Theory prediction of ϵ_K

Rome Theory Seminar
Roma III
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Based on Work in Collaboration with Joachim Brod

Martin Gorbahn
TU München
TUM-IAS &
Excellence Cluster `Universe´

Plan for the Talk

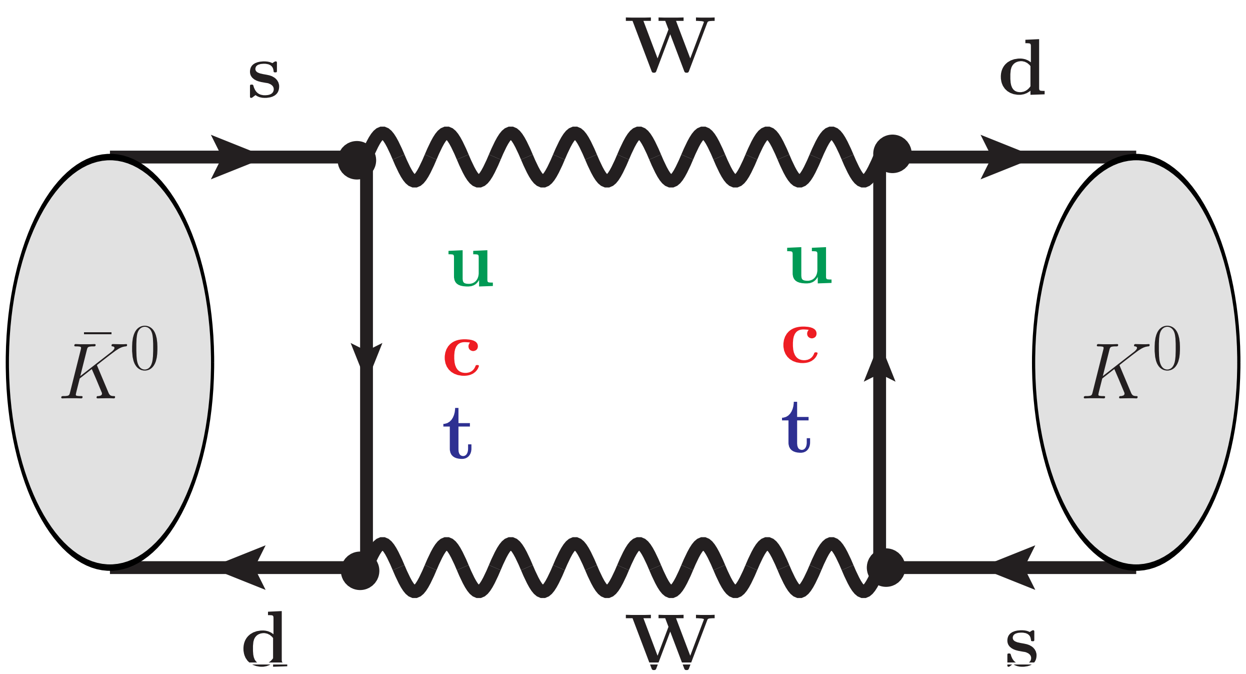
First look at ϵ_K:
standard model suppression & new physics sensitivity

Experimental and theory definition of ϵ_K

Long distance corrections

Perturbative calculation:
Status of the NNLO calculation

ϵ_K: Small in the Standard Model



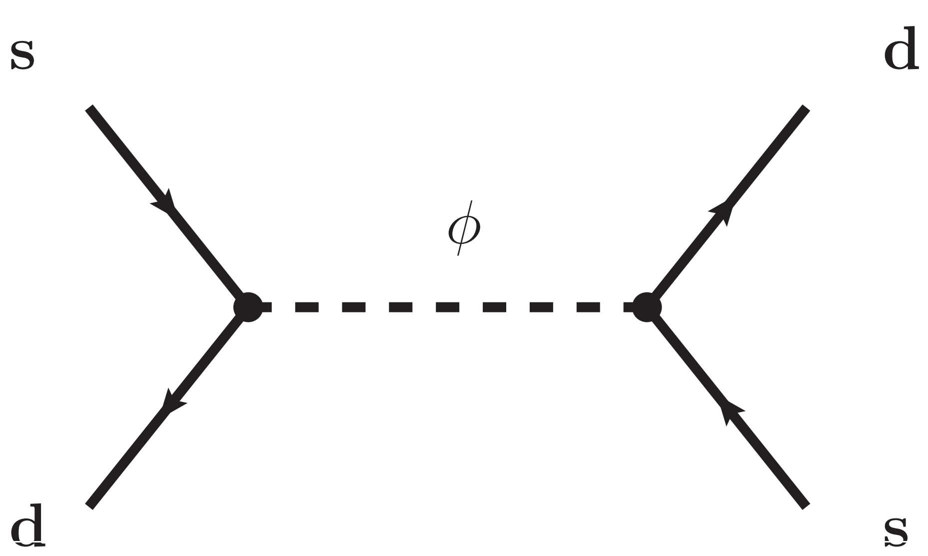
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SM → CP violation by Complex Phase in the CKM matrix

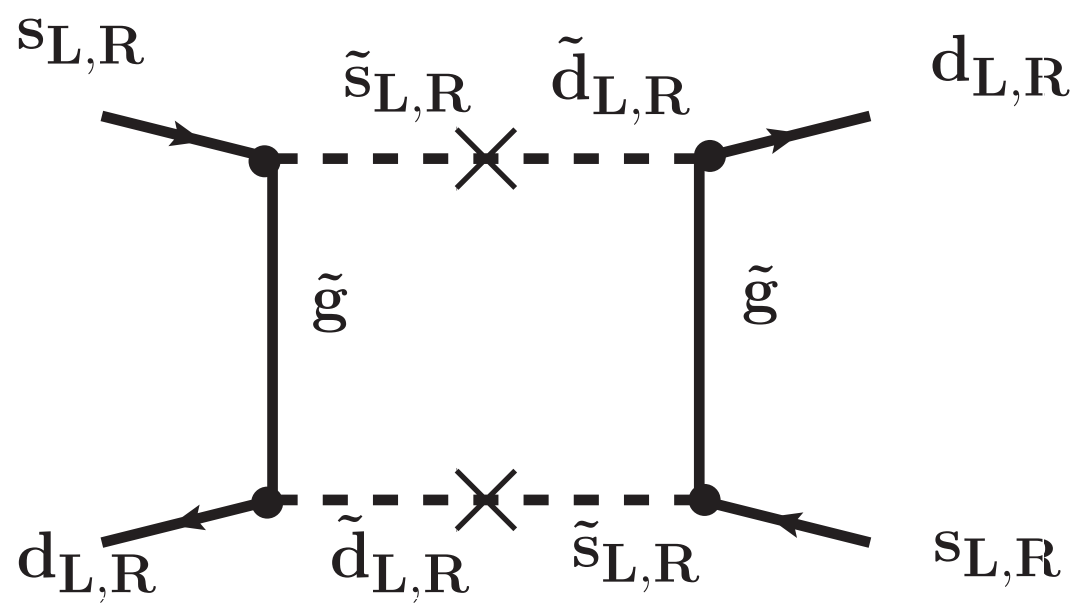
Top-quark contribution CKM suppressed
⇒ sensitivity to deviations from MFV

Sensitive to New Physics

Many more sources of CP violation:



2HDM Type III



SUSY models

+Technicolour, extra dimensions, ...

Strong constraints from ε_K!

Chiral Enhancement of non-SM operators

Experiment & Theory

Experiment:

ε_K is measured precisely:

$|\varepsilon_K| = 2.228(11)\times10^{-3}$.

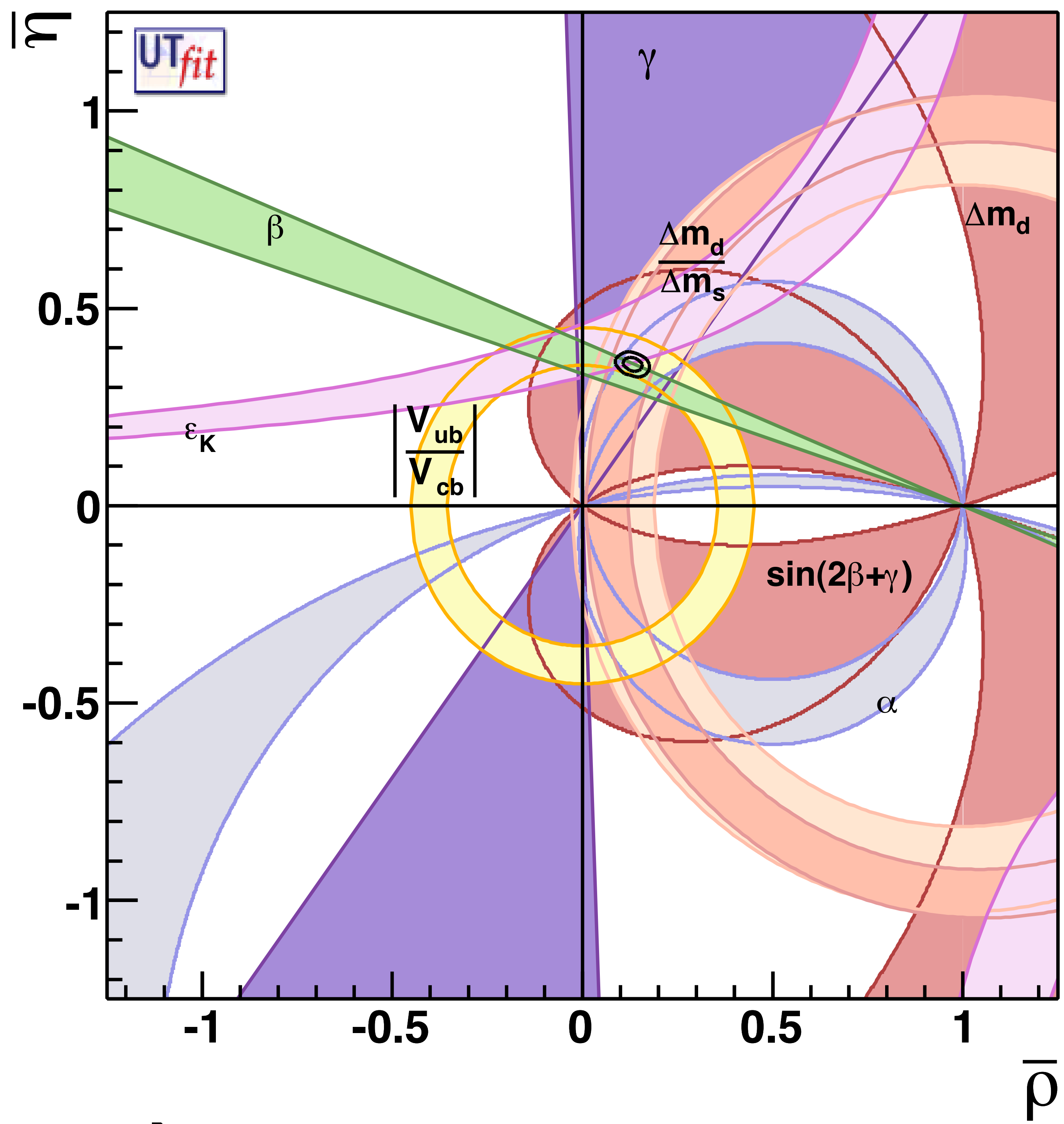
[PDG2010]

Theory:

ε_K can be calculated reliably:

$|\varepsilon_K| = 1.83(27)\times10^{-3}$.

[NLO SM prediction]



ϵ_K Measurement

CP violation in mixing, interference & decay → non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ϵ_K) & interference (Im ϵ_K)

$$\epsilon_K = \frac{\eta_{00} + 2\eta_{+-}}{3}$$

Isospin analysis ($|A_0| \approx 22\,|A_2|$):

$$\epsilon_K = \frac{\langle (\pi\pi)_{I=0} | K_L^0 \rangle}{\langle (\pi\pi)_{I=0} | K_S^0 \rangle} + \mathcal{O}\left(\frac{A_2^2}{A_0^2}\right)$$

$$\mathrm{A_I} = \langle (\pi\pi)_\mathrm{I} | \mathrm{K}^0 \rangle$$

$$\bar{\mathrm{A}}_\mathrm{I} = \langle (\pi\pi)_\mathrm{I} | \bar{\mathrm{K}}^0 \rangle$$

Mixing in the K System

Time evolution:

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix} = \left[\begin{pmatrix}M_{11} & M_{12} \\ M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11}\end{pmatrix}\right]\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix}$$

Diagonalise

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

CP is violated by the non-vanishing phase

$$\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) = \mathcal{O}\left(10^{-3}\right)$$

(in the Kaon system)

$$\phi \simeq \frac{\mathrm{Im}(M_{12})}{\mathrm{Re}(M_{12})} - \frac{\mathrm{Im}(\Gamma_{12})}{\mathrm{Re}(\Gamma_{12})} = \mathcal{O}\left(10^{-3}\right)$$

Three Types of CP Violation

In mixing

$$\left|\frac{q}{p}\right| \neq 1$$

In decay:

$$\left|\frac{\bar{A}_{\bar{I}}}{A_I}\right| \neq 1$$

Interference

$$\lambda_I = \frac{q}{p}\frac{\bar{A}_I}{A_I} \neq \pm 1$$

$$\epsilon_K = \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle} = \frac{1-\lambda_0}{1+\lambda_0} \qquad \simeq \frac{1}{2}\left(1 - \left|\frac{q}{p}\right| - i\mathrm{Im}\lambda_0\right)$$

Using isospin 1/2 saturates decay rate $\Gamma_{12} \approx A_0 \bar{A}_0$

$$\phi \approx 2\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \qquad \lambda_0 = 1 - i\phi\frac{\Delta m_K}{\Delta m_K + i\Delta\Gamma_K}$$

we find

$$\epsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right) \qquad \phi_\epsilon = \arctan\frac{\Delta m_K}{\Delta\Gamma_K/2}$$

Formula for ϵ_K

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right)$$

Δm_K, ϕ_ε: Directly from experiment:

$\mathrm{Im}(A_0)/\mathrm{Re}(A_0)$: from ϵ'/ϵ

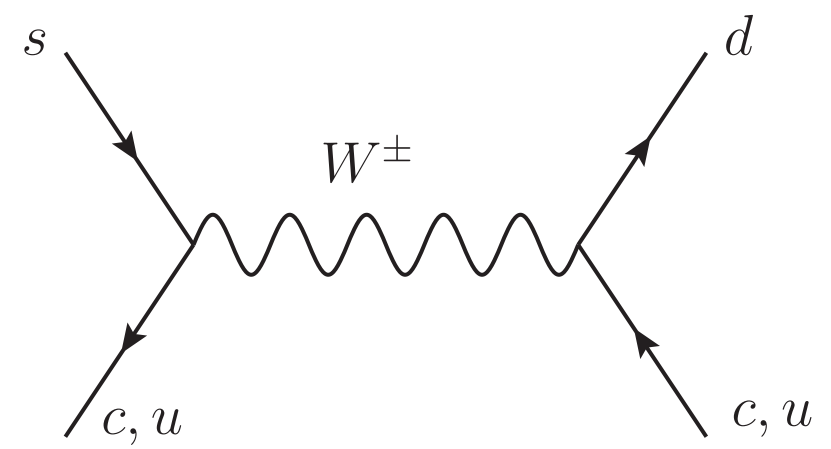
$\mathrm{Im}(M_{12}) = \mathrm{Im}(M_{12})_{SD} + \mathrm{Im}(M_{12})_{D=8} + \mathrm{Im}(M_{12})_{\text{Non Local}}$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle + \int d^4x \, \langle K^0 | H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) \, | \bar{K}^0 \rangle$$

Factories short and long distance: $H^{|\Delta S|=2} = C(\mu)\tilde{Q}$

From lattice: $\hat{B}_K = \dfrac{3b(\mu)}{2f_K^2 M_K^2} \langle K^0 | \tilde{Q} | \bar{K}^0 \rangle$ ($\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$)

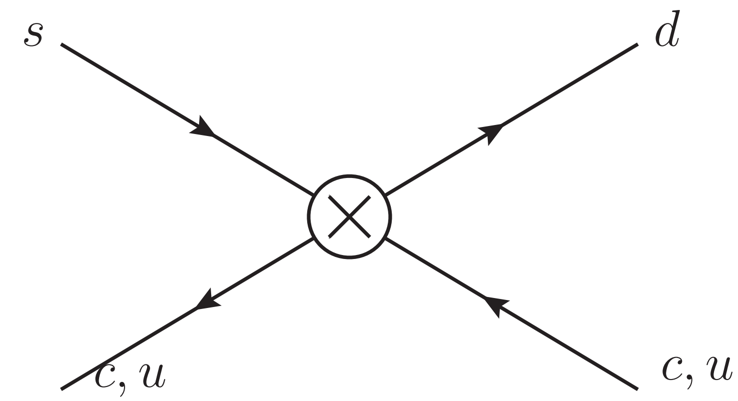
Long Distance Contribution



Origin of |ΔS|=1 Hamiltonian:

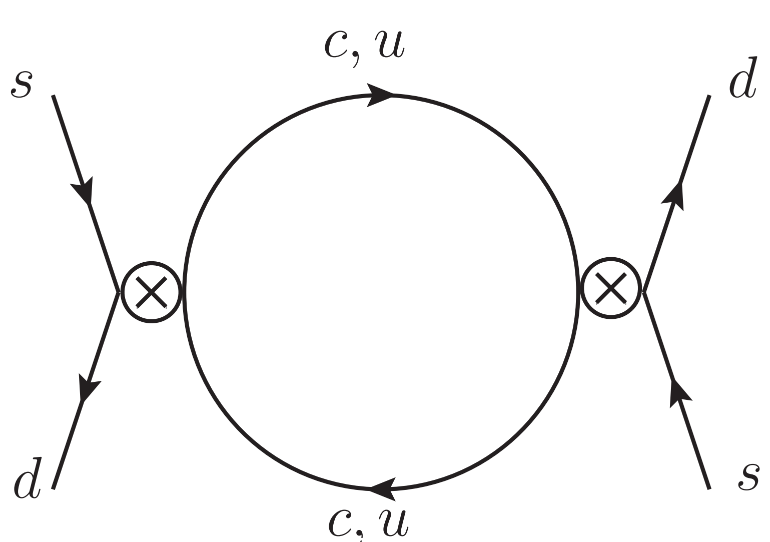
$$Q_2^{qq'} = (\bar{s}_L \gamma_\mu q_L) \otimes (\bar{q}'_L \gamma^\mu d_L)$$

$$Q_1^{qq'} = (\bar{s}_L \gamma_\mu T^a q_L) \otimes (\bar{q}'_L \gamma^\mu T^a d_L)$$



Like in Fermi theory + QCD

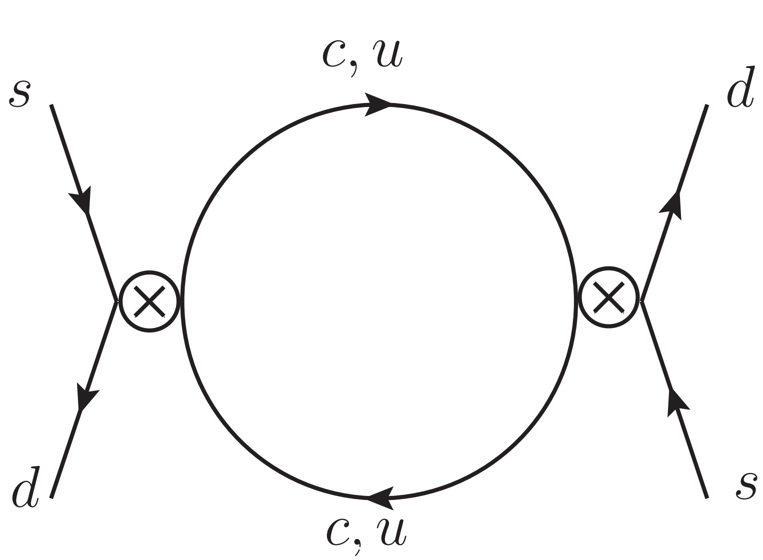
Below the charm quark mass scale:



$$\int d^4x \, \langle K^0 | \, H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) \, | \bar{K}^0 \rangle$$

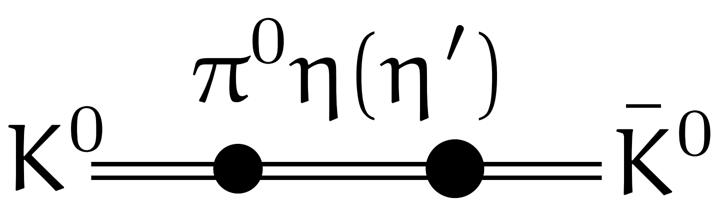
Higher dimensional operators and light quark contributions

Long Distance Contribution



$$\int d^4x \, \langle K^0 | \, H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) \, | \bar{K}^0 \rangle$$

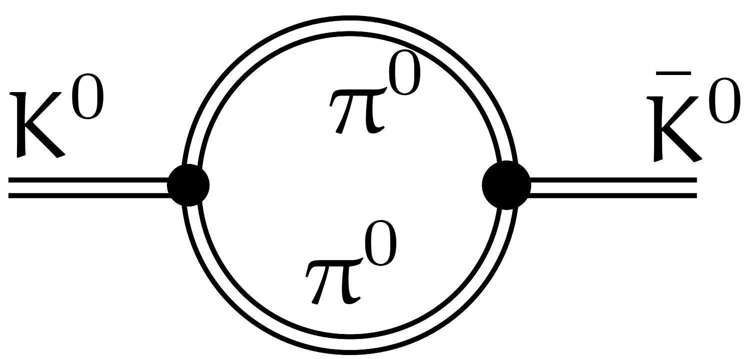
Higher dimensional operator [Cata Peris`04]



Light quark loops in CHPT:

π^0,η tree level vanishes (Gell-Mann-Okuba)

η´comes with zero phase [Gerard et.al. `05]



1-loop diagram divergent:

estimate from $\ln(m_\pi/m_\rho)$ [Buras et.al. `10]

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right)$$

absorptive part estimated form ϵ'

Long Distance Contribution

B_K now precisely known [Aubin, UKQCD, ETM]

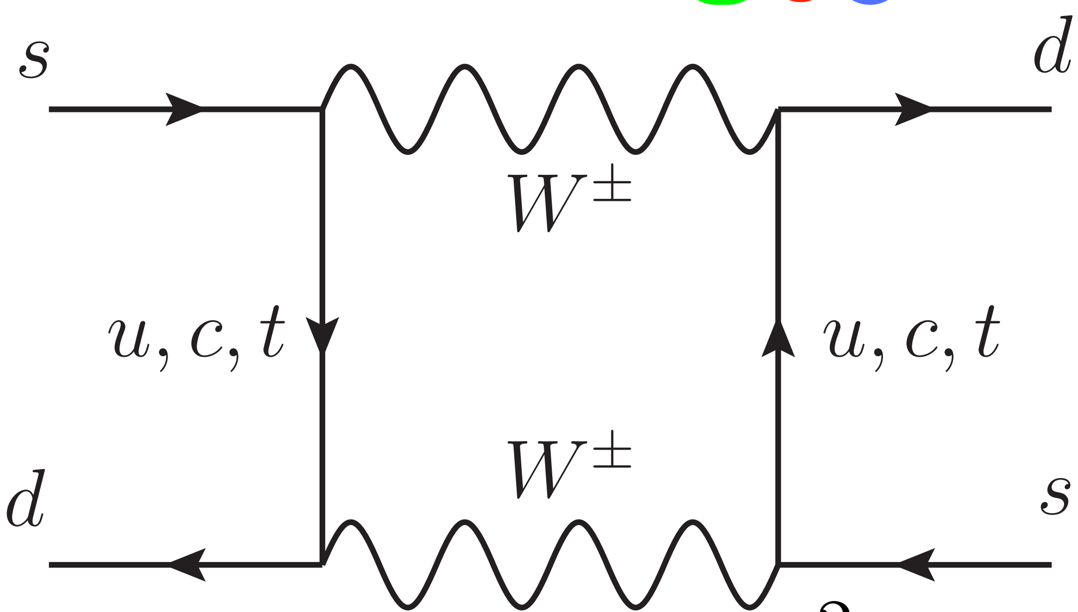
We use $B_K = 0.725\ (26)$

Put the other long distance contribution in $\kappa_\varepsilon = 0.94(2)$ [Buras, Isidori, Guadagnoli `10]

$$\epsilon_K = \kappa_\epsilon \frac{e^{i\pi/4}}{\sqrt{2}} \frac{\mathrm{Im}(M_{12}^{SD})}{\Delta M_K}$$

M_{12}: short distance

Box diagram with internal u,c,t



$$\lambda_i \lambda_j A\left(\frac{m_i^2}{M_W^2}, \frac{m_j^2}{M_W^2}\right)$$

$$\lambda_i = V_{is}^* V_{id}$$

plus GIM:

$$\lambda_c + \lambda_t = -\lambda_u$$

Gives three different contributions for

$$M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$$

$$\mathcal{H} \propto \Big[\lambda_t^2 \eta_t S(m_t^2/M_W^2) \quad \text{top}$$

$$+2\lambda_c \lambda_t \eta_{ct} S(m_c^2/M_W^2, m_t^2/M_W^2) \quad \text{charm top}$$

$$+2\lambda_c^2 \eta_c S(m_c^2/M_W^2) \Big] b(\mu) \tilde{Q} \quad \text{charm}$$

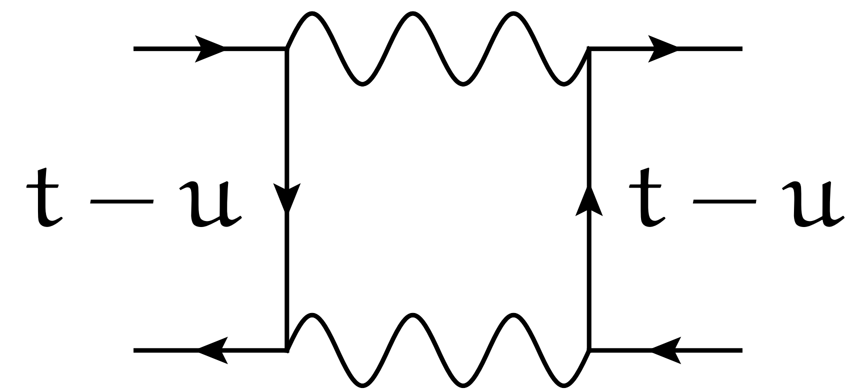
$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$$

Top Quark: Leading Order

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

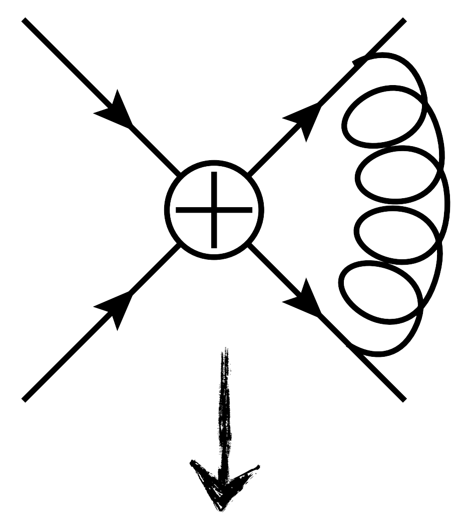
1-loop matching at ($\mu=\mu_t$)

$$\approx \lambda_{\text{Cabibbo}}^{10} \times \frac{m_t^2}{M_W^2} \tilde{Q}$$



1-loop renormalisation group ($\mu_t \rightarrow \mu_c$)

$$\approx \lambda_{\text{Cabibbo}}^{10} \times \frac{m_t^2}{M_W^2} \tilde{Q} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^i \ln^i \left(\frac{\mu_t}{\mu_c}\right)$$



tree-level matching (μ_c) & 1-loop RGI → $b(\mu)$ (which makes B_K renormalistion group independent

$$\approx \lambda_{\text{Cabibbo}}^{10} \times \frac{m_t^2}{M_W^2} \tilde{Q} \times \eta_{tt}$$

Top Quark: Beyond LO

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

@NLO: 2-loop matching at ($\mu=\mu_t$) [Buras `90]

@NNLO: 3-loop renormalisation group ($\mu_t \rightarrow \mu_c$)
2-loop matching (μ_b) & (μ_c)
3-loop RGI
[Brod, MG `10]

Complete NLO: $\eta_{tt} = 0.5765(65)$ at NLO QCD

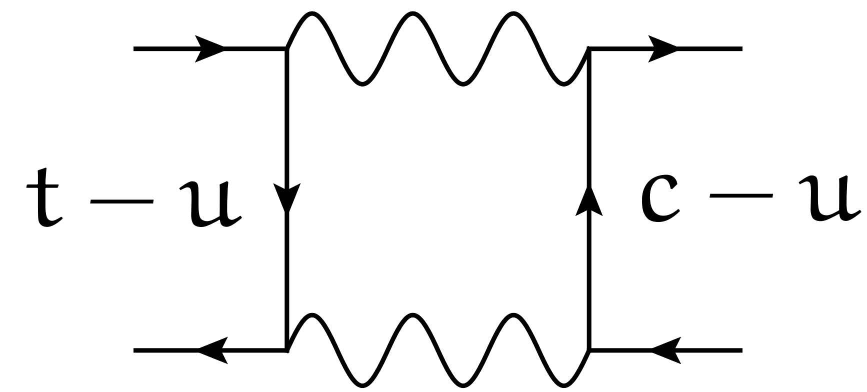
Gives the dominant (+70%) contribution to ϵ_K
(Uncertainty 0.8%)

Charm & Top Quark

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

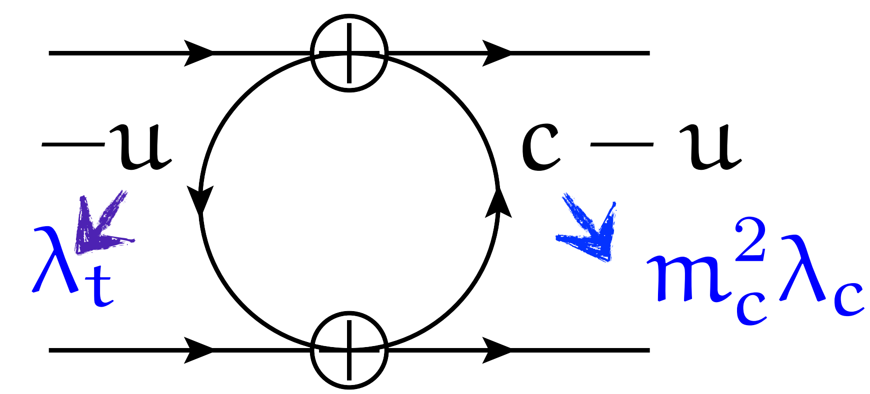
1-loop diagram: infrared divergent

$$\approx \lambda_{\text{Cabibbo}}^6 \times \frac{m_c^2}{M_W^2} \log\left(\frac{m_c^2}{M_W^2}\right) \tilde{Q}$$



Leading Log: 1-loop RGE

$$\approx \lambda_{\text{Cabibbo}}^6 \frac{m_t^2}{M_W^2} \tilde{Q} \sum_{i=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{(i-1)} \ln^i\left(\frac{m_c^2}{M_W^2}\right)$$



Leading order:
Tree-level matching &
1-loop RGE and 1-loop RGI

NLO: $\eta_{ct} = 0.457(73)$
+40% Contribution to ϵ_K
(Uncertainty 6%)

η_{ct}: NNLO Order

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

2-loop Matching at the high scale ($\mu = \mu_{t/W}$):
Current-Current Operators & Penguin Operators [Bobeth et. al. `00],
ΔS=2 Operator [Brod, MG `10]

3-loop renormalisation group equations

$$\mu \frac{d}{d \ln \mu} \tilde{C}(\mu) = \tilde{C}(\mu)\tilde{\gamma} + \sum_{k=1}^{2} \sum_{n=1}^{6} C_k(\mu) C_n(\mu) \hat{\gamma}_{kn,\sim}$$

[Brod, MG `10]

$$\mu \frac{d}{d \ln \mu} C_k(\mu) = \sum_{n=1}^{6} C_K(\mu) \gamma_{nk}$$

[MG, Haisch `05]

2-loop Matching at ($\mu = \mu_b$) and ($\mu = \mu_c$) [Brod, MG `10]

η_{ct}: the RGE

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

To solve these second order renormalisation group equations (RGEs)

$$\mu \frac{d}{d \ln \mu} \tilde{C}(\mu) = \tilde{C}(\mu) \tilde{\gamma} + \sum_{k=1}^{2} \sum_{n=1}^{6} C_k(\mu) C_n(\mu) \hat{\gamma}_{kn,\sim}$$

$$\mu \frac{d}{d \ln \mu} C_k(\mu) = \sum_{n=1}^{6} C_n(\mu) \gamma_{nk}$$

Use operator basis (and scheme) where γ is diagonal for n,k=1,2

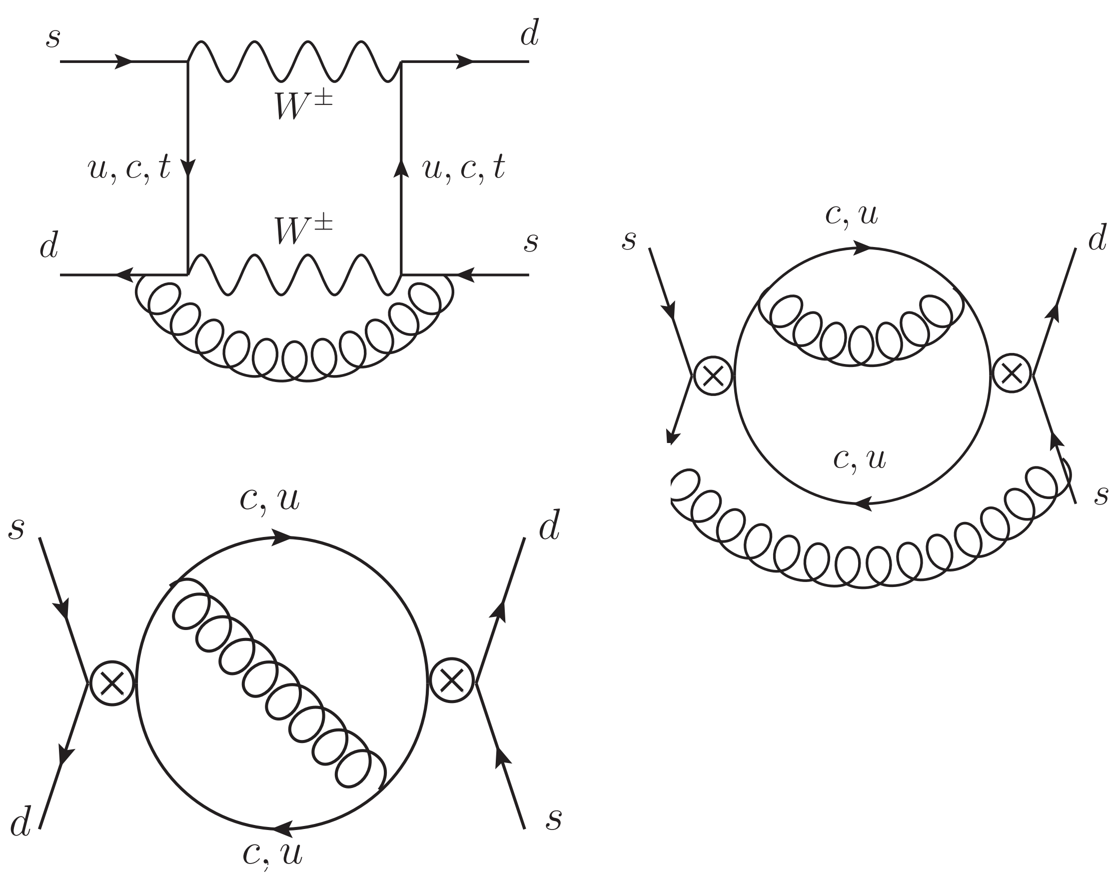
$C_i = (C_+, C_-, C_3, C_4, C_5, C_6)$

The RGE can then be cast into standard form

$$\mu \frac{d}{d \ln \mu} D_k(\mu) = \sum_{n=1}^{8} D_n(\mu) \gamma_{nk} \qquad \vec{D}(\mu) = \left(\vec{C}, \tilde{C}_+/C_+, \tilde{C}_-/C_-\right)^T$$

NNLO Calculation

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \boxed{\lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right)} + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$



O(10000) diagrams were calculated using Form, Matad and Mathematica [Brod, MG `10]

First order in effective interactions:
classic e.o.m. = quantum e.o.m. - BRST exact

Second order in effective interactions:
non-trivial contact interactions [Simma]

Uncertainty Estimation

Several Measures of Uncertainty:

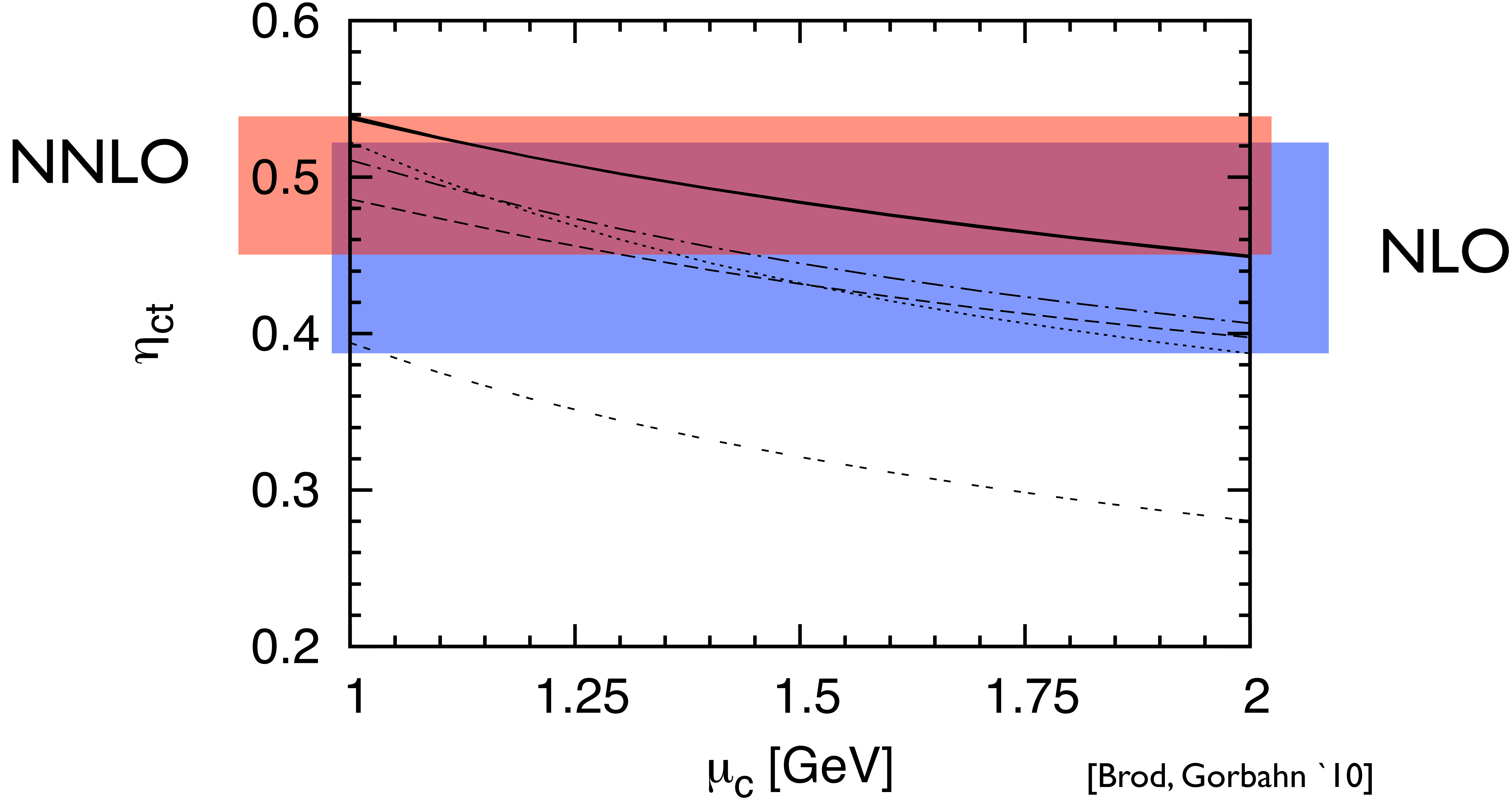
Calculation is formally renormalisation scheme and scale independent

Measure of higher order corrections:
the scale and scheme dependence

Size of the NNLO correction alternative measure

Scale Uncertainty

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$



[Brod, Gorbahn `10]

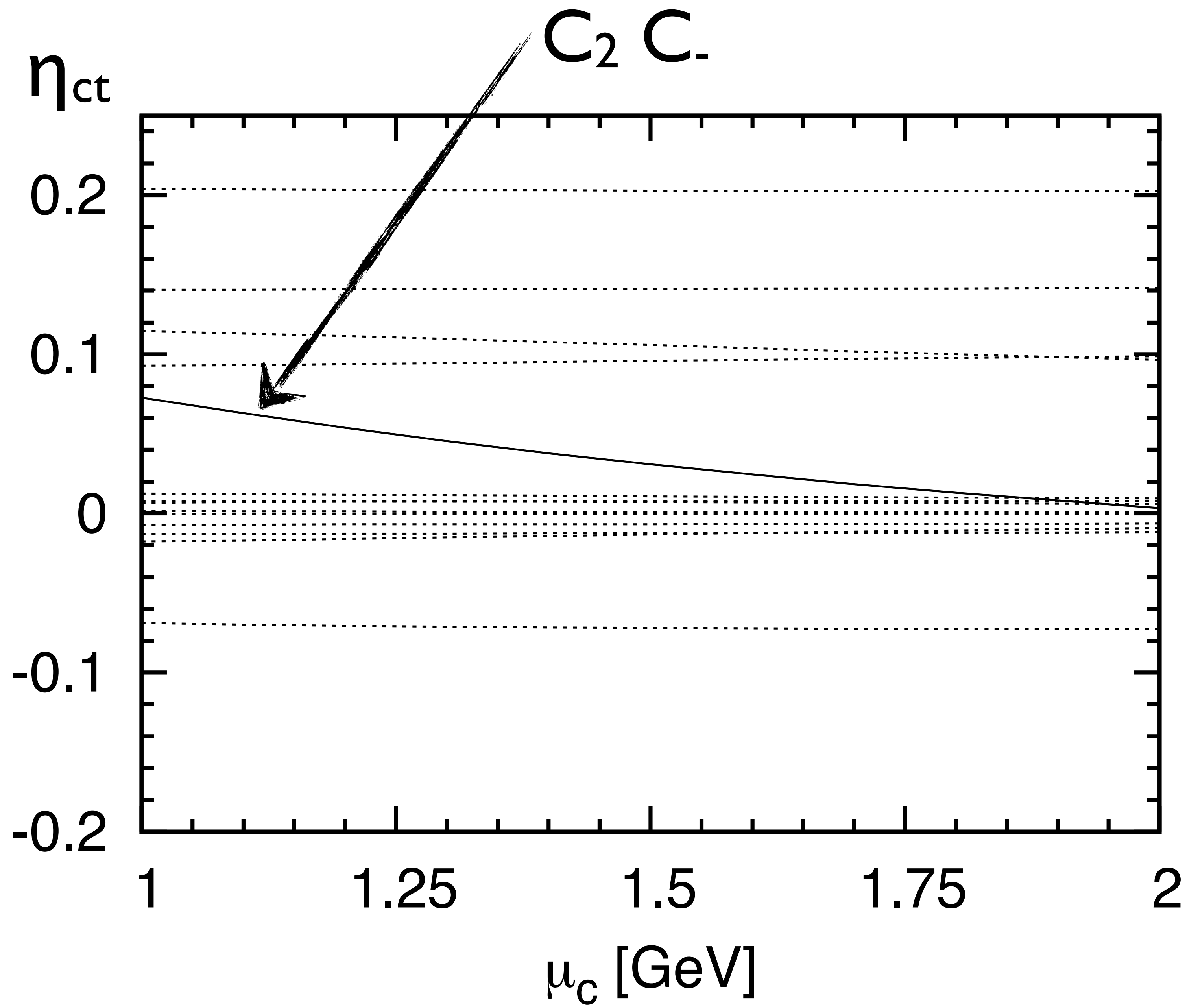
Separate Wilson Coefficients

At LO:
No Mixing of
$Q_2 Q_- \to \tilde{Q}$

2-loop matching ($\mu = \mu_c$) compensates log from 2-loop running

large 3-loop log behaves like NLL

Contributions of separate Wilson coefficient combinations



η_{ct}: Numbers

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)\right] \tilde{Q}$$

Uncertainty dominated by μ_c scale dependence

we use: 1 GeV < μ_c < 2 GeV

old NLO analysis used: 1.1 GeV < μ_c < 1.6 GeV

$$\eta_{ct}^{\mathrm{NLO}} = 0.457 \pm 0.072_{\mu_c} \pm 0.01_{\mu_W} \pm 0.0001_{\alpha_s} \pm 0.002_{m_c} \pm 0.0003_{m_t}$$

$$\eta_{ct}^{\mathrm{NNLO}} = 0.496 \pm 0.045_{\mu_c} \pm 0.013_{\mu_W} \pm 0.002_{\alpha_s} \pm 0.001_{m_c} \pm 0.0002_{m_t}$$

$\eta_{ct}^{\mathrm{NLO}} = 0.456(73) \rightarrow \eta_{ct}^{\mathrm{NNLO}} = 0.496(46)$

Charm Quark Contribution

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \boxed{\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)}\right] \tilde{Q}$$

Matching box diagrams at ($\mu=\mu_t$) and expand in m_c^2:

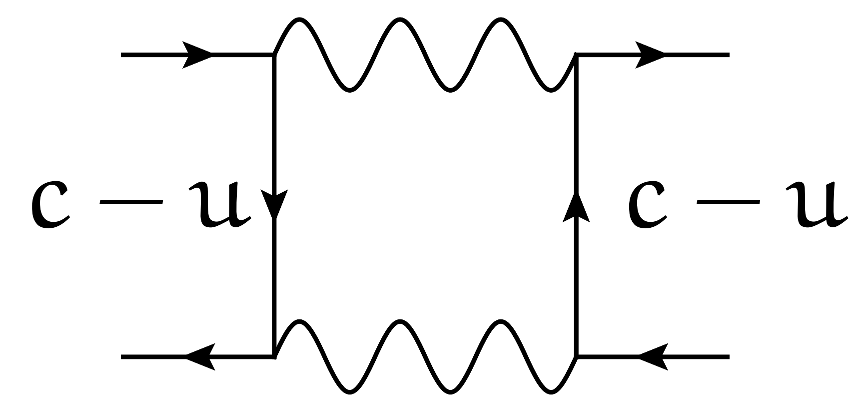
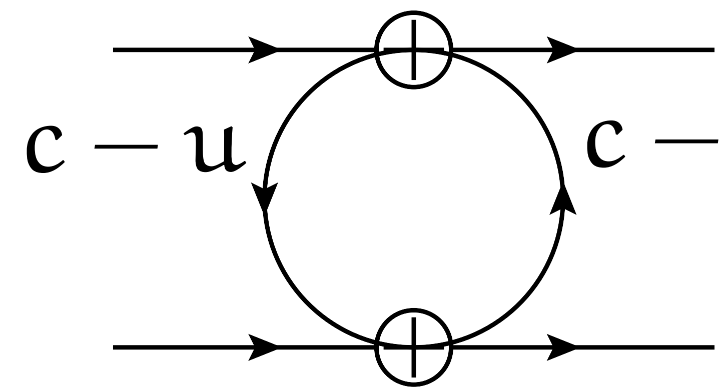


Diagram with 2 charm quarks cancels with 1 charm 1 up quark diagrams

→RGE & matching ($\mu=\mu_{t/W}$) only for: current-current operators



Leading order ($\mu=\mu_c$): 1-loop matching

$$\approx \lambda^6_{\text{Cabibbo}} \eta_{cc} \frac{m_c^2}{M_W^2} \tilde{Q}$$

η_{cc} = 1.43(23) at NLL QCD

[Herrlich, Nierste '94]

-10 % Contribution

η_{cc} at NNLO

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}\right) + \boxed{\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right)}\right] \tilde{Q}$$

η_{cc} = 1.43(23) at NLO QCD [Herrlich, Nierste '94]
(-10 % Contribution)

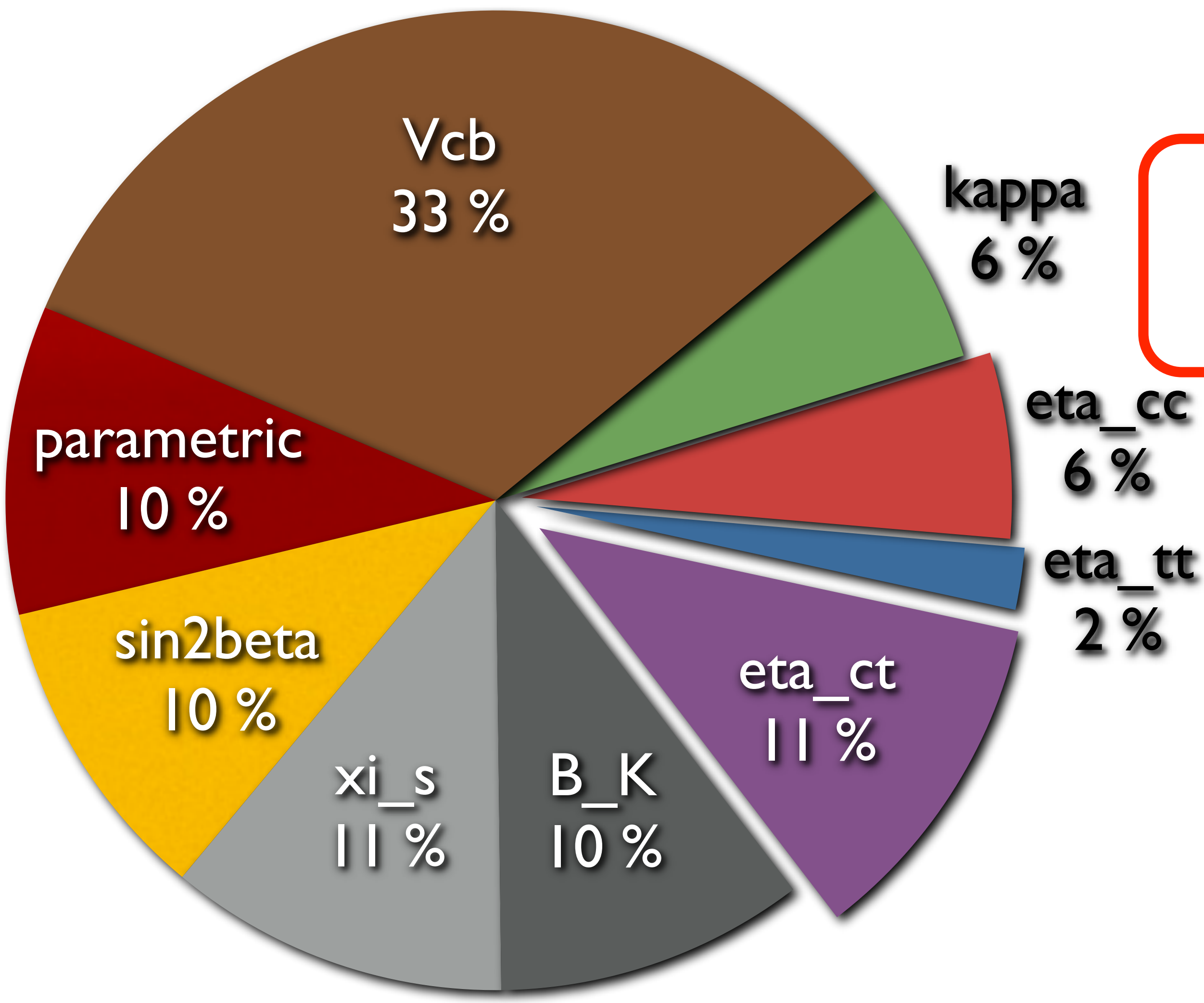
The NLO uncertainty was estimated using 1.1GeV < μ_c < 1.6 GeV

Finite shift of NNLO calculation will clarify the uncertainty

Three loop calculation & renormalisation done
Still: checks have to be performed

New value for η_{cc} coming soon

$|\varepsilon_K|$ and Error Budget



input [PDG `10]

$$|\epsilon_K| = 1.90(26) \times 10^{-3}$$

using

$$\eta_{ct} = 0.496 \pm 0.047$$

$$|V_{cb}| = 406(13) \times 10^{-4}$$

Experiment [PDG `10]:

$$|\epsilon_K| \stackrel{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

Conclusions

Experimental (CKM elements) and theoretical improvements

Tension in CP violating meson mixing parameters appear

NNLO calculation of η_{ct} relieves tension (a bit)

NNLO calculation of η_{cc} on the way