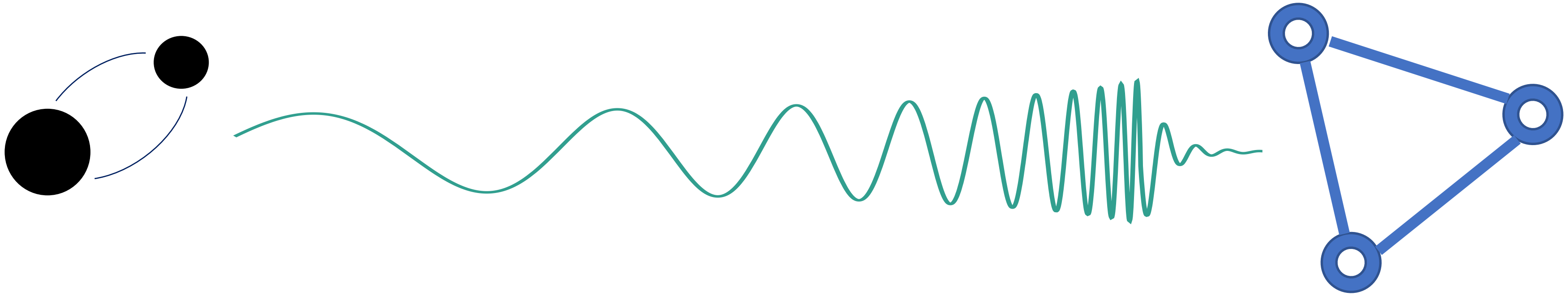
PERSPECTIVES ON FUNDAMENTAL PHYSICS AND EXOTIC COMPACT OBJECTS WITH CURRENT AND NEXT-GENERATION DTECTORS



GWADW2023, Isola d'Elba, 22-05-2023

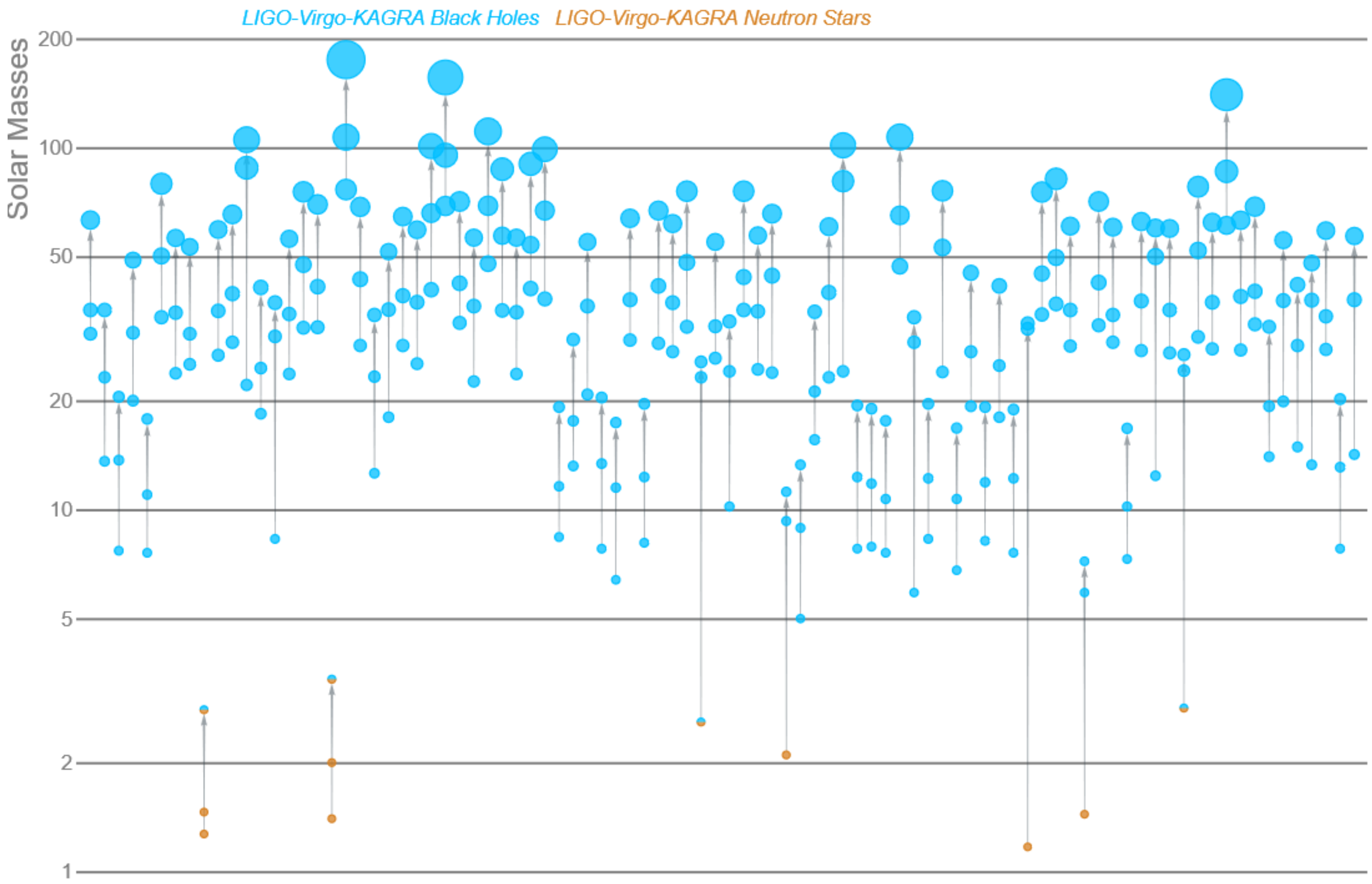
Massimo Vaglio (he/him/his) - ✉ massimo.vaglio@uniroma1.it

SAPIENZA UNIVERSITÀ DI ROMA

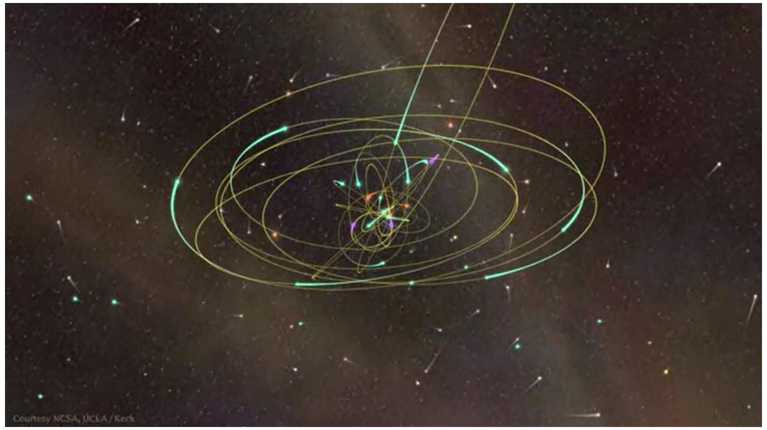
erc European Research Council

DarkGRA

Evidence for black holes

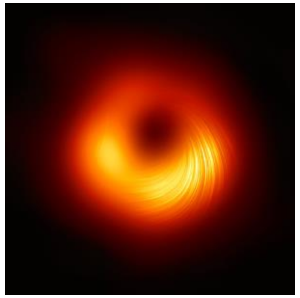


Mass distribution of the merger detected events sorted by date
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern



Tracking of orbits around the galactic center
Courtesy NCSA | UCLA / Keck

A view of M87* in polarized light
ETH Collaboration



Exotic Compact Objects in the Universe?

- Are ther compact objects other than BHs and NSs:

  LIGO/Virgo mass-gap (GW190814, GW190521) events.

  Supermassive BH seeds.

  (Dark) matter compact objects? (e.g. boson/axion stars).

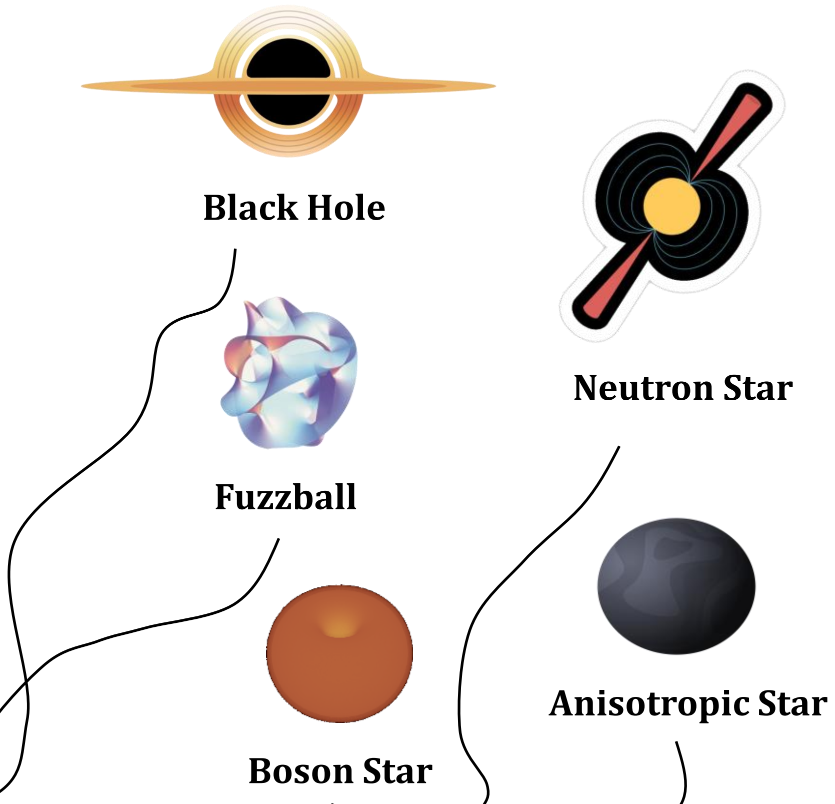
- Observational signatures of quantum black holes:

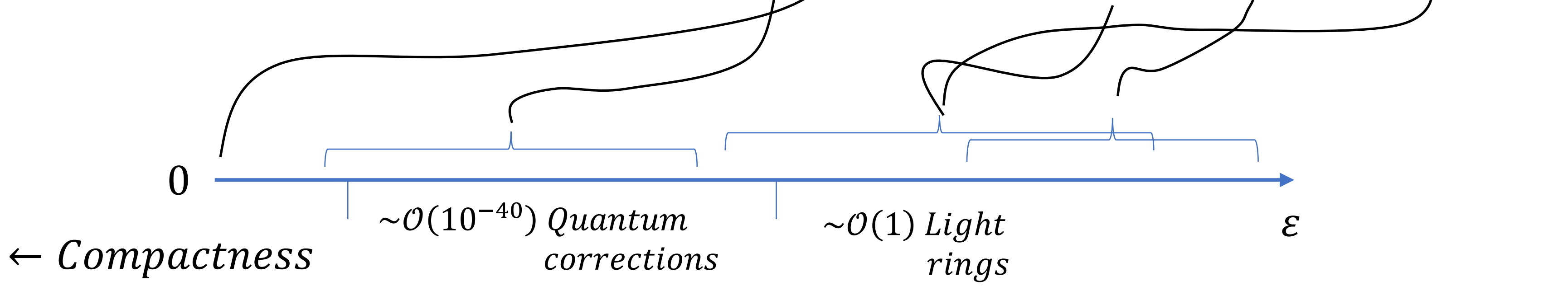
  Information loss, singularities, Cauchy horizons…

  New physics at the horizon (e.g. firewalls, nonlocality).

  Regular, horizonless compact objects (e.g. fuzzballs).

- Quantifying black hole-ness: $R_{ECO} = R_H(1+\varepsilon)$





How to construct ECOs

Solutions to GR
with exotic matter sources
(*e.g. anisotropic stars, boson stars, axion stars, gravastars, wormholes*)

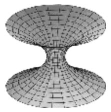
Solutions to modified gravity
(*e.g. fuzzballs/microstates, 2-2 holes, superspinars, wormholes*)

- In some case there is no sharp distinction.

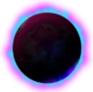
- Some models require modified gravity only in the interior / close to the horizon → assuming GR in the exterior is often a good approx.

- Some models are phenomenological (formation, dynamics, stability?).

Examples:



Wormholes



Dark stars

…

Any evidence for ECOs would imply evidence for new physics!

Some ECO models

- **Boson stars:** self-gravitating 'clumps' of energy. Can be described in classical field theory:

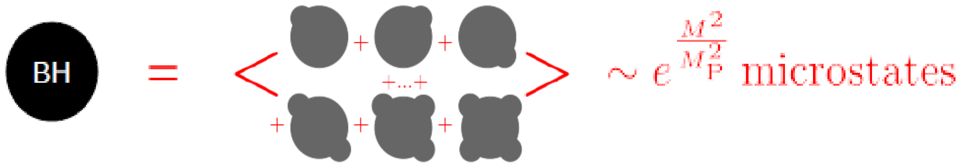
$$L = \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu}\partial_\mu \phi^* \partial_\nu \phi - \frac{1}{2} m^2 |\phi|^2 + \frac{\lambda}{4} |\phi|^4 + \cdots$$

- Can be seen as BE condensates as for $|p\rangle = (N, 0, \dots)$ and large N, the pure classical Klein-Gordon equation is recovered in the Hartree-Fock approximation.

- Self-interactions give rise to multiple branches of solutions which are dynamically stable.

-Numerical simulations available.

- **Fuzzballs:** classical BHs are ensembles of a huge number of regular, horizonless, microstategeometries



- BH entropy explained by the number of microstates.

- Motivated by (low energy truncation of) string theory.

GW Tests: Inspiral-merger

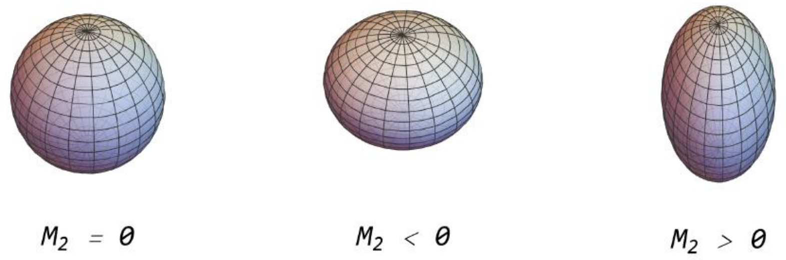
- **Multipolar structure:** Multipole moments analogous to their Newtonian counterpart can be defined in GR and characterize univocally a given matter-energy distribution.

$M = M_0$

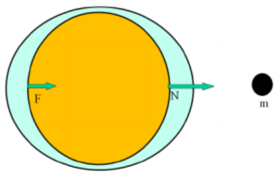
$J = S_1$

$Q = M_2$ quadrupole moment

...



- **Tidal deformability:** Tidal fields modify the shape of the bodies in binary systems affecting their dynamical evolution. The deformations are encoded in the "tidal Love Numbers".



$$g_{00} = -1 + \frac{2M}{r} + \frac{3Q_{ij}}{r^3}\left(n_i n_j - \frac{\delta_{ij}}{3}\right) + O\left(\frac{1}{r^4}\right) - \varepsilon_{ij} x_i x_j + O(r^3)$$

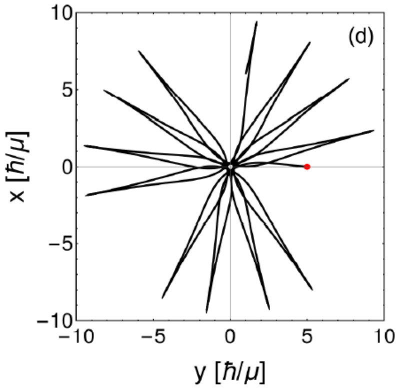
tidal deformability

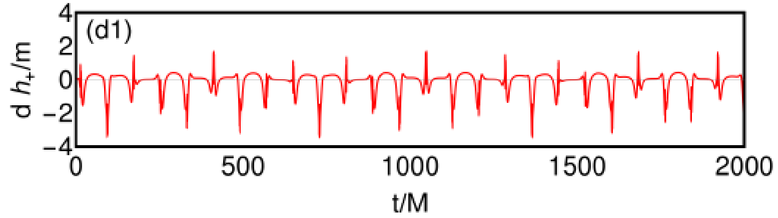
$$Q_{ij} = -\lambda_T \varepsilon_{ij}$$

- **Tidal heating:** Additional dissipative channels cause the tides to transfer rotational energy from the spin to the orbit and can accelerate the orbital evolution significantly.

GW Tests: Inspiral-merger

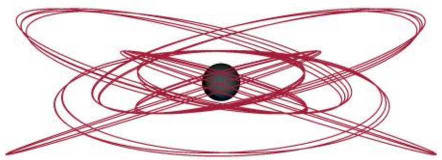
- **Extra emission channels:** Some ECOs possess charges under a scalar or gauge fields that could be detected through GWs.
- **Motion in ECOs interior:** If the ECO is lacking an event horizon or a hard surface, motion of test particles can take place inside the object and give rise to new type of orbits. *Grandclément, Somé+, PRD (2014)*



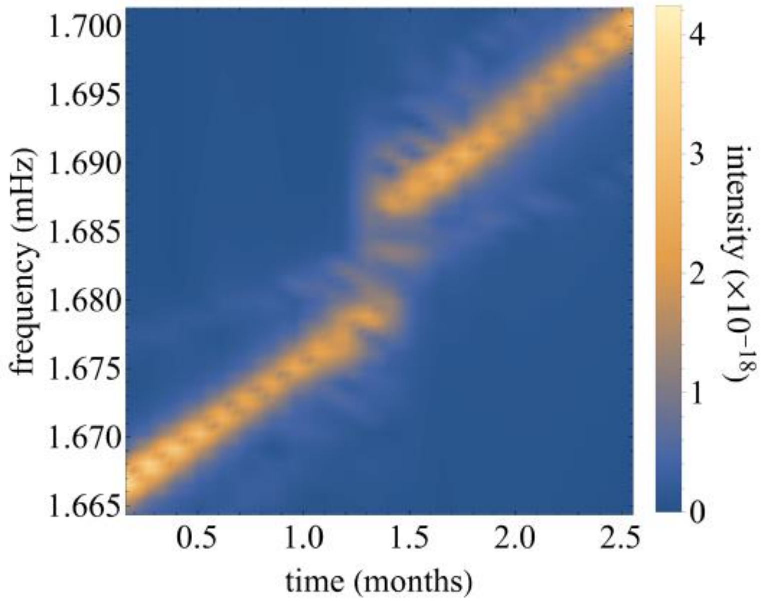


Zhang+ arXiv:2108.13170 (2021)

- **Chaos/integrability:** Some ECO models do not posses any equivalent of the Carter constant, resulting in chaotic behavior of the motion of small bodies orbiting around them, with observational consequences.



Destounis, Angeloni, Vaglio +, arXiv:2305.05691 (2023)

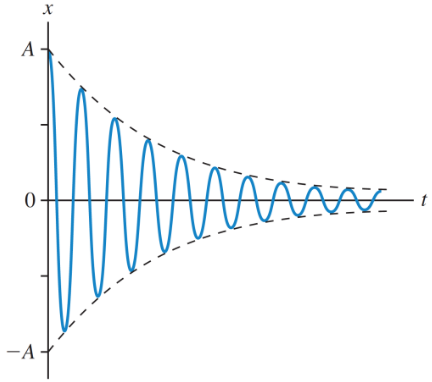


GW Tests: Ringdown

- QNMs Perturbed BHs and ECOs can oscillate with characteristic complex frequencies called quasi-normal-modes (QNMs).

Prompt ringdown: superposition of quasinormal modes (QNMs)

$$h_+ + i h_\times \sim \sum_i A_i \sin(\omega_i t + \varphi_i) e^{-t/\tau_i}$$



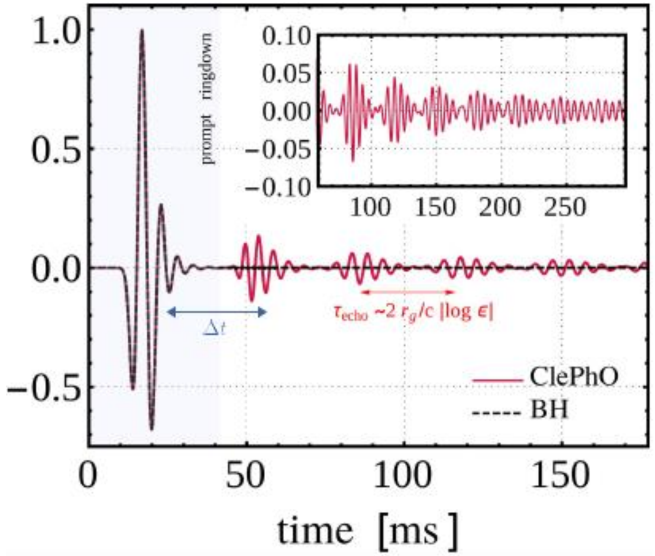
3G/LISA → O(100-1000) events/yr allowing for BH spectroscopy

ECO smoking guns:

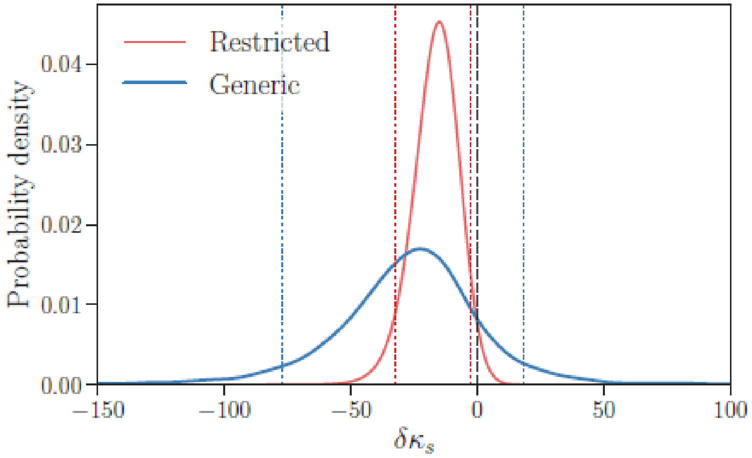
- Shift of the entire QNM spectrum
- Isospectrality breaking
- Extra ringdown modes (e.g., extra polarizations, matter modes)

Echoes: During the ringdown (if the remnant is compact enough to posses a light ring) after the prompt the ingoing part of the signal is partially reflected and produces a damped copy of the initial signal.

Cardoso, Pani, Nature Astronomy (2017)



Current/future constraints



Cumulative constraint on the quadrupole moment of compact objects from GWTC-3.

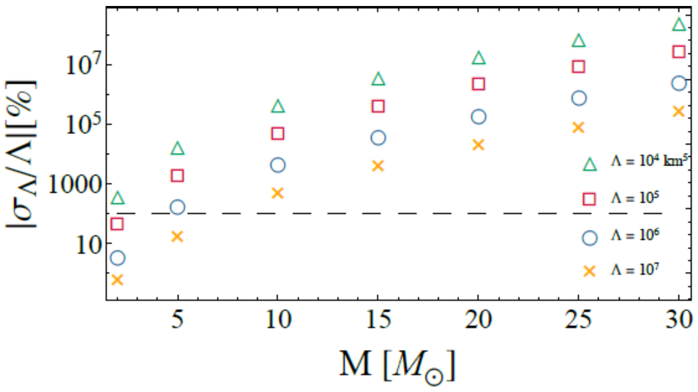
$$\kappa_s = 1/2(\kappa_1 + k_2), \qquad \kappa = M_2/(M^3\chi^2)$$

The red dashed line correspond to the constraint obtained by assuming a unique value for all the events.

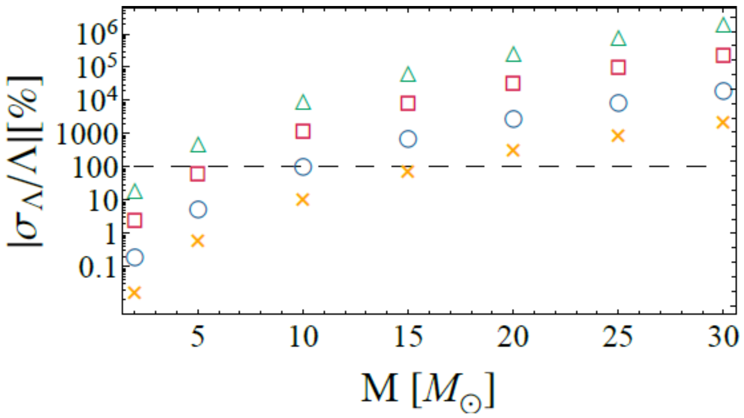
LIGO-Virgo Collaboration: arXiv:2112.06861 (2021)

Cardoso, Franzin +, PRD (2017)

AdLIGO



ET

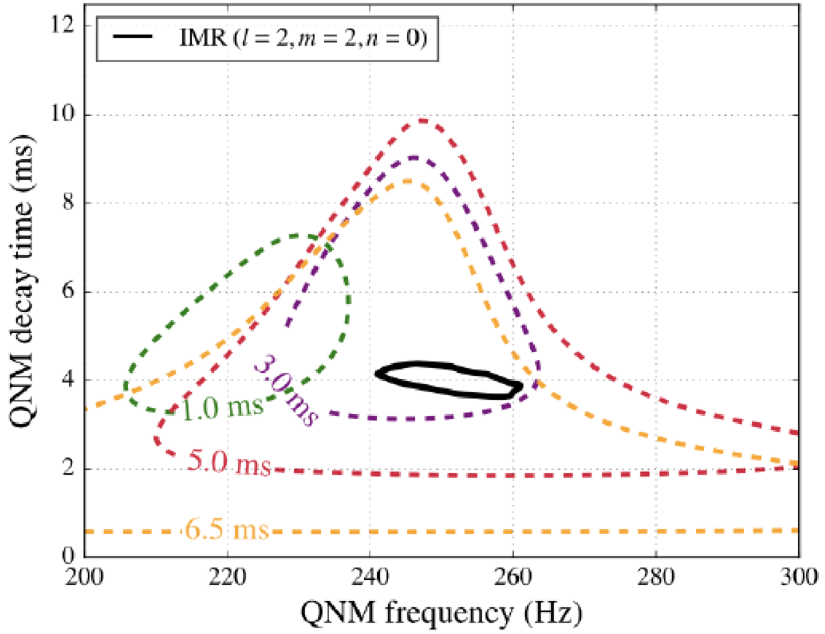


Relative percentage errors on the average tidal deformability

$$\Lambda = \frac{1}{26}\left[\left(1+\frac{12}{q}\right)\lambda_1 + (1+12q)\lambda_2\right]$$

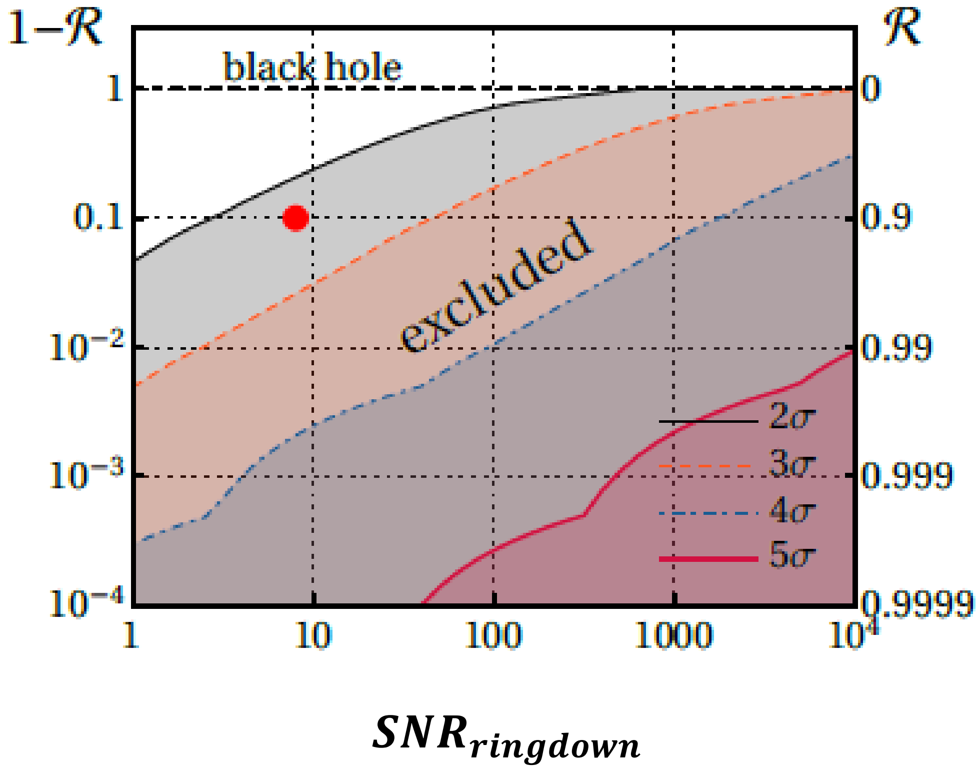
for equal-mass binaries at 100Mpc for AdLIGO and ET mass for different values of Λ.

Current/future constraints



Credible regions (90%) in the joint posterior distributions for the QNM frequency and decay time, for several values of the starting time of the ringdown. *LIGO-Virgo Collaboration PRD (2016)*

The solid black line shows the theoretical prediction for remnant's estimated mass and spin.



Projected exclusion plot for the ECO reflectivity due to the lack of echoes in future observations with ET. $\mathcal{R}$ is shown as a function of the SNR in the ringdown phase assuming $M = 30M_{\odot}$. *Testa, Pani, PRD (2018)*

The red marker corresponds to $SNR = 8$ and $\mathcal{R} = 0.9$

Quadrupole moment of boson stars (2PN)

- Stationary **axysimmetric** spacetime $\Rightarrow$
  scalar mass moments M_0, M_2... and current moments S_1, S_3...

for a Kerr black hole $$\boxed{M_l + iS_l = M^{l+1}(i\chi)^l}$$ $$\chi = \frac{J}{M^2}, \quad M = M_0$$

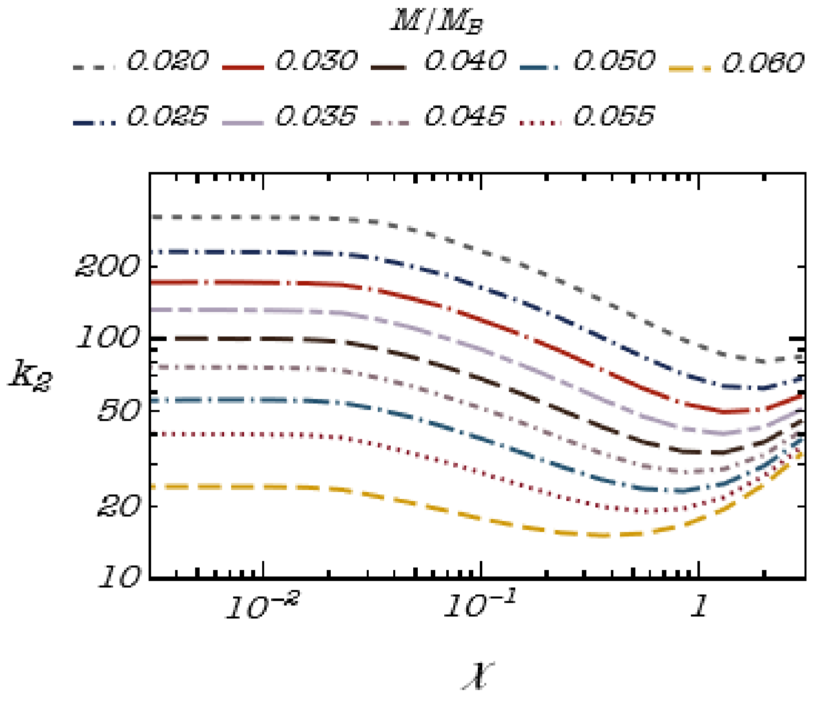
Not true for a generic compact object!

- The expression for the quadrupole moment can be obtained as:

$$\underbrace{M_2}_{\text{quadrupole moment}} = -\kappa_2(\chi, M/M_B)\chi^2 M^3$$

For a Kerr Black Hole
$\kappa_2 = 1$

$$\boldsymbol{M_B} = (\sqrt{\lambda\hbar}/m_S{}^2)\, M_p^3$$



Vaglio, Pacilio +, PRD (2022)

A coherent inspiral waveform model

- Post-Newtonian (PN) expanded waveform in $x = \frac{1}{c^2}(\pi M f)^{\frac{2}{3}} \sim (v/c)^2$ including finite size effects:

$$h_{\mathcal{H}} \sim \mathcal{A}(f)e^{i\psi(f)} \rightarrow \psi(f) = \psi_{pp}(f) + \psi_{tidal}(f) + \psi_{quad}(f) \propto x^{-\frac{5}{2}}\left(\sum_{n=0}^{7} \alpha_n \cdot x^{\frac{n}{2}}\right)$$

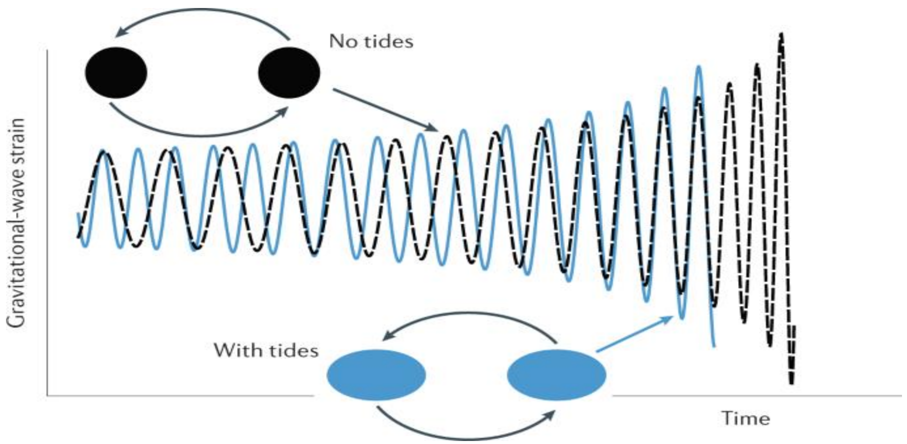
Post-Newtonian order

Point particle relative terms $\propto x^0 \ldots x^{\frac{7}{2}}$ (0PN, 3.5PN)

Tidal relative corrections $\propto x^5, x^6$ (5PN, 6PN)

Spin-quadrupole relative corrections $\propto x^2, x^3$ (2PN, 3PN)

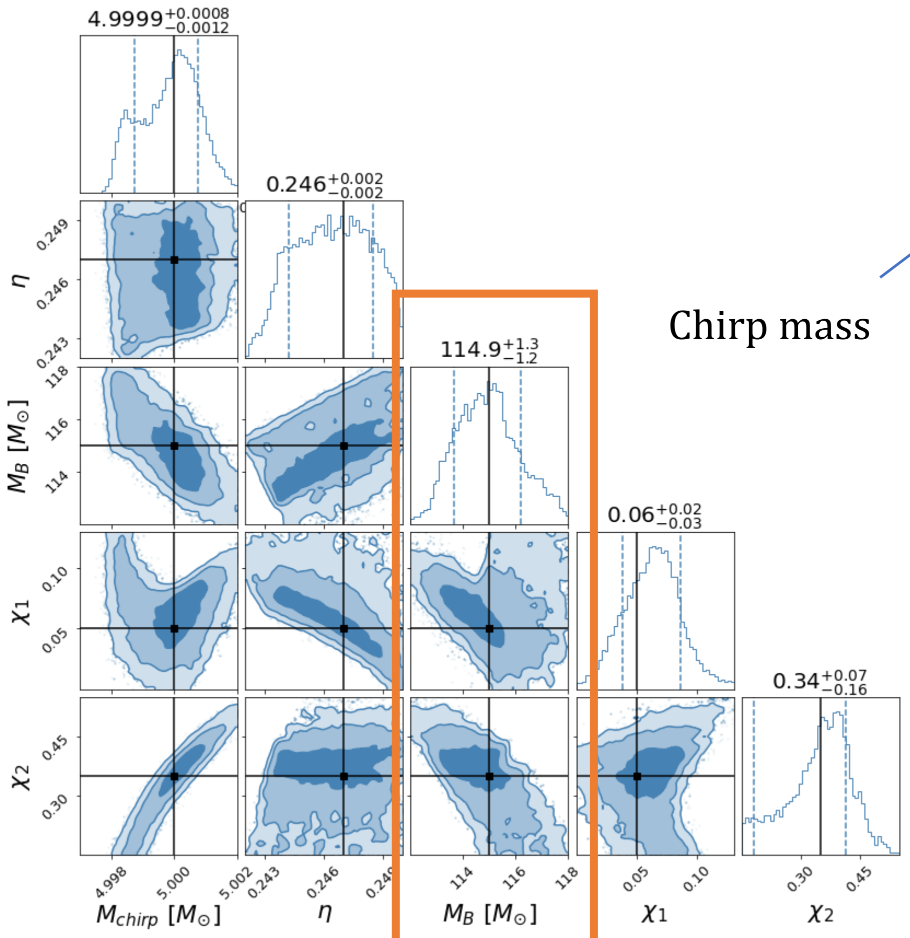
Pacilio, Vaglio+ PRD 2020



Example of an inspiral waveform with and without tidal effects with pronounced dephasing

Projected constraints on scalar interactions

Vaglio, Pacilio, + (2023) arXiv:2302.13954



Injection and recovery of a signal with ET

$(\mathcal{M} = 5M_\odot, q = 0.8, M_B = 115M_\odot, \chi_1 = 0.05, \chi_2 = 0.35, f_{Roche} = 127Hz)$

$$\vec{\theta} = (\mathcal{M}, \eta, \chi_1, \chi_1, M_B)$$

Chirp mass

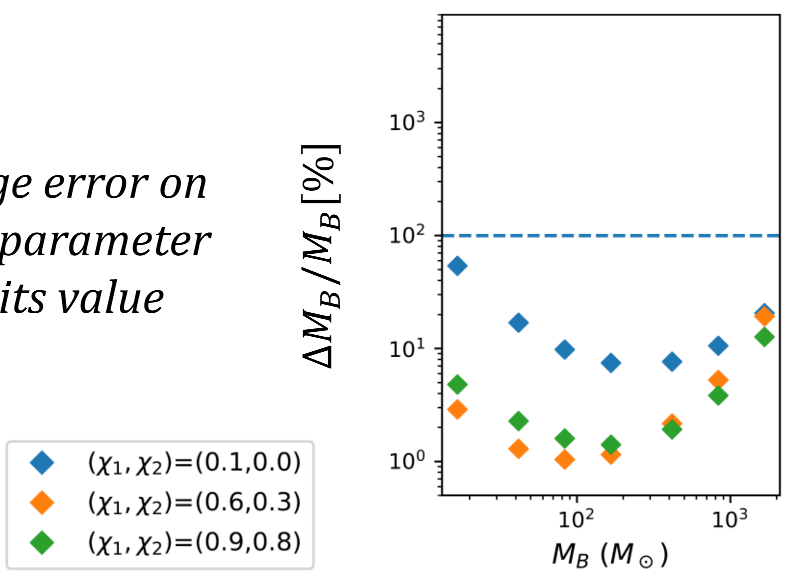
Symmetric mass ratio

Component spins

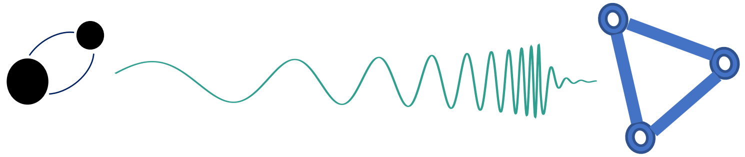
$$\boldsymbol{M_B} = (\lambda^{\frac{1}{2}}/m^2)\, M_p^3$$

Relative percentage error on the effective mass parameter as a function of its value

ET (D_L = 500 Mpc)



Conclusions



- *We are living the BH era* that came with discovery opportunities for new physics!
  (portal to observables quantum gravity effects? new fundamental physics?)

- There have been dramatic improvements on ECOs on all fronts in the last few years.

- Any signature of beyond-Kerrness would have striking consequences for fundamental physics.

- Exquisite constraints (percent and sub-percent level) on tidal deformability and multipole moments with 3G detectors.

- Evidence for light rings & measures of reflectivity with GWs/EM.

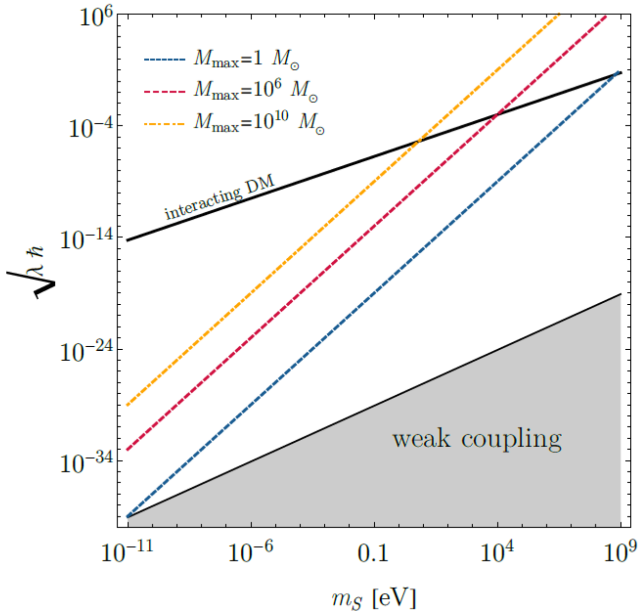
BSs parameter estimation main results

Boson stars are hypothetical compact astrophysical sources that can mimick within some extent the phenomenology of black holes:

- Finite-size tidal and monopole-quadrupole spin effects can leave a detectable imprint in binary waveforms and be interpreted as signatures of departures from the black hole binary behaviour.
- A detection with the Einstein Telescope could constraint the fundamental couplings of the scalar theory with percentage accuracy:

(m_1, m_2)	$\delta\mathcal{M}_{rel}$	$\delta\eta_{rel}$	$\delta M_{B\,rel}$	$\delta\chi_{1\,rel}$	$\delta\chi_{2\,rel}$
$(6.4, 5.2)M_\odot$	0.015%	0.8%	1.0%	48%	37%
$(12.8, 10.3)M_\odot$	0.05%	1.7%	1.5%	92%	43%

M. Vaglio, C. Pacilio, A. Maselli, P. Pani (2023)



Next steps and future works:

- Work on a full Inspiral-Merger-Ringdown waveform and generalize to different boson star models

Boson Stars

- As in General Relativity gravity couples with energy the idea that self-gravitating 'clumps' of energy could form was first considered by Wheeler.

  Thermal Geons - Edwin A. Power (University Coll. London), John A. Wheeler (Princeton U.)
  Rev.Mod.Phys. 29 (1957), 480-495

- It was realized later by Kaup that this was not possible for the electromagnetic field but so-called **Klein-Gordon geons** were found.

  Klein-Gordon Geon - David J. Kaup (Maryland U.)
  Phys.Rev. 172 (1968), 1331-1342

- Ruffini and Bonazzola using second quantization for the Klein-Gordon equation, showed that if all scalar particles are within the same ground state $|p\rangle = (N, 0, \dots)$, the pure classical Klein-Gordon equation of Kaup is recovered in the Hartree-Fock approximation.

  Systems of selfgravitating particles in general relativity and the concept of an equation of state - Remo Ruffini (Princeton U. and Princeton, Inst. Advanced Study), Silvano Bonazzola (Rome U.)
  Phys.Rev. 187 (1969), 1767-1783

Boson Stars: A possible candidate

- Boson stars are stationary configurations of a massive, complex scalar field, bound by gravity that:

  - Can be almost as compact as Black Holes and can mimick their phenomenology

  - Are regular (no singularity at the center, no event horizon)

  - Could stand for some fraction of the dark matter content of the Universe

- Can be described in classical field theory:

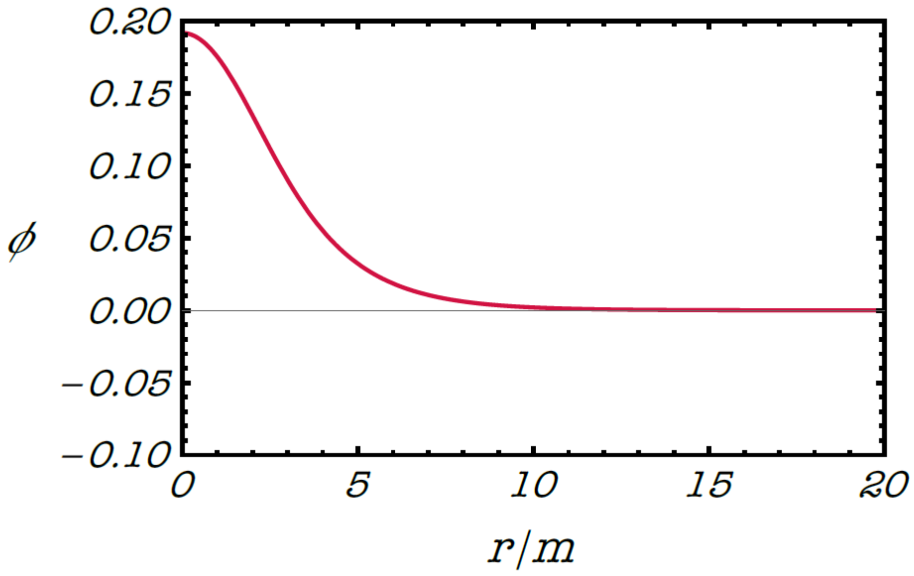
$$L = \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - \frac{1}{2} m^2 |\phi|^2 + \frac{\lambda}{4} |\phi|^4 + \cdots$$

The potential can contain interaction terms such as:

$$V(|\phi|^2) = m^2 |\phi|^2 + \lambda |\phi|^4$$

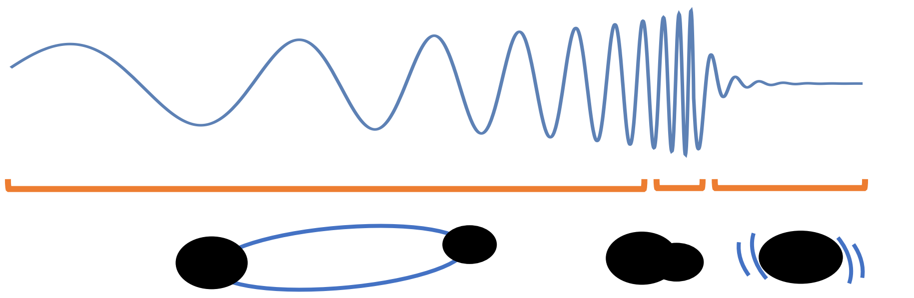
Scalar fields like these arise in many extensions of the Standard Model, String scenarios and comsological models!

Radial profile of a boson star



Stages of a binary coalescence

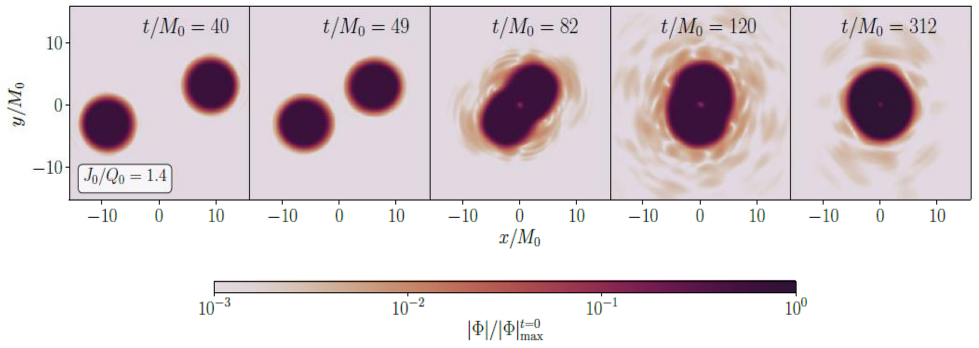
- The gravitational waveform from a binary coalescence can be thought as divided in three stages



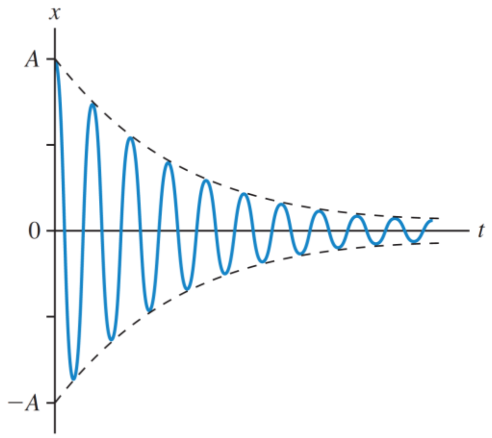
Inspiral **Merger** **Ringdown**

Post-Newtonian expansion in weak gravity $\frac{GM}{c^2 d}$ and small velocities $\frac{v}{c}$.

Numerical Relativity 3+1 non-linear simulations



Description of the final remnant as an oscillating perturbed object

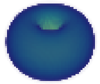


Some of the candidates

- The actual paradigm is that an astrophysical compact object, which is hevier than few solar masses, is a Black Hole.

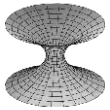
- *Are there other compact sources compatible with current observations?* ***-> Exotic Compact Objects***

Exotic Compact Objects

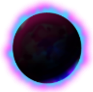


Boson stars

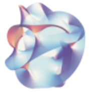
Proca stars



Wormholes



Hairy Black Holes



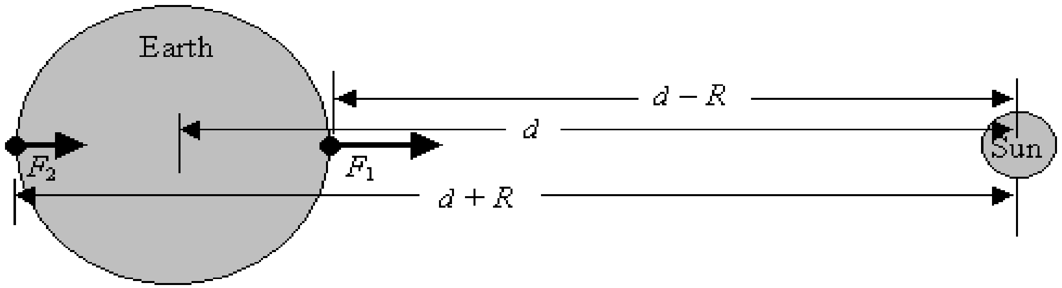
Fuzzballs

. . .

Astrophysical processes are not that sensitive to the geometry near the horizon: ***we need Gravitational Waves!***

Tidal deformability

- The presence of the companion induces a quadrupole moment in the star as response to the external tidal field:



$$F_2 = \frac{GMm}{(d+R)^2} \qquad F_1 = \frac{GMm}{(d-R)^2} \qquad \Delta F = F_2 - F_1 = \frac{GMm}{d^2}\left[\left(1+\frac{R}{d}\right)^{-2} - \left(1-\frac{R}{d}\right)^{-2}\right]$$

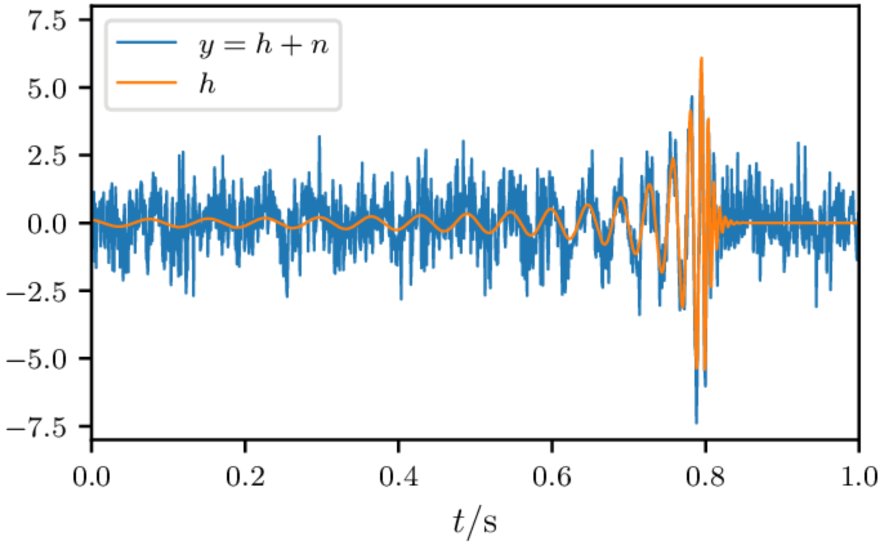
- The same happens in General Relativity and can be studied as a perturbation problem of the spherically symmetric background spacetime:

$$g_{00} = -1 + \frac{2M}{r} + \frac{3Q_{ij}}{r^3}\left(n_i n_j - \frac{\delta_{ij}}{3}\right) + O\left(\frac{1}{r^4}\right) - \varepsilon_{ij} x_i x_j + O(r^3)$$

$Q_{ij} = -\lambda_T \varepsilon_{ij}$ $\rightarrow$ $\boxed{\lambda_T \text{ is the tidal deformability}}$ $\xrightarrow{\text{for boson stars}}$ $\lambda_T(\mathrm{M}, M_B)$ $\rightarrow$ $\boldsymbol{M_B} = (\sqrt{\lambda}\hbar/{m_S}^2)\, M_p^3$

Parameter estimation

- Parameter estimation allows to reconstruct the template parameters from a given signal buried in noise.



- Spins alligned with $\vec{L}$ $\Rightarrow$ $\boxed{\vec{\theta} = (\mathcal{M}, \eta, \chi_1, \chi_1, M_B)}$

$\boldsymbol{M_B} = (\sqrt{\lambda\hbar}/{m_S}^2)\ M_p^3$

Chirp mass

$$\mathcal{M} = \frac{(M_1 M_2)^{\frac{3}{5}}}{(M_1 + M_2)^{\frac{1}{5}}}$$

Symmetric mass ratio

$$\eta = \frac{M_1 M_2}{(M_1 + M_2)^2}$$

Component spins

$$\chi_{1/2} = (\vec{J}_{1/2}/M_{1/2}^2) \cdot \hat{L}$$

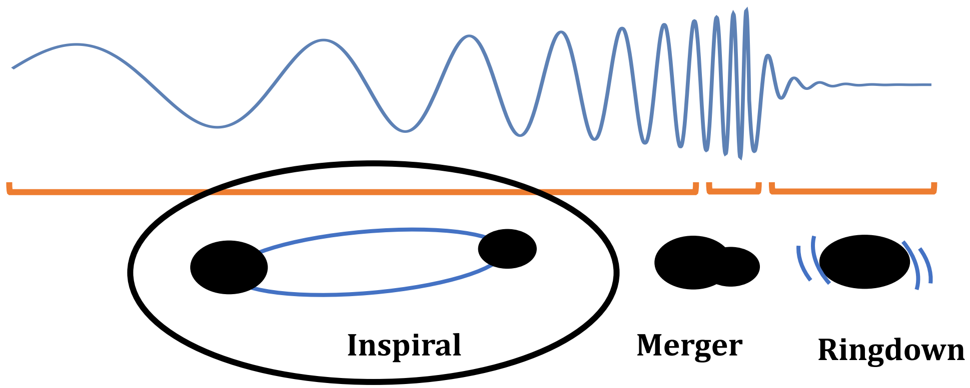
A coherent inspiral waveform model

- Post-Newtonian expanded waveform in $v = (\pi M f)^{\frac{1}{3}}$ consistently including finite size effects:

$$\mathcal{A}(f) = \frac{M_t^2}{D_L}\sqrt{\frac{\pi\eta}{30}}\,v^{-7/6}$$

(0PN)

$$h \sim \mathcal{A}(f)e^{i\psi(f)}$$



$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + v^{-5}\left(\sum_{n=0}^{7} \alpha_n v^n\right)$$

at 3.5PN

C.K. Mishra et.al, Phys. Rev. D, 93, 8 (2016), 084054

Point particle +spin

+ quadrupole corrections at 2PN, 3PN and 3.5PN

Krishnendu et.al, Phys. Rev. Lett.,119,9 (2017) 091101

+ tidal corrections at 5PN and 6PN

Lackey and L. Wade, Phys. Rev. D, 91, (2015) 4 043002

Finite size effects

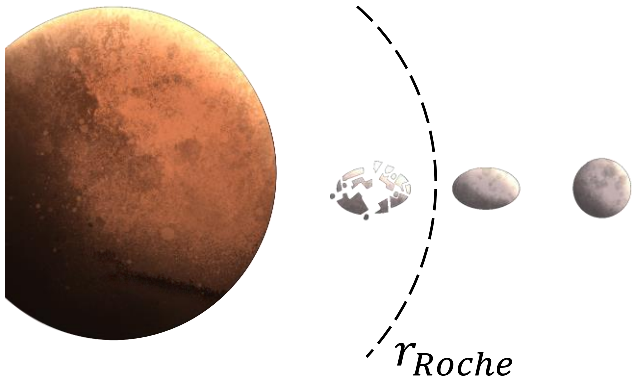
Tidal deformability of boson stars (5PN)

- The companion induces a quadrupole moment as response to the external tidal field:

$$\underline{Q_{ij} = -\lambda_T \varepsilon_{ij}} \qquad \lambda_T \text{ is the tidal deformability}$$

Dimensionless tidal $\Lambda = \lambda_T / M^5$ for BSs (*N. Sennett et al.,* Phys. Rev. D, 96, 2 (2017) 024002)

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}}\left[-0.828 + \frac{20.99}{\log\Lambda} - \frac{99.1}{(\log\Lambda)^2} + \frac{149.7}{(\log\Lambda)^3}\right] \longrightarrow \boxed{\Lambda = \Lambda\left(\frac{M}{M_B}\right)}$$

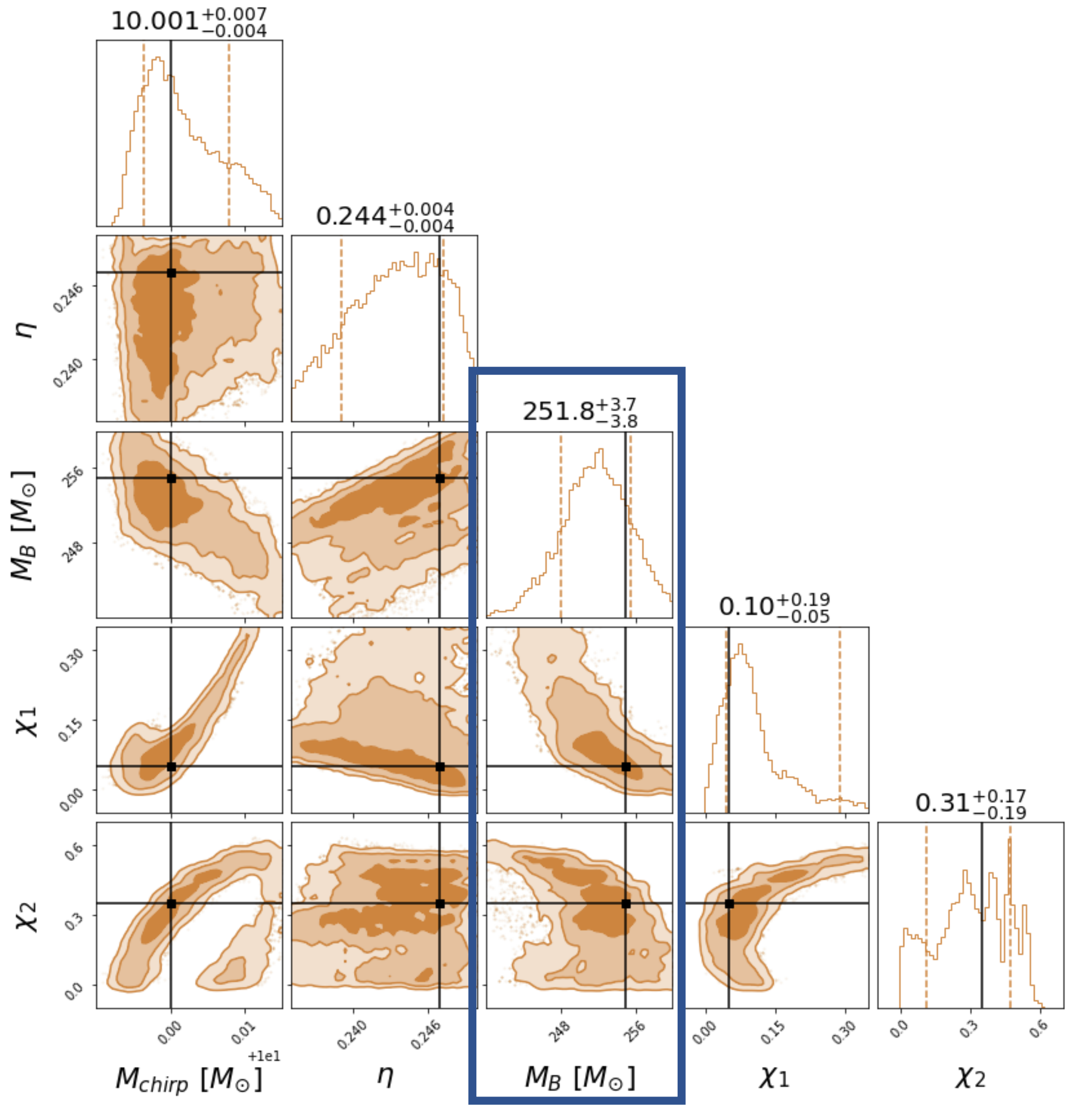


We fixed the maximum frequency of the waveform to that corresponding to the Roche radius:

$$f_{Roche} = \frac{c^3}{\pi G M M_\odot}\sqrt{\frac{1}{q^2} + q + 3\left(1 + \frac{1}{q}\right)}\left(\frac{C_2}{\gamma}\right)^{\frac{3}{2}} Hz$$

C_2 = Compactness of the secondary $\qquad$ γ = Numerical factor $\gamma \sim 2$

Parameter estimation - Results



Injection and recovery of a signal with the Einstein Telescope (SNR = 130):

- $\mathcal{M} = 10M_\odot$
- $q = 0.8$
- $M_B = 255M_\odot$
- $\chi_1 = 0.05$
- $\chi_2 = 0.35$
- $\boldsymbol{f_{Roche} = 50Hz}$ ←

Multipole moments in General Relativity

- The Newtonian potential satisfy the Laplace equation $\nabla^2 V = 0$,:

$$V(R,\theta,\varphi) = \sum_{k=0}^{\infty}\sum_{l=-k}^{k}\frac{M_{k,l}Y_k^l(\theta,\varphi)}{R^{k+1}} \xrightarrow[R\,\to\,1/r]{} \tilde{V}(r,\theta,\varphi) = \sum_{k=0}^{\infty}\sum_{l=-k}^{k} M_{k,l}Y_k^l(\theta,\varphi)r^k$$

Consider now the Taylor expansion of $\tilde{V}$:

$$\tilde{V}(x^a) = \sum_{k=0}^{\infty}\frac{x^{a_1}\dots x^{a_k}}{k!}\partial_{a_1}\dots\partial_{a_k}\tilde{V}\Big|_{r=0}$$

- $Y_k^l(\theta,\varphi)r^k$ polinomial of degree k in $(x^1,x^2,x^3) = (r\sin\theta\cos\varphi\,, r\sin\theta\sin\varphi\,, r\cos\theta) \Rightarrow$

$$M_{k,l} \leftrightarrow \partial_{a_1}\dots\partial_{a_k}\tilde{V}\Big|_{r=0}$$

There is a correspondence between multipole moments and these quantities at infinity!

- This definition have been generalized to stationary and asimptotically flat spacetimes in General Relativity [Geroch 1970, Hansen 1974.]

Parameter estimation

- Bayesian parameter estimation on injected signals with the inspiral template $h_{\mathcal{H}} \sim \mathcal{A}(f)e^{i\psi(f)}$

prior

likelyhood

posterior

$$p(\vec{\theta}|d) = \frac{\pi(\vec{\theta})\mathcal{L}(d|\vec{\theta},\mathcal{H})}{\int d^m\theta\pi(\vec{\theta})\mathcal{L}(d|\vec{\theta},\mathcal{H})}$$

evidence

The Likelihood is assumed to be a multivariate Gaussian in the signal parameters:

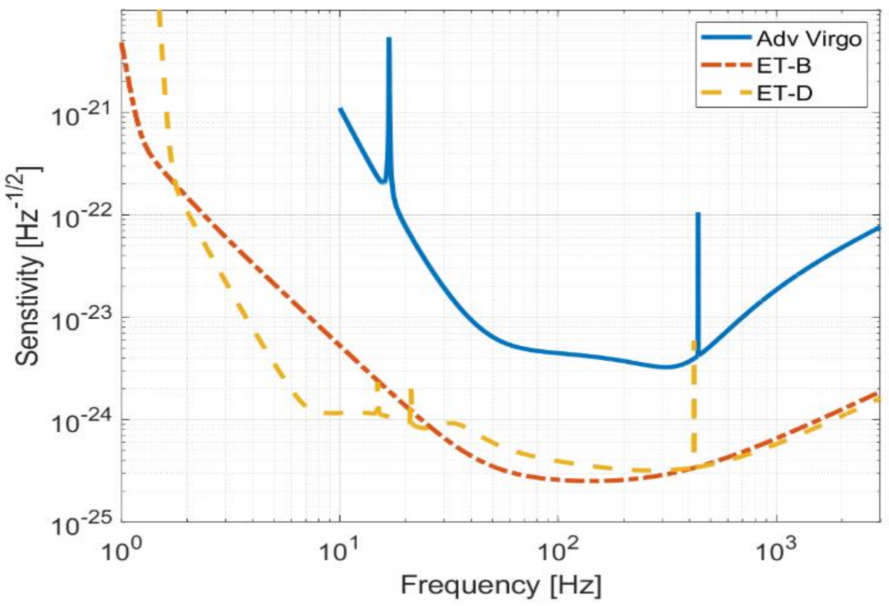
$$\mathcal{L}(d|\vec{\theta},\mathcal{H}) = \exp\left[-\frac{1}{2}(d - h_{\mathcal{H}}(\vec{\theta})|d - h_{\mathcal{H}}(\vec{\theta}))\right]$$

Where we have introduced the product:

$$(h_1|h_2) = 4Re\int_{f_{min}}^{f_{max}} \frac{h_1(f)h_2(f)}{S_n(f)}df$$

Detector Sensitivity

Sensitivity curves for Adv Virgo and the Einstein Telescope

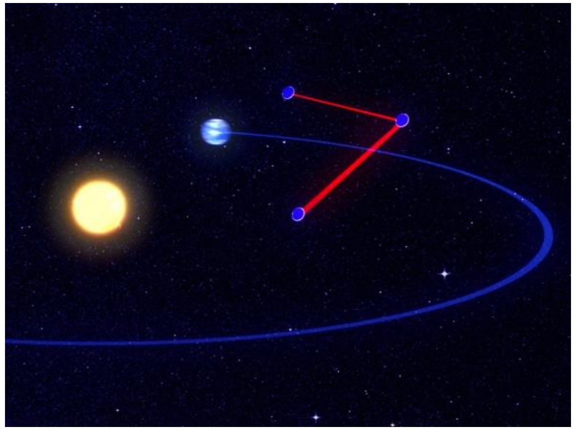


Gravitational-wave astronomy era

- Gravitational waves from binary systems are routinely observed by LIGO/Virgo.
- In the near future:

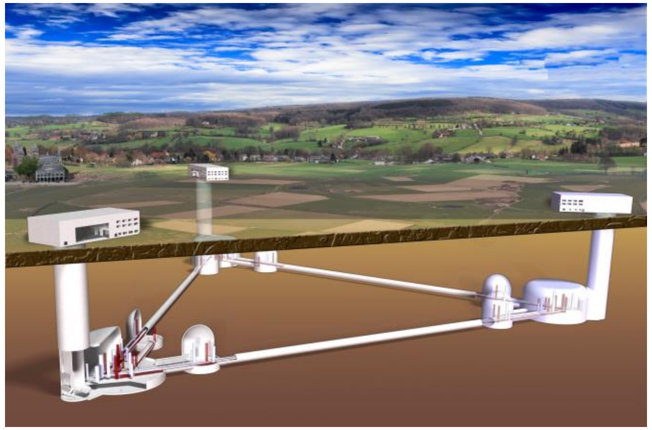
LISA (Laser Interferometer Space Antenna) – Launch expected in 2034

- Cluster of 3 spacecrafts in heliocentric orbit
- Equilater triangle with $5 \cdot 10^6 km$ arm lenght
- Sensitivity band $\sim$ 0.1 mHz $-$ 1 Hz



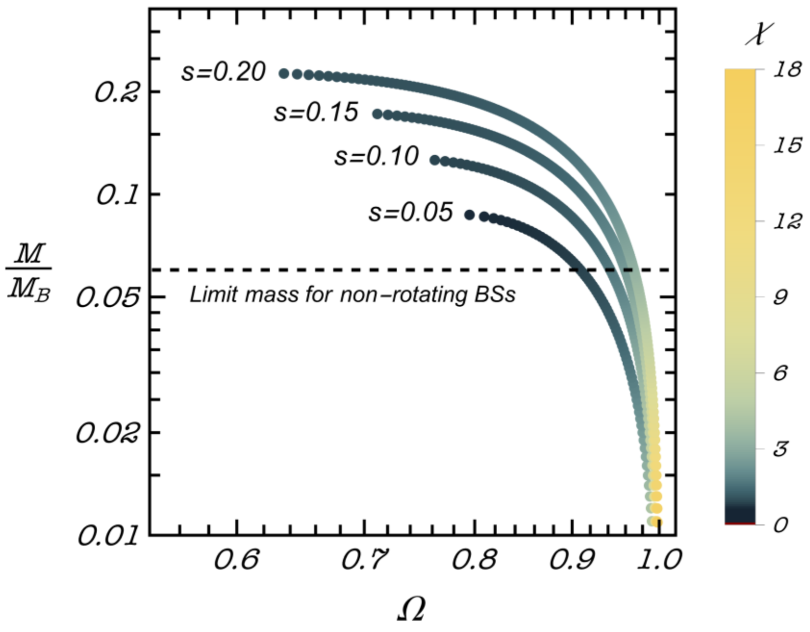
ET (Einstein Telescope) – Contruction will start in 2026

- Ground-based, triangle-shaped interferometer
- 10 km arm length (LIGO-Virgo $\sim$ 3 km)
- Sensitivity band $\sim$ 1 Hz $-$ 10^4Hz



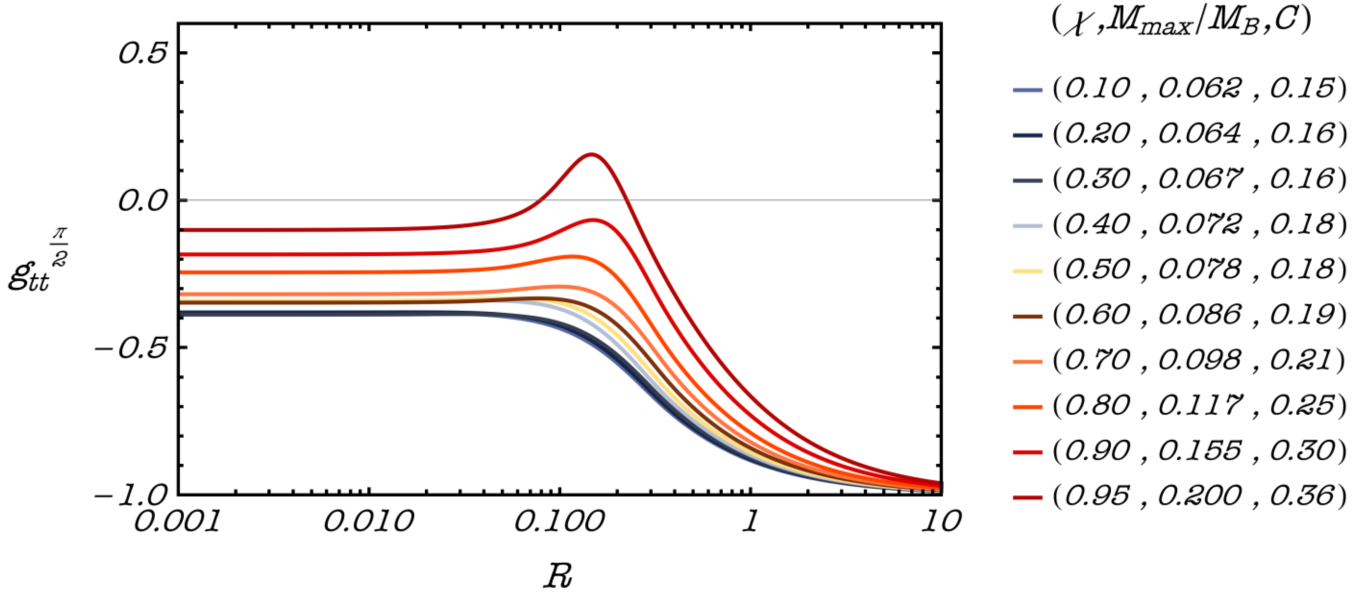
Opportunity to test the nature of compact objects with unprecedented accuracy!

Maximum mass and ergoregions



Increasing the vaue of the winding number $s = \left(\frac{\sqrt{\lambda}}{m}\right)^{-1} \times n_r$, it is possible to exceed significantly the non-spinning maximum mass limit $M \sim 0.06 M_B$

The model allows for configurations featuring ergoregions in the (linearly) stable branch.



The multipolar structure of fast rotating boson stars: *Massimo Vaglio, Costantino Pacilio, Andrea Maselli, Paolo Pani,* arXiv:2203.07442 (2022)

Families of (rotating) Boson Stars

- Different families of BSs, correspond to different potenatials in the lagrangian:

(Neutron Stars: Equation Of State $\longrightarrow$ *Boson Stars: Self-interactions* $V(|\phi|^2)$)

- Mini BSs $\quad V(|\phi|^2) = m^2|\phi|^2 \quad M_{max} \sim \frac{M_p^2}{m}$
- Massive BSs $\quad V(|\phi|^2) = m^2|\phi|^2 + \lambda|\phi|^4 \quad M_{max} \sim \frac{M_p^3}{m^2}\lambda^{\frac{1}{2}}$
- Solitonic BSs $\quad V(|\phi|)^2 = m^2|\phi|^2\left(1 - \frac{2|\phi|^2}{\sigma^2}\right) \quad M_{max} \sim \frac{M_p^4}{m\sigma^2}$

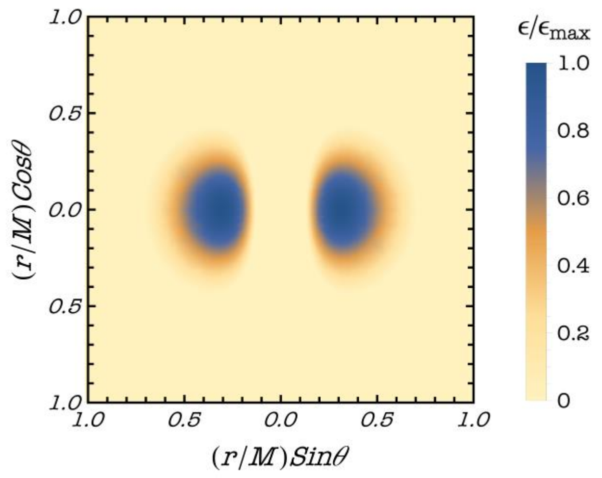
- To have stationarity and axysimmetry the field must satisfy:

azimuthal winding number

$$\phi = \phi_0(r,\theta)e^{i(n_r\varphi - \Omega t)}$$

frequency

$J = n_r N$ *The angular momentum is quantized!*



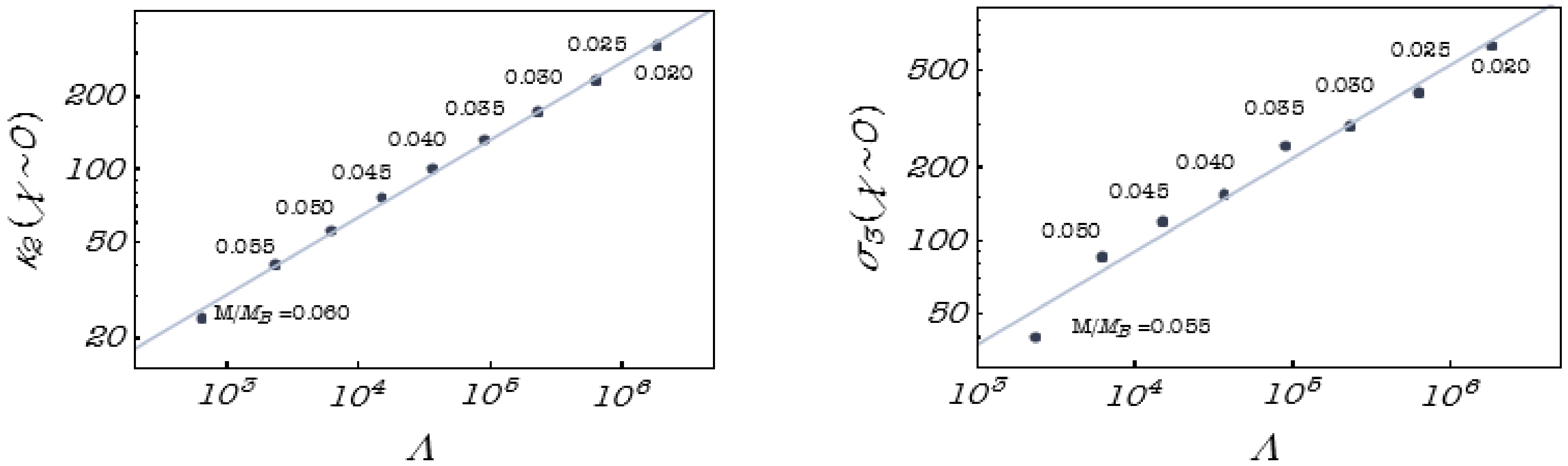
Normalized energy-density of a BS in a transversal section

Universal Relations for Boson Stars?

- Neutron Stars feature simple relations linking their moment of inertia, the tidal deformability and the quadrupole moment which do not depend sensitively on the star's internal structure.

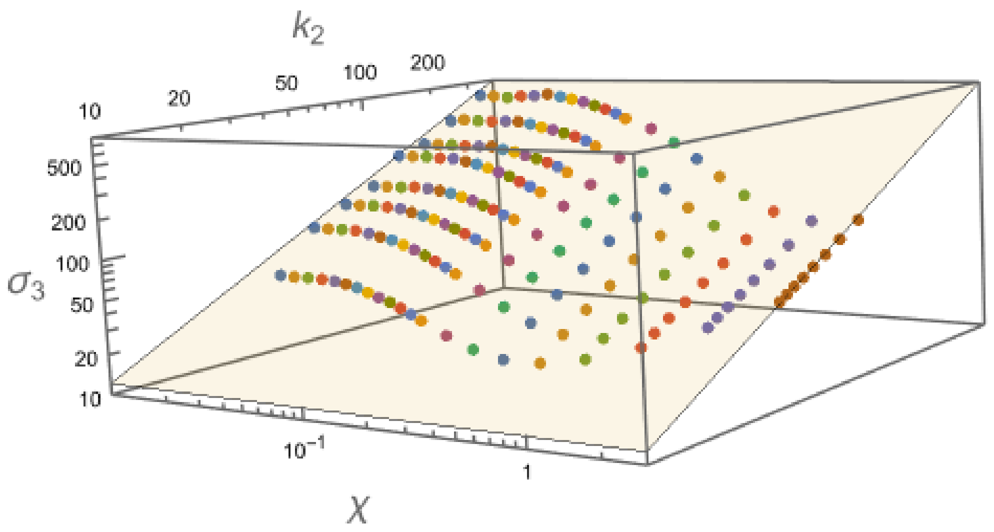
I-Love-Q Relations in Neutron Stars and their Applications to Astrophysics, Gravitational Waves and Fundamental Physics - *Kent Yagi and Nicolàs Yunes*

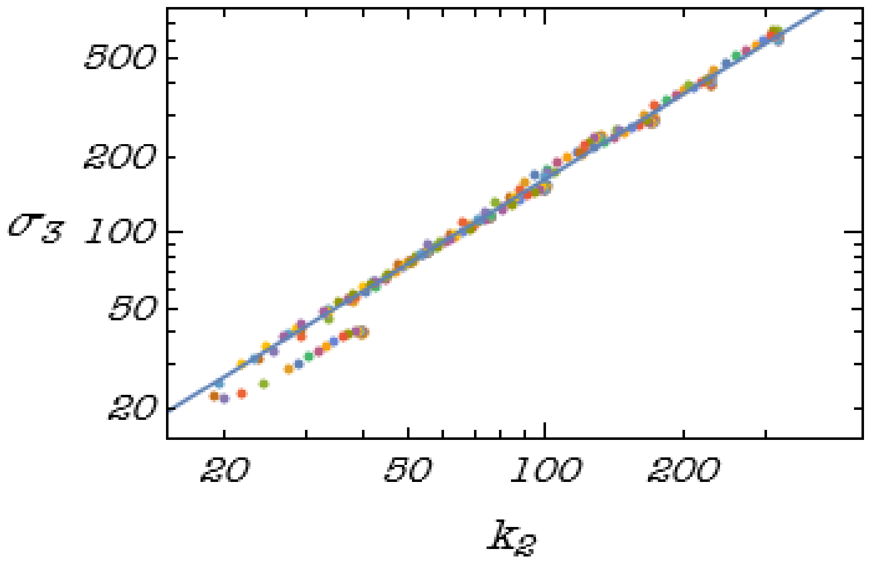
- We found the reduced quadrupole and octupole moments are simply connected to the tidal deformability of the boson star



Universal Relations for Boson Stars?

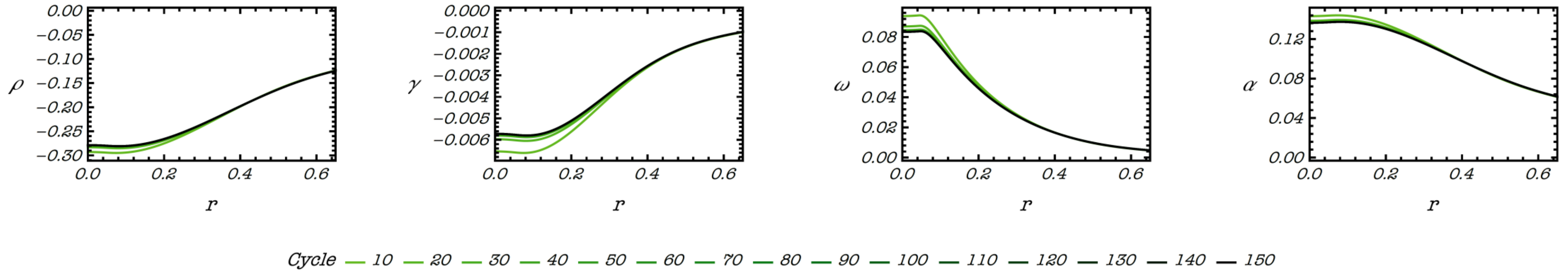
- The relation between κ_2 and σ_3 appears remarkably to be independent on the spin χ





- These relations have many applications and are especially useful to break degeneracies among parameters that characterize gravitational waveforms.

Integration and multipole moments



Coordinates	$q = r/(1+r)$, $\mu = \cos\theta$ $q,\mu \in [0,1]$	Compactified
Grid	$n_q \times n_\mu$	Fixed equally spaced
Derivatives	–	Five points central
Integration	–	Trapezoidal rule

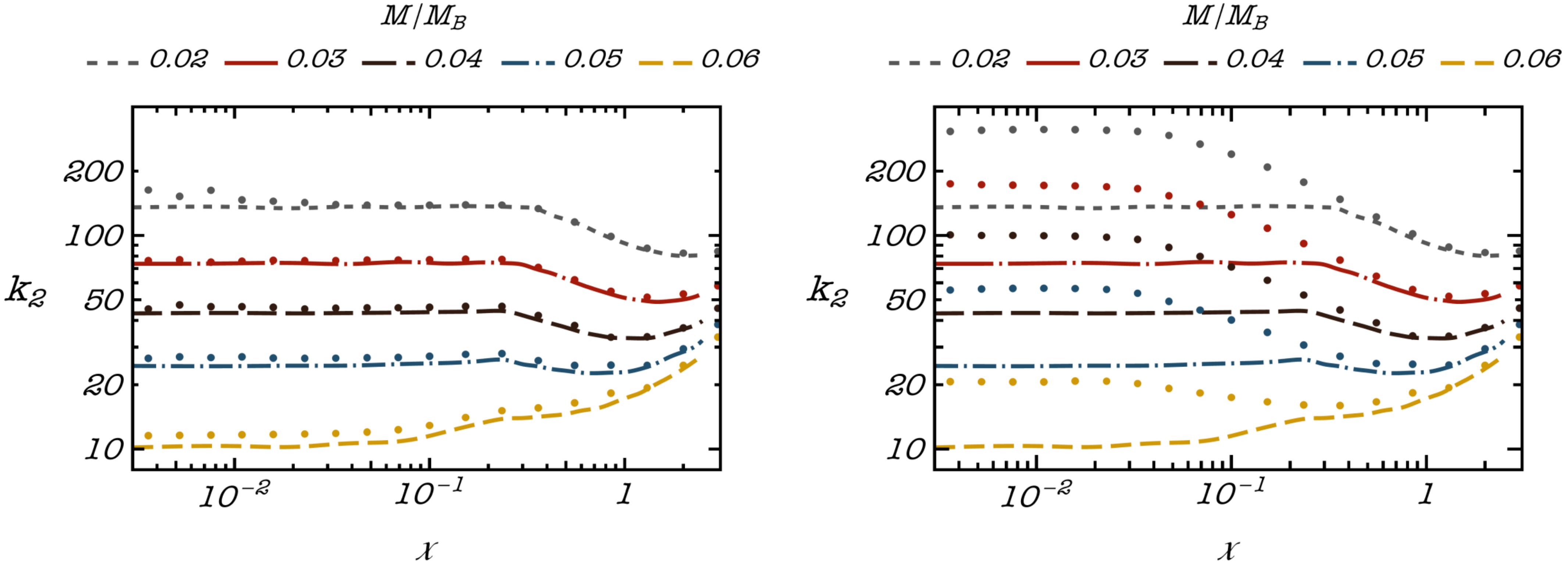
- Mass and current moments $\{M_0, M_2 \dots\}$, $\{S_1, S_3 \dots\}$ can be read off:

$$\rho(r,\mu) = \sum_{n=0}^{\infty} -2\frac{M_{2n}}{r^{2n+1}} P_2 n(\mu) + \text{higher orders}$$

$$\omega(r,\mu) = \sum_{n=1}^{\infty} -\frac{2}{2n-1}\frac{S_{2n-1}}{r^{2n+1}}\frac{P^1_{2n-1}(\mu)}{\sin\theta} + \text{higher orders}$$

Consistency with previous results

- Our findings about the quadrupole moments agree with previous results, when using the same grid $n_q \times n_\mu = 1600 \times 160$, but there is a deviation when n_μ is increased up to the saturation value $n_\mu \sim 20000$.



The dashed lines correspond to the values reported in ***F. D. Ryan, Phys. Rev. D 55, 6081 (1997)***

Dependence on the integration grid

- Due to numerical erros, we found a non-zero value of $M_2^{(off)} \equiv M_2(\chi = 0)$

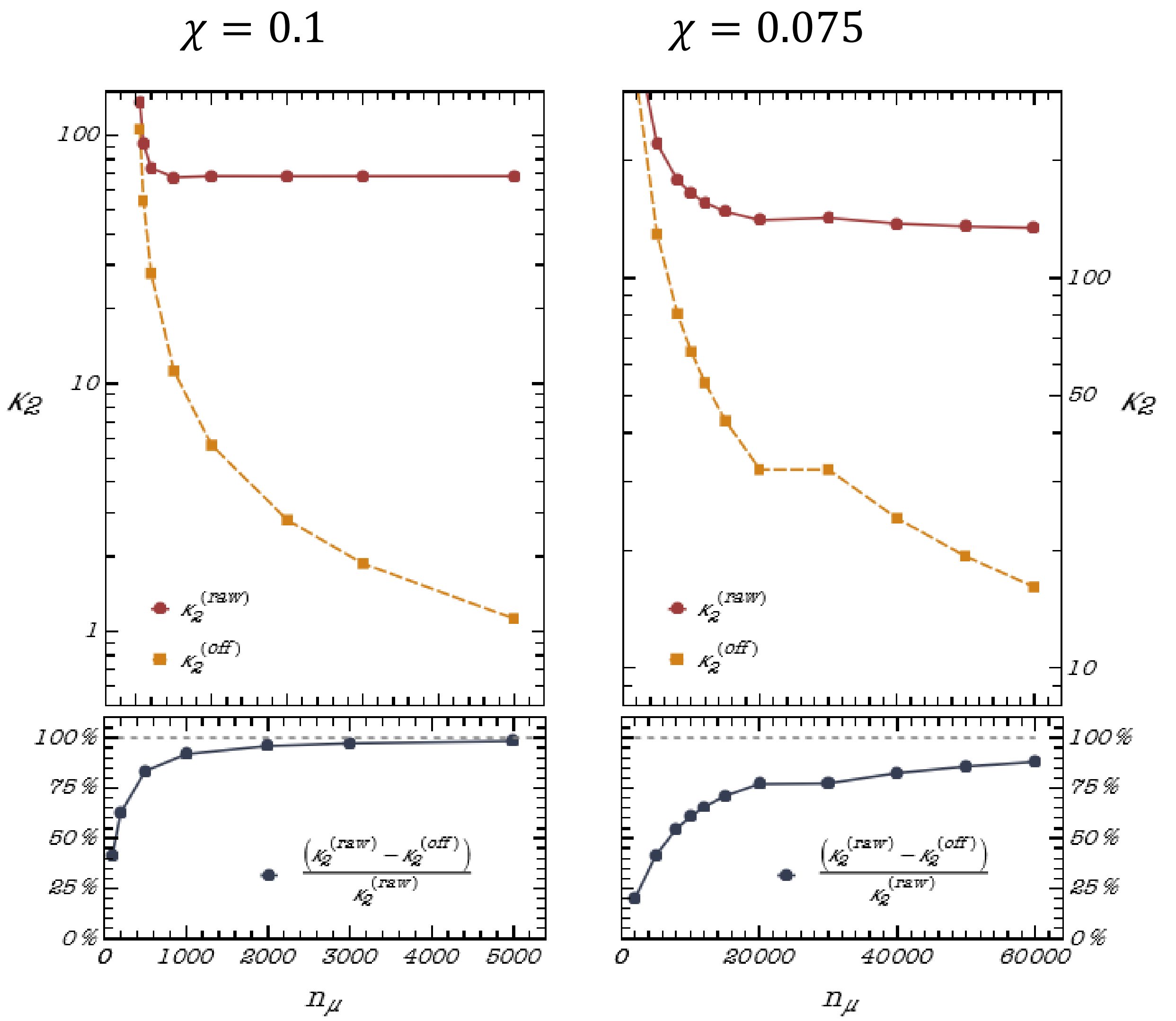
  In the plots (top panels):

  $$k_2^{(raw)} = M_2^{(raw)}/(\chi^2 M^3)$$

  $$k_2^{(off)} = M_2^{(off)}/(\chi^2 M^3)$$

  and their percentage difference (bottom panels), for fixed $M = 0.04 M_B$, $n_q = 1600$ and two values of χ.

- Extracting the quadrupole moments for slow spinning configurations requires more angular precision.

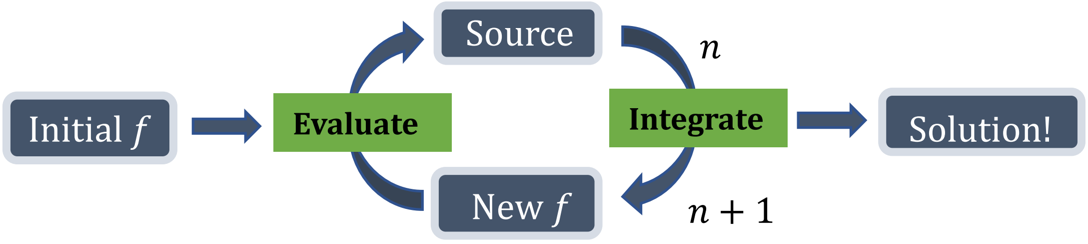


Self consistent field method

- The equations can be solved iteratively: $f(x) = \int G(x, x') S(f, \partial f, x') dx'$

(r', θ')

$(\rho, \gamma, \omega, \alpha)$



Es: $$\rho = -\frac{1}{4\pi} e^{-\frac{\gamma}{2}} \int_0^\infty dr' \int_{-1}^{1} d\mu' \int_0^{2\pi} d\phi' r' S_\rho(r', \mu') \frac{1}{|r - r'|}$$

Automatically satisfies aymptotic flatness conditions for reasonable sources!

Backup Slide – Coordinate rescaling

- It is possible to get rid of the coupling constants trought the following rescalings:

$$t = \frac{\lambda^{\frac{1}{2}}}{m^2}\tilde{t} \qquad s = \frac{\lambda^{\frac{1}{2}}}{m}\tilde{s} \qquad r = \frac{\lambda^{\frac{1}{2}}}{m^2}\tilde{r} \qquad \Omega = m\tilde{\Omega} \qquad \epsilon = \frac{m^4}{\lambda}\tilde{\epsilon} \qquad \omega = \frac{m^2}{\lambda^{\frac{1}{2}}}\tilde{\omega} \qquad P = \frac{m^4}{\lambda}\tilde{P} \qquad |\phi|^2 = \frac{m^2}{\lambda}\left|\tilde{\phi}\right|^2$$

- Consequently we have the following change in the relevant expressions:

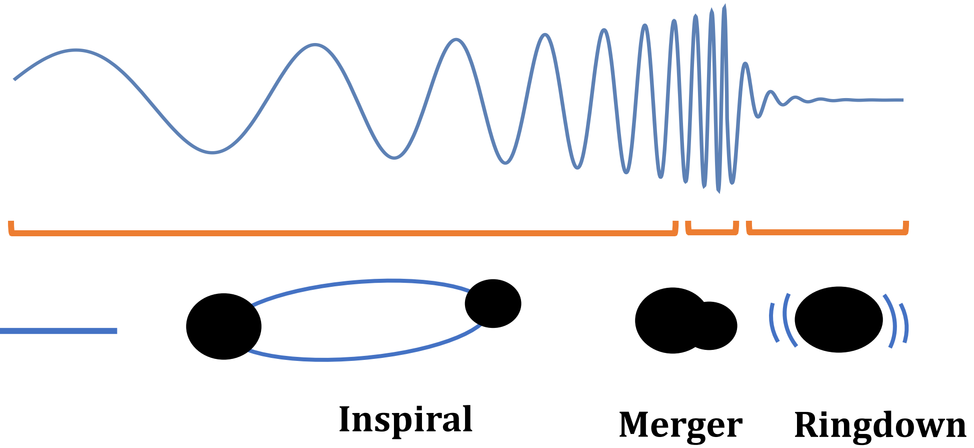
$$\tilde{P} = \frac{1}{4}\left|\tilde{\phi}\right|^4 \qquad \tilde{\epsilon} = \left|\tilde{\phi}\right|^2 + \frac{3}{4}\left|\tilde{\phi}\right|^4 \qquad \left|\tilde{\phi}\right|^2 = Max\left[0, \frac{\left(\tilde{\Omega} - s\tilde{\omega}\right)^2}{e^{\gamma+\rho}} - \frac{e^{\gamma-\rho}\widetilde{s^2}}{\widetilde{r^2}\sin\theta^2} - m^2\right]$$

$$d\widetilde{s^2} = -e^{\gamma+\rho}d\widetilde{t^2} + e^{2\alpha}\left(d\widetilde{r^2} + \widetilde{r^2}d\theta^2\right) + e^{\gamma-\rho}\widetilde{r^2}\sin\theta^2\,(d\phi - \tilde{\omega}d\tilde{t})^2$$

- Physical quantities can be derived multiplying the rescaled ones by: $\dfrac{\lambda^{\frac{1}{2}}}{m^2} \equiv M_B$

Binary Boson Star signal

- Multipole moments enter in the PN expansion in $v = (\pi M f)^{\frac{1}{3}}$ of the inspiral signal:



$$h \sim \mathcal{A}(f)e^{i\psi(f)}$$

$$\psi(f) = \psi_{BH}(f) + \psi_{\kappa_2}(f) + \psi_{\Lambda}(f)$$

Leading order

Reduced Quadrupole

$$\psi_{k_2} = -\frac{75}{64}\frac{(\kappa_2^1 M_1^2 \chi_1^2 + \kappa_2^2 M_2^2 \chi_2^2)}{M_1 M_2}(\pi M_t f)^{-1/3}$$

Gravitational-wave detectors as particle-physics laboratories: Constraining scalar interactions with a coherent inspiral model of boson star binaries, *Costantino Pacilio, Massimo Vaglio, Andrea Maselli, Paolo Pani. Phys.Rev. D* 102 (2020) 8, 083002

Backup Slide – Mass scale

- We want to explore the possibility of constraining the BS coupling with future observations:

$$M_{max} \approx 0.06(1+0.76\chi^2)M_B \quad \Rightarrow$$

$$M_{max}(\chi\sim 0) \approx 0.06 M_B \approx 0.06\frac{\sqrt{\lambda}}{m^2} \approx 0.06\frac{\sqrt{\lambda\hbar}}{m_S^2}M_P^3 \approx 10^5 M_{\odot}\sqrt{\lambda\hbar}\left(\frac{\mathrm{MeV}}{m_S}\right)^2$$

We can cover the whole spectrum of sources for LISA and ET varying λ and m_s

Backup Slide – Parameter Estimation

- The expression for the quadrupole moment as a funtion of mass, spin of the BS:

$$Q = -\kappa(\chi, M/M_B)\chi^2 M^3$$

can be used within parameter estimation to measure directly the effective coupling from GWs observation of BS binaries :

$$\vec{\theta} = (\mathcal{A}, t_c, \phi_c, \log \mathcal{M}\ , \log \eta\ , \chi_s, \chi_a, M_B)$$

- We used a Fisher matrix approach and a Post Newtonian expanded waveform to estimate the uncertainty with which M_B can be measured by LISA and ET in the following scenario:

Individual masses	Mass scale	Spins
$(M_1, M_2) \sim (0.05 M_B, 0.06 M_B)$	$0.06 M_B = \begin{cases} 1 - 100 M_\odot & ET \\ 10^4 - 10^6 M_\odot & LISA \end{cases}$	$(\chi_1, \chi_2) = \begin{cases} (0.1,0) \\ (0.6,0.3) \\ (0.9,0.8) \end{cases}$

Backup Slide – Tidal deformability

- To include the tidal deformability in the waveform we exploited the relation:

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}}\left[-0.828 + \frac{20.99}{\log\Lambda} - \frac{99.1}{(\log\Lambda)^2} + \frac{149.7}{(\log\Lambda)^3}\right]$$

N. Sennett et al., Phys. Rev. D, 96, 2 (2017) 024002

where $\Lambda = \lambda_T/M^5$ and λ_T is defined as $\mathcal{Q}_{ij} = -\lambda_T \varepsilon_{ij}$

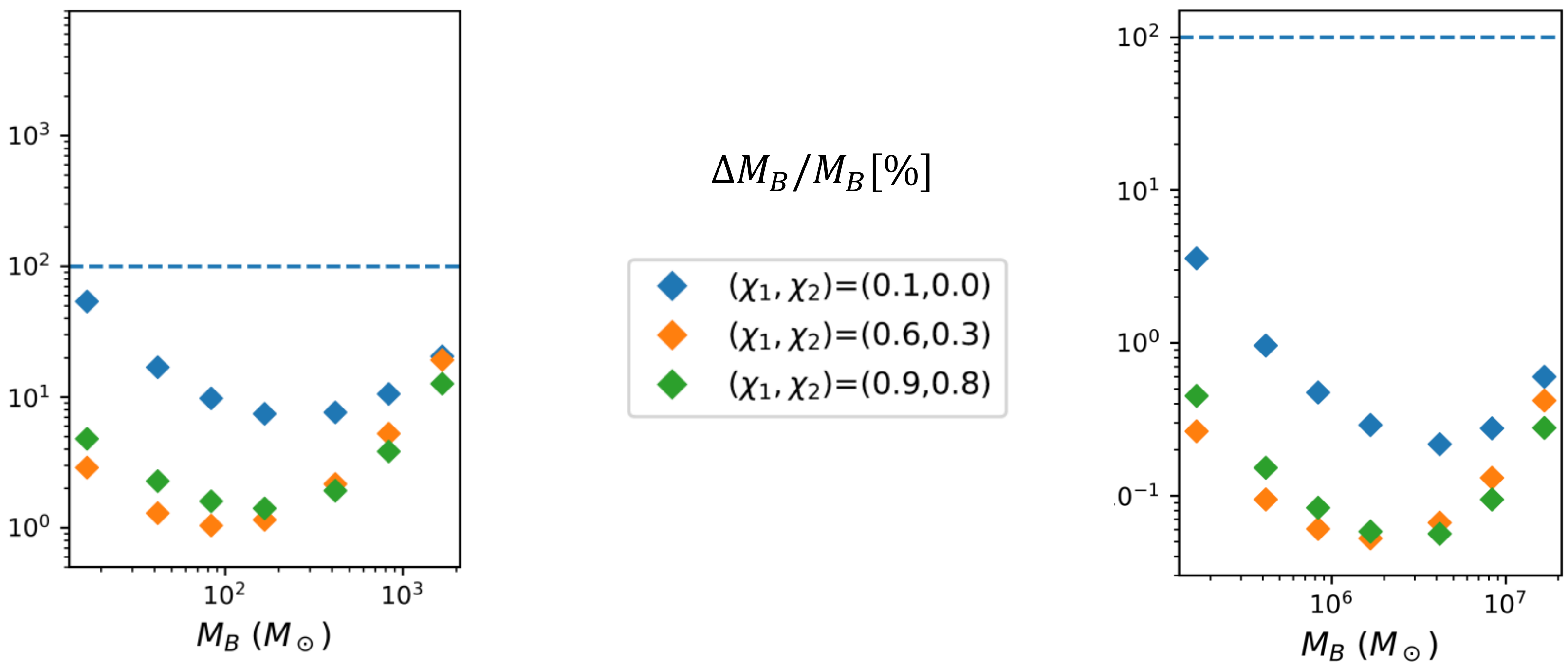
- Λ will affect the waveform through an effective combination of the values of each BS

$$\tilde{\Lambda} = \frac{16}{13}\left[\left(1 + \frac{12}{q}\right)\frac{M_1^5}{M_t^5}\Lambda_1 + (1 + 12q)\frac{M_2^5}{M_t^5}\Lambda_2\right]$$

Backup Slide – Constraining scalar interactions

- The errors on M_B for ET and LISA are at the percent and sub-percent level in the most optimistic configurations:

ET ($D_L = 500\ Mpc$) LISA ($D_L = 1\ Gpc$)



Backup Slide – Multipole moments

$$\rho(r,\mu) = \sum_{n=0}^{\infty} -2\frac{M_{2n}}{r^{2n+1}} P_2 n(\mu) + \text{higher orders} \quad \omega(r,\mu) = \sum_{n=1}^{\infty} -\frac{2}{2n-1}\frac{S_{2n-1}}{r^{2n+1}}\frac{P^1_{2n-1}(\mu)}{\sin\theta} + \text{higher orders}$$

$$\Rightarrow$$

$$\boxed{M_{2n} = \frac{1}{2}\int_0^r dr'(r')^{2n+2}\int_0^1 d\mu' P_{2n}(\mu')S_\rho(r',\mu')}$$

$$\boxed{S_{2n-1} = \frac{1}{4n}\int_0^r dr'(r')^{2n+2} \times \int_0^1 d\mu' \sin\theta' P^1_{2n-1}(\mu')S_\omega(r',\mu')}$$

- Correction factors to correctly match the Geroch-Hansen multipole moments

χ	κ_2	κ_2^{new}	$corr[\%]$
0.1	22.4	22.1	-1.4%
0.2	15.7	15.6	-0.5%
0.5	15.2	15.3	$\lesssim +0.1\%$
0.8	16.4	16.4	$\lesssim +0.1\%$
1,0	17.4	17.5	$\lesssim +0.1\%$
1.3	19.3	19.4	$\lesssim +0.1\%$
2.0	24.6	24.6	$\lesssim +0.05\%$

Table 1: Reduced quadrupole moment correction factors for different value of the spin χ and $M = 0.06$.