GRAN SASSO SCIENCE INSTITUTE
CENTER FOR ADVANCED STUDIES
INFN

To the Thermal Limit and Below

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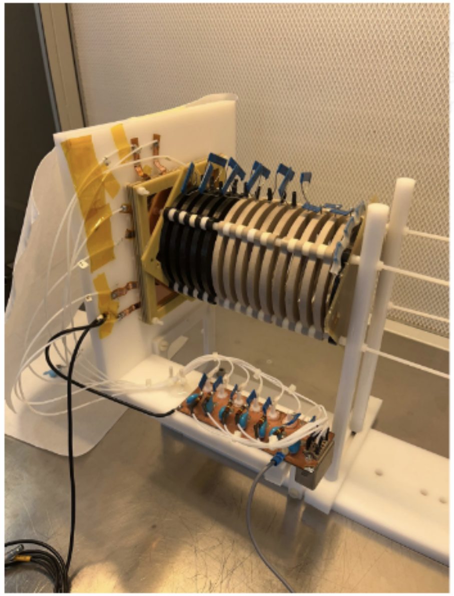
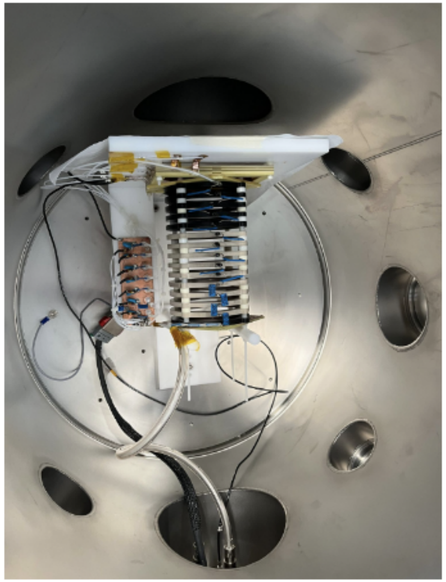
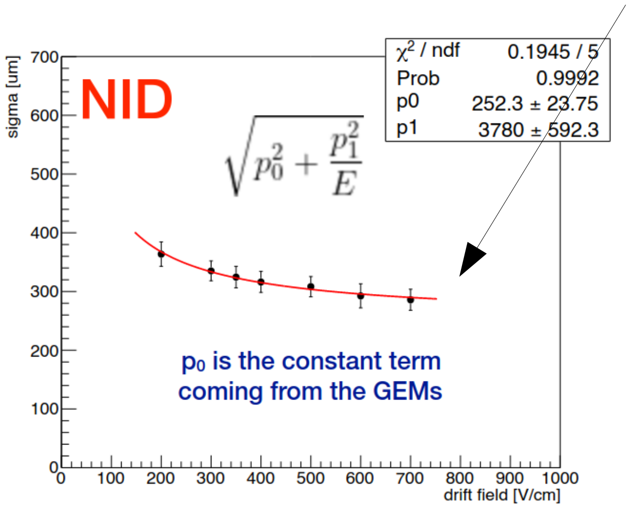
PREVIOUSLY

- MANGO was put in the keg to measure NID diffusion with more drift length to improve this

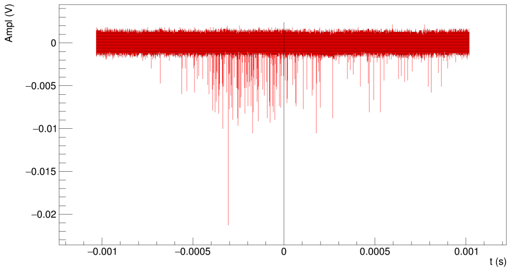


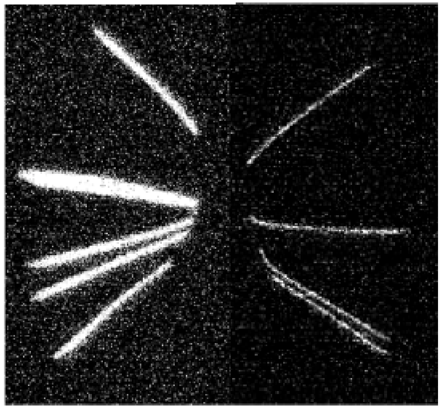




p_0 is the constant term coming from the GEMs

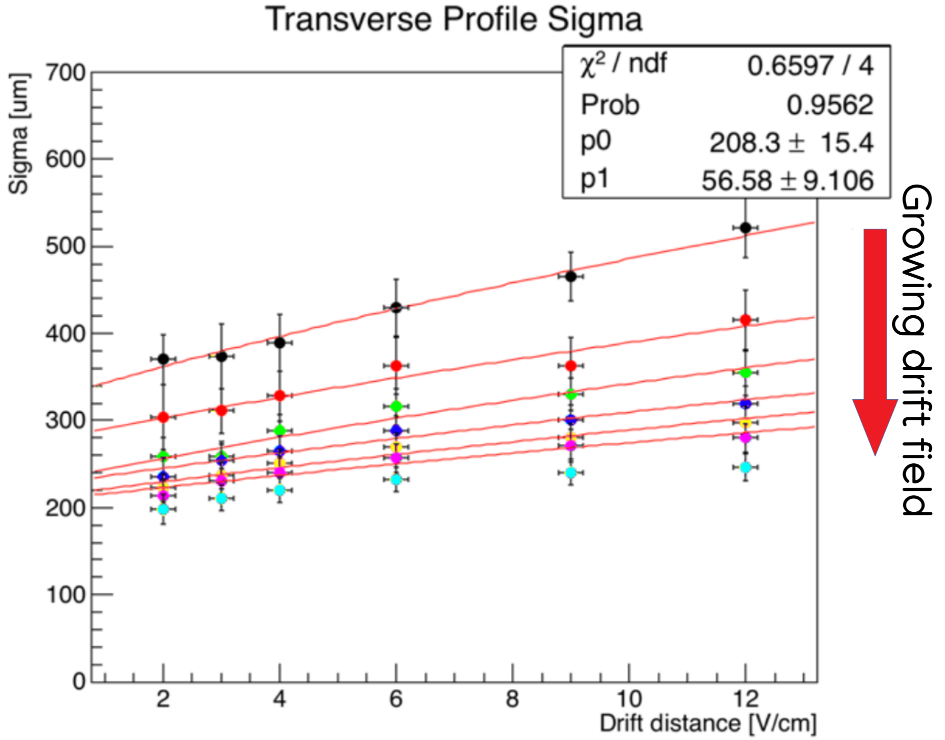
Pressure was reduced at 650 mbar for more stable operations



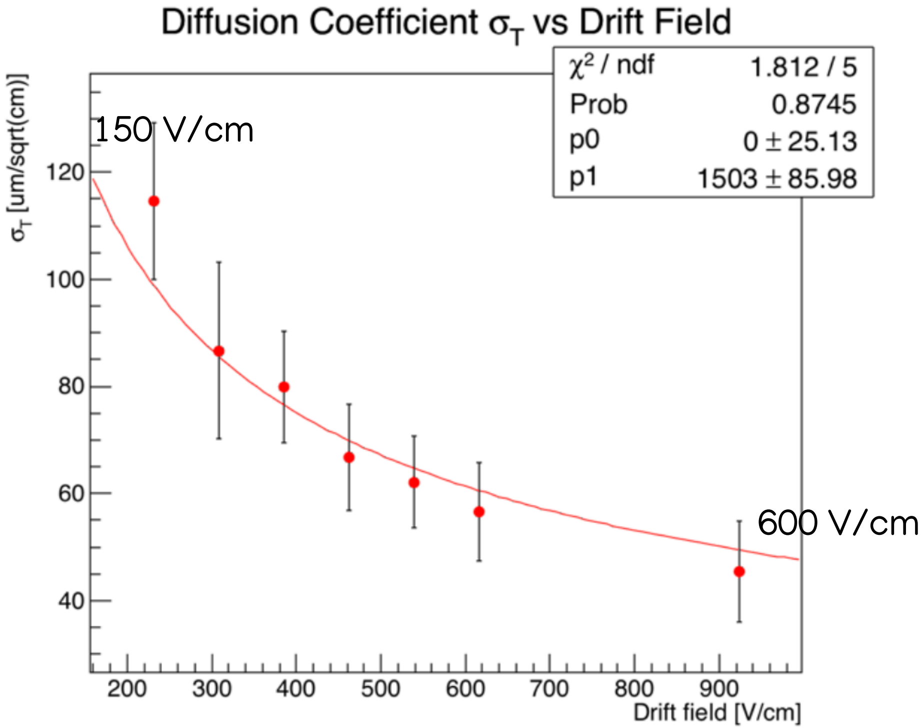


PREVIOUSLY

- This new configuration latest measurement presented on 16/6/2022



Same colour means fixed drift field



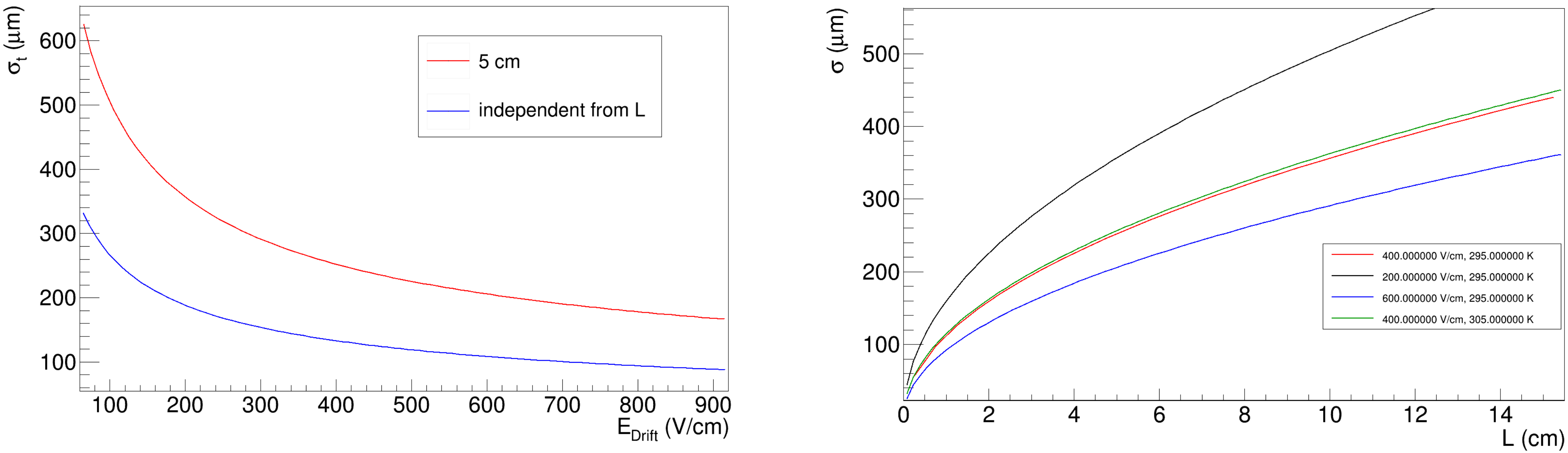
Drift fields are wrong

$$\sigma^2 = \frac{2 k_B T L}{eE}$$

Thermal limit of diffusion

Too Good to Be True?

- Our data suggest a diffusion coefficient smaller than what thermally possible



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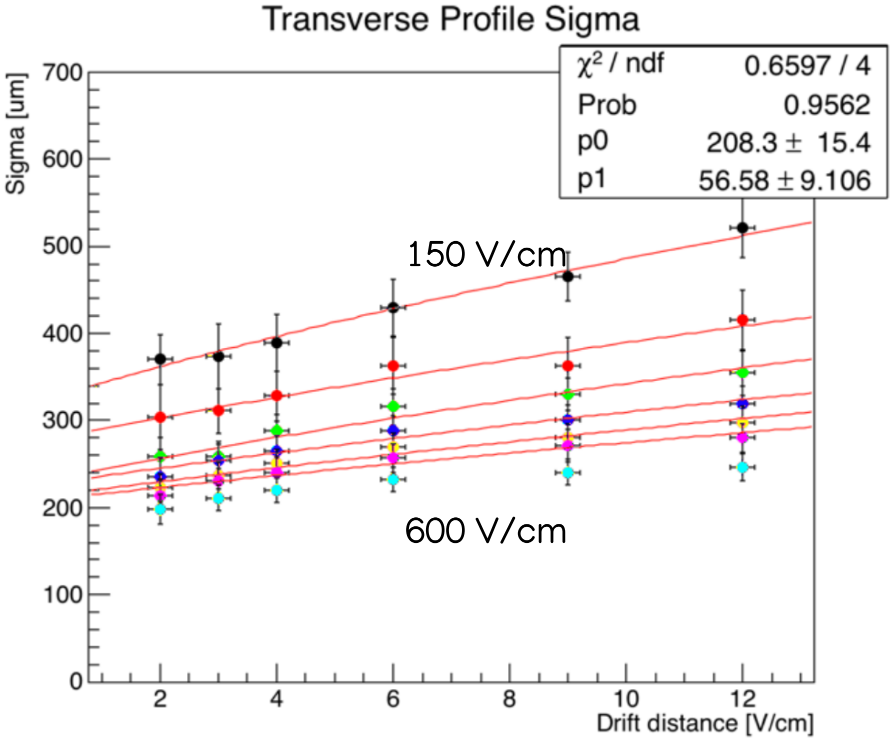
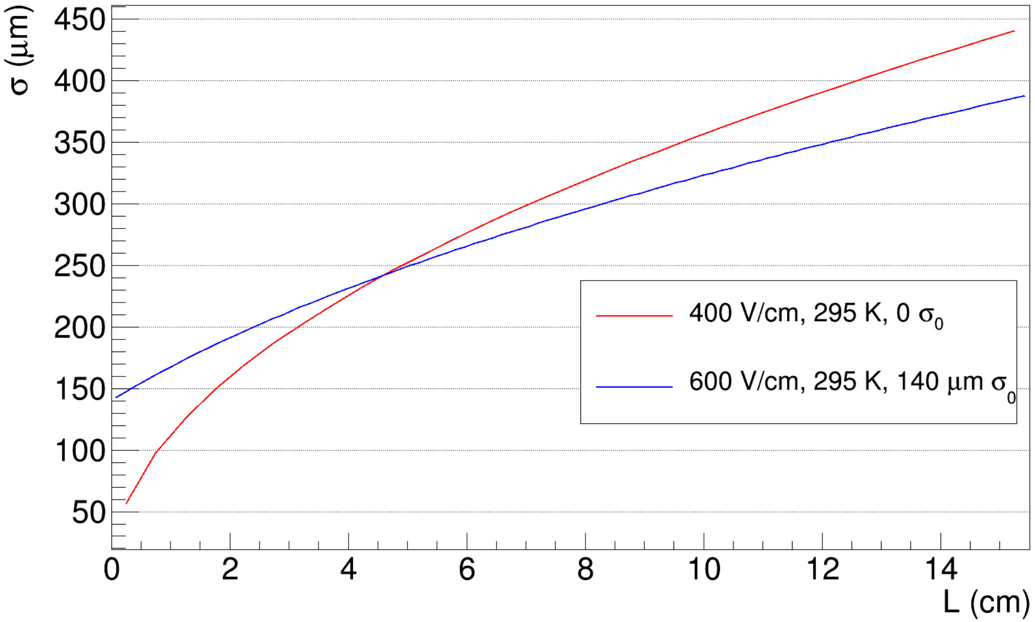




Contribution on diffusion by GEM is null in theoretical calculation

MATHEMATICAL PROVE: MOBILITY

- Starting from the Blum–Rolandi book calculation the w drift velocity is

$$w = v + c_d = \frac{eE}{m}\tau + c_d$$

  - E electric field
  - m mass of the target gas
  - τ average time between two collisions
  - c_d velocity in the direction of the drift as a result of the collision with molecules

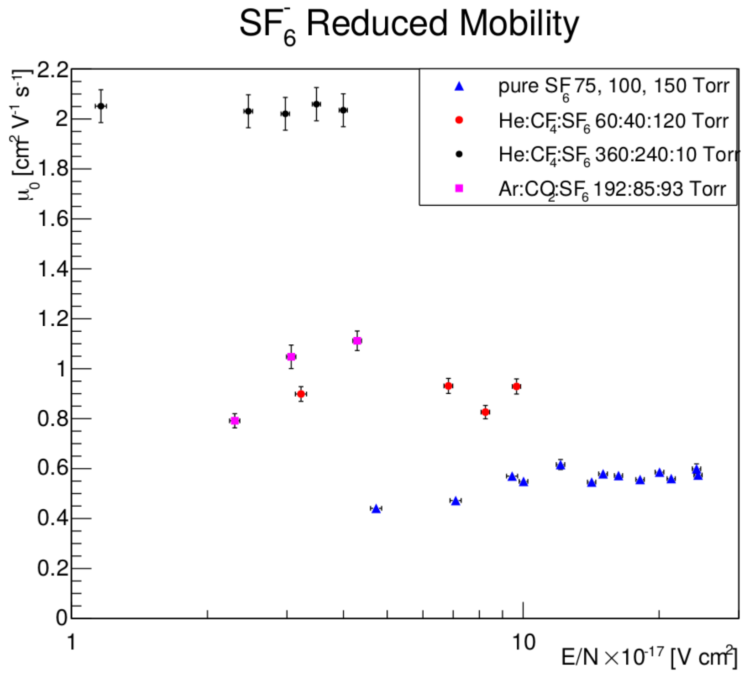
- The authors evaluate c_d and get the mobility

$$\mu = e\tau\left(\frac{1}{m} + \frac{1}{M}\right) \longrightarrow \mu = \sqrt{\frac{1}{m} + \frac{1}{M}}\sqrt{\frac{1}{3k_BT}}\frac{e}{N\sigma}$$

M mass of the negative ion drifting

MATHEMATICAL PROVE: MOBILITY

- This allows to correctly predict the relative difference of the mobilities for the gas mixtures of the NITEC paper, by averaging the gas components weighted on the average time between two collision of each molecule



$$\mu = \sqrt{\frac{1}{m}+\frac{1}{M}}\sqrt{\frac{1}{3k_BT}}\frac{e}{N\sigma}$$

When the gas is He dominated the NID are faster

MATHEMATICAL PROVE: DIFFUSION

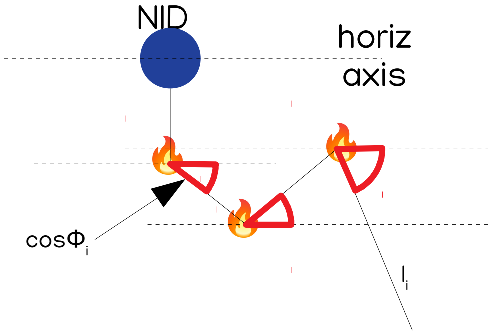
- For the diffusion the Blum–Rolandi writes the gaussian diffusion

$$\sigma^2_{tot} = 2Dt = 2\frac{D}{w}L = 2\frac{D}{\mu}\frac{L}{E} = \sigma^2_{diff}L$$

- But it can evaluate with the average quadratic dispersion

$$\sum_i^n \int (l_i \cos\phi_i)^2 \frac{1}{\lambda} e^{-\frac{l_i}{\lambda}} f(\cos\phi_i)\, d\lambda_i\, d\cos\phi_i$$

- l_i distance l traveled in the between the ith and ith-1 collision
- $\cos\phi_i$ angle between the ion direction and the chosen direction after the ith collision
- λ mean free path
- $f(\cos\varphi)$ probability distribution of the $\cos\varphi$ due to ion to ion collision
- n number of collision after a time t (large number of collisions)



CosΦ_i is not the scattering angle of the ion drifting

MATHEMATICAL PROVE: DIFFUSION

- Solving the integral, defining y the ratio of the masses M/m and F(y) the angular part of the integral

$$\sigma^2_{tot} = \sum_i^n 2\lambda^2 \int \cos^2\phi_i f(\cos\phi_i) d\cos\phi_i = \; 2\frac{2\epsilon}{m}\tau F(y) t$$

$$\sigma^2_{tot} = \; \frac{6k_B T\tau}{m\mu} F(y)\frac{L}{E}$$

- Inserting the mobility calculated before and dividing by L

$$\sigma^2_{diff} = \; \frac{6k_B T}{e}\cdot\frac{F(y)}{\left(1+\frac{1}{y}\right)}\frac{1}{E}$$

MATHEMATICAL PROVE: DIFFUSION

$$\sigma^2_{diff} = \frac{6k_BT}{e} \cdot \frac{F(y)}{\left(1+\frac{1}{y}\right)} \frac{1}{E}$$

If an isotropic distribution is chosen

- F(y) = 2/3

If also y=1 (ions on ions)

- The thermal limit formula is obtained

MATHEMATICAL PROVE: DIFFUSION

$$\sigma^2_{diff} = \frac{6k_BT}{e} \cdot \frac{F(y)}{\left(1+\frac{1}{y}\right)} \frac{1}{E}$$

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If an isotropic distribution is chosen

- F(y) = 2/3

Depending on the target y varies

- **y<1 the diffusion goes below thermal limit**

MATHEMATICAL PROVE: DIFFUSION

- Averaging on the different components of the gas (k_i percentage of gas in mixture)

$$\bar{\sigma}^2_{diff} = \frac{6k_BT}{eE}\frac{\sum_i^{gas}\delta_i F_i(y_i)\frac{\rho_i}{A_i}k_i(R_i+R_{SF_6})^2}{\sum_i^{gas}\frac{\rho_i}{A_i}k_i(R_i+R_{SF_6})^2}$$

k_i percentage of gas in mixture

δ_i average momentum loss =y/(y+1)

- Calculating the sigma with a field of 600 V/cm and using F(y)=2/3, I get

$$92\frac{\mu m}{\sqrt{cm}}$$

Thermal limit

$$65\frac{\mu m}{\sqrt{cm}}$$

Calculated value

This is only an upper limit as the distribution of the angle was considered isotropic

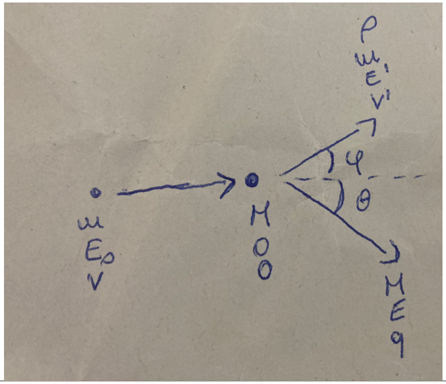
$$47\frac{\mu m}{\sqrt{cm}}$$

Measured value

Error under evaluation…
Could be consistent with the calculated one

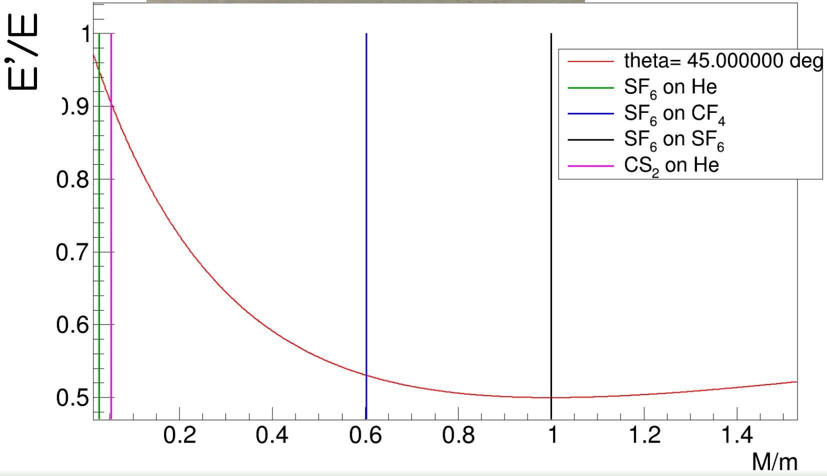
KINEMATICALLY

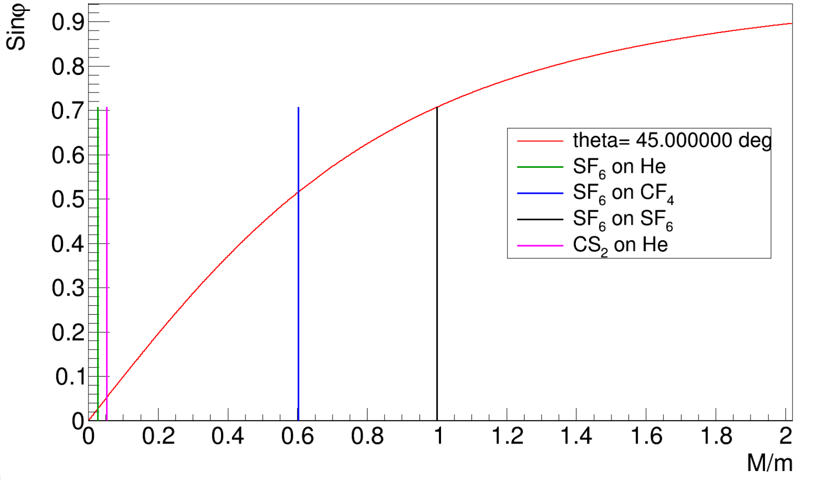
- The concept can be understood looking at the kinematics of the very heavy SF_6 on the lighter components of our gas



$$E' = E_0 \left[\frac{\left(\frac{M}{m}+1\right)^2 - 4\frac{M}{m}\cos^2\theta}{\left(\frac{M}{m}+1\right)^2} \right]$$

$$\sin\varphi = \sqrt{4\cos^2\theta(1-\cos^2\theta)\frac{\left(\frac{M}{m}\right)^2}{\left(\frac{M}{m}+1\right)^2 - 4\frac{M}{m}\cos^2\theta}}$$





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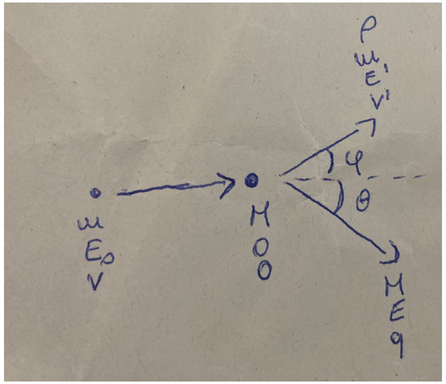

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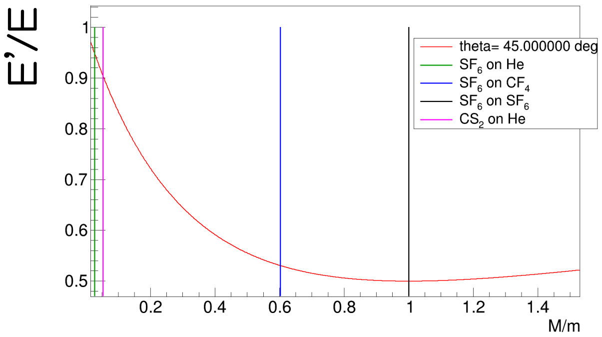
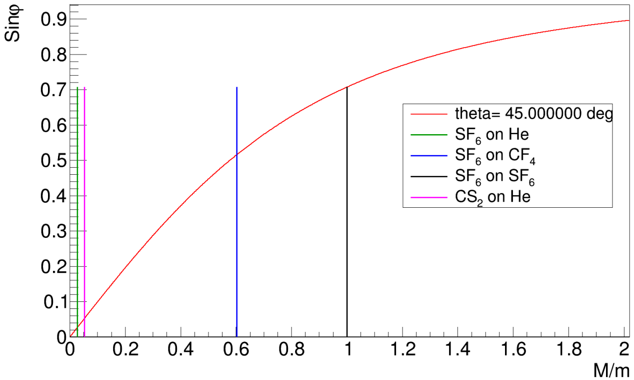


The distribution in angle for the diffusion could affect the result!

↓

Simulation needed



CONCLUSION

- The data on NID from the MANGO in a KEG campaign are being analyzed
- The measured diffusion seems to go below the best expectations of the thermal limt
- From prime principles it is possible to see that the large mass of SF_6 on the light He may be the reason for this behaviuor

  (Supergaseous helium-helped Anion drift??, Cold Anion Drift??...)
- A simulation of the collisions may be needed to obtain the actual values measured
- Some hard NID data taken with 50:50 He:CF_4 and 70:30 planned, to see a difference in the diffusion coefficient changing the helium content

Credit: Samuele Torelli