$B^0 ightarrow \pi^0 \pi^0$ first look towards LS1 data update M. Dorigo, <u>S. Raiz</u>, D. Tonelli

Trieste

BtoHadrons meeting Sep 21, 2022

Overview

BF and A_{CP} of $B^0 \rightarrow \pi^0 \pi^0$ decays: important measurements unique to Belle II.

Status: 189.9 fb⁻¹ analysis by Francis shown at ICHEP2022 and targeting PRD submission soon.

Start from Francis analysis, and try to improve for full LS1 data update.

Today focus on selection:

- revisit photonMVA
- revisit CSBDT
- Introduce specific BDT trained against continuum ho's



Samples and selections

Samples

GenericMC: MC15ri

SignalMC: MC15 locally produced (600000 events)

Data: Proc13 chunk1+chunk2

Off-res data: Proc13 (c1+c2) +Prompt

For data use "all" (no hadron skim).

Base selections (from Francis analysis)

 γ : E>0.03 GeV, |clusterTiming|<200, clusterNHits>1.5, 0.30<cluster θ <2.62 (very loose cuts)

π⁰: daughterAngle < 0.4,
|daughterDiffOfPhi| < 0.4,
|cosHelicityAngleMomentum| < 0.99,
p > 1.5 GeV/c, 0.115 < InvM < 0.150 GeV/c² (very loose cuts)

 B^0 : -0.3< ΔE <0.2 GeV, $M_{\rm bc}$ >5.26 GeV/c²

Will be optimised, large improvements unlikely

Can we improve γ and/or π^0 selections?

Apply all selections, CSMVA and photonMVA and check again distributions.

NEW SINCE LAST TIME

New possible γ selections?

γ : E>0.03 GeV, |clusterTiming|<200, clusterNHits>1.5, 0.30<cluster θ <2.62



Note: **clusterTiming** is not well

reproduced \rightarrow very loose cut



NEW SINCE LAST TIME

New possible π^0 selections?



New possible π^0 selections?

p > 1.5 GeV/c, 0.115 < InvM < 0.150 GeV/c²



Photon MVA

Photon MVA

Distinguish between signal photons and misreconstructed photons: beam backgrounds, energy releases from other particles...

Combine highly-discriminant cluster- and photon-variables in a MVA.

Mis-rec photons have low energies, $B^0 \rightarrow \pi^0 \pi^0$ photons have high-energy.

 \rightarrow Specialise MVA on high-energy photons: apply π^0 selections before training.



Photon MVA: inputs validation

Ideally would need a sample of true photons and a sample of mis-rec photons in data (difficult).

Use inclusive sample of photons from $D^* \to D^0(K\pi\pi^0)\pi$ decays: reweigh momenta to mirror $B^0 \to \pi^0\pi^0$ signal kinematics.

Sample is signal dominated \rightarrow ~all true photons (as in $B^0 \rightarrow \pi^0 \pi^0$).

Compare many and new possible ECL input variables using MC15ri (200 fb⁻¹) and Proc13c1(8 fb⁻¹).

0.045

0.04

0.035

0.03

0.025

0.02

0.015

0.01

0.005

0.25

0.2

0.15

0.1

0.05

0

0















0.03

0.025

0.02

0.015

0.01

0.005

0.3

0.25

0.2

0.15

0.1

0.05









Photon MVA discrimination in release-06



PhotonMVA output

Photon MVA validation

Apply photonMVA to $B^+ \rightarrow K^+ \pi^0$ proc13 sample (chunk1+chunk2 — 62fb⁻¹).



No photonMVA

PhotonMVA>0.2

Background: 740 ± 40 Signal: 260 ± 30 Background: 680 ± 40 (-8,5%) Signal: 260 ± 30 (-0%)

Small statistics. Modest but positive impact. Will repeat using 190 fb⁻¹.

Photon MVA comparison

Simulation **Look at photons**: reconstruct $B^0 \to \pi^0 \pi^0$ in genericMC and apply γ and π^0 selections. Consider as "signal" all real photons, and as "background" all misreconstructed photons. Use MC info to obtain photon signal efficiency and bkg rejection after photonMVA selection. For fixed ε_{sig} (=96%), compare bkg rejection.

Old bkg rejection: 67.4% My bkg rejection: 85.2%

Simulation **Look at** B^0 candidates: reconstruct $B^0 \rightarrow \pi^0 \pi^0$ candidates in genericMC and apply γ and π^0 selections. Consider as "signal" all signal $B^0 o \pi^0 \pi^0$ events, and everything else as "background". Use MC info to obtain signal efficiency and bkg rejection after photonMVA selection. For fixed ε_{sig} (=94.5%), compare bkg rejection.

Old bkg rejection: 14.8%

My bkg rejection: 15.7%

Data **Check on data**: reconstruct $D^{*+} \rightarrow D^0(K^-\pi^+\pi^0)\pi^+$ candidates in data and apply $\gamma^$ and π^0 selections. Reweigh using $p(\pi^0)$. Consider as "signal" all signal $D^{*+} \rightarrow D^0 \pi^+$ events, and everything else as bkg. Obtain $\varepsilon_{\rm sig}$ and bkg rejection as $N_{\text{pass}}/(N_{\text{pass}} + N_{\text{not pass}})$ from fit. For fixed ε_{sig} (=96.6%), compare bkg rejection. My bkg rejection: 9.4±0.1% Old bkg rejection: 5.1±0.1%

Photon MVA comparison

190 fb⁻¹ analysis

- 10 ECL variables
- Inputs are validated using MC14 and Proc12.
- Validation on $D^{*+} \rightarrow D^0(K^0_S \pi^0) \pi^+$ with no reweighing.
- AUC = 0.94

This analysis

- 11 ECL variables including beamBackgroundSuppressionMVA.
- Apply π^0 selections prior to training.
- All inputs are validated using MC15 and Proc13.
- Validation on $D^* \to D^0(K\pi\pi^0)\pi$ reweighed using $p(\pi^0)$.
- AUC = 0.97

Photon MVA discrimination in release-06



Remove beamBackgroundSuppression



clusterAbsZernikeMoment51, E, clusterE9E21

Classifier Output

PhotonMVA output

CSBDT

CSBDT

Develop continuum-suppression BDT. Main difference wrt Francis analysis: include $B_{\rm Tag}$ variables, avoiding large correlations (<10% — was 5% in old analysis) and/or sculpting.

Check if B_{Tag} variables sculpts or introduces large correlations in flavour tagger.

Start from Δr and ΔZ (distance of vertex from IP).

Note: 6.7% of the signal events don't have a B_{Tag} vertex \rightarrow remove these events (bkg: -9.4%).

New possible inputs:



CSBDT: inputs validation

Validate **signal** distributions: compare $B \rightarrow D(K\pi\pi^0)\pi$ sideband-subtracted data and sideband-subtracted MC.

Do not use $B \rightarrow D(K\pi\pi^0)\pi$ for bkg because of the different compositions

Validate **background** distributions: compare $B^0 \rightarrow \pi^0 \pi^0$ sideband data and sideband MC15.



Inputs validation — Signal

Use $B \rightarrow D(K\pi\pi^0)\pi$ sideband-subtracted data and sideband-subtracted MC.







Better training CSBDT using data only.

CSMVA using data to train bkg

Two options:



Pro: describes well the background in the signal-region in all the variables

Cons: very small amount of data



Cons: doesn't describe well bkg in the signal region in two distributions (angles between pions)

Poorly described variables in sideband data

Only two distributions not describing well bkg in signal region:



CSMVA using data to train bkg

Two options:





CSMVA using data to train bkg



AUC curves show same performance.

CSMVA using off-resonance data: k-fold cross validation

Split sample in k = 10 folds. Use each combination of 9 folds to train BDT and remaining fold to test it \rightarrow obtain 10 ROC curves. Remove possible statistical effects.



Performance is the same. Still deciding final choice.

Check on flavor tagger: are Δr and ΔZ in CSBDT biasing the FT?

Flavor tagging check

Compare flavour tagger parameters obtained in $B^0 \to \pi^0 \pi^0$ after applying CSBDT selection and in control channel $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$ (largest BF btw those used by Sato-san in FT calibration).

						$D \rightarrow \pi^{\circ}\pi^{\circ}$
r- Interval	$arepsilon_i$	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$
0.000 - 0.100	17.1	0.22	47.54 ± 0.26	2.60 ± 0.52	0.0414 ± 0.0087	-0.0866 ± 0.0212
0.100 - 0.250	16.6	0.06	40.97 ± 0.26	1.01 ± 0.52	0.5417 ± 0.0311	-0.1196 ± 0.0631
0.250 - 0.500	21.1	0.85	30.02 ± 0.21	0.34 ± 0.43	3.3760 ± 0.0737	0.0203 ± 0.1471
0.500 - 0.625	11.8	-0.38	20.87 ± 0.25	1.84 ± 0.51	3.9966 ± 0.0732	-0.6337 ± 0.1479
0.625 - 0.750	11.5	-0.07	15.08 ± 0.23	0.60 ± 0.45	5.5858 ± 0.0797	-0.2268 ± 0.1598
0.750 - 0.875	8.6	0.06	8.31 ± 0.20	0.56 ± 0.40	5.9602 ± 0.0712	-0.1181 ± 0.1425
0.875 - 1.000	13.4	-0.74	1.67 ± 0.07	0.24 ± 0.15	12.5174 ± 0.0785	-0.8171 ± 0.1573
Total			$\varepsilon_{eff} = \Sigma$	$\sum_i \varepsilon_i \cdot \langle 1 - 2w_i \rangle^2$	$2^{2} = 32.02 \pm 0.17 \Delta \epsilon$	$z_{eff} = -1.98 \pm 0.34$
						$B^0 \rightarrow D^-(K^+\pi^-\pi)$
<i>r</i> -Interval	ε_i	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \overline{\varepsilon_{eff,i}}$
0.000 - 0.100	17.7	0.27	47.27 ± 0.26	2.58 ± 0.51	0.0526 ± 0.0099	-0.0986 ± 0.0217
0.100 - 0.250	16.8	0.26	41.09 ± 0.26	1.07 ± 0.52	0.5322 ± 0.0311	-0.1189 ± 0.0623
0.250 - 0.500	21.1	0.60	30.37 ± 0.22	0.24 ± 0.43	3.2461 ± 0.0728	0.0138 ± 0.1457
0.500 - 0.625	11.5	0.06	21.40 ± 0.26	1.66 ± 0.52	3.7607 ± 0.0723	-0.4175 ± 0.1445
0.625 - 0.750	11.2	-0.13	15.23 ± 0.23	0.97 ± 0.46	5.4192 ± 0.0794	-0.3649 ± 0.1588
0.750 - 0.875	8.6	-0.22	8.46 ± 0.20	0.46 ± 0.41	5.9675 ± 0.0721	-0.2828 ± 0.1442
0.875 - 1.000	13.1	-0.85	1.62 ± 0.08	0.47 ± 0.15	12.2983 ± 0.0783	-1.0309 ± 0.1565
			<u> </u>	$1/1 \circ 1^2$	0100 ± 0.17 Å	0.00 + 0.01

Results are compatible. Waiting for the official numbers from Sato-san (end of month).

Flavor tagging check

Compare parameters using pulls.



Add PID info

NEW SINCE LAST TIME All MC15

Compare flavour tagger parameters obtained in $B^0 \to \pi^0 \pi^0$ after applying CSBDT selection and in control channel $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$ (largest BF btw those used by Sato-san in FT calibration).

<i>r</i> - Interval	$arepsilon_{i}$	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$
0.000 - 0.100	17.1	0.22	47.54 ± 0.26	2.60 ± 0.52	0.0414 ± 0.0087	-0.0866 ± 0.0212
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0.250 - 0.500	21.1	0.85	30.02 ± 0.21	0.34 ± 0.43	3.3760 ± 0.0737	0.0203 ± 0.1471
0.500 - 0.625	11.8	-0.38	20.87 ± 0.25	1.84 ± 0.51	3.9966 ± 0.0732	-0.6337 ± 0.1479
0.625 - 0.750	11.5	-0.07	15.08 ± 0.23	0.60 ± 0.45	5.5858 ± 0.0797	-0.2268 ± 0.1598
0.750 - 0.875	8.6	0.06	8.31 ± 0.20	0.56 ± 0.40	5.9602 ± 0.0712	-0.1181 ± 0.1425
0.875 - 1.000	13.4	-0.74	1.67 ± 0.07	0.24 ± 0.15	12.5174 ± 0.0785	-0.8171 ± 0.1573
Total			$\varepsilon_{eff} = \sum$	$\sum_i \varepsilon_i \cdot \langle 1 - 2w_i \rangle^2$	$= 32.02 \pm 0.17 \Delta \varepsilon$	$_{eff} = -1.98 \pm 0.34$
						DID
<i>r</i> - Interval	$arepsilon_i$	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$
0.000 - 0.100	16.5	0.22	47.63 ± 0.26	2.52 ± 0.53	0.0373 ± 0.0083	-0.0787 ± 0.0202
0.100 - 0.250	16.2	0.06	40.99 ± 0.26	0.90 ± 0.52	0.5265 ± 0.0307	-0.1038 ± 0.0620
0.250 - 0.500	20.9	0.80	30.03 ± 0.22	0.30 ± 0.43	3.3337 ± 0.0731	0.0288 ± 0.1459
0.500 - 0.625	11 🗖	0.91	00.01 ± 0.05	1 FO + 0 F1	0.0000 + 0.0700	0 = 0 = 0 + 0 + 4 = 1
0.000 0.010	11.1	-0.31	20.91 ± 0.25	1.59 ± 0.51	3.9692 ± 0.0729	-0.5378 ± 0.1471
0.625 - 0.750	11.7 11.6	$-0.31 \\ -0.07$	20.91 ± 0.25 15.07 ± 0.22	1.59 ± 0.51 0.63 ± 0.45	3.9692 ± 0.0729 5.6428 ± 0.0800	-0.5378 ± 0.1471 -0.2368 ± 0.1603
0.625 - 0.750 0.750 - 0.875	11.7 11.6 8.8	-0.31 -0.07 0.03	20.91 ± 0.25 15.07 ± 0.22 8.31 ± 0.20	1.59 ± 0.51 0.63 ± 0.45 0.50 ± 0.40	$\begin{array}{c} 3.9692 \pm 0.0729 \\ 5.6428 \pm 0.0800 \\ 6.1016 \pm 0.0719 \end{array}$	-0.5378 ± 0.1471 -0.2368 ± 0.1603 -0.1233 ± 0.1439
0.625 - 0.750 0.750 - 0.875 0.875 - 1.000	$ 11.7 \\ 11.6 \\ 8.8 \\ 14.3 $	-0.31 -0.07 0.03 -0.74	20.91 ± 0.25 15.07 ± 0.22 8.31 ± 0.20 1.67 ± 0.07	1.59 ± 0.51 0.63 ± 0.45 0.50 ± 0.40 0.20 ± 0.14	$\begin{array}{c} 3.9692 \pm 0.0729 \\ 5.6428 \pm 0.0800 \\ 6.1016 \pm 0.0719 \\ 13.3349 \pm 0.0806 \end{array}$	-0.5378 ± 0.1471 -0.2368 ± 0.1603 -0.1233 ± 0.1439 -0.8013 ± 0.1615

Add PID info

NEW SINCE LAST TIME All MC15

Compare flavour tagger parameters obtained in $B^0 \to \pi^0 \pi^0$ after applying CSBDT selection and in control channel $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$ (largest BF btw those used by Sato-san in FT calibration).



NEW SINCE LAST TIME

CS validation

Validate CS on control channel $B^- \to D^0(K^+\pi^-\pi^0)\pi^-$. Obtain binomial efficiency of CS selection using Francis CS and mine.

Tried using $B^0 \to D^0(K^+\pi^-\pi^0)\pi^0$ but small BF.

Still some problems in the fit.

At the current precision, using Δr and ΔZ as inputs does not bias the result after CS selection.

Need to check this again when official numbers will be available.

CSBDT comparison

190 fb⁻¹ analysis

- 24 variables.
- Training using sideband data.
- Validate only CSBDT output in data and MC.
- AUC = 0.948.

This analysis

- 19 variables including Δr and ΔZ: no bias on flavour tagger.
- Validation of input variables.
- Include variables with larger correlation with fit variables (but no sculpting).
- Training using sideband data or offresonance data.
- AUC = 0.96

1% improvement in AUC.

Next step: compare performance in data using control channel.

ho MVA

ho MVA

Beyond the CS: identify the principal bkg offenders.

	Events that have at least a π^0 from
ρ(770)+	47.1%
Z ^o (direct from e+e-)	75.0%

Large number of continuum π^0 's come from a ρ . Develop a specific BDT to identify them (in addition to the default CSBDT).

Combine each track in the event with each π^0 .

Use kinematic and angular variables to distinguish between ho's and other particles.

Challenge: the number of combinations (ρ^+ candidates) is large. Not obvious how to exploit it efficiently in bkg rejection strategy.



ρ variables

Use kinematic and angular variables to distinguish between ρ 's and other particles.



 ρ mass and helicity angle have large discriminating power.

Max hoMVA distribution

Each candidate has for example 20 ρ sub-candidates. Take the one with largest rhoMVA (the one more similar to a ρ).



Variable shows separation, modelling discrepancy is acceptable. Additional discriminating power may be available from using multiple ρ^+ candidates.

Use $\rho {\rm MVA}$ as input of the CSBDT



Inclusion of ρ MVA gives no improvement

Summary

First steps towards LS1 update of $B^0 \rightarrow \pi^0 \pi^0$ analysis. Use Francis result as reference.

Today focus on selection:

- revisited photonMVA (use new variables with good data/MC agreement)

 \rightarrow 3% improvement. At given $\varepsilon_{\rm sig'}$ always better bkg rejection.

- revisited CSBDT: add $B_{\rm Tag}$ variables (no bias on FT). Use data to train BDT, validate inputs \rightarrow 1% improvement.

- introduced ρ BDT: improvement is negligible, do not add it in analysis \rightarrow no improvement.

Don't expect major breakthrough wrt 190fb⁻¹ analysis, but obtained various small improvements and refinements.

Backup

ClusterTiming (rel-06)











Inputs validation — Signal only



Inputs validation — Signal only



Inputs validation — Signal only



CSMVA inputs

Inputs (after pruning)

7 Kakuno-Super-Fox-Wolfram moments

cosTBTO

1 CleoCone

cosTheta*

R2

thrustOm

ΔZ (BTag)

∆r (BTag)

thrustAxisCosTheta

angle between $\pi^{_{0}\prime}\!s$

cosHelicityAngle

KSFWVariableset

KSFWVariablesmm2

Flavor tagging check: no ΔZ , no Δr

Compare flavour tagger parameters obtained in $B^0 \to \pi^0 \pi^0$ after applying CSBDT selection and in control channel $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$ (largest BF btw those used by Sato-san in FT calibration). Use MC15.

 $B^0 \rightarrow -0.0$

							10
<i>r</i> -Interval	ε_i	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$	
0.000 - 0.100	17.1	0.23	47.64 ± 0.26	2.35 ± 0.53	0.0382 ± 0.0085	-0.0752 ± 0.0204	-
0.100 - 0.250	16.6	0.09	41.07 ± 0.26	1.13 ± 0.53	0.5287 ± 0.0313	-0.1304 ± 0.0634	
0.250 - 0.500	21.2	0.81	30.04 ± 0.22	0.36 ± 0.43	3.3799 ± 0.0749	0.0071 ± 0.1495	
0.500 - 0.625	11.8	-0.36	20.94 ± 0.26	1.95 ± 0.51	3.9692 ± 0.0742	-0.6524 ± 0.1499	
0.625 - 0.750	11.4	-0.08	15.17 ± 0.23	0.72 ± 0.46	5.5267 ± 0.0806	-0.2681 ± 0.1618	
0.750 - 0.875	8.5	0.07	8.27 ± 0.20	0.44 ± 0.41	5.9513 ± 0.0722	-0.0788 ± 0.1445	
0.875 - 1.000	13.4	-0.76	1.68 ± 0.08	0.19 ± 0.15	12.5437 ± 0.0799	-0.8107 ± 0.1601	-
Total			$\varepsilon_{eff} = \sum$	$\sum_{i} \varepsilon_i \cdot \langle 1 - 2w_i \rangle^2$	$^{2} = 31.94 \pm 0.17 \Delta \varepsilon$	$c_{eff} = -2.01 \pm 0.35$	-
T / 1				A 5 A		$B^0 \rightarrow D^-(K^+)$	$ au^-\pi$
<i>r</i> -Interval	$arepsilon_i$	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$	
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0.625 - 0.750	11.2	-0.13	15.23 ± 0.23	0.97 ± 0.46	5.4192 ± 0.0794	-0.3649 ± 0.1588	
0.750 - 0.875	8.6	-0.22	8.46 ± 0.20	0.46 ± 0.41	5.9675 ± 0.0721	-0.2828 ± 0.1442	
0.875 - 1.000	13.1	-0.85	1.62 ± 0.08	0.47 ± 0.15	12.2983 ± 0.0783	-1.0309 ± 0.1565	
Total			$\varepsilon_{eff} = \sum$	$\sum_{i} \varepsilon_i \cdot \langle 1 - 2w_i \rangle^2$	$= 31.28 \pm 0.17 \Delta \varepsilon_e$	$e_{ff} = -2.30 \pm 0.34$	

Results are compatible. Waiting for the official numbers from Sato-san (end of month).

Flavor tagging check: no ΔZ , no Δr

Compare flavour tagger parameters obtained in $B^0 \rightarrow \pi^0 \pi^0$ after applying CSBDT selection including and excluding ΔZ and Δr .

Τ, 1							
<i>r</i> -Interval	ε_i	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$	
0.000 - 0.100	17.1	0.23	47.64 ± 0.26	2.35 ± 0.53	0.0382 ± 0.0085	-0.0752 ± 0.0204	
0.100 - 0.250	16.6	0.09	41.07 ± 0.26	1.13 ± 0.53	0.5287 ± 0.0313	-0.1304 ± 0.0634	
0.250 - 0.500	21.2	0.81	30.04 ± 0.22	0.36 ± 0.43	3.3799 ± 0.0749	0.0071 ± 0.1495	
0.500 - 0.625	11.8	-0.36	20.94 ± 0.26	1.95 ± 0.51	3.9692 ± 0.0742	-0.6524 ± 0.1499	$B^0 \to \pi^0 \pi$
0.625 - 0.750	11.4	-0.08	15.17 ± 0.23	0.72 ± 0.46	5.5267 ± 0.0806	-0.2681 ± 0.1618	
0.750 - 0.875	8.5	0.07	8.27 ± 0.20	0.44 ± 0.41	5.9513 ± 0.0722	-0.0788 ± 0.1445	
0.875 - 1.000	13.4	-0.76	1.68 ± 0.08	0.19 ± 0.15	12.5437 ± 0.0799	-0.8107 ± 0.1601	
Total			$\varepsilon_{eff} = \sum$	$\sum_{i} \varepsilon_i \cdot \langle 1 - 2w_i \rangle^2$	$= 31.94 \pm 0.17 \Delta \varepsilon_{e}$	$_{eff} = -2.01 \pm 0.35$	
				_ 0			
		Δ	1 5			A S A	
<i>r</i> -Interval	$arepsilon_i$	$\Delta \varepsilon_i$	$w_i \pm \delta w_i$	$\Delta w_i \pm \delta \Delta w_i$	$\varepsilon_{eff,i} \pm \delta \varepsilon_{eff,i}$	$\Delta \varepsilon_{eff,i} \pm \delta \Delta \varepsilon_{eff,i}$	
0.000 - 0.100	17.1	0.22	47.54 ± 0.26	2.60 ± 0.52			
0.100 - 0.250	100			2.00 ± 0.02	0.0414 ± 0.0087	-0.0866 ± 0.0212	
0.100 0.200	10.0	0.06	40.97 ± 0.26	1.01 ± 0.52	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \end{array}$	
0.250 - 0.500	$\frac{16.6}{21.1}$	$\begin{array}{c} 0.06 \\ 0.85 \end{array}$	40.97 ± 0.26 30.02 ± 0.21	1.01 ± 0.52 0.34 ± 0.43	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \\ 3.3760 \pm 0.0737 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \\ 0.0203 \pm 0.1471 \end{array}$	$R^0 \rightarrow \pi^0 \tau$
0.250 - 0.500 0.500 - 0.625	16.6 21.1 11.8	$0.06 \\ 0.85 \\ -0.38$	40.97 ± 0.26 30.02 ± 0.21 20.87 ± 0.25	1.01 ± 0.52 0.34 ± 0.43 1.84 ± 0.51	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \\ 3.3760 \pm 0.0737 \\ 3.9966 \pm 0.0732 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \\ 0.0203 \pm 0.1471 \\ -0.6337 \pm 0.1479 \end{array}$	$B^0 o \pi^0 \pi$
$\begin{array}{c} 0.250 - 0.250 \\ 0.250 - 0.500 \\ 0.500 - 0.625 \\ 0.625 - 0.750 \end{array}$	$ \begin{array}{r} 16.6 \\ 21.1 \\ 11.8 \\ 11.5 \end{array} $	$0.06 \\ 0.85 \\ -0.38 \\ -0.07$	$\begin{array}{c} 40.97 \pm 0.26 \\ 30.02 \pm 0.21 \\ 20.87 \pm 0.25 \\ 15.08 \pm 0.23 \end{array}$	$\begin{array}{c} 1.01 \pm 0.52 \\ 1.01 \pm 0.52 \\ 0.34 \pm 0.43 \\ 1.84 \pm 0.51 \\ 0.60 \pm 0.45 \end{array}$	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \\ 3.3760 \pm 0.0737 \\ 3.9966 \pm 0.0732 \\ 5.5858 \pm 0.0797 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \\ 0.0203 \pm 0.1471 \\ -0.6337 \pm 0.1479 \\ -0.2268 \pm 0.1598 \end{array}$	$B^0 o \pi^0 \pi$ with Δ r and
$\begin{array}{c} 0.250 - 0.200\\ 0.250 - 0.500\\ 0.500 - 0.625\\ 0.625 - 0.750\\ 0.750 - 0.875\end{array}$	$ \begin{array}{r} 16.6 \\ 21.1 \\ 11.8 \\ 11.5 \\ 8.6 \end{array} $	$0.06 \\ 0.85 \\ -0.38 \\ -0.07 \\ 0.06$	$\begin{array}{c} 40.97 \pm 0.26 \\ 30.02 \pm 0.21 \\ 20.87 \pm 0.25 \\ 15.08 \pm 0.23 \\ 8.31 \pm 0.20 \end{array}$	$\begin{array}{c} 1.01 \pm 0.52 \\ 1.01 \pm 0.52 \\ 0.34 \pm 0.43 \\ 1.84 \pm 0.51 \\ 0.60 \pm 0.45 \\ 0.56 \pm 0.40 \end{array}$	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \\ 3.3760 \pm 0.0737 \\ 3.9966 \pm 0.0732 \\ 5.5858 \pm 0.0797 \\ 5.9602 \pm 0.0712 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \\ 0.0203 \pm 0.1471 \\ -0.6337 \pm 0.1479 \\ -0.2268 \pm 0.1598 \\ -0.1181 \pm 0.1425 \end{array}$	$B^0 o \pi^0 \pi$ with Δ r and
$\begin{array}{c} 0.250 - 0.200\\ 0.250 - 0.500\\ 0.500 - 0.625\\ 0.625 - 0.750\\ 0.750 - 0.875\\ 0.875 - 1.000 \end{array}$	$ \begin{array}{r} 16.6 \\ 21.1 \\ 11.8 \\ 11.5 \\ 8.6 \\ 13.4 \end{array} $	0.06 0.85 -0.38 -0.07 0.06 -0.74	$\begin{array}{c} 40.97 \pm 0.26 \\ 30.02 \pm 0.21 \\ 20.87 \pm 0.25 \\ 15.08 \pm 0.23 \\ 8.31 \pm 0.20 \\ 1.67 \pm 0.07 \end{array}$	$\begin{array}{c} 2.00 \pm 0.02 \\ 1.01 \pm 0.52 \\ 0.34 \pm 0.43 \\ 1.84 \pm 0.51 \\ 0.60 \pm 0.45 \\ 0.56 \pm 0.40 \\ 0.24 \pm 0.15 \end{array}$	$\begin{array}{c} 0.0414 \pm 0.0087 \\ 0.5417 \pm 0.0311 \\ 3.3760 \pm 0.0737 \\ 3.9966 \pm 0.0732 \\ 5.5858 \pm 0.0797 \\ 5.9602 \pm 0.0712 \\ 12.5174 \pm 0.0785 \end{array}$	$\begin{array}{c} -0.0866 \pm 0.0212 \\ -0.1196 \pm 0.0631 \\ 0.0203 \pm 0.1471 \\ -0.6337 \pm 0.1479 \\ -0.2268 \pm 0.1598 \\ -0.1181 \pm 0.1425 \\ -0.8171 \pm 0.1573 \end{array}$	$B^0 o \pi^0 \pi$ with Δ r and

Results are compatible. Inclusion of ΔZ and Δr doesn't bias FT.

Photon MVA validation

Apply photonMVA to $D^* \to D^0(K\pi\pi^0)\pi$ data after applying signal π^0 selections and reweighting candidates based on signal $p(\pi^0)$.

Compare background rejection at given signal efficiency.

Obtain signal efficiency as

