Mode matching sensing through RF Higher Order Modulation method

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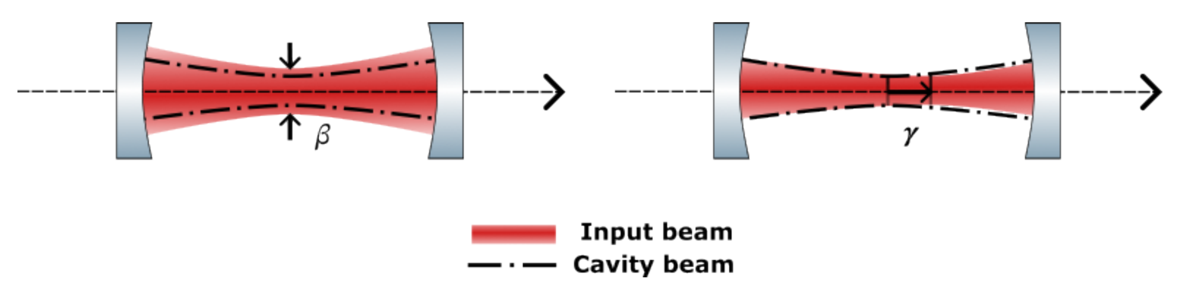
The mode-mismatch problem and the RF-Higher Order Modulation method

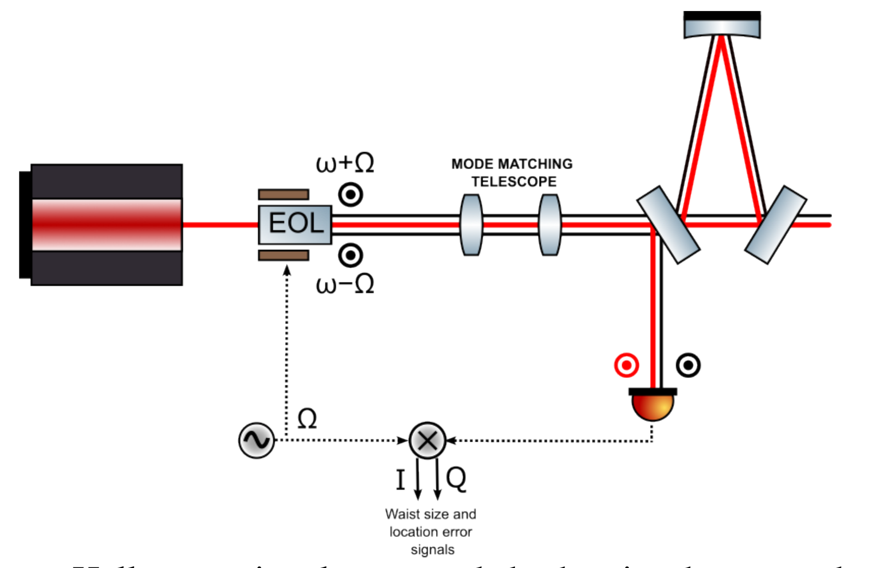
Mode Mismatch (MM) occurs when there is a difference in either the waist size or position between the fundamental mode of the input beam and the one supported by a cavity. In modern and future GW detectors, mode mismatch is a dangerous source of degradation for the vacuum squeezing injected in the interferometer to reduce quantum noise.
For moderate mismatch values, we can represent the beam in the cavity base, as a combination of the fundamental Gaussian mode and the LG_{10} mode. By examining the complex coefficient on the LG_{10} mode, we can determine the nature of the mismatch: the real part indicates a size mismatch, while the complex part indicates a position mismatch. If the cavity is locked to the fundamental mode, the LG_{10} mode will be reflected: we can use this reflection to gather information regarding the type of mismatch.

$$LG_{00}(w_0+\delta w, z_0+\delta z) = LG^C_{00}(w_0, z_0) - \left(\frac{\delta w}{w_0} - i\frac{\delta z}{2z_R}\right) LG^C_{10}(w_0, z_0) \qquad MM = \frac{LG_{10}}{LG_{00}} = \gamma^2+\beta^2$$

$\rightarrow \beta + i\gamma$



We propose a technique consisting in using a modulated lensing element (Electro-optical lens, EOL) to insert sidebands in the LG_{10} mode. The modulation frequency needs to be equal to twice the higher-order modes (HOM) spacing frequency of the cavity: in this way when the cavity is locked to the fundamental mode only one sideband will be reflected alongside the LG_{10} carrier, while the other will resonate within the cavity. This breaks the symmetry between sidebands, and coverts the phase modulation obtained by them beating with the carrier LG10 into amplitude modulation, that can be readily sensed.



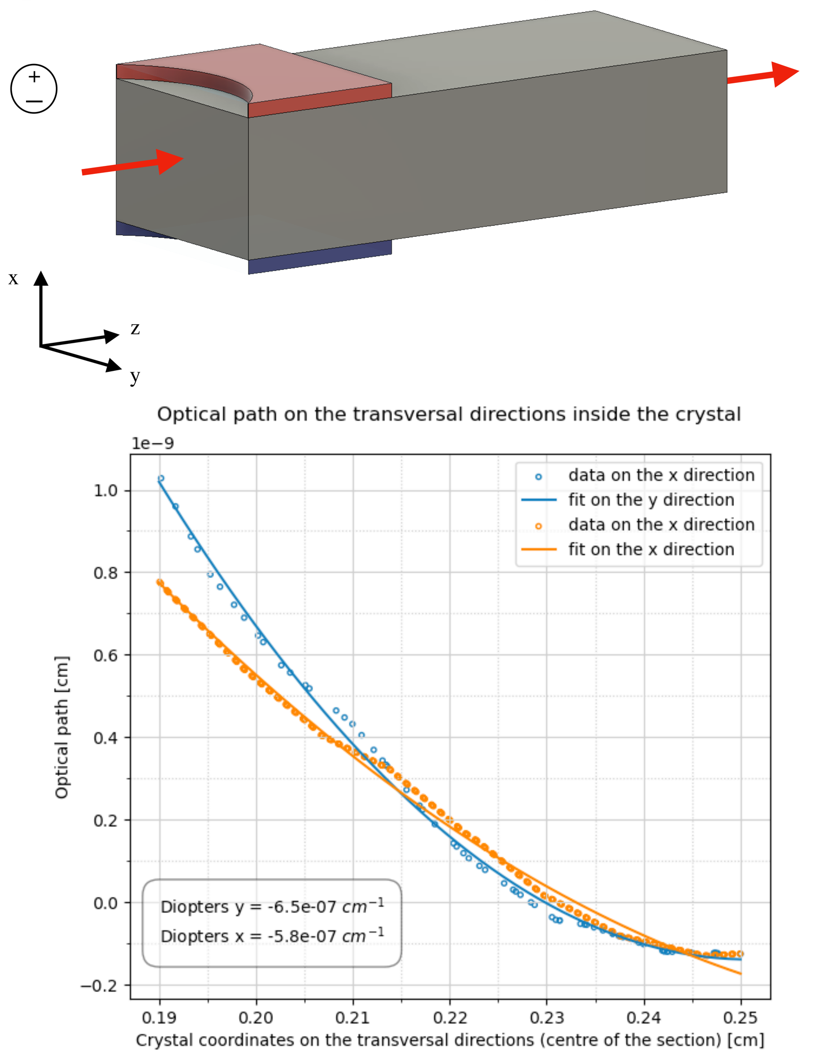
We can then use the same photodiode of the Pound-Drever-Hall error signal to record the beating between the LG10 carrier and the LG10 reflected sideband. Demodulating this signal with I/Q demodulation allow us to obtain two signals containing γ and β:

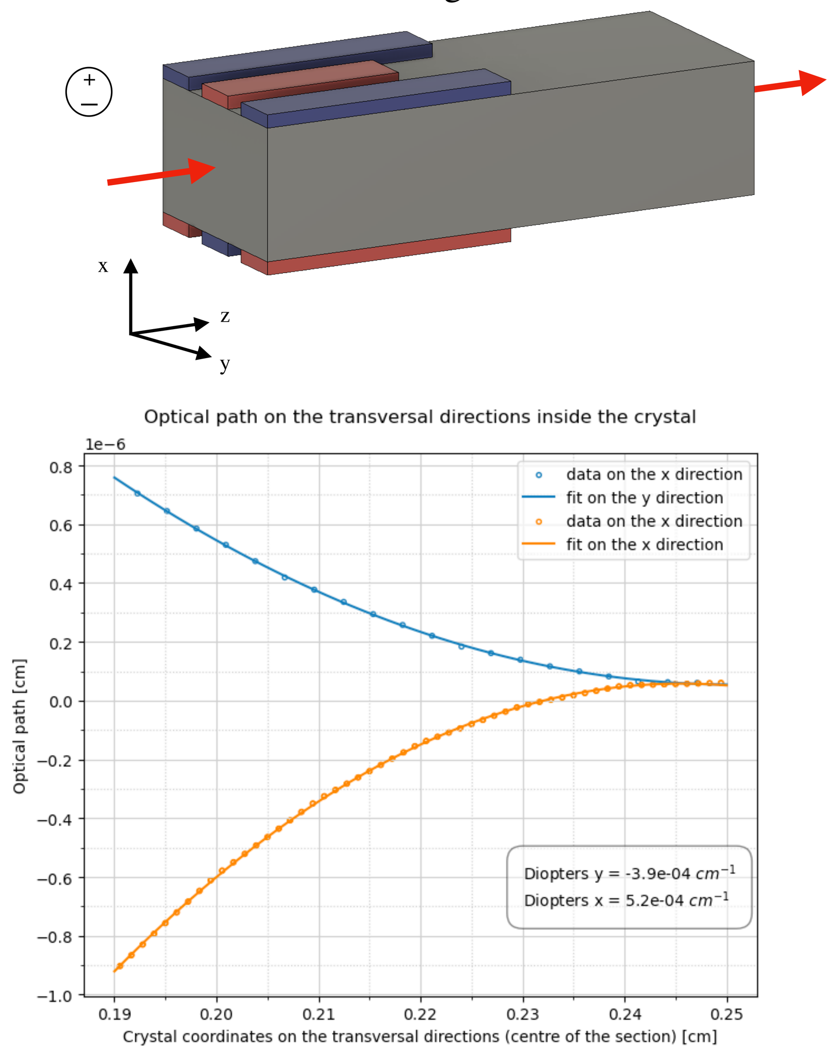
$$I = \gamma m_\gamma + \beta m_\beta \qquad Q = \beta m_\gamma - \gamma m_\beta$$

Where m_β and m_γ are the amplitude modulation of of waist size and position (whose relative magnitude depends on the position of the EOL along the input beam).

Electro-optical lens project

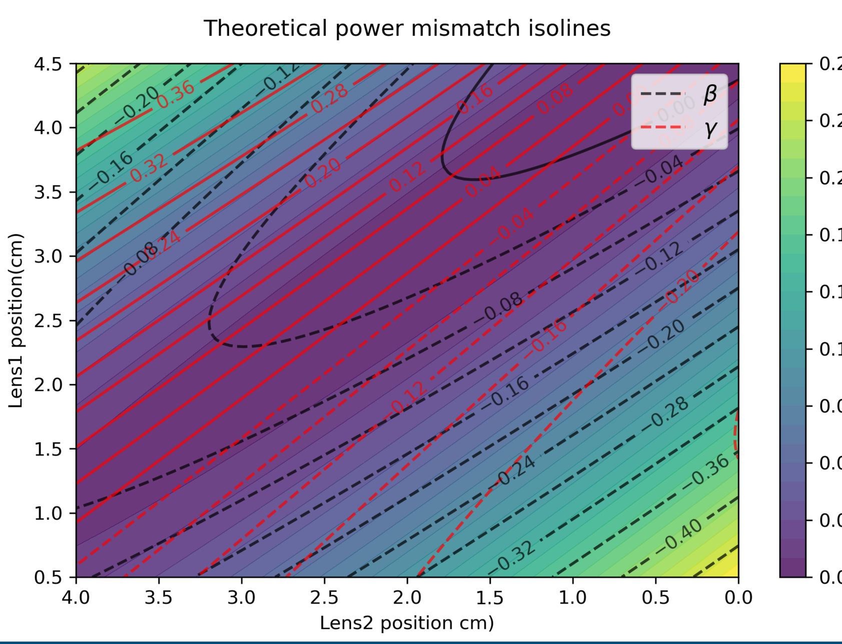
The Electro-optical lens (EOL) comprises a lithium niobate crystal ($x = 5$mm, $y = 8$mm, $z = 10$mm) with electrodes applied on top of it in order to use the Pockels effect. The electrodes' design was developed to create a different change of the refractive index across of the crystal we manage to obtain a lensing effect.
We designed two EOLs with COMSOL simulations: the one we call EOLpd, on the left, acts as a spherical lens but with a focal length of $\simeq 10^6 cm$ for a voltage on the electrodes of $0-400V$; the one we call EOLfl (its design has been proposed by the LIGO group at the University of Florida) acts on the two transversal planes as two different lenses with inverse focal length of $\simeq \pm 10^3 cm$ (for the same voltage) that create a phase shift of π between the two planes. Therefore, in this, cylindrical telescope is used to regain a spherical deformation, adding a π phase to one of the axis. Eventually, we decided to work with the EOLfl due to its stronger effect.

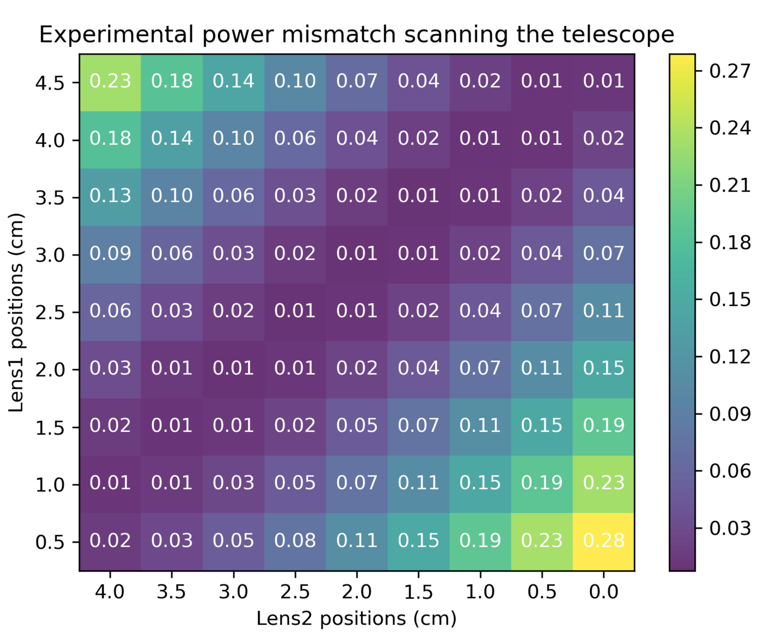




Mode-matching telescope

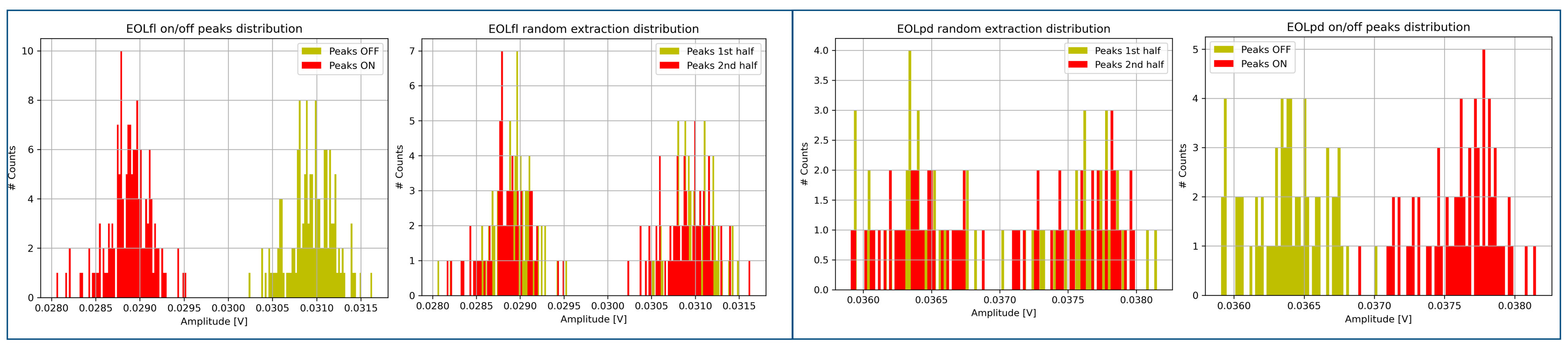
In our setup, a two-lens telescope is used to mode (mis)match the input beam to a reference cavity. In order to test the technique and explore the output signals in relation to the values of gamma and beta defining the mismatch, it is important to simulate and verify the type of mismatch induced by different position of the lenses.
The plot on the left shows the calculated overall mismatch (color gradient) and the β (black) and γ(red) isolines (dashed is <0, solid >0) for different position of the lenses. The plot on the right shows the mismatch at different lenses position measured by scanning the cavity, and shows a good agreement with the calculation.





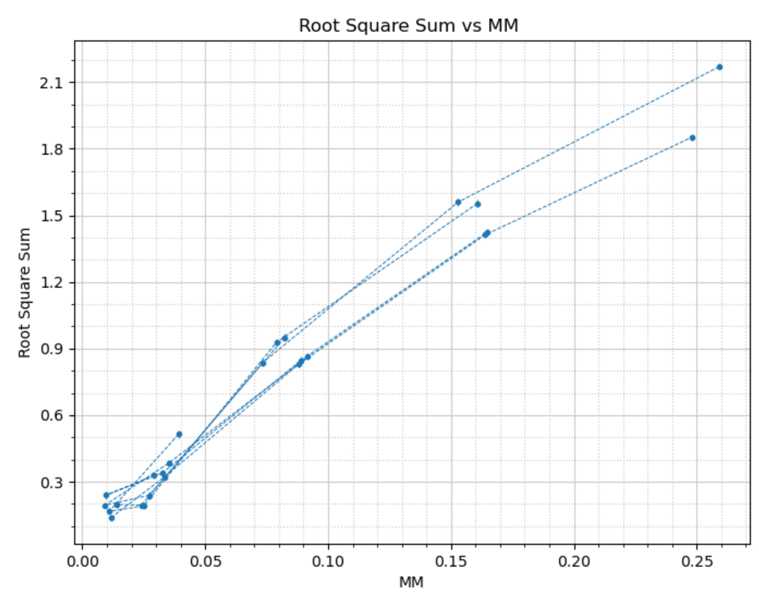
Results

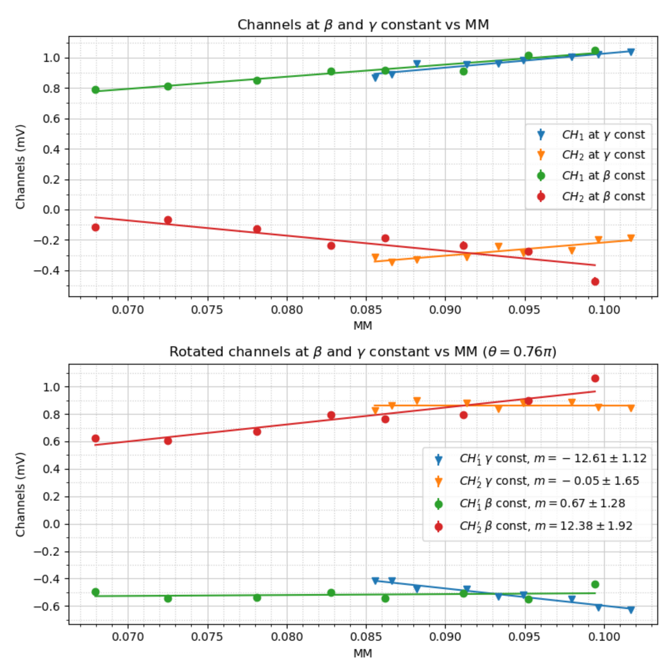
Since we expect the effect to be small, we performed a slow lock-in measurement. We sent a 0-400V, 1Hz square wave to the EOL electrodes and monitored the modulation of the LG10 peak measured by scanning the cavity continuously at an higher rate. Below are represented the plot (EOLfl on the left and EOLpd on the right) which show a clear difference in the two populations of peaks measured when the applied voltage was on or off, as well as a control test in which we see not difference if we select the peaks at random. Moreover we can see the stronger effect of EOLfl with respect to EOLpd.



After proving that the EOL does modulate the amplitude of the LG_{10} mode, we focused on the RF measurements. We sent to the EOL a sine wave at frequency 2HOMs and, with the cavity locked on the fundamental mode, we demodulated the two quadratures, that we call CH_1 and CH_2. Since the demodulation angle and gain are arbitrary, CH_1/CH_2 are related to I/Q defined above again by a rotation and rescaling. This means that also the overall transformation between CH_1/CH_2 and β/γ is of the same type.
The plot on the left shows the proportionality of the mismatch in power measured by the sum of the square of CH_1 and CH_2.
To identify the rotation angle, we used the mode-matching telescope to make measurements alongside the isolines of β and γ, and then applied a rotation to make CH_1 and CH_2 sensitive to only gamma or beta, respectively. The plots on the right shows CH_1 and CH_2 before (top) and after (bottom) the rotation: the signals obtained in this way allow us not only to to measure the amount of mismatch but also its origin (beam waist size or position).

$$I^2 + Q^2 = (m_\beta^2 + m_\gamma^2)(\gamma^2+\beta^2) \propto MM$$





Future work

In order to improve the sensitivity of the technique and be able to measure at lower values of mismatch, a number of improvement are planned:

- Improve the electronics to reduce noise and pickups, and reduce or better characterise offsets.
- Use a Shack-Hartmann sensor to measure directly the EOL effect on the beam, verify the purity of the LG10 modulation and optimize the design if necessary
- Installation of a mode-cleaner
- A longer term development could be the installation of a second test cavity downstream of the first, to demonstrate that a second cavity mismatch can be measured exploiting the resonant sideband transmitted by the first cavity, without adding any hardware except for the demodulation electronics.

Dana Z. Anderson, "Alignment of resonant optical cavities", Apple.Opt. 23, 2944-2949 (1984)
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