A MIMO approach for Advanced Virgo controls

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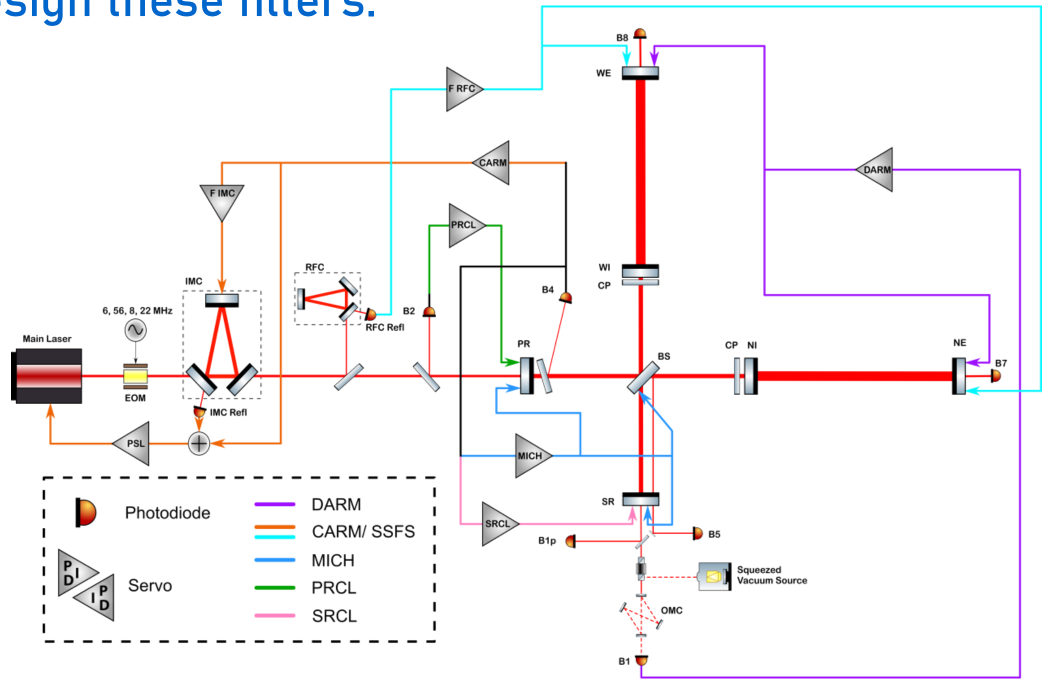
on behalf of the Virgo collaboration

Interferometer

Cross-coupled system

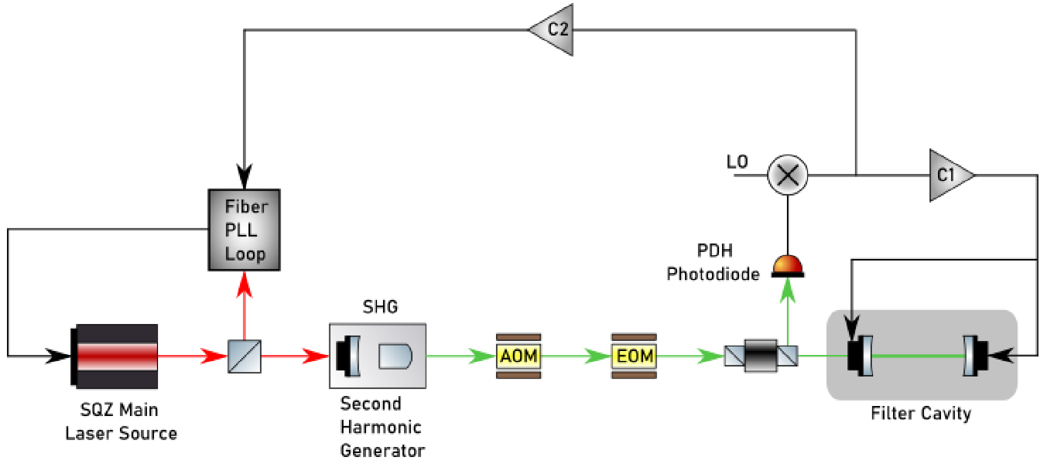
Interferometer O4 configuration has 5 cross-coupled longitudinal DoF. Decoupling filters have been implemented both in the simulation tool and for the real interferometer. Precise measurement of the cross-coupling terms of the optical plant are required to design these filters.



Filter Cavity (Green laser)

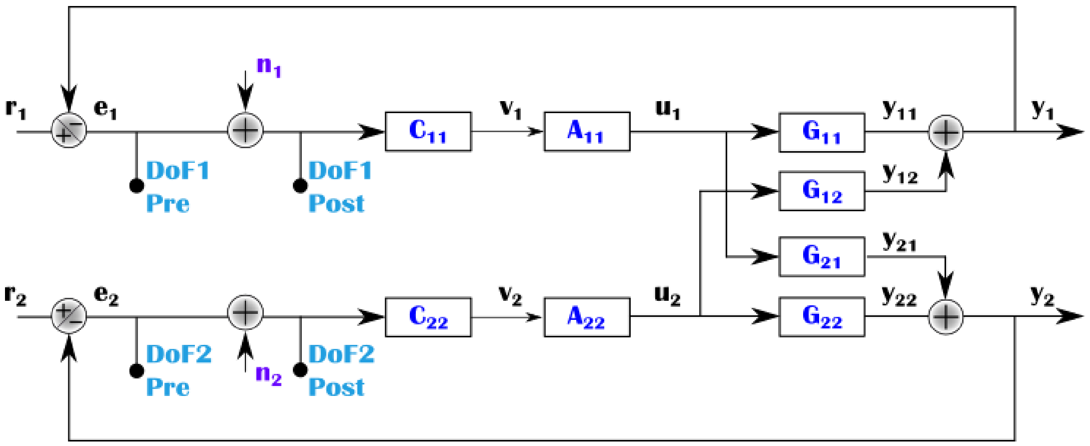
Strong cross-coupled system

The FC (Filter cavity) has 2 loops working together to achieve a good locking precision for the green laser lock. This part of the FDS (Frequency dependent Squeezing) system can be represented as a 2x2 Multiple Input Multiple Output (MIMO) system. A well-chosen system identification method must be used to characterize the optical plant from the Open Loop Transfer Function (OLTF).



OLTF and Cross-coupling measurements

OLTFs can be estimated using the usual approach, which is taking the ratio of the cross-power spectrum, of the 'Signal_PRE' and 'Signal_POST' over the auto-power spectrum of the 'Signal_POST'. But with this method one gets a biased Transfer Function (TF) due to the cross-coupling terms of the system.



Scheme for a control matrix 'C', an actuator matrix 'A' and an optical plant matrix 'G'.

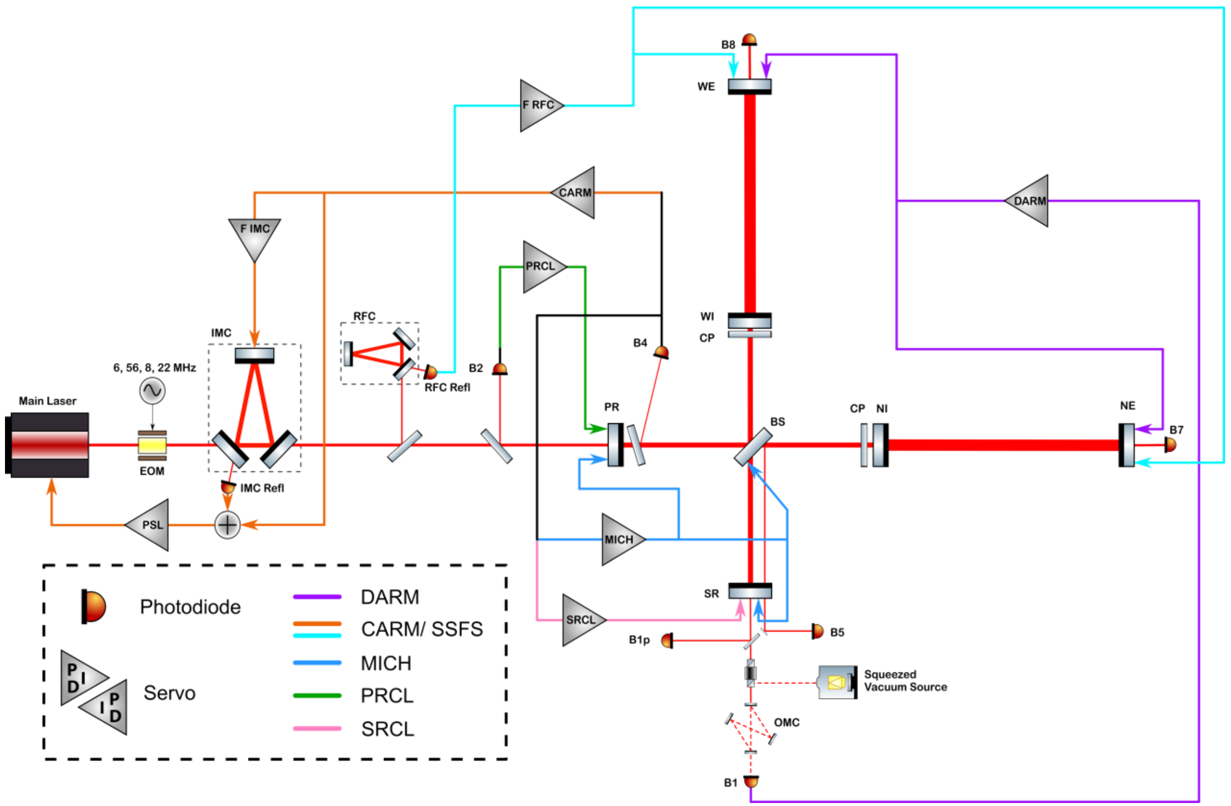
Use of a MIMO system identification approach. One obtain two matrices. "Sensitivity function matrix" (S) and "Complementary sensitivity function matrix" (T).
One can treat the elements of the block diagram as multidimensional matrices.

$S(s)$ is the closed loop TF from 'noise' to 'DoF post' node.
$T(s)$ is the closed loop TF from 'noise' to 'DoF pre' node

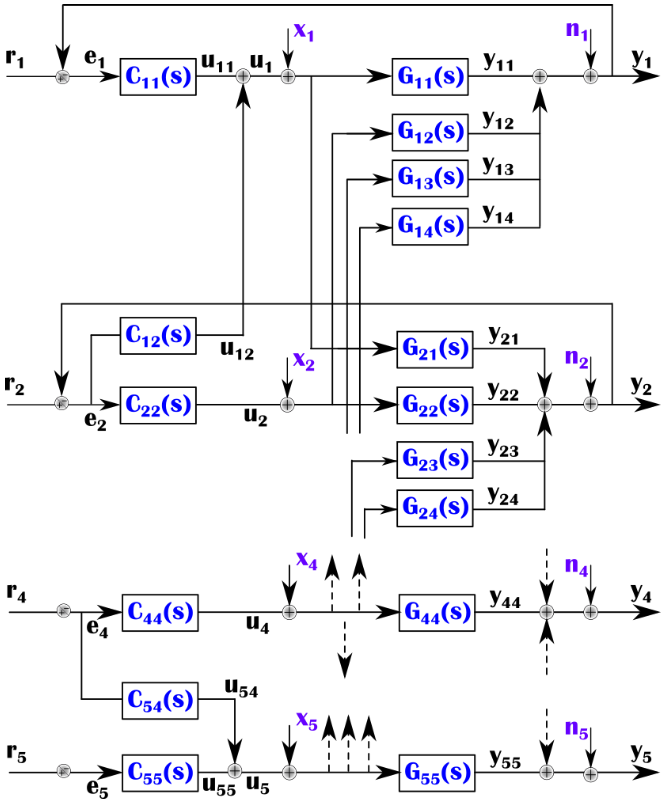
- $L(s) = G(s) \cdot A(s) \cdot C(s)$
- $S(s) = \left(I + L(s)\right)^{-1}$
- $T(s) = -L(s)S(s)$

Then the product of $-T(s) \cdot \left(S(s)\right)^{-1} = L(s)$.
Gives the unbiased OLTF (L) of the system.

Interferometer:



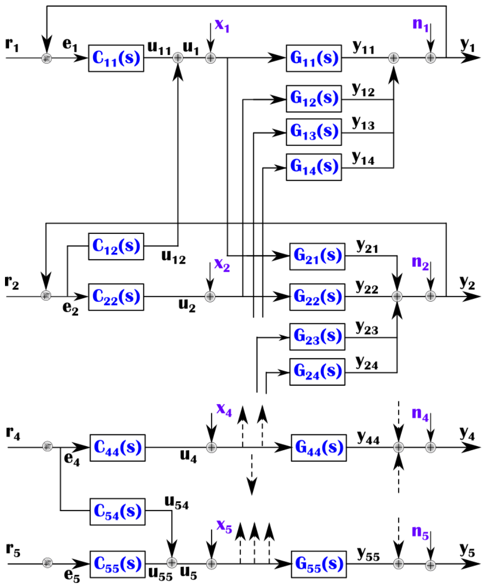
- **DARM** $= L_N - L_W$: Differential **ARM** length.
- **CARM** $= \frac{L_N + L_W}{2}$: Common **ARM** length.
- **MICH** $= l_N - l_W$: Short arms of the **MICH**elson interferometer.
- **PRCL** $= l_{PR} - \frac{l_N + l_W}{2}$: **P**ower **R**ecycling **C**avity **L**ength.
- **SRCL** $= l_{SR} - \frac{l_N + l_W}{2}$: **S**ignal **R**ecycling **C**avity **L**ength.

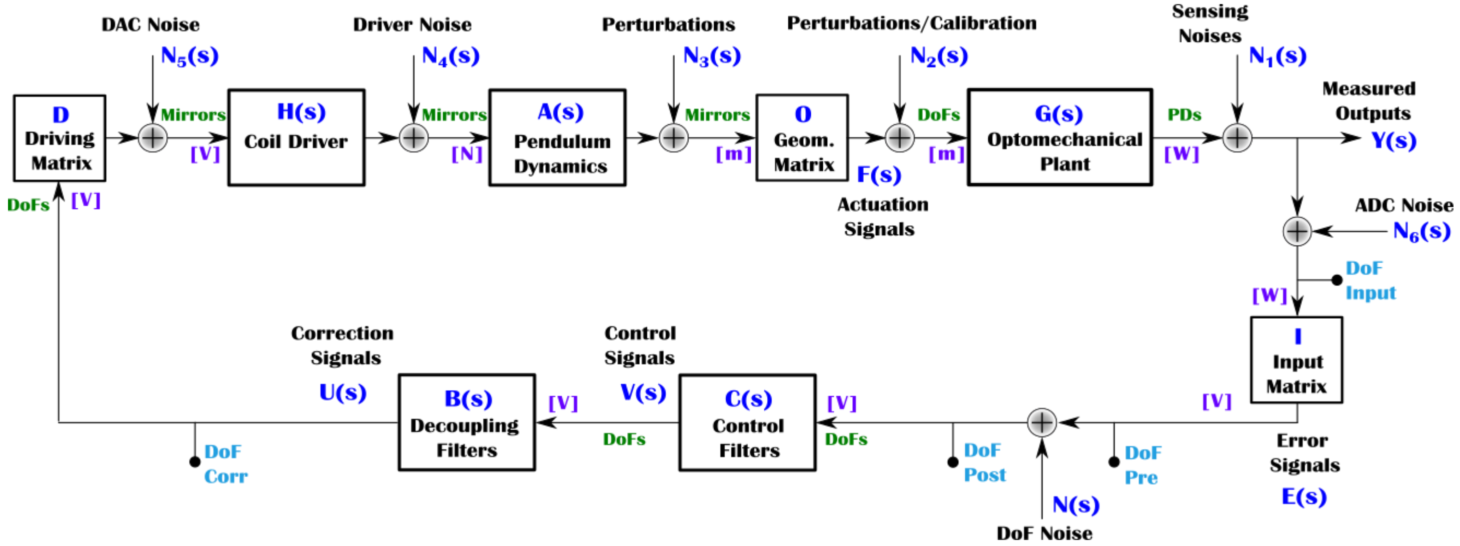


We have represented the interferometer configuration for O4 as an optical plant of 5 cross-coupled longitudinal DoFs with sensors, inputs, outputs, drivers, controllers and actuators matrices with a MIMO scheme.

Interferometer:

Here: $p = 5$, $q = 5$ and $n = 10e3$.





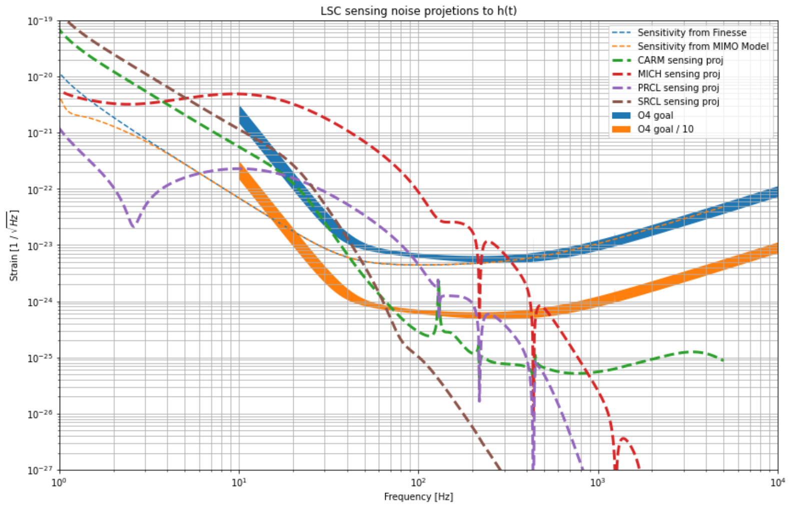
To represent the interferometer in a MIMO scheme, one has to define the matrices $\boldsymbol{D}$, $\boldsymbol{H}(s)$, $\boldsymbol{A}(s)$, $\boldsymbol{O}$, $\boldsymbol{B}(s)$, $\boldsymbol{C}(s)$, $\boldsymbol{I}$ and $\boldsymbol{G}(s)$.
These are $p \times q \times n$ dimensional matrices.
Where p is the number of inputs, q number of outputs and n is the length of the frequency vector.

$$L(s) : OLTF$$

$$S(s) = \left(I + L(s)\right)^{-1}$$

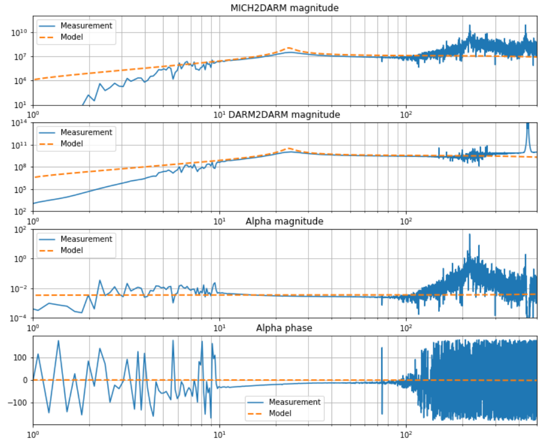
$$T(s) = \left(I + L(s)\right)^{-1} L(s)$$

$$N_1^{Proj}(f) = \frac{|S_{DoF}(f)| N_1(f)}{|T_{DARM}(f)|}$$

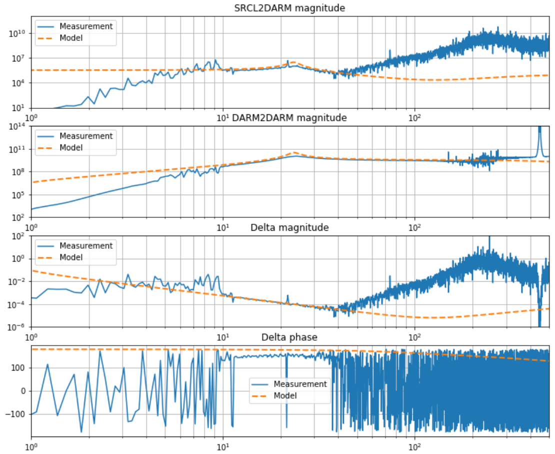


Interferometer:

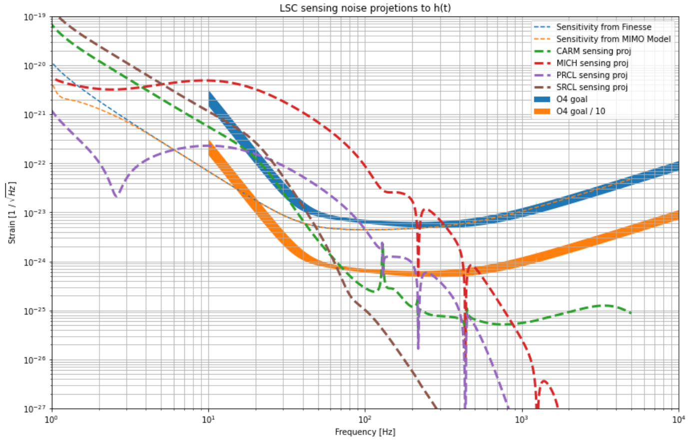
Alpha: MICH to DARM decoupling



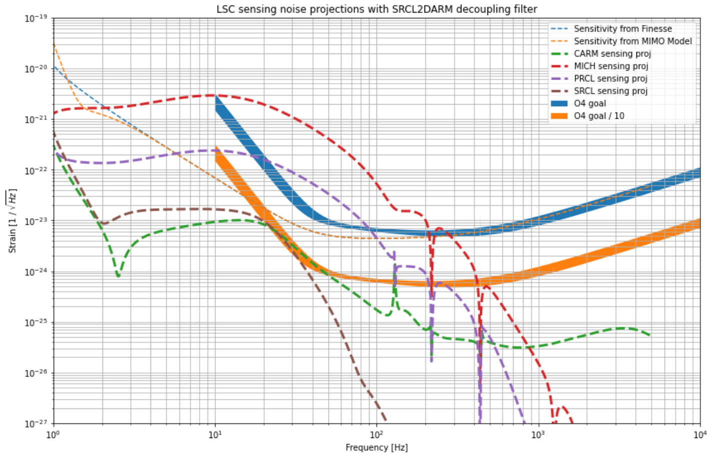
Delta: SRCL to DARM decoupling



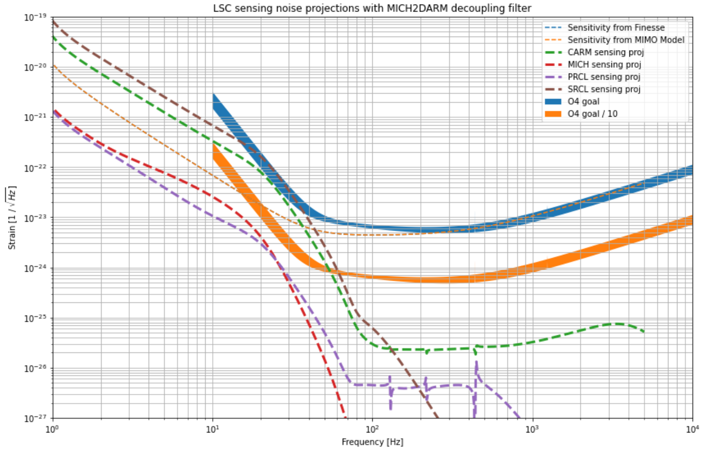
Noise projection, no decoupling:



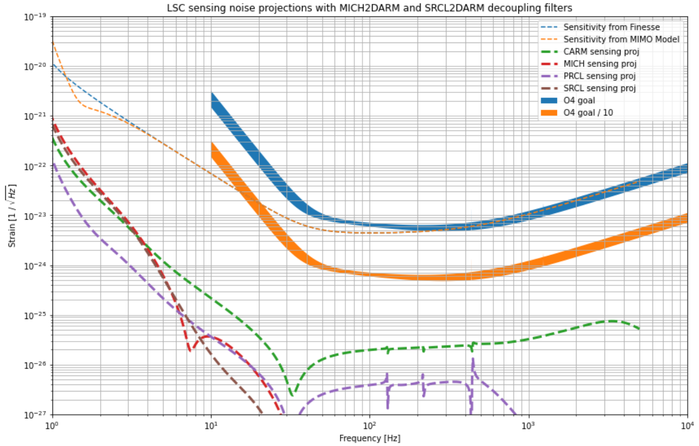
Noise projection Delta ON:



Noise projection Alpha ON:



Noise projection Alpha & Delta ON:

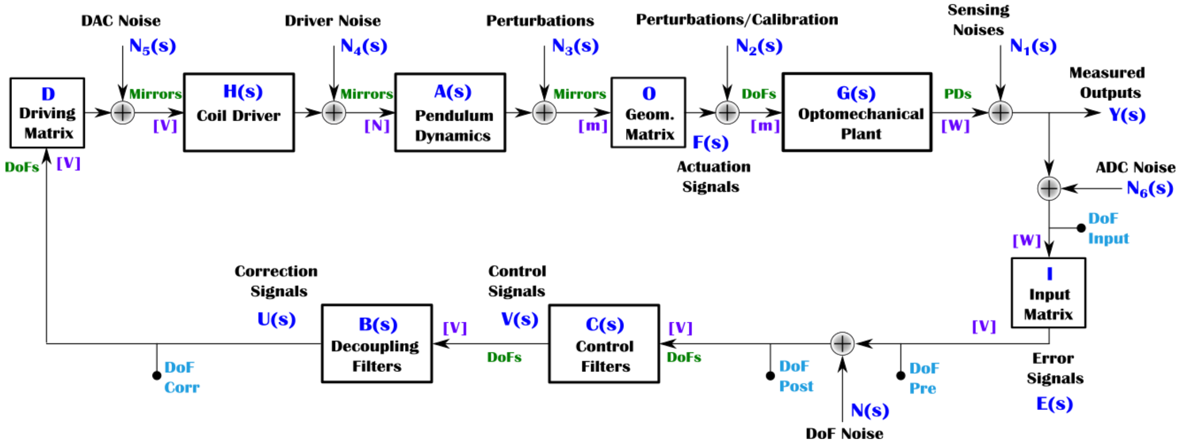


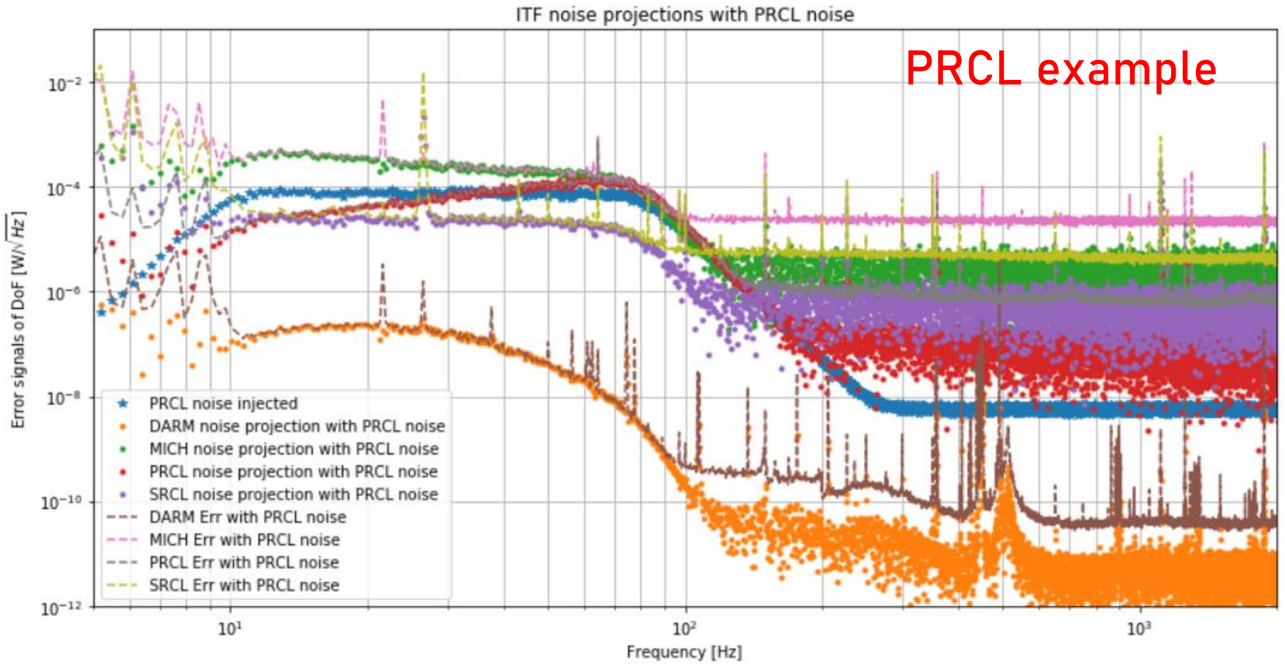
Decoupling filters have been implemented both in the simulation tool and for the real interferometer.

Paper submitted to CQG.

Towards a new Noise Budget tool:

In order to consolidate the understanding of noises limiting the detector, a new Noise budget tool is being prepared following the MIMO method described before. This should facilitate the work on noise projections.





Blue curve is the PRCL noise injected.
Dashed plots are the spectra of the longitudinal DoFs when the noise of PRCL is injected.
Dot plots are the longitudinal noise projections of the PRCL noise on the different DoFs error signals.

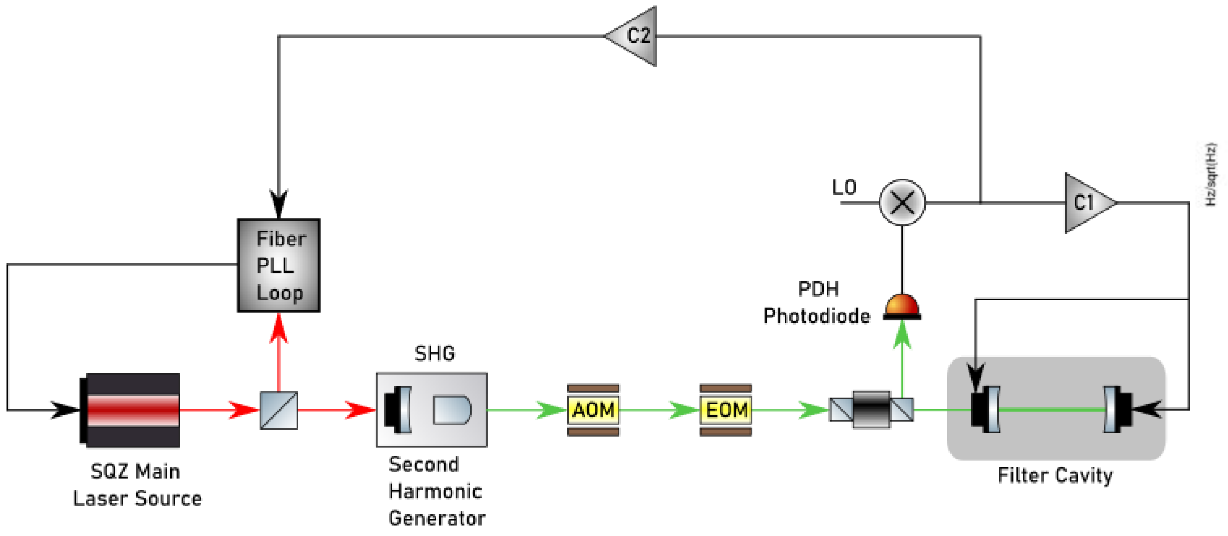
Clearly more noise sources are present and need to be added to the different nodes of the model.

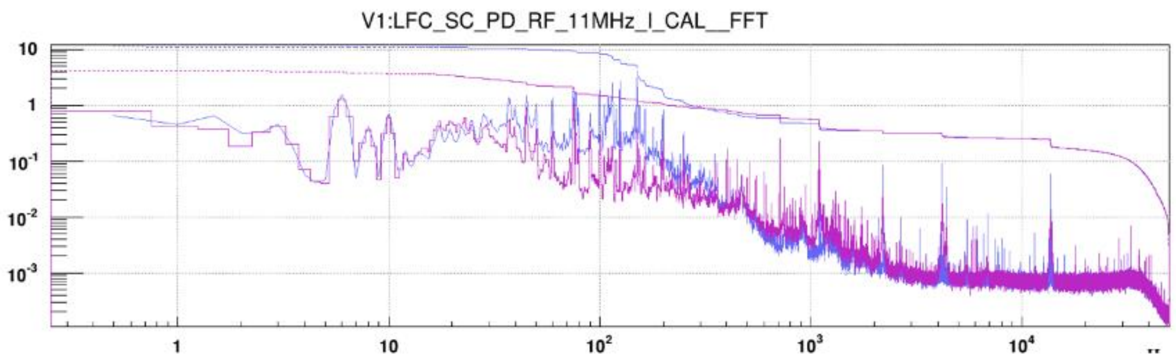
Filter Cavity (Green laser)

This set-up uses a PLL (Phase-locked loop) for the SQZ main laser to have the same frequency of the main laser source of Advanced Virgo.

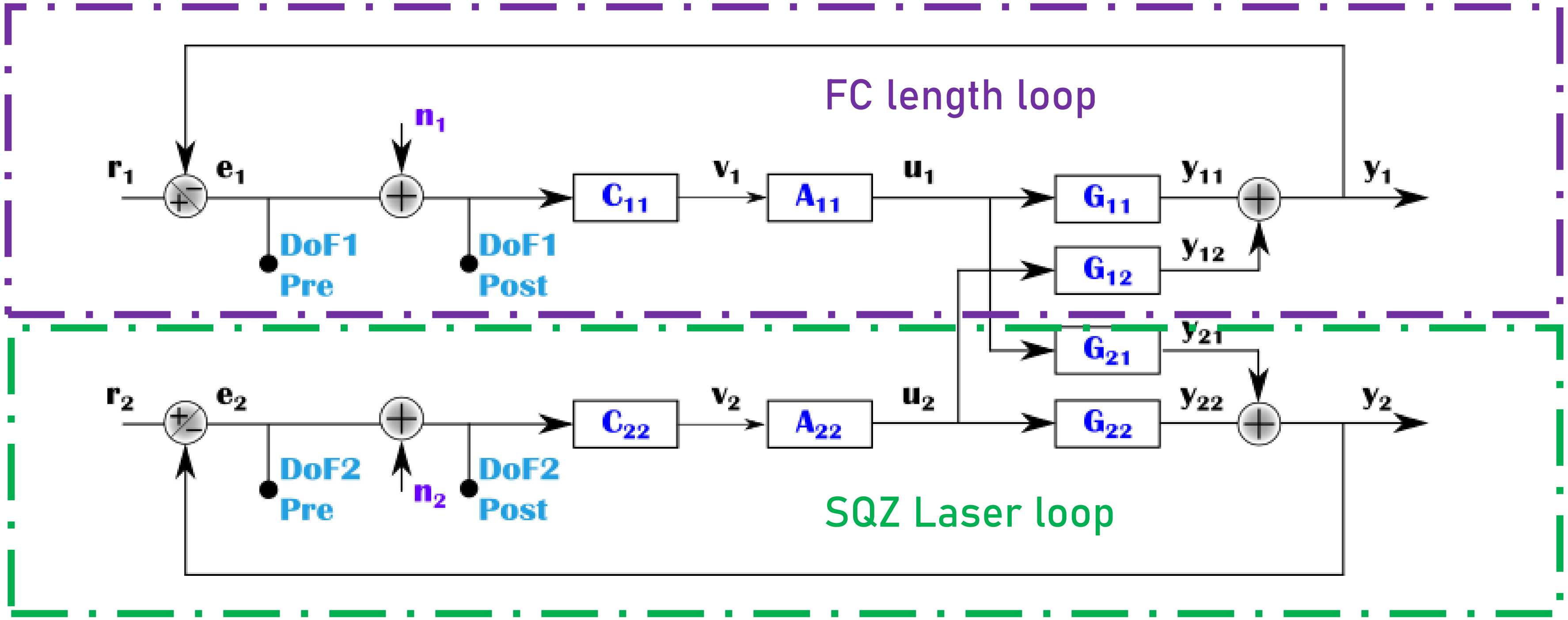
With a loop actuating only on the mirrors of the FC it was impossible to suppress excess of noise around 100 Hz. Therefore a secondary loop was implemented to act on the AC part of the SQZ main laser through the PLL servo.

In this way the FC has now 2 loops working together to achieve the required locking accuracy using the green laser. However, during the commissioning phase, the TFs estimated using the usual system identification approach did not match the model TFs.





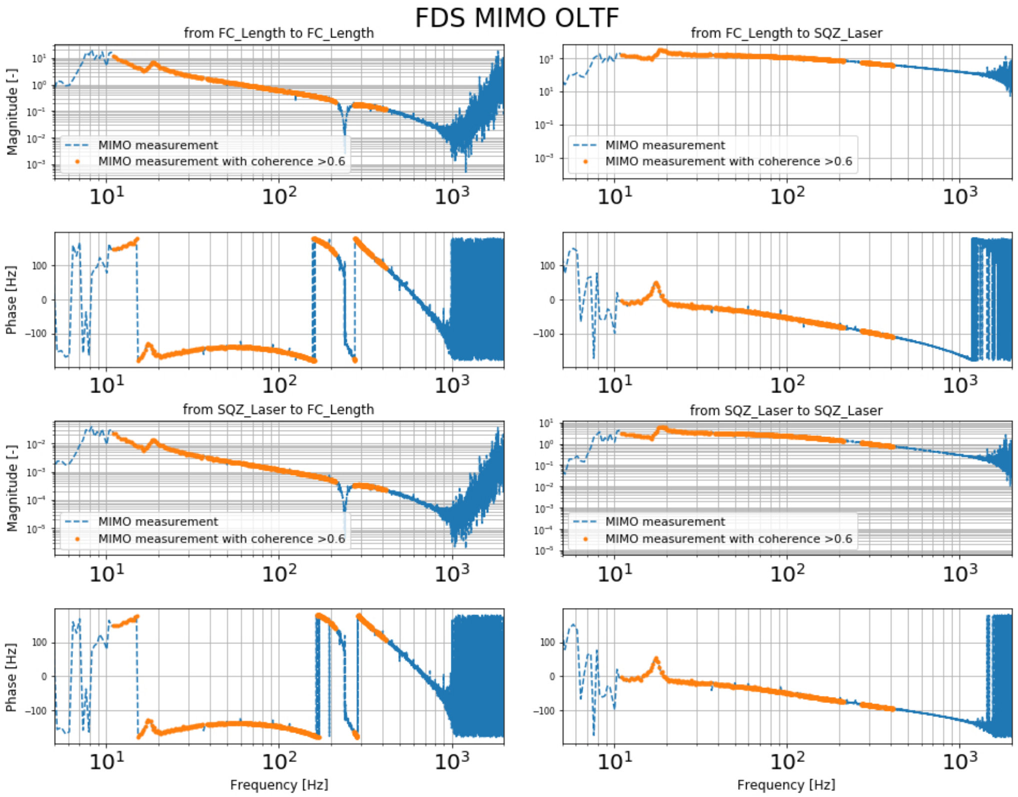
FC green control TF measurements:

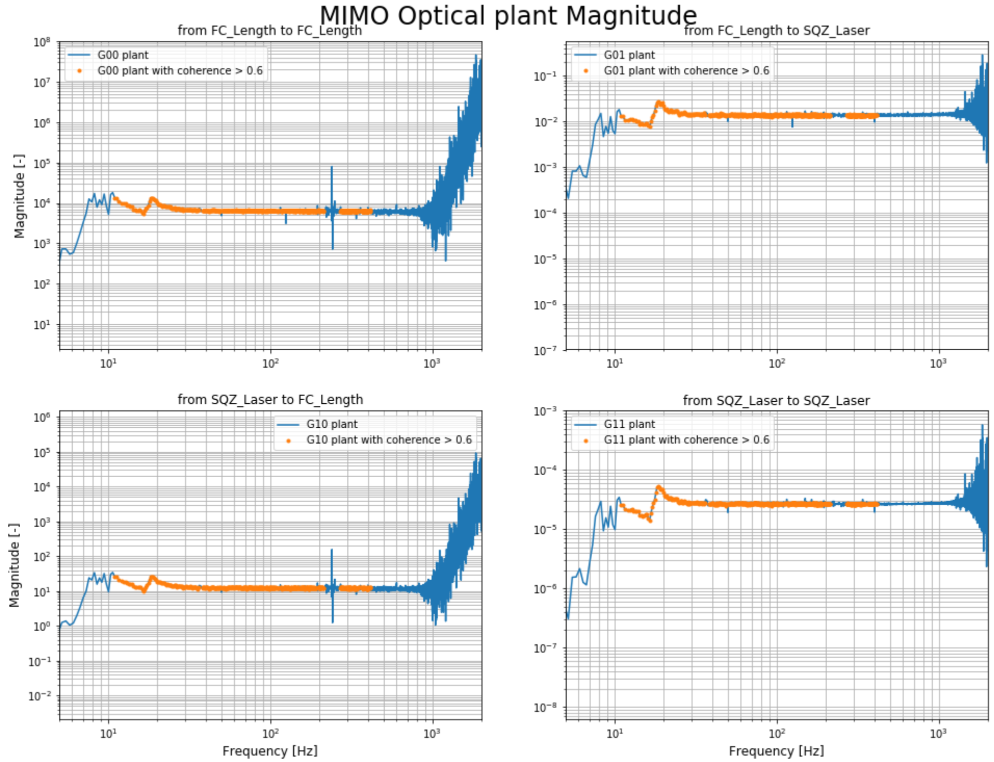


First, using a proper MIMO system identification approach we aim to understand the full TF matrix of the optical plant “G(s)”.

Secondly, since the system exhibits strong cross-coupling terms, one should use a MIMO system control strategy.

FC green control OLTF measurements:

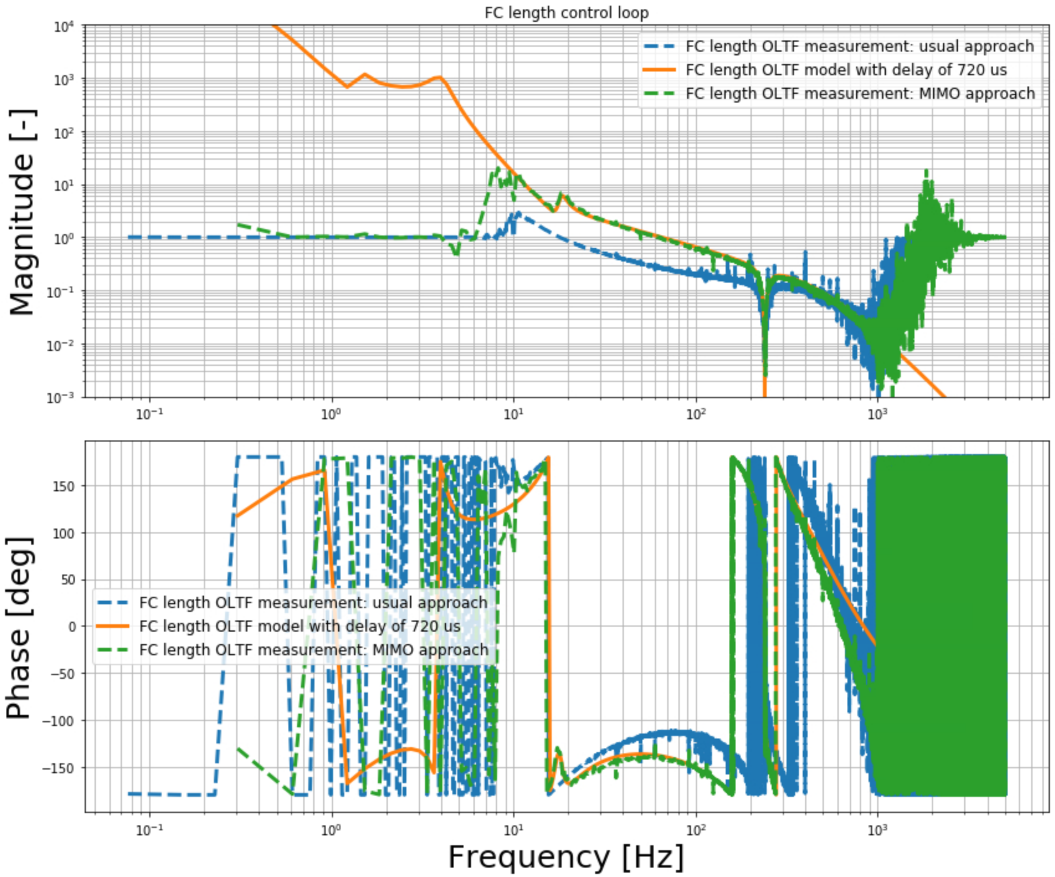


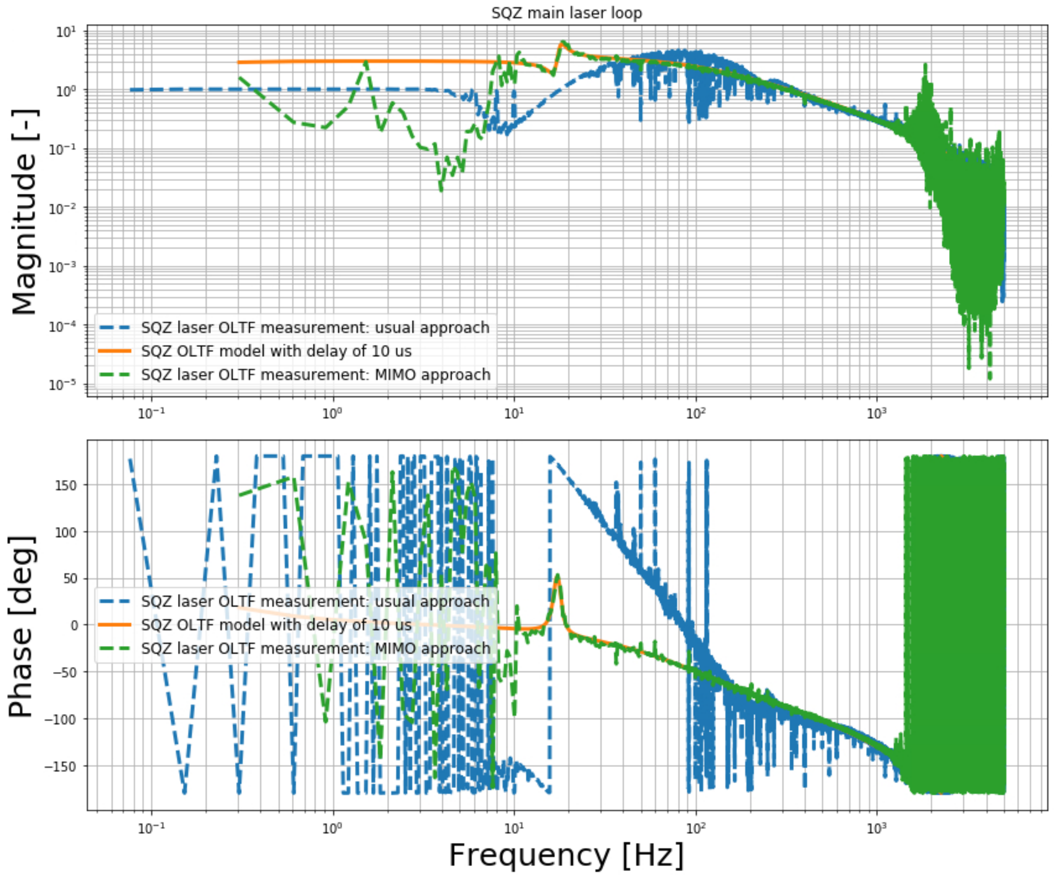


Measurement of the OLTF matrix of the FC system using a MIMO approach.

Optical plant “G(s)” reconstructed from the measurements.

FC green control model vs measurements:

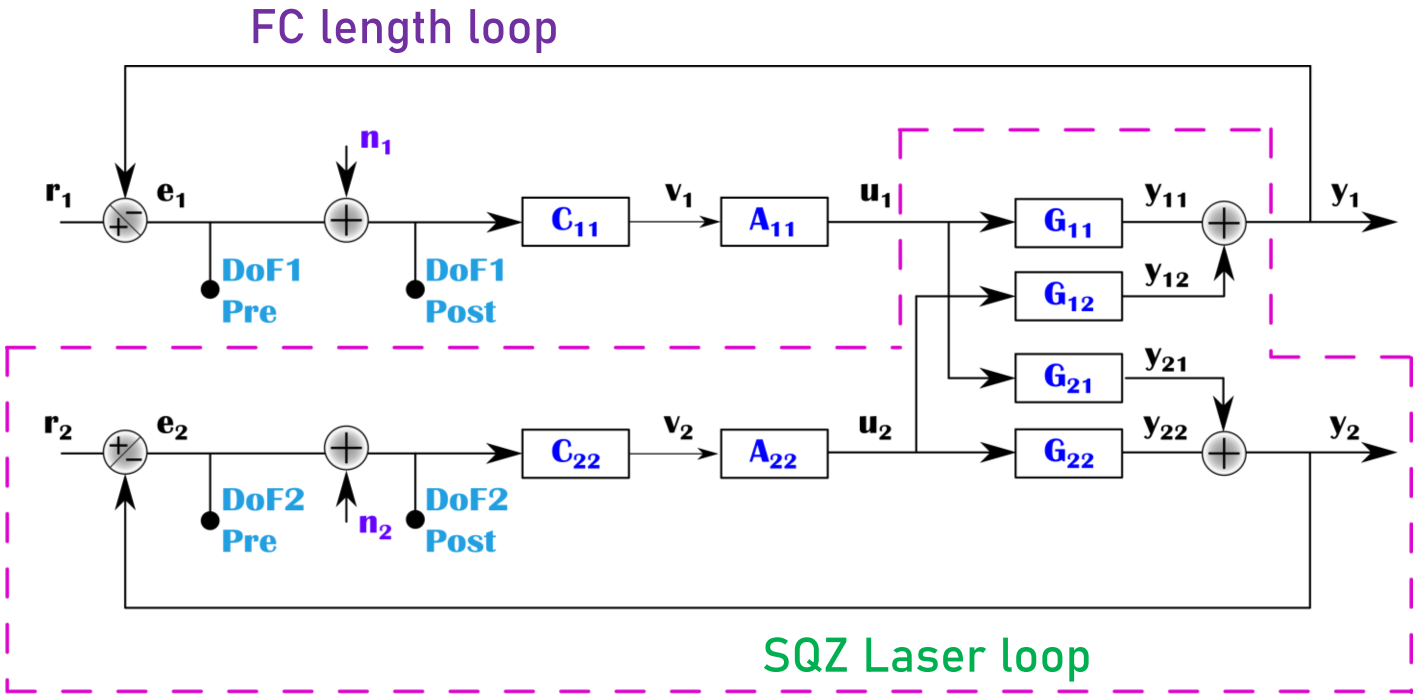




Orange plot: Analytical model of the system. Green plot: OLTF reconstructed using this MIMO approach.
Blue plot: OLTF obtained using the usual SISO approach.
Noticeable difference between the OLTF estimated using the two approaches, this can lead to misinterpretation of stability margins.

FC green control equivalent plant:

Why does the usual TF estimation differ from the model of the system?



Because with this method one can not decouple the effects of the two closed loops in the estimation of the OLTFs of the system.

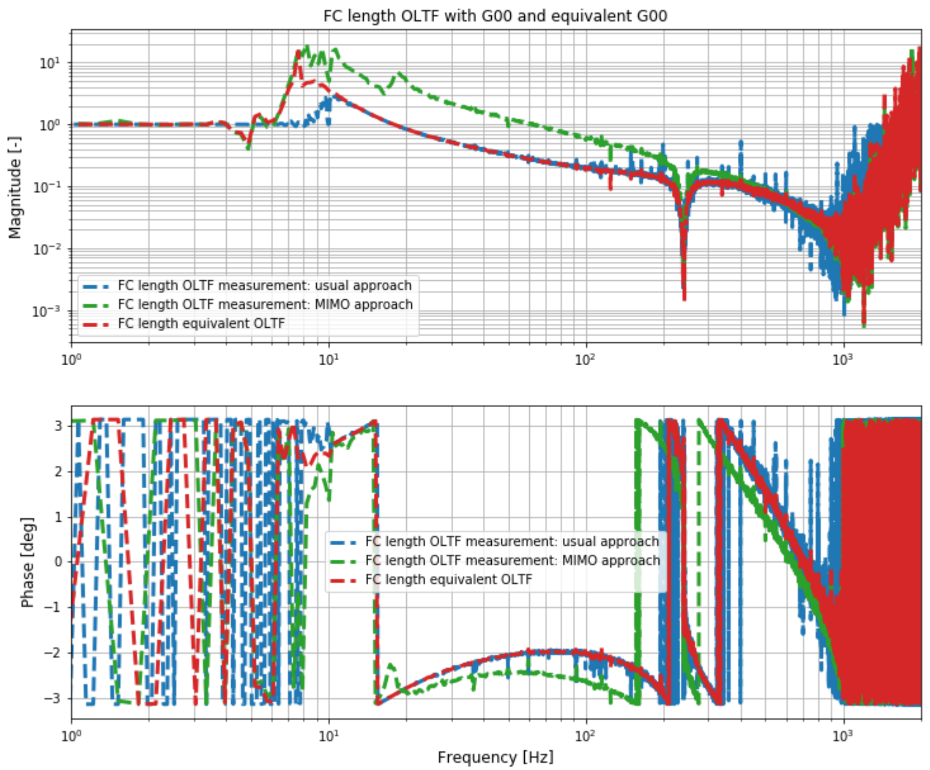
The estimated OLTF of one DoF is biased by the closed loop of the other DoF.

$$G_{11}^{eq}(s) = G_{11}(s) - \frac{G_{12}(s)C_{22}(s)G_{21}(s)}{1 + C_{22}(s)G_{22}(s)}$$

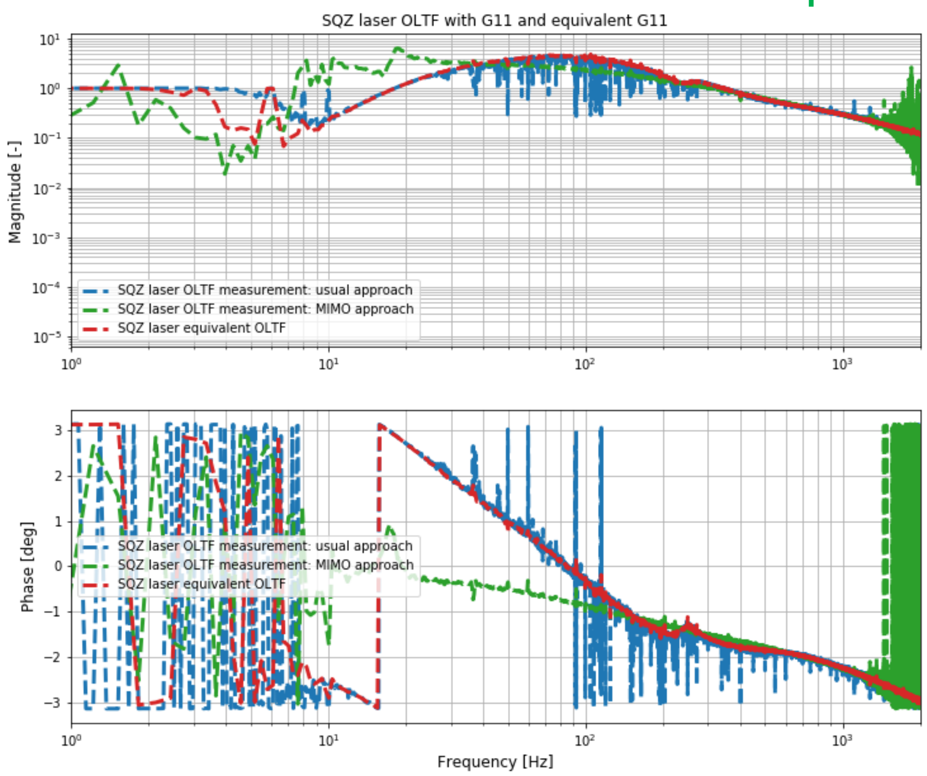
$$OLTF_{11}^{eq}(s) = G_{11}^{eq}(s)\, A_{11}(s)\, C_{11}(s)$$

FC green control equivalent plant:

FC length loop



SQZ Laser loop



Green plot: OLTF reconstructed using this MIMO approach.
Blue plot: OLTF obtained using the usual SISO approach.
Red plot: Equivalent OLTF.
Using the usual OLTF estimation approach one is taking into account the couplings among the loops.

FC green MIMO control strategy:

The system can be further optimized, however with the current implemented filters, the system suffers several unlocks, which is a clear indication of system instabilities.
There are several ways to tackle this problem. Here we give a short description of two proposals.

Sequential loop closing:

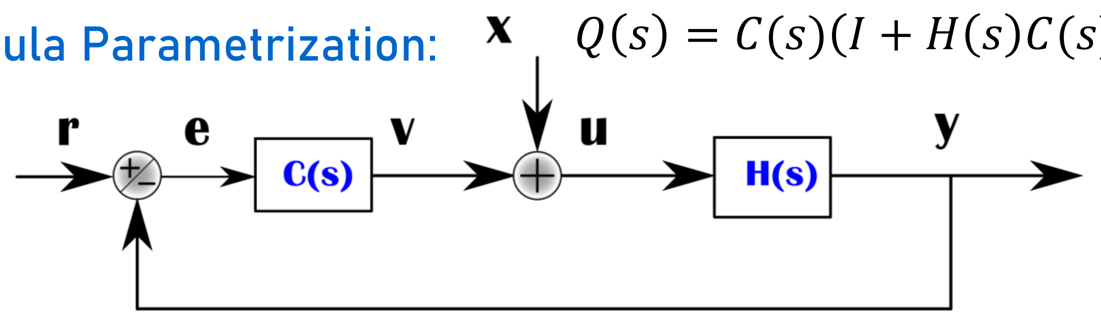
Close the fast loop, i.e. $C_{22}(s)$ (SQZ Laser loop). Then the equivalent plant seen from the FC length loop is:

$$H_{11}^{eq}(s) = H_{11}(s) - \frac{H_{12}(s)C_{22}(s)H_{21}(s)}{1 + C_{22}(s)G_{22}(s)}$$

$$OLTF_{11}^{eq}(s) = H_{11}^{eq}(s)\, C_{11}(s)$$

From here, design a controller $C_{11}(s)$ for the equivalent plant $H_{11}^{eq}(s)$, afterwards design a controller for the equivalent plant seen from the "other" loop i.e. $H_{22}^{eq}(s)$ when $C_{11}(s)$ is closed.

Youla Parametrization: $Q(s) = C(s)(I + H(s)C(s))^{-1}$



Study of internal stability of the system.

$$\begin{bmatrix} Y(s) \\ V(s) \end{bmatrix} = \begin{bmatrix} (I+HC)^{-1}HC & (I+HC)^{-1}H \\ (I+CH)^{-1}C & -(I+CH)^{-1}CH \end{bmatrix} \begin{bmatrix} R(s) \\ X(s) \end{bmatrix}$$

$$= \begin{bmatrix} T_O & S_O H \\ S_I C & -T_I \end{bmatrix} \begin{bmatrix} R(s) \\ X(s) \end{bmatrix}$$

Internal stability if T_O , $S_O H$, $S_I C$ and T_I are stable.

Summary

For a system with no strong cross couplings, it is possible to use SISO techniques to design its controllers. But it is still necessary to characterize cross-coupling terms if the goal is to implement partial decoupling i.e. MICH2DARM, SRCL2DARM among others.

For the Interferometer, decoupling filters are needed to achieve a cleaner DARM signal. Using this concept, a new noise budget tool is being prepared to consolidate the understanding of noises limiting the detector.

For a system with strong cross-coupling terms one not only need a well-chosen system identification tool, but also should revisit the control strategy. Depending on the goals and understanding of the system one can choose among different MIMO control techniques.

For the FC we have presented just a glance of two techniques that can help in the case of system instabilities.

Thank you for your attention!

References:

1. Control System Design, chapter 20. Goodwin, Graebe, Salgado. 2000.
2. On frequency response function identification for advanced motion control. E. Evers, R. Voorhoeve, T. Oomen.
3. Interferometer Sensing and Control for the Advanced Virgo Experiment I the O3 Scientific Run. ISC team.
4. Conceptual design of second stage of frequency stabilization for Advanced Virgo. VIR-0013C-12. E.Calloni, G. Vajente.