Theory and Phenomenology of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA

Workshop
Quantum Computing @ INFN
November 14, 2022

Bayesian Optimization for QAOA

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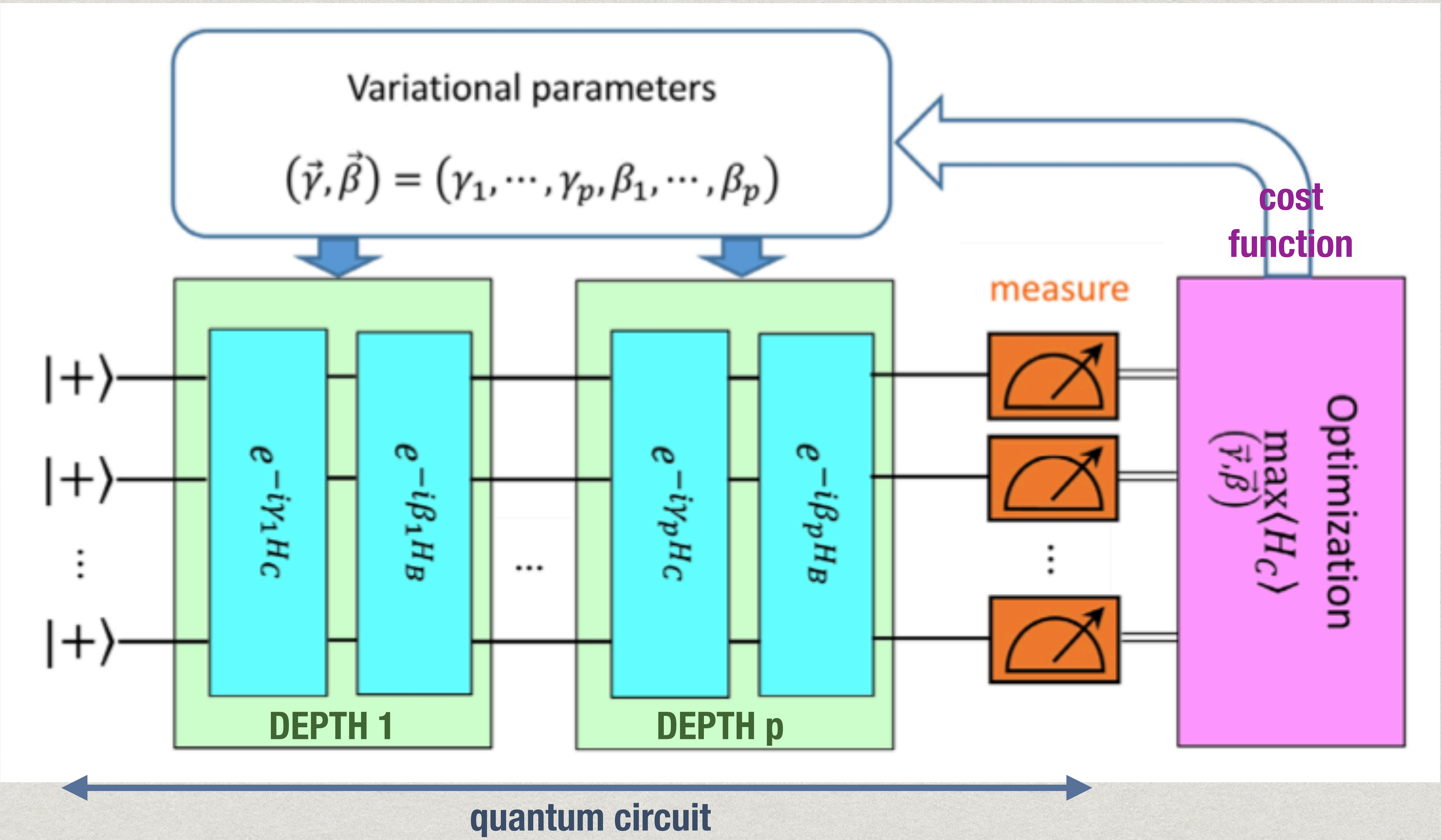
• Hybrid classical-quantum protocol: QAOA

(Quantum Approximate Optimisation Algorithm) Farhi et al.- arXiv:1411.4028 (2014)



quantum

- Objective function to be minimised
- Quantum evolution (circuit) with variational parameters
- Measurement scheme for average Hamiltonian to get cost function

classical

- Optimisation algorithm to get the best values of the parameters

Ground State Preparation for Nontrivial Quantum Hamiltonians

Z2 Lattice (pure) Gauge Theory $H(h) = H_E + h\,H_B$

Variational Ground State

$$|\psi_P(\gamma,\beta)\rangle = \boxed{\left(\prod_{m=1}^{P} e^{-i\beta_m H_E} e^{-i\gamma_m H_B}\right)} |\psi_0\rangle$$

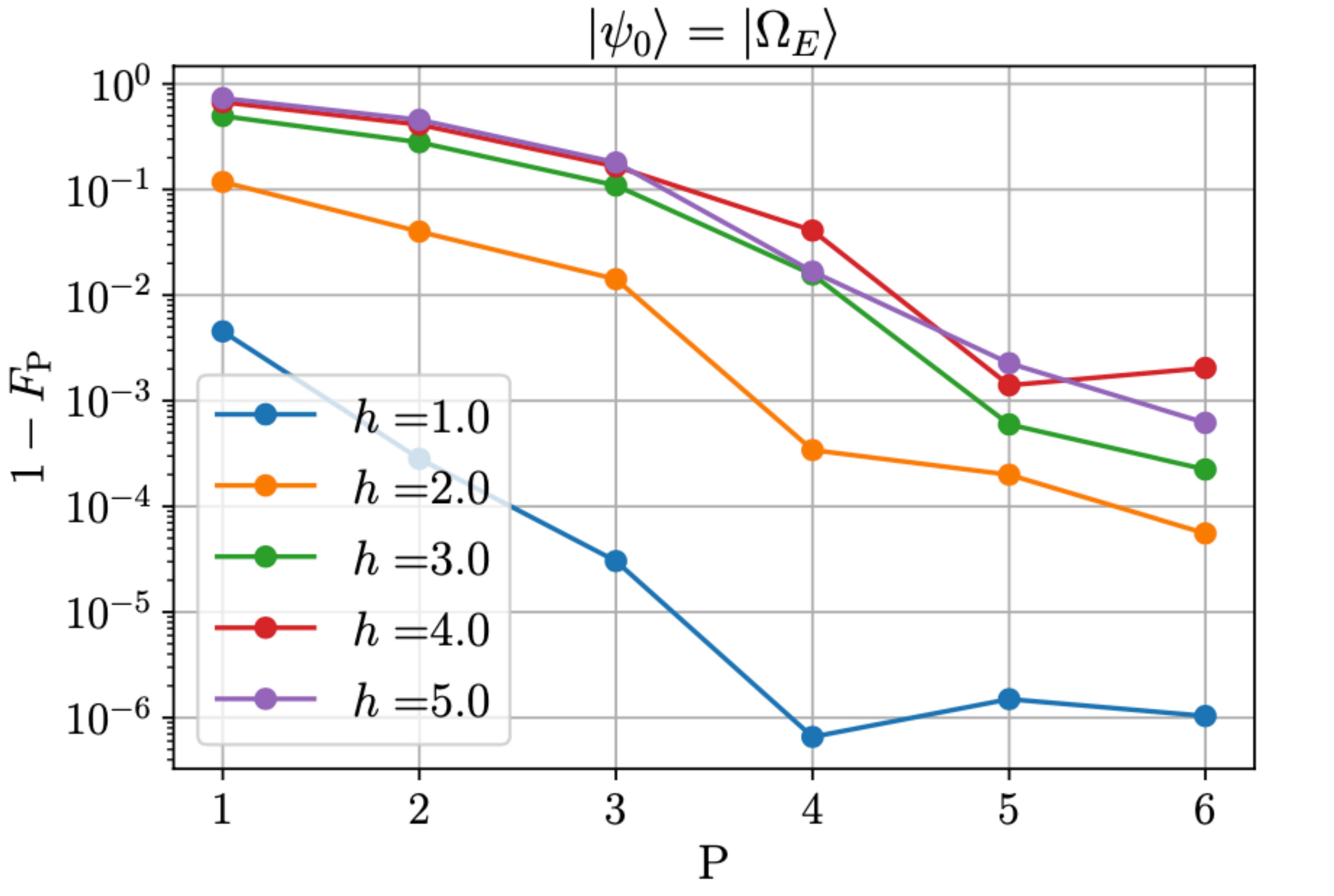
via quantum circuit

Cost function to be minimized

by classical optimiser:

$$E_P(\gamma,\beta) = \langle \psi_P(\gamma,\beta)\,|\,H(h)\,|\,\psi_P(\gamma,\beta)\rangle$$

PRX Quantum 3, 020320 (2022)

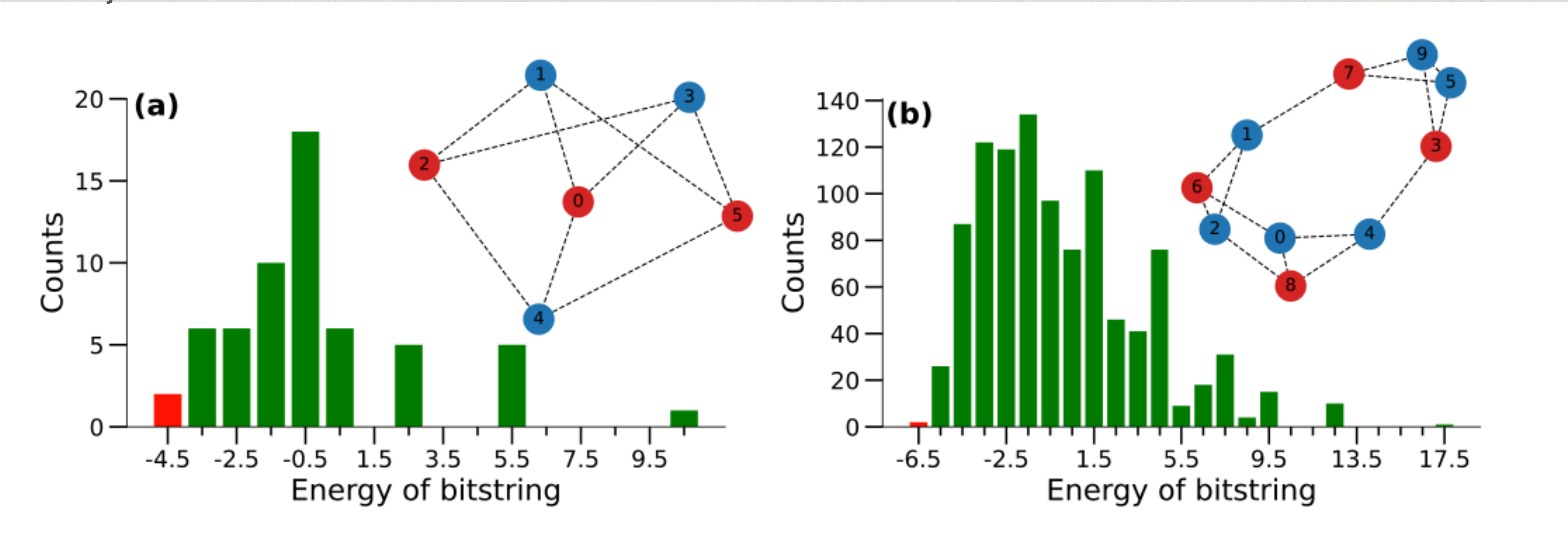


(within: QuantERA 2020 Project “QuantHEP”, with BA and PD
& Project in INFN-Quantum Computing HUB CERN-IBM)

• Solution of Combinatorial problems on Graphs

MIS problem: find the largest set of nodes not adjacent

Max-Cut problem: partition the graph in two sets of nodes interconnected by the largest number of links



GS of $H_C^{\mathrm{MIS}} = \sum_i Z_i + \omega \sum_{(i,j)\in E} Z_i Z_j,$

GS of $H_C^{\mathrm{MC}} = -\sum_{(i,j)\in E} (1 - Z_i Z_j)/2.$

$H_M = \sum_i X_i$

where Z_j, X_j are the Pauli matrices acting on the qubit representing node j

The Quantum circuit now prepares the state

$$|\boldsymbol{\theta}\rangle = \prod_{l=1}^{p} e^{-i\beta_l H_M} e^{-i\gamma_l H_C} |+\rangle,$$

with $\theta = (\gamma_1\beta_1, \cdots, \gamma_p\beta_p)$

The energy cost has a minumum for γ^*, β^*
to be find by means of a classical protocol

$$E(\boldsymbol{\theta}) = \langle \boldsymbol{\theta} | H_C | \boldsymbol{\theta} \rangle$$

Issue:

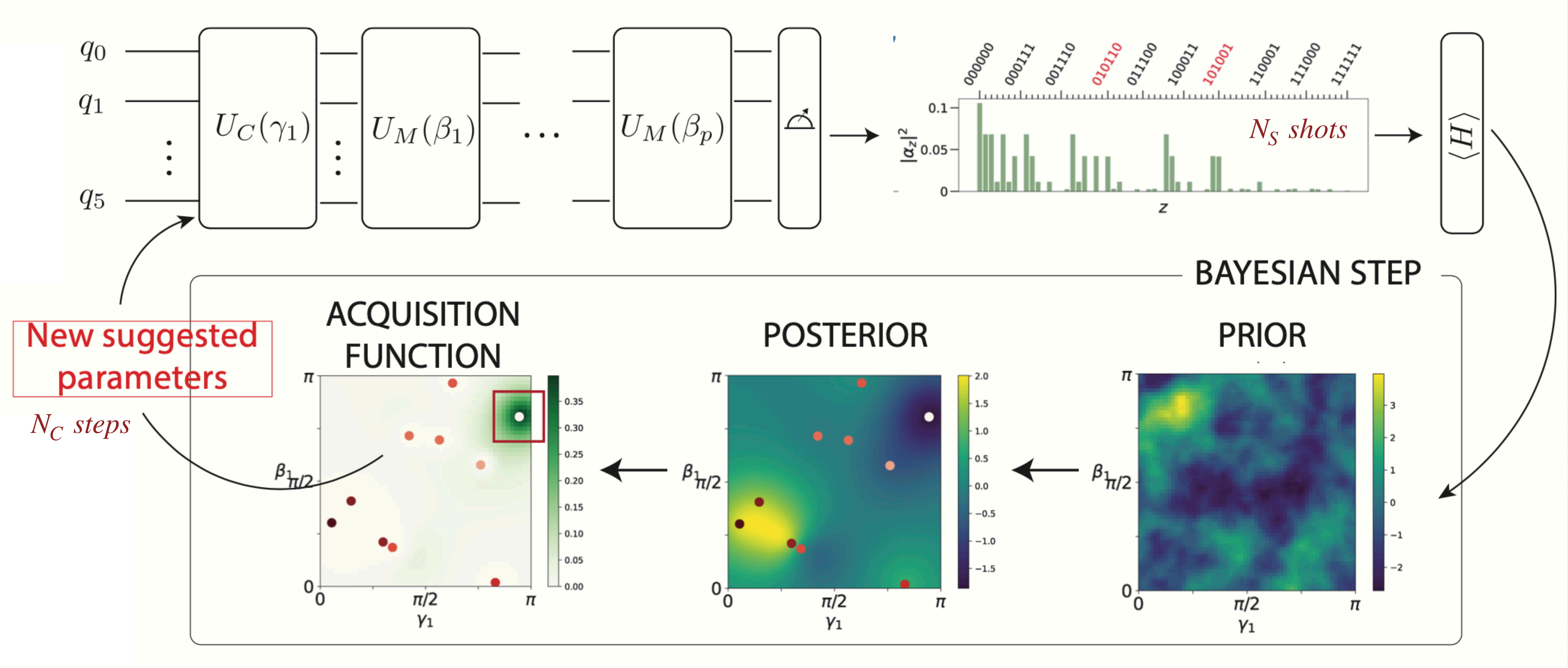
- **the landscape of the energy can become very rough and develop barren plateaus**
- **convergence can become hard (global techniques are needed) and a large number of points are to be tested**

but, on a real device, we have to consider also problems related to

- **the measure of the cost function, which requires many measurements (N_S) to give a good estimate of the average, a procedure that have to be repeated for all points (N_C) necessary to reach the minimum with the desired precision**

Bayesian algorithm for classical optimization

warm up: we sample $N_W = 10$ values of the parameters θ_j and measure the cost function $\langle \theta_j | H_C | \theta_j \rangle$



subroutine: a prior distribution is updated by means of the observed data & the posterior distribution is acquired through an acquisition function whose maximum indicates the new value of the parameter where to perform the run (Gaussian process)

Results of the simulations

for the Max-Cut problem
of the 10-node graph

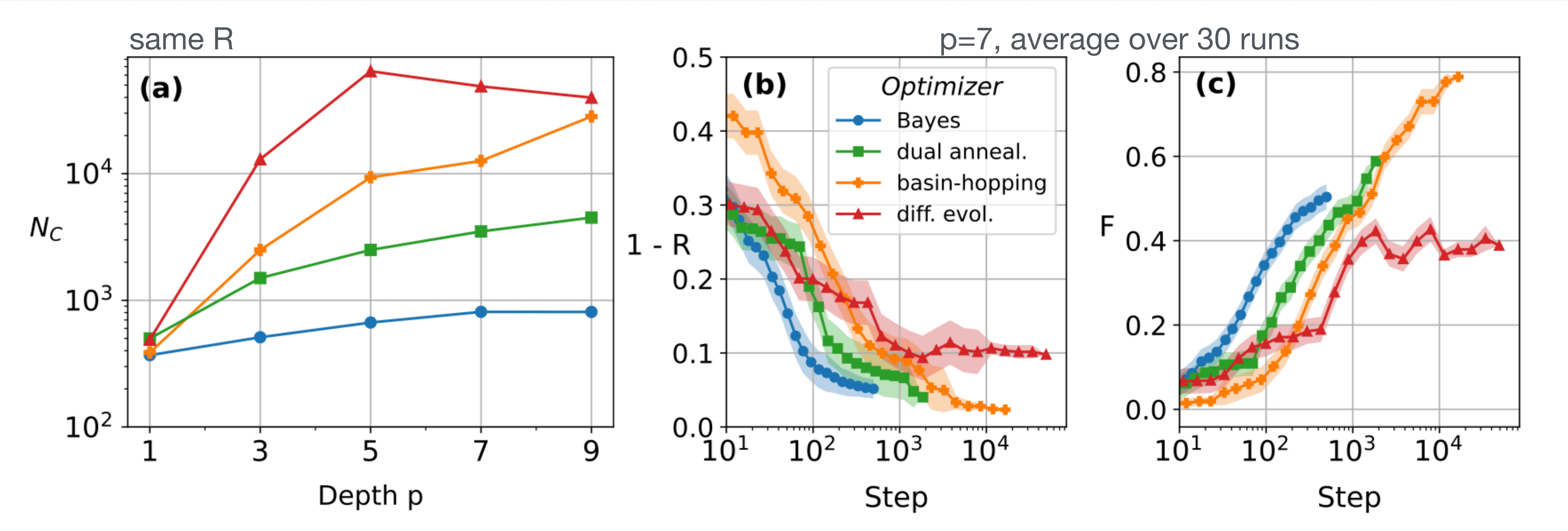
approximation ratio

$$R = E(\boldsymbol{\theta})/E_{GS}$$

fidelity

$$F = |\langle \boldsymbol{\theta} | z^{\star} \rangle|^2$$

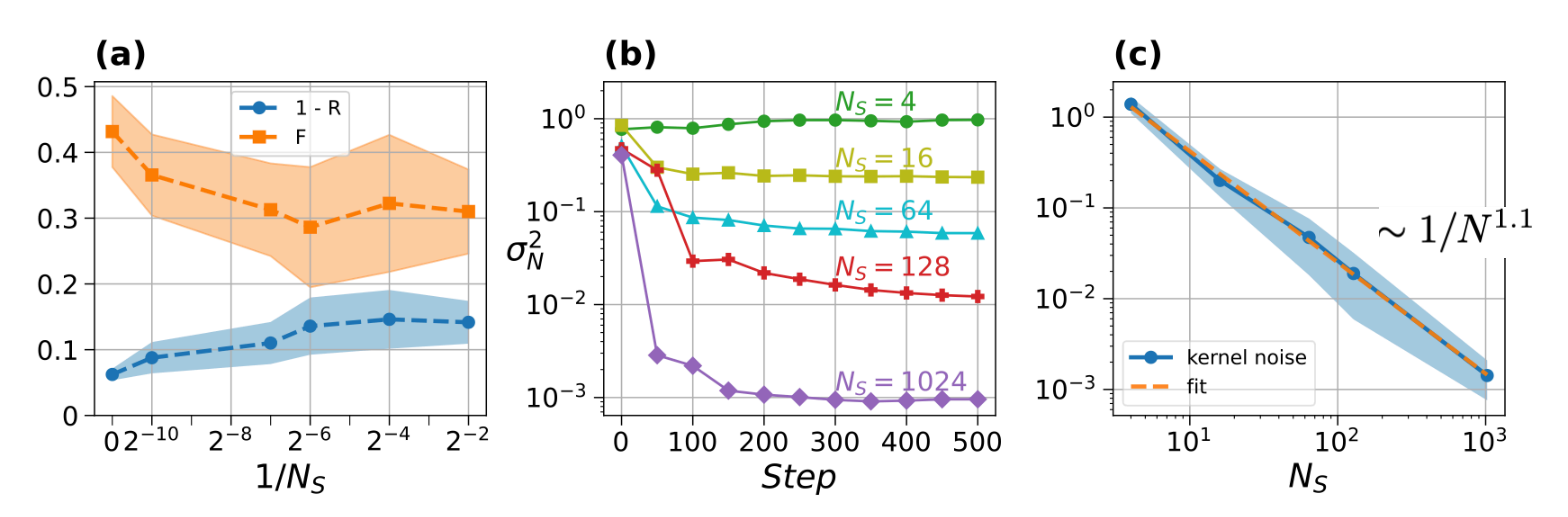
comparison among optimisers



slow measurements: $N_s = 1024, 128, 64, 16, 4$

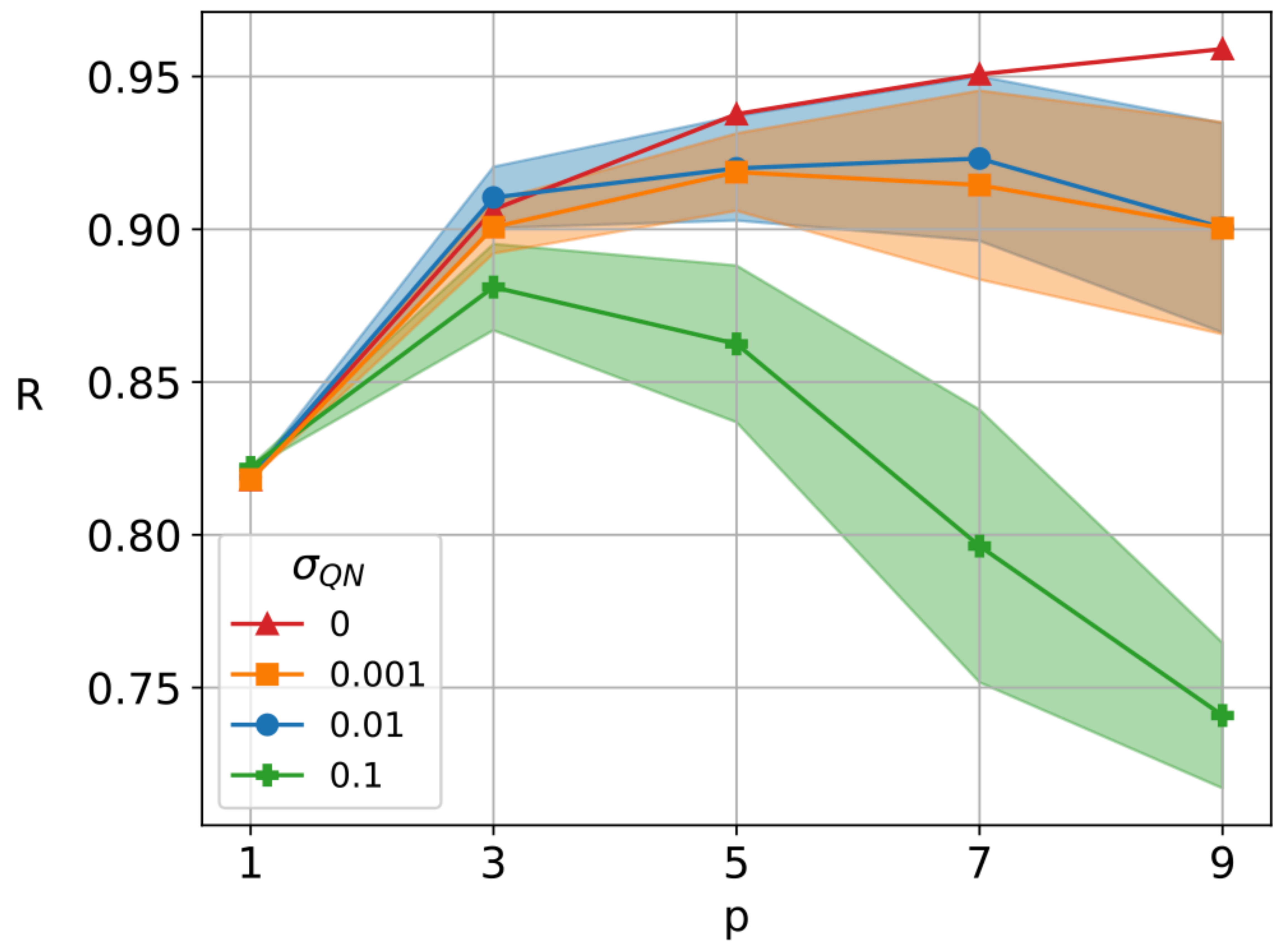
reducing the numbers of measurements, decreases the precision of the energy landscape reconstruction

this is a multinomial sampling problem, thus variance of energy should depend on N_S^{-1}



quantum noise:
quantum circuits are subject to different kinds of noise (device dependent)

$$|\boldsymbol{\theta}\rangle = \prod_{l=1}^{p} e^{-i\sum_i \beta_l^i X_i} \; e^{i\sum_{\langle i,j\rangle} \gamma_l^{(i,j)} Z_i Z_j} |+\rangle,$$



R is expected to decrease with depth p (that increases the number of evolutions) and by increasing the q. noise variance σ_{QN}

However, up to p=3, R increases, signalling that Bayesian optimization is robust

S. Tibaldi, D. Vodola, E. Tignone. E.E.
arXiv:2209.03824

AND NOW?

- Collaboration, via CINECA, with PASQAL
  to test our Bayesian approach on their Rydberg atom platform

  ✓ Run simulations on Pulser with their real optical pulses to generate gates and with (different kinds of) noise

  ★ To be run on real atomic platform

- Applications to different problems:
  quantum chemistry, many body hamiltonians

Thank you for your attention