Noisy Gates for Quantum Computing

Quantum Computing @INFN

Bologna, 14-15 November 2022

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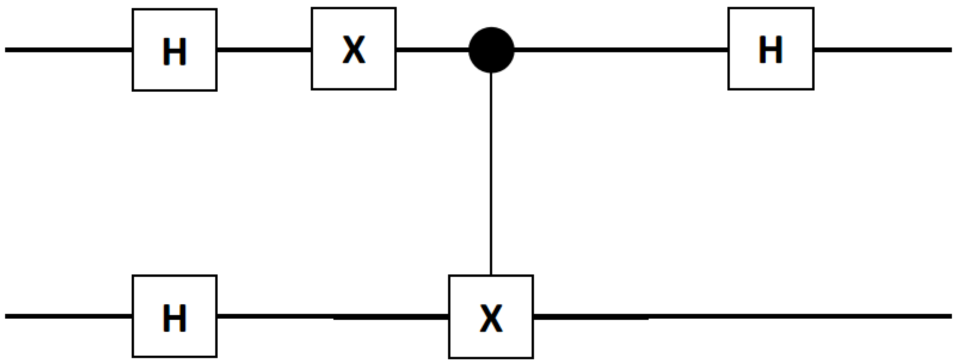
Real Quantum Computers



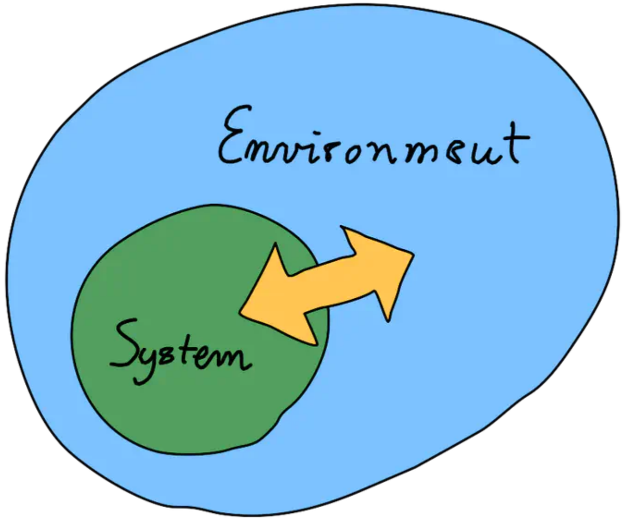
In an ideal (= isolated) world, quantum computers run beautifully

In real life, they are subject to **noise**

- Quantum Error Correction, but even more qubits are needed
- NISQ (Noise Intermediate-Scale Quantum) devices

Study the noise

A **proper theoretical modelling** of the effect of the environment on a quantum systems allows to:



- Have a **physical understanding** of the sources of noise
- Suggest strategies to **mitigate errors**
- Perform **accurate simulations** to predict how the performances scale with the number of qubits/gates.

Georgopoulos, K., Emary, C., & Zuliani, P. (2021). Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, *104*(6), 062432.

Sun, J., Yuan, X., Tsunoda, T., Vedral, V., Benjamin, S. C., & Endo, S. (2021). Mitigating realistic noise in practical noisy intermediate-scale quantum devices. *Physical Review Applied*, *15*(3), 034026.

Guerreschi, G. G., & Matsuura, A. Y. (2019). QAOA for Max-Cut requires hundreds of qubits for quantum speed-up. *Scientific reports*, *9*(1), 1-7.

Xue, C., Chen, Z. Y., Wu, Y. C., & Guo, G. P. (2021). Effects of quantum noise on quantum approximate optimization algorithm. *Chinese Physics Letters*, *38*(3), 030302.

Resch, S., & Karpuzcu, U. R. (2021). Benchmarking quantum computers and the impact of quantum noise. *ACM Computing Surveys (CSUR)*, *54*(7), 1-35.

Standard noise model

Breuer and Petruccione: *The Theory of Open Quantum Systems*, Oxford University Press (2002)

Theory of **open quantum systems**

$$|\psi\rangle \;\rightarrow\; \rho = |\psi\rangle\langle\psi|$$

State vector Density matrix

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\} \right]$$

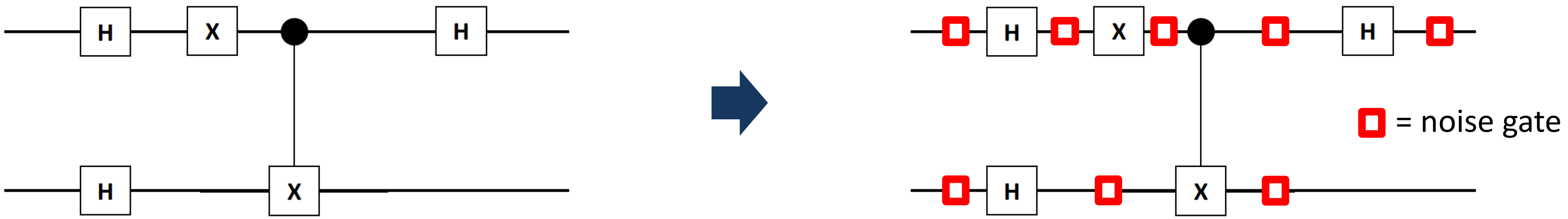
Internal evolution Effect of the environment

Issues to deal with:

- More complicated dynamics; how to model the environment efficiently
- With the density matrix, the problem scales quadratically with the size of the system.

How to describe noises

Noises (like gates) formally act instantly: Lindblad → Kraus $\rho \to \sum_i K_i \rho K_i^\dagger \qquad \sum_i K_i^\dagger K_i = 1$



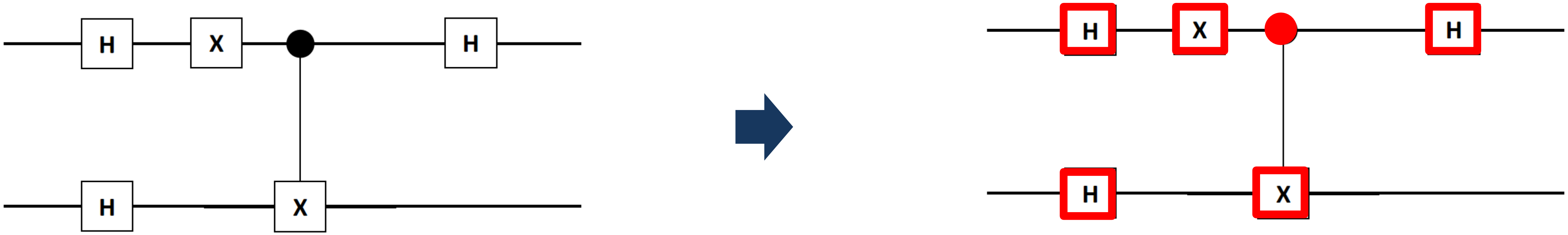
Standard noise simulation (e.g. in Qiskit)

- Gates and noise are formally **decoupled** (a sort of Trotterizzation), because time scales are small (IBM: gate time ~ 10^{-8} s, decoherence times ~ 10^{-4} s) ?

- Use the **quantum-jump-like approach** to replace the density matrix with (stochastic) state vector → stochastic dynamics ✓

Noisy Gates

Our approach: provide a more accurate description of the noisy behaviour of a quantum computer



- Noises are **embedded** in the gate → more realistic picture ✓
- State vector (stochastic) description ✓

Noisy Gates

Bassi, A., & Deckert, D. A. (2008). Noise gates for decoherent quantum circuits. *Physical Review A*, *77*(3), 032323.

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\}\right]$$

Gate Noise

$$\mathrm{d}\left|\psi_s\right\rangle = \left[-\frac{i}{\hbar}\mathrm{H}_s \mathrm{d}s + \sum_{k=1}^{N^2-1}\left[i\epsilon \mathrm{dW}_{k,s} - \frac{\epsilon^2}{2}\mathrm{d}s \mathrm{L}_k^\dagger\right]\mathrm{L}_k\right]\left|\psi_s\right\rangle$$

* - reparemetrized time in gate time units
- diagonalized Lindblad in canonical form

Stochastic evolution for the state vector (stochastic unravelling)

1) Formal equivalence: $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$

2) The dynamics is **linear**, therefore it can be represented as a gate (noisy gate)

$$\left|\psi_{s=1}(\boldsymbol{\xi})\right\rangle = \bar{\mathrm{N}}(\boldsymbol{\xi})\left|\psi_0\right\rangle$$

*Due to the noises **ξ**, the gate is not unitary and norm preserving. But at the statistical level the trace is preserved, and one recovers the standard (Lindblad) behaviour.

Solution of the SDE

Gardiner, C. W. (1985). *Handbook of stochastic methods* (Vol. 3, pp. 2-20). Berlin: Springer.
Arnold, L. (1974). Stochastic differential equations. *New York:* John Wiley & Sons

$$\bar{\mathrm{N}}(\boldsymbol{\xi}) = \mathrm{U}_g e^{\Lambda} e^{\bar{\Xi}(\boldsymbol{\xi})}$$

U_g = noiseless gate $\quad s \in [0, 1]$

$$\Lambda = -\frac{\epsilon^2}{2}\int_0^1 \mathrm{d}s \sum_{k=1}^{N^2-1} [\mathrm{L}_{k,s}^{\dagger}\mathrm{L}_{k,s} - \mathrm{L}_{k,s}^2]$$

Deterministic contribution of the noise, to order O(ϵ^2)

$$\Xi(\boldsymbol{\xi}) := i\epsilon \sum_{k=1}^{N^2-1} \int_0^1 \mathrm{dW_{k,s}} \mathrm{L}_{k,s}$$

Stochastic contribution of the noise, to order O(ϵ^2)

Operators in the interaction picture

- Find $H(U_g)$ and L_k for a given device
- Compute $L_{k,s}$ and Λ
- Compute stochastic properties of Itô processes in $\Xi(\boldsymbol{\xi})$
- Sample to get a realization of $\bar{N}(\boldsymbol{\xi})$

IBMQ devices

Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (2019). A quantum engineer's guide to superconducting qubits. *Applied Physics Reviews*, *6*(2), 021318.
Georgopoulos, K., Emary, C., & Zuliani, P. (2021). Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, *104*(6), 062432.

Gates

Native gate set $\{RZ(\phi)$, X, SX, CNOT}

θ : rotation angle

ϕ : phase, realizes virtual Z gates

$$H(\theta,\phi) = \frac{\theta\hbar}{2}R_{xy}(\phi)$$
$$R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

+

Cross resonance (CR) gate

$$\mathrm{H}^{(1,2)}(\theta,\phi) = \frac{\hbar\theta}{2}\mathrm{Z}^{(1)} \otimes \mathrm{R}_{xy}^{(2)}$$
$$R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

Note: how to implement the pulse

Noises

-Single qubit depolarization: γ_d

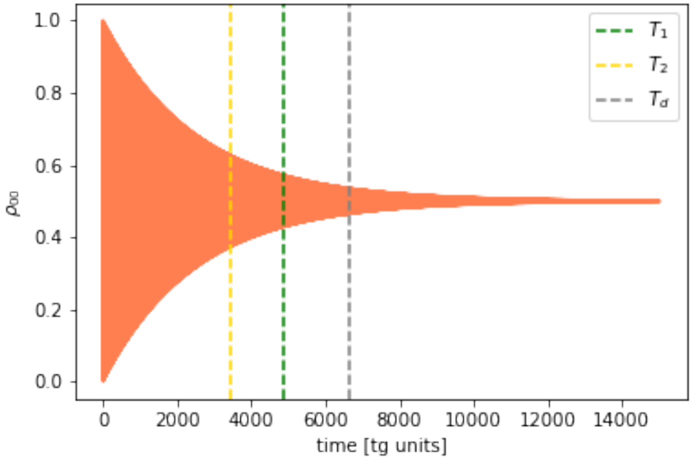
-Single qubit amplitude and phase damping: γ_1, γ_z

$$\mathrm{L}_1 = \sqrt{\frac{\lambda_1}{\lambda}}\sigma^-, \quad \mathrm{L}_2 = \sqrt{\frac{\lambda_2}{\lambda}}\sigma^+, \quad \mathrm{L}_3 = \sqrt{\frac{\lambda_3}{\lambda}}\mathrm{Z}$$
$$\lambda_1 = 2\gamma_d, \ \lambda_2 = 2\gamma_d + \gamma_1, \ \lambda_3 = \gamma_d + \gamma_z$$
$$\lambda = \lambda_1 + \lambda_2 + \lambda_3$$

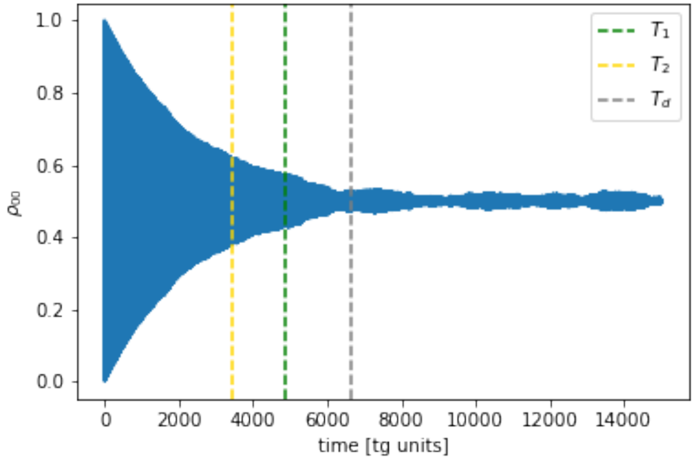
$$\lambda_k \sim 10^4\mathrm{Hz}$$
$$t_g \sim 10^{-8}s$$
$$\epsilon = \sqrt{\lambda t_g} \ll 1$$

Simulation of the noisy X gate

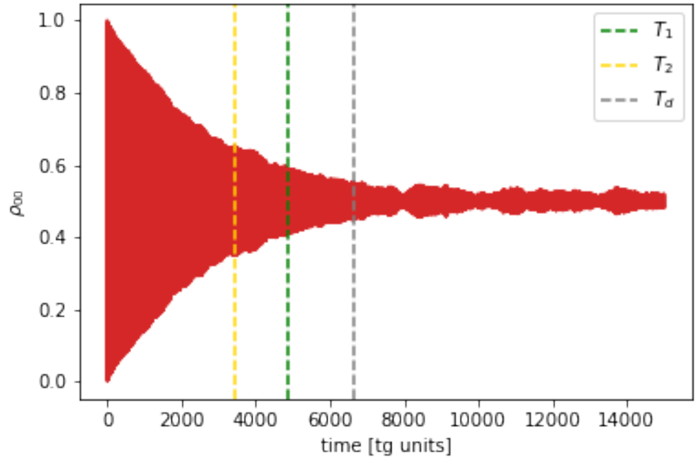
Lindblad equation



Noisy gates

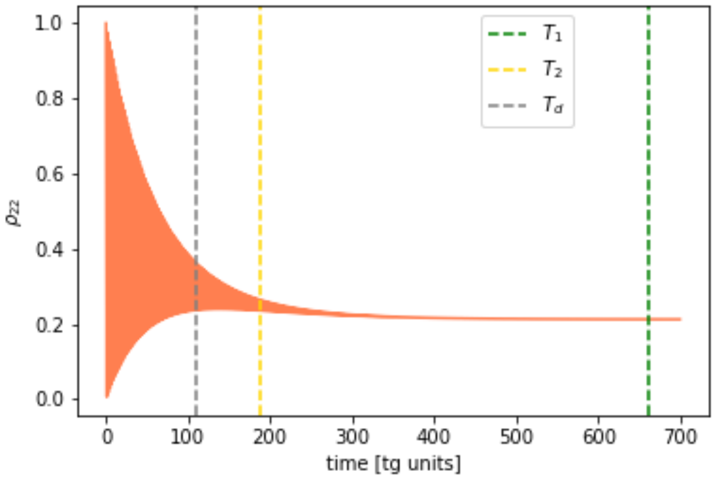


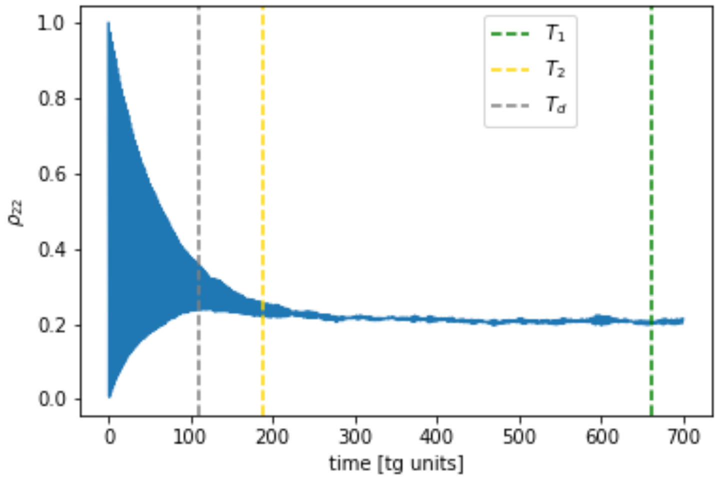
Qiskit simulator

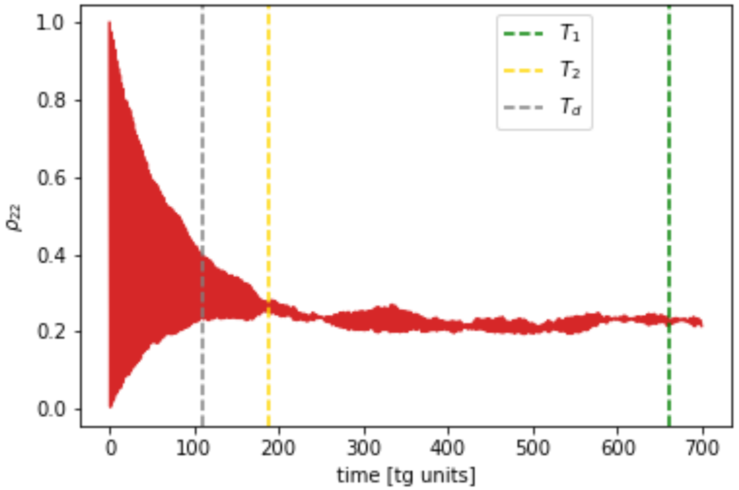


Initial state = $|0\rangle$ Shown: $\langle 0|\rho|0\rangle$

Simulation of the noisy CR gate







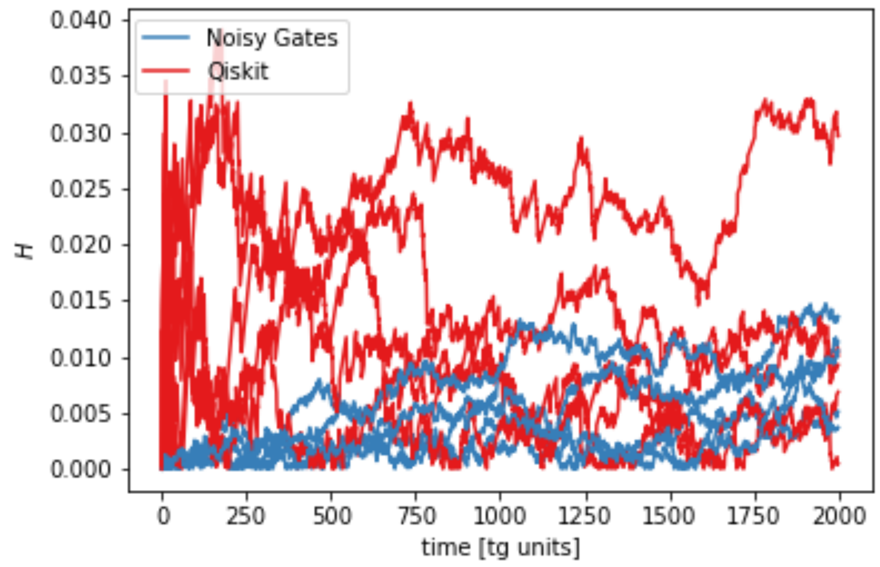
Initial state = $|10\rangle$ Shown: $\langle 10|\rho|10\rangle$

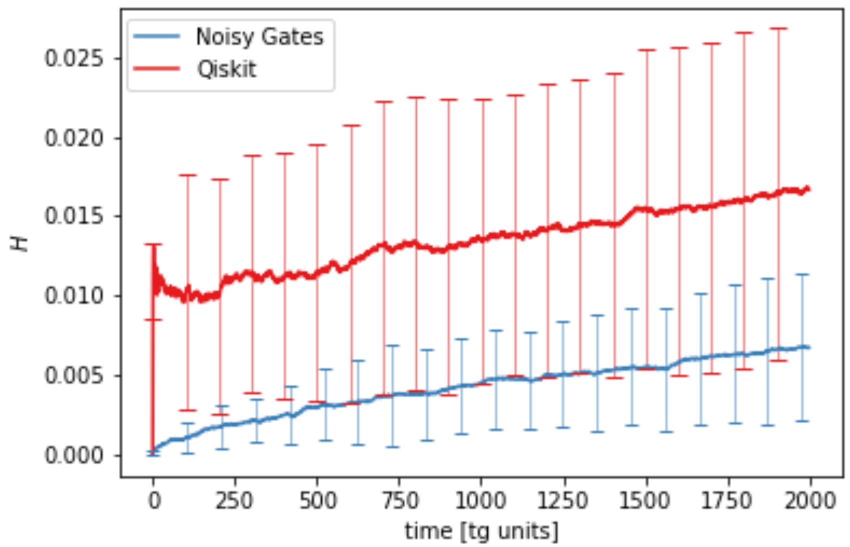
Numerical solution

Average over 1000 realizations

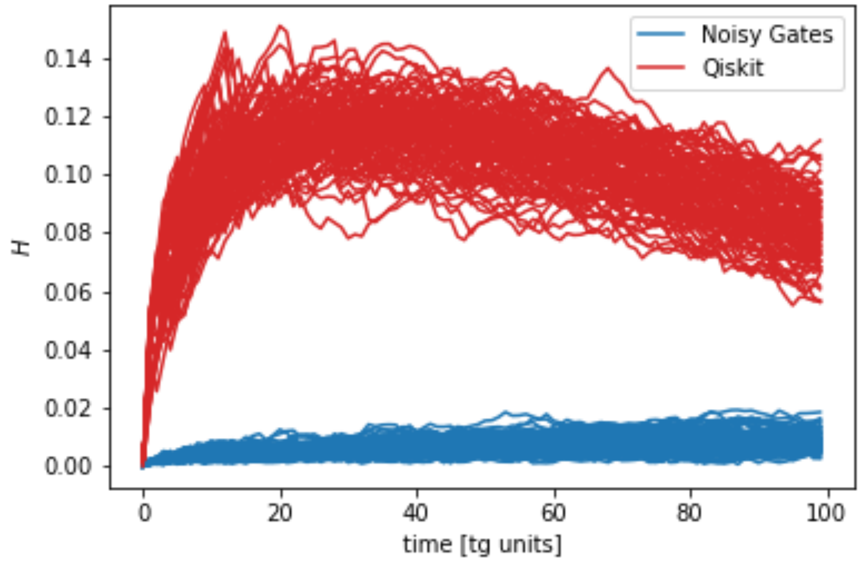
Average over 1000 realizations

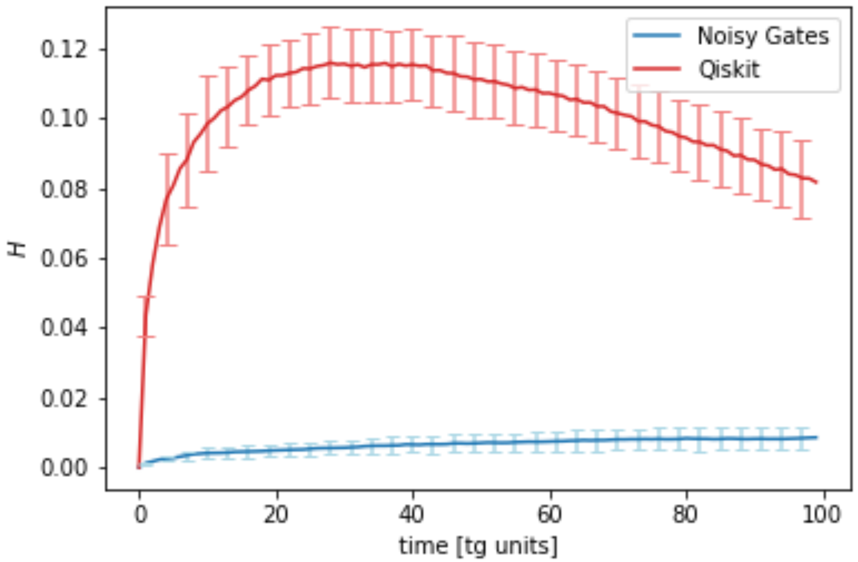
Simulation of the noisy X gate





Simulation of the noisy CR gate





Hellinger Distance

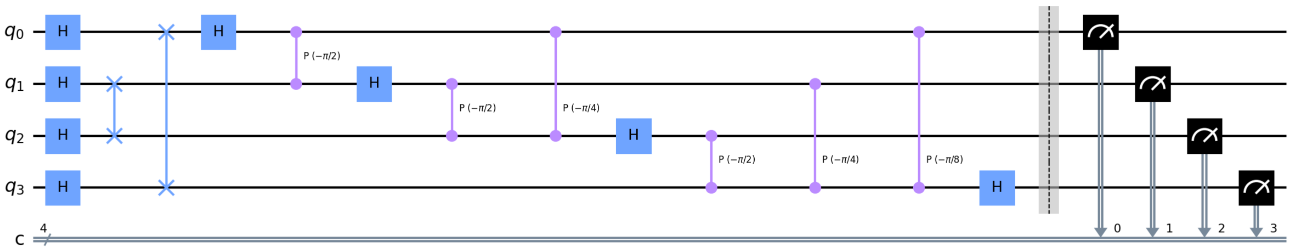
Lindblad and noisy gates

Lindblad and Qiskit simulator

$$H(P,Q) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{n} (\sqrt{p_i} - \sqrt{q_i})^2}$$

100 independent simulations each including 1000 runs

The inverse QFT algorithm



$$|0\rangle^{\otimes N} \rightarrow H^{\otimes N}|0\rangle^{\otimes N} \rightarrow |+\rangle^{\otimes N} \rightarrow QFT^{-1}|+\rangle^{\otimes N} \rightarrow |0\rangle^{\otimes N}$$

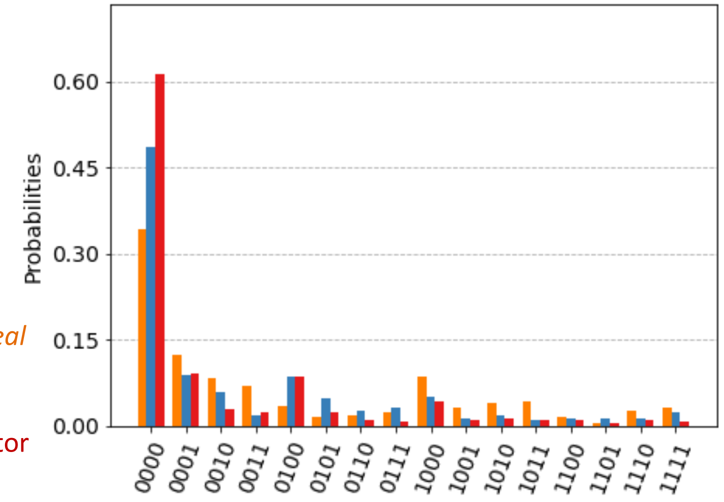
Transpile

Run

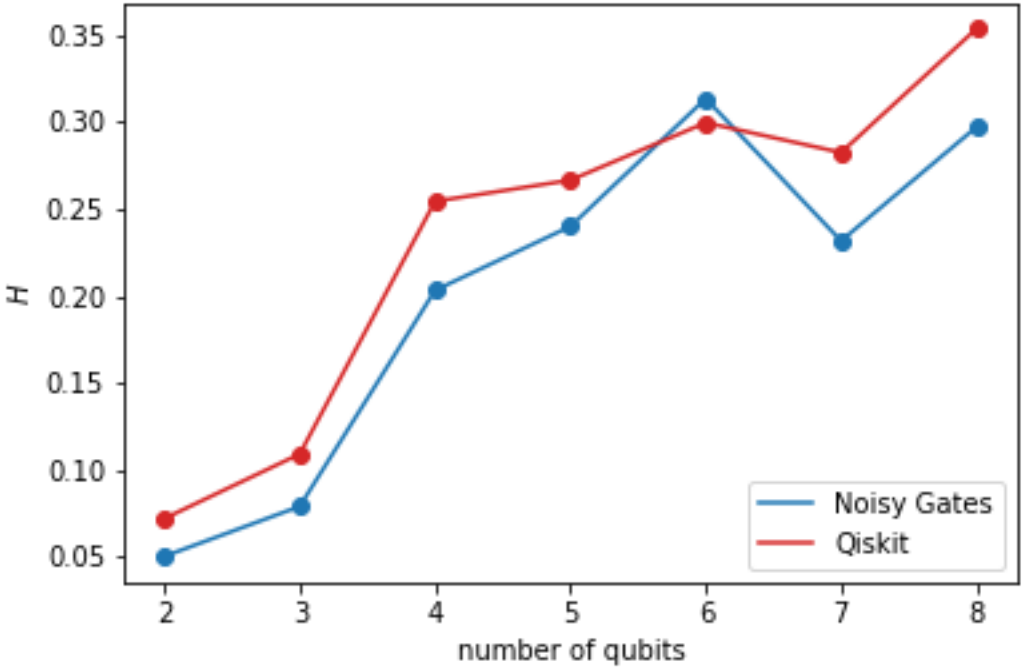
ibmq_montreal

Noisy gates

Qiskit simulator



Note: same noise model (depolarization + relaxation) during (after) gates + relaxation on idle qubits and bit-flip error before measurements



Hellinger Distance

ibmq_montreal and noisy gates

ibmq_montreal and Qiskit simulator

Future work

Better analysis of noises, especially for two qubit gates

Application to algorithm of interest, to extract "long-time" behaviour

The Team & Collaborators

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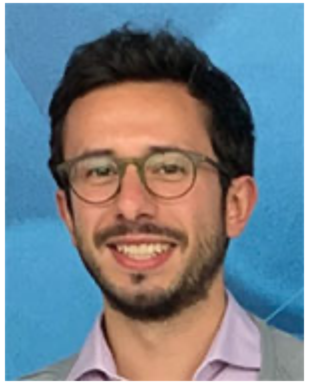
Sandro Donadi

Giovanni Di Bartolomeo

Francesco Cesa



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Thank you for your attention