A study of suspensions with flexures in compression for cryogenic mirrors

Ezra Zerina Appavuravther[1], Francesca Badaracco[2], Alessandro Bertolini[3], Christian Chavez[3], Andy Damas[3], Richard Damas[3], Riccardo DeSalvo[4], Eric Hennes[3], Britney Gallego[3], Joris van Heijningen[5], Seth Linker[3], Ettore Majorana[6], Marina Mondin[3], Luca Naticchioni[7], Kevin Pang[3], Stefano Selleri[8], Harry Themann[3], Flavio Travasso[1], Morgane Zeoli[3,9]

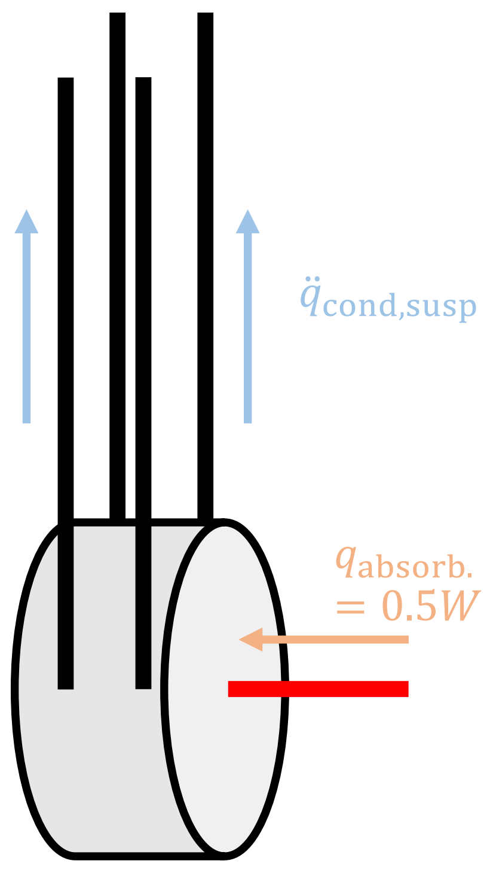
Università di Camerino[1], INFN Sezione di Genova[2], Nikhef[3], California State University[3], Riclab LLC[4], Université Catholique de Louvain-la-Neuve[5], Istituto Nazionale di Fisica Nucleare[6], INFN Roma[7], Università di Firenze[8], Université de Liège[9]

E-mail: morgane.zeoli@uclouvain.be

Abstract

The Einstein Telescope will operate with cryogenic mirrors at temperatures as low as 4K [1]. This temperature needs to be maintained by extracting heat coming from absorption in the substrate and coating. We present a study of an alternative to tensile suspensions of the Einstein Telescope mirrors. The mirror suspensions presented here tackle the conflicting requirements of being good heat conductors while remaining soft to preserve low thermal noise vibration isolation. Additionally, using small-diameter silicon flexures in compression deals with the low tensile strength of crystalline silicon [2].

The design consists of large cross-section suspension beams, connected to a marionette and a mirror, by thin, small flexures. The architecture is organized such that the flexure sustains compressive load only. The bending strain due to mirror motion is concentrated in this flexible part, and because of the lower rigidity of these thinner flexures, we expect a low suspension thermal noise. Here, we show the trade-offs induced by this suspension design, focusing on the flexure design, and how it affects the frequency and thermal response of the system.



Objective

Suspend effectively ET’s cryogenic mirrors while:
- minimizing thermal noise in the suspensions;
- extracting the heat deposited by the stored light beam;
- isolating the test mass from the suspension thermal noise of the warmer chain above it
- isolating the test mass from the cryocooler mechanical noise;
- providing control forces necessary to acquire and maintain the Fabry-Perot optical lock.

Conflicting requirements

Test mass at cryogenic temperature to decrease the thermal noise that limits ET low frequency bandwidth

Cooling power $P = k\frac{A}{L}\Delta T$

Short, fat rod
favor conduction to extract heat from mirror

Vibration isolation requires low resonance frequency suspensions

Long, soft rod
Flexibility to decrease the suspension resonance freq.

+ Avoid thermal noise in the suspensions!

Strategy: divide and conquer

- The suspensions are made of **crystalline silicon**
  - ✓ Large Q-factor
  - ✓ Good heat-conductor
  - ✓ $\sigma_{\text{compression}} > \sigma_{\text{tension}}$ [2][3]

The suspension is composed of thick suspension beams that sit on flexures located inside their cross-section, on the marionette, and under the mirror. Since crystalline silicon is stronger in compression, the flexures can be made thin. Note that R&D is actively conducted on tensile strength of similar rod cross-sections while only the bulk compressive limit of silicon is known for now.

- **Suspension beams** with a large cross-section act as **heat links for heat extraction**
  - ✓ Thermal conduction
  - ✓ Reduce tensional stress
- **Flexures** in compression provide **a high-Q compliance**
  - ✓ Thin → soft, good for thermal noise
  - ✓ Thermal conduction
- Counterweights are included in the design to nullify the center of percussion effect.
- The beams are suspended by silicon cantilever springs that provide vertical compliance

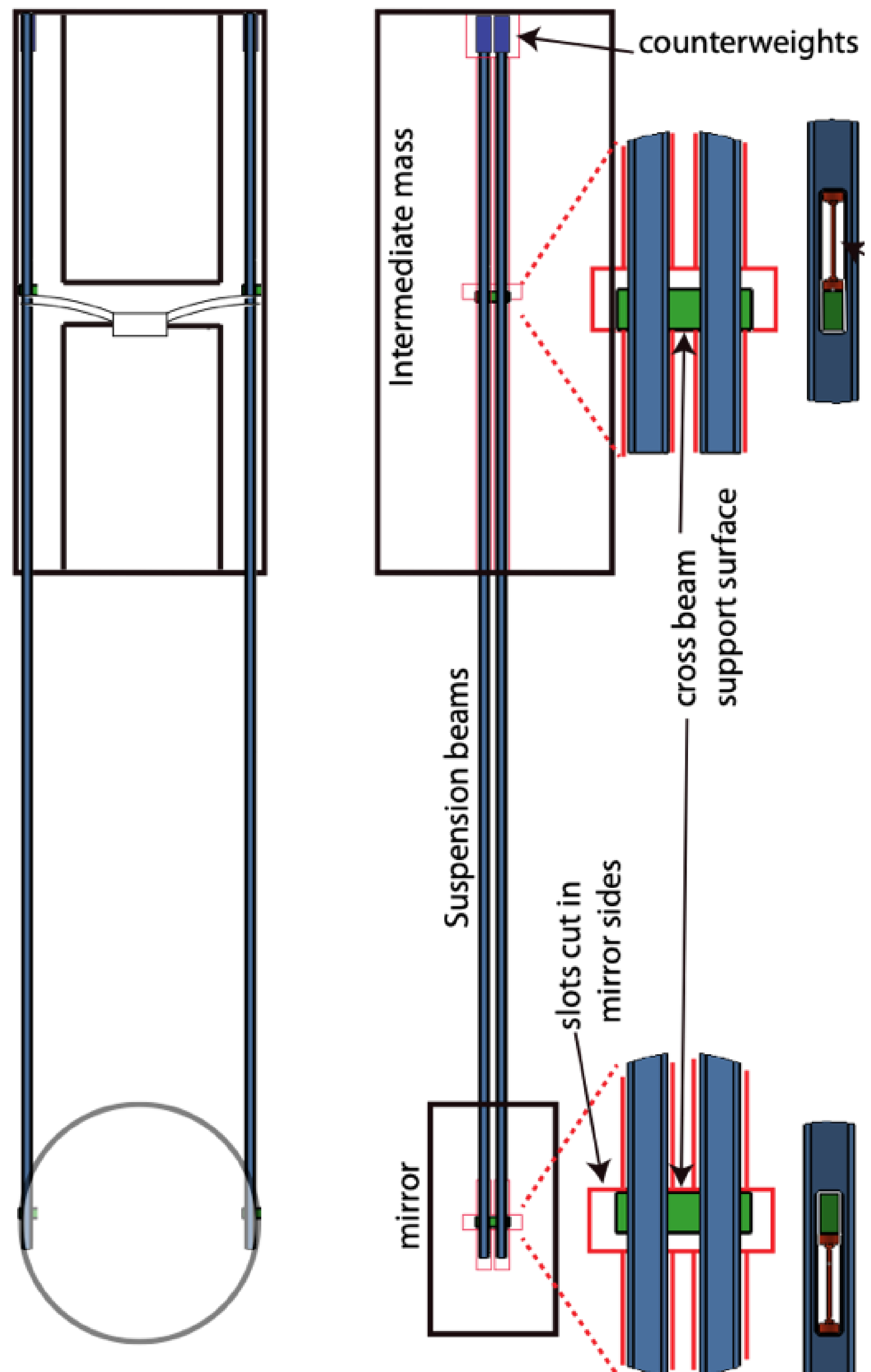


Figure 1: Scheme made by CSULA team of the suspension solution

References

[1] ET design report update 2020 ("long ESFRI document"), ET-0007A-20 , 2020

[2] A. V. Cumming, *et al.*, Silicon mirror suspensions for gravitational wave detectors, Class. Quantum Grav. 31 (2014) 025017 (16pp)

[3] Villa, F.F. (2022). Silicon Properties and Crystal Growth. In: Vigna, B., Ferrari, P., Villa, F.F., Lasalandra, E., Zerbini, S. (eds) Silicon Sensors and Actuators. Springer, Cham.

[4] Y. Levin (1998), Internal thermal noise in the LIGO test masses: A direct approach, Phys. Rev. D 57, 659

Thermal analysis

The suspension chain needs 14 days to reach thermal equilibrium and the expected temperature drop is 1.8 K.

Figure 4: Temperature distribution of isotropic silicon flexure after 8h. The flexure is the limiting part for heat transferring: the larger r²:l, the better

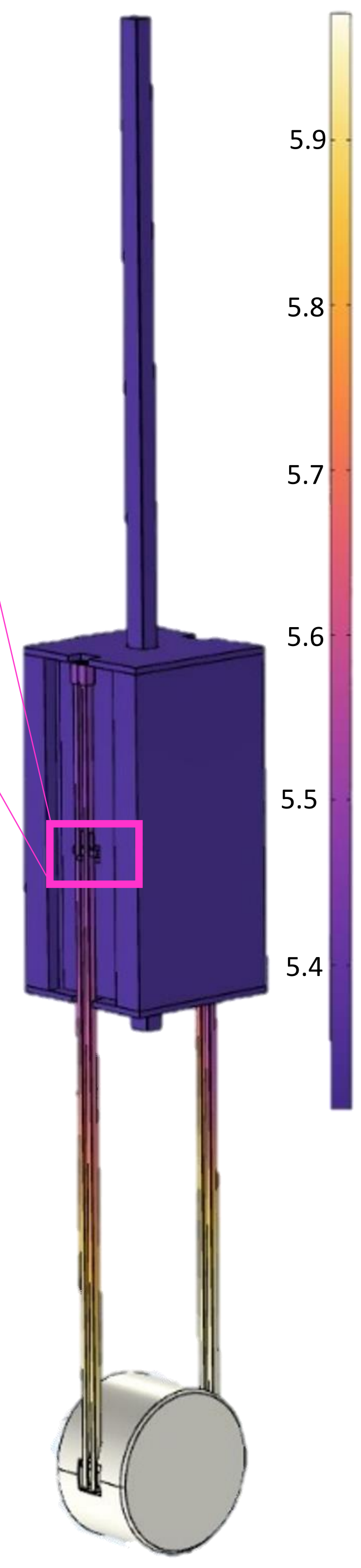


Figure 5: Temperature distribution of the suspension chain after 14 days.

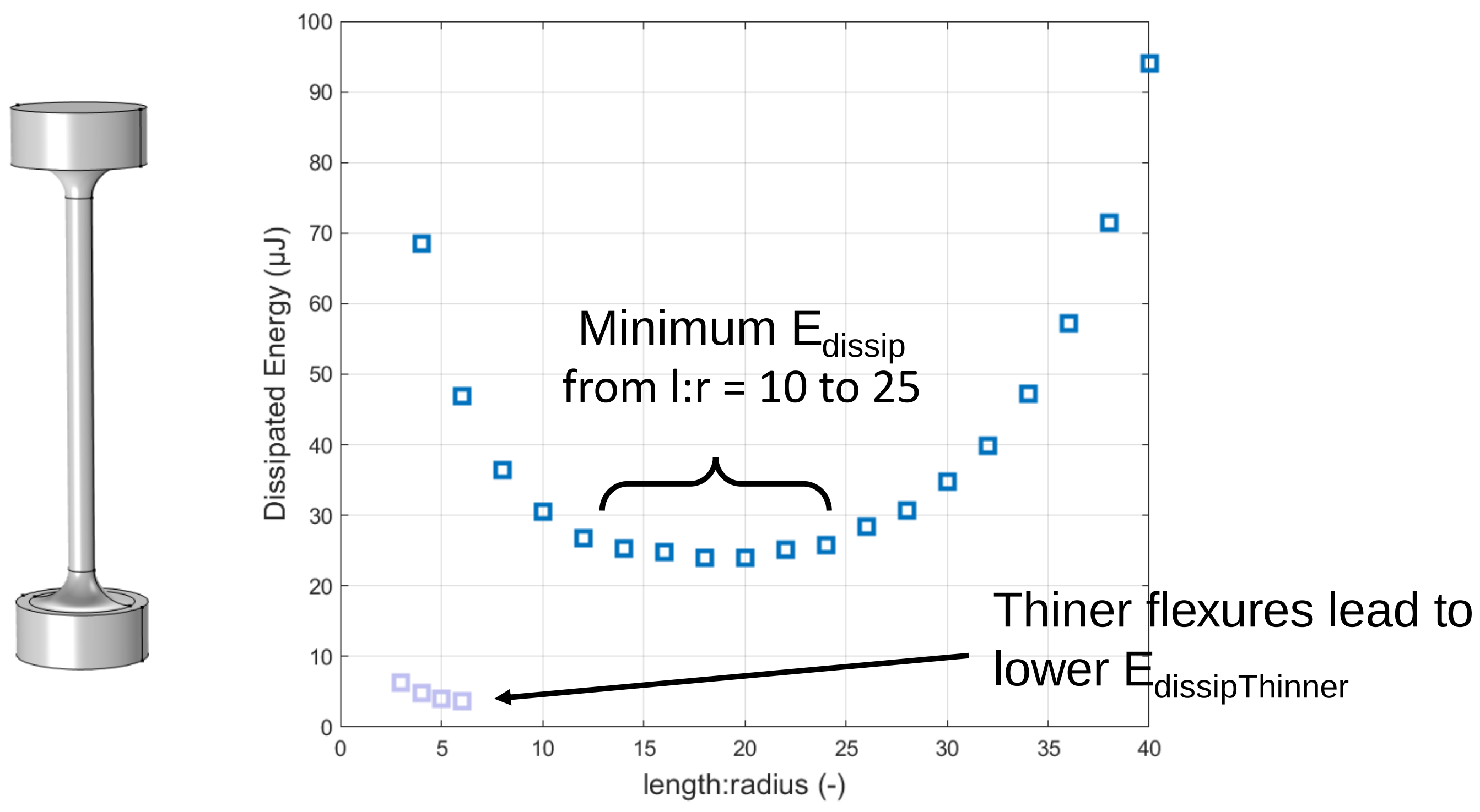
Flexure design: optimal length:radius ratio

The optimal flexure design is based on the minimum radius needed to sustain the compressive mirror load while remaining within the compressive limit. The maximum length is limited by buckling. This optimal flexure is compared to a thicker one (1 mm radius).

Thermal noise

According to Levin’s theorem [4], the thermal noise linearly depends on the power dissipated, and thus on the dissipated energy, when the system is subject to forces oscillation.

$$S_q(f) = \frac{k_B T}{\pi^2 f^2}\,\frac{2\,W_{\text{dissip}}}{F_{\text{load}}^2}$$



Minimum E_{dissip} from l:r = 10 to 25

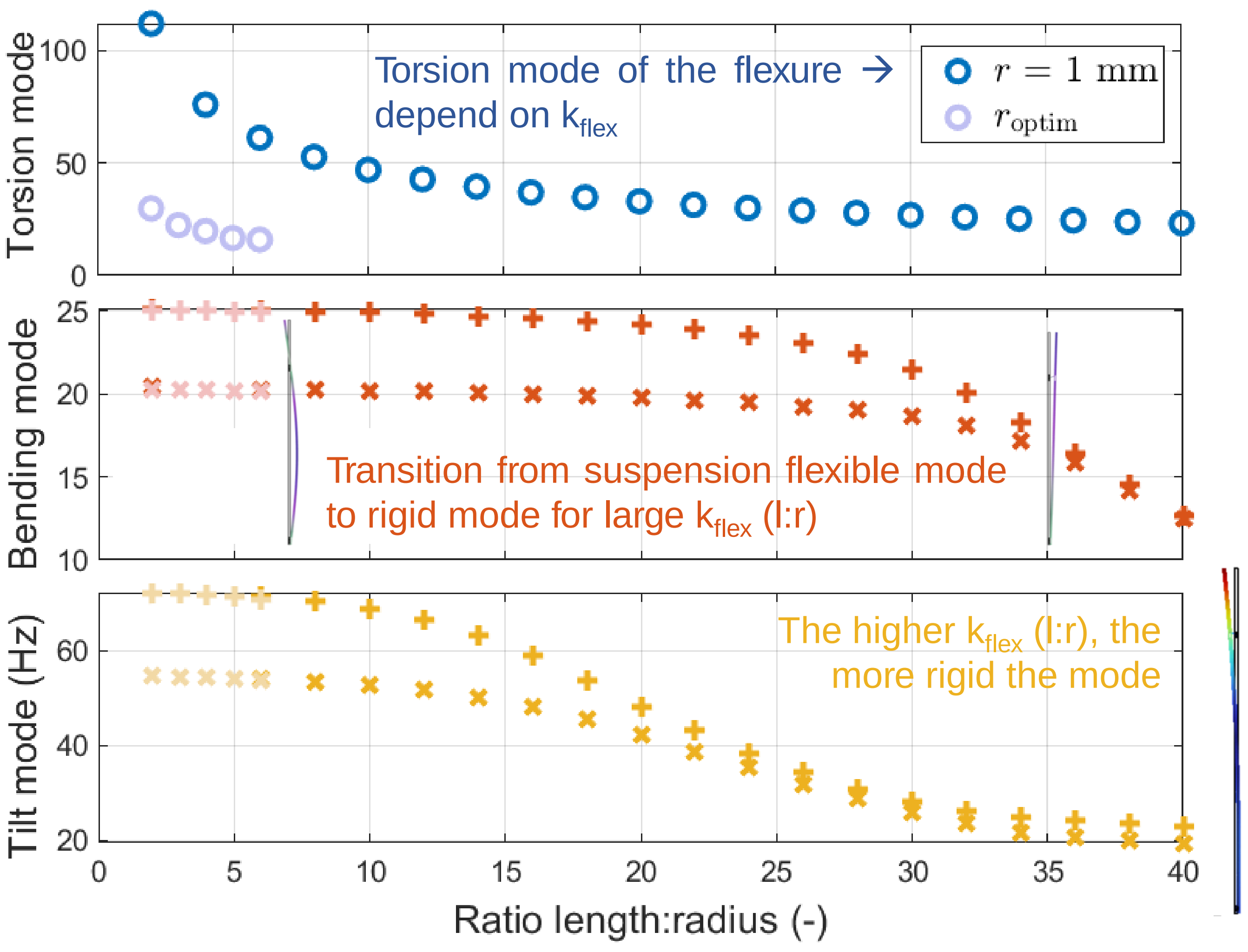
Thiner flexures lead to lower $E_{\text{dissipThinner}}$

Figure 2: Evolution of energy dissipated in the flexures w.r.t the length:radius ratio.

Suspension mode

Ratios < 10 should be chosen to avoid the suspension modes spoiling the detector sensitivity.

$$k_{\text{flexure}} = \frac{12EI}{L^3} + \frac{6\pi^2 N}{5L}$$



Torsion mode of the flexure → depend on k_{flex}

Transition from suspension flexible mode to rigid mode for large k_{flex} (l:r)

The higher k_{flex} (l:r), the more rigid the mode

Figure 3: Evolution of the suspension modes w.r.t the length:radius ratio.