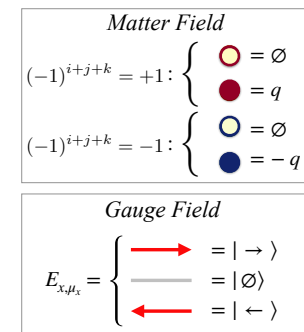
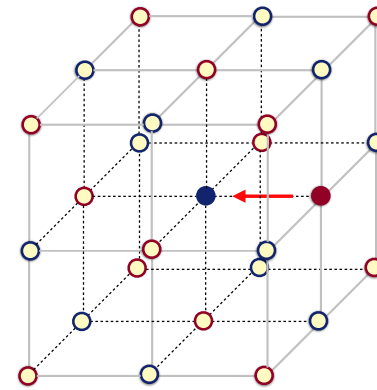
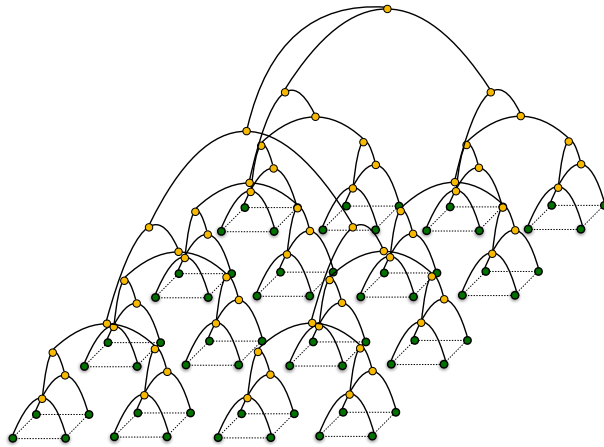


Tensor Networks methods for Lattice Gauge Theories



Giuseppe Magnifico

University of Padua & INFN

Padua Quantum Technologies Research Center

quantum.dfa.unipd.it

qtech.unipd.it

Quantum Computing @ INFN
14–15 November 2022



Dipartimento
di Fisica
e Astronomia
Galileo Galilei



Nature Communications 12, 3600 (2021)



TIMO FELSER
Ulm University



PIETRO SILVI
University of Padua



SIMONE MONTANGERO
University of Padua



Lattice Gauge Theories (LGT)

- 1) Quantum Matter and Quantum Fields
- 2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$

$-e$



Lattice Gauge Theories (LGT)

- 1) Quantum Matter and Quantum Fields
- 2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$



Lattice Gauge Theories (LGT)

- 1) Quantum Matter and Quantum Fields
- 2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$



Lattice Gauge Theories (LGT)

- 1) Quantum Matter and Quantum Fields
- 2) Local symmetries, e.g. Gauss's law in QED

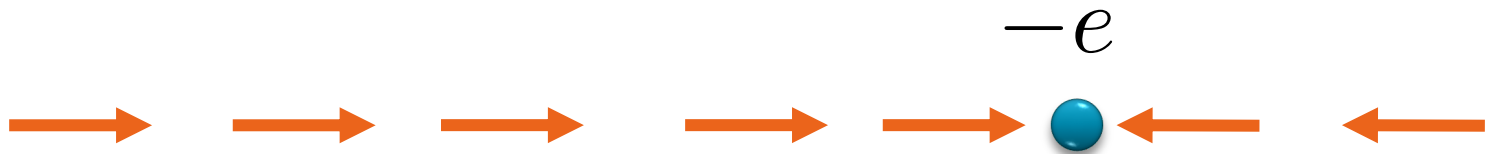
$$\nabla \cdot \mathbf{E} = \rho$$



Lattice Gauge Theories (LGT)

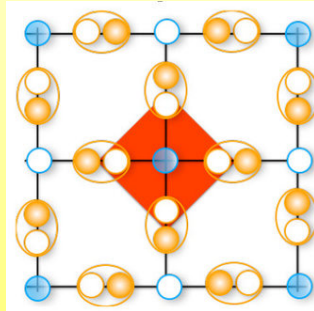
- 1) Quantum Matter and Quantum Fields
- 2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$



LGT are almost everywhere in theoretical physics!

As emergent theories in condensed matter: **high-T_c superconductors, frustrated systems, spin liquids.**



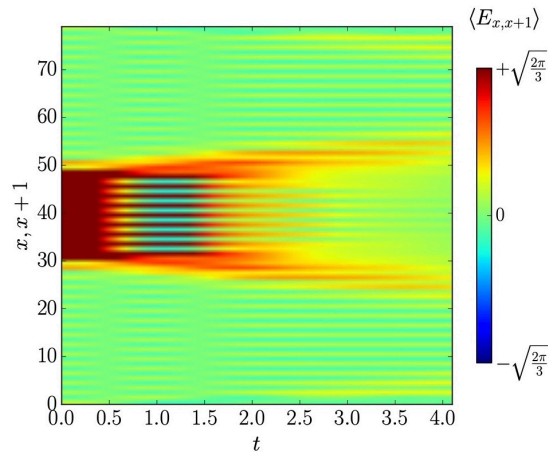
As fundamental description in particle physics: **Standard Model**

mass →	~2.3 MeV/c ²	~1.275 GeV/c ²	~173.07 GeV/c ²	0	~126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
	d down	s strange	b bottom	γ photon	
QUARKS	~4.8 MeV/c ²	~95 MeV/c ²	~4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
				GAUGE BOSONS	

- They are extremely demanding from a numerical point of view.
- Powerful numerical methods, such as Monte Carlo, fail in several regimes of finite-density or for non-equilibrium phenomena (sign-problem).
- Ideal goal for quantum-inspired efficient algorithms and quantum simulation/computation!

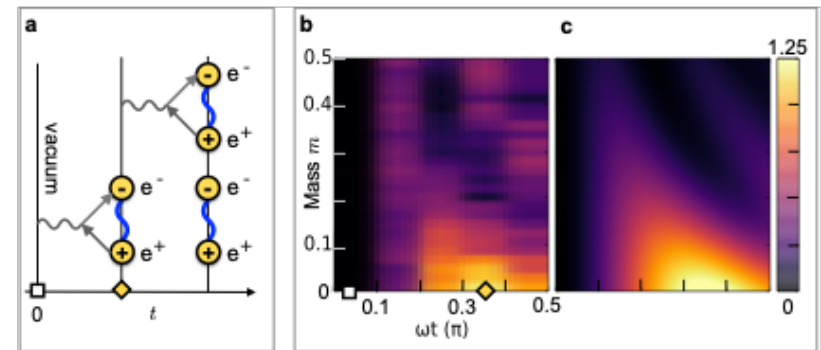
Quantum Technologies for LGT

Efficient quantum-inspired algorithms:
Tensor Networks (no sign-problem)



Quantum 4, 281 (2020)

First implementation of U(1) LGT on
digital quantum computer



Nature 534, 516–519 (2016).

Simulating Lattice Gauge Theories within Quantum Technologies

M.C. Bañuls^{1,2}, R. Blatt^{3,4}, J. Catani^{5,6,7}, A. Celi^{3,8}, J.I. Cirac^{1,2}, M. Dalmonte^{9,10}, L. Fallani^{5,6,7}, K. Jansen¹¹, M. Lewenstein^{8,12,13}, S. Montangero^{7,14} ^a, C.A. Muschik³, B. Reznik¹⁵, E. Rico^{16,17} ^b, L. Tagliacozzo¹⁸, K. Van Acoleyen¹⁹, F. Verstraete^{19,20}, U.-J. Wiese²¹, M. Wingate²², J. Zakrzewski^{23,24}, and P. Zoller³

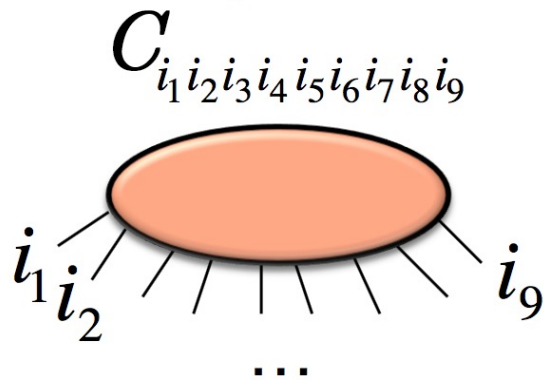
Eur. Phys. J. D 74, 165 (2020)

Tensor Networks

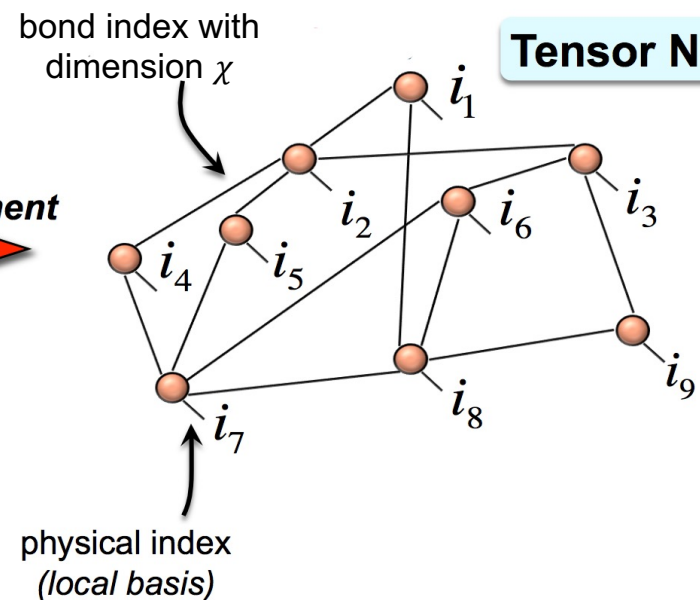
$$|\Psi\rangle = \sum_{i_1 i_2 \dots i_N} C_{i_1 i_2 \dots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)



replacement



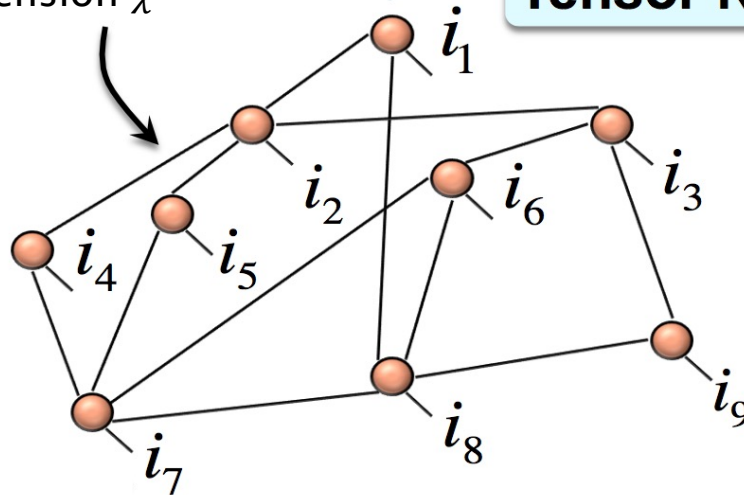
Tensor Network

$O(d^N)$ representation, exponentially large in the system size. Inefficient.

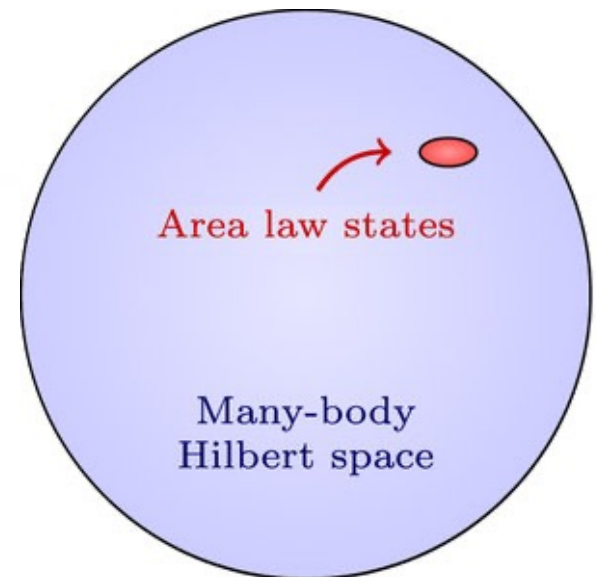
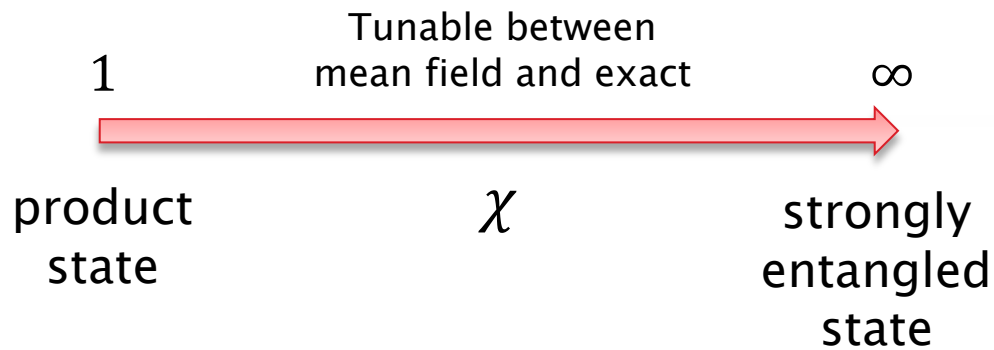
$O(Nd\chi^c)$ representation, polynomial in the system size. Efficient.

bond index with
dimension χ

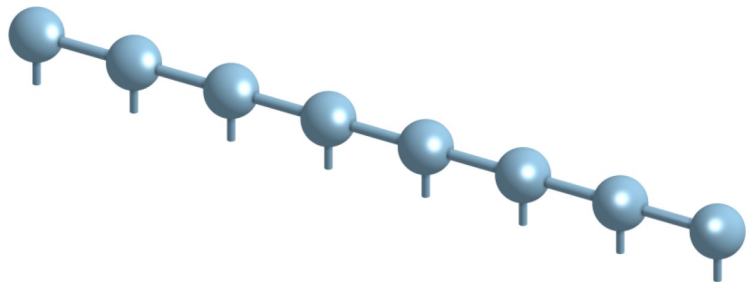
Tensor Network



Low-Energy States



Examples



Matrix Product States (MPS)

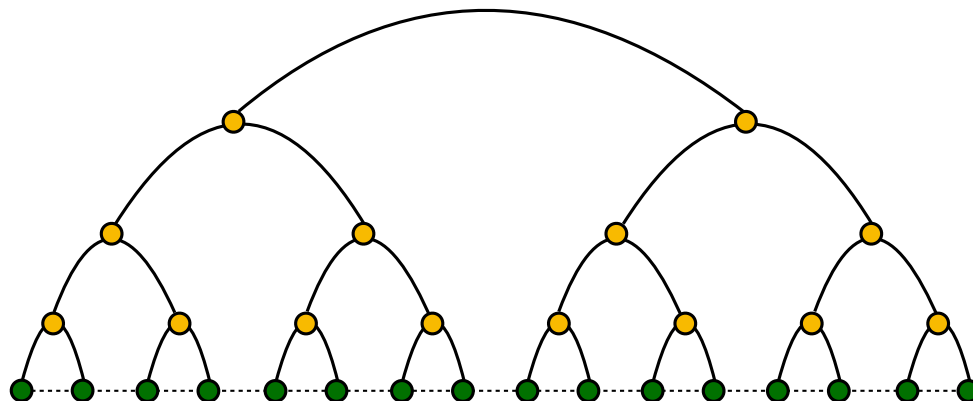
$$\text{minimize } E = \langle \psi | H | \psi \rangle$$

$$O(\chi^3)$$

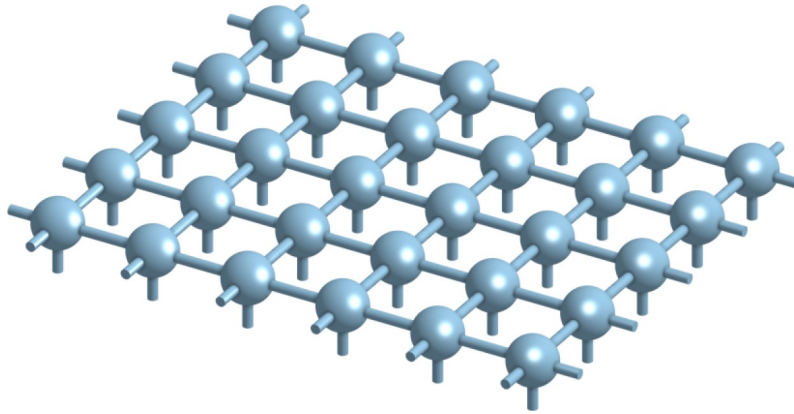
Tree Tensor Networks (TTN)

$$O(\chi^4)$$

- strong connectivity
- distance between two lattice sites scales logarithmically within the network



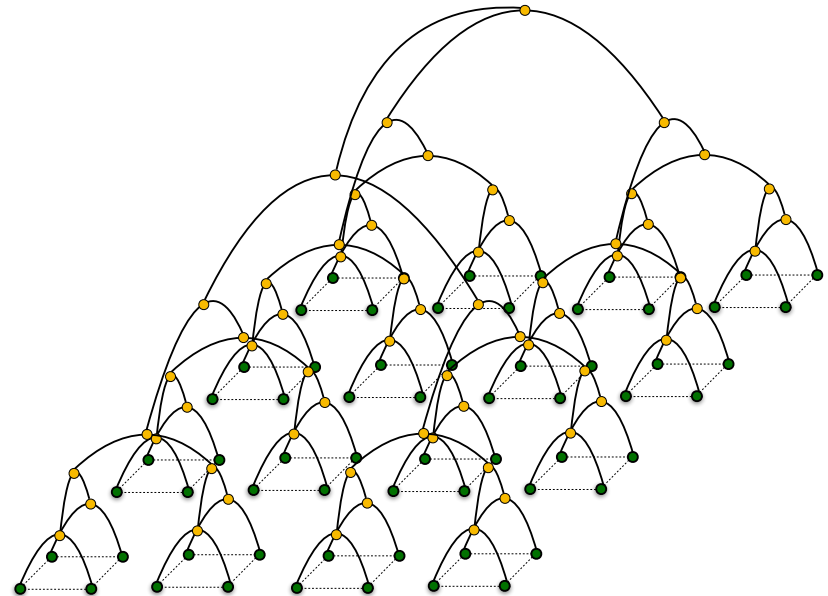
Projected Entangled Pair States (PEPS)



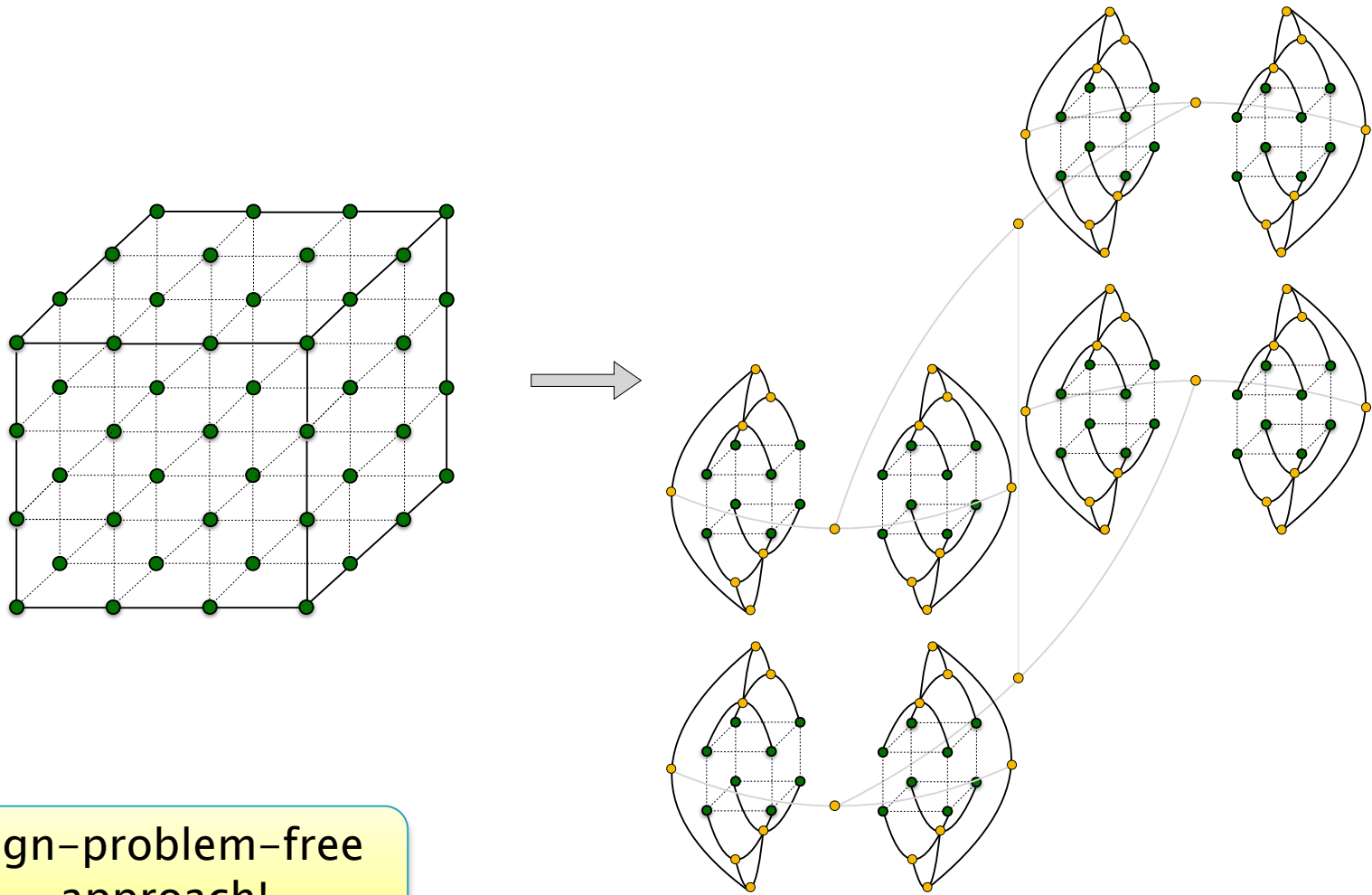
- they automatically reproduce the area-law of entanglement
- the optimization has a complexity $O(\chi^{10})$

Tree Tensor Networks (TTN)

- they do not automatically reproduce the area-law of entanglement
- the optimization has a complexity $O(\chi^4)$



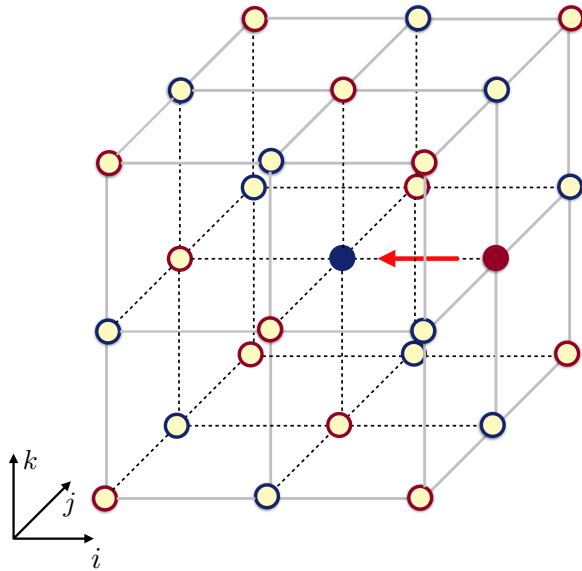
Tree Tensor Networks in 3D



Sign-problem-free
approach!

The optimization still
has a complexity $O(\chi^4)$

Lattice QED in (3+1)D

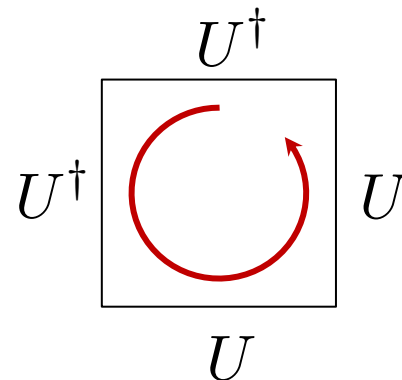


Matter Field	
$(-1)^{i+j+k} = +1$:	$\begin{cases} \text{red circle} = \emptyset \\ \text{dark red circle} = q \end{cases}$
$(-1)^{i+j+k} = -1$:	$\begin{cases} \text{blue circle} = \emptyset \\ \text{dark blue circle} = -q \end{cases}$

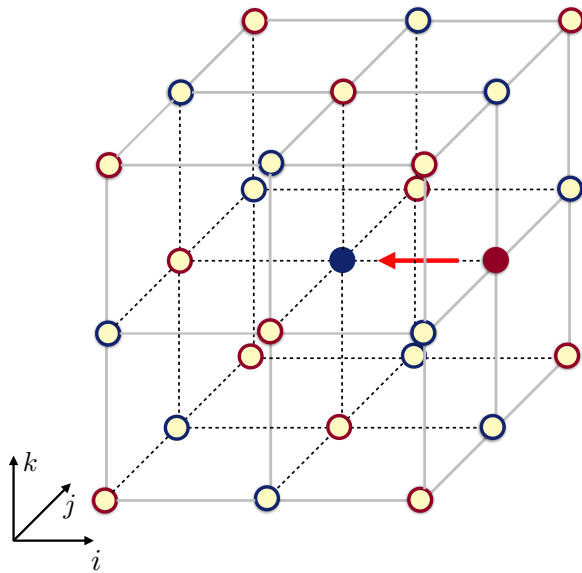
Gauge Field	
E_{x,μ_x} :	$\begin{cases} \text{red arrow pointing right} = \rightarrow\rangle \\ \text{grey line} = \emptyset\rangle \\ \text{red arrow pointing left} = \leftarrow\rangle \end{cases}$

$$\begin{aligned} \hat{H} = & -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \hat{U}_{x,\mu} \hat{\psi}_{x+\mu} + \text{H.c.} \right) \\ & + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 \\ & - \frac{g_m^2}{2} \sum_x \left(\square_{\mu_x, \mu_y} + \square_{\mu_x, \mu_z} + \square_{\mu_y, \mu_z} + \text{H.c.} \right) \end{aligned}$$

$$\square_{\mu_x, \mu_y} = U_{x, \mu_x} U_{x+\mu_x, \mu_y} U_{x+\mu_y, \mu_x}^\dagger U_{x, \mu_y}^\dagger$$



Lattice QED in (3+1)D



Matter Field	
$(-1)^{i+j+k} = +1$:	$\begin{cases} \text{red circle} = \emptyset \\ \text{dark red circle} = q \end{cases}$
$(-1)^{i+j+k} = -1$:	$\begin{cases} \text{blue circle} = \emptyset \\ \text{dark blue circle} = -q \end{cases}$

Gauge Field	
E_{x,μ_x}	$\begin{cases} \text{red arrow} = \rightarrow\rangle \\ \text{grey line} = \emptyset\rangle \\ \text{dark red arrow} = \leftarrow\rangle \end{cases}$

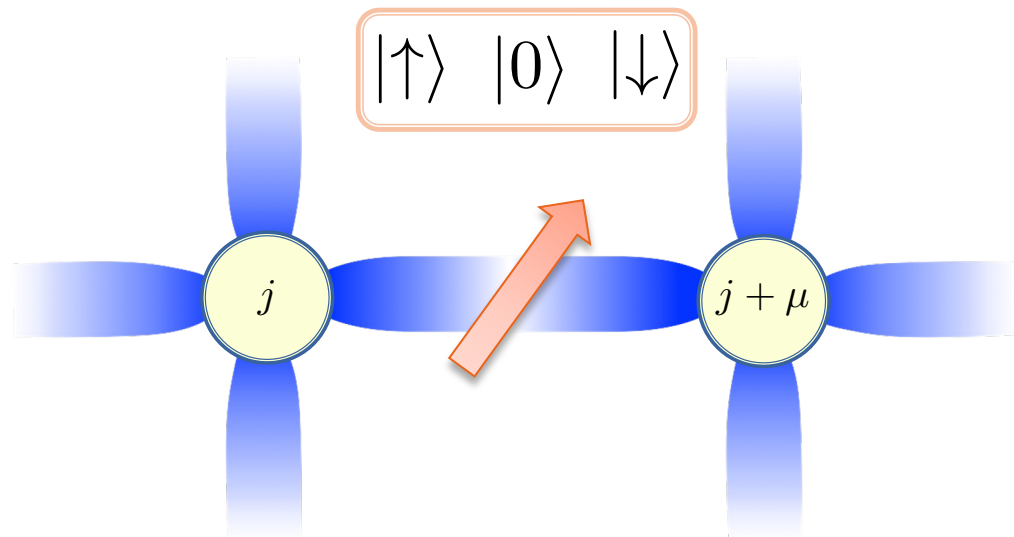
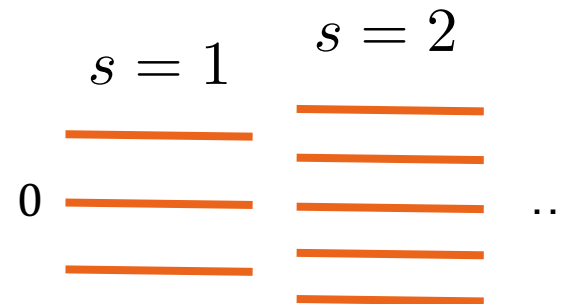
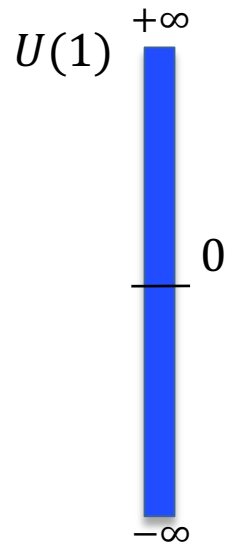
$$\begin{aligned} \hat{H} = & -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \hat{U}_{x,\mu} \hat{\psi}_{x+\mu} + \text{H.c.} \right) \\ & + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 \\ & - \frac{g_m^2}{2} \sum_x \left(\square_{\mu_x, \mu_y} + \square_{\mu_x, \mu_z} + \square_{\mu_y, \mu_z} + \text{H.c.} \right) \end{aligned}$$

Quantum Link Model
discretization of Gauge Fields

$$\begin{aligned} \hat{E}_{x,\mu} & \rightarrow \hat{S}_{x,\mu}^z \\ \hat{U}_{x,\mu} & \rightarrow \hat{S}_{x,\mu}^+ / s, \end{aligned}$$

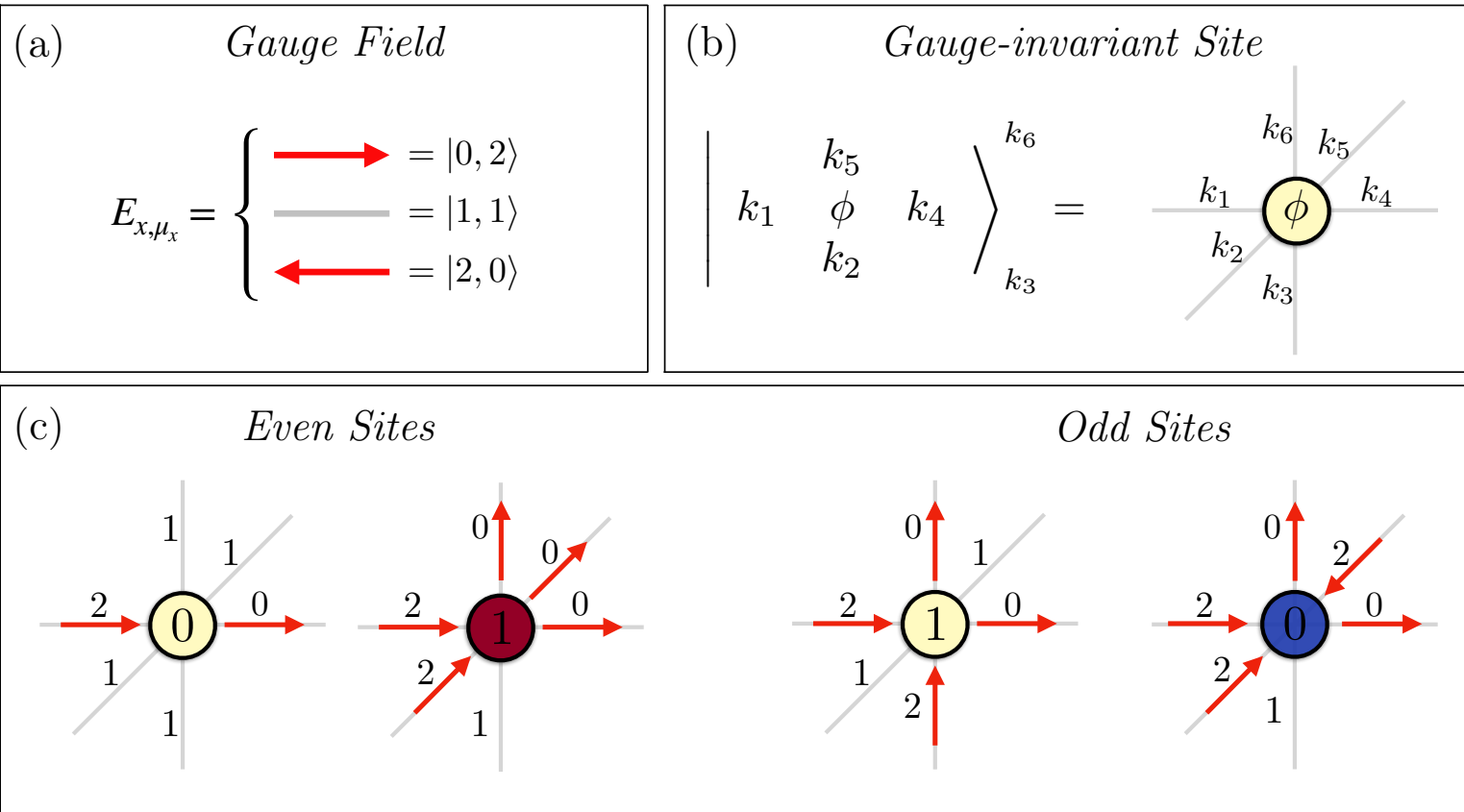
$$\hat{E}_{x,\mu} \rightarrow \hat{S}_{x,\mu}^z$$

$$\hat{U}_{x,\mu} \rightarrow \hat{S}_{x,\mu}^+ / s,$$



$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1 - p_x}{2} - \sum_{\mu} \hat{E}_{x,\mu}$$

$$\hat{G}_x |\Phi\rangle = 0 \quad \forall x$$



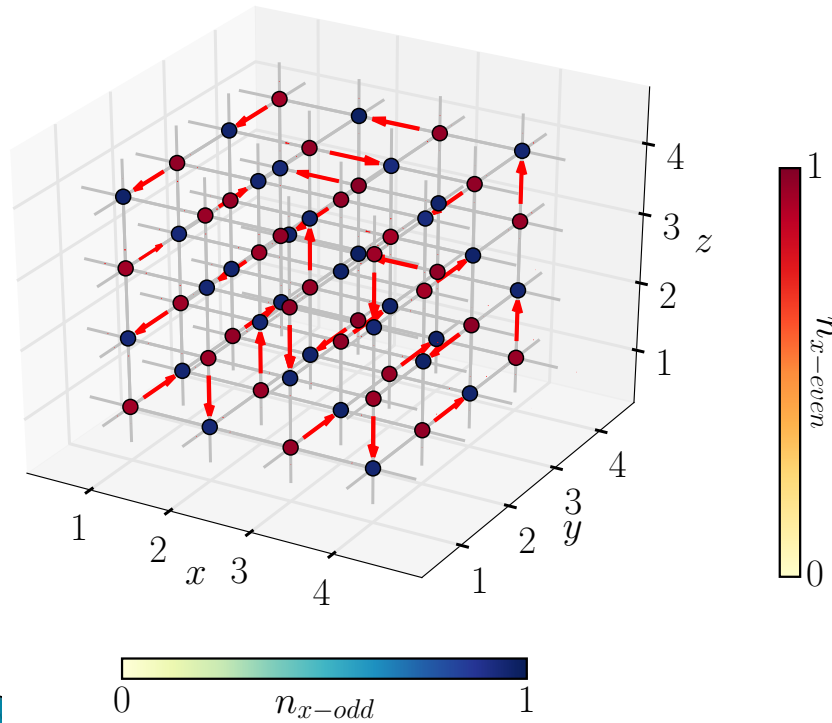
$$\dim H_x = 267$$

just for comparison, the $8 \times 8 \times 8$ system
corresponds to $64 \times 64 \times 64$ qubits

Local configurations of matter and gauge fields

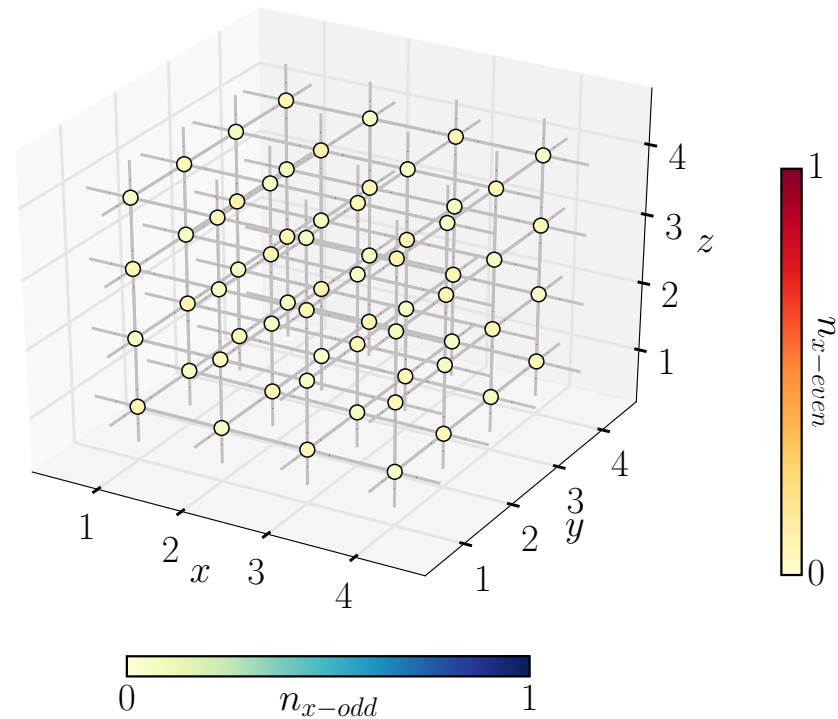
$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase:
particle-antiparticle dimers



$$g_e^2/2 \gg 2|m|$$

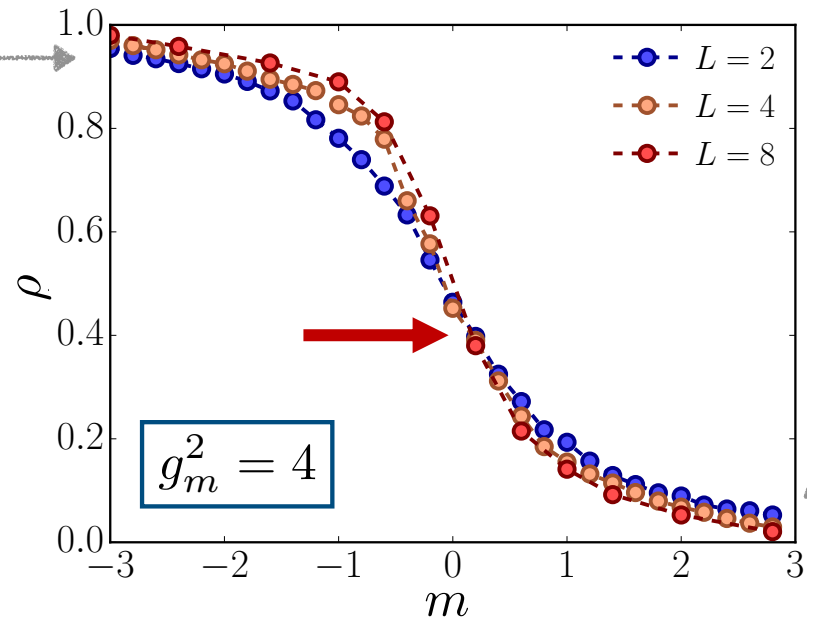
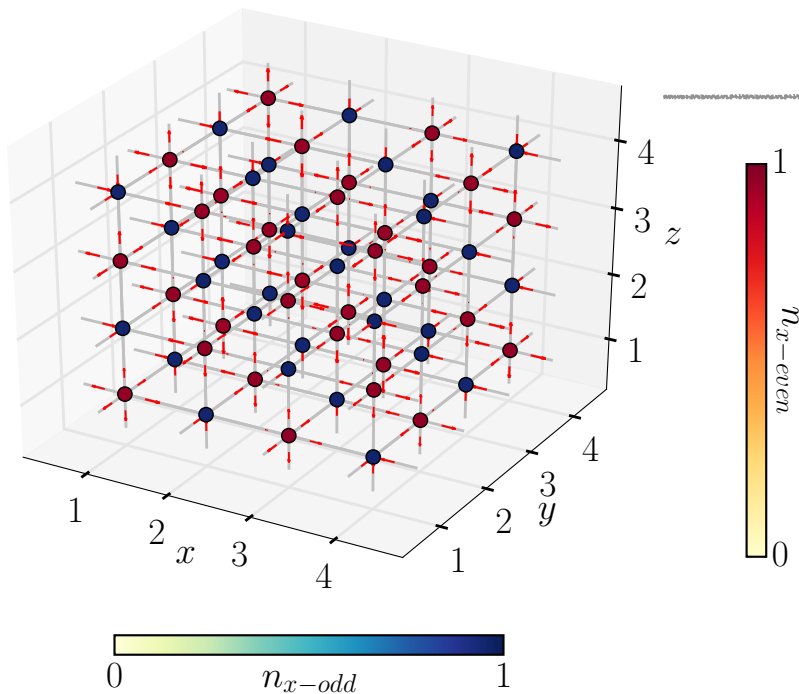
Vacuum phase: no particles, no
field excitations



Ground state properties for $g_m^2 = \frac{8}{g_e^2} = 4$

$$\rho = \frac{1}{L^3} \sum_x \langle GS | \hat{n}_x | GS \rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda \left(L^{\frac{1}{\nu}} (m - m_c) \right)$$



$$m_c = +0.22$$

$$\beta = 0.16 \quad \nu = 0.22$$

Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \hat{U}_{x,\mu} \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$

$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$

$$- \frac{g_m^2}{2} \sum_x \left(\square_{\mu_x, \mu_y} + \square_{\mu_x, \mu_z} + \square_{\mu_y, \mu_z} + \text{H.c.} \right)$$

Plaquette terms

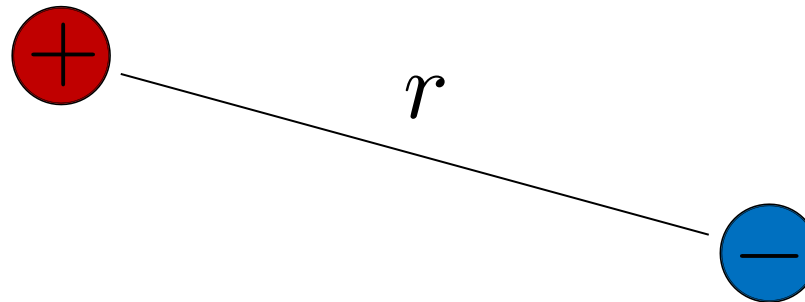
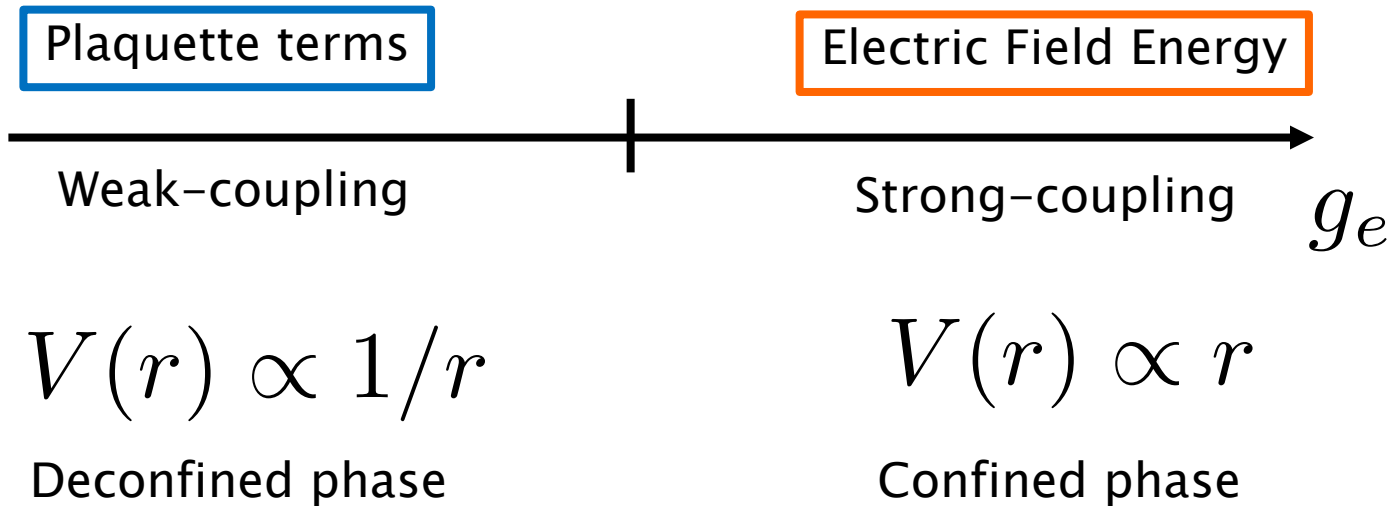
Electric Field Energy

Weak-coupling

Strong-coupling

g_e

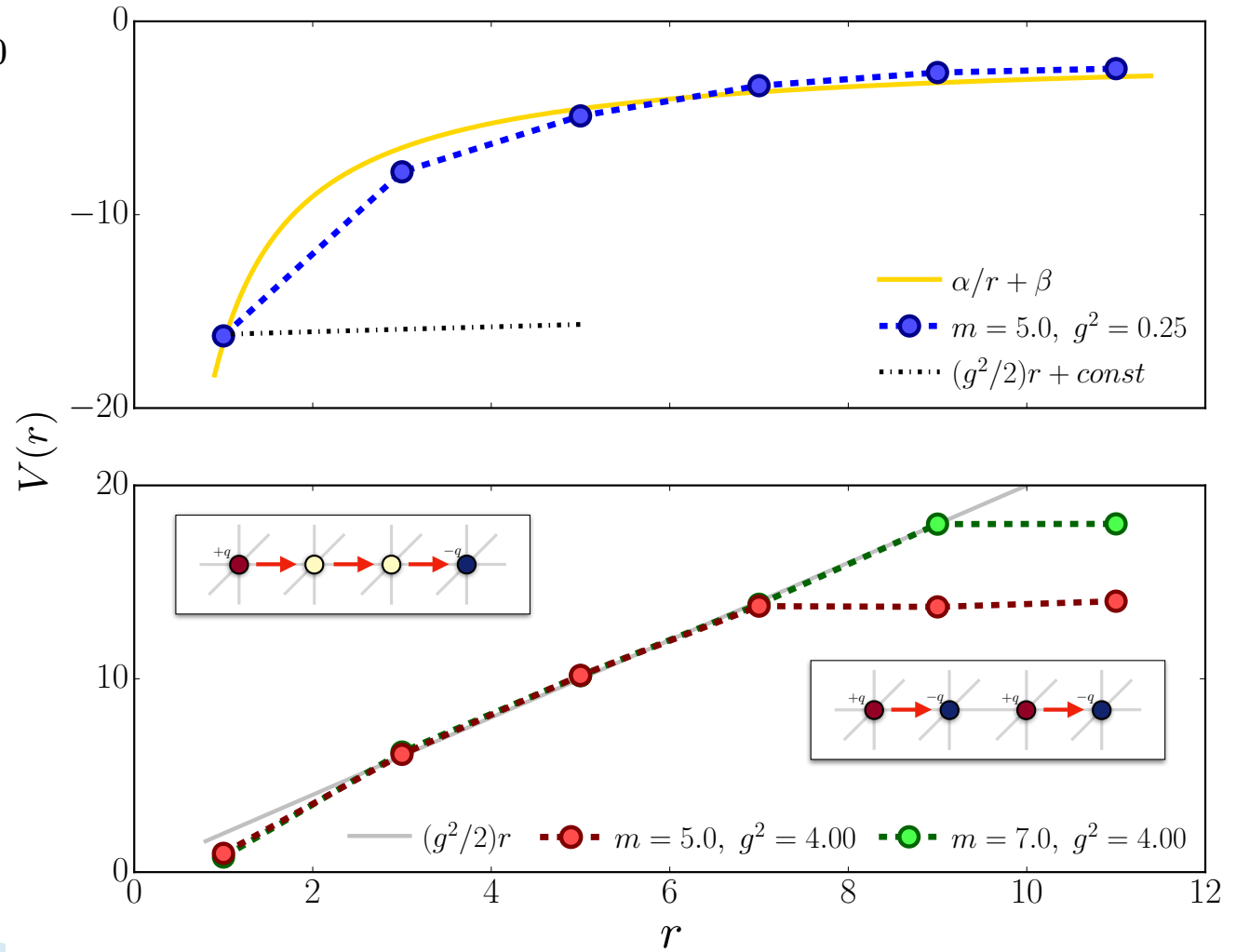
Confinement Properties



Confinement Properties

$$V(r) = E(r) - E_0$$

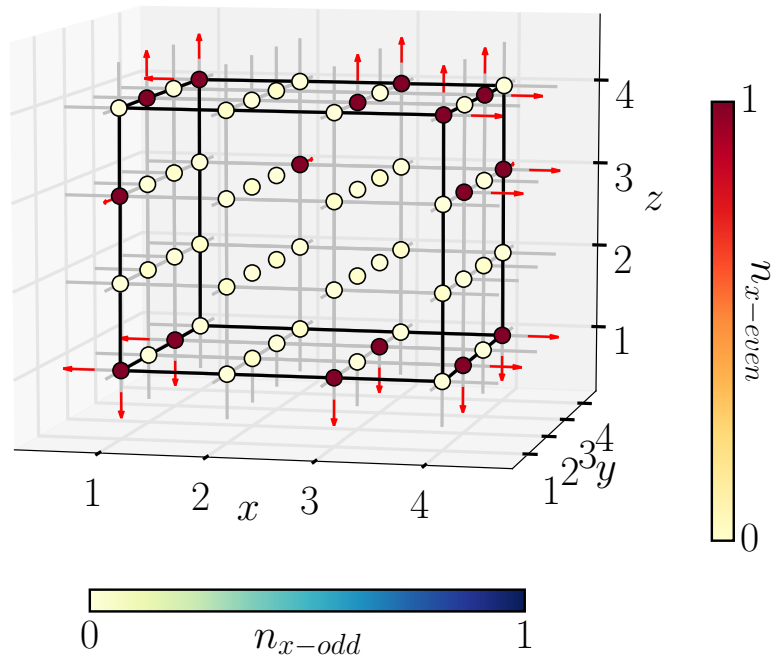
Weak-coupling



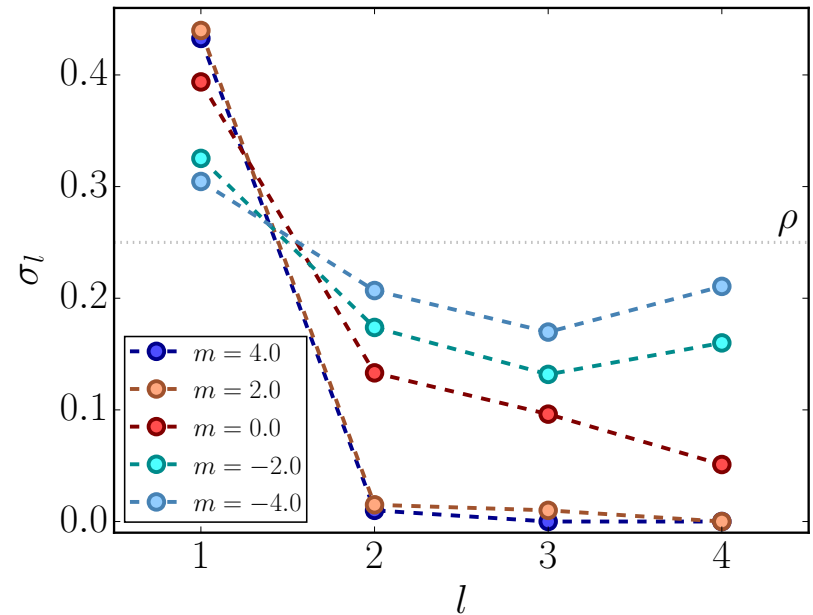
Strong-coupling

Finite Density

$$L = 4, Q = 16, \rho = 1/4$$



$$L = 8, Q = 128, \rho = 1/4$$



$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \langle \hat{\psi}_x^\dagger \hat{\psi}_x \rangle$$

Tensor Networks – Scattering dynamics



MARCO RIGOBELLO
University of Padua

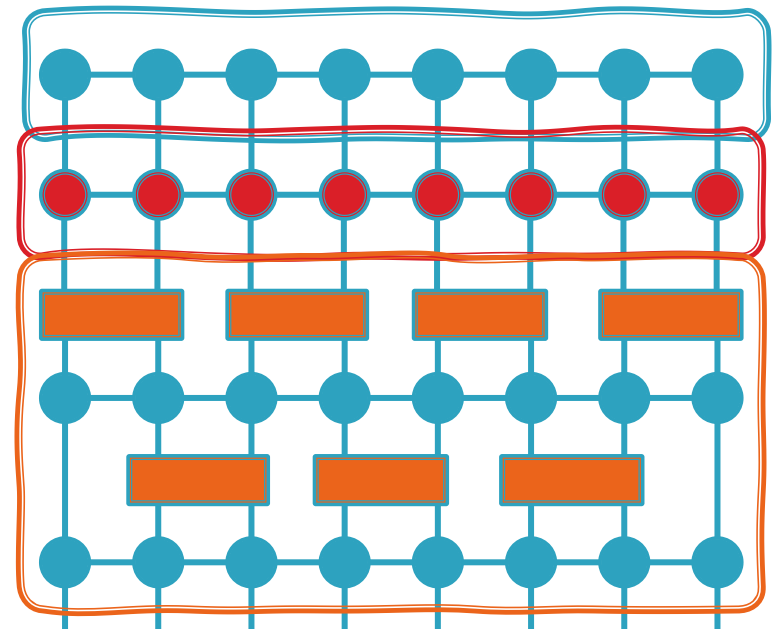
$$\hat{H} = -t \sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1} \hat{\psi}_{x+1} + \text{h.c.} + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2} \sum_x \hat{E}_{x,x+1}^2$$



interacting vacuum MPS via DMRG

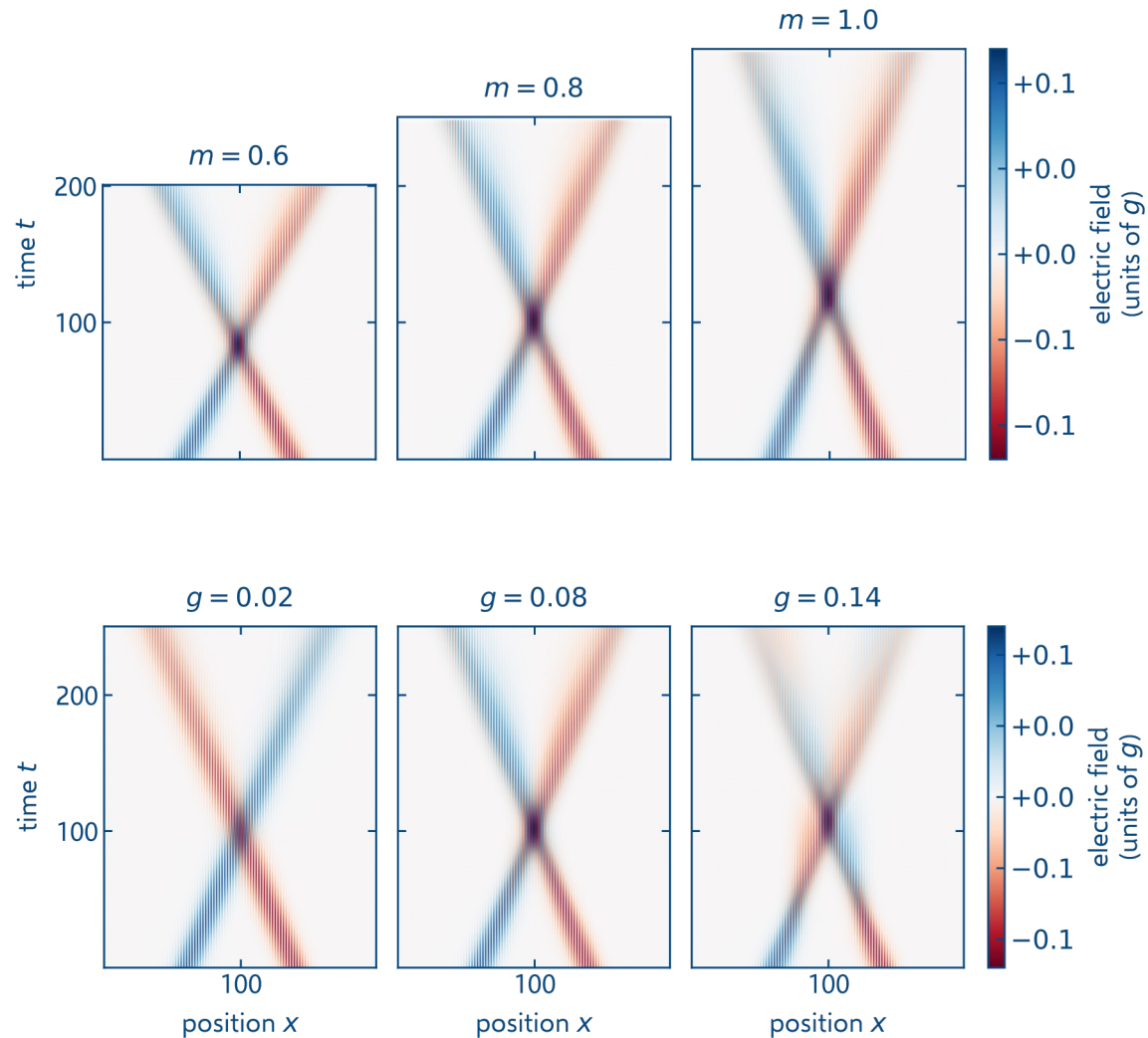
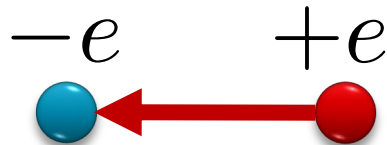
initial state via wave packet creation MPOs

time evolution via TEBD & observables

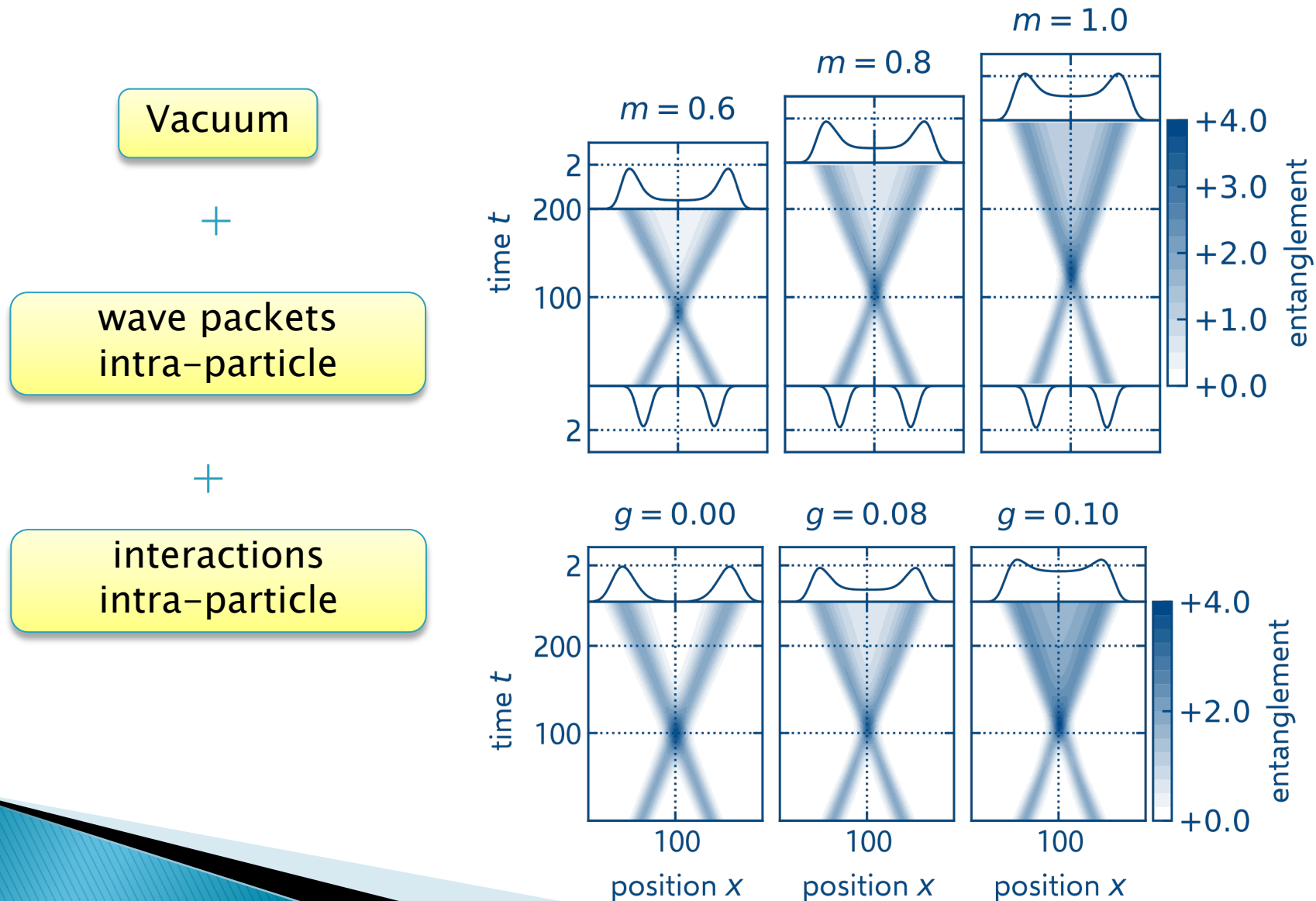


Tensor Networks – Scattering dynamics

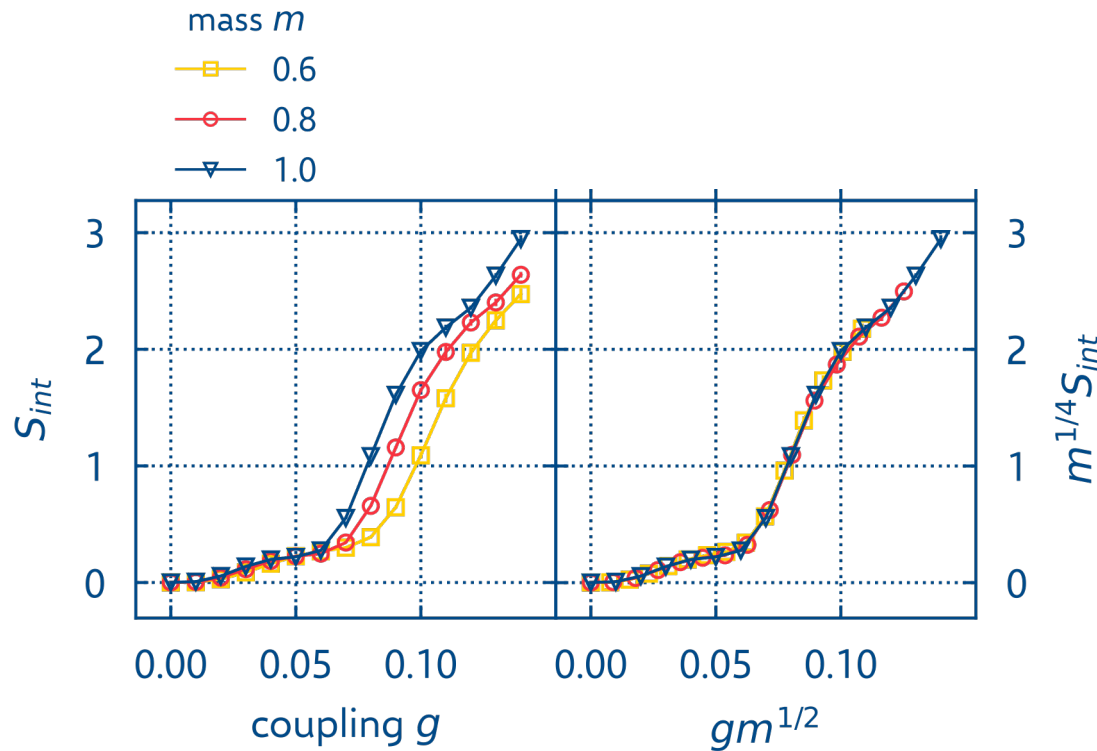
Mesons with opposite momenta and internal electric fields



Tensor Networks – Scattering dynamics



Tensor Networks – Scattering dynamics



scaling relation

$$m^{1/4} S_{int}(m, g/\sqrt{m}) = S_{int}(1, g)$$

is m independent

two regimes

$$g \leq g^* \approx 0.06/\sqrt{m}$$

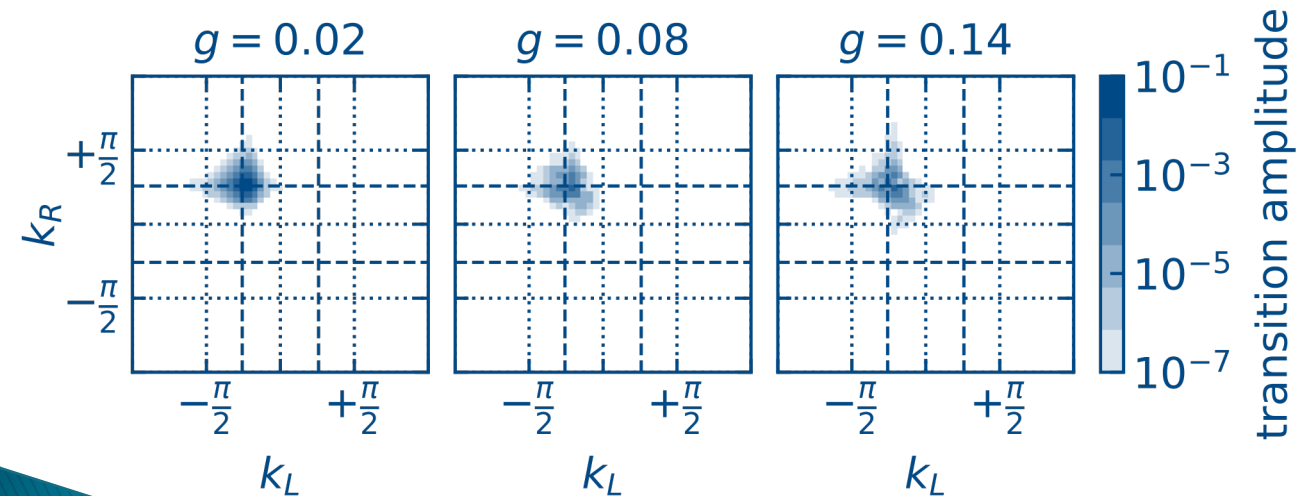
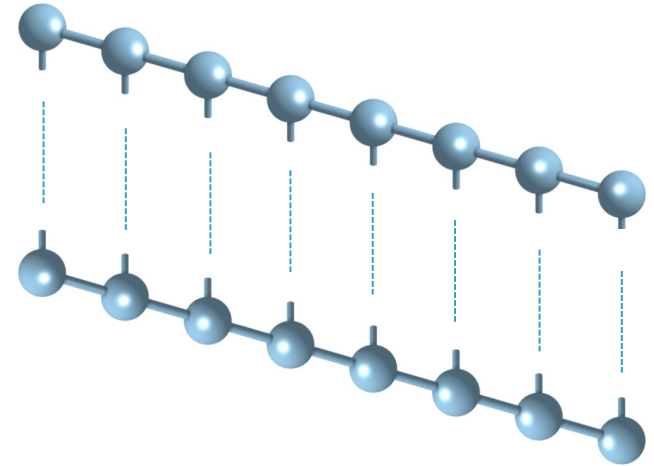
appearance of new effective
d.o.f.

Tensor Networks – Scattering dynamics

overlap of final state with
pair of meson wave packet

$$|\psi(t_f)\rangle$$

$$\langle\psi(t_i)|$$



~ S-matrix elements

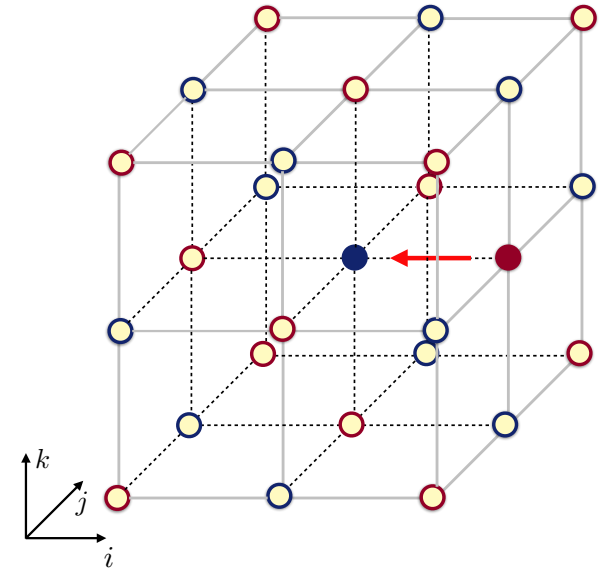
What's next?

Quantum
Many-Body
Physics



High-Energy
Physics
domain

- Tensor networks can be used to develop, support, validate, quantum simulation and quantum computation protocols.
- Relevant interactions with High-Energy Physics.
- Long-term goal: tensor networks and quantum simulation of QCD.



THANK YOU