The RBC/UKQCD g-2 program

Christoph Lehner
(Regensburg)

February 15, 2023 – Pisa, NePSi 23

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

University of Bern & Lund
Nils Hermansson Truedsson

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Nobuyuki Matsumoto
Shigemi Ohta (KEK)
Amarjit Soni
Tianle Wang

CERN
Andreas Jüttner (Southampton)
Tobias Tsang

Columbia University
Norman Christ
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Bigeng Wang (Kentucky)
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Douglas Stewart
Joshua Swaim
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Maxwell T. Hansen
Tim Harris
Ryan Hill
Raoul Hodgson
Nelson Lachini
Zi Yan Li
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

Liverpool Hope/Uni. of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

University of Milano Bicocca
Mattia Bruno

Nara Women's University
Hiroshi Ohki

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

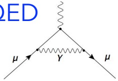
University of Southampton
Alessandro Barone
Jonathan Flynn
Nikolai Husung
Rajnandini Mukherjee
Callum Radley-Scott
Chris Sachrajda

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Contributions from known particles: The Standard Model

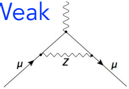
$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

QED



$+\ldots$ $\quad 116\,584\,718.9\,(1) \times 10^{-11}$ $\quad$ 0.001 ppm

Weak

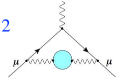


$+\ldots$ $\quad 153.6\,(1.0) \times 10^{-11}$ $\quad$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

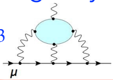
α^2



$+\ldots$ $\quad 6845\,(40) \times 10^{-11}$ [0.6%] $\quad$ 0.37 ppm

…Light-by-Light (HLbL)

α^3

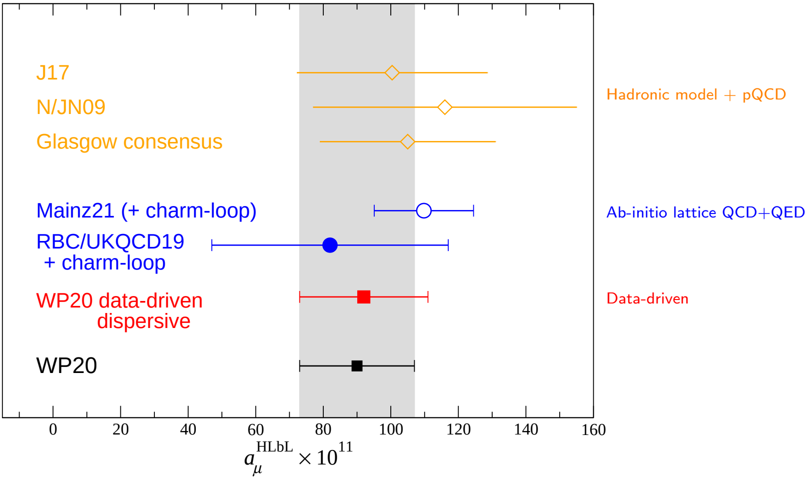


$+\ldots$ $\quad 92\,(18) \times 10^{-11}$ [20%] $\quad$ 0.15 ppm

Numbers from Theory Initiative Whitepaper

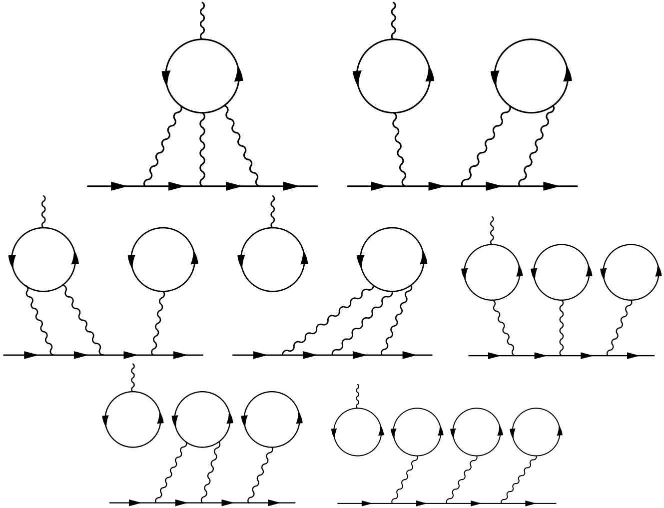
Uncertainty dominated by hadronic contributions

Status of hadronic light-by-light contribution



Systematically improvable methods are maturing; uncertainty to a_μ controlled at 0.15ppm; cross-checks detailed in Theory Initiative whitepaper

Diagrams to calculate



The first complete ab-initio LQCD result for HLbL

Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD

Thomas Blum,[1,2] Norman Christ,[3] Masashi Hayakawa,[4,5] Taku Izubuchi,[6,2]
Luchang Jin,[1,2,*] Chulwoo Jung,[6] and Christoph Lehner[7,6]
[1]*Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, Connecticut 06269-3046, USA*
[2]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[3]*Physics Department, Columbia University, New York, New York 10027, USA*
[4]*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*
[5]*Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*
[6]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[7]*Universität Regensburg, Fakultät für Physik, 93040 Regensburg, Germany*

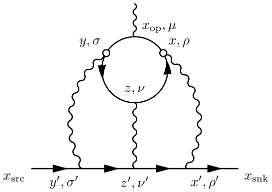
(Received 18 December 2019; accepted 27 February 2020; published 1 April 2020)

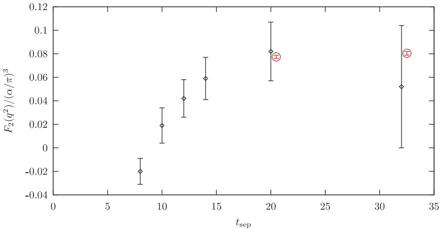
We report the first result for the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment with all errors systematically controlled. Several ensembles using $2+1$ flavors of physical mass Möbius domain-wall fermions, generated by the RBC and UKQCD collaborations, are employed to take the continuum and infinite volume limits of finite volume lattice QED + QCD. We find $a_\mu^{\mathrm{HLbL}} = 7.87(3.06)_{\mathrm{stat}}(1.77)_{\mathrm{sys}} \times 10^{-10}$. Our value is consistent with previous model results and leaves little room for this notoriously difficult hadronic contribution to explain the difference between the standard model and the BNL experiment.

It took many years and improvements to get there . . .

PRD93(2015)014503:

We introduce a new sampling strategy with 10x reduced noise for same cost (red versus black):



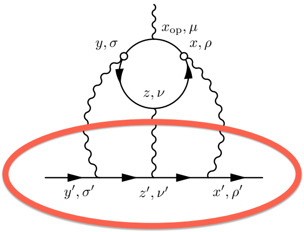


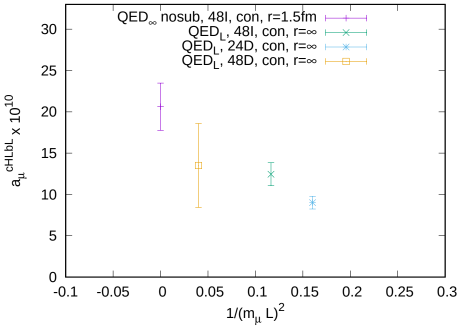
Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

PRD96(2017)034515:

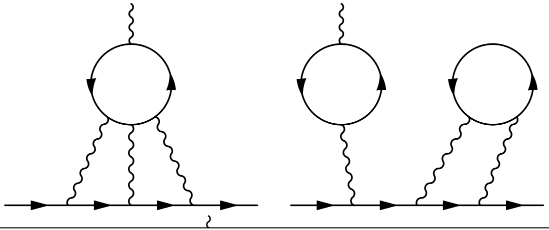
We remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (similar idea proposed by Mainz group)





Infinite-volume kernel projects to larger four-point function ⇒ large noise at physical pion mass

Paper in preparation using explicit pion-pole cancellation



$$\lim_{R\to\infty} \frac{a_\mu^{\text{discon}}(R_{\text{max}} > R)}{a_\mu^{\text{con}}(R_{\text{max}} > R)} = -\frac{1}{2} \cdot \frac{(e_u^2 + e_d^2)^2}{e_u^4 + e_d^4} = -\frac{25}{34}.$$

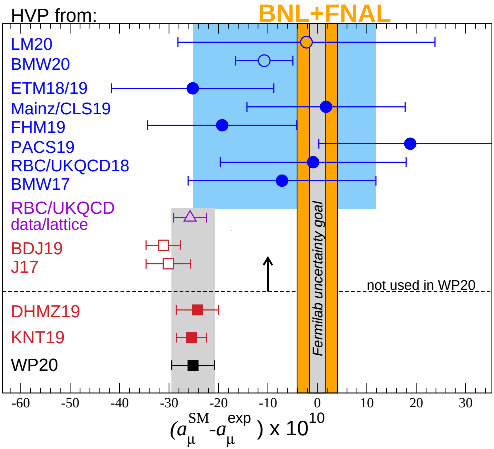
$$a_\mu^{\text{no-pion}} = a_\mu^{\text{discon}} + \frac{25}{34} a_\mu^{\text{con}},$$

$$a_\mu^{\text{discon}} = a_\mu^{\text{no-pion}} - \frac{25}{34} a_\mu^{\text{con}},$$

$$a_\mu^{\text{total}} = a_\mu^{\text{no-pion}} + \frac{9}{34} a_\mu^{\text{con}}.$$

Long-distance pion-pole cancels in $a_\mu^{\text{no}-\text{pion}}$ such that it can be evaluated at shorter distances. Only less noisy connected diagram remains.

Status and impact of hadronic vacuum polarization contribution



Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

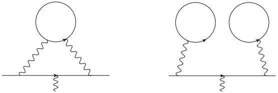
Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments)

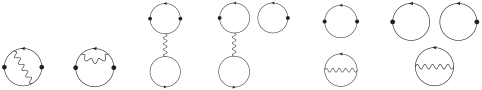
Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology (same situation as for HLbL)

Diagrams to calculate

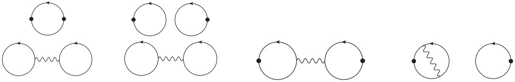
Isospin limit



QED corrections

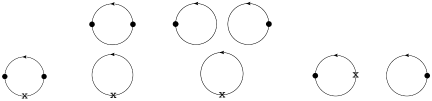


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

RBC/UKQCD status 2018 (Iso+QED+SIB)

PHYSICAL REVIEW LETTERS **121,** 022003 (2018)

Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

T. Blum,[1] P. A. Boyle,[2] V. Gülpers,[3] T. Izubuchi,[4,5] L. Jin,[1,5] C. Jung,[4] A. Jüttner,[3] C. Lehner,[4,*] A. Portelli,[2] and J. T. Tsang[2]

(RBC and UKQCD Collaborations)

[1]*Physics Department, University of Connecticut, Storrs, Connecticut 06269-3046, USA*
[2]*School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3FD, United Kingdom*
[3]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[4]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[5]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 25 January 2018; published 12 July 2018)

We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\mathrm{HVP\ LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\mathrm{HVP\ LO}}$.

Pure lattice result and dispersive result with reduced $\pi\pi$ dependence (window method)

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Window method (introduced in RBC/UKQCD 2018)

We also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

with

$$\begin{aligned} a_\mu^{\rm SD} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] \,, \\ a_\mu^{\rm W} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \,, \\ a_\mu^{\rm LD} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta) \,, \\ \Theta(t, t', \Delta) &= \left[1 + \tanh\left[(t - t')/\Delta\right]\right]/2 \,. \end{aligned}$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to {\rm had})$.

$a_\mu^{\rm W}$ has small statistical and systematic errors on lattice!

An update of Euclidean windows of the hadronic vacuum polarization

T. Blum,[1] P. A. Boyle,[2,3] M. Bruno,[4,5] D. Giusti,[6] V. Gülpers,[3] R. C. Hill,[3] T. Izubuchi,[2,7] Y.-C. Jang,[8,9] L. Jin,[1,7] C. Jung,[2] A. Jüttner,[10,11] C. Kelly,[12] C. Lehner,[6,*] N. Matsumoto,[7] R. D. Mawhinney,[9] A. S. Meyer,[13,14] and J. T. Tsang[10,15]

(RBC and UKQCD Collaborations)

[1] *Physics Department, University of Connecticut, Storrs, CT 06269-3046, USA*
[2] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[3] *School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3FD, UK*
[4] *Dipartimento di Fisica, Universitá di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*
[5] *INFN, Sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*
[6] *Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*
[7] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*
[8] *Electron-Ion Collider, Brookhaven National Laboratory, Upton, NY 11973, USA*
[9] *Physics Department, Columbia University, New York, NY 10027, USA*
[10] *CERN, Theoretical Physics Department, Geneva, Switzerland*
[11] *School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK*
[12] *Computational Science Initiative, Brookhaven National Laboratory, Upton, NY 11973, USA*
[13] *University of California, Berkeley, CA 94720, USA*
[14] *Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
[15] *CP^3-Origins & Department of Mathematics and Computer Science, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*

(Dated: February 14, 2023)

We compute the standard Euclidean window of the hadronic vacuum polarization using multiple independent blinded analyses. We improve the continuum and infinite-volume extrapolations of the dominant quark-connected light-quark isospin-symmetric contribution and address additional sub-leading systematic effects from sea-charm quarks and residual chiral-symmetry breaking from first principles. We find $a_\mu^{\rm W} = 235.56(65)(50) \times 10^{-10}$, which is in 3.8σ tension with the recently published dispersive result of $a_\mu^{\rm W} = 229.4(1.4) \times 10^{-10}$ [1] and in agreement with other recent lattice determinations. We also provide a result for the standard short-distance window. The results reported here are unchanged compared to our presentation at the Edinburgh workshop of the g-2 Theory Initiative in 2022 [2].

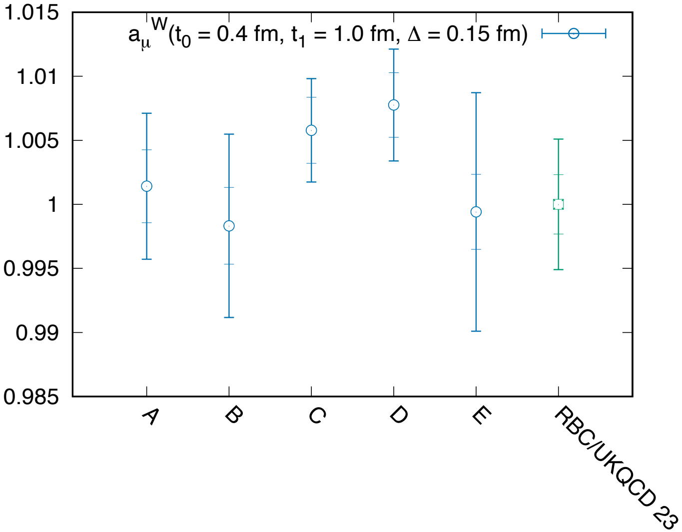
Blinding

- 2 analysis groups for ensemble parameters (not blinded)
- 5 analysis groups for vector-vector correlators (blinded, to avoid bias towards other lattice/R-ratio results)
- Blinded vector correlator $C_b(t)$ relates to true correlator $C_0(t)$ by

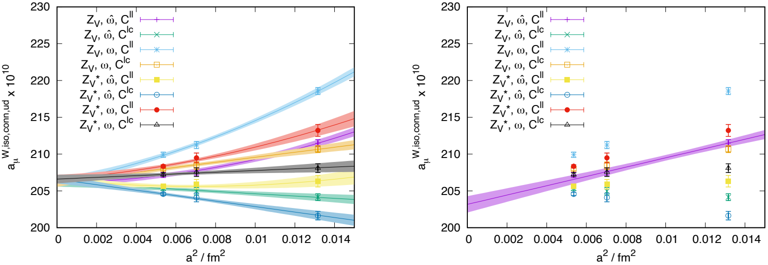
$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t) \tag{1}$$

with appropriate random b_0, b_1, b_2, different for each analysis group. This prevents complete unblinding based on previously shared data on coarser ensembles.

Relative unblinding



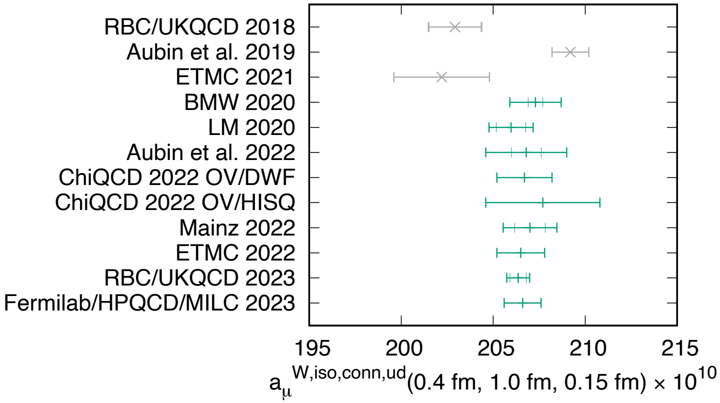
Full unblinding on August 31st, 2022



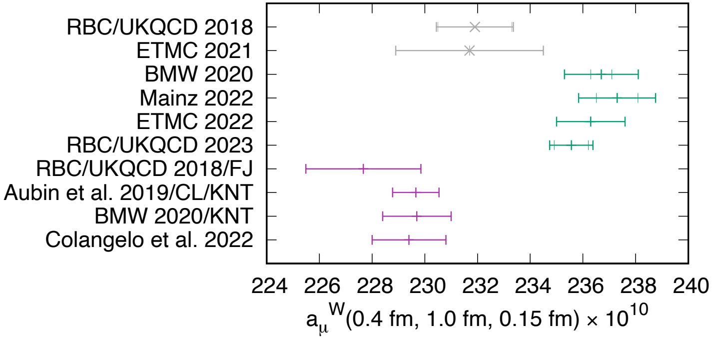
24 data points enter new continuum extrapolation, in 2018 had only 2 data points

Already generating data at two finer lattice spacings ($a^2/\mathrm{fm}^2 = 0.0017, 0.003$)

Isospin symmetric standard window in context



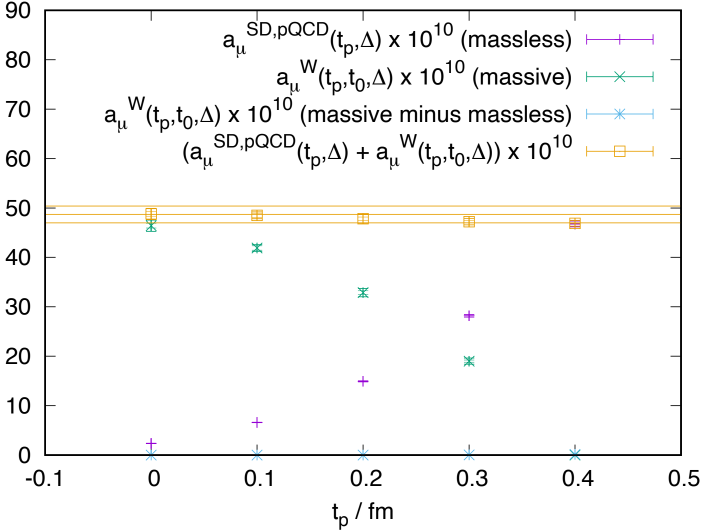
Adding back the QED+SIB parts calculated in RBC/UKQCD2018



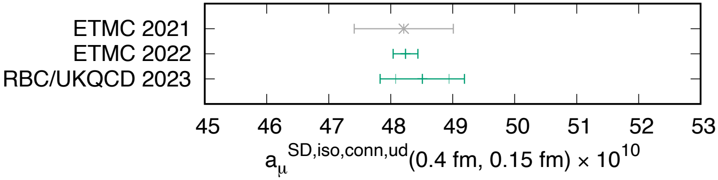
3.8σ tension between lattice and $e^+e^- \to$ hadrons

Short-distance window also computed

Stability tested against α_s^4 massless QCD calculation Chetyrkin & Maier, 2011:



Short-distance window in context



Summary:

- In isospin symmetric limit SD and standard windows are converging within lattice QCD.
- Tension for standard window with dispersive approach.
- Working on LD update. Possibly available this summer.
- Working on update to QED+SIB as well, possibly available this fall.
- Also: τ decays; on-going project for needed QED+SIB corrections
- Our final goal: match or surpass final FNAL experimental precision