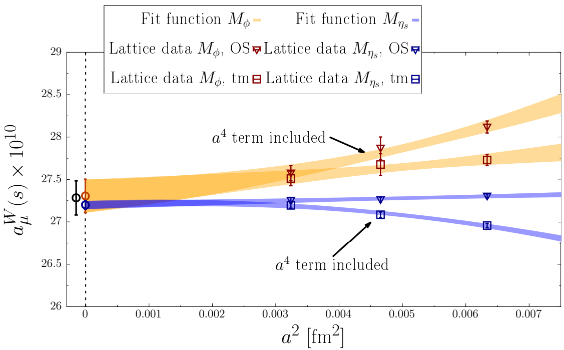
Charm contributions

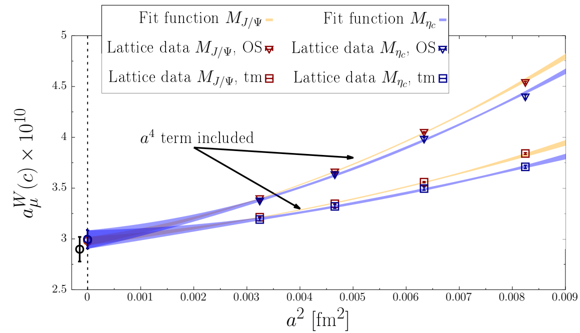
- Valence c quark mass tuned alternatively using M_{η_c} or $M_{J/\Psi}$ as input.
- Both determinations included in final analysis of systematics.
- Added a fourth (coarser) lattice spacing $a \sim 0.09$ fm with pion masses $M_\pi^{sea} \in [250 - 350]$ MeV to improve continuum limit extrapolation.
- No M_π^{sea} dependence observed, negligible finite size effects.
- Subtraction of perturbative $\mathcal{O}(\alpha_s^0)$ cut-off effects in $a_\mu^{SD}(c)$. More effective if evaluated with $m_q = m_c^{bare}$.

Strange and charm connected contributions

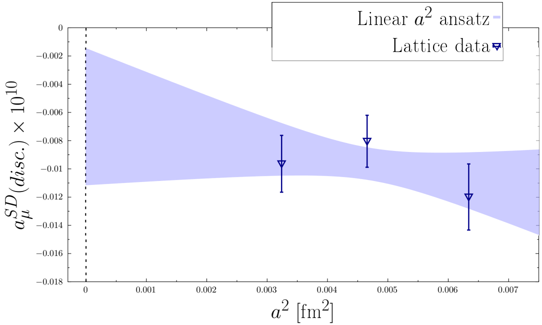


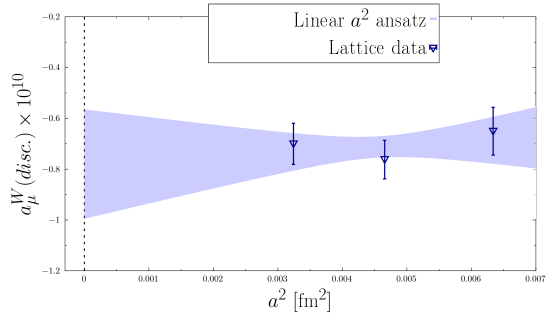






Disconnected contribution to a_μ^W and a_μ^{SD}





- Noise-reduction techniques: one-end-trick, exact deflation of low-modes, hierarchical probing.
- Small cut-off effects within accuracy. No study of Finite Size Effect.
- $a_\mu^{SD}(disco)$ completely negligible, $a_\mu^W(disco) \sim 0.3\%\ a_\mu^W$.

$$a_\mu^{SD}(disco) = -0.006\ (5) \times 10^{-10}\ , \qquad a_\mu^{W}(disco) = -0.77\ (17) \times 10^{-10}$$

	ETMC-22	BMW-20	CLS/MAINZ-22	RBC/UKQCD-18
$a_\mu^W(disco) \times 10^{10}$	$-0.77\ (17)$	$-0.85\ (6)$	$-0.81\ (9)$	$-1.00\ (10)$

Summary

Unbridged results

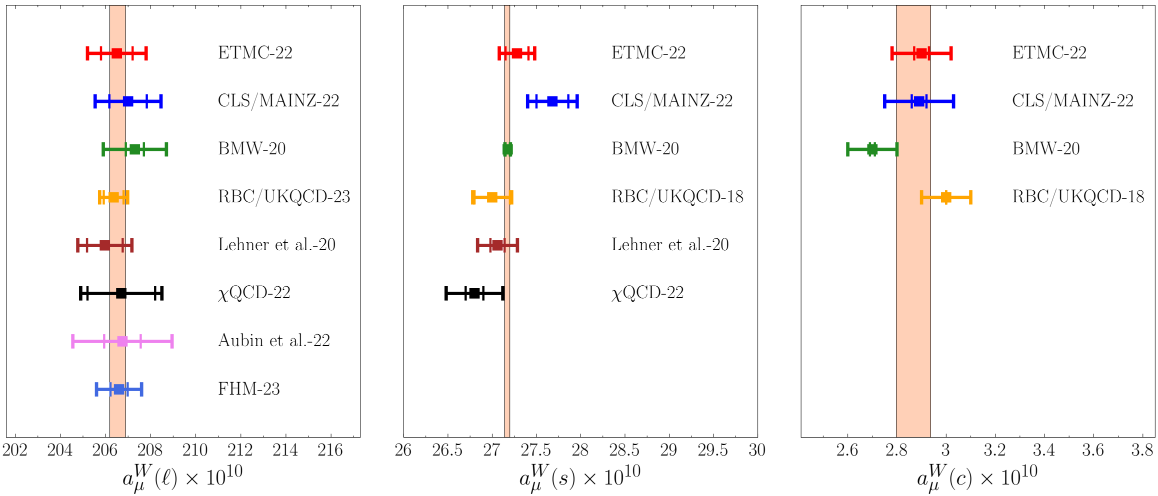
	$a_\mu^{SD} \times 10^{10}$	$a_\mu^{W} \times 10^{10}$
ℓ	48.24 (20)	206.5 (1.3)
s	9.074 (64)	27.28 (20)
c	11.61 (27)	2.90 (12)
disco	−0.006 (5)	−0.78 (21)
IB	0.03*	0.43 (4)**
b	0.32 (2)***	–
total	69.27 (34)	236.3 (1.3)

- * *rhad software package.* 0.04% *of the total* a_μ^{SD} *(or* 0.1σ*).*
- ** *From Borsanyi et al. (Nature, 2021).*
  0.18% *of the total* a_μ^{W} *(or* 0.4σ*).*
- *** *rhad & lattice.* 0.46% *of total* a_μ^{SD} *(or* 1.1σ*).*

Precision achieved on a_μ^{W} and a_μ^{SD} is $\sim 0.5\%$.

Per-flavour lattice comparisons...

...include only results from at least 3 lattice spacings and 1 phys. point ensemble.

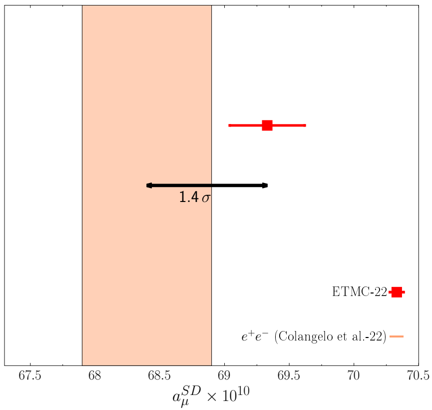


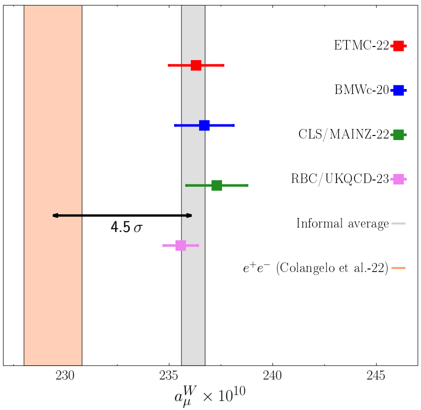
$[a_\mu^{SD} + a_\mu^W] \times 10^{10}$	ℓ	s	c	total, incl. disc., IB, b
ETMC-22*	254.74 (1.5)	36.4 (0.3)	14.51 (4)	305.65 (1.5)
Fermilab/HPQCD/MILC-22	253.5 (0.9)	36.3 (0.2)	14.63 (5)	303.8 (1.1)

*Preliminary: conservative error (Assuming 100% correlation between a_μ^{SD} and a_μ^W).

Comparison with $e^+e^- \to$ hadrons results

$e^+e^- \to$ hadrons from Colangelo et al. arXiv:2205.12963 (2022).





Tension in a_μ^W rises to 4.5σ if we combine ETMC '22, BMW '20 and CLS/Mainz '22 (informal average $\to$ next WP).

Deviation of $e^+e^- \to$ hadrons data w.r.t. the SM

- in the **intermediate** energy regions more **pronounced**,
- in the **low energy very mild**,
- but **not in the high** energy region.

Conclusions

The $g_\mu - 2$ puzzle

- Dipsersive approach disagree with experimental measurement
- Lattice calculation substantially agree with the experimental measurement

Slicing the comparison

- Comparing $R(E)$ energy per energy would be highly interesting
- And we are making progress in this directions, see this morning's talk
- Meanwhile we refer to WINDOW OBSERVABLES, with interesting phenomenology:
  - deviation concentrated in the **intermediate** energy regions,
  - agreement **in the low/high** energy region.

Perspective

- Including isospin breaking & QED
- Improving statistics (also for $R(E)$)
- Including more volumes
- Produce results for the total a_μ

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THANKS!