A phase-insensitive table-top quantum filter

A. Dmitriev[1], J. Smetana[1], H. Miao[1,2], D. Martynov[1]

1. Institute for Gravitational Wave Astronomy
University of Birmingham, UK

2. Tsinghua University, Beijing, China

May 23, 2023

GWADW 2023, Elba

UNIVERSITY OF BIRMINGHAM | GRAVITATIONAL WAVE ASTRONOMY

Motivation

- The modern and future planned gravitational-wave detectors are limited by the photon shot noise within their most sensitive range
- Tackling the shot noise is vital for the high frequency detector upgrades

Motivation

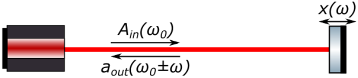
- The modern and future planned gravitational-wave detectors are limited by the photon shot noise within their most sensitive range
- Tackling the shot noise is vital for the high frequency detector upgrades
- Phase-sensitive techniques for quantum noise mitigation (use of squeezed states) are already in use and pushed to technological limits – unlike phase-insensitive amplification

Motivation

- The modern and future planned gravitational-wave detectors are limited by the photon shot noise within their most sensitive range
- Tackling the shot noise is vital for the high frequency detector upgrades
- Phase-sensitive techniques for quantum noise mitigation (use of squeezed states) are already in use and pushed to technological limits – unlike phase-insensitive amplification
- Compared to phase-sensitive techniques, phase-insensitive amplification is relatively less understood both experimentally and theoretically

Shot noise

Position sensing:

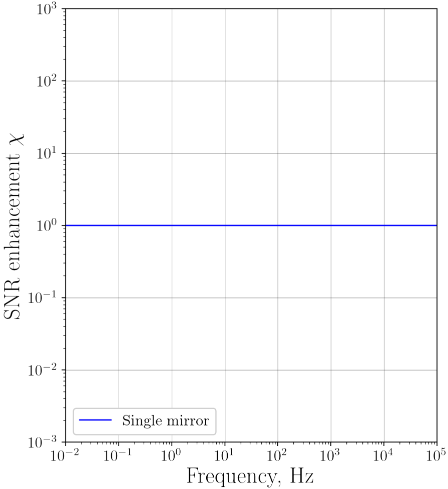


Shot noise

Position sensing:

Shot noise

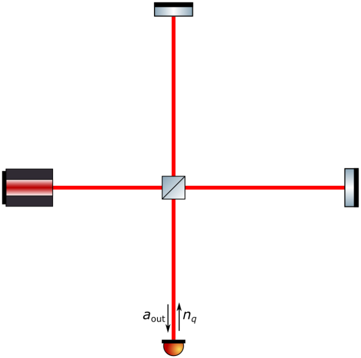
Position sensing:

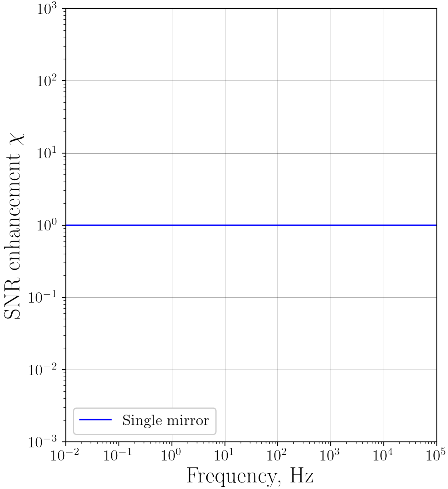


Shot noise

Position sensing:

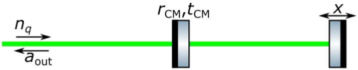
Differential sensing:

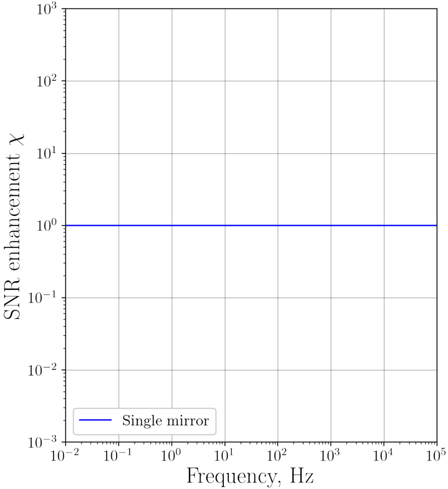




Shot noise

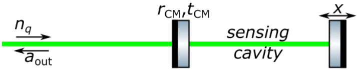
Cavity-assisted sensing:

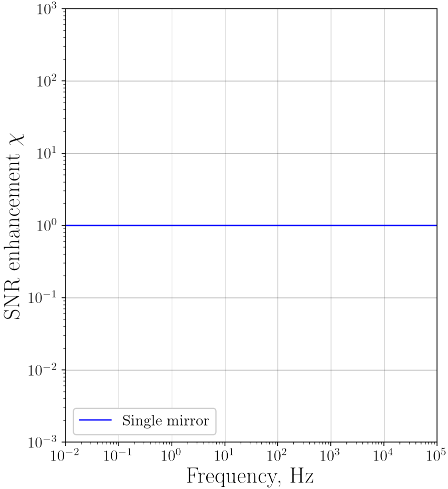




Shot noise

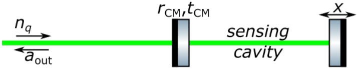
Cavity-assisted sensing:

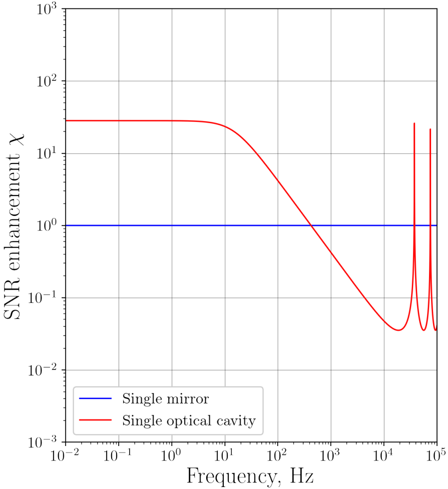




Shot noise

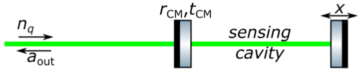
Cavity-assisted sensing:



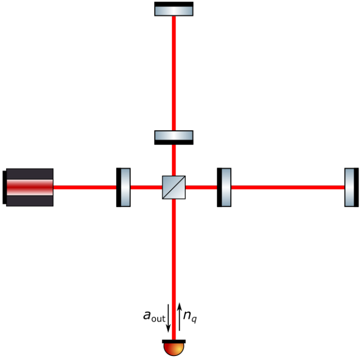


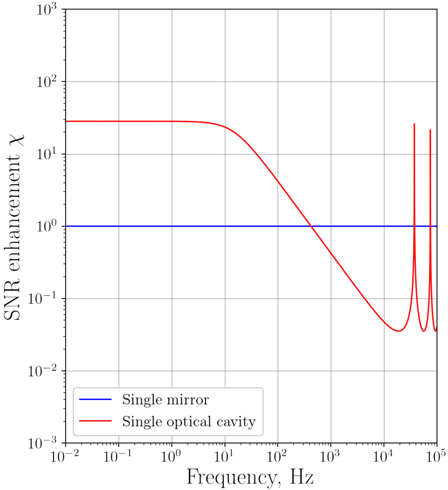
Shot noise

Cavity-assisted sensing:



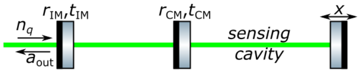
Cavities in arms:

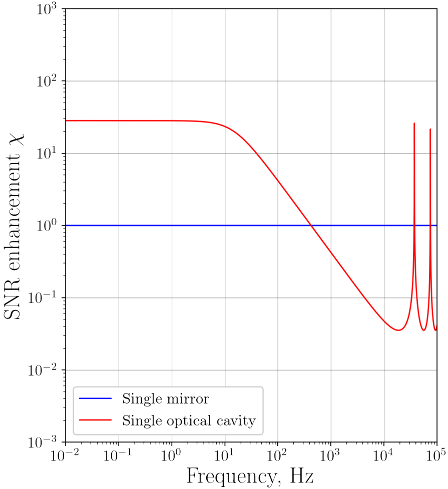




Shot noise

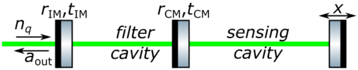
Filter cavity for signal recycling:

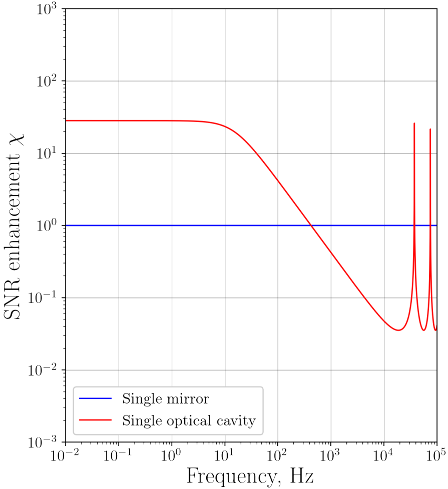




Shot noise

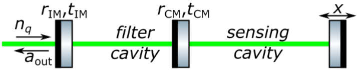
Filter cavity for signal recycling:

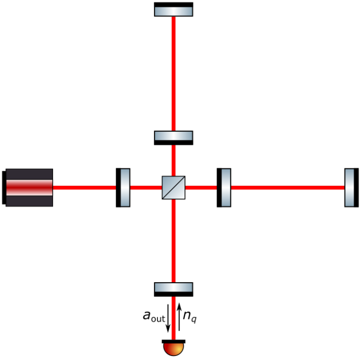


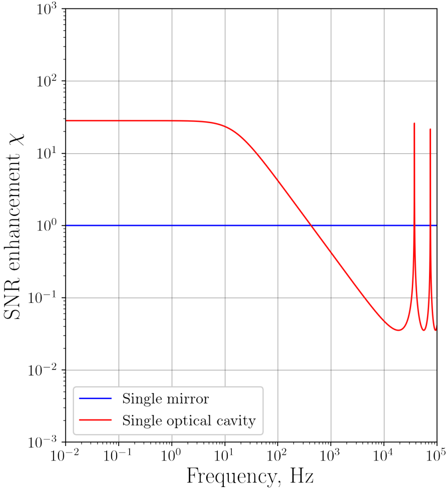


Shot noise

Filter cavity for signal recycling:

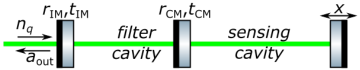




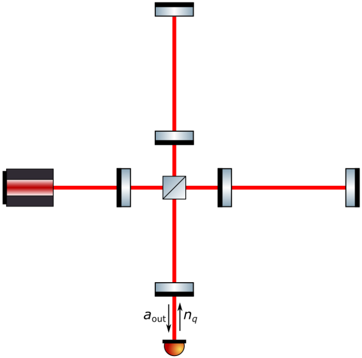


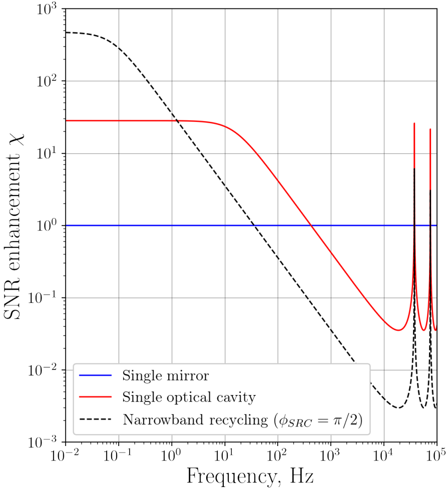
Shot noise

Filter cavity for signal recycling:



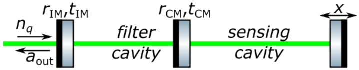
Signal recycling cavity:



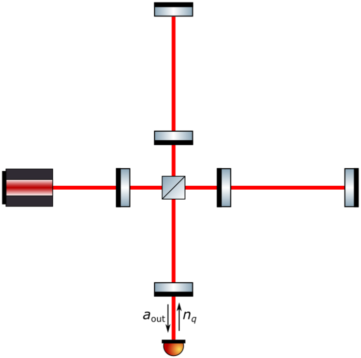


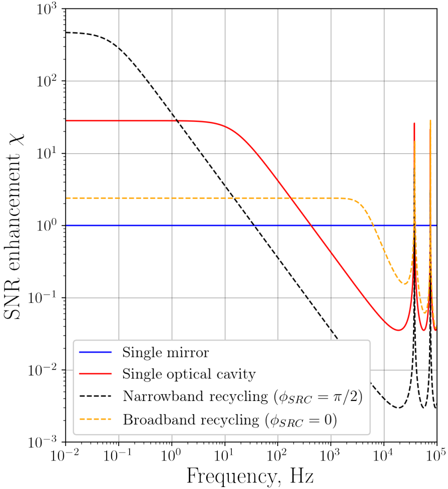
Shot noise

Filter cavity for signal recycling:



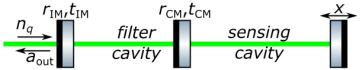
Signal recycling cavity:



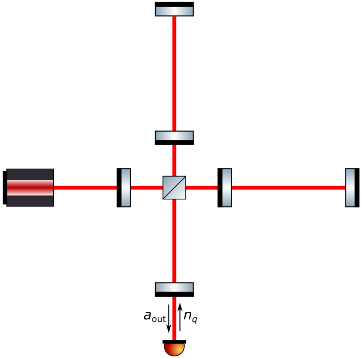


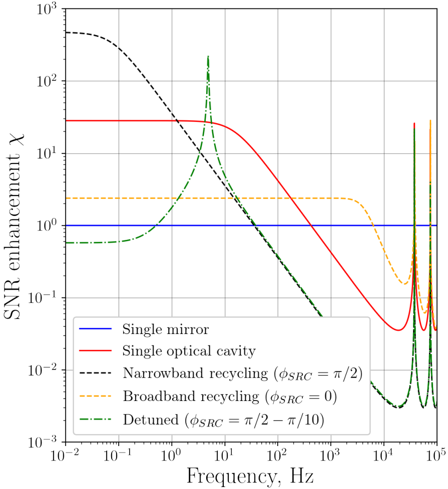
Shot noise

Filter cavity for signal recycling:



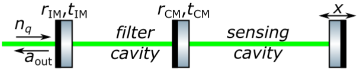
Signal recycling cavity:



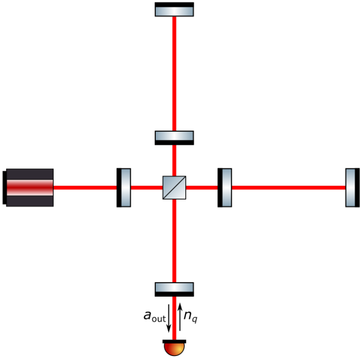


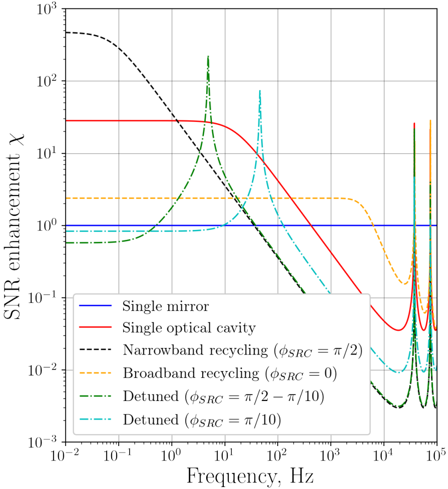
Shot noise

Filter cavity for signal recycling:



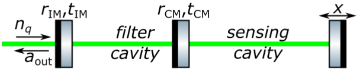
Signal recycling cavity:



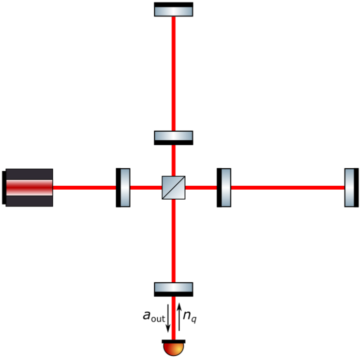


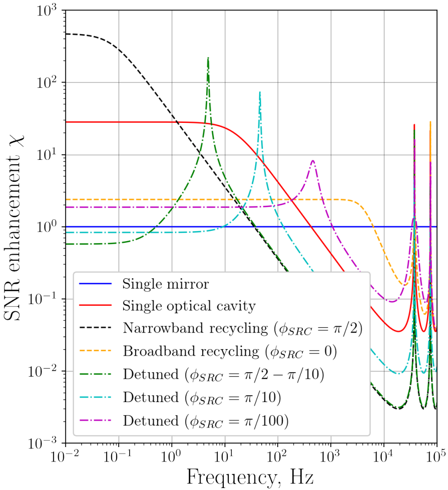
Shot noise

Filter cavity for signal recycling:



Signal recycling cavity:



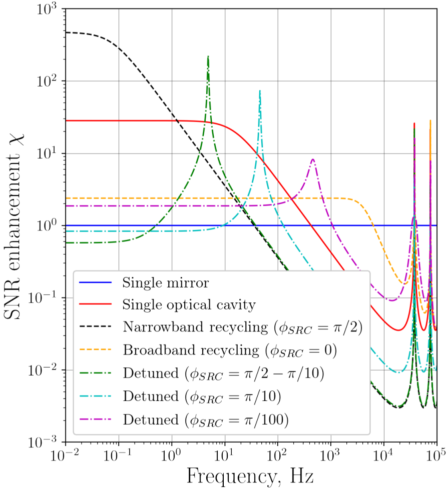


- Gain-bandwidth product (or integral sensitivity enhancement) remains constant:

$$\int_0^{\mathsf{FSR}} \chi^2(\omega)\,\mathrm{d}\omega = \text{const.}$$

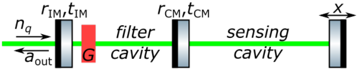
- The reason is the energetic quantum limit or quantum Cramer-Rao bound
- Can be overcome only with quantum techniques
- Quantum amplification can be phase-sensitive (squeezing) or phase-insensitive (this talk!)

Shot noise



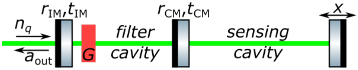
Phase-insensitive amplification in the filter

Filter cavity for signal recycling:

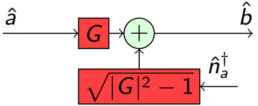


Phase-insensitive amplification in the filter

Filter cavity for signal recycling:



- Phase-insensitive quantum amplifier G added to the filter cavity:

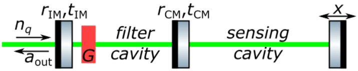


$$\hat{b} = G\hat{a} + \left(\sqrt{|G|^2 - 1}\right)\hat{n}_a^\dagger$$

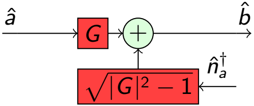
Carlton M. Caves (1982). "Quantum limits on noise in linear amplifiers". In: *Physical Review D* 26.8, pp. 1817–1839

Phase-insensitive amplification in the filter

Filter cavity for signal recycling:



- Phase-insensitive quantum amplifier G added to the filter cavity:



$$\hat{b} = G\hat{a} + \left(\sqrt{|G|^2 - 1}\right)\hat{n}_a^\dagger$$

Carlton M. Caves (1982). "Quantum limits on noise in linear amplifiers". In: *Physical Review D* 26.8, pp. 1817–1839

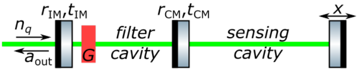
- Optimal "amplifier" G_{opt} compensates for the phase shift introduced by the main cavity with bandwidth γ_0:

$$G_{\mathrm{opt}}(i\omega) = \sqrt{\frac{i\omega + \gamma_0}{i\omega - \gamma_0}}$$

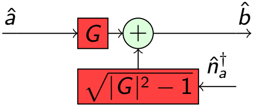
- No added noise because $|G_{\mathrm{opt}}| \equiv 1$ (no amplification!)
- Similar to the detuned filter cavity case but works at all frequencies

Phase-insensitive amplification in the filter

Filter cavity for signal recycling:

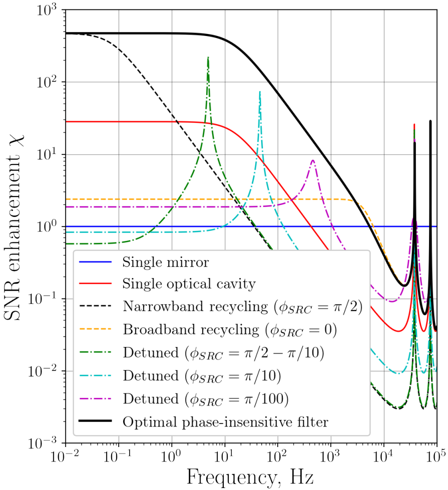


- Phase-insensitive quantum amplifier G added to the filter cavity:



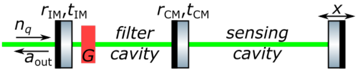
$$\hat{b} = G\hat{a} + \left(\sqrt{|G|^2 - 1}\right)\hat{n}_a^\dagger$$

Carlton M. Caves (1982). "Quantum limits on noise in linear amplifiers". In: *Physical Review D* 26.8, pp. 1817–1839

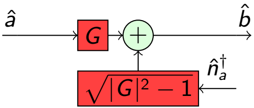


Phase-insensitive amplification in the filter

Filter cavity for signal recycling:



- Phase-insensitive quantum amplifier G added to the filter cavity:



$$\hat{b} = G\hat{a} + \left(\sqrt{|G|^2 - 1}\right)\hat{n}_a^\dagger$$

Carlton M. Caves (1982). "Quantum limits on noise in linear amplifiers". In: *Physical Review D* 26.8, pp. 1817–1839

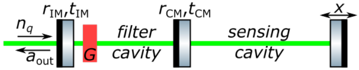
- Optimal "amplifier" G_{opt} compensates for the phase shift introduced by the main cavity with bandwidth γ_0:

$$G_{\text{opt}}(i\omega) = \sqrt{\frac{i\omega + \gamma_0}{i\omega - \gamma_0}}$$

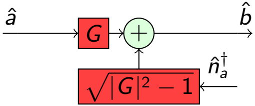
- No added noise because $|G_{\text{opt}}| \equiv 1$ (no amplification!)
- Similar to the detuned filter cavity case but works at all frequencies
- G_{opt} is unstable

Phase-insensitive amplification in the filter

Filter cavity for signal recycling:



- Phase-insensitive quantum amplifier G added to the filter cavity:



$$\hat{b} = G\hat{a} + \left(\sqrt{|G|^2 - 1}\right)\hat{n}_a^\dagger$$

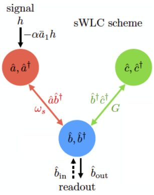
Carlton M. Caves (1982). "Quantum limits on noise in linear amplifiers". In: *Physical Review D* 26.8, pp. 1817–1839

- Optimal "amplifier" G_{opt} compensates for the phase shift introduced by the main cavity with bandwidth γ_0:

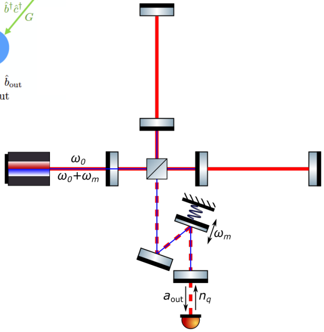
$$G_{\mathrm{opt}}(i\omega) = \sqrt{\frac{i\omega + \gamma_0}{i\omega - \gamma_0}}$$

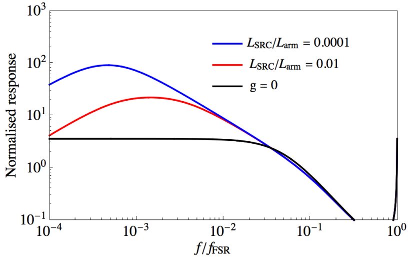
- No added noise because $|G_{\mathrm{opt}}| \equiv 1$ (no amplification!)
- Similar to the detuned filter cavity case but works at all frequencies
- G_{opt} is unstable
- In a *finite* frequency band, G_{opt} can be approximated with a stable function to any precision.

Artemiy Dmitriev, Haixing Miao, and Denis Martynov (2022). "Enhancing the sensitivity of interferometers with stable phase-insensitive quantum filters". In: *Phys. Rev. D* 106, p. 022007



PT-symmetric optomechanical realization





- A mechanical oscillator is coupled to the recycling cavity
- Blue-detuned pump field turns it into an amplifier for the signal
- Special tuning of the coupling strength ("PT symmetry"):

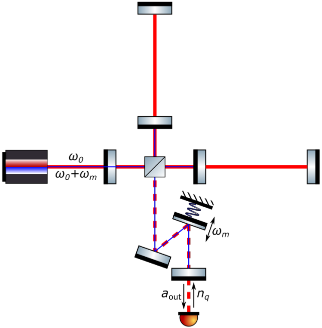
$$\underset{\text{Mechanical}\leftrightarrow\text{SRC}}{\text{coupling rate}} = \underset{\text{SRC}\leftrightarrow\text{Arm}}{\text{coupling rate}}$$

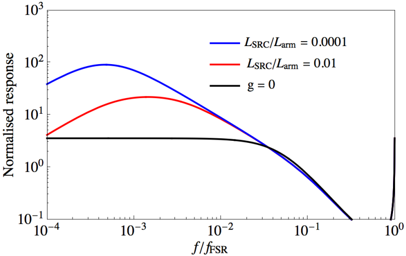
- SNR is enhanced as compared to the passive case
- System is fully causal and stable

Xiang Li, Maxim Goryachev, et al. (2020). "Broadband sensitivity improvement via coherent quantum feedback with PT symmetry". arXiv: 2012.00836 [quant-ph]

Xiang Li, Jiri Smetana, et al. (2021). "Enhancing interferometer sensitivity without sacrificing bandwidth and stability: Beyond single-mode and resolved-sideband approximation". In: *Physical Review D* 103.12

How to scale it down to table-top?

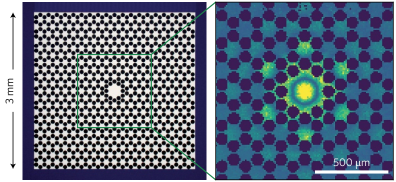




Scaling the design down to the table-top version is challenging!

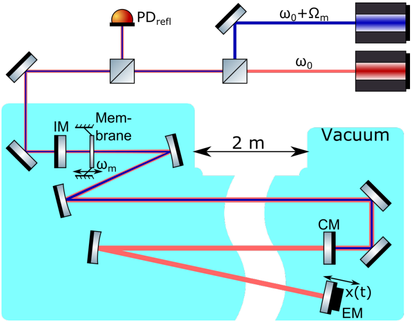
- Optical coupling rate increases with the length of the main cavity decreasing
- Additional thermal noise
- Locking
- Mode matching

Solution: use Si_3N_4 membranes



Yeghishe Tsaturyan et al. (2017). "Ultracoherent nanomechanical resonators via soft clamping and dissipation dilution". In: *Nature nanotechnology* 12.8, pp. 776–783

- $Q_m \approx 10^9$ @ 10 K
- $m = (10^{-12} \ldots 10^{-10})$ kg
- Low reflectivity
- Need to use membrane-in-the-middle layout

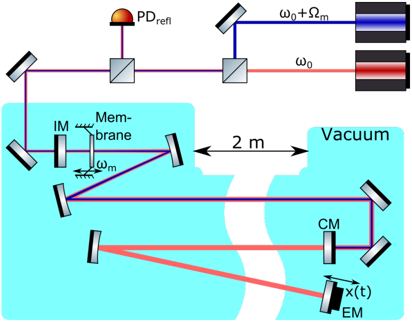


- Mode-matching telescope in the filter cavity
- Advanced LIGO-style locking
- Works for $T/Q_m \lesssim 10^{-7}$ K

Jiri Smetana et al. (2022). "Design of a tabletop interferometer with quantum amplification". arXiv: 2210.04566 [quant-ph]

Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity band-width	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmis-sivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigen-frequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thick-ness	h	50 nm
Membrane trans-missivity	$\mathcal{T}_m$	0.8
Membrane temper-ature	T	10 K
Input pump power	P_{in}	70 mW
Filter cavity power	P_{f}	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

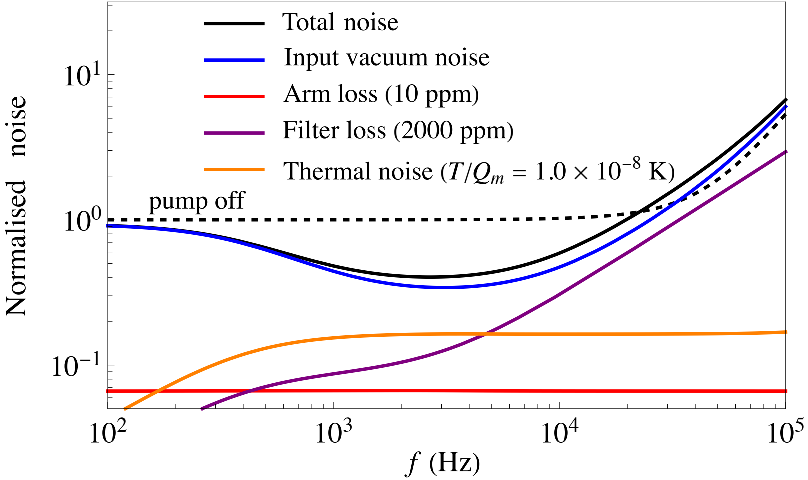
Parameters



- Mode-matching telescope in the filter cavity
- Advanced LIGO-style locking
- Works for $T/Q_m \lesssim 10^{-7}$ K

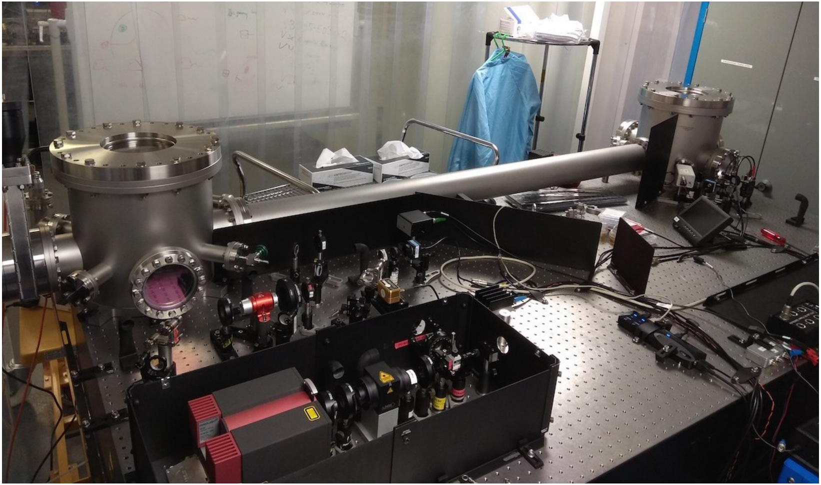
Jiri Smetana et al. (2022). "Design of a tabletop interferometer with quantum amplification". arXiv: 2210.04566 [quant-ph]

Noise budget



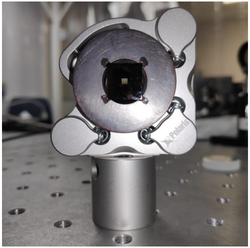
Experimental progress

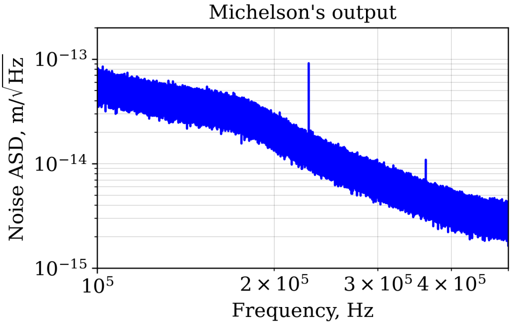
- Classical demonstration of the effect at room temperature is underway



Experimental progress

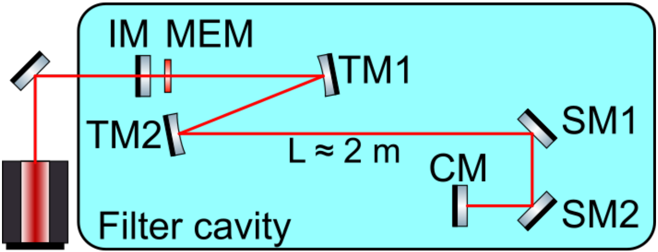
- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz





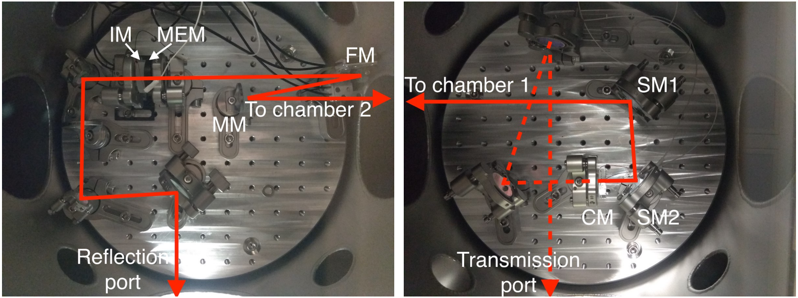
Experimental progress

- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked



Experimental progress

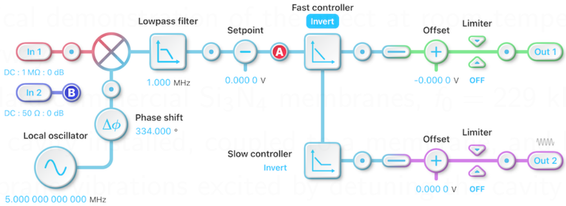
- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked

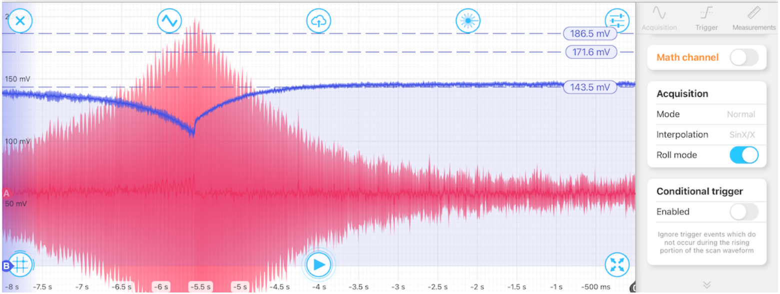


Experimental progress

- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked
- Membrane vibrations excited by detuning the cavity resonance, measured $Q \approx 8 \times 10^5$ for f_0.

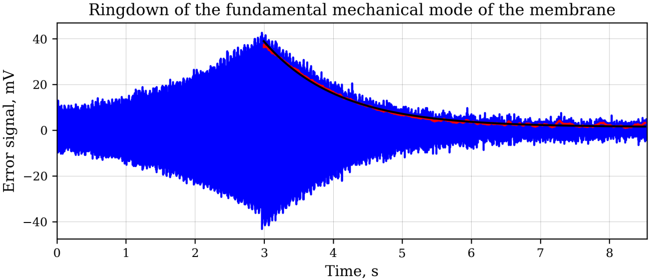
Experimental progress





Experimental progress

- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked
- Membrane vibrations excited by detuning the cavity resonance, measured $Q \approx 8 \times 10^5$ for f_0.



Experimental progress

- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked
- Membrane vibrations excited by detuning the cavity resonance, measured $Q \approx 8 \times 10^5$ for f_0.

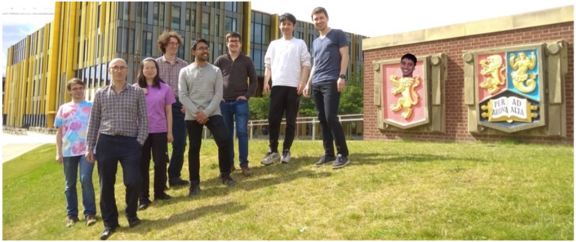
Experimental progress

- Classical demonstration of the effect at room temperature is underway
- Standard commercial Si_3N_4 membranes, $f_0 = 229$ kHz
- Filter cavity installed, coupled to a membrane, and locked
- Membrane vibrations excited by detuning the cavity resonance, measured $Q \approx 8 \times 10^5$ for f_0.

Next steps:

- Add the sensing cavity
- Demonstrate stability and sensitivity enhancement
- Proceed to the cryogenic quantum version

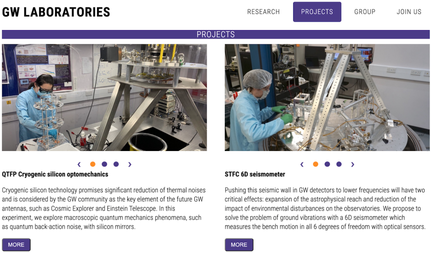
Acknowledgments



https://www.sr.bham.ac.uk/instrumental/

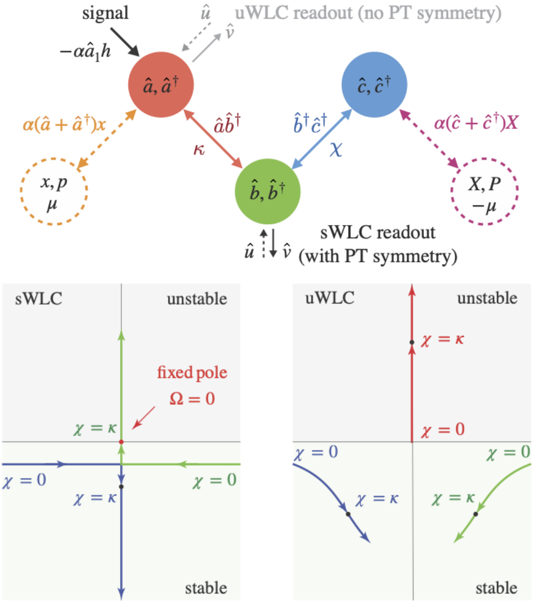
We do other stuff!

- Axion interferometer (see Alex's poster on LIDA)
- New detector topologies (Teng's talk on Thursday)
- Cryo-silicon optomechanics
- 6D and Compact 6D
- Optical coatings etc.

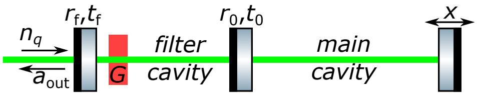


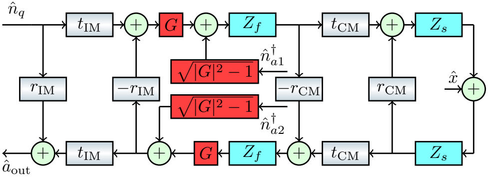
Auxiliary slides

PT symmetry


signal
$\hat{u}$ uWLC readout (no PT symmetry)
$\hat{v}$
$-\alpha\hat{a}_1 h$
$\hat{a}, \hat{a}^\dagger$
$\hat{c}, \hat{c}^\dagger$
$\alpha(\hat{a}+\hat{a}^\dagger)x$
$\hat{a}\hat{b}^\dagger$
$\hat{b}^\dagger\hat{c}^\dagger$
$\alpha(\hat{c}+\hat{c}^\dagger)X$
κ
χ
x, p
μ
$\hat{b}, \hat{b}^\dagger$
X, P
$-\mu$
sWLC readout
$\hat{u}$ $\hat{v}$ (with PT symmetry)
sWLC
unstable
fixed pole
$\Omega = 0$
$\chi = \kappa$
$\chi = 0$
$\chi = \kappa$
$\chi = 0$
stable
uWLC
unstable
$\chi = \kappa$
$\chi = 0$
$\chi = 0$
$\chi = 0$
$\chi = \kappa$
$\chi = \kappa$
stable

Optimal gain





- χ depends on G
- What G would maximise χ?

Double mode approximation:

$$Z_s(s) = e^{-s\tau_s} \approx 1 - s\tau_s + s^2\tau_s^2/2,$$

$$Z_f(s) = e^{-s\tau_f} \approx 1 - s\tau_f + s^2\tau_f^2/2,$$

$$G_{\rm opt}(s) = \sqrt{\frac{s+\gamma_s}{s-\gamma_s}}$$

$$|G_{\rm opt}(s)| \equiv 1$$

γ_s — bandwidth of the sensing cavity

Transfer functions:

$$T[\hat{x} \to \hat{a}_{\rm out}](s) = T_{\hat{x}}(s),$$

$$T[\hat{n}_q \to \hat{a}_{\rm out}](s) = T_{\hat{n}_q}(s),$$

etc.

Spectral density:

$$\hat{x} \leftrightarrow S_{xx},\ \hat{n}_q \leftrightarrow S_{vv},$$

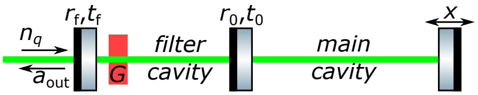
$$\hat{n}^\dagger_{a1} \leftrightarrow S_{vv},\ \hat{n}^\dagger_{a1} \leftrightarrow S_{vv},$$

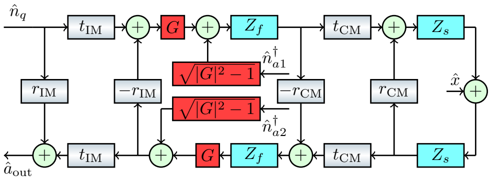
all uncorrelated

$$\mathrm{SNR}(\omega) = \chi^2(\omega)\ S_{xx}(\omega)/S_{vv}(\omega),$$

$$\chi^2(\omega) = \frac{|T_{\hat{x}}(i\omega)|^2}{|T_{\hat{n}_q}(i\omega)|^2 + |T_{\hat{n}_{a1}}(i\omega)|^2 + |T_{\hat{n}_{a2}}(i\omega)|^2}$$

Optimal gain





- Reflection off the sensing cavity:

$$r_s(s) = \frac{Z_s^2(s) - r_{CM}}{1 - r_{CM} Z_s^2(s)} \approx \frac{s - \gamma_s}{s + \gamma_s}$$

- Round-trip in the filter cavity:

$$T_{RT_f}(s) = r_{IM} G^2(s) Z_f^2(s) r_s(s)$$

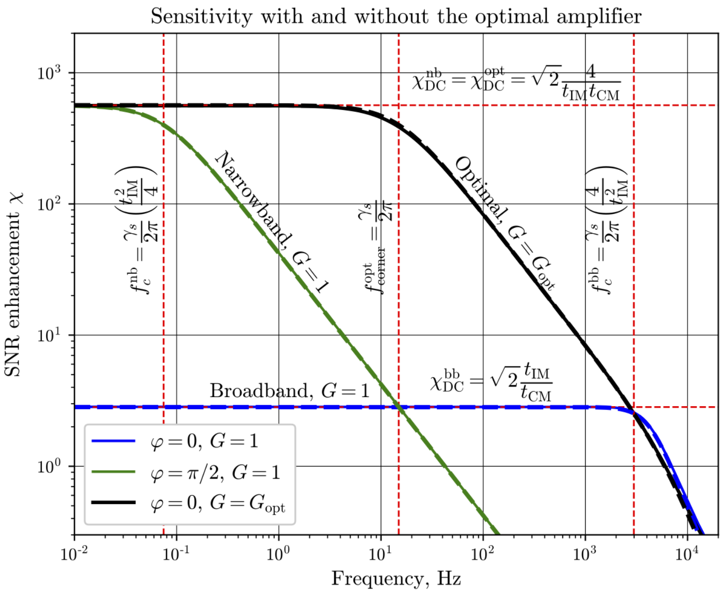
$$G_{opt}(s) = \sqrt{\frac{s + \gamma_s}{s - \gamma_s}}$$

$$|G_{opt}(s)| \equiv 1$$

- What does it mean?

- Optimal amplifier G_{opt} compensates for the phase shift introduced by the arm cavity.
- No added noise because $|G_{opt}| \equiv 1$

Sensitivity levels

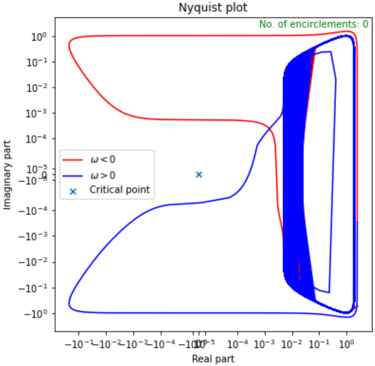


PT-symmetric optomechanical filters

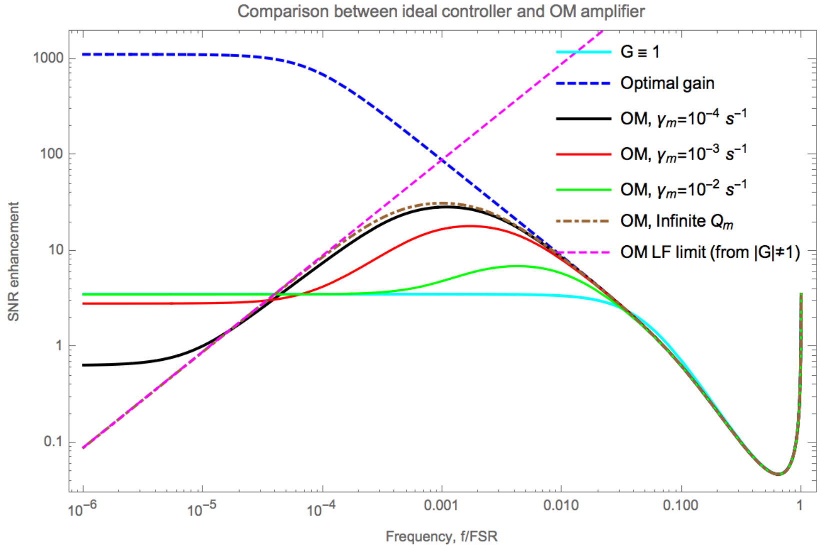
- Optomechanical amplifier:

$$G_{\mathrm{OM}}(s) = 1 - \frac{4g^2\tau_f\omega_m}{(s+\gamma_m)(s+\gamma_m-2i\omega_m)}$$

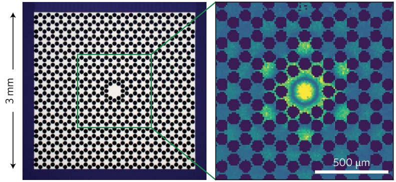
- System and all parts are causal and stable. However, MIMO analysis required as the amplifier's mode can no longer be ignored.



Sensitivity levels, PT filter

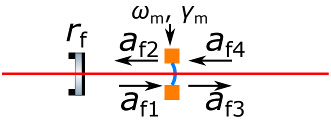


Silicon nitride (Si_3N_4) membranes



Yeghishe Tsaturyan et al. (2017). "Ultracoherent nanomechanical resonators via soft clamping and dissipation dilution". In: *Nature nanotechnology* 12.8, pp. 776–783

- $Q_m \approx 10^9$ @ 10 K
- $m = (10^{-12} \ldots 10^{-10})$ kg

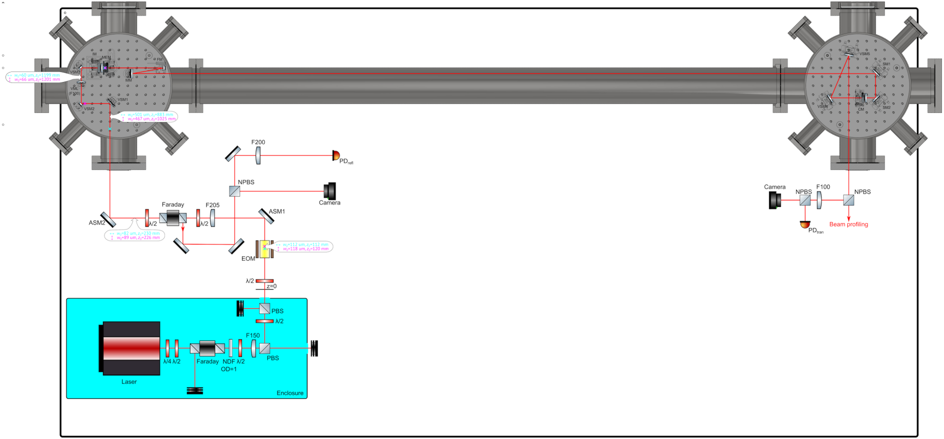


$$G(\omega) = a_{f3}/a_{f4}$$

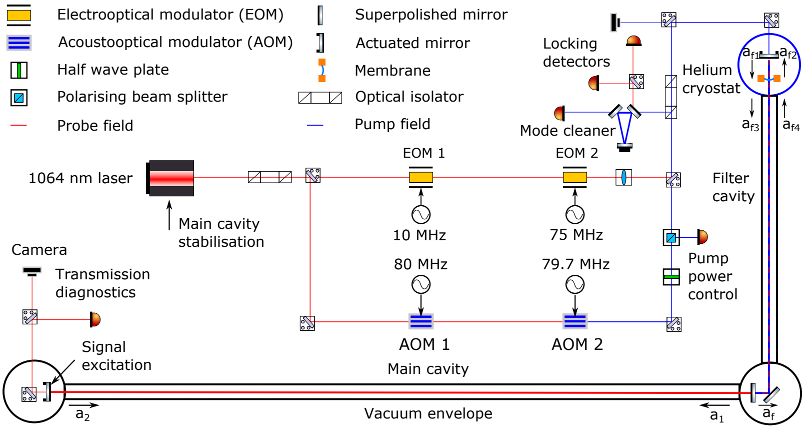
$$\approx 1 + \frac{2i\, g^2 \omega_m t_f}{\omega^2 - i\gamma_m \omega - \omega_m^2}$$

$$g \approx \left(\frac{r_m t_m^2 P_f \omega_0}{(1 - r_f r_m)^2 m c L_f \omega_m} \right)^{1/2}$$

Full experimental layout



Future experimental layout



Parameters

Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity bandwidth	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmissivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigenfrequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thickness	h	50 nm
Membrane transmissivity	$\mathcal{T}_m$	0.8
Membrane temperature	T	10 K
Input pump power	P_{in}	70 mW
Filter cavity power	P_{f}	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

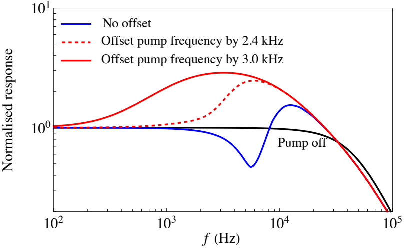
Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity bandwidth	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmissivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigenfrequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thickness	h	50 nm
Membrane transmissivity	$\mathcal{T}_m$	0.8
Membrane temperature	T	10 K
Input pump power	$P_{\rm in}$	70 mW
Filter cavity power	$P_{\rm f}$	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

Optical stiffening

- Pump field must be tuned to the mechanical resonant frequency
- Including frequency shift due to the optical spring

$$\Delta\omega_{\rm OS} = \frac{r_m t_m^2 g^2 \omega_m}{4(1 - r_f r_m)^2(\gamma_f^2 + \omega_m^2)}$$

$$\Delta\omega_{\rm OS} \approx 2\pi \times 3.0 \text{ kHz}$$



Locking

Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity bandwidth	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmissivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigenfrequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thickness	h	50 nm
Membrane transmissivity	$\mathcal{T}_m$	0.8
Membrane temperature	T	10 K
Input pump power	$P_{\rm in}$	70 mW
Filter cavity power	$P_{\rm f}$	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

- aLIGO-style locking
- Resonating a sideband at the first FSR of the filter cavity (75 MHz)
- Non-resonant sideband at 10 MHz

Sensing matrix:

Demodulation frequency	Main cavity	Filter cavity
75 MHz	1	-1/3
10 MHz	-1	0

Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity bandwidth	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmissivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigenfrequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thickness	h	50 nm
Membrane transmissivity	$\mathcal{T}_m$	0.8
Membrane temperature	T	10 K
Input pump power	$P_{\rm in}$	70 mW
Filter cavity power	$P_{\rm f}$	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

Loss-induced noise

- Noise induced by loss in the main cavity:

$$S^{\epsilon}_{\rm main}(\Omega) = \frac{4}{\mathcal{T}_{\rm eff}} Y_0$$

- Noise induced by loss in the filter cavity:

$$S^{\epsilon}_{\rm filter}(\Omega) = \frac{2(\gamma_0^2+\Omega^2)\tau}{\mathcal{T}_{\rm eff}\,\gamma_0} Y_{\rm f}$$

Parameter	Symbol	Value
Main cavity length	L_0	4.1 m
Coupling mirror transmissivity	T_0	30 ppm
Main cavity loss	Y_0	10 ppm
Filter cavity length	L_f	2 m
Filter cavity bandwidth	$\gamma_f/2\pi$	30 kHz
Filter cavity input coupler transmissivity	T_f	0.5 %
Filter cavity loss	Y_f	2000 ppm
Membrane eigenfrequency	$\omega_m/2\pi$	300 kHz
Motional mass	m	40 ng
Membrane thickness	h	50 nm
Membrane transmissivity	$\mathcal{T}_m$	0.8
Membrane temperature	T	10 K
Input pump power	P_{in}	70 mW
Filter cavity power	P_{f}	3.4 W
Pump frequency offset	$\omega_p/2\pi$	303 kHz

Thermal noise

- Thermal noise can be mapped to an equivalent optical loss in the main cavity:

$$Y_{\text{main}}^{\text{eq}} = \frac{k_B\gamma_0}{\hbar g^2}\left(\frac{T}{Q_m}\right)$$

- for $T/Q = 10^{-8}$ K, we get

$$Y_{\text{main}}^{\text{eq}} \approx 70 \text{ ppm}$$