Pseudoscalar Transition Form Factors and the Hadronic Light-by-Light Contribution to a_μ

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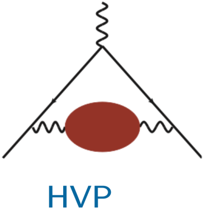
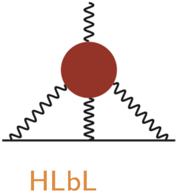
Introduction

- The different contributions (Aoyama et al., 2020):

Contributions	**Value** $\times 10^{11}$
Experiment	116 592 089(63)
QED	116 584 718.931(0.104)
Electroweak	153.6(1.0)
HVP	6845(40)
HLbL	92(19)
Total SM value	116 591 810(43)
Difference: $\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)



- Theory error on a_μ is currently dominated by two hadronic loop corrections:
  1. Hadronic Vacuum Polarization (HVP) [$\mathcal{O}(\alpha_e^2)$]
  2. Hadronic Light-by-Light (HLbL) scattering [$\mathcal{O}(\alpha_e^3)$].



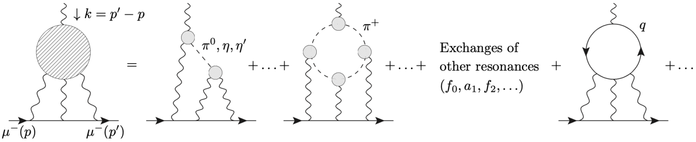
→ HLbL is a small contribution *but* with a relatively large error.

→ Theory uncertainty needs to be reduced by a factor of two to meet future experimental precision.

Hadronic Light-by-Light Contribution

- Two ways to compute the HLbL contribution: direct lattice calculation (Chao et al., 2021; Blum et al., 2020) & dispersive framework (Colangelo et al., 2014b,a, 2015).
- In dispersive framwork: the HLbL scattering has different sub-contributions (Aoyama et al., 2020):

Contributions	**Value** $\times 10^{11}$
π^0, η, η'-poles	93.8(4.0)
π, K-loops/boxes	-16.4(0.2)
$\pi\pi$ scattering	-8(1)
scalars + tensors	-1(3)
axial vectors	6(6)
u, d, s-loops / short distance	15(10)
c-loop	3(1)
Total	92(19)



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→ Pseudoscalar (π^0, η, η') poles form the largest contribution to HLbL diagram.

→ π^0-pole estimated using lattice (Gérardin et al., 2019) + dispersive framework (data-driven) (Hoferichter et al., 2018).

→ ETM has recently shown a result for the η-pole at finite lattice spacing (Alexandrou et al., 2022).

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Project Goal: Calculate the sum of π^0, η, η'-pole contributions from lattice QCD methods with $< 10\%$ precision in the continuum limit.

Motivation

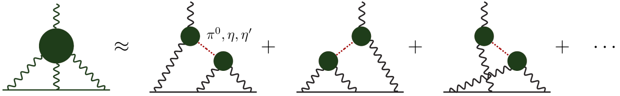

Figure 1: HLbL diagram and its leading contributions resulting from π^0, η, η' pseudoscalar exchanges.

In the dispersive framework, the 'master equation' relates the Pseudoscalar Transition Form Factors (**TFF**s) to pseudoscalar (p) pole contributions to a_μ^{p-pole} (Knecht and Nyffeler, 2002)

$$a_\mu^{p-pole} = \left(\frac{\alpha_e}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \, \Big[w_1(Q_1, Q_2, \tau) \mathcal{F}_{p\gamma^*\gamma^*}(-Q_1^2, -Q_3^2) \mathcal{F}_{p\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1, Q_2, \tau) \mathcal{F}_{p\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \mathcal{F}_{p\gamma^*\gamma^*}(-Q_3^2, 0) \Big]$$

- $Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_2$
- $\tau = \cos\theta$
- θ angle between Q_1 & Q_2

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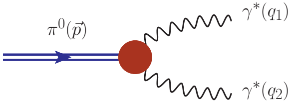
$$\left. + w_2(Q_1, Q_2, \tau)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_3^2, 0)\right]$$

We recognize two main objects

1. The TFFs $\mathscr{F}_{p\gamma^*\gamma^*}(q_1^2, q_2^2)$
2. The weight functions $w_i(q_1, q_2, \tau)$

$\mathscr{F}_{p\gamma^*\gamma^*}(q_1^2, q_2^2)$ encodes the interaction between a pseudoscalar and two virtual photons. E.g. for the pion

$\mathscr{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) =$ 

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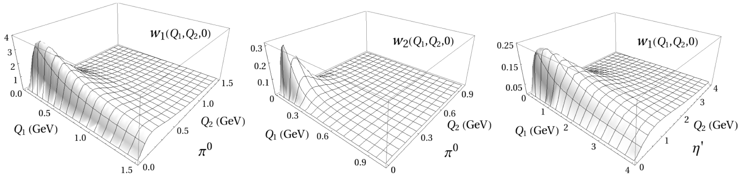
1. The TFFs $\mathscr{F}_{p\gamma^*\gamma^*}(q_1^2,q_2^2)$
2. The weight functions $w_i(q_1,q_2,\tau)$

Weight functions are peaked at low spacelike Q^2 so lattice QCD is the perfect method.



Transition Form Factor from the Lattice

The TFF for a pseudoscalar meson is defined by the matrix elements $M_{\mu\nu}$ (Ji and Jung, 2001) (Gérardin et al., 2016)

$$M_{\mu\nu}(p,q_1) = i\int d^4x\, e^{iq_1\cdot x}\,\langle\Omega|\,T\{J_\mu(x)J_\nu(0)\}\,|P(\vec{p})\rangle = \varepsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta\,\mathscr{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$$

where J_μ is the EM current. (Euclidean) Matrix elements are related to 3-point correlation function $C^{(3)}_{\mu\nu}$ on lattice

$$\boxed{C^{(3)}_{\mu\nu}(\tau,t_P) = a^6\sum_{\vec{x},\vec{z}}\langle J_\mu(\vec{z},\tau+t_P)J_\nu(\vec{0},t_P)P^\dagger(\vec{x},0)\rangle e^{i\vec{p}\cdot\vec{x}}e^{-i\vec{q}_1\cdot\vec{z}}}$$

where τ is the time-separation between the two EM currents and

1. In the Euclidean:

$$M^E_{\mu\nu} = \frac{2E_P}{Z_P}\int_{-\infty}^{\infty} d\tau\, e^{\omega_1\tau}\tilde{A}_{\mu\nu}(\tau) \qquad \text{with } \tilde{A}_{\mu\nu}\sim C^{(3)}_{\mu\nu}$$

2. E_P, Z_P energy and overlap of the pseudoscalar that are extracted from two-point correlations functions.
3. $q_1 = (\omega_1,\vec{q}_1)$ and $q_2 = (E_P-\omega_1,\vec{p}-\vec{q}_1)$

Wick Contractions Correlation Function

$$C^{(3)}_{\mu\nu}(\tau, t_{PS}) = a^6 \sum_{\vec{x},\vec{z}} \langle J_\mu(\vec{z}, t_i) J_\nu(\vec{0}, t_f) P^\dagger(\vec{x}, t_0) \rangle e^{i\vec{p}\cdot\vec{x}} e^{-i\vec{q}_1\cdot\vec{z}}.$$

Correlation function receives contributions from (potentially) four different Wick contractions

1. - **For the π^0**

   $$P_{\pi^0}(x) = \frac{1}{\sqrt{2}} \left(\overline{u}\gamma_5 u(x) - \overline{d}\gamma_5 d(x)\right).$$

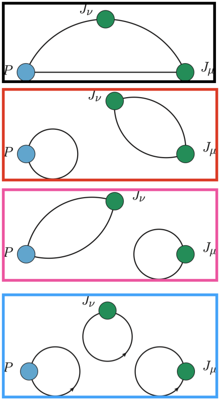
   - We work in the isospin limit.
     Consequently disconnected pseudoscalar loop is formally zero.
   - Two diagrams contribute.
   - Disconnected contribution is small $\mathcal{O}(1-2\%)$.
     (Gérardin et al., 2019).

2. - **For the η, η'**

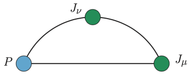
   $$P_{\eta_8}(x) = \frac{1}{\sqrt{6}} \left(\overline{u}\gamma_5 u(x) + \overline{d}\gamma_5 d(x) - 2\overline{s}\gamma_5 s(x)\right),$$

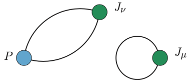
   $$P_{\eta_0}(x) = \frac{1}{\sqrt{3}} \left(\overline{u}\gamma_5 u(x) + \overline{d}\gamma_5 d(x) + \overline{s}\gamma_5 s(x)\right).$$

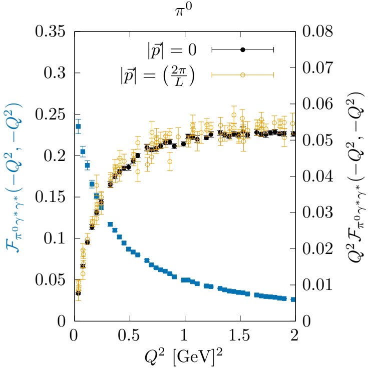
   - All four diagrams contribute.
   - η^8 and η^0 mix to create physical η, η'.

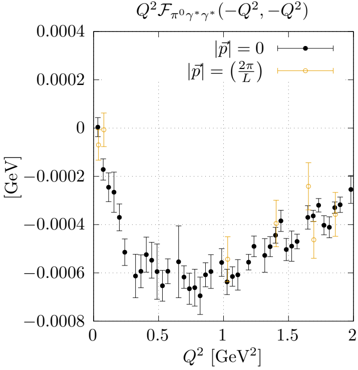


Results Transition Form Factor π^0 Single Ensemble



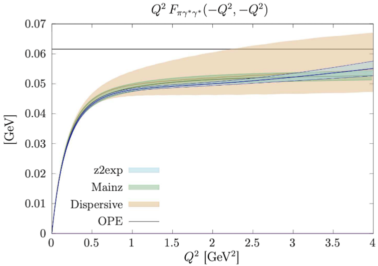


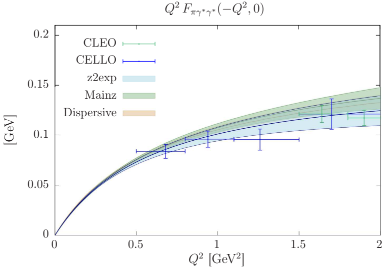




- High precision data, also for the disconnected contribution.
- Good agreement between two momentum frames of the pion.

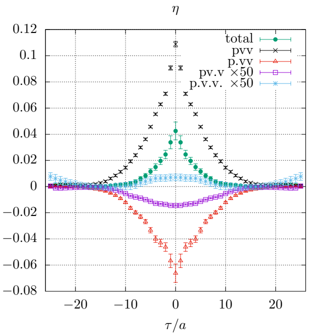
Results Transition Form Factor π^0 Continuum

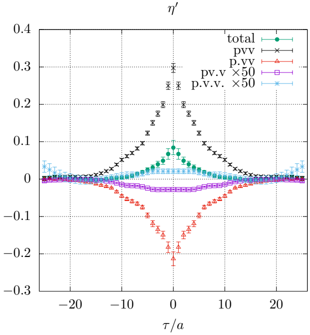


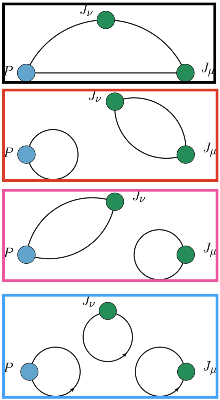


- Our result agrees with previous lattice computation (Mainz) and available experimental data.

Integrand Double Virtual regime η, η' TFF



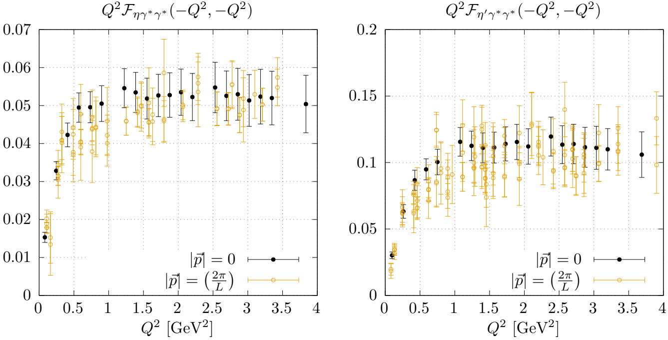




- We have a good signal for all four different Wick contractions.
- Bulk of signal formed by pvv (fully connected) and p.vv (vv disconnected)
- pv.v (pv disconnected) and p.v.v. (fully disconnected) are subdominant.

Results Form Factor η, η' Single Enemble

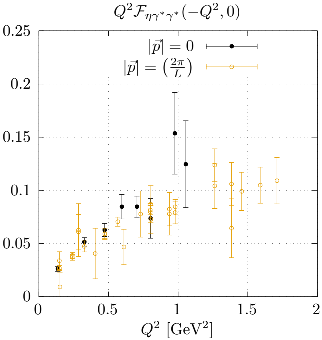
- Integrating over this leads to one point for the TFF.

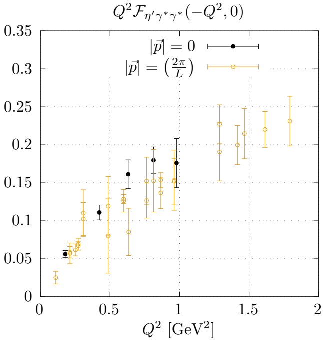


- We find a good agreement between two momentum frames of the η, η' mesons.
- Errors are bigger than for the π^0 due to sizeable disconnected diagram and mixing.

Results Form Factor η, η' Single Enemble

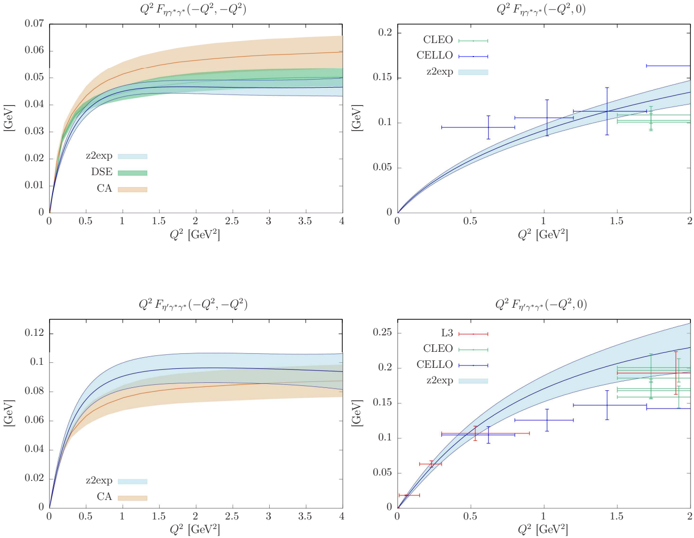
- Single Virtual regime





- We find a good agreement between two momentum frames of the η, η' mesons.
- $|\vec{p}| = \left(\frac{2\pi}{L}\right)$ frame helps at large Q^2.
- Errors are bigger than for the π^0 due to sizeable disconnected diagram and mixing.

Results Form Factor η, η' Continuum



- Relatively good agreement experimental data and our lattice result.
- Tension at low Q^2 double virtual TFF η between our result and other estimates.

Preliminary Results $a_\mu^{\text{pseudoscalar}-\text{pole}}$

Reminder:

$$a_\mu^{p-pole} = \left(\frac{\alpha_e}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \, \Big[w_1(Q_1,Q_2,\tau)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_1^2,-Q_3^2)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_2^2,0)$$
$$+w_2(Q_1,Q_2,\tau)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\mathscr{F}_{p\gamma^*\gamma^*}(-Q_3^2,0)\Big]$$

- In the end we find

$$a_\mu^{\eta-\text{pole}} = (11 \pm 2)\cdot 10^{-11}$$
$$a_\mu^{\eta'-\text{pole}} = (15.5 \pm 4.0)\cdot 10^{-11}$$

- Errors are on statistics only.
- Confirms that η, η' pole contributions are about $1/2$ the size of the π^0 pole contribution (e.g. Mainz: $a_\mu^{\pi^0-\text{pole}} = 59.7(3.6)\cdot 10^{-11}$).
- Systematics still need to be fully understood so values may shift slightly.

Conclusions

- We performed the first ab-initio computation of the π^0, η, η' TFFs.
- Systematics need to be finalized.
- Pre-print will be out soon!
- New experimental data for π^0 TFF by BES-III (preliminary data already shown) and for η, η' TFF by JLab (expected) $\rightarrow$ comparison with our result.

References

Alexandrou, C. et al. (2022). The $\eta \to \gamma^* \gamma^*$ transition form factor and the hadronic light-by-light η-pole contribution to the muon $g-2$ from lattice QCD.

Aoyama, T. et al. (2020). The anomalous magnetic moment of the muon in the standard model. *Physics Reports*, 887:1–166.

Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., Jin, L., Jung, C., and Lehner, C. (2020). Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD. *Physical Review Letters*, 124(13).

Chao, E.-H., Hudspith, R. J., Gérardin, A., Green, J. R., Meyer, H. B., and Ottnad, K. (2021). Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation. *Eur. Phys. J. C*, 81(7):651.

Colangelo, G., Hoferichter, M., Kubis, B., Procura, M., and Stoffer, P. (2014a). Towards a data-driven analysis of hadronic light-by-light scattering. *Phys. Lett. B*, 738:6–12.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2014b). Dispersive approach to hadronic light-by-light scattering. *JHEP*, 09:091.

References

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2015). Dispersion relation for hadronic light-by-light scattering: theoretical foundations. *JHEP*, 09:074.

Gérardin, A., Meyer, H. B., and Nyffeler, A. (2016). Lattice calculation of the pion transition form factor $\pi^0 \to \gamma^* \gamma^*$. *Phys. Rev. D*, 94(7):074507.

Gérardin, A., Meyer, H. B., and Nyffeler, A. (2019). Lattice calculation of the pion transition form factor with $N_f = 2+1$ Wilson quarks. *Phys. Rev. D*, 100(3):034520.

Hoferichter, M., Hoid, B.-L., Kubis, B., Leupold, S., and Schneider, S. P. (2018). Dispersion relation for hadronic light-by-light scattering: pion pole. *Journal of High Energy Physics*, 2018(10).

Ji, X. and Jung, C. (2001). Studying hadronic structure of the photon in lattice QCD. *Physical Review Letters*, 86(2):208–211.

Knecht, M. and Nyffeler, A. (2002). Hadronic light-by-light corrections to the muon $g-2$: The pion-pole contribution. *Physical Review D*, 65(7).