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Evaluation quantum gradient through quantum non-demolition measurements

Paolo Solinas

Department of Physics and INFN

Collaboration

- Simone Caletti, Dept. Physics, Univ. Genova and INFN
- Giovanni Minuto, Dept. Informatics, Univ. Genova
- Ernesto De Vito, Dept. Mathematics, Univ. Genova
- Alessandro Verri, Dept. Informatics, Univ. Genova

Variational problem

- Find the minimum of a function $\hat{H}(\vec{\theta})$ $\quad \vec{\theta} = \text{parameter vector}$

- We know that for any state

$$\langle H(\vec{\theta})\rangle = \langle \psi|\hat{H}(\vec{\theta})|\psi\rangle \geq E_g \qquad E_g = \text{minimum energy}$$

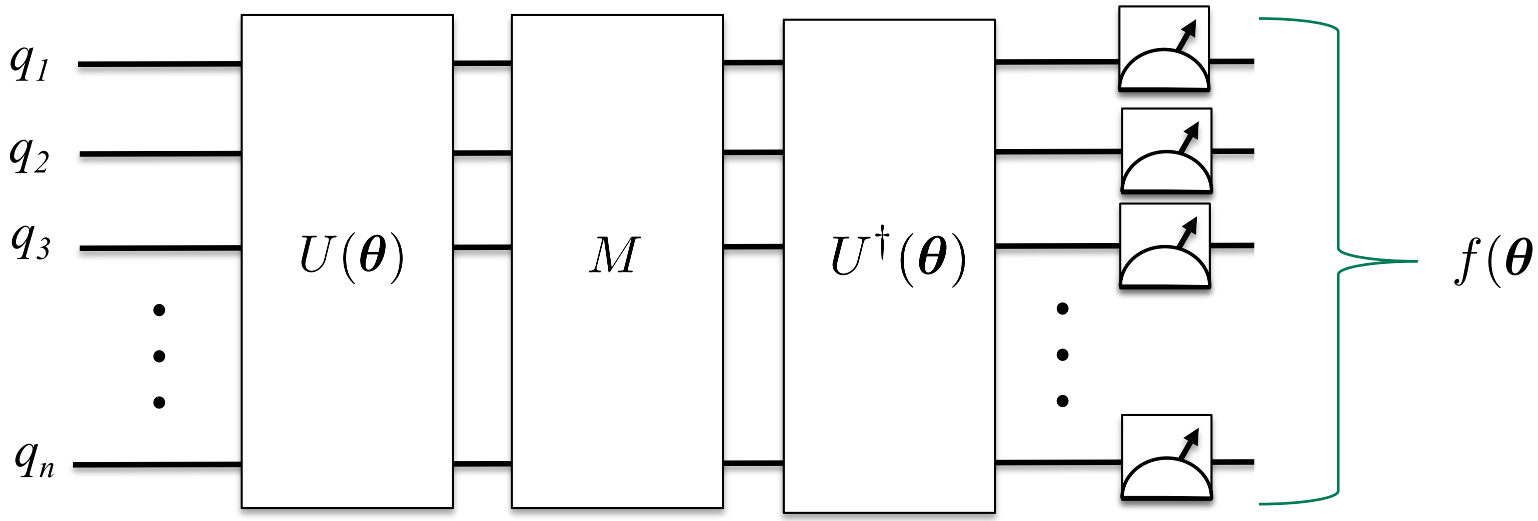
- For a generic operator we define a cost function that is the associated to its average value

$$f(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta})|\hat{M}|\psi(\boldsymbol{\theta})\rangle = \langle 0|U^\dagger(\boldsymbol{\theta})\hat{M}U(\boldsymbol{\theta})|0\rangle$$

Variational problem

$$f(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta}) | \hat{M} | \psi(\boldsymbol{\theta}) \rangle = \langle 0 | U^\dagger(\boldsymbol{\theta}) \hat{M} U(\boldsymbol{\theta}) | 0 \rangle$$

- **Circuit in a quantum computer**

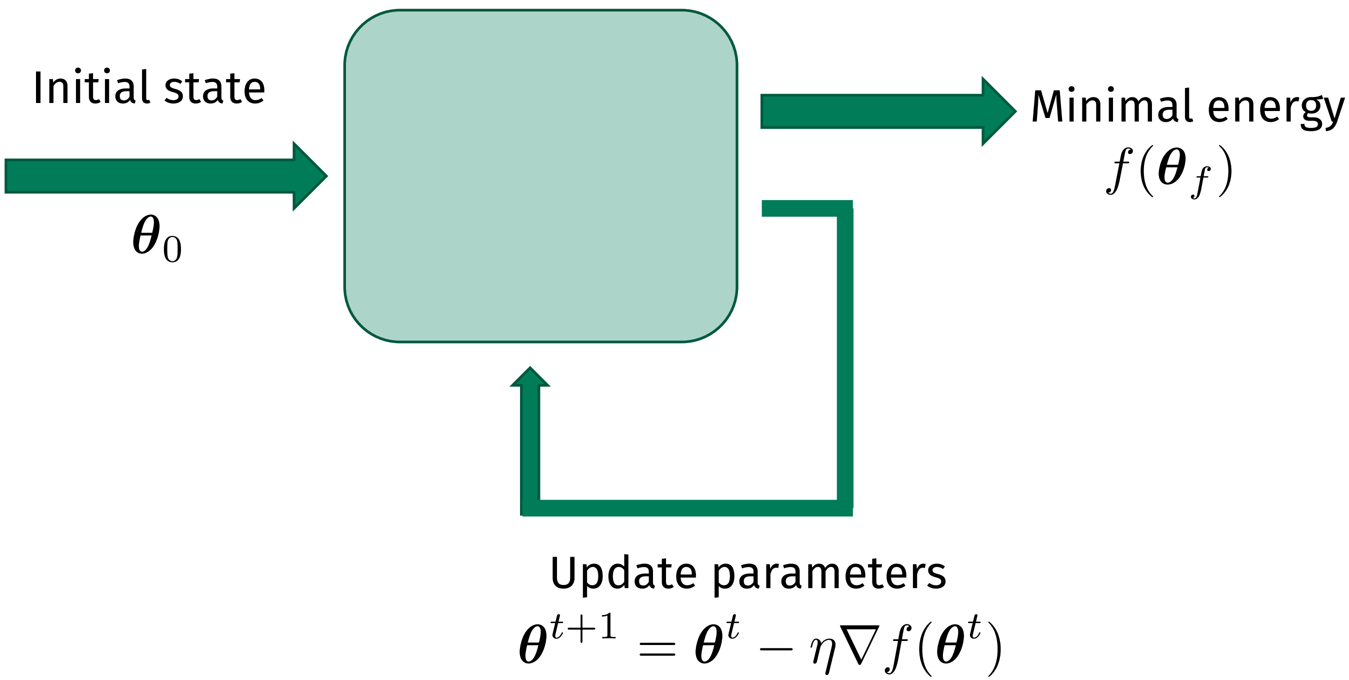


- **Cost**

  m measurements

  $2k \longrightarrow 2km$ total logical operators

Variational problem: classical



- Gradient allows us to gain more information and speed up the convergence
- It needs the *first derivative* of f
- The Newton optmizer uses *second derivatives*

Variational quantum algorithm

- **For the quantum case the first derivative can be obtained as**

$$g_j(\boldsymbol{\theta}) = \frac{f(\boldsymbol{\theta} + s\mathbf{e}_j) - f(\boldsymbol{\theta} - s\mathbf{e}_j)}{2\sin(s)}$$

- We need to estimate the *f* function in two points
- This means to measure twice the quantum observable
- Multiplication of preparation of the system, evolution and measurements

- For the second derivative we need four ponts

$$\begin{aligned} g_{j_1,j_2}(\boldsymbol{\theta}) = \big[& f\big(\boldsymbol{\theta} + s(\mathbf{e}_{j_1} + \mathbf{e}_{j_2})\big) - f\big(\boldsymbol{\theta} + s(-\mathbf{e}_{j_1} + \mathbf{e}_{j_2})\big) \\ & - f\big(\boldsymbol{\theta} + s(\mathbf{e}_{j_1} - \mathbf{e}_{j_2})\big) + f\big(\boldsymbol{\theta} - s(\mathbf{e}_{j_1} + \mathbf{e}_{j_2})\big)\big] \\ & \times [2\sin(s)]^{-2}. \end{aligned} \tag{11}$$

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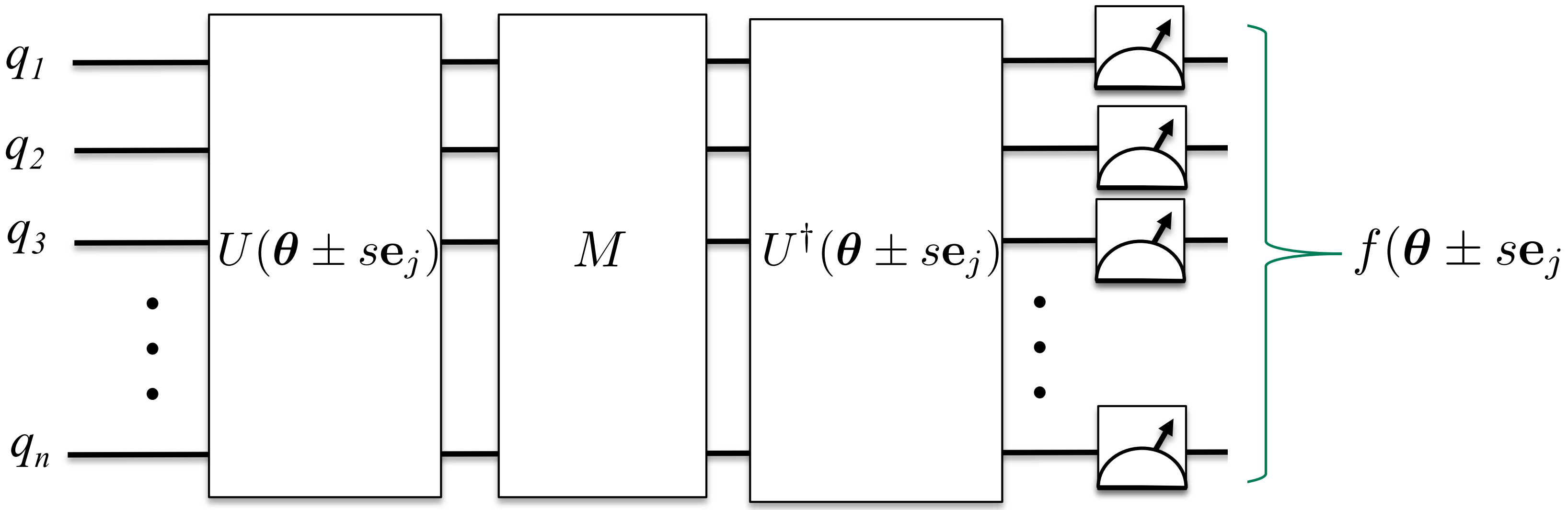
Hybrid quantum-classical algorithms

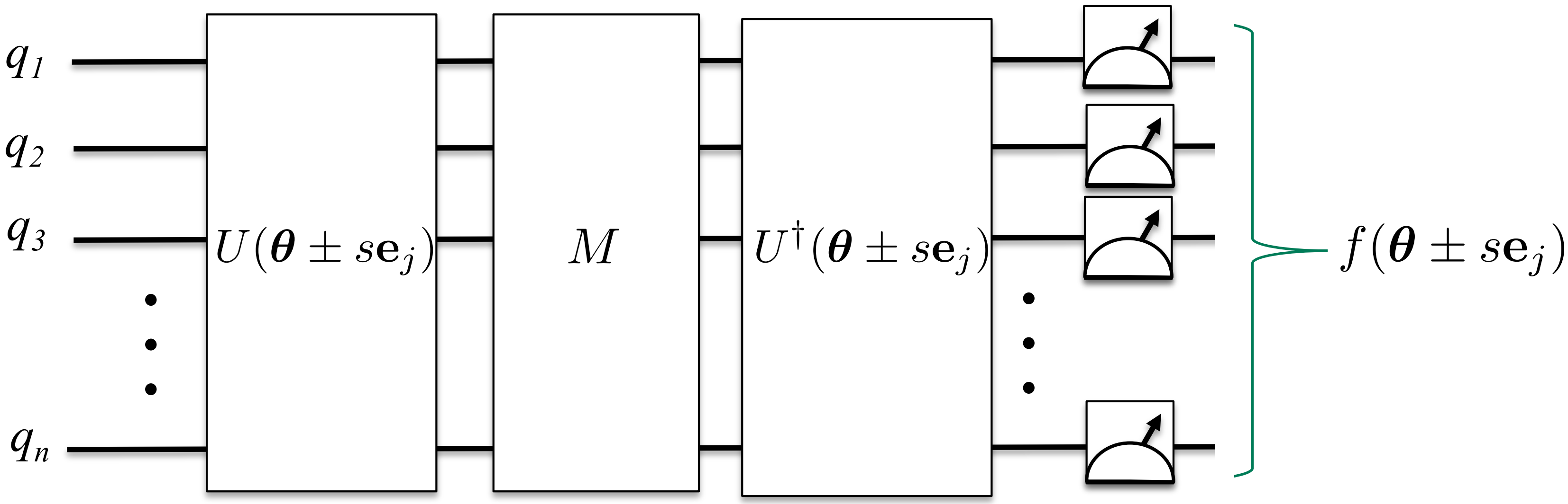


- The two average values are used by a classical computer to update θ (hybrid architecture)

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \eta \nabla f(\boldsymbol{\theta}^t)$$

- Gradient update

Hybrid quantum-classical algorithms



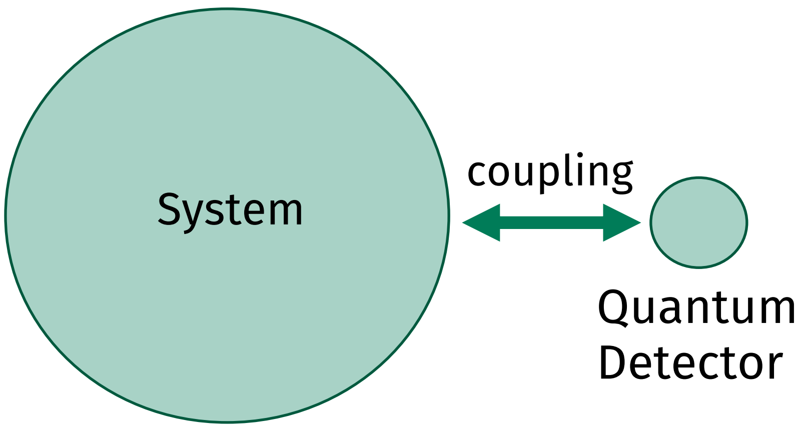
- **Cost**

  $2m$ measurements

  $4k \longrightarrow 4km$ total logical operators
- **For large quantum system the cost is too high**
- **Any cost reduction is of crucial importance**

Alternative extraction of information

- Using a quantum detector, we can have direct information about the gradient if *f*
- We store this information in the phase which is eventually measured *a single time*

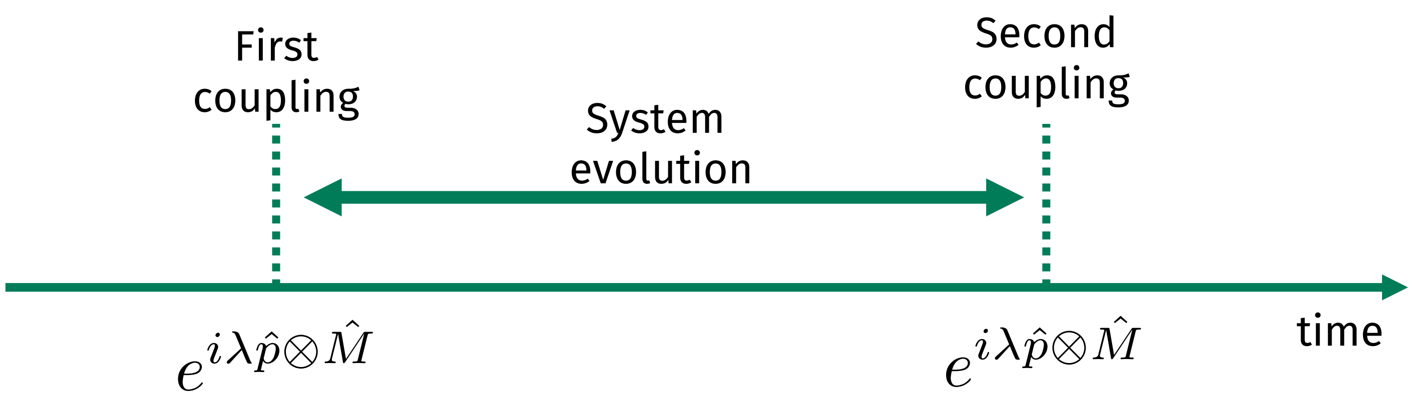


Alternative extraction of information

- If we want to measurement the gradient of *M*, we couple the system and the detector twice with $e^{i\lambda\hat{p}\otimes\hat{M}}$

$\lambda = \text{coupling parameter}$

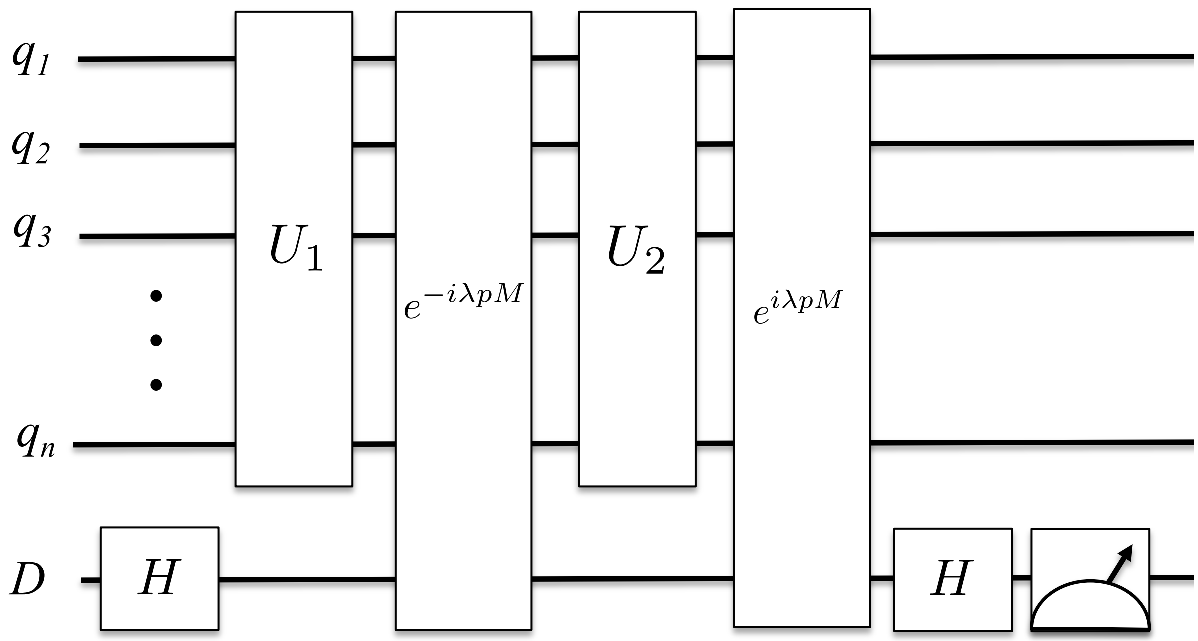
$\hat{p} = \text{detector operator}$



$$U_{tot} = e^{i\lambda\hat{p}\otimes\hat{M}} U_2 e^{-i\lambda\hat{p}\otimes\hat{M}} U_1$$

Alternative extraction of information

- Quantum circuit



Alternative extraction of information

- It can be shown that the phase of the detector is a characteristic function for the gradient
- The moments can be obtained from its derivatives

First derivative

$$
\begin{aligned}
-i\partial_\lambda \mathcal{G}_\lambda \Big|_{\lambda=0} \quad &\propto \quad \mathrm{Tr}_S[U_1^\dagger U_2^\dagger \hat{M} U_2 U_1 \rho_S^0 - U_1^\dagger \hat{M} U_1^\dagger \rho_S^0] \\
&= f(\boldsymbol{\theta} + s\mathbf{e}_{j_1}) - f(\boldsymbol{\theta} - s\mathbf{e}_{j_1})
\end{aligned}
$$

Gradient

Notes
- λ can be taken small
- Close to $\lambda=0$ we need only one evaluation of the phase

Advantages

- We perform only a measurement on detector instead of two
- We reduce the number of repetition and the logical gate to evolve the system

	$qubits$	$iterations$	$gates$
Direct measurement	n	$2m$	$\approx 4mk$
quantum non-demolition	$n+1$	m	$\approx mk$

$m =$ number of repetitions

$k =$ number of logical operations

Advantages

- For the higher derivatives, the advantage is exponential because we perform a single measurement against an exponential number of measurements

	$qubits$	$iterations$	$gates$
Direct measurement	n	$4m$	$\approx 8mk$
quantum non-demolition	$n+1$	m	$\approx mk$

$m =$ number of repetitions

$k =$ number of logical operations

Advantages

- For complex operators we must perform additional measurements of the single terms

$$H = h_1 P_1 + h_2 P_2 + h_3 P_3 + ... + h_J P_J$$

$$\Big\downarrow$$

m measurements

	$qubit$	$iterations$	$gates$
Direct measurement	n	$2mJ$	$\approx 4mJk$
quantum non-demolition	$n+1$	m	$\approx (k+2J)m$

$m =$ number of repetitions

$k =$ number of logical operations

Summary

- We have shown that using a quantum detector the number of operations and repetitions to estimate a quantum gradient can be reduced
- The advantages increase for the higher derivatives and complex operators
- Possible uses for variational quantum algorithms
- Implementation in noisy quantum computers where the cost reduction is of paramount importance

Thank you