Lattice Flavour Physics

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outline

- predictions vs postdictions
- an excursion in b-physics
- what do we know?
- how do we improve the accuracy on quantities that we already know with good precision?
- QCD isospin breaking corrections in kaon decays:
  - a theoretical problem and its solution
  - some (preliminary) results
  - lessons for other observables
- outlooks

prediction vs. postdiction

we do have a lot of precise experimental measurements in the quark flavour sector of the standard model that, combined with CKM unitarity (first row), allow us to **measure** hadronic matrix elements

a simple example from FLAVIAnet kaon working group **M.Antonelli et al. Eur.Phys.J.C69**

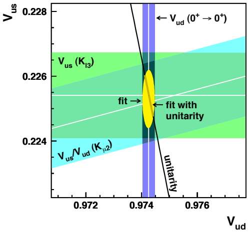
$$\begin{cases} \left|\frac{V_{us}F_K}{V_{ud}F_\pi}\right| = 0.27599(59) \\ \\ \left|V_{us}F_+^{K\pi}(0)\right| = 0.21661(47) \end{cases} \qquad \begin{cases} |V_{ud}|^2 + |V_{us}|^2 = 1 \\ \\ |V_{ud}| = 0.97425(22) \end{cases}$$

where $|V_{ud}|$ comes by combining 20 super-allowed nuclear β-decays and $|V_{ub}|$ has been neglected because smaller than the uncertainty on the other terms, combine to give

$$|V_{us}| = 0.22544(95)$$

$$F_+^{K\pi}(0) = 0.9608(46)$$

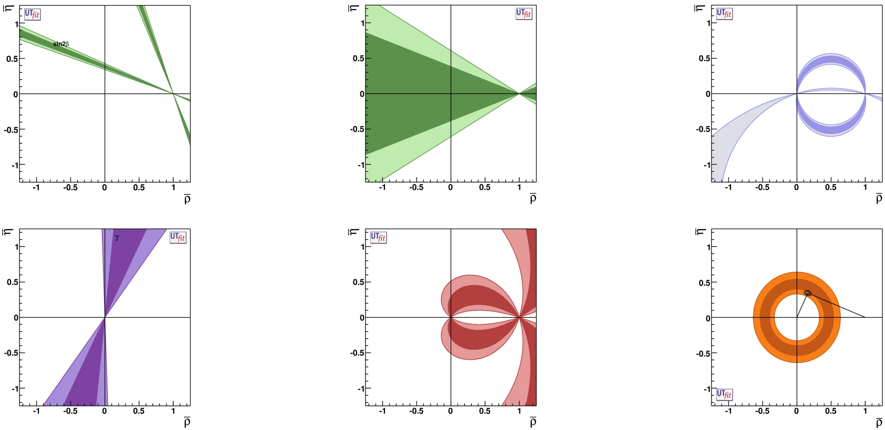
$$\frac{F_K}{F_\pi} = 1.1927(59)$$



lattice QCD is **still** needed to **postdict** these quantities and, in case, to falsify the standard model

prediction vs postdiction

the previous one is a particular example of what one can do by using CKM unitarity. our friends of the UTfit collaboration have been the first to realize that, after the B-factories, the unitarity triangle is over constrained



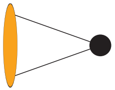
as we said, the lattice community has **lost the opportunity to predict** but **must postdict** hadronic matrix elements. . .

prediction vs postdiction

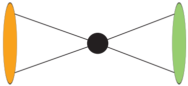
in the postdiction era, we don't calculate to feed the UT analysis but, we "simply" try to compare with experiment at the same level of precision

here a short list of observables:

- concerning the light quarks sector

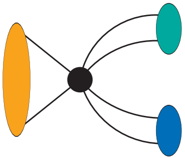


  - F_π, F_K, ...
  - $K \to \pi \ell \nu$, $K \to \rho \ell \nu$, ...
  - $K \to \pi \ell \ell$, $K \to \pi \nu \nu$, ...
  - B_K, ...
  - $K \to (\pi\pi)_{\Delta I=3/2}$, $K \to (\pi\pi)_{\Delta I=1/2}$, ...
  - $\pi\pi \to \pi\pi$, $\rho \to \pi\pi$, ...
  - ...



- concerning the heavy quarks sector

  - F_{D_q}, F_{B_q}, ...
  - $B_q \to D_q \ell \nu$, $B_q \to D_q^\star \ell \nu$, ...
  - $B_q \to \pi \ell \nu$, $B_q \to \rho \ell \nu$, ...
  - $D \to K \ell \nu$, ...
  - B_{B_q}, ...
  - $B \to \pi\pi$
  - ...



even $K \to \pi\pi$ is coming...

the RBC-UKQCD collaboration is putting a huge effort in the calculation of $K \to \pi\pi$ amplitudes

the key ingredients are the theoretical developments of the last few years

L.Lellouch, M.Lüscher Commun.Math.Phys.219 (2001)
D.Lin et al. Nucl.Phys.B619 (2001)
G.M.de Divitiis, N.T. hep-lat/0409154
C.h.Kim, C.T.Sachrajda, S.R.Sharpe Nucl.Phys.B727 (2005)
...

$$|A|^2 = 8\pi V^2 \frac{M_K^2}{q_\star^2} \left[\delta'(q_\star) + \phi'(q_\star)\right] |M|^2$$

from Sachrajda's talk at LATTICE 2010

C.T. Sachrajda PoS LATTICE2010:018,2010

$M_\pi = 145 MeV \qquad M_K = 519 MeV$

$\Re A_2 = 1.56(07)(25) \times 10^{-8} GeV$

$\Im A_2 = -9.6(04)(2.4) \times 10^{-13} GeV$

$M_\pi = 420 MeV$ unphysical kinematics!

$\Re A_0 = 3.0(9) \times 10^{-7} GeV$

$\Im A_0 = -2.9(2.2) \times 10^{-11} GeV$

of course, one needs to be a bit careful...

we do already know many observables with high precision, but in grading lattice simulations some care is needed

Q: would you choose a lattice calculation performed with $n_f = 2 + 1 + 1 + 1 + 1$ dynamical quarks or a quenched calculation?

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Q: ok, but the dynamical one has been performed at finite lattice spacing $a = 1$ **meter**, while the quenched one takes into properly account all the remaining systematics. . .

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Q: ok, but the dynamical one has been performed at finite lattice spacing $a = 1$ **meter**, while the quenched one takes into properly account all the remaining systematics...

A: no debate, the dynamical one!

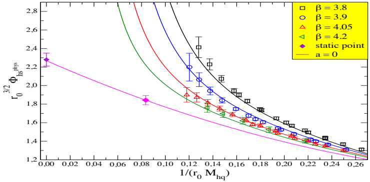
luckily we are not in such an **extreme** and **ridiculous** situation but some care is needed

the way (methodology) results are extracted may have a deep impact on the systematics...

first an excursion in the heavy flavour sector: extrapolating

let's take the simplest example, $\Phi_{B_s} = f_{B_s}\sqrt{M_{B_q}}$

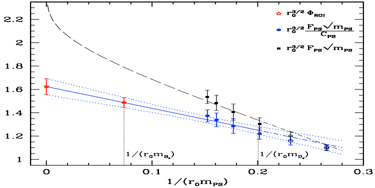
B.Blossier et al. PoS LAT2009 151



the standard approach to b-physics consists in:

- making simulations at "not so heavy" quark masses ($m_h \sim m_c$)
- extrapolating at the physical point ($m_h^{phys} = m_b$)
- constraining extrapolations with HQET (possibly non-perturbatively renormalized and matched)

J. Heitger and R. Sommer JHEP 0402:022,2004
M. Della Morte et al. JHEP 0802:07,2008



$$\frac{\Phi_{B_q}}{C_{PS}} = f_q^0\left[1 + \frac{f_q^1}{m_b} + \dots\right]$$

first an excursion in the heavy flavour sector: go on a small volume

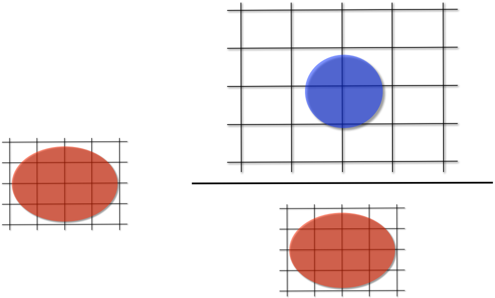
consider the following simple identity...

M.Guagnelli, F.Palombi, R.Petronzio N.T. Phys.Lett.B546:237-246,2002

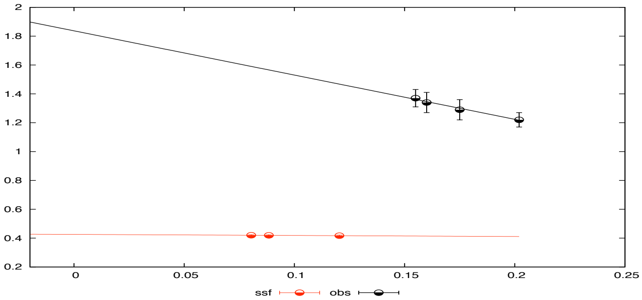
$$\mathcal{O}(E_h, E_l) \quad = \quad \mathcal{O}(E_h, E_l; \mathbf{L_0}) \quad \overbrace{\frac{\mathcal{O}(E_h, E_l; \mathbf{2L_0})}{\mathcal{O}(E_h, E_l; \mathbf{L_0})}}^{\sigma(\mathbf{E_h}, \mathbf{E_l}; \mathbf{L_0})} \quad \frac{\mathcal{O}(E_h, E_l; \mathbf{4L_0})}{\mathcal{O}(E_h, E_l; \mathbf{2L_0})} \quad \ldots$$

this is the starting point of a "finite size scaling" calculation

$$\mathrm{O}(\mathrm{E}_h, E_l; 2L_0) =$$



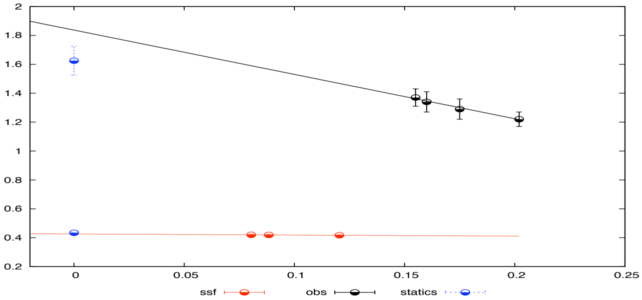
Let's take again $\mathcal{O} = f_{B_s}\sqrt{M_{B_s}}$



G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003

first an excursion in the heavy flavour sector: step scaling functions / observables

Let's take again $\mathcal{O} = f_{B_s}\sqrt{M_{B_s}}$



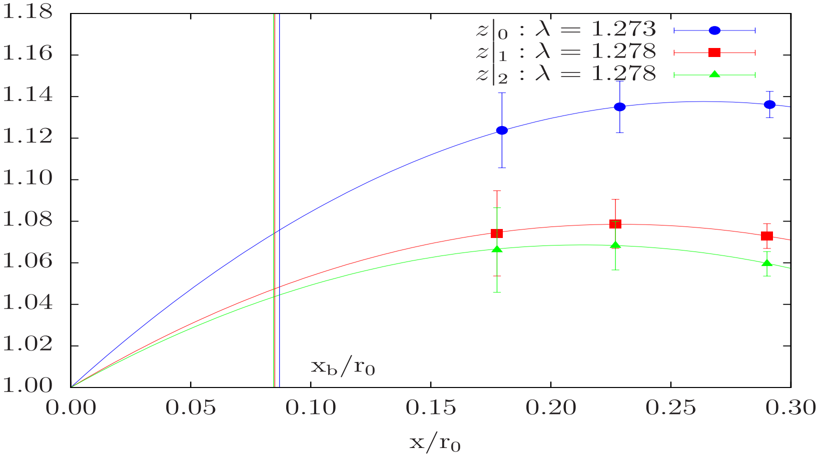
G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003
D.Guazzini, R.Sommer, N.T. JHEP 0801:076 (2008)

first an excursion in the heavy flavour sector: mass ratios

similar ideas have been developed in

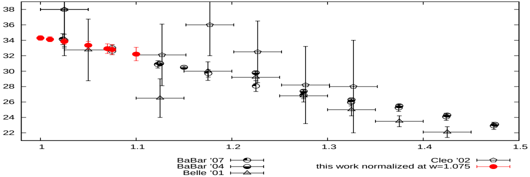
B.Blossier et al. JHEP 1004:049 (2010)

one considers ratios of observables at fixed volume but at different values of the heavy quark masses in such a way that the static limit is exactly known:

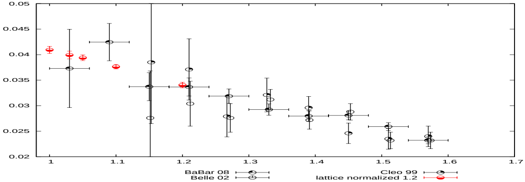


first an excursion in the heavy flavour sector: $B \to D^{(\star)}\ell\nu$

de Divitiis,Petronzio,N.T. Nucl.Phys.B807:373,2009
de Divitiis,Molinaro,Petronzio,N.T. Phys.Lett.B655:45,2007



$V_{cb}(@w = 1.075) \ = \ 3.74(8)(5) \times 10^{-2}$



$V_{cb}(@w = 1.2) \ = \ 3.84(9)(42) \times 10^{-2}$

B_K summary from FLAG

there are observables that we know how to calculate since many years and that are currently known with good precision...

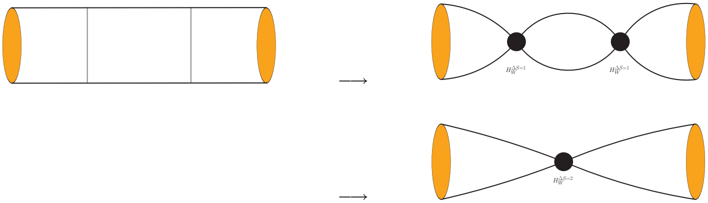
G.Colangelo et al. arXiv:1011.4408

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	B_K	$\hat{B}_K$
Kim 09	[252]	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
Aubin 09	[240]	2+1	A	●	★□	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	[253]	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07A, 08	[84, 254]	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	[255]	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	[256]	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08	[250]	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	[257]	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	[258]	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

the average is obtained by considering $n_f = 2+1$ results only (no debate!) and is

$$B_K(2GeV) = 0.527(6)(21) \qquad \hat{B}_K = 0.724(8)(29) \qquad \sim 4\%$$

can we do better?



B_K parametrizes the mixing of the neutral Kaons in the effective theory in which both the W bosons and the up-type quarks have been integrated out,

$$B_K(\mu) = \frac{\langle \bar{K} |\, H_W^{\Delta S=2}(\mu)\, | K \rangle}{\frac{8}{3} F_K^2 M_K^2}$$

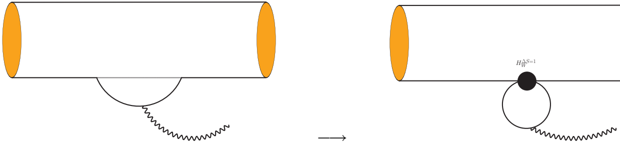
in order to do better on this process, we should be able to make a step backward and compute the **long distance contributions**,

$$\langle \bar{K} |\, \mathsf{T} \left\{ \int d^4x \; H_W^{\Delta S=1}(x;\mu)\; H_W^{\Delta S=1}(0;\mu) \right\} | K \rangle$$

to this end, we should be able to **make sense of the previous quantity in euclidean space**

see N. Crist arXiv:1012.6034

same kind of correlation functions in rare kaon decays



similar problems have to be faced in order to calculate the branching ratios of the rare semileptonic decay processes $K \longrightarrow \pi\nu\nu$ and $K \longrightarrow \pi\ell\ell$

$$\langle\pi|\ \mathsf{T}\ \left\{\int d^4x\ H_W^{\Delta S=1}(x;\mu)\ J_W(0)\right\}|K\rangle$$

the first discussion on how to compute on the lattice long distance contributions has been done in the case of these processes in

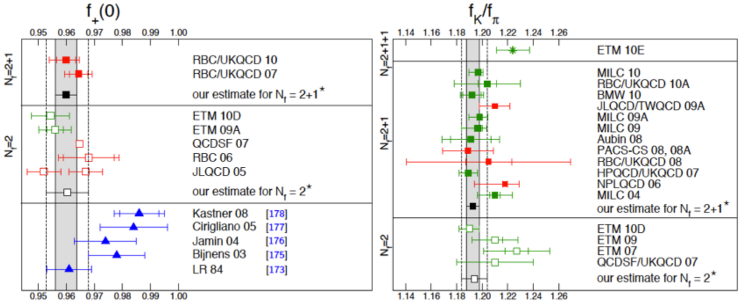
G.Isidori, G.Martinelli, P.Turchetti Phys.Lett. B633

we shall come back to this kind of matrix elements in a few slides when discussing the calculation of isospin breaking corrections. . .

F_K/F_π & $F_+^{K\pi}(0)$ summary from FLAG

matrix elements entering leptonic and (frequent) semileptonic kaon decays are other examples of well known quantities

G.Colangelo et al. arXiv:1011.4408



$$F_+^{K\pi}(0) = 0.956(8) \qquad \sim 0.8\% \qquad\qquad \frac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\%$$

also here to do better we should include effects that have been neglected up to now. . .

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gauge configurations generated and provided by the ETMC collaboration

F_K/F_π & $F_+^{K\pi}(q^2)$ beyond the isospin limit $m_u \neq m_d$

there are two sources of isospin breaking effects,

$$\underbrace{m_u \neq m_d}_{\textbf{QCD}} \qquad\qquad \underbrace{q_u \neq q_d}_{\textbf{QED}}$$

in the particular and (lucky) case of these observables, the correction to the isospin symmetric limit due to the difference of the up and down quark masses (**QCD**) can be estimated in *chiral perturbation theory*,

$$\begin{cases} F_+^{K\pi}(0) = 0.956(8) \qquad \sim 0.8\% \\ \\ \left(\dfrac{F_+^{K^+\pi^0}(q^2)}{F_+^{K^0\pi^-}(q^2)} - 1\right)_{QCD} = 0.029(4) \end{cases}$$

A. Kastner, H. Neufeld Eur.Phys.J.C57 (2008)

$$\begin{cases} \dfrac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\% \\ \\ \left(\dfrac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1\right)_{QCD} = -0.0022(6) \end{cases}$$

V. Cirigliano, H. Neufeld arXiv:1102.0563

reducing the error on these quantities without taking into account isospin breaking is useless. . .

isospin breaking on the lattice

- the calculation of QED isospin breaking effects on the lattice it has been don for the first time in
  Duncan, Eichten, Thacker, Phys. Rev. Lett. 76 (1996)

- QED is treated in the quenched approximation in its “compact” formulation

- because the photons are massless and unconfined this approach may introduce large finite volume effects...

- the calculation of QCD isospin breaking effects on the lattice poses a theoretical problem

$$
\begin{aligned}
Z &= \int DU D\psi \; e^{-S_g[U]+S_f[U;m_u,m_d]} \\
&= \int DU \; e^{-S_g[U]} \underbrace{\det(D[U]+m_u)\;\det(D[U]+m_d)}_{\text{must be } >0}
\end{aligned}
$$

- if $m_u \neq m_d$ this can be only achieved by recurring to very expensive fermion formulations (overlap)
- furthermore the effect is very small and it can be extremely difficult to see it with limited statistical accuracy

our QCD isospin breaking on the lattice

- our idea is to calculate QCD isospin corrections **at first order** in $m_d - m_u$:

$$
\begin{aligned}
S_f &= \bar{u}\,(D[U] + m_u)\,u + \bar{d}\,(D[U] + m_d)\,d \\
&= \underbrace{\bar{u}\,(D[U] + \bar{m})\,u + \bar{d}\,(D[U] + \bar{m})\,d}_{S_f^0} - \underbrace{(\mathbf{m_d} - \mathbf{m_u})\frac{\bar{\mathbf{u}}\mathbf{u} - \bar{\mathbf{d}}\mathbf{d}}{\mathbf{2}}}_{\boldsymbol{\Delta}\mathbf{m}\mathbf{S^3}}
\end{aligned}
$$

- the calculation of an observable proceeds as follows

$$
\begin{aligned}
\langle \mathcal{O} \rangle + \boldsymbol{\Delta}\langle \mathcal{O} \rangle &= \frac{\int DU\ e^{-S_g[U]-S_f^0[U]+\Delta m S^3}\ \mathcal{O}}{\int DU\ e^{-S_g[U]-S_f^0[U]+\Delta m S^3}} = \frac{\int DU\ e^{-S_g[U]-S_f^0[U]}\ (1+\Delta m S^3)\ \mathcal{O}}{\int DU\ e^{-S_g[U]-S_f^0[U]}\ (1+\Delta m S^3)} \\
&= \langle \mathcal{O} \rangle + \boldsymbol{\Delta}\mathbf{m}\langle \mathbf{S^3}\ \mathcal{O} \rangle - \underbrace{\Delta m \langle S^3 \rangle}_{=0}
\end{aligned}
$$

in what follows we shall neglect the "interference" between S^3 and isospin breaking effects due to the Twisted Mass term because these vanish in the continuum limit

our QCD isospin breaking on the lattice

- inserting $\bar{u}u - \bar{d}d$ within a correlation function amounts (after Wick contractions) to calculate **the same observables** but with **light propagators squared**

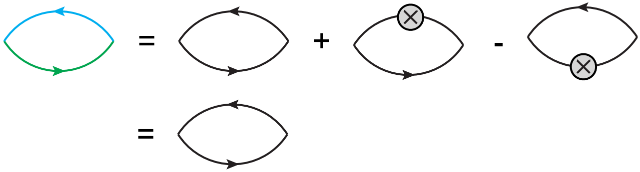
$$\mathcal{S}_u = \frac{1}{D[U]+\bar{m}-\Delta m} = \frac{1}{D[U]+\bar{m}} + \frac{\Delta m}{(D[U]+\bar{m})^2}$$

$$\mathcal{S}_D = \frac{1}{D[U]+\bar{m}+\Delta m} = \frac{1}{D[u]+\bar{m}} - \frac{\Delta m}{(D[U]+\bar{m})^2}$$

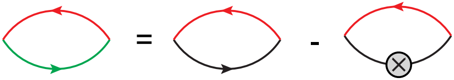
- relations that can be represented diagrammatically as

our QCD isospin breaking on the lattice: two point functions

- at first order in Δm the pion's mass and decay constants don't get a correction (here $\pi^{\pm}$ but it works also for π^0)



- the kaons do get a correction

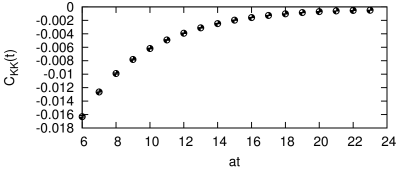


- this means that at first order (δ stays for relative error while Δ for absolute error),

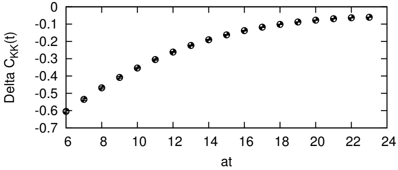
$$\delta\left(\frac{F_{K^+}}{F_{\pi^+}}\right) = \frac{\Delta F_{K^+}}{F_{K^+}} - \frac{\Delta F_{\pi^+}}{F_{\pi^+}} = \frac{\Delta F_{K^+}}{F_{K^+}}$$

what do we expect from corrected two point correlation functions?

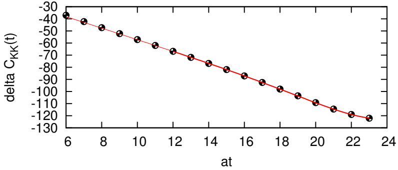
$$C_{KK}(t) = \langle K(t)\ K^\dagger(0)\rangle = \mathbf{Z_K\ e^{-E_K t}} + \ldots$$



$$\begin{aligned}\Delta C_{KK}(t) &= T\left\{\langle K(t)\ \int d^4x\ S^3(x)\ K^\dagger(0)\rangle\right\} \\ &= \mathbf{(\Delta Z_K - Z_K \Delta E_K t)\ e^{-E_K t}} + \ldots\end{aligned}$$

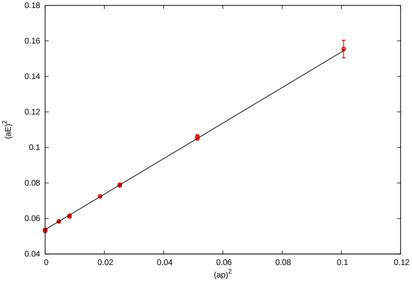


$$\begin{aligned}\delta C_{KK}(t) &= \frac{\Delta C_{KK}(t)}{C_{KK}(t)} \\ &= \mathbf{\delta Z_K - \Delta E_K t} + \ldots\end{aligned}$$

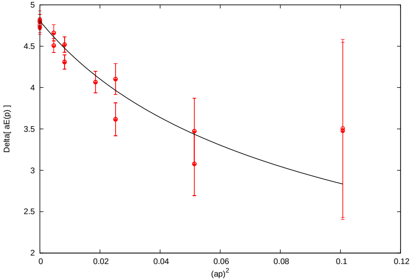


our QCD isospin breaking on the lattice: kaon's two point functions

are we sure that the slopes correspond to ΔE_K?



$$E_K^2(p) = M_K^2 + p^2$$



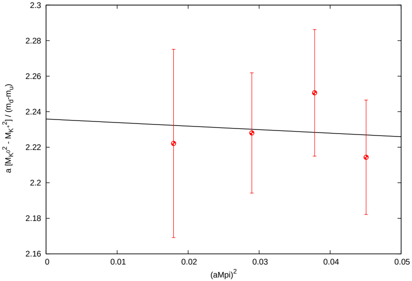
$$\Delta E_K(p) = \frac{M_K \Delta M_K}{\sqrt{M_K^2 + p^2}}$$

- the solid lines are not fitted, but theoretically predicted by using calculated M and ΔM
- this kind of accuracy on kinematics at $p \neq 0$ is possible thanks to the use of twisted boundary conditions

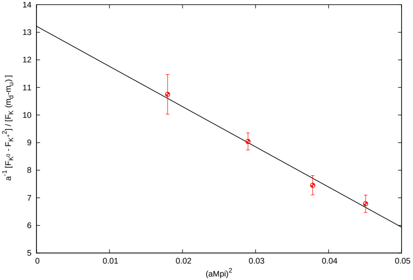
G.M. de Divitiis, R. Petronzio, N.T. Phys.Lett. B595 (2004)

$$\psi(x+L) = e^{i\theta}\psi(x) \qquad \longrightarrow \qquad p = \frac{\theta}{L} + \frac{2\pi n}{L}$$

here we see the light quark mass dependence of the corrections to M_K and F_K



$$a\,\frac{M^2_{K^0}-M^2_{K^+}}{m_d-m_u} = -2a\,\frac{\Delta M_{K^+}}{m_d-m_u}$$

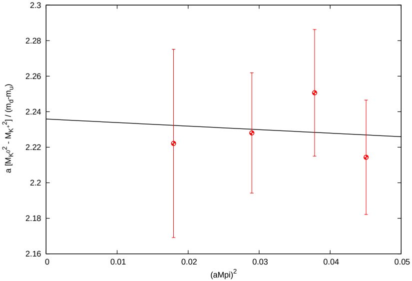


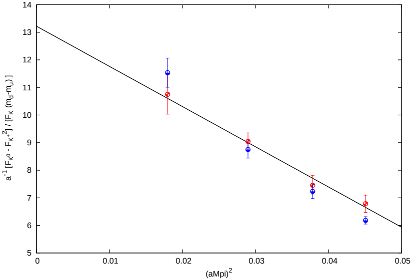
$$\frac{1}{a}\,\frac{F_{K^0}-F_{K^+}}{F_K(m_d-m_u)} = -\frac{2}{a}\,\frac{\delta F_{K^+}}{m_d-m_u}$$

- the results are **VERY PRELIMINARY**
- at present we have performed simple linear extrapolations but NLO chiral perturbation theory formulae are known. . .
- **remember:** these are **QCD isospin breaking effects**
- in order to extract $m_d - m_u$ from these numbers we must **subtract** from the physical values of $M^2_{K^0} - M^2_{K^+}$ the **QED isospin breaking effects**
- do we know something about that?

our QCD isospin breaking on the lattice: kaon's two point functions

here we see the light quark mass dependence of the corrections to M_K and F_K





$$a\,\frac{M_{K^0}^2 - M_{K^+}^2}{m_d - m_u} = -2a\,\frac{\Delta M_{K^+}}{m_d - m_u}$$

$$\frac{1}{a}\,\frac{F_{K^0} - F_{K^+}}{F_K\,(m_d - m_u)} = -\frac{2}{a}\,\frac{\delta F_{K^+}}{m_d - m_u}$$

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our QCD isospin breaking on the lattice: kaon's two point functions

for the kaons and pions we can define

$$\underbrace{M_P^2}_{\text{physical mass}} = \underbrace{\hat{M}_P^2}_{QCD\ \text{mass}} + \underbrace{\Delta_P}_{QED\ \text{correction}}$$

at LO in the chiral expansion one has (**Dashen's theorem**)

$$\Delta_P = A(e_q + e_{\bar{q}}) + O(m_q) \qquad \longrightarrow \qquad \begin{cases} \Delta_{\pi^0} = \Delta_{K^0} = 0 \\ \Delta_{\pi^+} = \Delta_{K^+} \neq 0 \\ (\Delta_{K^+} - \Delta_{K^0}) - (\Delta_{\pi^+} - \Delta_{\pi^0}) = 0 \end{cases}$$

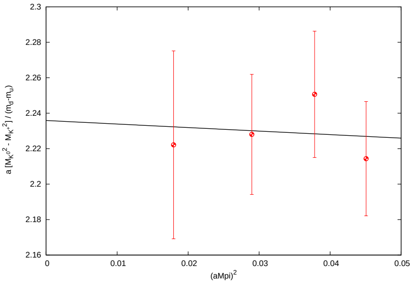
corrections to the Dashen's theorem are parametrized in terms of small parameters ($\delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2 = 1260 MeV^2$)

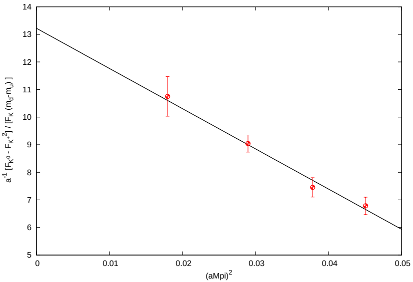
$$\begin{cases} \Delta_{\pi^0} = \epsilon_{\pi^0}\delta_\pi \\ \Delta_{K^0} = \epsilon_{K^0}\delta_\pi \\ \Delta_{\pi^+} = (1 + \epsilon_{\pi^0} - \epsilon_m)\delta_\pi \\ \Delta_{K^+} = (1 + \epsilon_{K^0} + \epsilon - \epsilon_m)\delta_\pi \end{cases} \qquad \begin{cases} \epsilon_{\pi^0} = 0.07(7) \\ \epsilon_{K^0} = 0.3(3) \\ \epsilon_m = \hat{M}_{\pi^+}^2 - \hat{M}_{\pi^0}^2 = 0.04(2) \\ \epsilon = (\Delta_{K^+} - \Delta_{K^0}) - (\Delta_{\pi^+} - \Delta_{\pi^0})/\delta_\pi = 0.7(5) \end{cases}$$

the numbers for the ϵ-parameters are from FLAG:

G.Colangelo et al. arXiv:1011.4408

by using previous numbers we get,





$$a\,\frac{M^2_{K^0} - M^2_{K^+}}{m_d - m_u} = -2a\,\frac{\Delta M_{K^+}}{m_d - m_u}$$

$$(M^2_{K^0} - M^2_{K^+})_{QCD} = 609(61) \times 10 MeV^2$$

$$\mathbf{(m_d - m_u)^{\overline{MS},2GeV} = 2.69(9)(29)MeV}$$

$$\frac{1}{a}\,\frac{F_{K^0} - F_{K^+}}{F_K(m_d - m_u)} = -\frac{2}{a}\,\frac{\delta F_{K^+}}{m_d - m_u}$$

$$\delta\left(\frac{F_{K^+}}{F_{\pi^+}}\right)_{\chi pt} = -0.0022(6)$$

$$\delta\left(\frac{\mathbf{F_{K^+}}}{\mathbf{F_{\pi^+}}}\right) = \mathbf{-0.0033(2)}$$

VERY PRELIMINARY

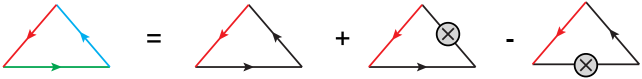
our QCD isospin breaking on the lattice: form factors

in order to calculate QCD isospin breaking corrections to $K \to \pi \ell \nu$ form factors one needs to calculate,

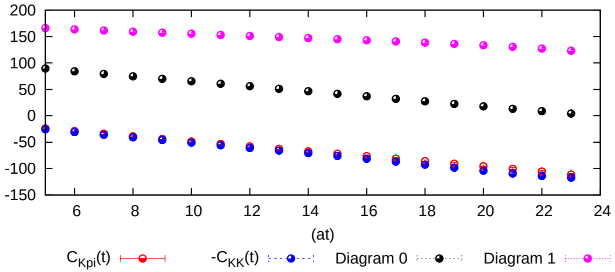
$$\langle \pi | \, \mathsf{T} \, \left\{ \int d^4x \; S^3(x;\mu) \; J_W(0) \right\} |K\rangle \quad \longrightarrow \quad \begin{cases} \langle \bar{K} | \, \mathsf{T} \, \left\{ \int d^4x \; H_W^{\Delta S=1}(x;\mu) \; H_W^{\Delta S=1}(0;\mu) \right\} |K\rangle \\ \\ \langle \pi | \, \mathsf{T} \, \left\{ \int d^4x \; H_W^{\Delta S=1}(x;\mu) \; J_W(0) \right\} |K\rangle \end{cases}$$

a key difference with respect to the calculation of long distance effects for $K \to \pi \nu \nu$ and K-$\bar{K}$ mixing is that the isospin breaking correction does not induce the decay of the Kaon. . .

diagrammatically, the $K^0 \to \pi^+$ case looks like



our QCD isospin breaking on the lattice: form factors

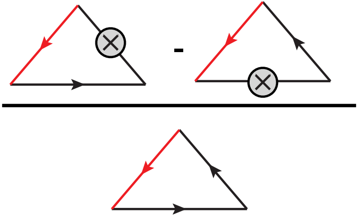


what do we expect from corrected three point correlation functions?

$$C^{\mu}_{K\pi}(t) = Z^{\mu}_{K\pi}\ e^{-E_K t}\ e^{-E_\pi (T-t)}$$

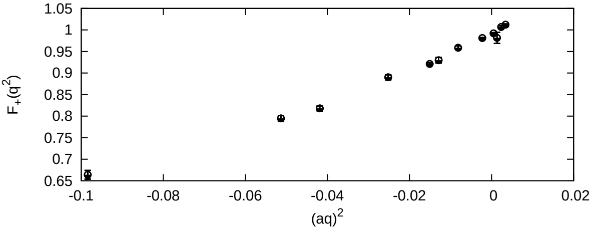
$$\Delta C^{\mu}_{K\pi}(t) = \left(\Delta Z^{\mu}_{K\pi} - Z^{\mu}_{K\pi} \Delta E_K t\right)\ e^{-E_K t}\ e^{-E_\pi (T-t)}$$

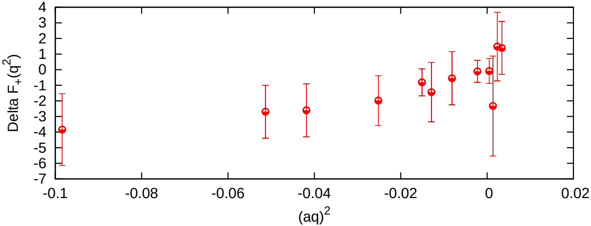
$$\delta C^{\mu}_{K\pi}(t) = \left(\delta \mathbf{Z}^{\mu}_{\mathbf{K}\pi} - \mathbf{\Delta E_K t}\right)$$



putting everything together we get

$F_+^{K^0\pi^+}(q^2)$





$\Delta F_+^{K^0\pi^+}(q^2)$

VERY PRELIMINARY

outlooks

- some "trivial" statements:
  - in the postdiction era we should calculate everything
  - give priority to quantities that have been (can be) measured with high precision
  - to improve the accuracy of very well known observable we should add effects that have been neglected up to now (long distance, isospin breaking, etc.)
- QCD isospin breaking effects can be calculated on the lattice at first order
- preliminary results are encouraging. . .
- first small steps toward the calculation of long distance contributions to rare semileptonic kaon decays and K-$\bar{K}$ mixing. . .