

Semileptonic B decays

Paolo Gambino
Università di Torino



Outline

- General motivation
- Inclusive semileptonic decays and the OPE
- NNLO pert calculation and other higher orders
- Exclusive V_{cb} determination: a unitarity bound
- Heavy quark masses
- The inclusive V_{ub} determination
- Conclusions

Why precision CKM studies?

- The SM accomodates flavour & CP violation, but **we have no theory of flavour**
- We expect New Physics at the EW scale, and most models predict additional flavour and CP violation.
- The CKM mechanism is very successful \Rightarrow **flavour and CP problem** (NP must preserve agreement with data)
- To uncover small signals of physics beyond CKM, we need precision tests, in many ways a challenge for our QCD understanding

The CKM matrix

Weak and mass eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

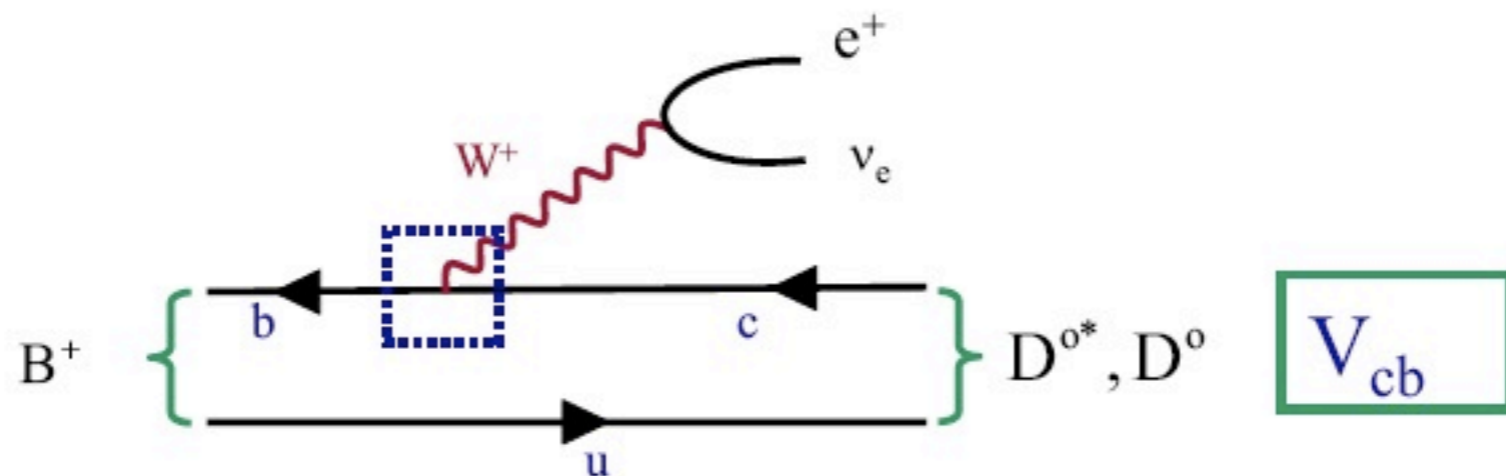
Wolfenstein parameterization $\lambda \sim 0.22$, A , ρ , η are $\mathcal{O}(1)$

We can improve the accuracy, defining some element to all orders in λ

Determination of A

$$\hat{V} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

A can be determined from either V_{cb} or V_{ts}



Two roads to V_{cb} : inclusive and exclusive semilept B decays

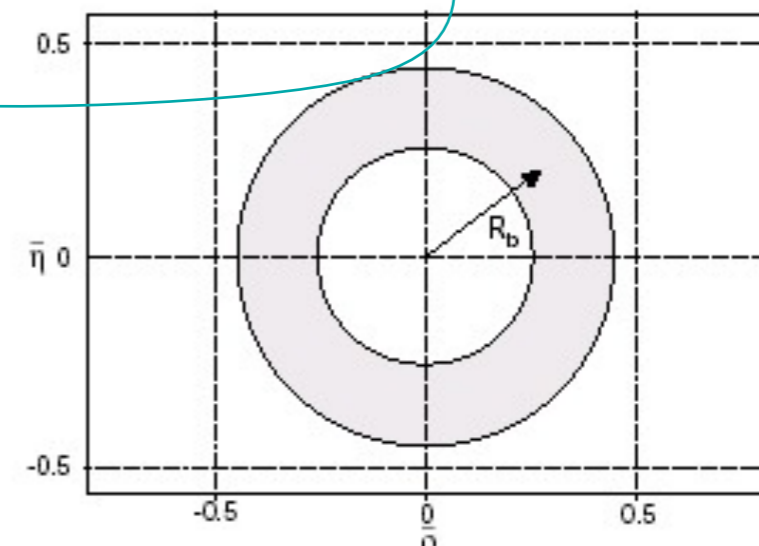
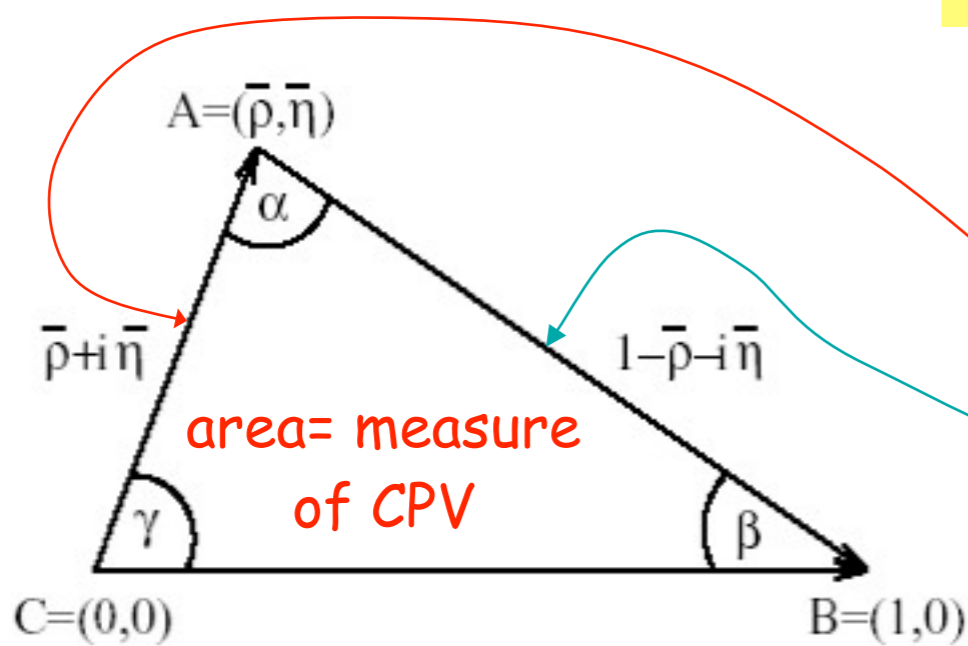
The Unitarity Triangle

$$V_{ji} V_{jk}^* = \delta_{ik}$$

Unitarity determines several triangles in complex plane

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \quad \mathcal{O}(\lambda^3)$$

$$1 + \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0$$

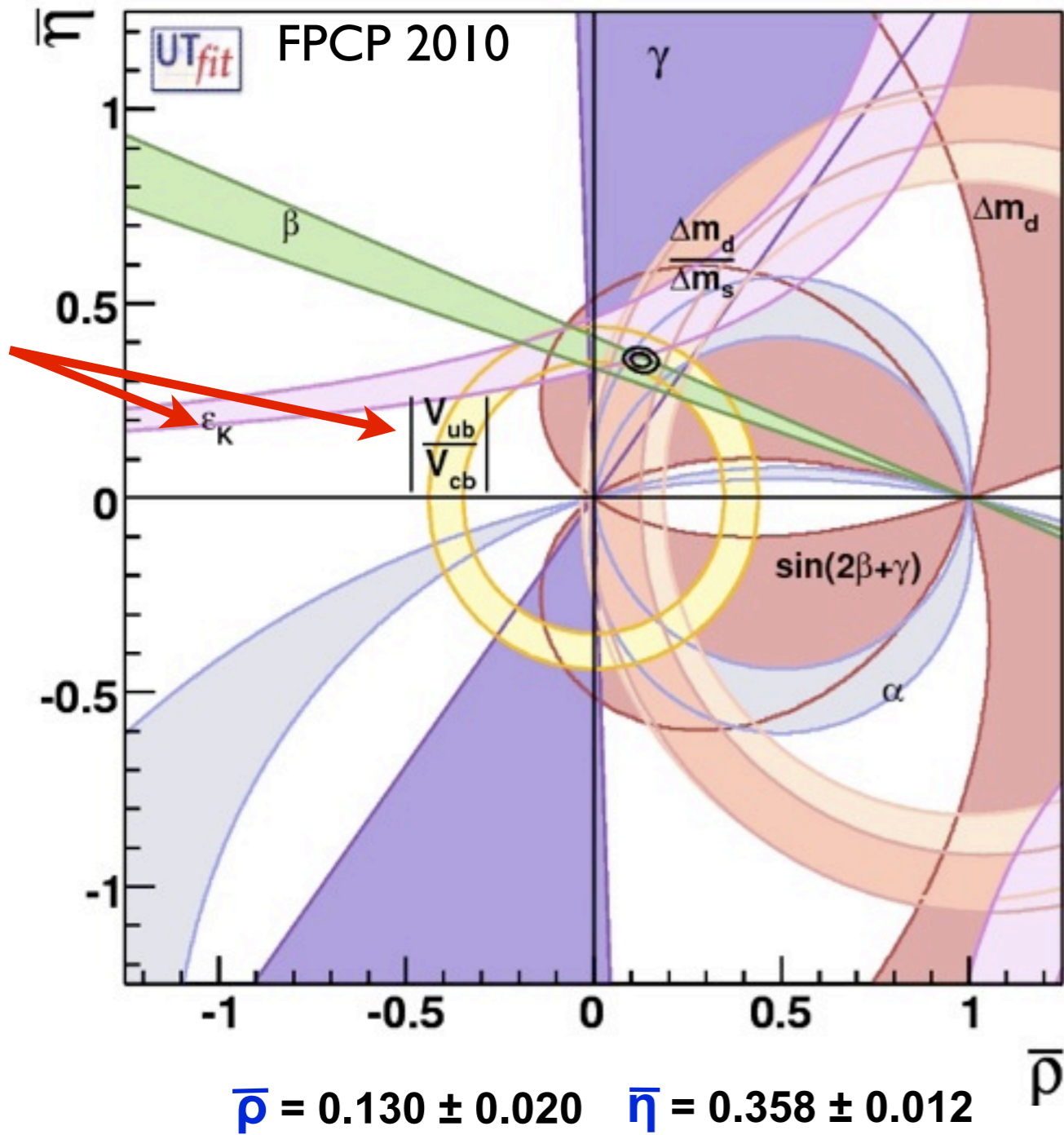


$$\left| \frac{V_{ub}}{V_{cb}} \right|$$

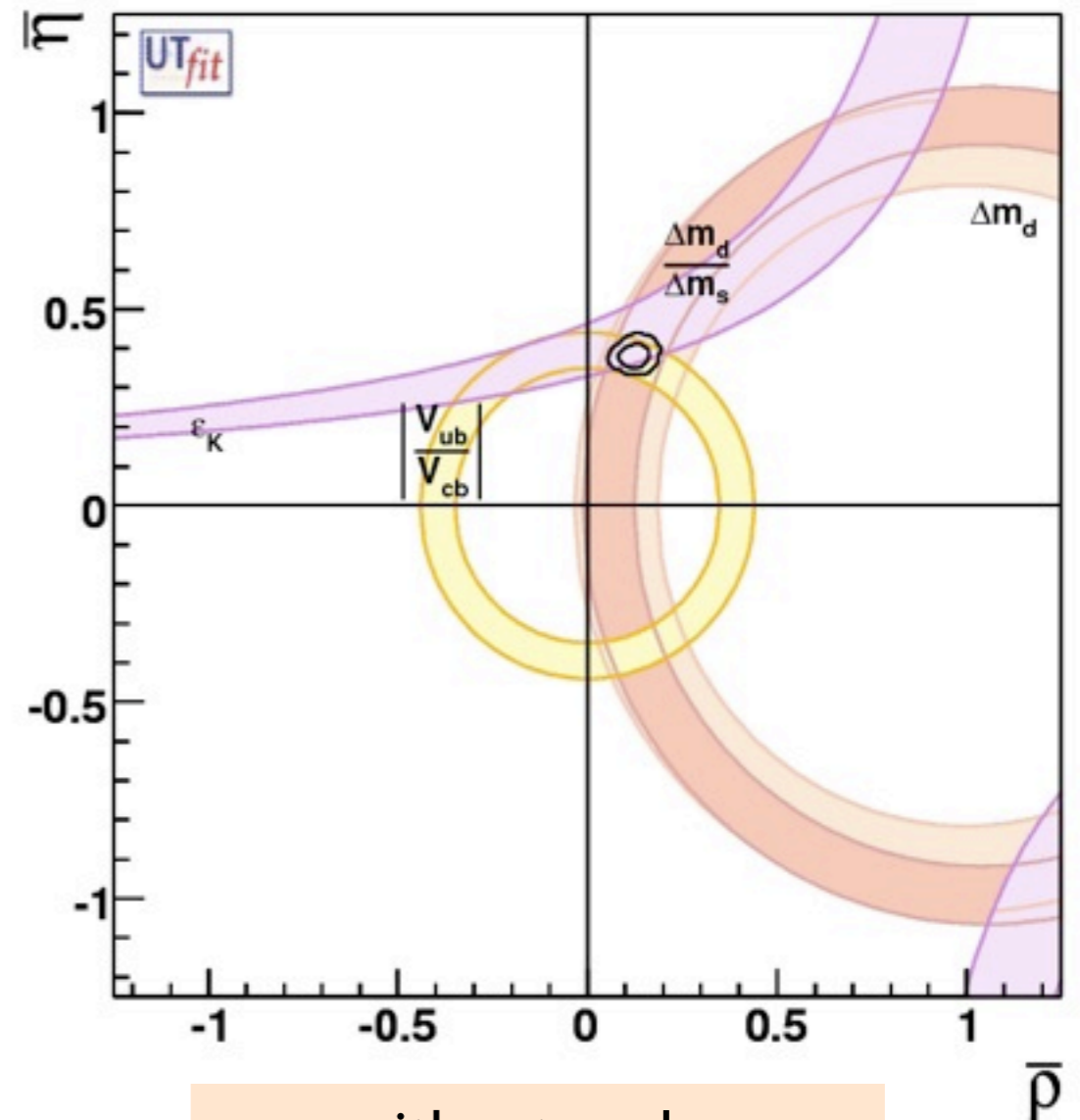
V_{td} cannot be accessed directly:
need FCNC loops sensitive to
new physics eg B_d , B_s mixing

$|V_{ub}|$ can be measured in tree level
semileptonic B decays

The UT and V_{ub}, V_{cb}



$\sin 2\beta_{\text{charmionium}} = 0.655 \pm 0.024$



without angles
 UTfit inputs:
 $\xi = 1.24(3)$ $B_K = 0.731(36)$
 recent results $B_K = 0.72(3)$
 getting to 5% accuracy

Tensions in the UTfit

compatibility plots in the SM

measure the agreement of a single measurement with the indirect determination from the fit using all the other inputs

$$\sin 2\beta_{\text{exp}} = 0.655 \pm 0.024$$

$$\sin 2\beta_{\text{UTfit}} = 0.753 \pm 0.034$$

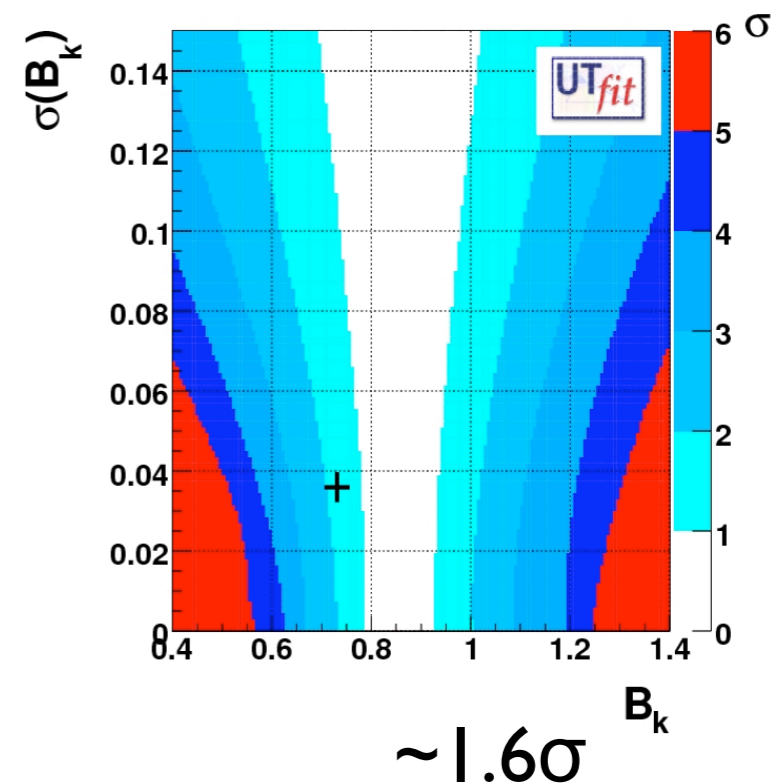
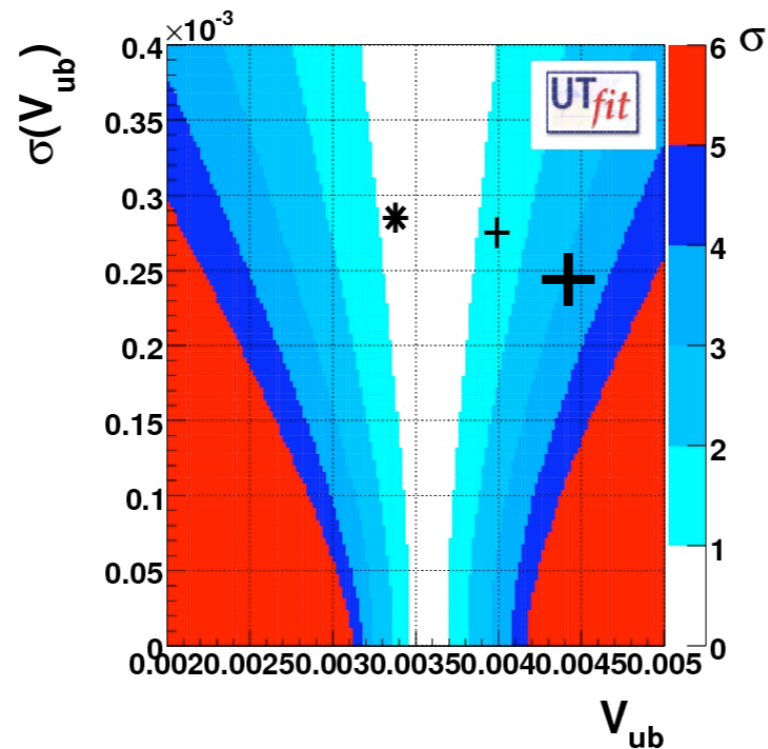
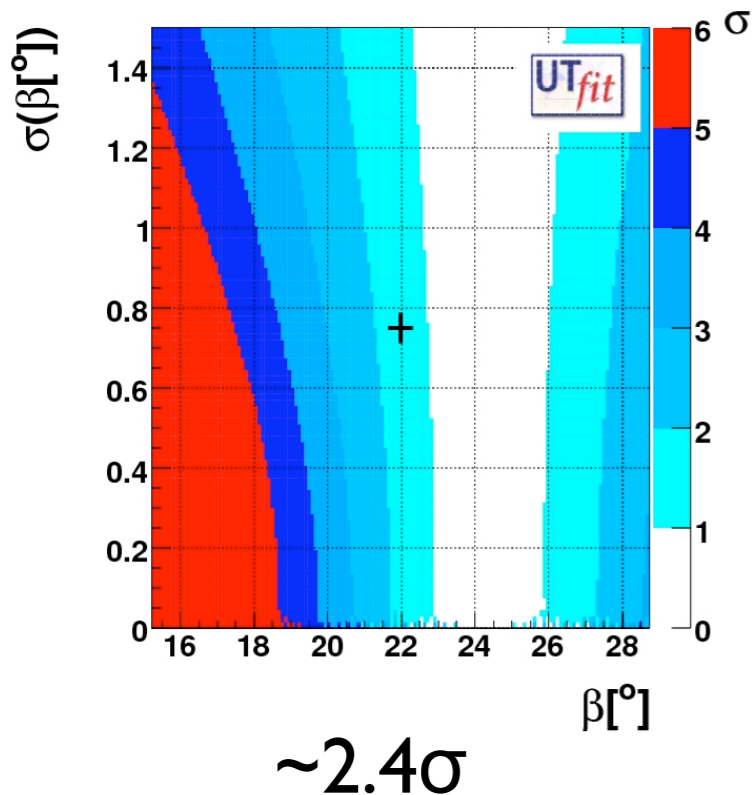
$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{incl}} = 0.101 \pm 0.006$$

$$\left| \frac{V_{ub}}{V_{cb}} \right|_{\text{excl}} = 0.084 \pm 0.008$$

$$B_{K_{\text{exp}}} = 0.731 \pm 0.036$$

$$B_{K_{\text{UTfit}}} = 0.855 \pm 0.069$$

$$B_{K_{\text{no lattice}}} = 0.869 \pm 0.079$$



$$V_{ub_{\text{exp}}} = (37.2 \pm 2.1) \cdot 10^{-4}$$

$$V_{ub_{\text{UTfit}}} = (35.8 \pm 1.1) \cdot 10^{-4}$$

! $< 1\sigma$ (incl $\sim 2.5\sigma$)

M. Bona
UTfit@FPCP 2010

Tensions in the UTfit

compatibility plots in the SM

measure the agreement of a single measurement with the indirect determination from the fit using all the other inputs

NEW 2011: 0.676 ± 0.020

~~$\sin 2\beta_{\text{exp}} = 0.655 \pm 0.024$~~
 $\sin 2\beta_{\text{UTfit}} = 0.753 \pm 0.034$

$$\frac{V_{ub}}{V_{cb}^{\text{incl}}} = 0.101 \pm 0.006$$

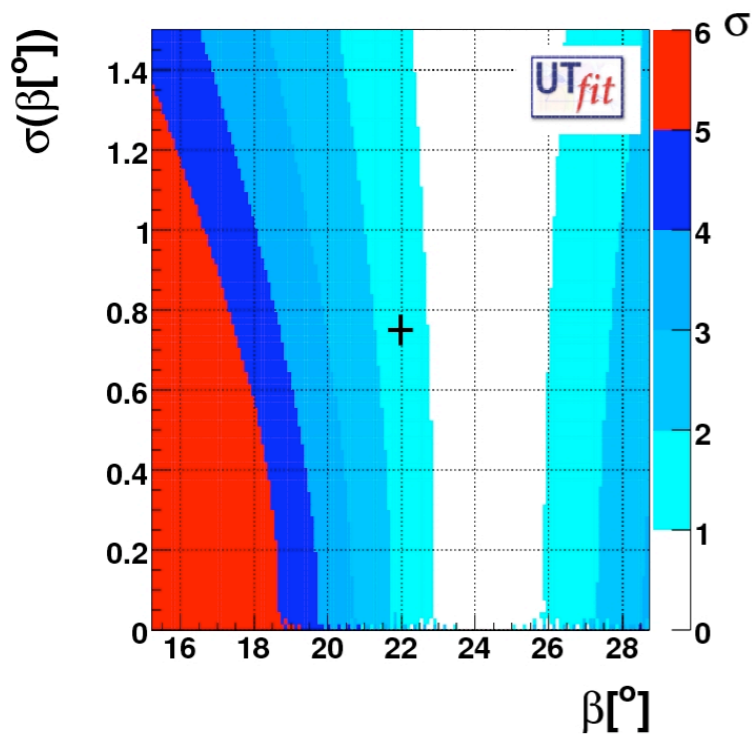
$$\frac{V_{ub}}{V_{cb}^{\text{excl}}} = 0.084 \pm 0.008$$

$B_{K_{\text{exp}}} = 0.731 \pm 0.036$

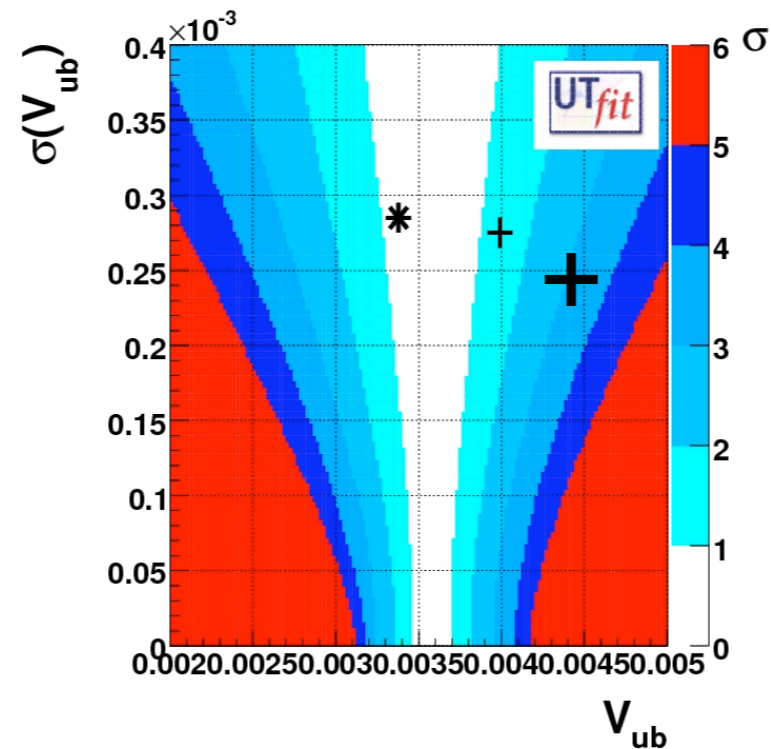
$B_{K_{\text{UTfit}}} = 0.855 \pm 0.069$

$B_{K_{\text{no lattice}}} = 0.869 \pm 0.079$

??



$\sim 2.4\sigma$

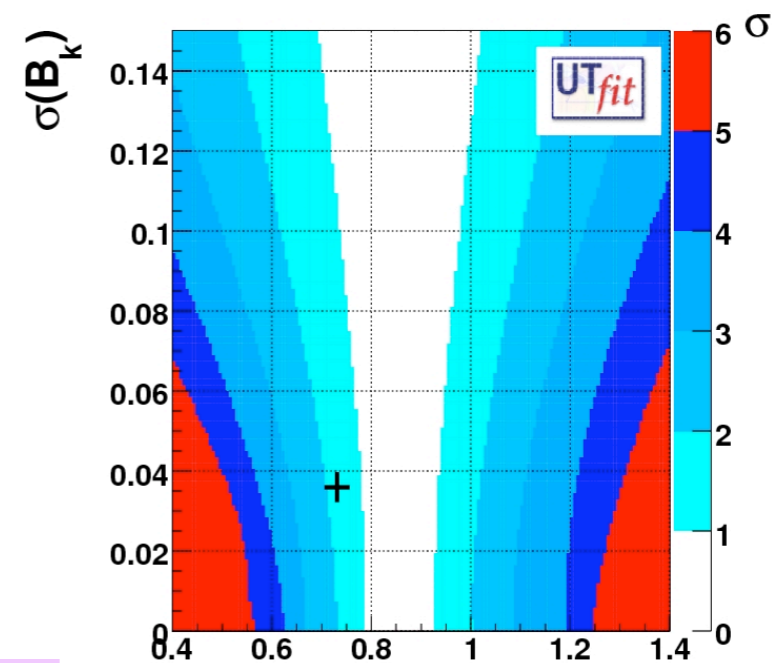


$V_{ub_{\text{exp}}} = (37.2 \pm 2.1) \cdot 10^{-4}$

$V_{ub_{\text{UTfit}}} = (35.8 \pm 1.1) \cdot 10^{-4}$

??

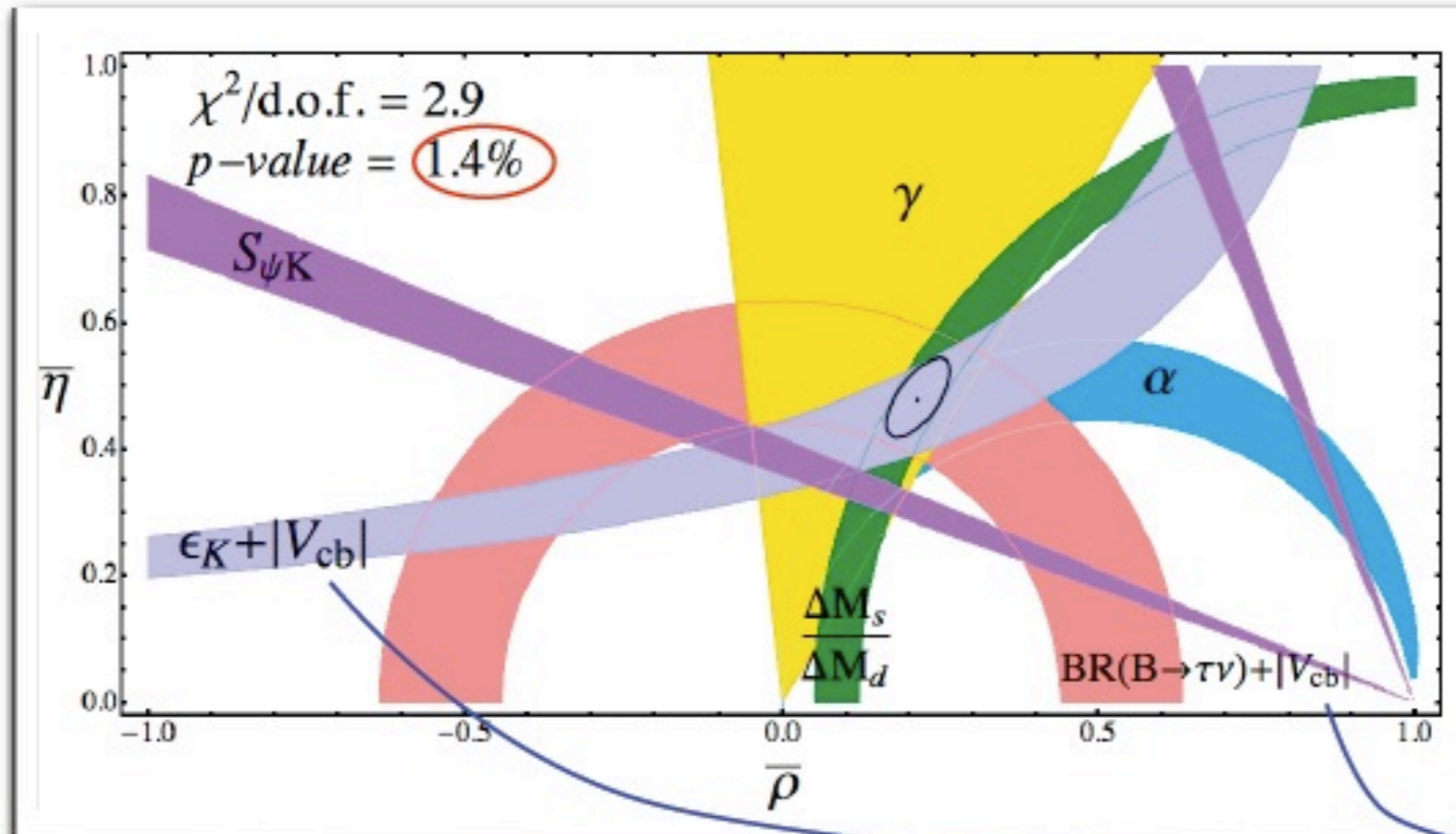
$< 1\sigma$ (incl $\sim 2.5\sigma$)



$\sim 1.6\sigma$

M. Bona
 UTfit@FPCP 2010

- V_{ub} is the *most controversial* input



$$[\sin 2\beta]_{\text{fit}} = 0.862 \pm 0.045 \Rightarrow 3.3 \sigma$$

$$[BR(B \rightarrow \tau\nu)]_{\text{fit}} = (0.784 \pm 0.098) \times 10^{-4} \Rightarrow 2.6 \sigma \quad \text{1.7}\sigma \text{ problem}$$

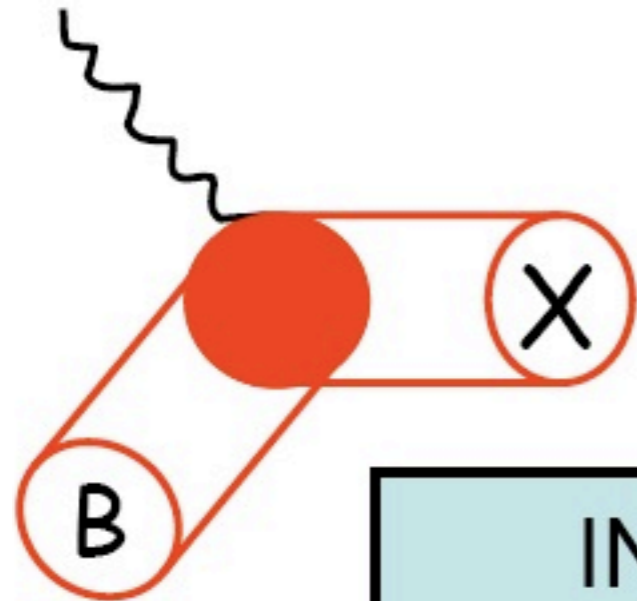
$$[\hat{B}_K]_{\text{fit}} = 0.914 \pm 0.086 \Rightarrow 2.4 \sigma$$

?
disturbing

Without V_{ub} the situation is actually worse.

But there is indeed a problem

Inclusive vs exclusive B decays



Simplicity: ew or em currents probe the B dynamics

INCLUSIVE	EXCLUSIVE
OPE : non-pert physics described by B matrix elements of local operators can be extracted by exp suppressed by $1/m_b^2$	Form factors : in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

As we aim at high precision, things are not at all simple...

Inclusive semileptonic B decays: basic features

- **Simple idea:** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators

$$T J(x) J(0) \approx c_1 \bar{b} b + c_2 \bar{b} \overrightarrow{D}^2 b + c_3 \bar{b} \boldsymbol{\sigma} \cdot \mathbf{G} b + \dots$$

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: **double series in $\alpha_s, \Lambda/m_b$**
- Lowest order: decay of a free b , linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} (i\overrightarrow{D})^2 b \right| B \right\rangle_\mu \quad \mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} \frac{i}{2} \boldsymbol{\sigma}_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_\mu$$

The total s.l. width in the OPE

$$\Gamma[\bar{B} \rightarrow X_c e \bar{\nu}] = \frac{G_F^2 m_b^5 |V_{cb}|^2 g(r)}{192\pi^3} \left[1 + \frac{\alpha_s}{\pi} p_c^{(1)}(r, \mu) + \frac{\alpha_s^2}{\pi^2} p_c^{(2)}(r, \mu) \right. \\ \left. - \frac{\mu_\pi^2}{2m_b^2} + \left(\frac{1}{2} - \frac{2(1-r)^4}{g(r)} \right) \frac{\mu_G^2 - \frac{\rho_{LS}^3 + \rho_D^3}{m_b}}{m_b^2} \right. \\ \left. + \left(8 \ln r - \frac{10r^4}{3} + \frac{32r^3}{3} - 8r^2 - \frac{32r}{3} + \frac{34}{3} \right) \frac{\rho_D^3}{g(r) m_b^3} \right] \\ + O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

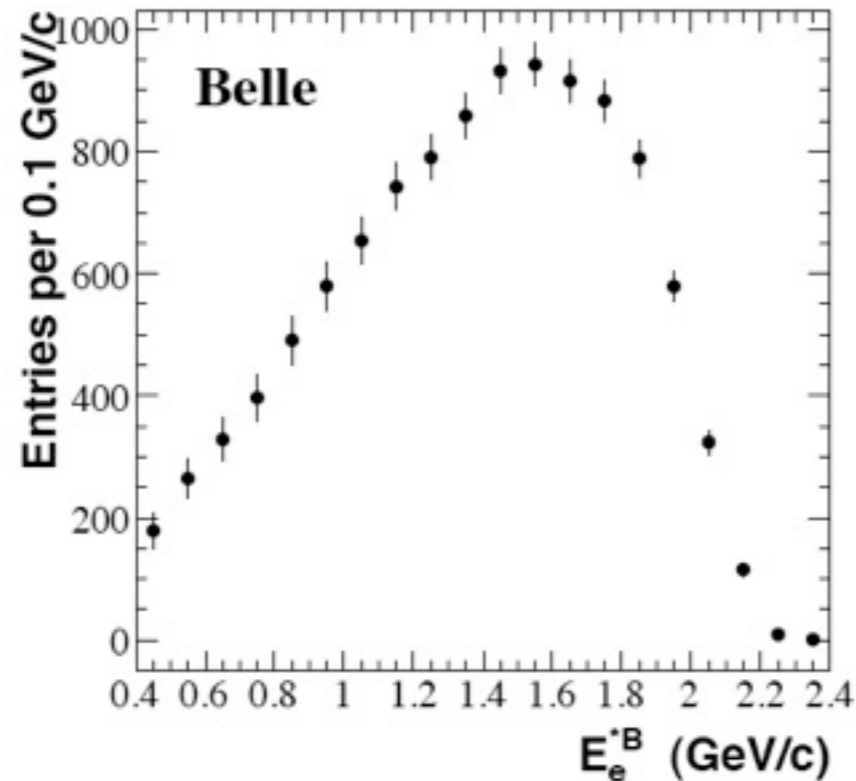
$$r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities \Rightarrow moments

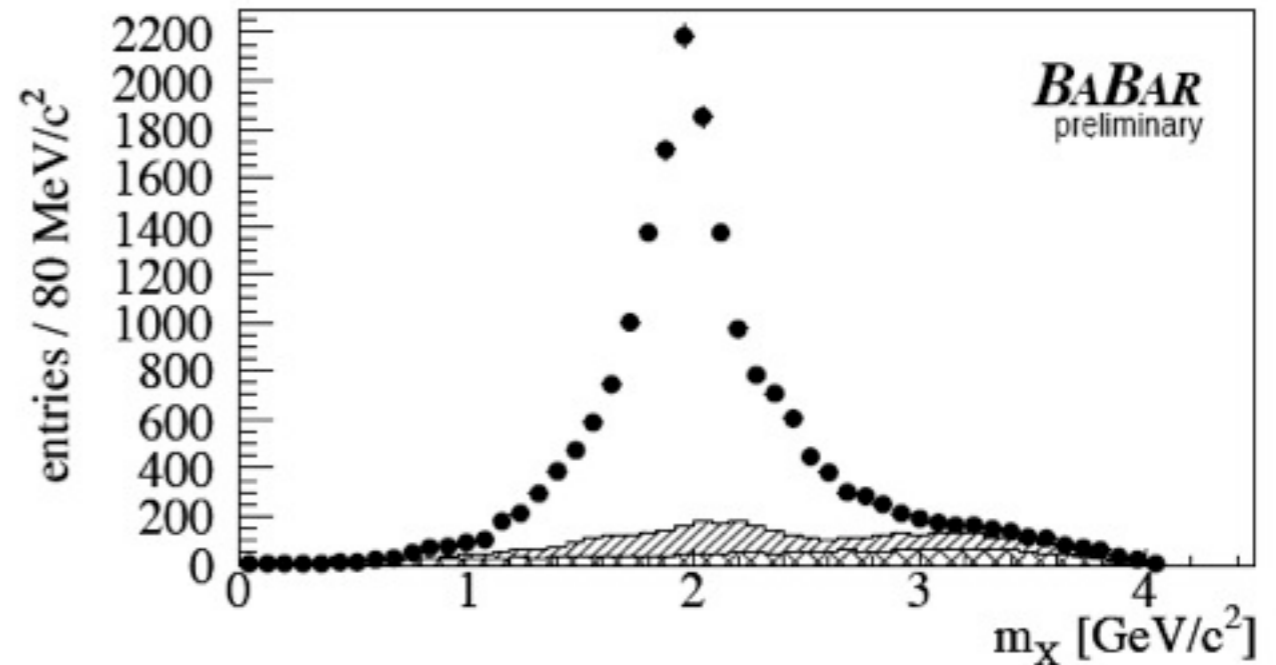
Present implementations include all terms through $O(\alpha_s^2 \beta_0, 1/m_b^3)$: $m_{b,c}, \mu_{\pi,G}^2, \rho_{D,LS}^3$ 6 parameters

Fitting OPE parameters to the moments

E_l spectrum



m_x spectrum



Total **rate** gives $|V_{cb}|$, global **shape** parameters (moments of the distributions) tell us about B structure, m_b and m_c

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications

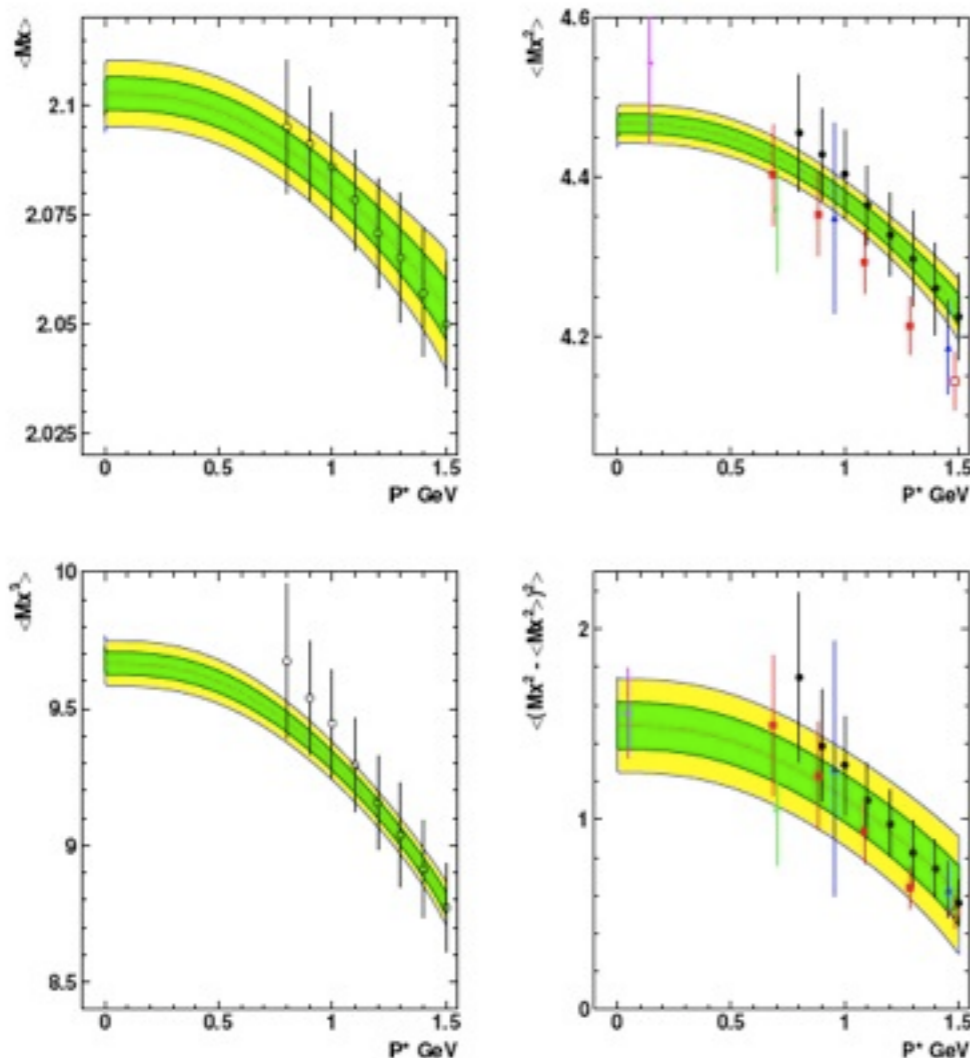
Global HFAG fit (kinetic scheme)

Inputs	$ V_{cb} \cdot 10^3$	m_b^{kin}	χ^2/ndf
$b \rightarrow c$ & $b \rightarrow s\gamma$	41.85(44)(58)	4.590(31)	29.7/59
$b \rightarrow c$ only	41.68(48)(58)	4.646(47)	24.2/48

Based on PG, Uraltsev, Benson et al

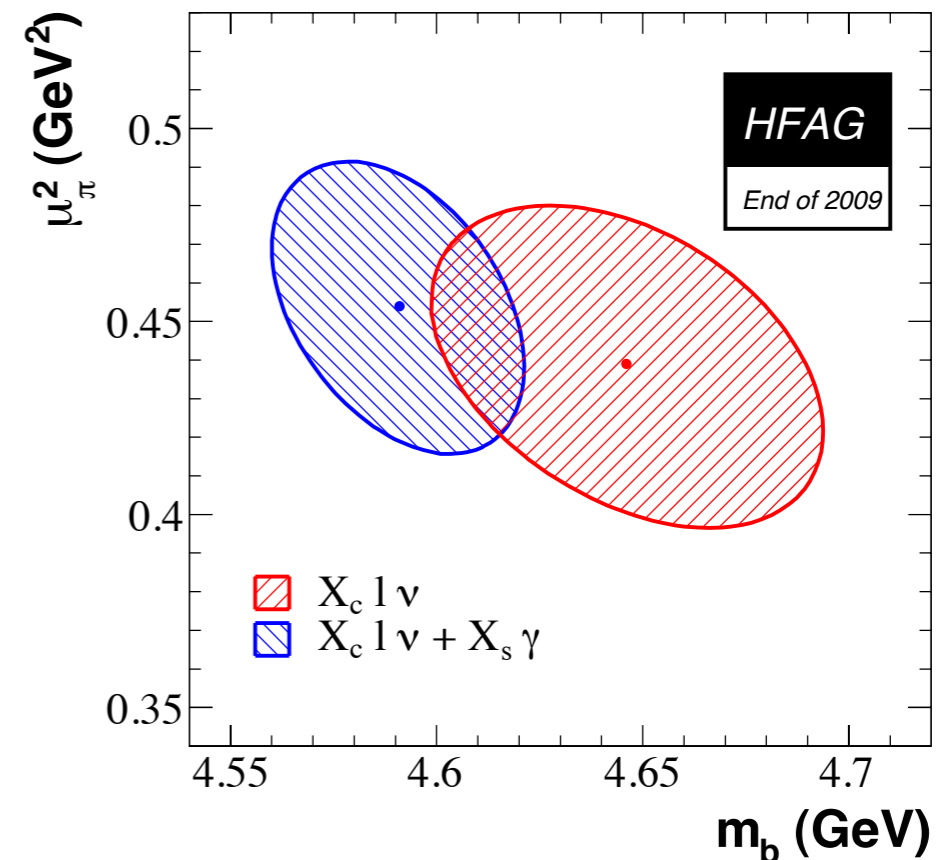
In the kinetic scheme the contributions of gluons with energy below $\mu \approx 1 \text{ GeV}$ are absorbed in the OPE parameters

Here scheme means also a number of different assumptions, inclusion of different data, and a recipe for theory errors

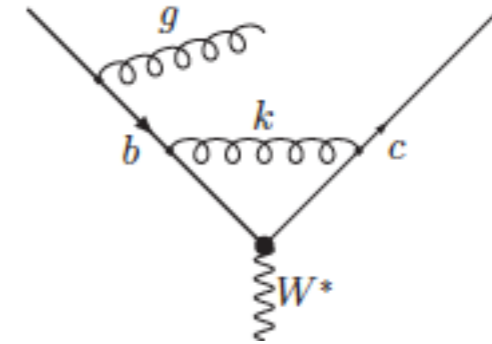


Very close result for $|V_{cb}|$ in $1S$ scheme

Bauer Ligeti Luke Manohar Trott



OPE at NNLO



- * Complete 2loop corrections to width and moments with cuts are now known, either in expansion m_c/m_b or numerically Biswas-Melnikov, Pak, Czarnecki

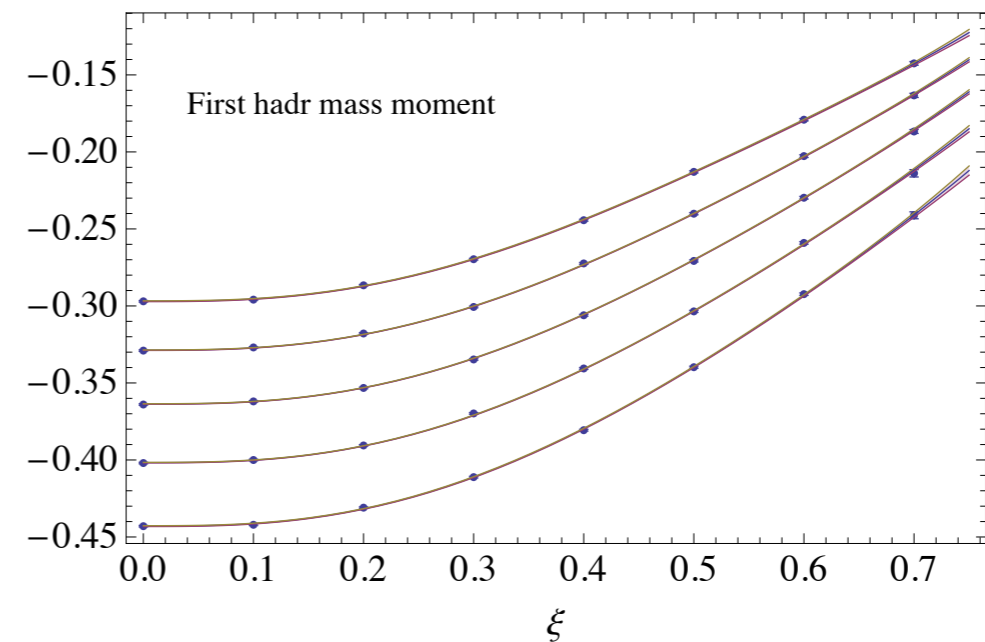
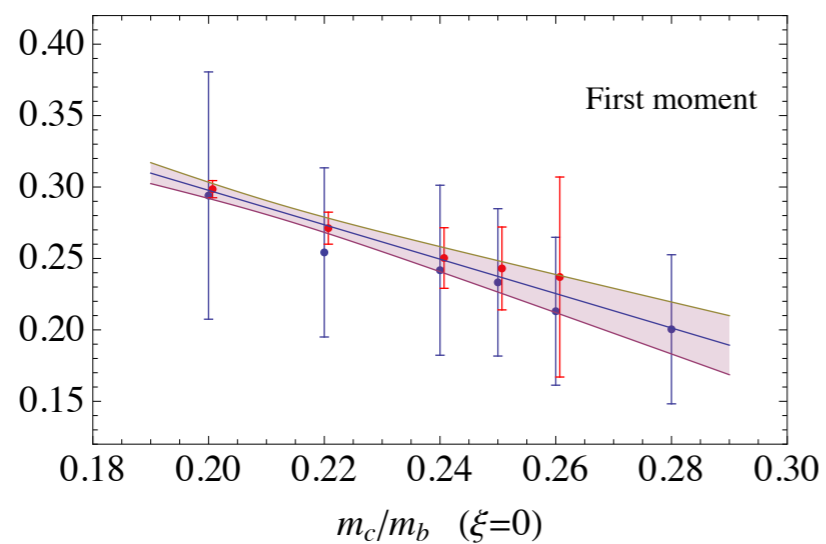
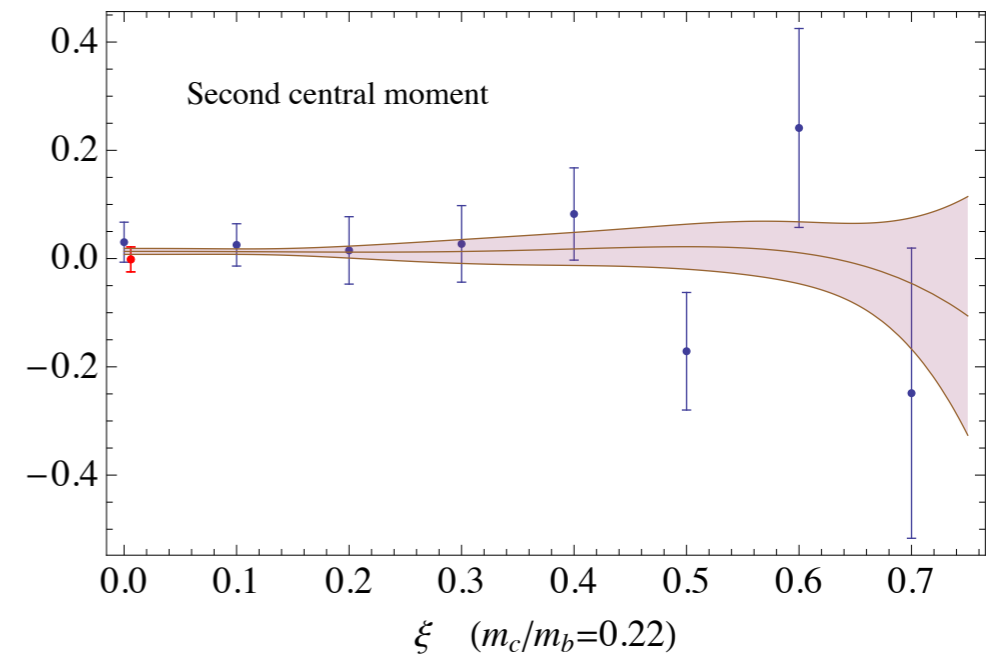
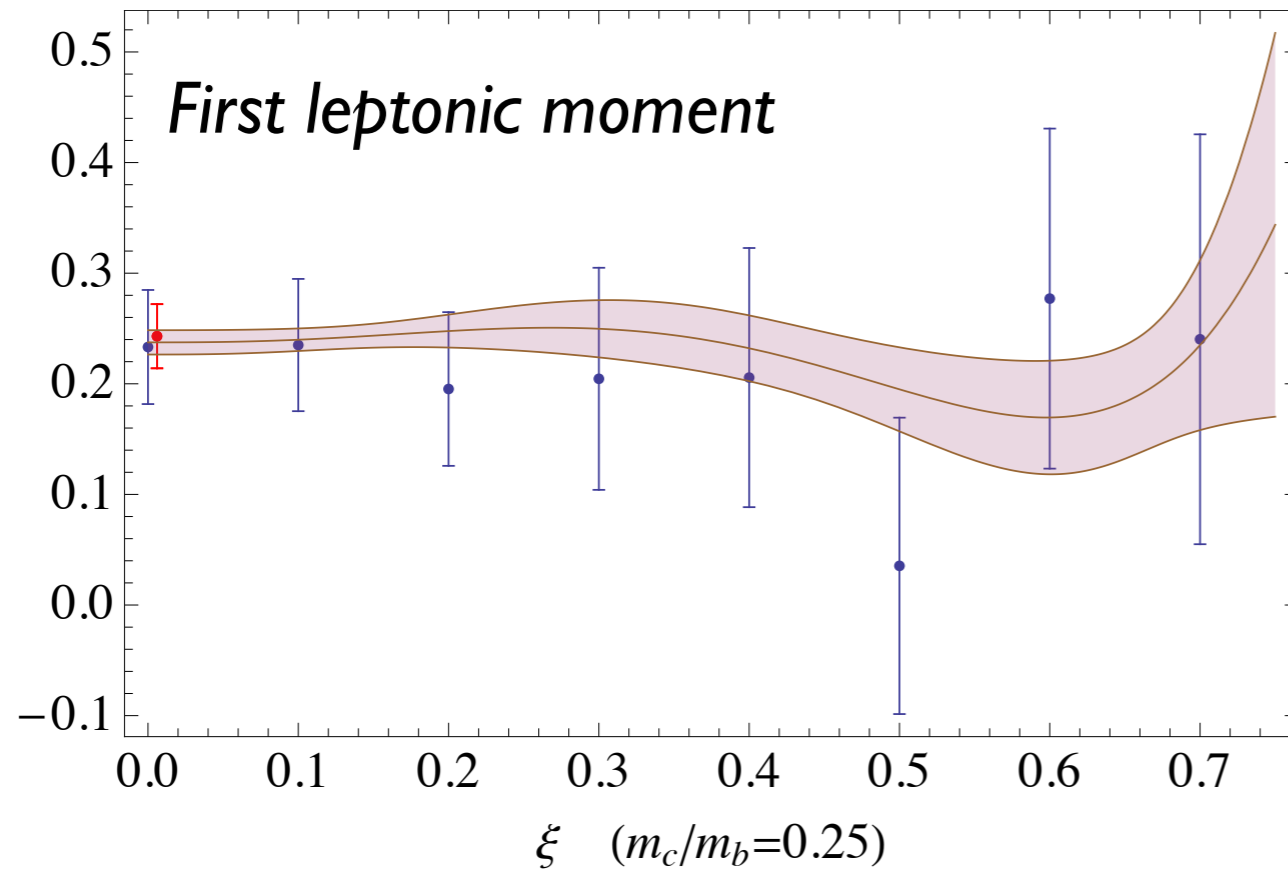
$$d\Gamma = \Gamma_0 \left[dF_0 + \frac{\alpha_s(m_b)}{\pi} dF_1 + \left(\frac{\alpha_s}{\pi}\right)^2 (\beta_0 dF_{\text{BLM}} + dF_2) + \dots \right]$$

- * Non-BLM minor corrections to BLM, residual th error on V_{cb} $O(0.5\%)$.
- * Strong cancellations between different contributions make NNLO to moments: non-accidental, numerical accuracy crucial

$$\begin{aligned} \langle E_l \rangle_{E_l > 1\text{GeV}} &= 1.54 \text{ GeV} \left[1 + (0.96_{den} - 0.93) \frac{\alpha_s}{\pi} + (0.48_{den} - 0.46) \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 \right. \\ &\quad \left. + [1.69(7) - 1.75(9)_{den}] \left(\frac{\alpha_s}{\pi}\right)^2 + O(1/m_b^2, \alpha_s^3) \right] \end{aligned}$$

$$\ell_2 = \langle E_\ell^2 \rangle - \langle E_\ell \rangle^2 = (2.479 - 2.393) \text{ GeV}^2 = 0.087 \text{ GeV}^2.$$

NNLO corrections to moments



NNLO results

	l_1	l_2	l_3	R^*
	$\mu = 0$			
tree	1.5674	0.0864	-0.0027	0.8148
$1/m_b^3$	1.5426	0.0848	-0.0010	0.8003
$O(\alpha_s)$	1.5398	0.0835	-0.0010	0.8009
$O(\beta_0\alpha_s^2)$	1.5343	0.0818	-0.0009	0.7992
$O(\alpha_s^2)$	1.5357(2)	0.0821(6)	-0.0011(16)	0.7992(1)
	$\mu = 1\text{GeV}$			
$O(\alpha_s)$	1.5455	0.0858	-0.0003	0.8029
$O(\beta_0\alpha_s^2)$	1.5468	0.0868	0.0005	0.8035
$O(\alpha_s^2)$	1.5466(2)	0.0866(6)	0.0002(16)	0.8028(1)
$O(\alpha_s^2)^*$	–	0.0865	0.0004	–
tot error [6]	0.0113	0.0051	0.0022	

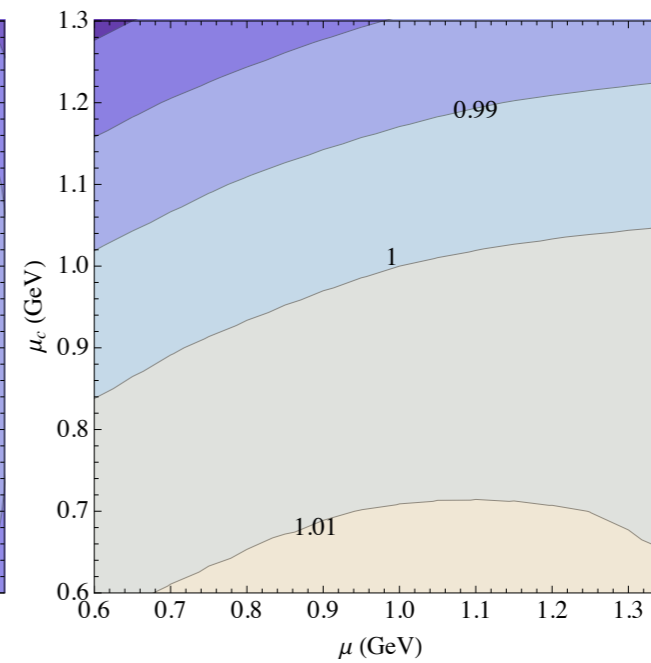
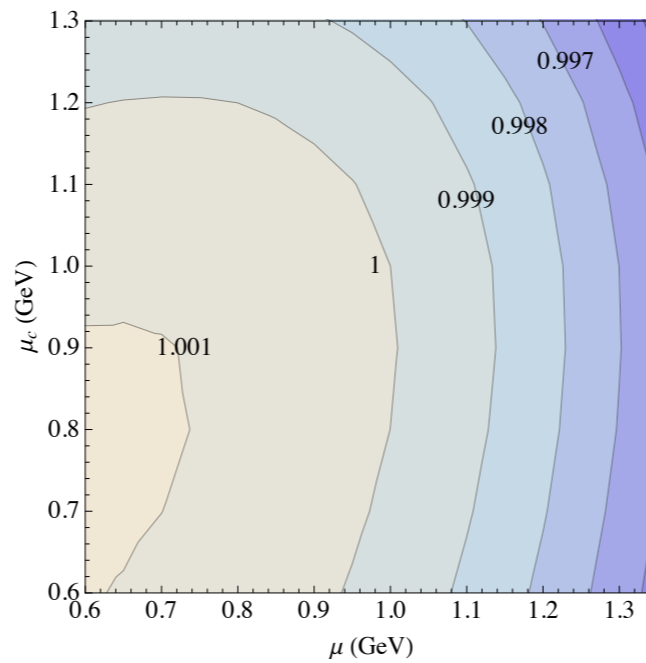
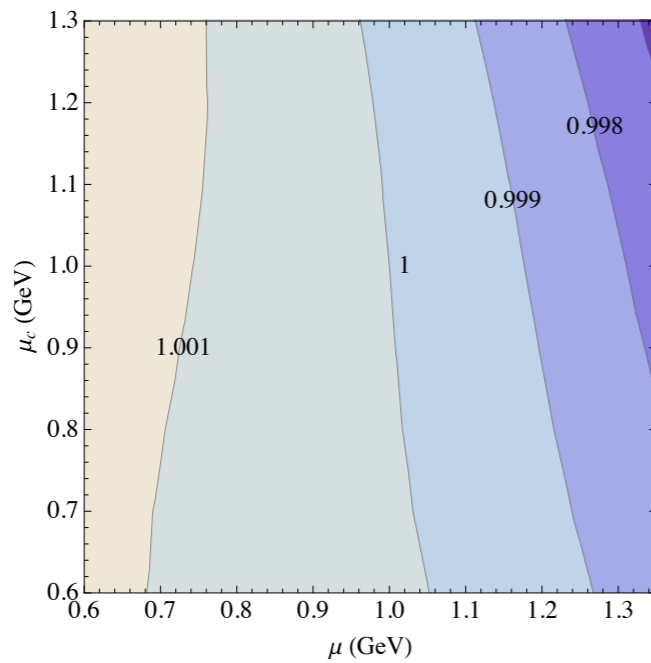
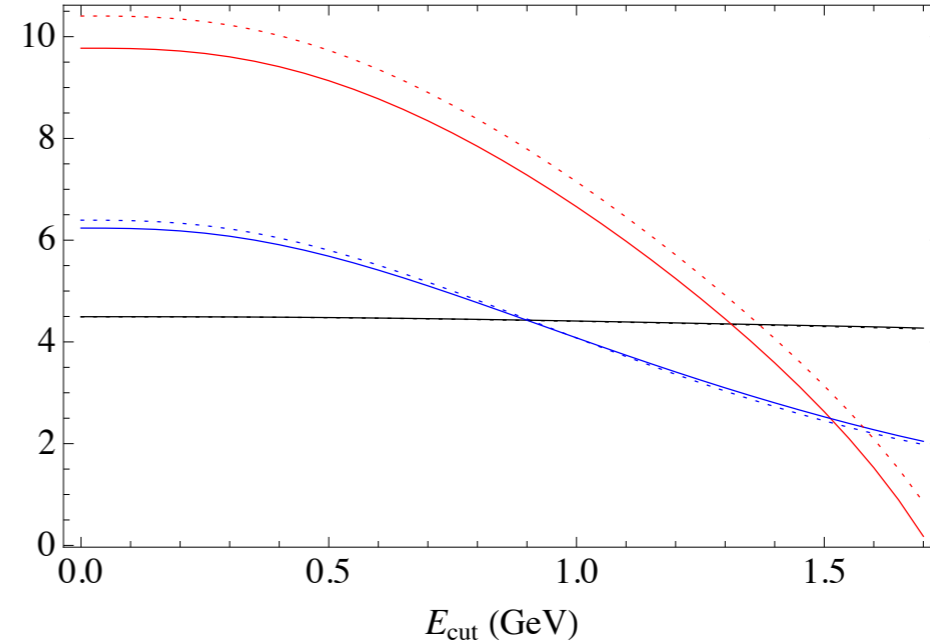
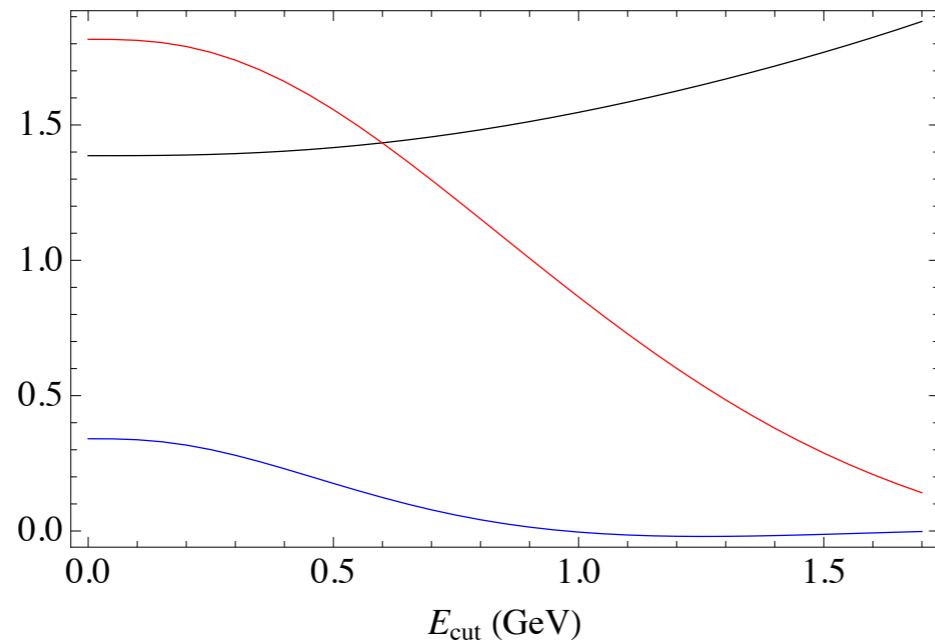
	$\mu = 1\text{GeV}, m_c^{\overline{\text{MS}}}(3\text{GeV})$			
	l_1	l_2	l_3	R^*
tree	1.6021	0.0940	-0.0043	0.8296
$1/m_b^3$	1.5748	0.0922	-0.0020	0.8159
$O(\alpha_s)$	1.5613	0.0894	-0.0004	0.8118
$O(\beta_0\alpha_s^2)$	1.5629	0.0904	0.0004	0.8125
$O(\alpha_s^2)$	1.5571(4)	0.0890(9)	-0.0008(25)	0.8090(2)
$O(\alpha_s^2)^*$	–	0.0889	0.0006	–

$E_{\text{cut}}=1\text{GeV}, m_c/m_b=0.25$

Small corrections. Cancellations may be partially spoiled by choice of scheme

	$\mu = 0$			$\mu = 1\text{GeV}$		
	h_1	h_2	h_3	h_1	h_2	h_3
LO	4.345	0.198	-0.02	4.345	0.198	-0.02
$1/m_b^3$	4.452	0.515	4.90	4.452	0.515	4.90
$O(\alpha_s)$	4.563	0.814	5.96	4.426	0.723	4.50
$O(\beta_0\alpha_s^2)$	4.701	1.105	6.85	4.404	0.894	4.08
$O(\alpha_s^2)$	4.682(1)	1.066(3)	6.69(4)	4.411(1)	0.832(4)	4.08(4)
tot error [6]				0.149	0.501	1.20

NNLO results



Kin. Cutoff dependence

$O(\alpha_s/m_b^2)$ effects in $B \rightarrow X_s \gamma$

Ewerth, Nandi, PG arXiv:0911.2175

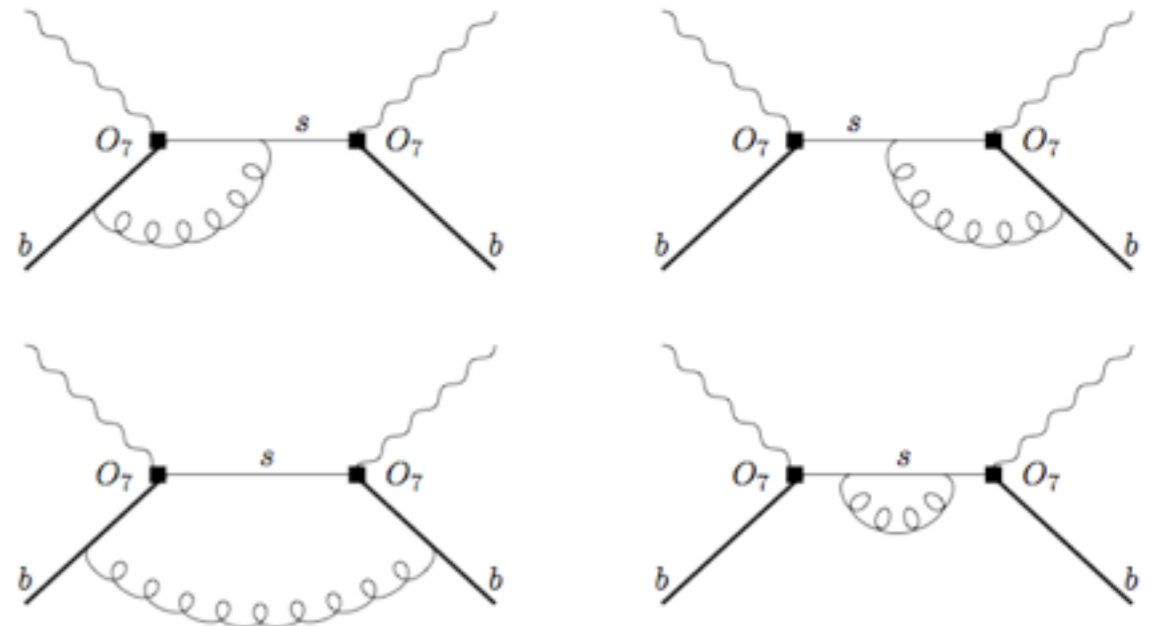
$$T\{\bar{b}(x)\sigma_{\mu\nu}P_L s(x)\bar{s}(0)\sigma_{\alpha\beta}P_R b(0)\} = c_{\text{dim } 3} O_{\text{dim } 3} + \frac{1}{m_b} c_{\text{dim } 4} O_{\text{dim } 4} + \frac{1}{m_b^2} c_{\text{dim } 5} O_{\text{dim } 5} + \dots$$

$$O_b^\mu = \bar{b}\gamma^\mu b,$$

$$O_2^{\mu\nu} = \bar{b}_v \frac{1}{2} \{iD^\mu, iD^\nu\} b_v,$$

$$O_1^\mu = \bar{b}_v iD^\mu b_v,$$

$$O_3^{\mu\nu} = \bar{b}_v \frac{g_s}{2} G^{a\mu}{}_\alpha \sigma^{\alpha\nu} T^a b_v,$$



One-loop matching onto local operators with HQET fields in dim reg

$$\frac{d\Gamma_{77}}{dz} = \Gamma_{77}^{(0)} \left[c_0^{(0)} + c_{\lambda_1}^{(0)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)} \frac{\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{4\pi} \left(c_0^{(1)} + c_{\lambda_1}^{(1)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(1)} \frac{\lambda_2(\mu)}{2m_b^2} \right) \right]$$

$\lambda_{1,2}$ are HQET analogues of $\mu_{\pi,G}^2$

The NLO effect 10-20% in coefficients of first few moments, leading to $\delta m_b \sim 10 \text{ MeV}$, $\delta \mu_\pi^2 \sim 0.04 \text{ GeV}^2$

Extension to semileptonic case almost complete: these corrections likely more important than non-BLM ones.

$O(\alpha_s \mu_\pi^2 / m_b^2)$ to moments known numerically Becher, Boos, Lunghi

Higher power corrections

Mannel,Turczyk,Uraltsev 1009.4622
see also Bigi,Mannel,Turczyk,Uraltsev

Proliferation of non-pert parameters: for ex at $1/m_b^4$ Bigi,Uraltsev,Zwicky

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$

$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$

$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$

$$2M_B m_4 = g \langle \vec{p} \cdot \text{rot } \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$

$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$

$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$

$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$

$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by Ground State Saturation

$$\langle \Omega_0 | \bar{Q} iD_j iD_k iD_l iD_m Q | \Omega_0 \rangle = \langle \Omega_0 | \bar{Q} iD_j iD_k Q | \Omega_0 \rangle \langle \Omega_0 | \bar{Q} iD_l iD_m Q | \Omega_0 \rangle$$

$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma} \approx 0.013 \quad \frac{\delta V_{cb}}{V_{cb}} \approx +0.4\%$$

after inclusion of the corrections in the moments. While this might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.*

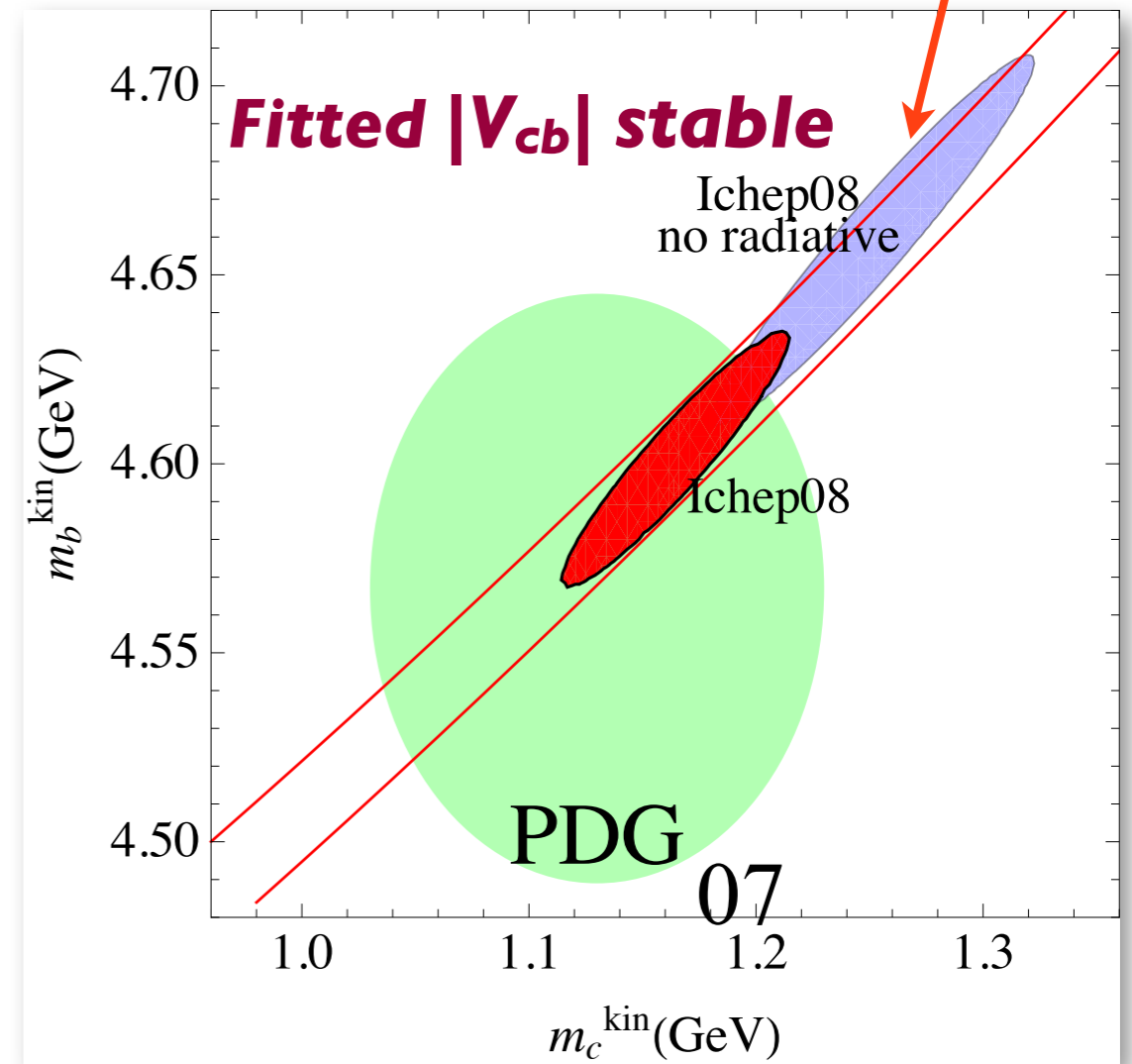
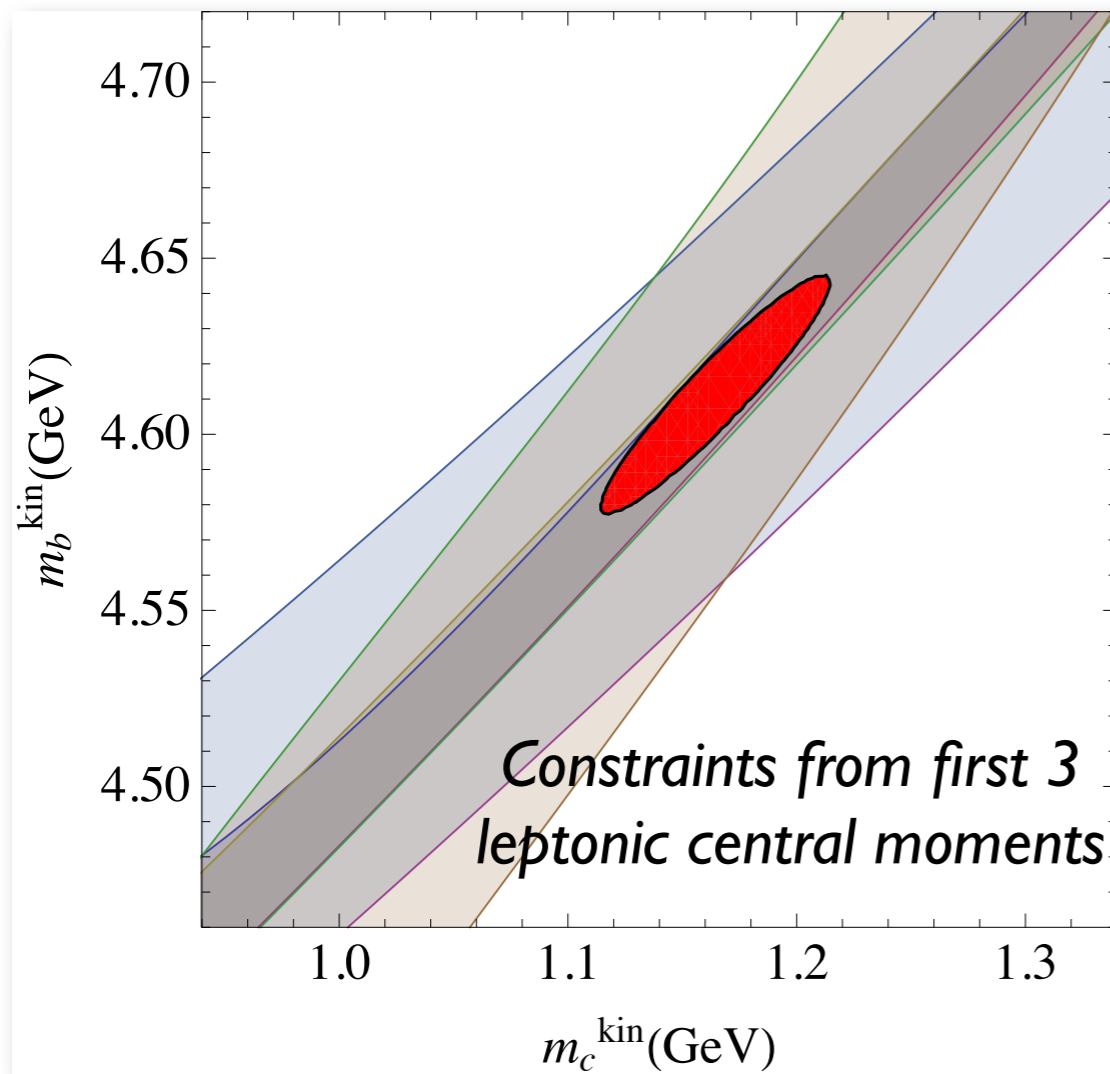
Semileptonic fits and heavy quark masses

- Results of fits to semileptonic & radiative moments are crucial input in inclusive $|V_{ub}|$ determination (mostly m_b and μ_π^2) and in normalizing $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$
- b quark mass determinations from e^+e^- have recently improved significantly: how do they compare with fits? do we understand/trust theory errors?
- Work in progress to use additional inputs (masses) in the fits, to control problems due to highly correlated theoretical inputs, to understand better various uncertainties. Role of radiative moments equivalent to loose PDG m_b constraint.

C.Schwanda, PG

A strip in the m_b - m_c plane

Constant values
of s.l. width
at fixed V_{cb}



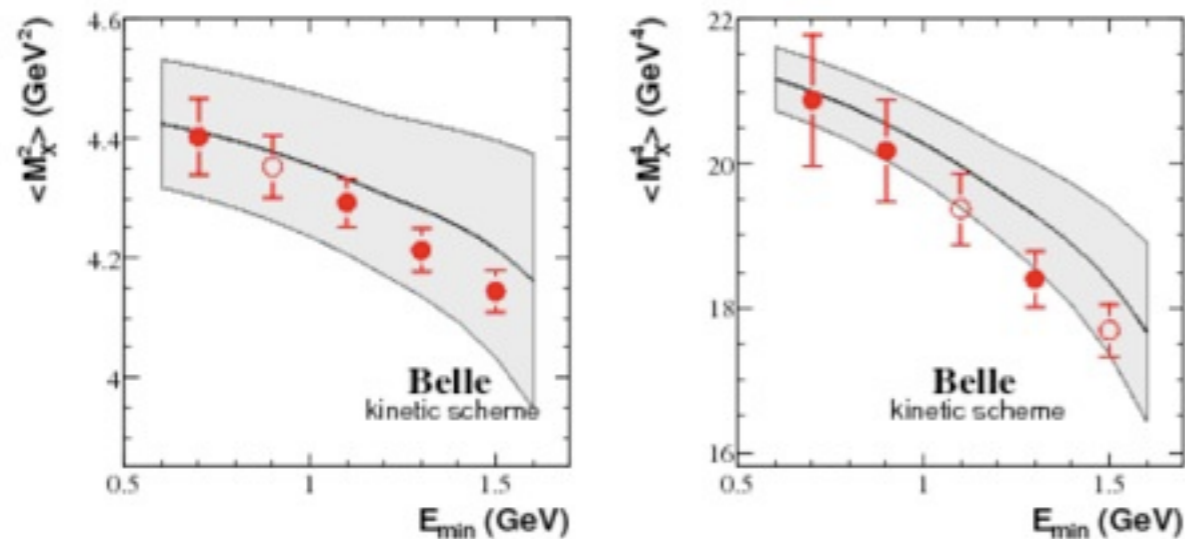
Semileptonic moments do not measure m_b well. They rather identify a strip in (m_b, m_c) plane along which the minimum is shallow.

Unknown non-pert $O(\alpha_s/m_b)$ effects in radiative moments. Possibly irrelevant here but must be studied. But role of radiative moments in the fits is equivalent to using loose bound $m_b(m_b) = 4.20(7) \text{ GeV}$

How reliable are mass determinations?

Collaboration with C. Schwanda, in progress

I. Theoretical correlations

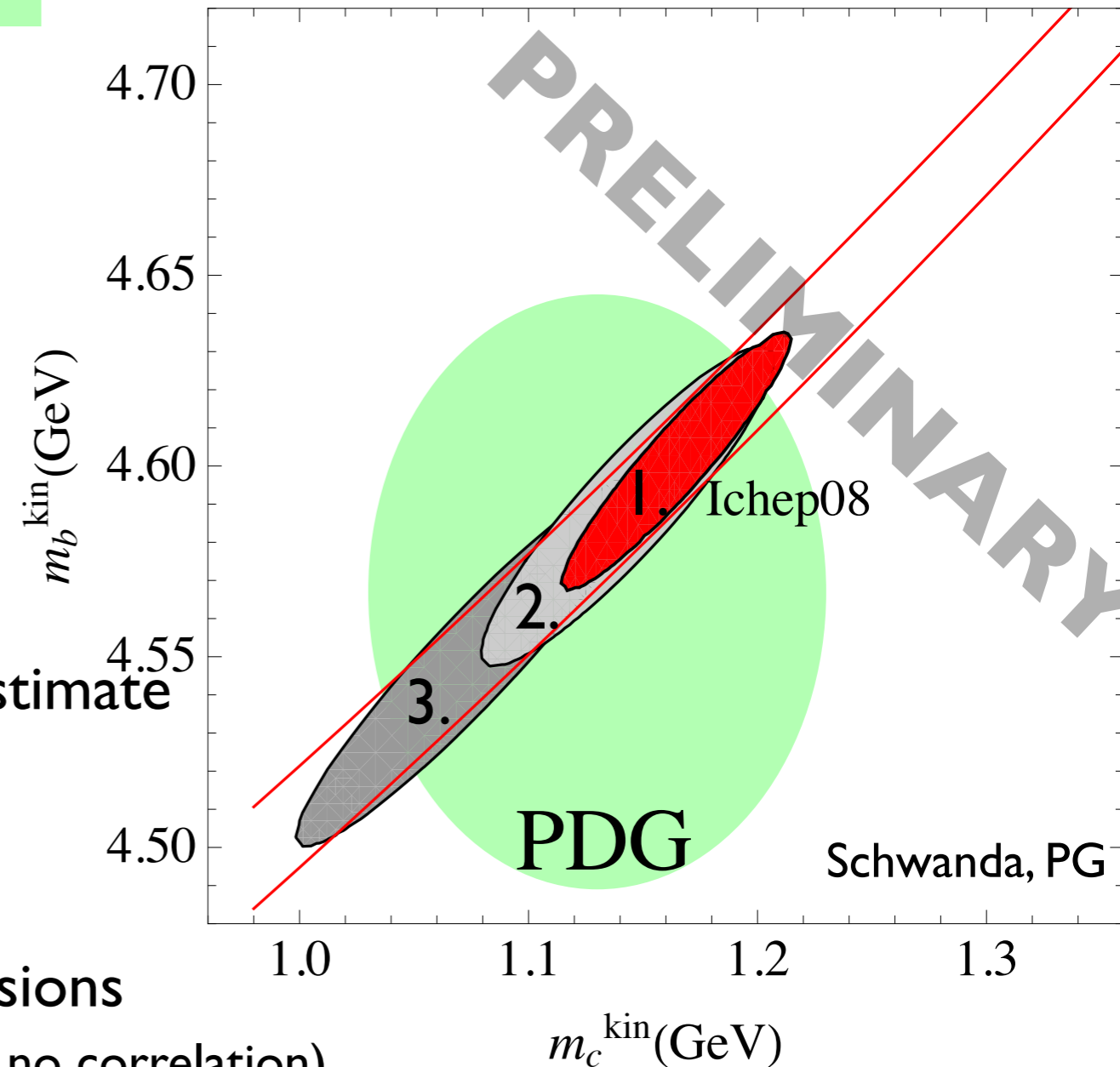


Correlations between theory errors of moments with different cuts difficult to estimate

Examples:

1. 100% correlations
2. corr. computed from low-order expressions
3. experimental correlations (very similar to no correlation)

always assume different central moments uncorrelated



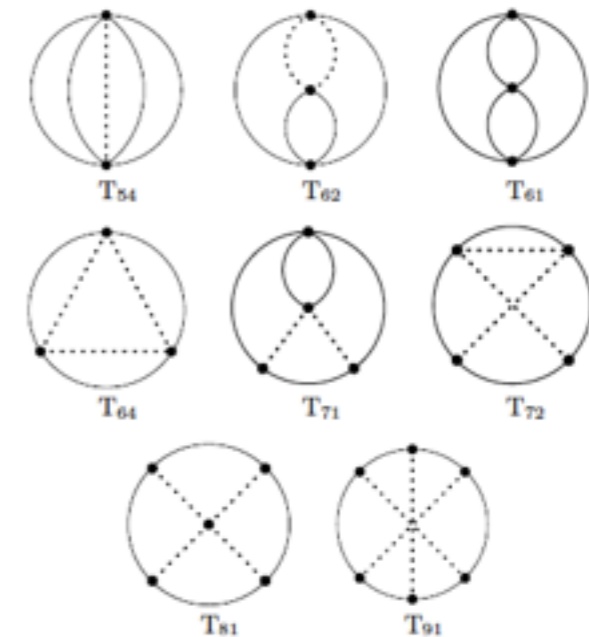
c,b masses from SVZ sum rules

$$R(s) = 12\pi \text{Im} [\Pi(q^2 = s + i\epsilon)] \quad (-q^2 g_{\mu\nu} + q_\mu q_\nu) \Pi(q^2) \equiv i \int dx e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\text{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_S}{\pi} \bar{C}_n^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 \bar{C}_n^{(3)} + \dots$$



Boughezal, Czakon, Schutzmeier (2006)
Kuhn, Steinhauser, Sturm (2006)

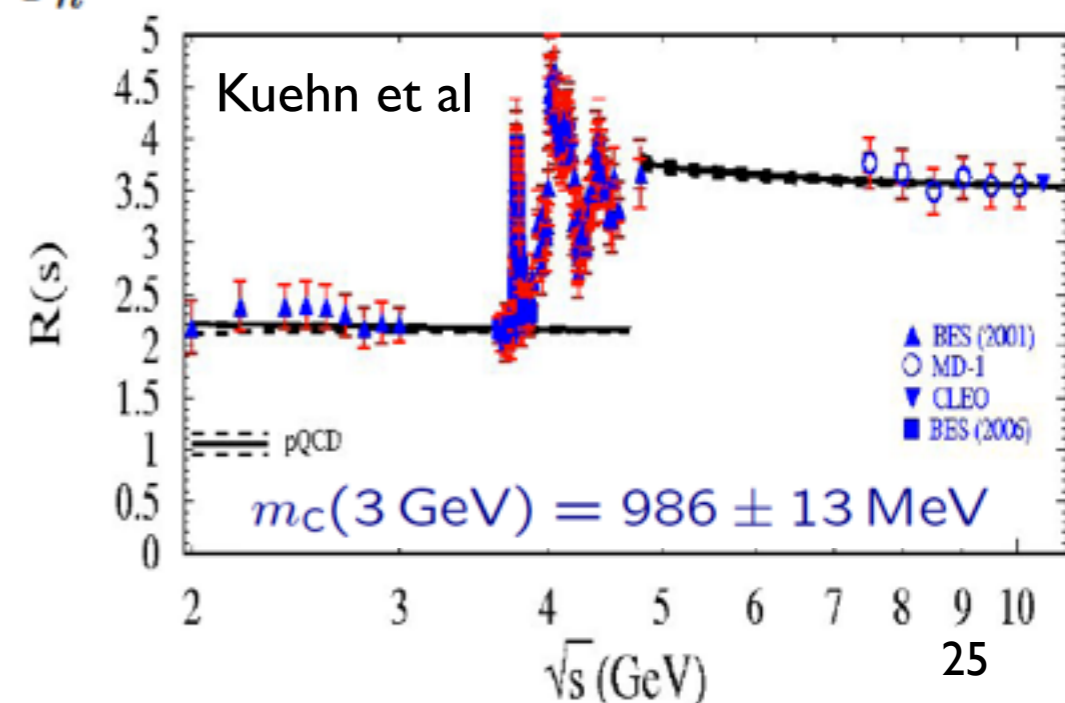
Relation to measurements

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_c(q^2) \Big|_{q^2=0} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2} \right)^n \bar{C}_n$$

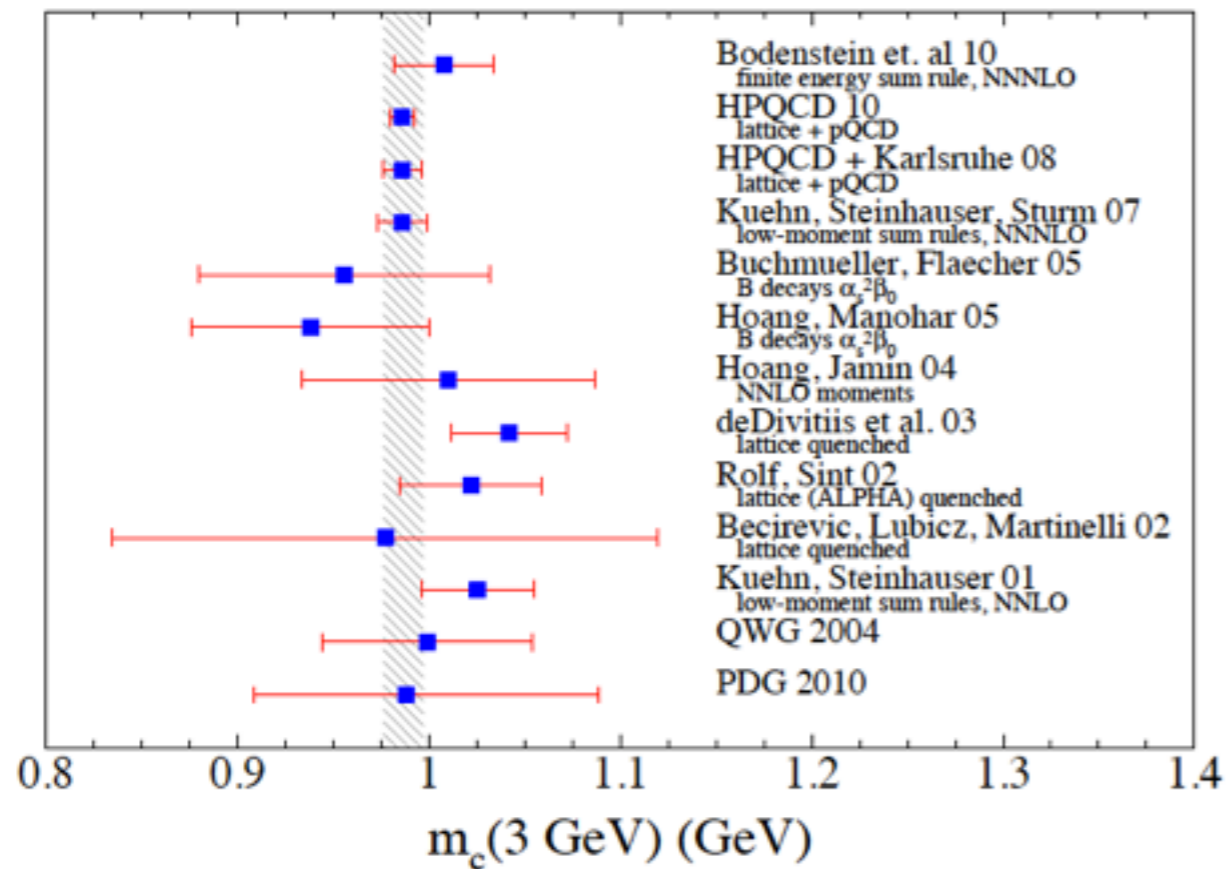
$$\Pi_c(q^2) = \frac{q^2}{12\pi^2} \int ds \frac{R_c(s)}{s(s - q^2)} + \text{subtraction}$$

$$\Leftrightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R_c(s) = \mathcal{M}_n^{\text{th}}$$

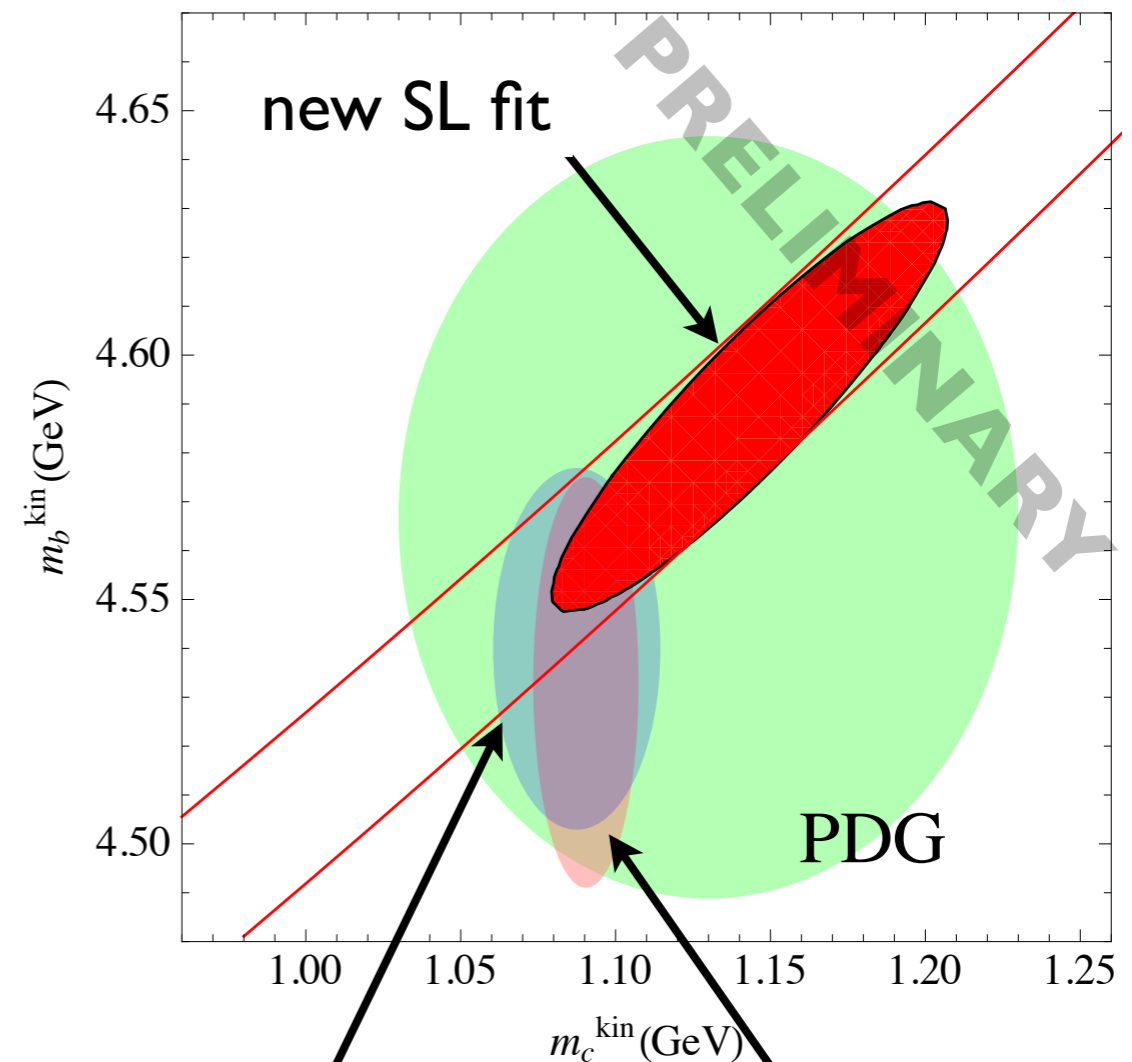
moments can also be *measured* on the lattice!



Using mass determinations



Recent sum rules determinations converted to kin scheme



Comparisons and combinations for $m_{b,c}$ penalized by changes of scheme.

Direct fit to $m_c(3\text{GeV})$ with Karlsruhe constraint on m_c leads to

$$m_b^{\text{kin}} = 4.535(21)\text{GeV}$$

$$\Rightarrow m_b(m_b) = 4.165(45)\text{GeV} \text{ Consistent!}$$

Hoang et al 2010
Hoang (m_b)

Kuhn et al 2009

Exclusive decays: $B \rightarrow D^* l \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A(1 + \delta_{1/m^2})$$

Recent progress in the measurement of slopes and shape parameters *Despite extrapolation, exp error is only ~2%*

Main problem is the ff $F(1)$: cannot be experimentally determined or constrained

New unquenched Lattice QCD (only group):

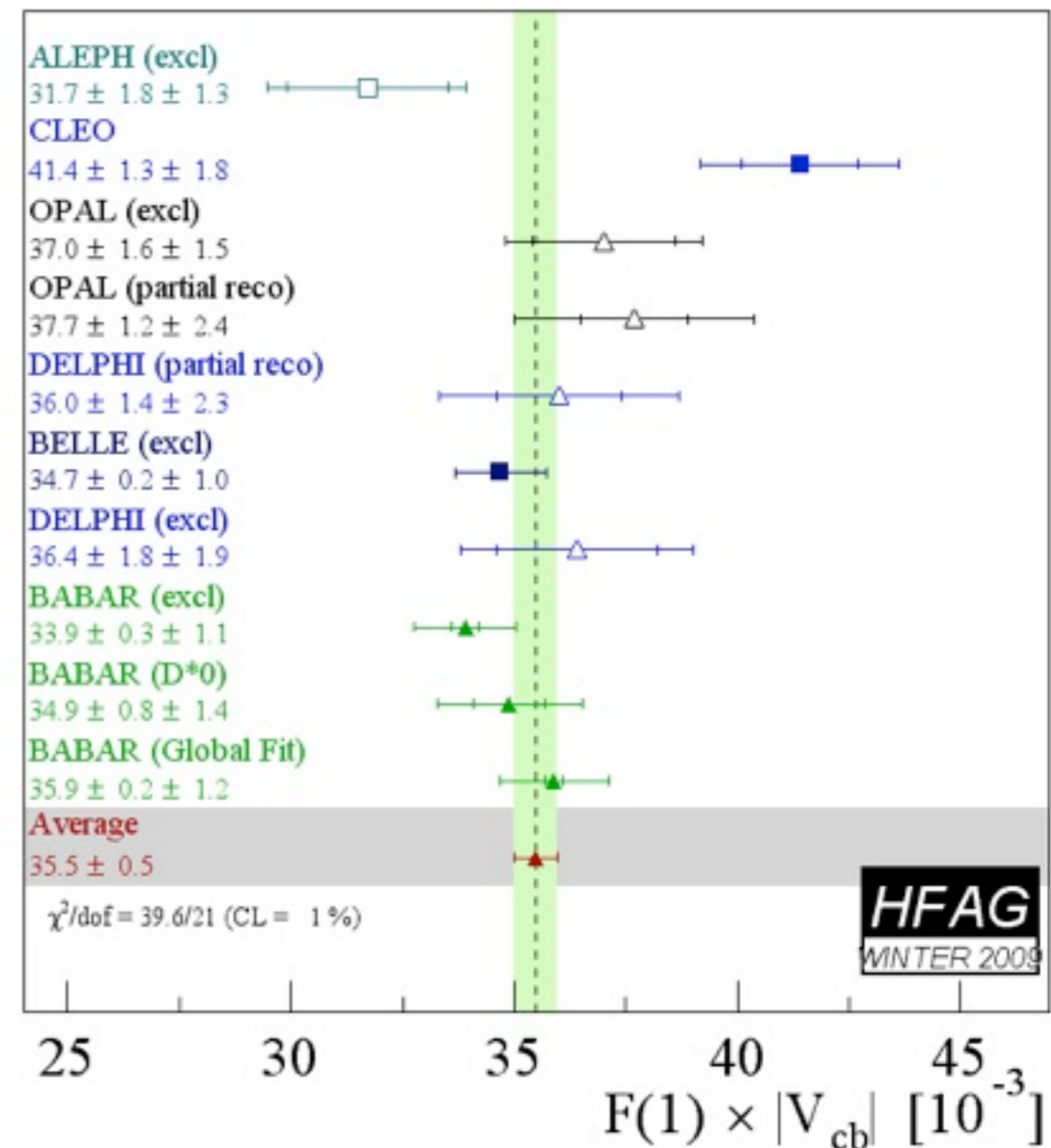
$$F(1) = 0.908(17) \quad \text{Laiho et al 2010}$$

$$|V_{cb}| = 39.1(1.1)(0.7) \times 10^{-3}$$

$\sim 1.8\sigma$ from inclusive determination

2.5% error

$B \rightarrow D l \nu$ gives consistent result with larger errors $|V_{cb}| = 39.1(1.4)(1.3) \times 10^{-3}$



Promising alternative: w dependence, only quenched *de Divitiis et al*

Heavy Quark Sum Rule for $B \rightarrow D^* l \nu$

The local OPE for inclusive B decays provides a (unitarity) bound on $F(I)$:

$$F_{D^*}^2 + \sum_{f \neq D^*} |F_{B \rightarrow f}|^2 = \xi_A^{\text{pert}} - \frac{\mu_G^2}{3m_c^2} - \underbrace{\frac{\mu_\pi^2 - \mu_G^2}{4}}_{>0} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) - \Delta_{\frac{1}{m_Q^3}} + \Delta_{\frac{1}{m_Q^4}} + \dots$$

inelastic >0

A strict bound follows for zero inelastic contributions.

$$\sqrt{\xi_A^{\text{pert}}(0.75 \text{ GeV})} = 0.985 \pm 0.01 \quad \Delta_{\text{power}}(0.75 \text{ GeV}) = 0.09 + 0.03 - 0.02 \approx 0.10$$

$$\mathbf{F(I) < 0.93}$$

Uraltsev, Mannel, PG arXiv:1004.2859

Also the inelastic piece can be estimated, although with large uncertainty. It typically leads to $\mathbf{F(I) \approx 0.86}$, in agreement with V_{cb} inclusive.

The total $B \rightarrow X_u l \nu$ width in the OPE

$$\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}] = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3} \left[1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r, \mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \right. \\ \left. + \left(\frac{77}{6} + 8 \ln \frac{\mu_{\text{WA}}^2}{m_b^2} \right) \frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\text{WA}}(\mu_{\text{WA}}) \right] \\ + O\left(\alpha_s \frac{\mu_{\pi, G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

Life could be relatively easy
with the total width...

Weak Annihilation

Weak Annihilation

Spectator dependent non-pert contribution localized at max E_l (or max q^2)

Bigi, Uraltsev 1993

In principle affects B^+ only but WA mixes with Darwin operator at $O(1)$.

Isosinglet component can be as large as isotriplet

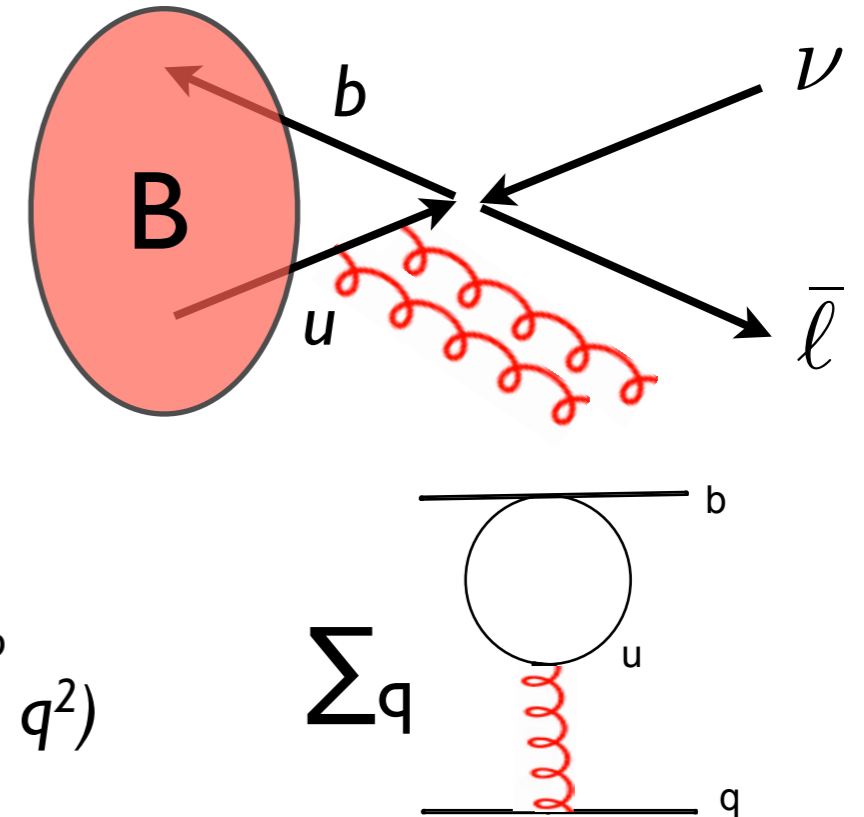
Difficult to study on the lattice, can be constrained experimentally Rosner et al [Cleo coll]

WA may pollute all present inclusive determinations of V_{ub} and more severely the less inclusive ones (E_l endpoint, high q^2)

D_s and D_0 rates differ significantly, (Cleo-c [arXiv:0912.4232](https://arxiv.org/abs/0912.4232))

$$\Gamma(D^+ \rightarrow Xe^+\nu)/\Gamma(D^0 \rightarrow Xe^+\nu) = 0.985(28)$$

$$\Gamma(D_s^+ \rightarrow Xe^+\nu)/\Gamma(D^0 \rightarrow Xe^+\nu) = 0.828(57)$$



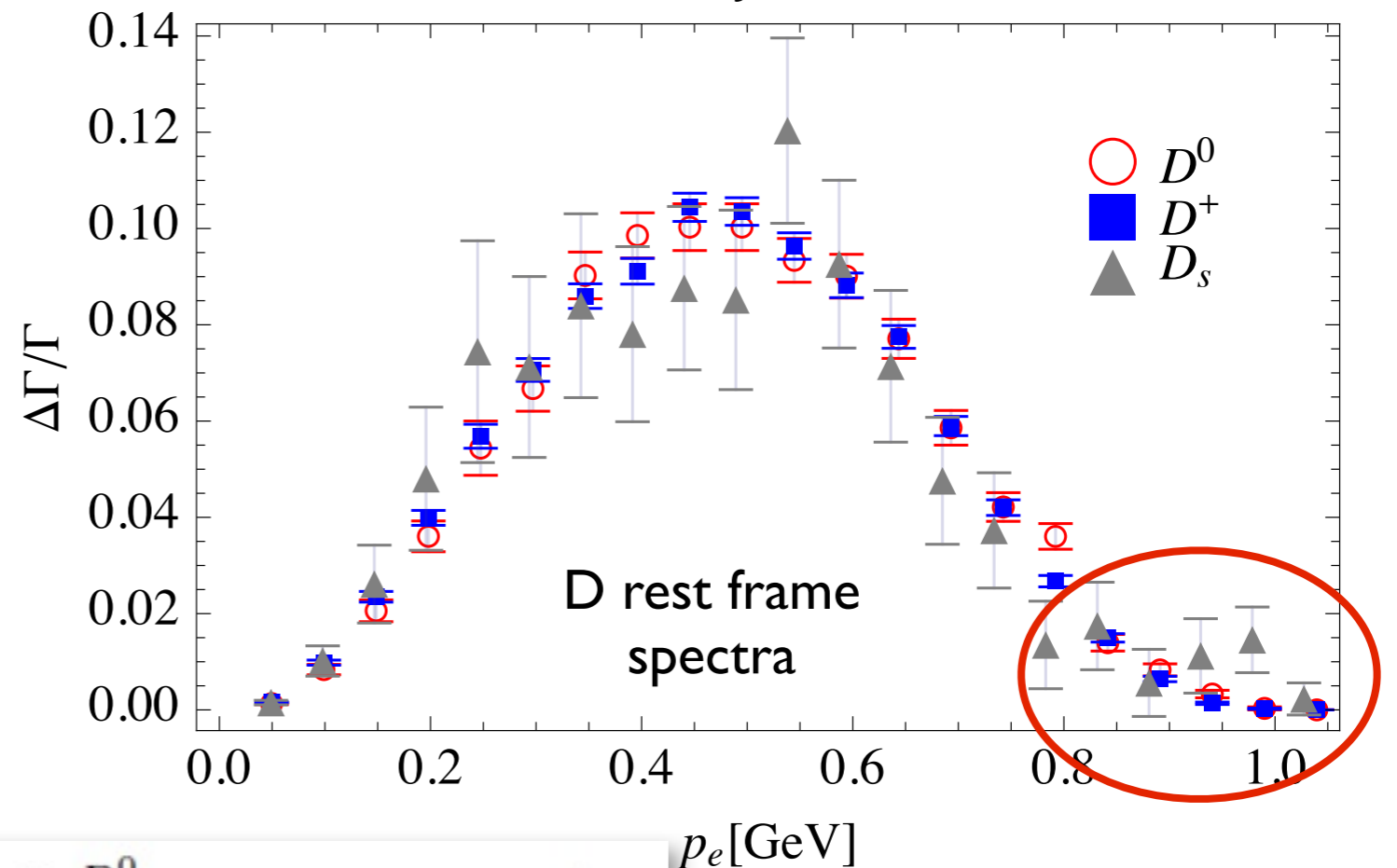
Valence WA Cabibbo suppressed in D^+ , absent in D^0 , is it a sign of WA?

Bigi, Mannel, Turczyk, Uraltsev 0911.3322 Ligeti, Luke, Manohar 1003.1351

Cleo-c electron spectra

JF Kamenik, PG, arXiv:1004.0114

- Cleo also measured the electron spectra for $p > 0.2 \text{ GeV}$. We extrapolated them to $p=0$, computed their first moments, and boosted to the D rest frame
- Moments should follow OPE, but are less sensitive to power and pert corrections than widths



$$\langle E_\ell \rangle_{exp}^{D^0} = 0.459(3) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D^0} = 0.240(2) \text{ GeV}^2,$$

$$\langle E_\ell \rangle_{exp}^{D^+} = 0.455(1) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D^+} = 0.236(1) \text{ GeV}^2,$$

$$\langle E_\ell \rangle_{exp}^{D_s} = 0.456(11) \text{ GeV},$$

$$\langle E_\ell^2 \rangle_{exp}^{D_s} = 0.239(12) \text{ GeV}^2,$$

Moments

No evidence for spectator effects! Is there really evidence for WA?

SU(3) violation in charm

- Decay constants on the lattice:

$$f_D=260(10)\text{MeV} \quad \text{vs} \quad f_{D_s}=217(10)\text{MeV} \quad \text{Bazavov et al}$$

- Hyperfine splittings

$$\Delta_{D_q}^{hf} = 3(m_{D_q^*}^2 - m_{D_q}^2)/4 \approx \mu_G^2$$

$$\Delta_{D^+}^{hf} = 0.409(1)\text{GeV}^2, \quad \Delta_{D^0}^{hf} = 0.413(1)\text{GeV}^2, \quad \Delta_{D_s}^{hf} = 0.440(2)\text{GeV}^2$$

- SU(3) violation can be as large as 20%. Widths get much larger power corrections than moments and this might partially explain the observed width difference without WA

Results and implications J Kamenik, PG 1004.0114

Allowing for 20% SU(3) violation in the OPE parameters

$$\Delta B_{\text{WA}}^{(0),s} \equiv B_{\text{WA}}^{(0),s}(D_s) - B_{\text{WA}}^{(0),s}(D^0) \quad \text{Valence component}$$
$$\Delta B_{\text{WA}}^{(0),s} = 0.0000(12)(3)\text{GeV}^3 \quad \text{always compatible with zero}$$

$$B_{\text{WA}}^s = -0.0003(25)\text{GeV}^3 \quad \text{Singlet component}$$

*In worst dilution scenario from the moments alone (linearly adding errors)
equivalent to 30% error on rate*

A factor ~2.5 in going to B: $|B_{\text{WA}}^b(\mu_{\text{WA}} = 0.8\text{GeV})| \lesssim 0.006\text{GeV}^3$ **Singlet**

$$-0.004\text{GeV}^3 \lesssim \Delta B_{\text{WA}}^b(\mu_{\text{WA}} = 0.8\text{GeV}) \lesssim 0.002\text{GeV}^3 \quad \text{Valence}$$

Max 1% effect on V_{ub} for most inclusive analyses
 B^0 and B^+ inclusive widths should not differ more than about 1%

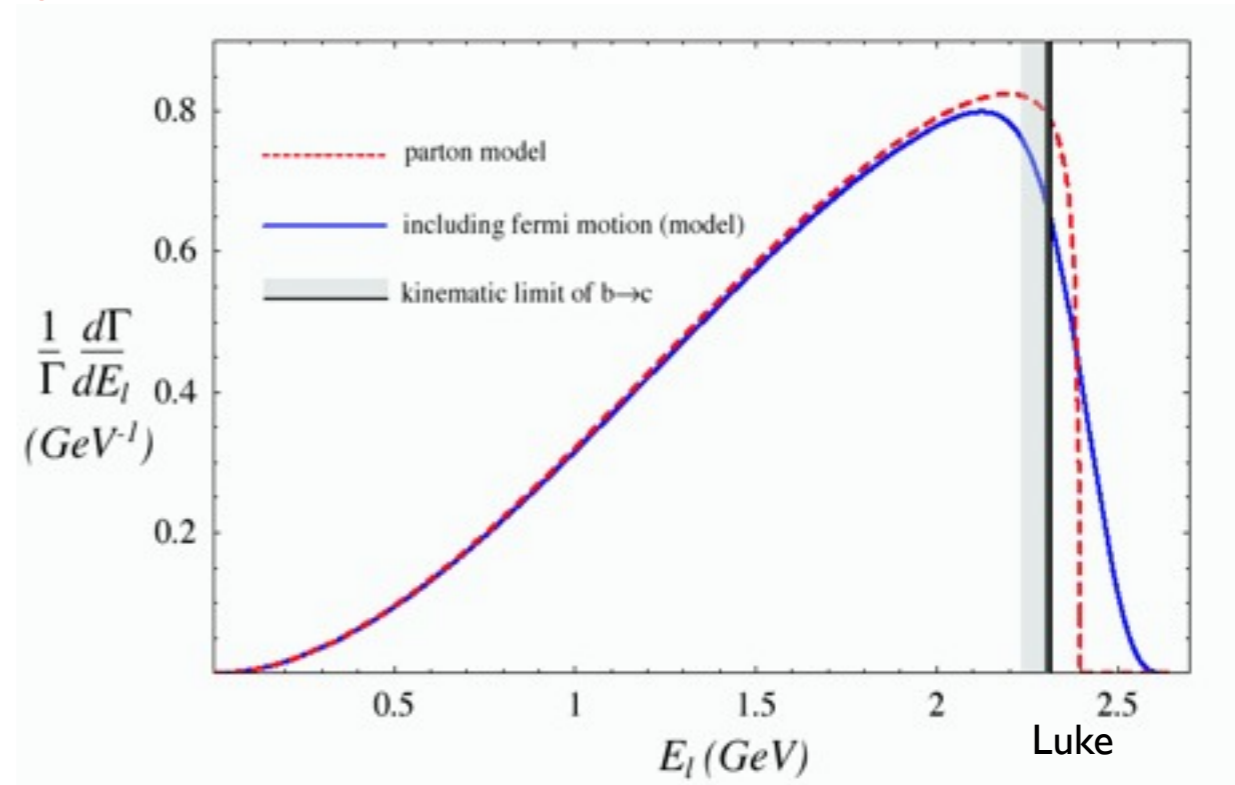
The problems with cuts

Experiments generally need kinematic cuts to avoid the $\sim 100x$ larger $b \rightarrow cl\nu$ background:

$$m_X < M_D \quad E_l > (M_B^2 - M_D^2)/2M_B \quad q^2 > (M_B - M_D)^2 \dots$$

The cuts destroy convergence of the OPE that works so well in $b \rightarrow c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** $f(k_+)$



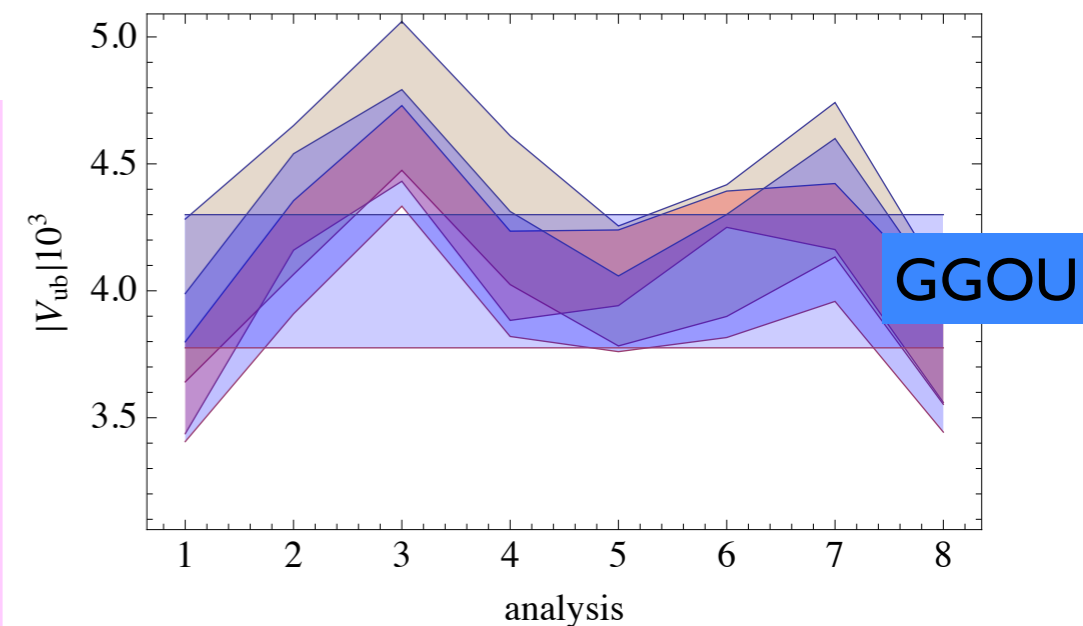
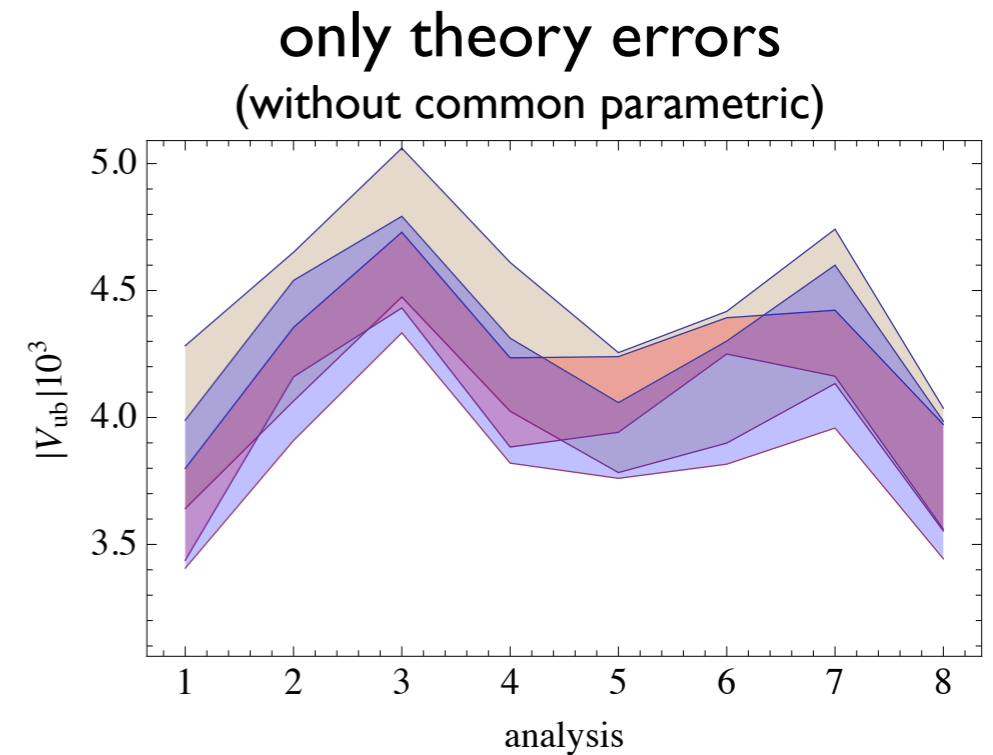
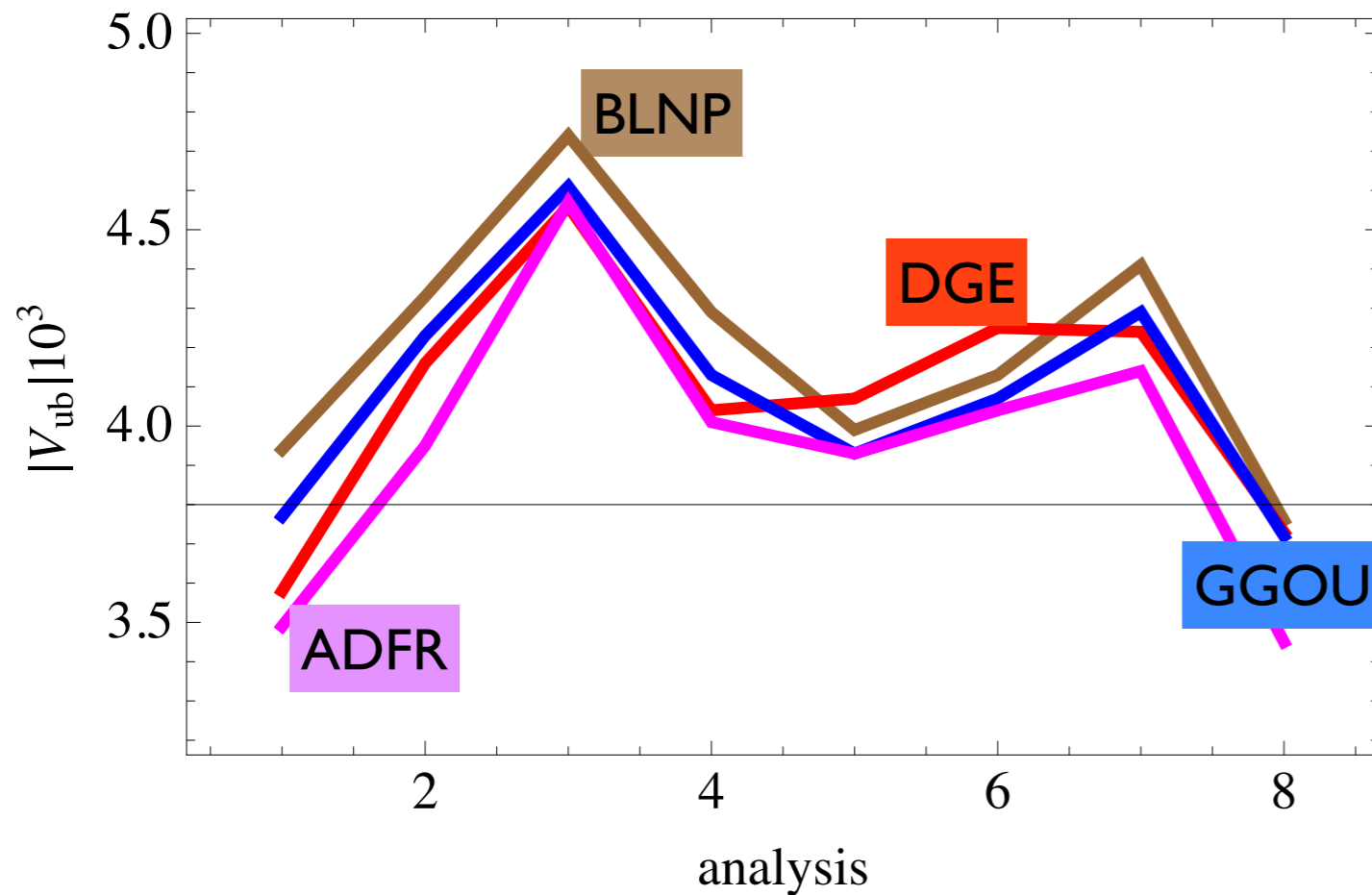
How to access the SF?

Prediction <i>based</i> on resummed pQCD DGE, ADFR	OPE constraints + parameterization without/with resummation GGOU, BLNP
---	---

SIMBA fits radiative data for leading SF & m_b , parameterizes subleading only. No V_{ub} yet Bernlocher et al

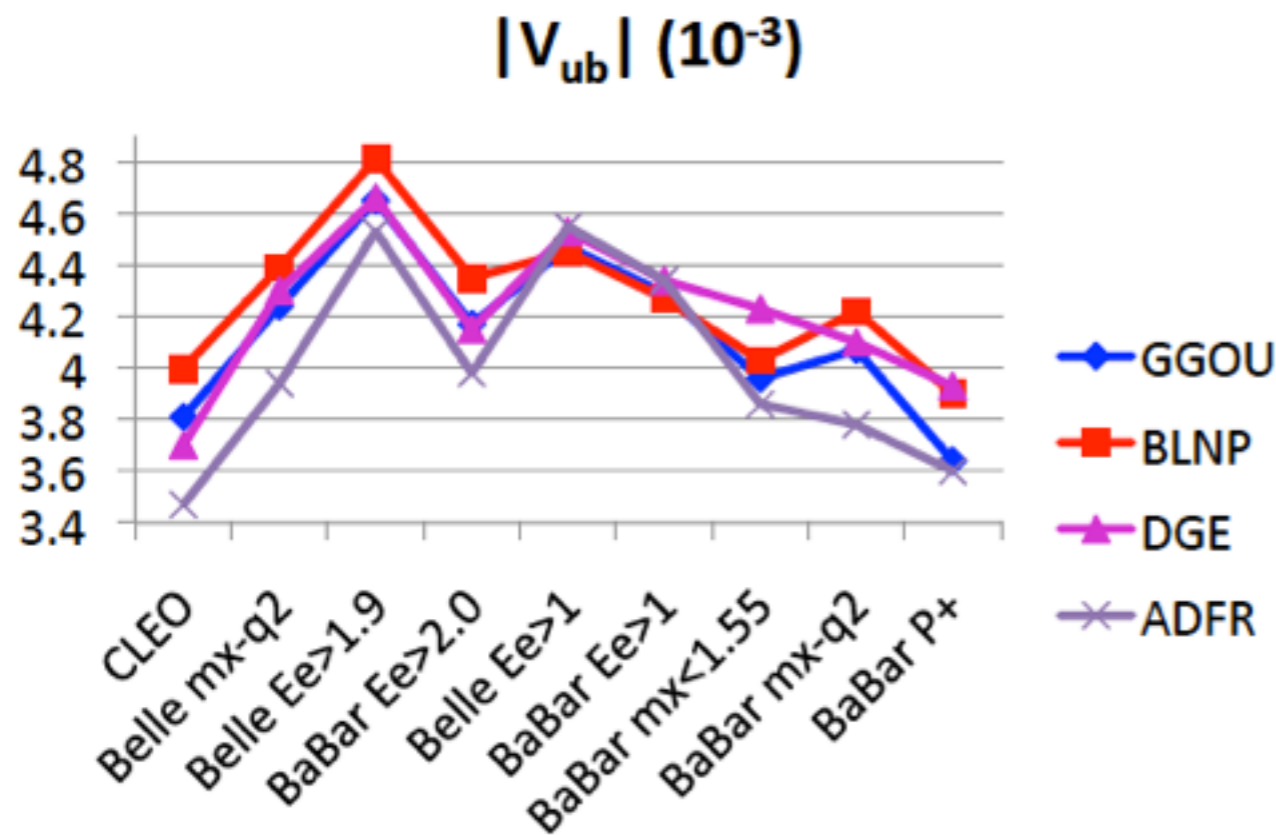
A global comparison

0907.5386, Phys Rept



- * Overall good agreement with common inputs
SPREAD WITHIN TH ERRORS
- * Recent BLNP at NNLO (in SF region) Asatrian, Greub, Neubert, Pecjak, Bonciani, Ferroglia Beneke, Huber, Li, Bell *Strong impact in BLNP (+10%), not yet included, unlikely in other approaches. $O(\alpha_s^2)$ calculation in the full phase space necessary*
- * Not all observables are equally clean.

Inclusive $|V_{ub}|$ averages



HFAG 2010	Average $ V_{ub} \times 10^3$
DGE	$4.44(16)_{\text{ex}}^{+18}_{-17}$
BLNP	$4.31(16)_{\text{ex}}^{+22}_{-23}$
GGOU	$4.33(16)_{\text{ex}}^{+15}_{-22}$

2.1 σ from $B \rightarrow \pi l \nu$ (MILC-FNAL)
 2.5 σ from UTFit 2010 (because of $\sin 2\beta$)

Belle+Babar multivariate analysis, $E_l > 1 \text{ GeV}$

$$|V_{ub}| \approx (4.35 \pm 0.18_{-0.17}^{+0.13}) \times 10^{-3}$$

Includes about 90% of the rate: really inclusive measurement, no need for SF.

Crucial input m_b

- **7-8% total error**
- More inclusive measurements, less dependence on m_b

V_{ub} exclusive

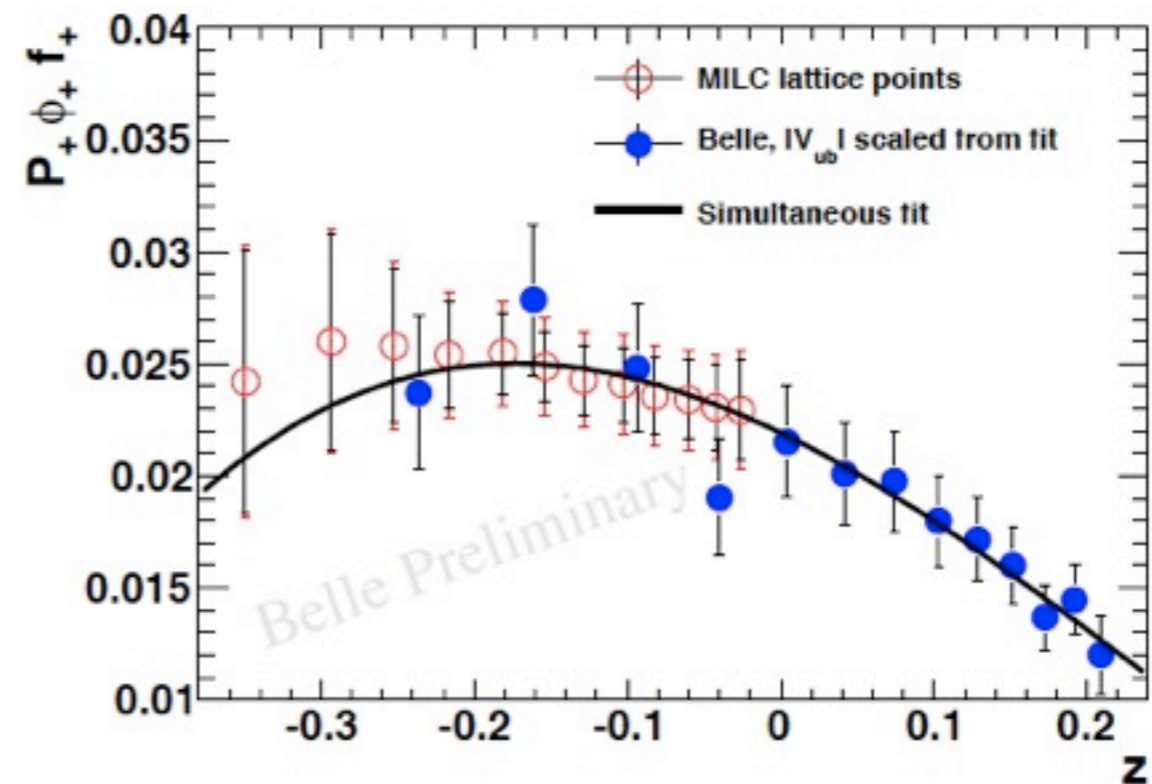
$B \rightarrow \pi \ell \nu$

Results for different models

	q^2 (GeV ²)	$\Delta\zeta$ (ps ⁻¹)	$ V_{ub} (10^{-3})$
HPQCD	> 16	2.07 ± 0.57	$3.55 \pm 0.09 \pm 0.09$ ^{+0.62} _{-0.41}
FNAL	> 16	1.83 ± 0.50	$3.78 \pm 0.10 \pm 0.10$ ^{+0.65} _{-0.43}
LCSR	< 16	5.44 ± 1.43	$3.64 \pm 0.06 \pm 0.09$ ^{+0.60} _{-0.40}

Recent method employed in PRD **79** 054507 (2009) (MILC)

- $z = z(q^2)$
- Lattice points:
 $f_+(q^2)$
- Experiment:
 $|V_{ub}| \times f_+(q^2)$
- Simultaneous fit $\Rightarrow |V_{ub}|$



Result:

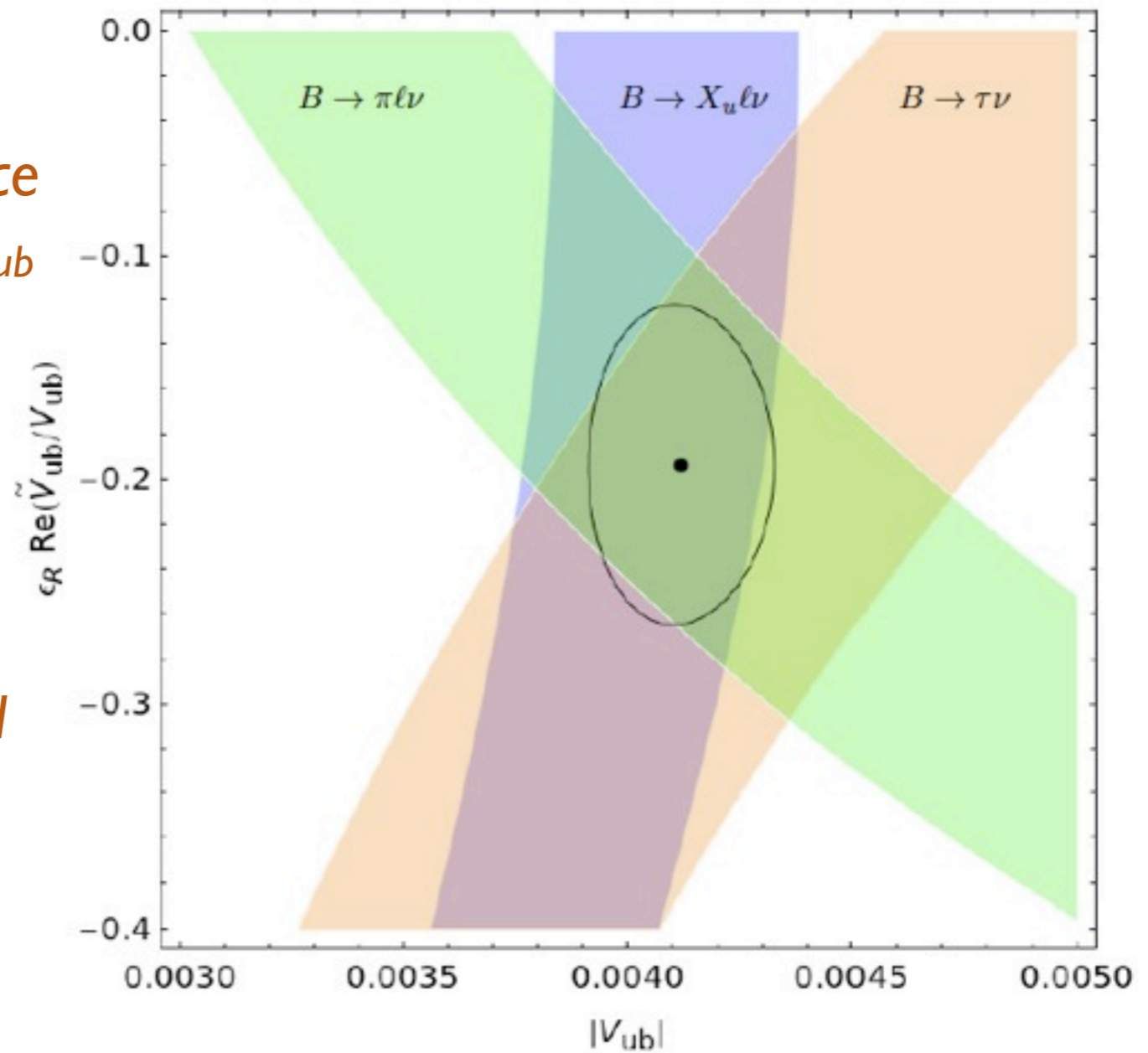
$$|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3} \quad (\text{Error stat. and syst. combined})$$

New physics?

LR models can explain a difference between inclusive and exclusive V_{ub} determinations Chen,Nam

Also in MSSM Crivellin

BUT the RH currents affect predominantly the exclusive V_{ub} , making the conflict between V_{ub} and $\sin 2\beta$ (ψK_S) stronger...



Buras, Gemmler, Isidori 1007.1993

Conclusions

- *Semileptonic B decays provide us with a lot of information: V_{cb}, V_{ub} , constraints on $m_{b,c}$ (consistent with sum rules)*
- *Slow but steady progress in inclusive $|V_{cb}|$, NNLO and higher power corrections, good prospects for the error reduction*
- *Some tension persists between exclusive and inclusive $|V_{cb}|$*
- *Inclusive V_{ub} moves slightly up, $\sim 2\sigma$ clash with exclusive one and UT fit, but latest results should be described by local OPE.
it can be either experimental problem or new physics*