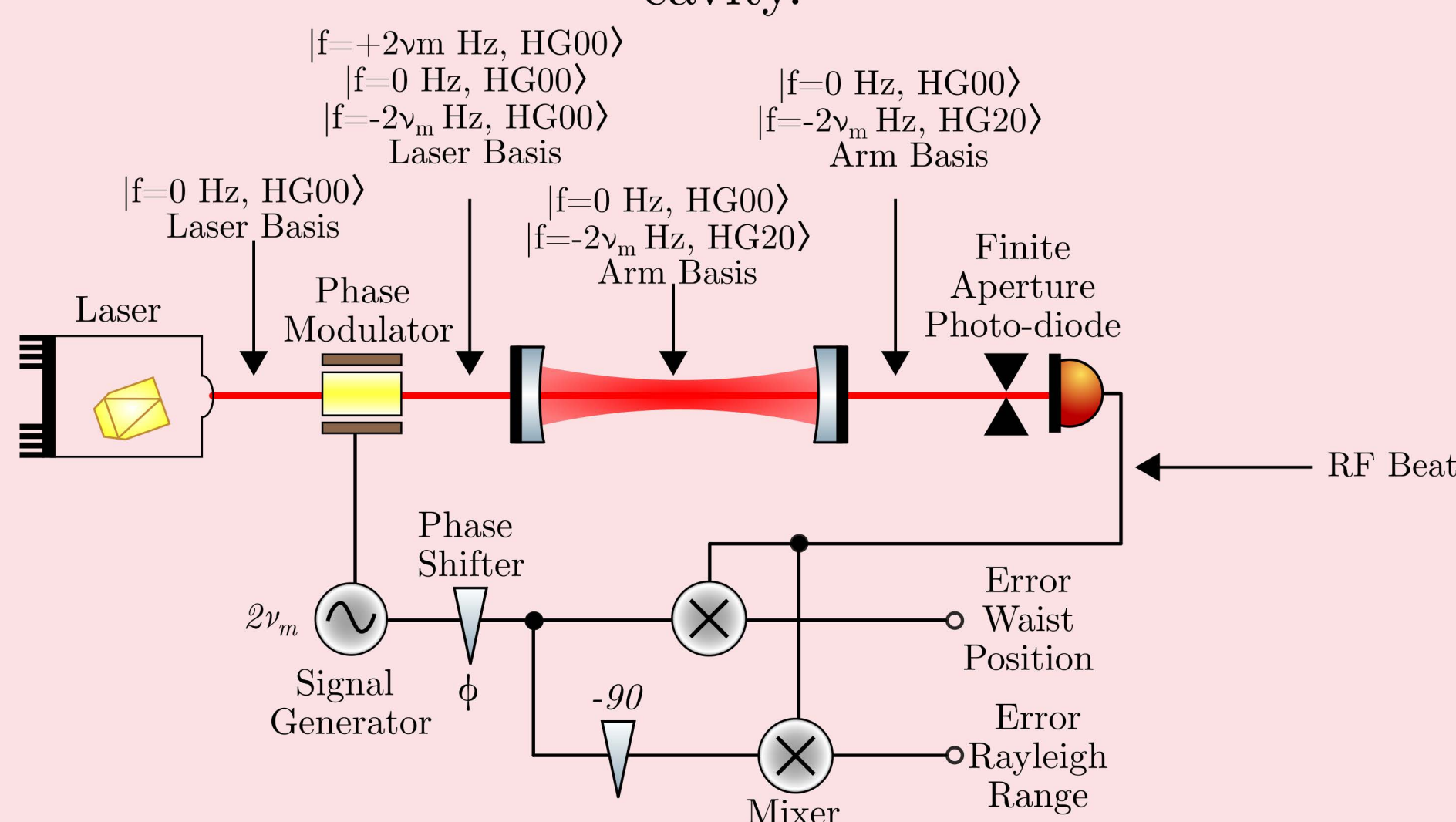


As shown by Töyrä, McCuller [1,2] and others, optical mode mismatches can have a severe limiting effect in quantum enhanced gravitational wave detectors. Firstly, mode-mismatches cause losses, degrading the vacuum state. Secondly, mode mismatches can mix quantum states, shaping the instruments frequency response.

We propose a scheme, that directly interrogates the mismatch between the recycling cavities and the arms. By adding appropriate modulation sidebands, we make the second order modes resonate in the coupled cavity. The sensing is achieved on transmission, by mixing the frequency offset higher order modes with the carrier.

## Example - Simple Cavity Case

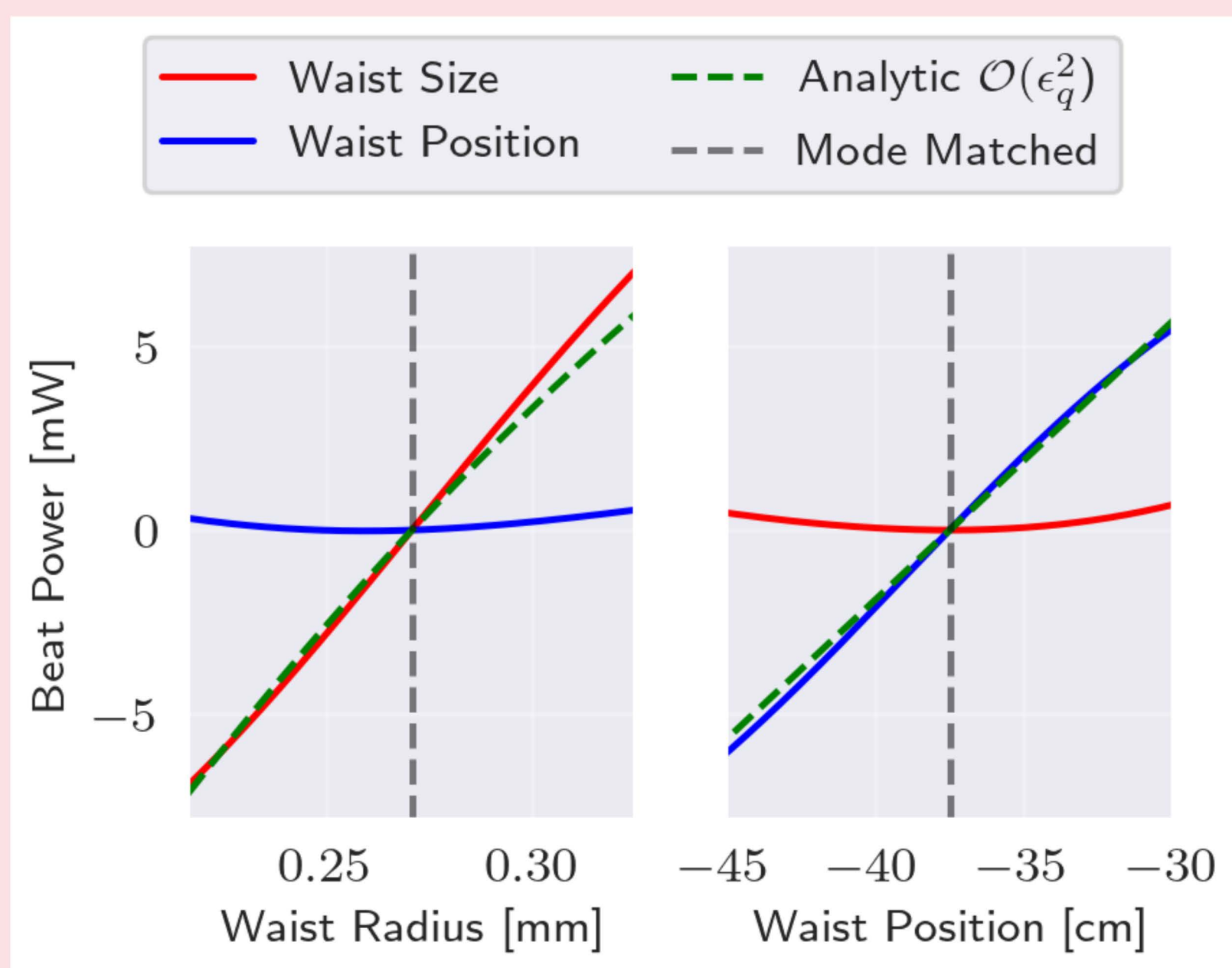
For a simple cavity, the proposed scheme is similar to Anderson's proposal [3]. A sideband exists at twice the mode separation frequency. Ordinarily this is reflected. When a mode mismatch is present, light is scattered into the second order mode (e.g. the HG20). The HG20 then resonates in the cavity.



The cavity filters out unwanted modulations. On transmission there are two modes: the HG20 and the carrier. The amplitude and phase of the HG20 are dependant on the nature of the mismatch.

These two modes beat together on a finite aperture photo-detector. A lock-in amplifier can then be used to demodulate in both quadratures.

By setting the global demodulation phase, the two quadratures will produce voltages that are directly proportional to the waist position and Rayleigh range mismatch. The paper contains a formula for the correct phase.



The plot above shows simulated error signals, for a simple cavity. The simulation is in Finesse. The non-linearity occurs for three reasons. Firstly, at high mismatch, the accumulated Gouy phase changes, thus changing the demodulation phase. As a result, cross coupling occurs. Secondly, we approximate the power scattered into HG20 is linear with mode mismatch. Thirdly, leakage from the finite cavity finesse sets a lower bound on the detectable mismatch.

## Limitations and Further Work

This approach has several limitations. Firstly the mode separation frequency needs to be known ahead of time. One approach trialled at LIGO Livingston (see LLO aLOG 61000) was to scan the field back and forth, until the resonance is found. Additionally, there is a limit to the detectable mismatch by non-resonant modes passing through the coupled cavity. This requires a full design to estimate. Lastly, when this was trialled with the Audio Diagnostic Field at LLO, range was severely degraded. We are working to understand the mechanism.

Co-author HZ is working on an experiment at UWA to verify the simple cavity. Co-author AG-J is working on a coupled cavity verification, see poster #44 (DCC G230123).

## References

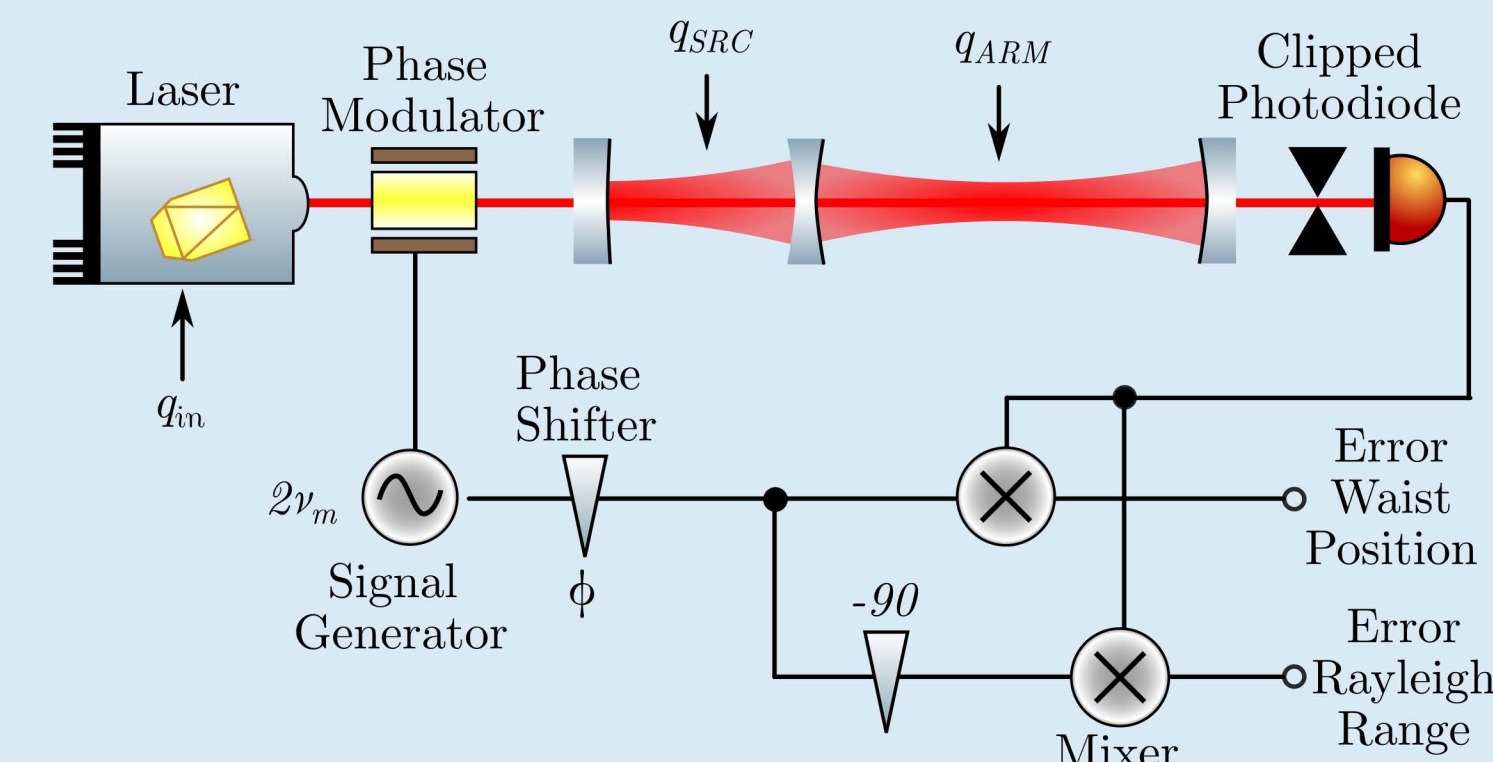
- [1] Töyrä, D., et al Multi-spatial-mode effects in squeezed-light-enhanced interferometric gravitational wave detectors, Phys. Rev D, 96, no. 2 (2017).
- [2] McCuller et al, LIGO's quantum response to squeezed states, Phys. Rev. D 104, 062006 (2021).
- [3] D. Z. Anderson, Alignment of resonant optical cavities, Appl. Opt. 23, 2944 (1984).
- [4] A. A. Ciobanu, et. al., Mode matching error signals using radio-frequency beam shape modulation, Applied Optics, 59, 9884 (2020)
- [6] S. Rowlinson, Feasibility study of beam-expanding telescopes in the interferometer arms for the Einstein Telescope, Physical Review D 103, 02300 (2021)

## Example - Recycling Cavity Case

The core interferometer may be approximated as a single coupled cavity.

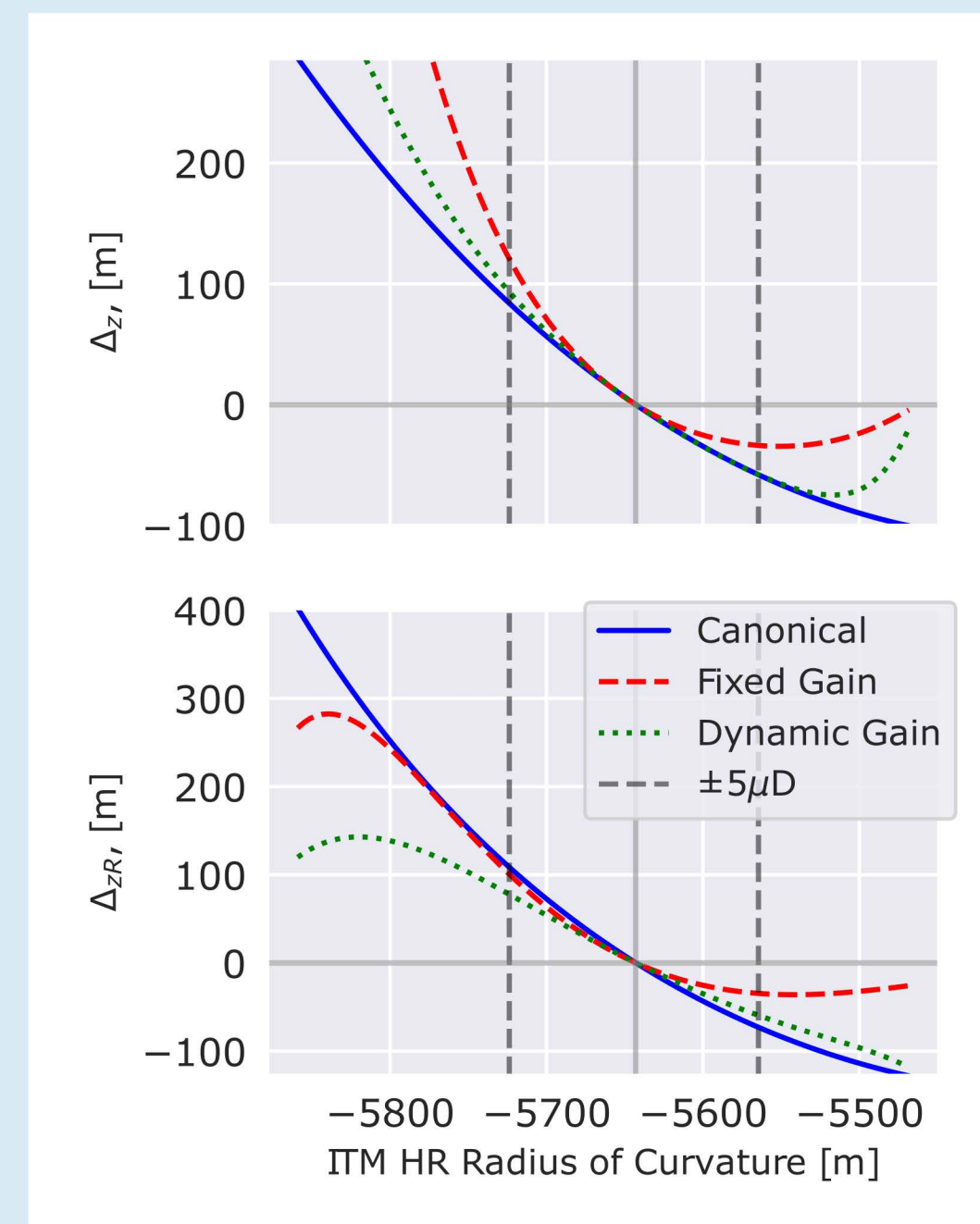
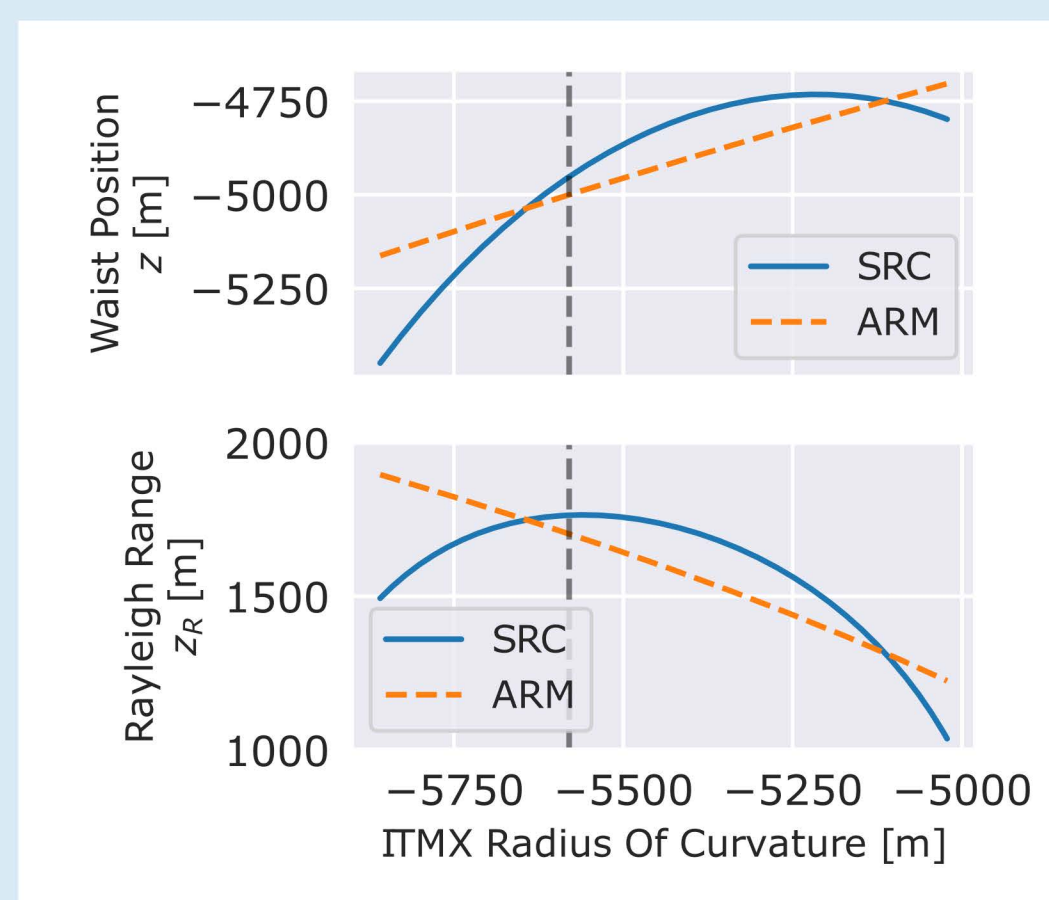
The assumption is made that the recycling cavities and input/output optics are mode matched. This could be achieved by some scheme with a unity gain frequency much higher than this loop. One option is to use the Simple Cavity scheme on the left panel.

The assumption is also made that the recycling cavity is much shorter and also of lower finesse than the arm.



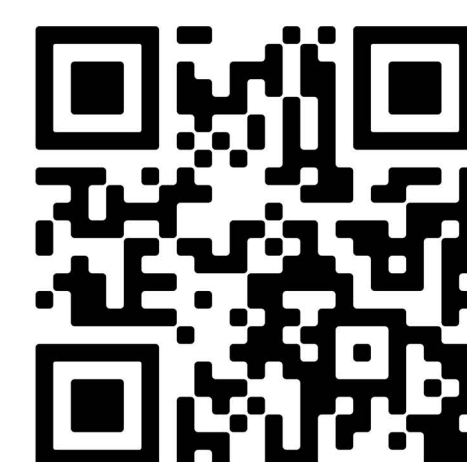
We drive the modulator at twice the arm mode separation frequency. Our assumptions mean no HG20 is excited at the input/recycling cavity boundary, therefore the sideband is not resonant in the recycling cavity.

Some sideband light will enter the recycling cavity. At the ITM, there exists some mismatch. This scatters light into the HG20 mode, which resonates in the arm. The detection is as in the simple cavity.



In the simple cavity, it is possible to generate a pure waist size or waist position mismatch. In a coupled cavity both eigenmodes are a function of ITM radius of curvature. The above left plot, shows the waist position and waist size mismatch for a ET-LF nominal design [6].

The top right plot shows the canonical mismatch inferred from known mirror curvatures alongside the error signals obtained in simulation. ITM thermal lensing is generally of the microdiopter scale, within this region the scheme produces reasonable error signals.



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