La QCD sul reticolo a una svolta: dalle simulazioni ai calcoli di precisione

Guido Martinelli

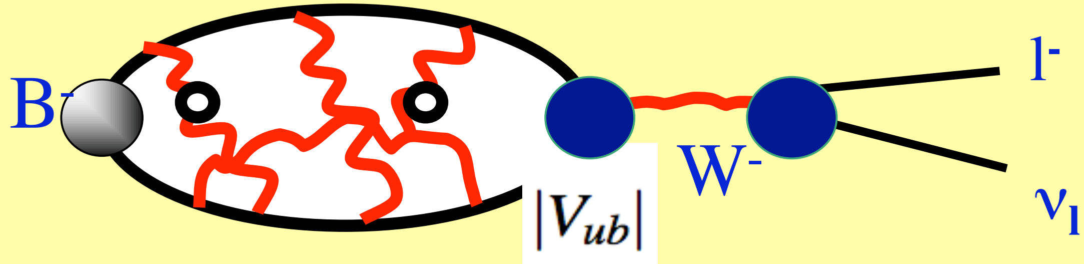
Legnaro 16/2/2011

Piano del mio intervento

1) *Generalita`*
2) *Masse e costanti di decadimento*
3) *Un caso emblematico: Kl3 decays*
4) B_K
5) *Conclusioni e prospettive*

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

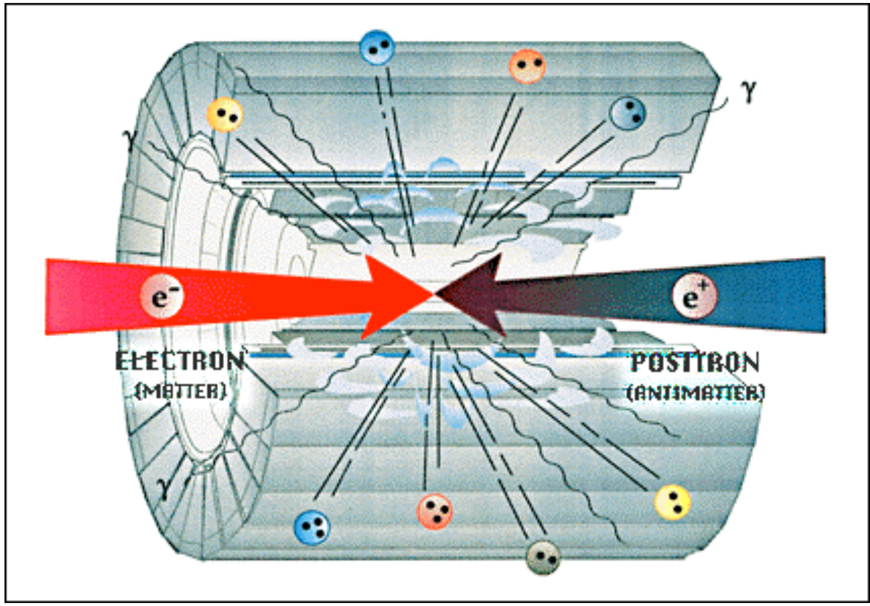


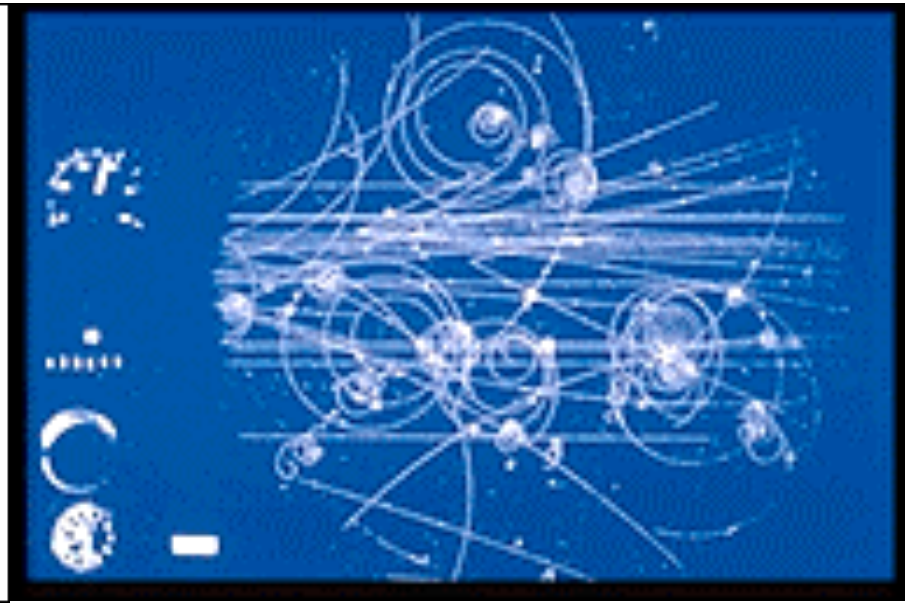
$$BR(B^- \to \tau^- \bar{\nu}_\tau) = f_B^2 |V_{ub}|^2 \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_B$$

$$f_B^2 \, |V_{ub}|^2$$

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 d | B^0(p) \rangle = i f_B p_\mu$$

COLOUD WE COMPUTE THIS PROCESS WITH
SUFFICIENT COMPUTER POWER ?





THE ANSWER IS: NO

IT IS NOT ONLY A QUESTION OF COMPUTER POWER
BECAUSE THERE ARE COMPLICATED
FIELD THEORETICAL PROBLEMS

LATTICE FIELD THEORY IN FEW SLIDES

La teoria di campo sul reticolo in una trasparenza:

Tutte le quantita` fisiche possono essere derivate da un integrale numerico eseguito con metodi Montecarlo su un reticolo di passo reticolare *a* finito (cutoff UVioletto) e volume totale L=N*a* finito (cutoff Infrarosso).

Bisogna poi eseguire il limite $a \to 0$ e $L \to \infty$

$$Z^{-1} \int [d\,\phi]\ \phi(x_1)\,\phi(x_2)\ \ldots\phi(x_{n-1})\,\phi(x_n)\ e^{-S(\phi)}$$

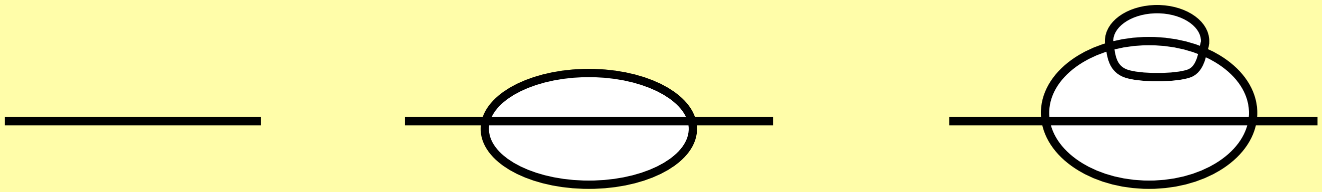
Determination of hadron masses and simple matrix elements

An example from the $\lambda\,\phi^4$ theory

$$G(t,\vec{q}) = \int d^3x\, e^{i\vec{q}\cdot\vec{x}} \langle 0|\phi^\dagger(\vec{x},t)\phi(\vec{0},0)|0\rangle$$

$$= \sum_n \langle 0|\phi^\dagger|n\rangle\langle n|\phi|0\rangle \frac{e^{-E_n t}}{2E_n} \quad t>0$$

$$\langle n|m\rangle = (2\pi)^3 2E_n \delta(\vec{q}_n - \vec{q}_m)$$



The field ϕ can excite one-particle, 3-particle etc. states

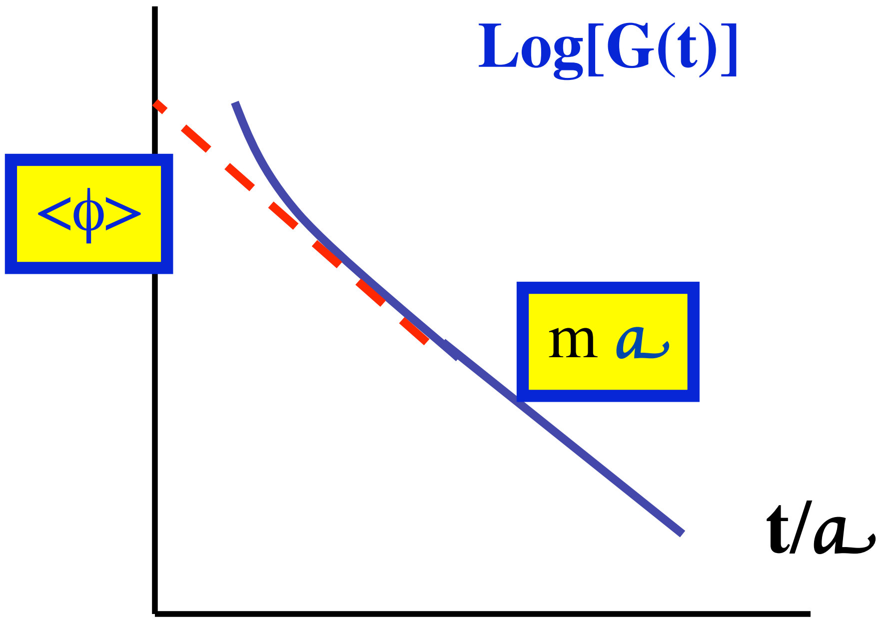
At large time distances the lightest (one particle) states dominate :

$$G(t,\vec{q})=\sum_n \langle 0|\phi^\dagger|n\rangle\langle n|\phi|0\rangle \frac{e^{-E_n t}}{2E_n} \rightarrow \langle 0|\phi^\dagger|\vec{q}\rangle\langle \vec{q}|\phi|0\rangle \frac{e^{-E_q t}}{2E_q}$$

For a particle at rest we have

$$G(t)=|\langle \vec{q}=0|\phi|0\rangle|^2 \frac{e^{-ma(t/a)}}{2m}$$

$\xi = 1/\ m\,a$ is the dimensionless correlation length (and the size of the physical excitations)



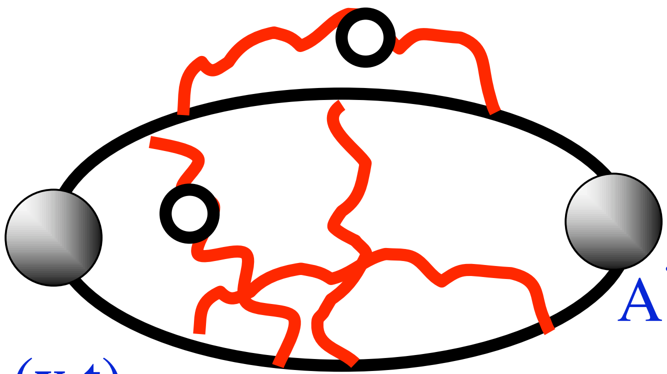
HADRON SPECTRUM AND DECAY CONSTANTS IN QCD

Define a source with the correct quantum numbers :

$$\text{“}\pi\text{”} \equiv A_0(\mathbf{x},t) = u^{\mathbf{a}}_{\alpha}(\mathbf{x},t)\,(\gamma_0\gamma_5)^{\alpha\beta}\,d^{\mathbf{a}}_{\beta}(\mathbf{x},t)$$

a=colour

β=spin



$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^{\dagger}_0(\mathbf{x},t)\rangle$$

$$= \sum_n \frac{|\langle 0 | A_0 | n\rangle|^2}{2E_n} \exp[-E_n t]$$

$$\rightarrow \frac{|\langle 0 | A_0 | \pi\rangle|^2}{2M_\pi} \exp[-M_\pi t]$$

$$\rightarrow \frac{f_\pi^{\,2} M_\pi}{2} \exp[-M_\pi t]$$

$$f_\pi M_\pi \sim Z_\pi$$

Mass and decay constant in lattice units $M_\pi = m_\pi\, \mathbf{a}$

$$A_\mu^{\eta'} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$$

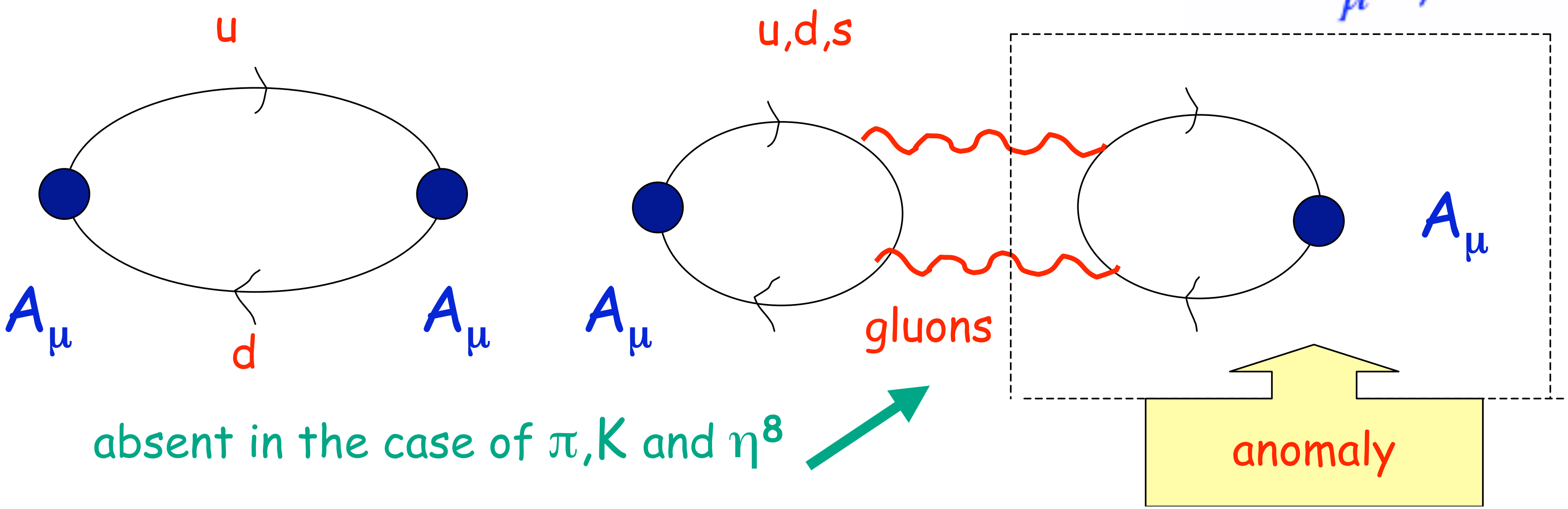
$$\partial^\mu A_\mu^{\eta'} = \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu} + 2m_q P^{\eta'}$$

$$P^{\eta'} = \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s \quad \tilde{G}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

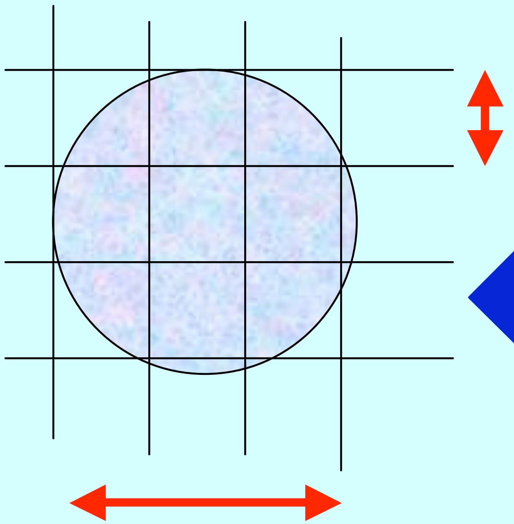
In the chiral limit $m_q \to 0$

$$\partial^\mu A_\mu^{\pi} = 0$$

$$\partial^\mu A_\mu^{\eta'} \neq 0$$



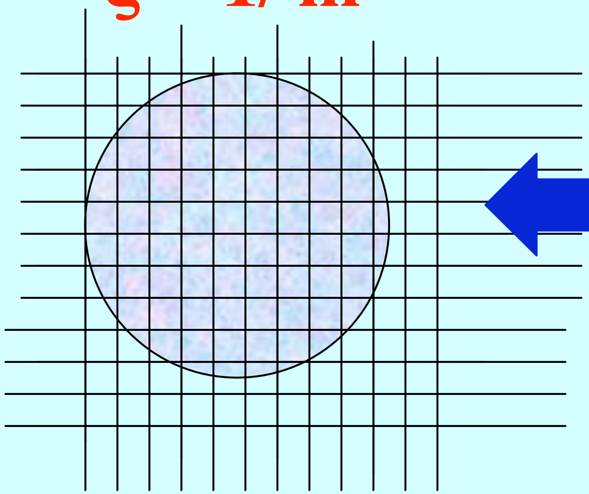
Continuum limit



a Formal $\lim_{a \to 0} S_{Lattice}(\phi) \to S_{Continuum}(\phi)$

$a/\xi = m\,a \sim 1$ The size of the object is comparable to the lattice spacing

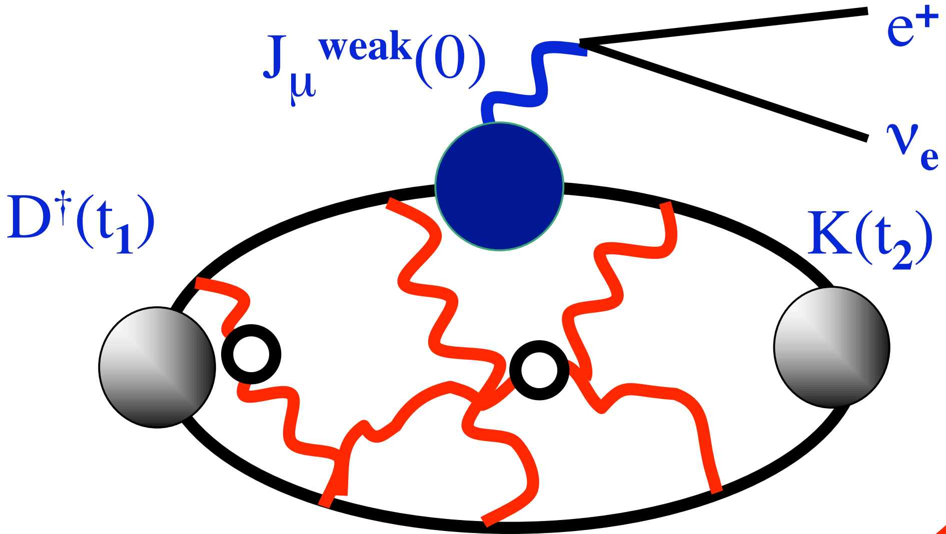
$\xi = 1/\,m$



a/ξ **<<1** i.e. **m a -> 0** The size of the object is much larger than the lattice spacing

Similar to $a \sum_n \to \int dx$

3-point functions



$D^\dagger(t_1) = \sum_x D^\dagger(\mathbf{x}, t_1) \exp[-i\, \mathbf{p_D}\, \mathbf{x}]$
$K(t_2) = \sum_x K(\mathbf{x}, t_2) \exp[+i\, \mathbf{p_K}\, \mathbf{x}]$

$$\langle K(t_2,\vec{p}_K) J_\mu^{weak}(0) D^\dagger(t_1,\vec{p}_D)\rangle \rightarrow \left[\frac{\langle 0|K|K\rangle\langle D|D^\dagger|0\rangle}{2E_D\, 2E_K} e^{-E_D t_1 - E_K t_2}\right] \times \langle K(\vec{p}_K)|J_\mu^{weak}|D(\vec{p}_D)\rangle$$

from the 2-point functions

$< K \mid J_\mu^{weak}(0) \mid D >$
also electromagnetic form factors, structure functions, dipole moment of the neutron, g_a/g_v, etc.

1) Kl3 namely $K \rightarrow \pi\, l\, \nu_l$

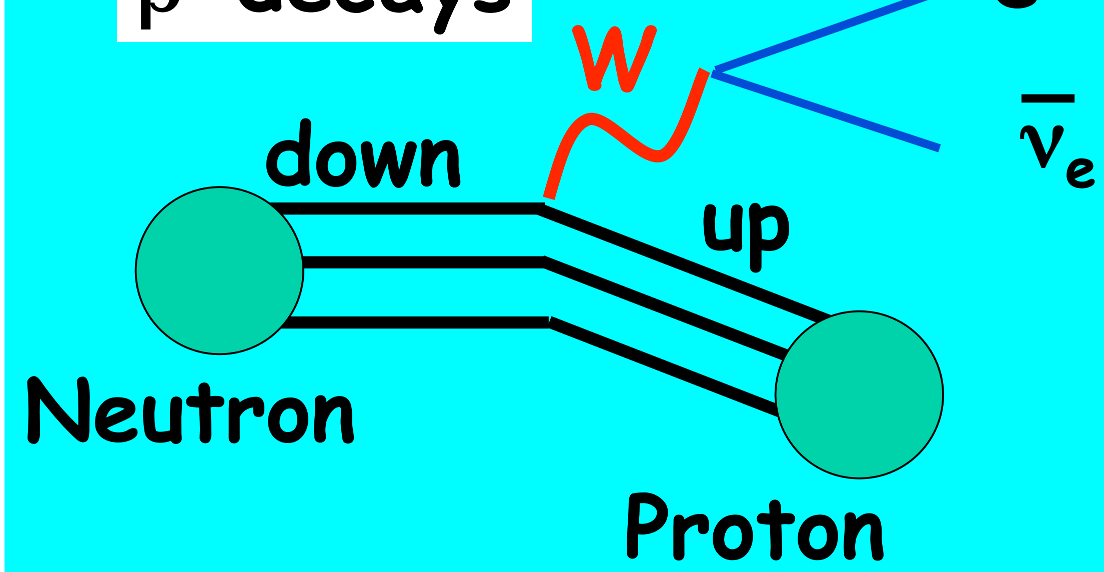
2) $D \rightarrow (K,\pi)\, l\, \nu_l$

3) $B \rightarrow (\pi,\rho)\, l\, \nu_l$

Quark masses & Generation Mixing

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{tb}	V_{ts}	V_{tb}

β-decays



$|V_{ud}|$

$|V_{ud}| = 0.9735(8)$

$|V_{us}| = 0.2196(23)$

$|V_{cd}| = 0.224(16)$

$|V_{cs}| = 0.970(9)(70)$

$|V_{cb}| = 0.0406(8)$

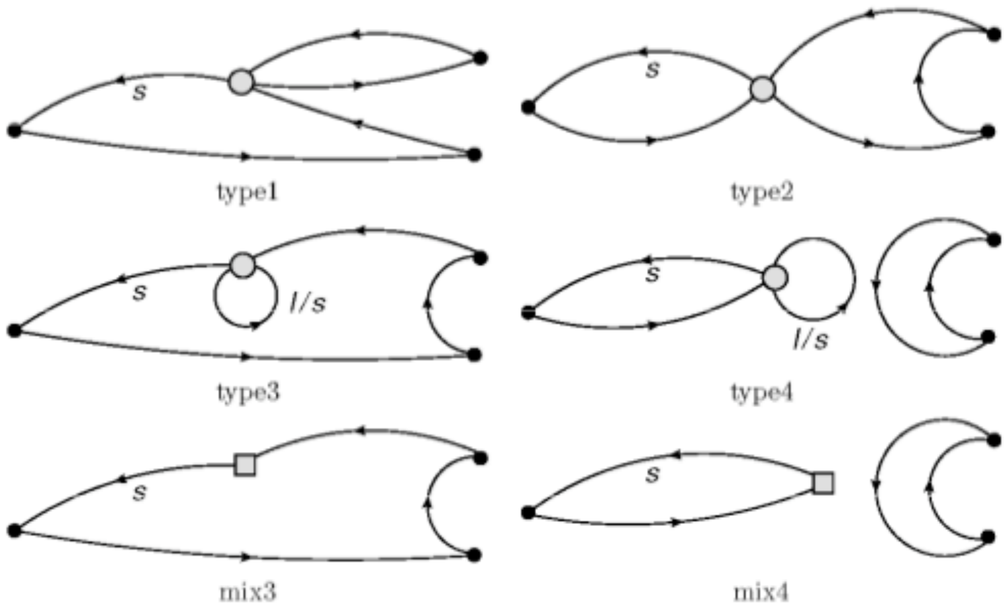
$|V_{ub}| = 0.00363(32)$

$|V_{tb}| = 0.99(29)$

(0.999)

$\Delta I = 1/2 \; K \rightarrow \pi\pi$
(Qi Liu)

- Code 50 different contractions
- For each of 400 configurations invert with source at each of 32 times.
- Use Ran Zhou's deflation code



3. Direct Calculations of $K \to \pi\pi$ Decay Amplitudes

- We need to be able to calculate $K \to \pi\pi$ matrix elements directly.
- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are now understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

where $E^2 = 4(m_\pi^2 + q^{*2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function.

M.Lüscher, 1986, 1991, $\cdots$.

- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2 \frac{m_K E^2}{q^{*2}} \left\{\delta'(q^*) + \phi^{P\prime}(q^*)\right\} |M|^2\,,$$

where $\prime$ denotes differentiation w.r.t. q^* .

L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006; N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

C.-J.D. Lin, G. Martinelli, C.T. Sachrajda, M. Testa
hep-lat/0104006v2

- Computation of $K \to (\pi\pi)_{I=2}$ matrix elements does not require the subtraction of power divergences or the evaluation of disconnected diagrams.
- In principle, we understand how to calculate the $\Delta I = 3/2$ $K \to \pi\pi$ matrix elements.
- Our aim is to calculate the matrix elements with as good a precision as we can.

General consideration on non-perturbative methods/approaches/models

Models a) bag-model b) quark model

not based on the fundamental theory; at most QCD "inspired"; cannot be systematically improved

Effective theories c) chiral lagrangians d) Wilson Operator Product Expansion (OPE) e) Heavy quark effective theory (HQET)

based on the fundamental theory; limited range of applicability; problems with power corrections (higher twists), power divergences & renormalons; need non perturbative inputs (f_π , $< x >$, λ_1, $\underline{\Lambda}$)

Methods of effective theories used also by QCD sum rules and Lattice QCD

f) QCD sum rules

based on the fundamental theory **+ "condensates"** (non-perturbative matrix elements of higher twist operators, which must be determined phenomenologically; very difficult to improve; share with other approaches the problem of renormalons etc.

LATTICE QCD

Started by Kenneth Wilson in 1974

- Based on the fundamental theory [Minimum number of free parameters, namely Λ_{QCD} and m_q]

- Systematically improvable [errors can me measured and corrected, see below]

- Lattice QCD is not at all numerical simulations and computer programmes only. A real understanding of the underline Field Theory, Symmetries, Ward identities, Renormalization properties is needed.

LATTICE QCD IS REALLY EXPERIMENTAL FIELD THEORY

Major fields of investigation

QCD

- QCD thermodynamics
- Hadron spectrum
- Hadronic matrix elements

(K -> $\pi\pi$, structure functions, etc. see below)

EW

- Strong interacting Higgs Models
- Strong interacting chiral models

- Surface dynamics
- Quantum gravity

LATTICE QCD

α_S and the Quark Masses

Leptonic decay constants f_π , f_K , f_D , f_{D_s}, f_B , f_{B_s}, f_ρ .

Electromagnetic form factors $F_\pi(Q^2)$, $G_M(Q^2)$, ...

Semileptonic form factors $f^{+,0}(Q^2)$, $A_{0,..3}(Q^2)$, $V(Q^2)$

$K \to \pi$, $D \to K, K^*, \pi, \rho$, $B \to D, D^*, \pi, \rho$ $B \to K^* \gamma$

The Isgur-Wise function

B-parameters : $\langle \overline{K^0} | Q^{\Delta S=2} | K^0 \rangle$ and $\langle \overline{B^0} | Q^{\Delta B=2} | B^0 \rangle$

Weak decays : $\langle \pi | Q^{\Delta S=1} | K \rangle$ and $\langle \pi\, \pi | Q^{\Delta S=1} | K \rangle$

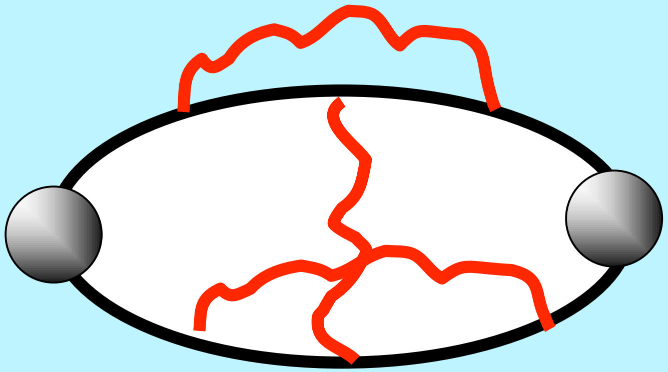
Matrix elements of leading twist operators

Lattice QCD is really a powerful approach

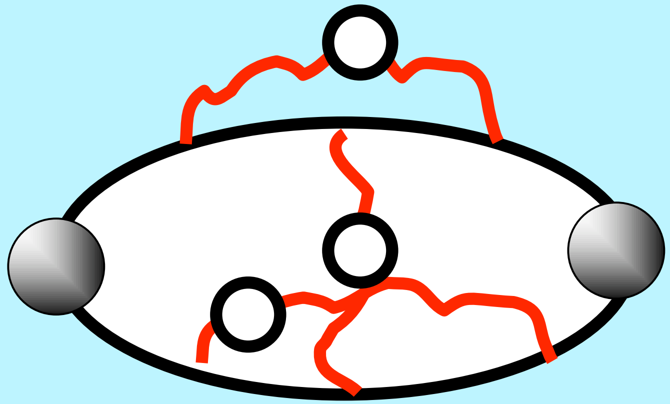
↓

SYSTEMATIC ERRORS

Quenching errors



QUENCHED



UNQUENCHED

(partially,two-flavours, three?, etc.)

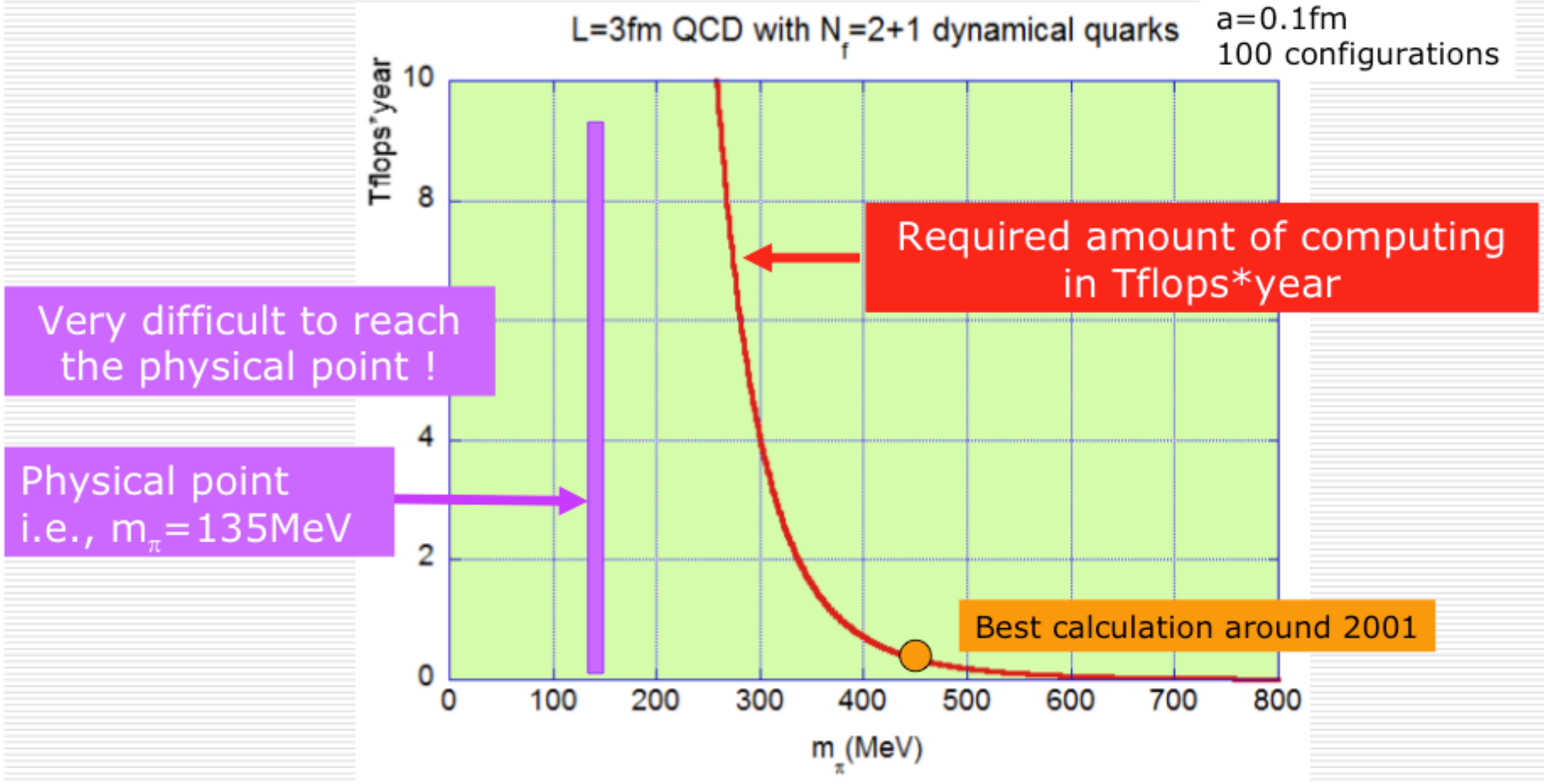
ALL MODERN LATTICE CALCULATIONS ARE UNQUENCHED:

Nf=2, 2+1 or 2+1+1

Many slides from he Workshop Future Directions in lattice gauge theory LGT10 July 19th- August 13th CERN

"Berlin wall" at Lattice 2001@Berlin

A. Ukawa for CP-PACS and JLQCD

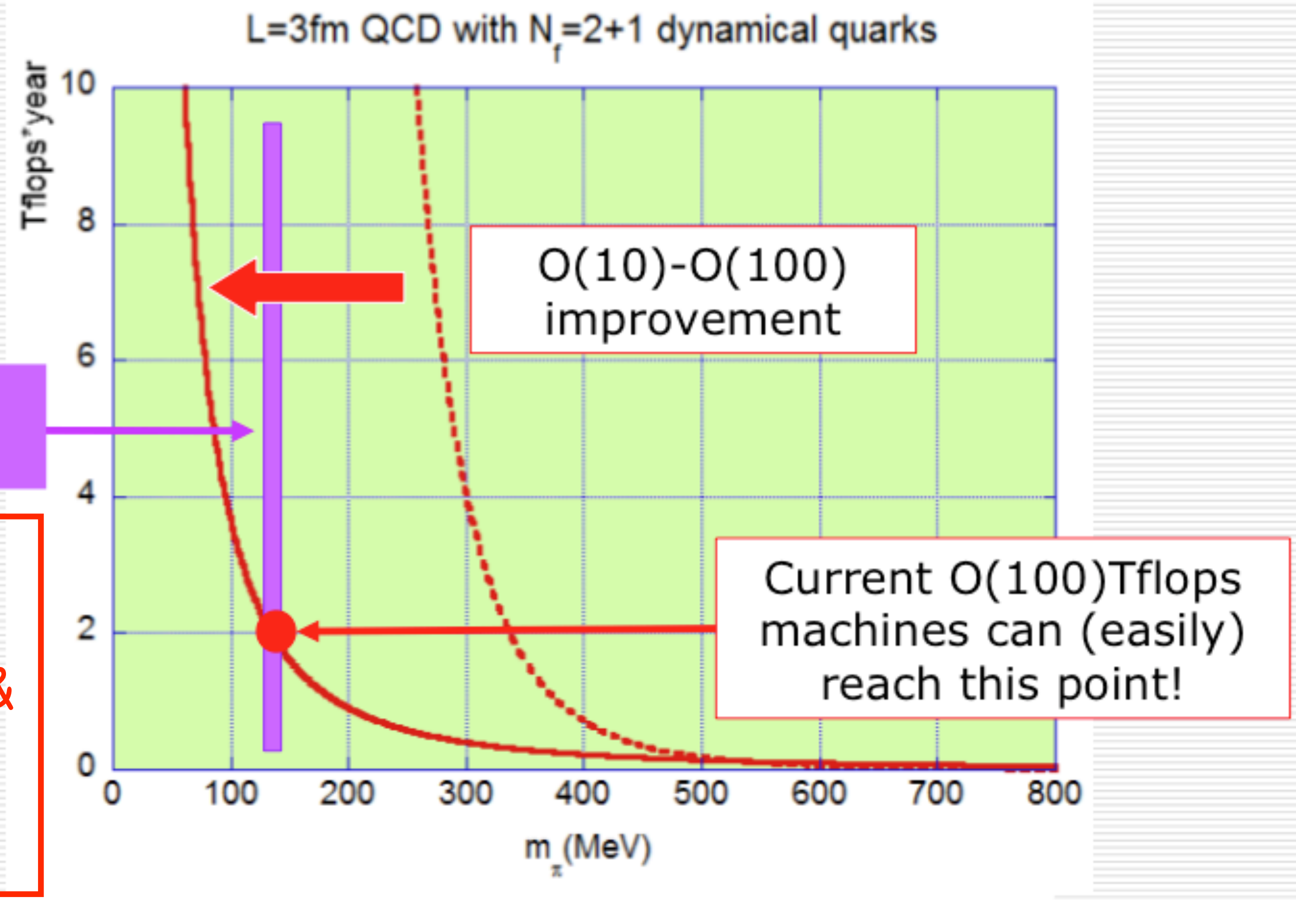


Revolutionary progress since 2005 ; beating the critical slowing down

Physical point
i.e., m_π=135MeV

Still
Extrapolation
to continuum &

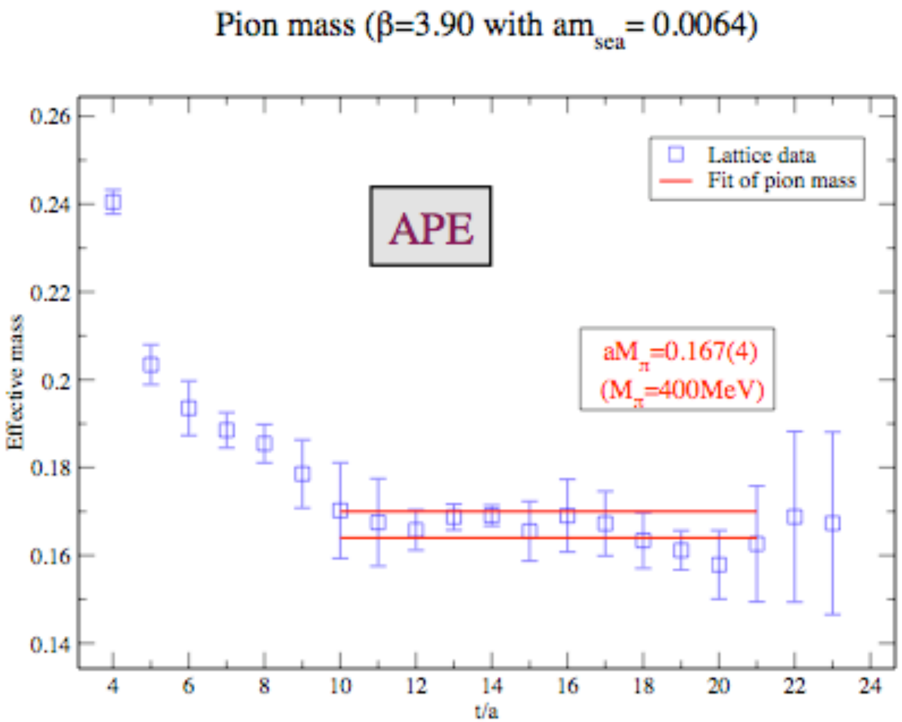
Finite volume
effects

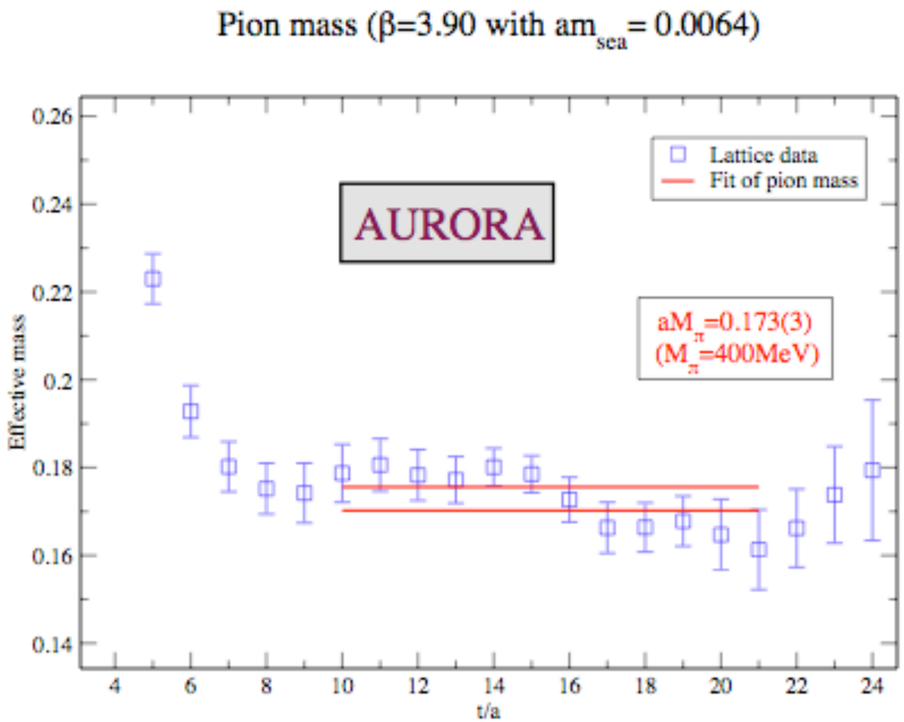


Physical Point Simulation has become reality

-il lattice spacing e' circa 85/1000 di fermi
-la massa del quark e' circa 30 MeV
-il reticolo ha estensione 24^3x48

$$K \longrightarrow \pi\nu\bar{\nu}$$





Key observation

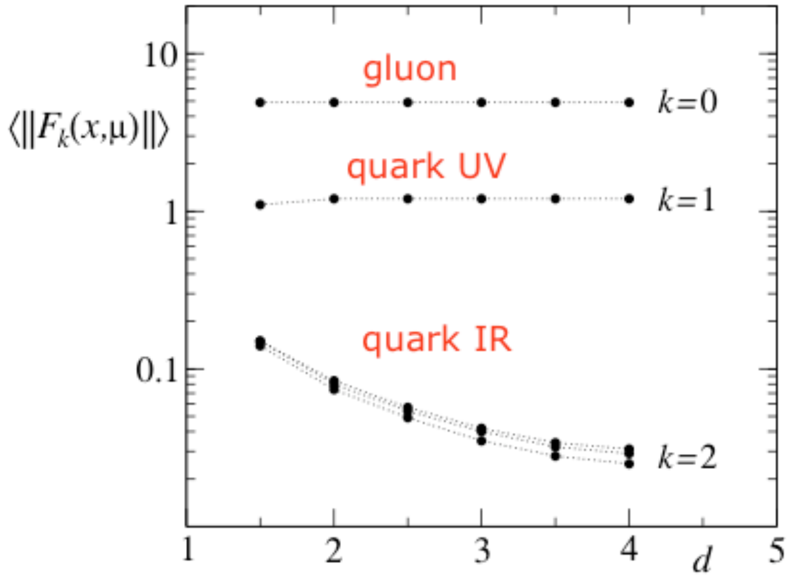
M. Luescher, C

- Separate UV and IR modes of quark fluctuations
- gluon:UV:IR forces are order of magnitude different!

$$F_{gluon} >> F_{quark,UV} >> F_{quark,I}$$

- This invites a multi-time step integration:

$$\delta\tau_{gluon} << \delta\tau_{quark,UV} << \delta\tau_{quark,}$$



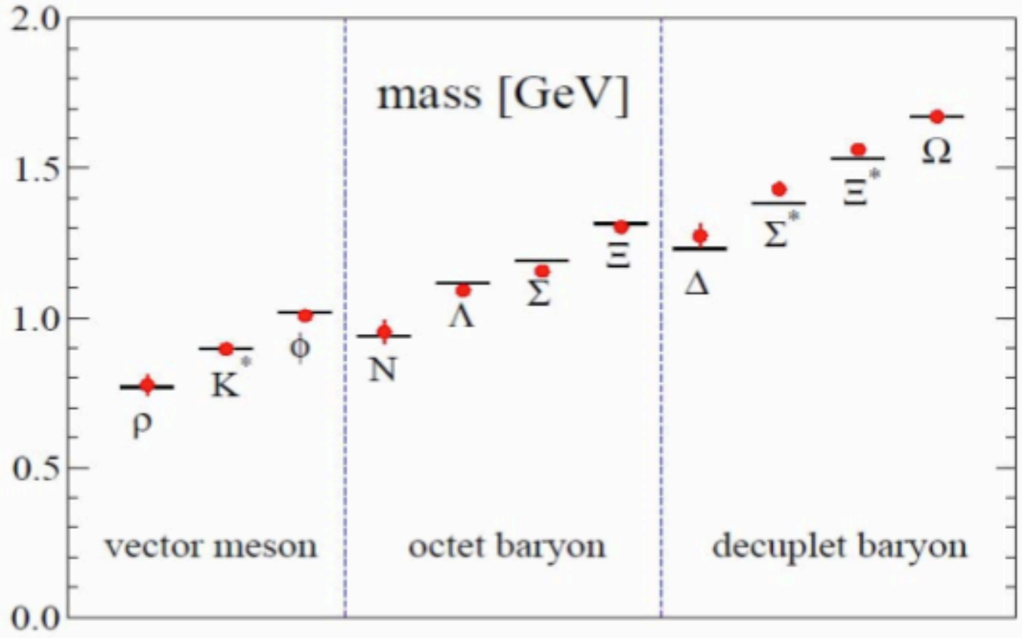
i.e., one can enlarge the time step for the most compute intensive IR quark force, *leading to a large reduction in the computing requirement.*

This is physics!

Our conscious effort toward physical pion mass (II)

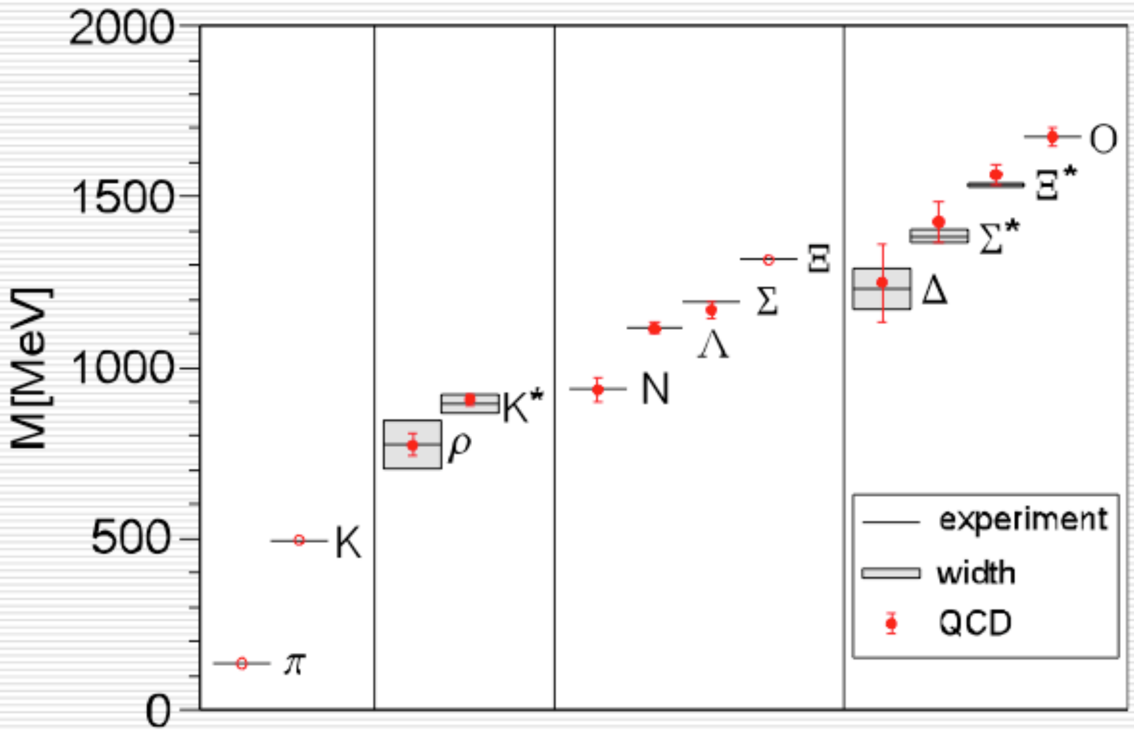
PACS-CS Collaboration Phys. Rev. D79 034504 (2008)

pion mass down to $m_\pi \approx 156MeV$ $32^3 \times 64, \quad a = 0.907(13)\,fm$



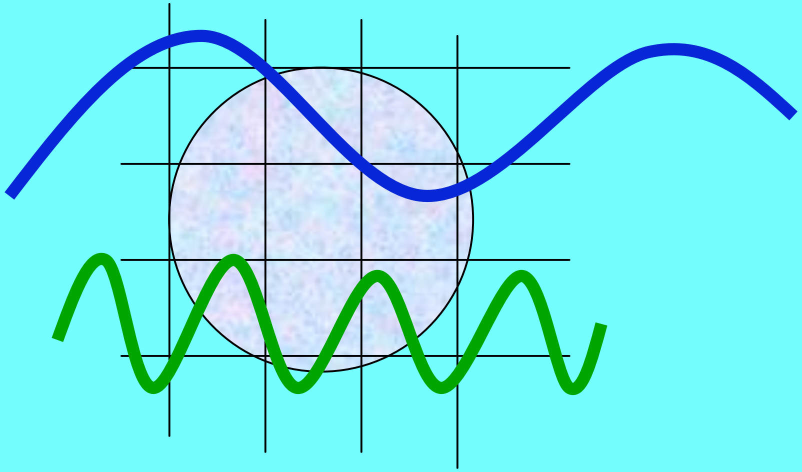
In the mean time,
came along the BMW Collaboration

BMW Collaboration (Butapest-Marseille-Wuppertal)
Science 322(2008) 1224
m_π>200MeV but large lattices ($m_\pi L$>4) and continuum extrapolated!



Extrapolation in the heavy quark mass

DISCRETIZATION ERRORS



THE ULTRAVIOLET PROBLEM

$1/M_H \;>>\; a$

$O(a)$ errors $\begin{cases} m_q\, \mathbf{a} << 1 \\ p\, \mathbf{a} << 1 \end{cases}$

For a good approximation of the continuum

Typically $\mathbf{a}^{-1} \sim 2 \div 5$ GeV

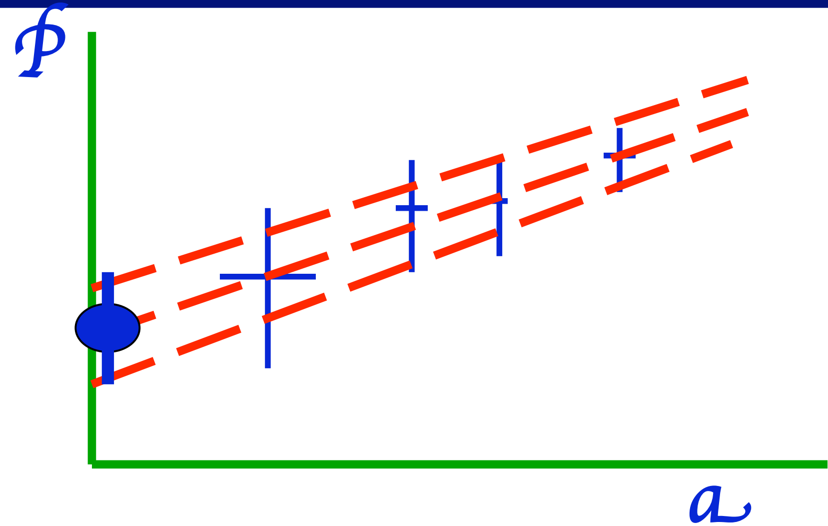
$m_{charm} \sim 1.3$ GeV $\quad m_{charm}\, a \sim 0.3$

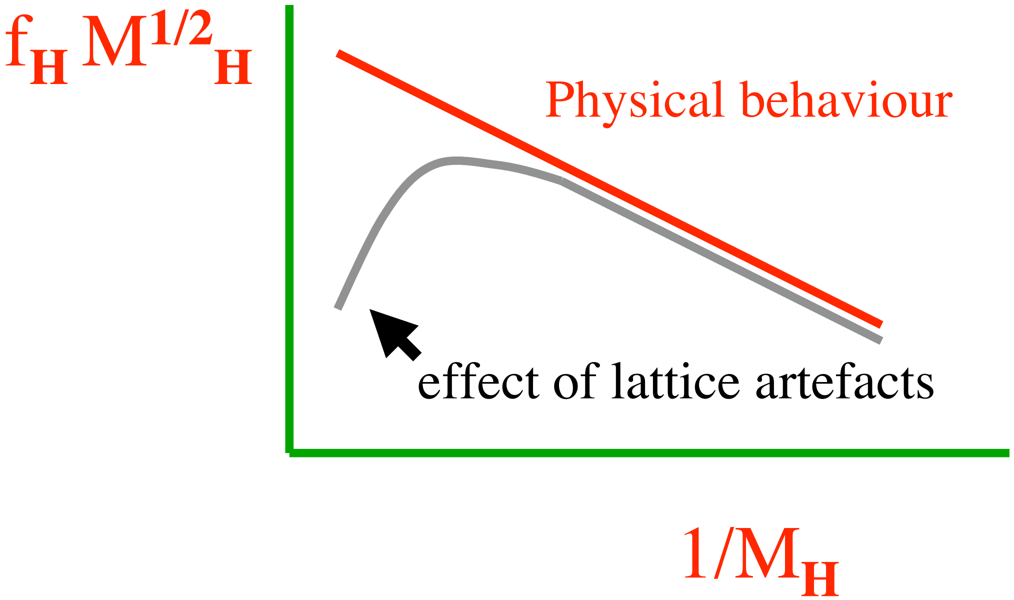
$m_{bottom} \sim 4.5$ GeV $\quad m_{bottom}\, a \sim 1$

SYSTEMATIC ERRORS

DISCRETIZATION ERRORS

Naïve solution: extrapolate measures performed at different values of the lattice spacing. Price: the error increases



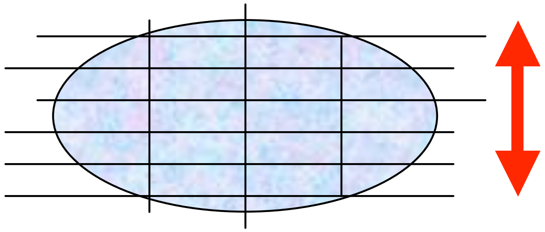


IMPROVEMENT

SYSTEMATIC ERRORS

FINITE VOLUME EFFECTS

THE INFRARED PROBLEM



BOX SIZE

$$L >> \xi = 1/M_H >> a$$

To avoid finite size effects

For a good approximation of the continuum

Finite size effects were not really a problem for quenched calculations; potentially more problematic for the unquenched case

$O(\exp[- \xi /L])$

Is $L \geq 4 \div 5 \ \xi$ sufficient ?

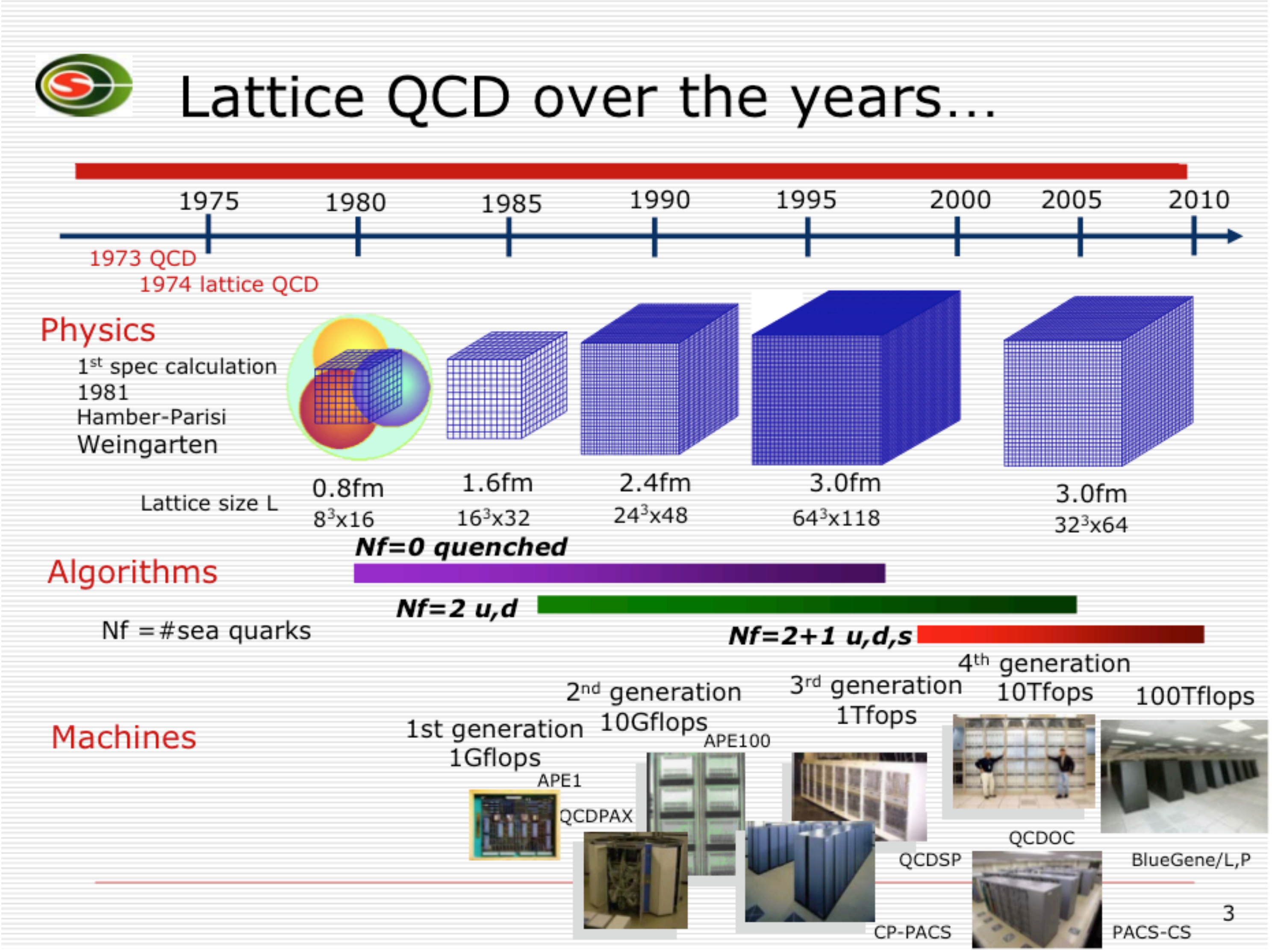
an extrapolation in m_{light} to the physical point is in many cases still necessary

Test if the quark mass dependence is described by Chiral perturbation Theory (χPT),
Then the extrapolation with the functional form suggested by χPT is justified

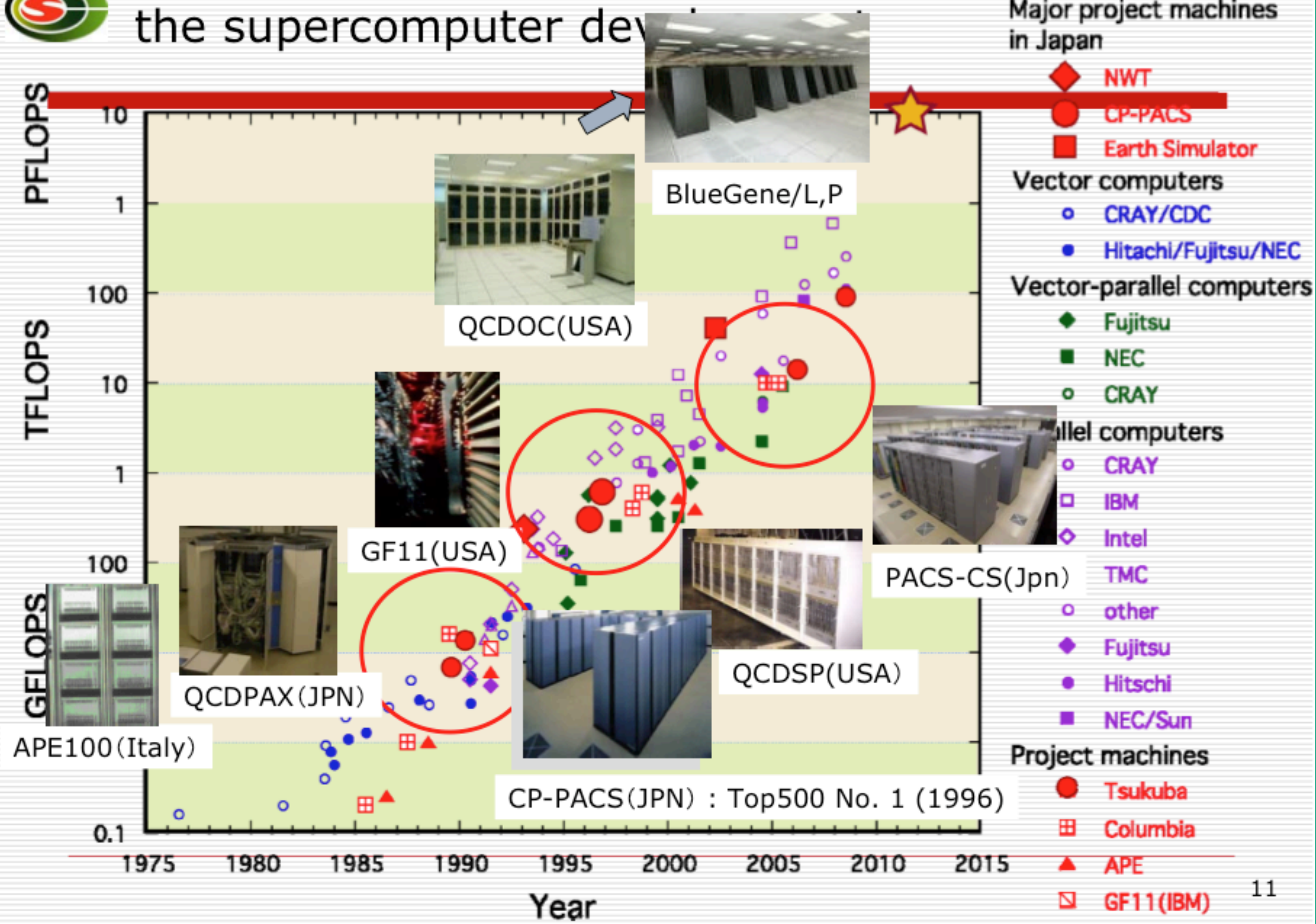
For heavy quark the extrapolation is suggested by the Heavy quark effective theory (HQET)

Precision Lattice QCD: from simulations to calculations

1) Better theoretical understanding
2) Better Algorithms
3) More powerful machines



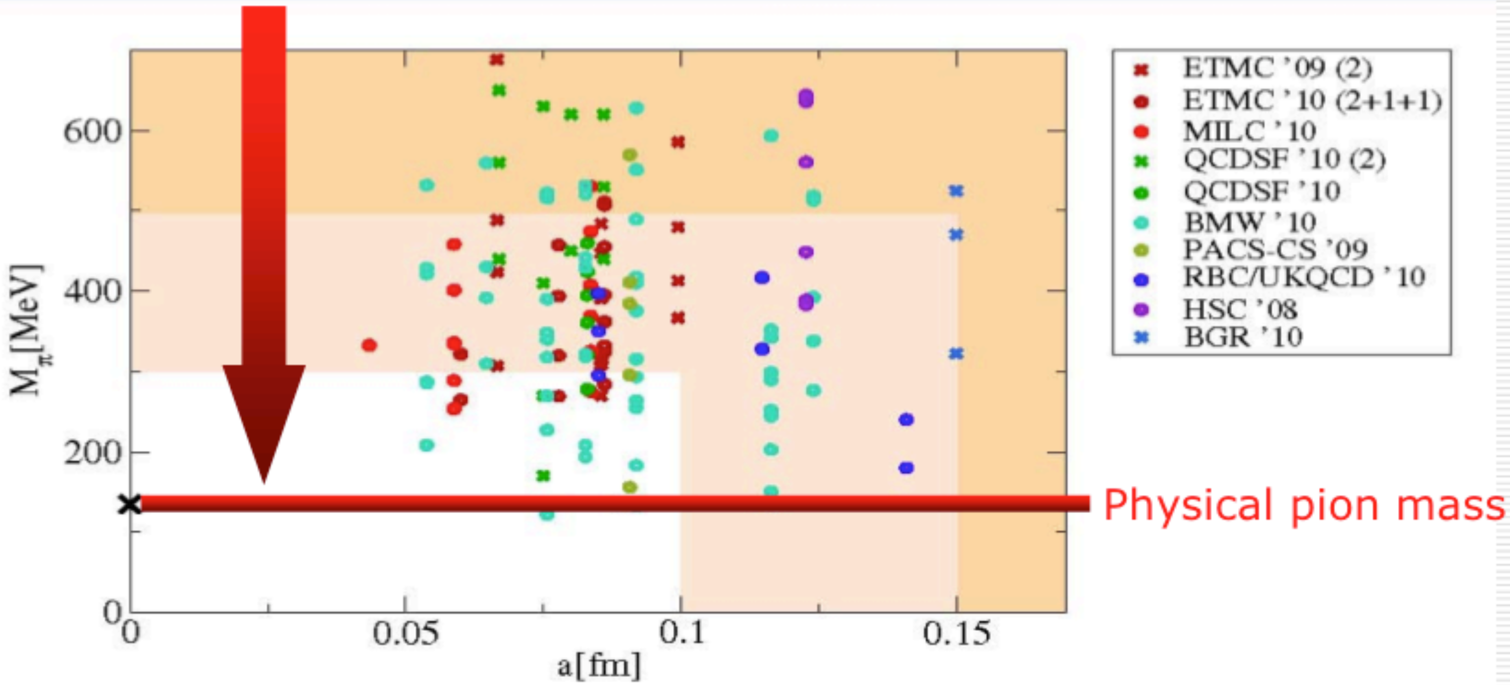
Impact of lattice QCD machines on the supercomputer dev



Status this year:
pion mass vs lattice spacing

Continuum Landscape

From C. Hoelbling's review at Lattice 2010



Quality Criteria

FLAG: Flavianet Lattice Averaging Group

FlaviA net

A. Vladikas

Quality Criteria

- chiral extrapolation:
  - ★ $M_{\pi,min} < 250$ MeV
  - ● 250 MeV $\leq M_{\pi,min} \leq$ 400 MeV
  - ■ 400 MeV $\leq M_{\pi,min}$

  NB: at least 3 points requested (otherwise there is a "special mention")
- continuum extrapolation:
  - ★ at least 3 lattice spacings, at least two below 0.1 fm
  - ● 2 or more lattice spacings, at least one below 0.1 fm
  - ■ otherwise
- finite volume effects:
  - ★ $[M_{\pi} L]_{min} > 4$ or at least 3 volumes
  - ● $[M_{\pi} L]_{min} > 3$ and at least 2 volumes
  - ■ otherwise, and in any case if $L < 2$ fm

  NB: p-regime
- renormalization (where applicable):
  - ★ non perturbative
  - ● 2-loop perturbation theory
  - ■ otherwise
- renormalization group running (where applicable):
  - ★ non perturbative
  - ● otherwise

Only:
Decay constants
KL3 Form Factors
BK for Neutral Kaon Mixing

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\ (36) \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47)$$

form factor @ zero momentum transfer $K^0 \rightarrow \pi^- \nu l^+$

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\ (36)\ \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47)$$
$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$
- 3 expressions, 4 unknowns; need one more input

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\ (36)\ \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47)$$
  $$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$
- 3 expressions, 4 unknowns; need one more input

Form factor, decay constants and unitarity

$$f_+(0) = 0.964\ (3)\ (4) \qquad (N_f = 2+1)$$
$$f_+(0) = 0.956\ (6)\ (6) \qquad (N_f = 2)$$

Precision at the per mille level !!

most systematics OK

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	2+1	A	●	★	■	0.9644(33)(34)(14)
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	2	A	■	★	■	0.968(9)(6)
JLQCD 05	2	C	■	★	■	0.967(6)

Table 1: Colour code for the data on $f_+(0)$.

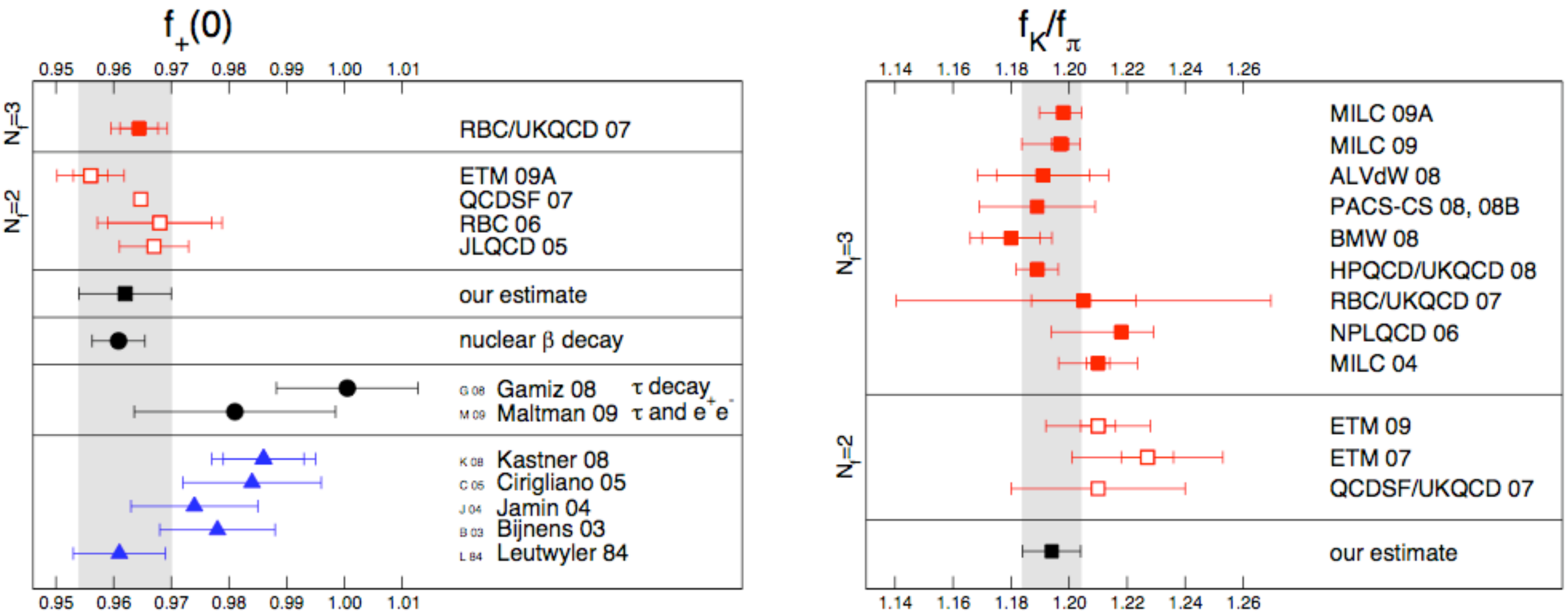
$$f_K/f_\pi = 1.190\ (2)\ (10) \qquad (N_f = 2+1)$$
$$f_K/f_\pi = 1.210\ (6)\ (17) \qquad (N_f = 2)$$

most systematics OK

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	f_K/f_π
MILC 09A	2+1	C	★	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	P	★	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	2+1	C	★	●	●	1.191(16)(17)
PACS-CS 08, 08B	2+1	A	★	■	■	1.189(20)
BMW 08	2+1	C	★	★	★	1.18(1)(1)
HPQCD/UKQCD 08	2+1	A	★	●	★	1.189(2)(7)
RBC/UKQCD 08	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	★	●	1.21(3)

Table 1: Colour code for the data on f_K/f_π

Form factor, decay constants and unitarity



- lattice agrees with nuclear β decay
- disagrees with semi-inclusive τ decay
- "our estimate" explained later
- from χPT:

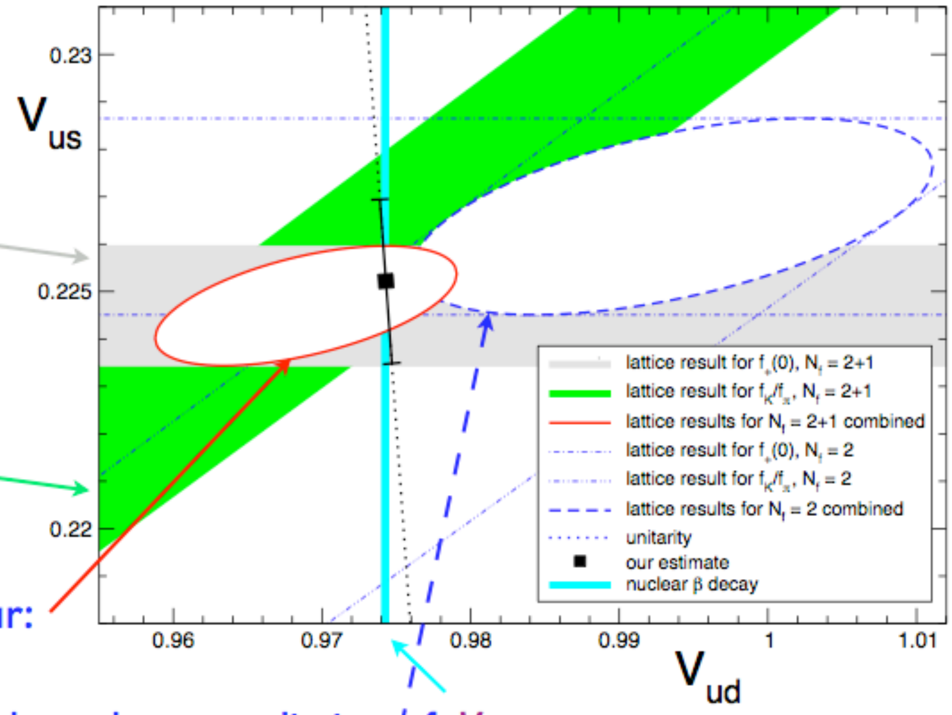
$$\Delta f \;\equiv\; f_+(0) - 1 - f_2 \;=\; f_+(0) - 0.977$$

- lattice suggests $\Delta f < 0$
- results from various model estimates vary; Δf sign unclear

Form factor, decay constants and unitarity

- use: $|V_{us}|\ f_+(0) \ = \ 0.21661\ (47)$
- $N_f = 3$ result of $f_+(0)$ gives:
- use: $\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| \ = \ 0.27599\ (59)$
- $N_f = 3$ result of f_K/f_π gives:

treating these two results as independent measurements gives the 68% likelihood contour:



- $N_f = 3$ lattice data consistent with nuclear beta decay prediction of V_{ud}:
- $N_f = 2$ lattice data consistent with $N_f = 3$ data within errors (just!!):

note the scale of the errors: this is really precision physics.

Unquenched calculations, nf=2 at smaller quark masses and more accurate continuum limit.

Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

- from Kaon decays we have:

$$|V_{us}| \; f_+(0) \; = \; 0.21661 \; (47) \qquad \left| \frac{V_{us} f_K}{V_{ud} f_\pi} \right| \; = \; 0.27599 \; (59)$$

- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$
- take $|V_{ub}|$ from experiment; the unitarity constraint is well satisfied:

$$|V_{ud}|^2 \; + \; |V_{us}|^2 \; + \; |V_{ub}|^2 \; = \; 0.989 \; (20) \qquad N_f = 2 + 1$$

- $N_f = 2$ 1.038(35) - OKish

- now use V_{ud} from β decays and $f_+(0)$ from $N_f = 3$ lattice:

$$|V_{ud}|^2 \; + \; |V_{us}|^2 \; + \; |V_{ub}|^2 \; = \; 0.9997 \; (7)$$

- $N_f = 2$ 1.0005(10) - OK

- now use V_{ud} from β decays and f_K / f_π from $N_f = 3$ lattice:

- $N_f = 2$ 0.9986(16) - OK

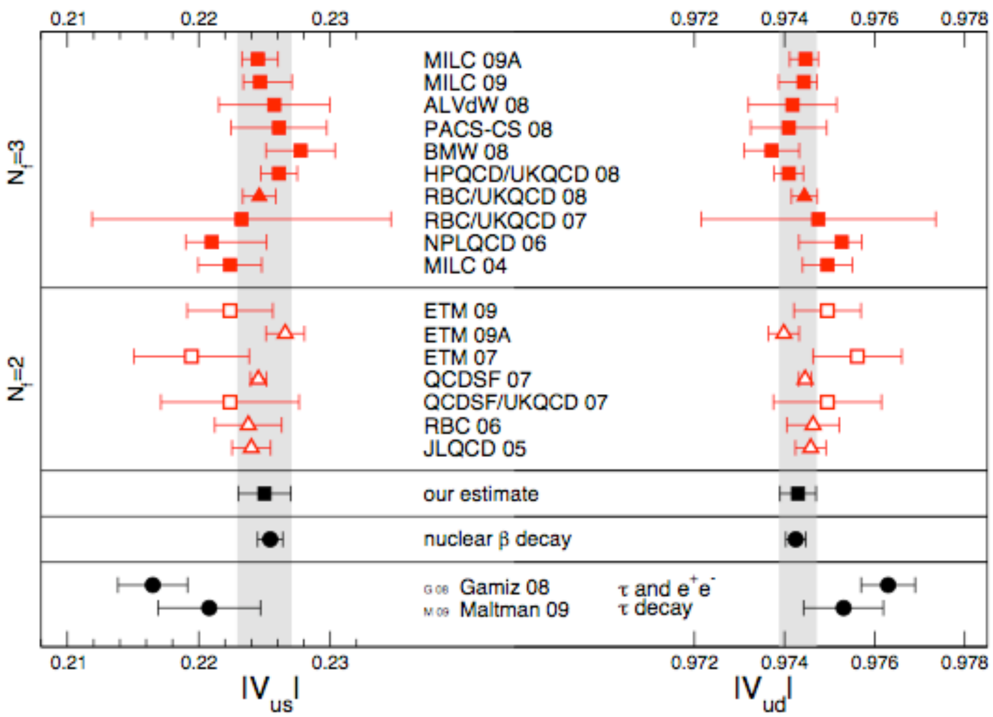
$$|V_{ud}|^2 \; + \; |V_{us}|^2 \; + \; |V_{ub}|^2 \; = \; 1.0002 \; (10)$$

Form factor, decay constants and unitarity

- Analysis based on Standard Model:

	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$	$f_+(0)$	f_K/f_π
$N_f = 2+1$	0.2251(11)	0.97433(24)	0.9626(43)	1.1944(61)
$N_f = 2$	0.2253(17)	0.97428(40)	0.9608(73)	1.1934(98)
our estimate	0.225(2)	0.9743(4)	0.962(8)	1.194(10)

Table 1: Final results for the analysis of the lattice data within the Standard Model

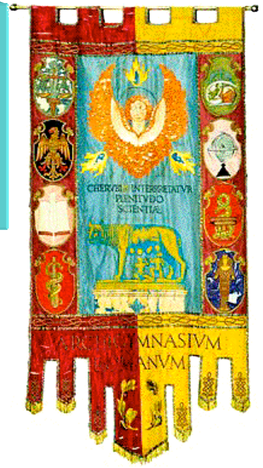


- combine data from direct f_K/f_π measurements with f_K/f_π results obtained from direct $f_+(0)$ measurements, to get **best f_K/f_π result** at a given N_f

- vice versus get **best f_K/f_π result**

- extremely close agreement between N_f=2 and N_f=2+1 results; take biggest uncertainty into account to obtain "our estimate"

Heavy-Light Semileptonic Decays



D -> K,K* DECAYS PROBE LATTICE (or model) RESULTS BY COMPARISON WITH EXPERIMENTAL DATA:

$\Gamma(D \to K) = \text{known constant } |V_{cs}|^2 \, |A|^2$

Also $\Gamma(D \to K^*)_L / \Gamma(D \to K)_T$

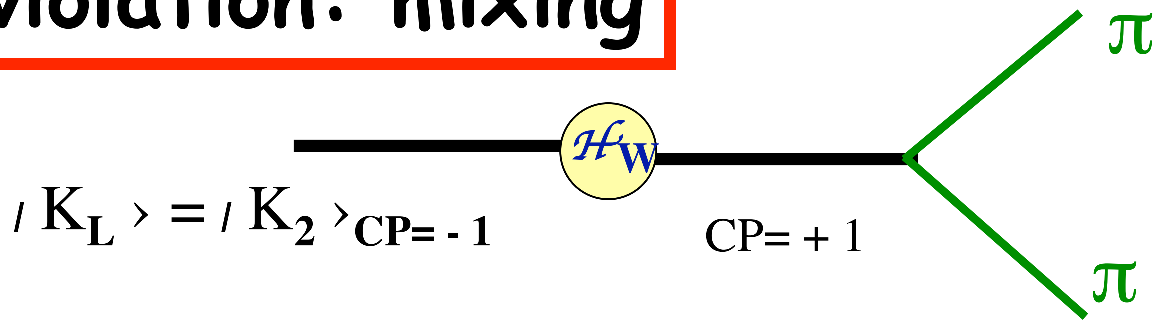
$$f_+^{D\to\pi}(0) = 0.64(3)(6) \quad f_+^{D\to K}(0) = 0.73(3)(7) \quad \frac{f_+^{D\to\pi}(0)}{f_+^{D\to K}(0)} = 0.87(3)(9)$$ *theory*

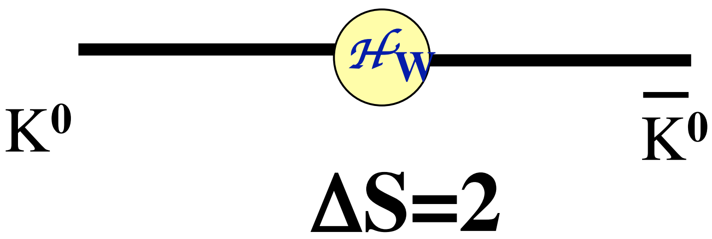
$$f_+^{D\to\pi}(0) = 0.73(15) \quad f_+^{D\to K}(0) = 0.78(5) \quad \frac{f_+^{D\to\pi}(0)}{f_+^{D\to K}(0)} = 0.86(9)$$ *experiment hep-ex/0406028*

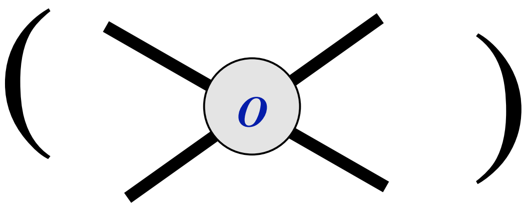
or provide and independent determination of the CKM matrix elements

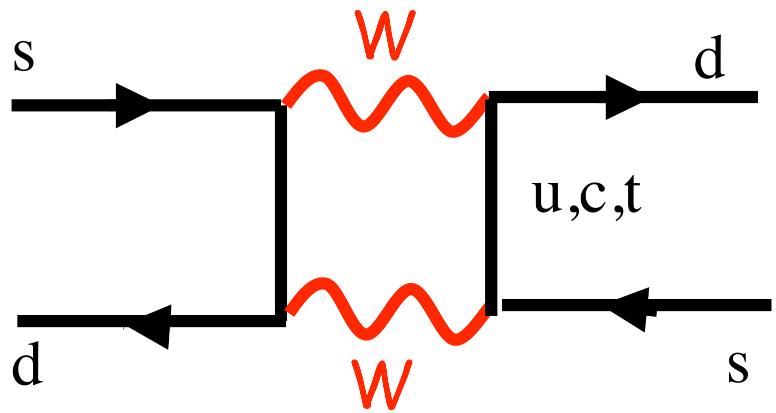
$$|V_{cd}| = 0.239(10)(24)(20) \quad |V_{cs}| = 0.969(39)(94)(24)$$

Indirect CP violation: mixing







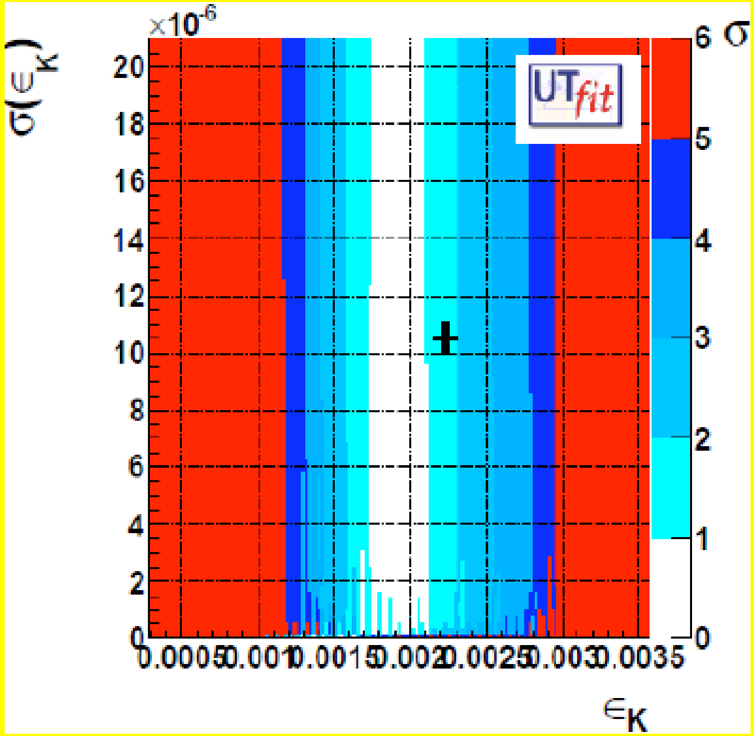


Box diagrams:
They are also responsible for B^0 - $\overline{B}^0$ mixing
$\Delta m_{d,s}$

Complex ΔS=2 effective coupling

Progresses in the long distance calculation? See N. Christ at Lattice 2010

ε_K



Three "news" ingredients

1) Buras&Guadagnoli BG&Isidori corrections

$$\varepsilon_K = \sin\phi_\varepsilon e^{i\phi_\varepsilon}\left[\frac{\mathrm{Im}\, M_{12}^{(6)}}{\Delta m_K} + \beta_\xi\right]$$

→ Decrease the SM prediction by 6%

2) Improved value for BK

→ BK=0.731±0.07±0.35

3) Brod&Gorbhan charm-top contribution at NNLO

→ enhancement of 3%

(not included yet)

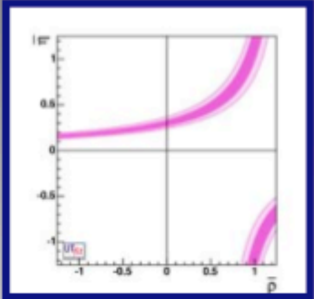
Lattice '96	$\hat{B}_K = 0.90 \pm 0.03 \pm 0.15$
Lattice '00	$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$
Lattice '05	$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$
Lattice '08	$\hat{B}_K = 0.723 \pm 0.037$

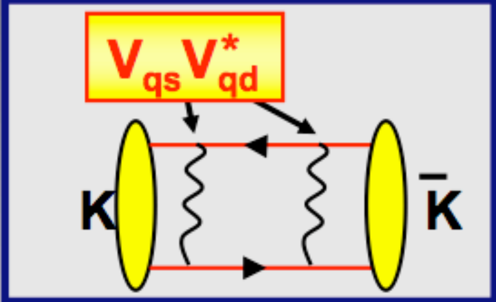
$\hat{B}_K = 0.731(7)(35)$

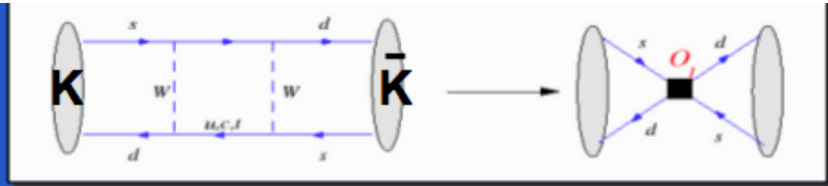
-1.7σ devation

K^0-$\bar{K}^0$ mixing: B_K

V. Lubicz SuperB meeting nov. 2009







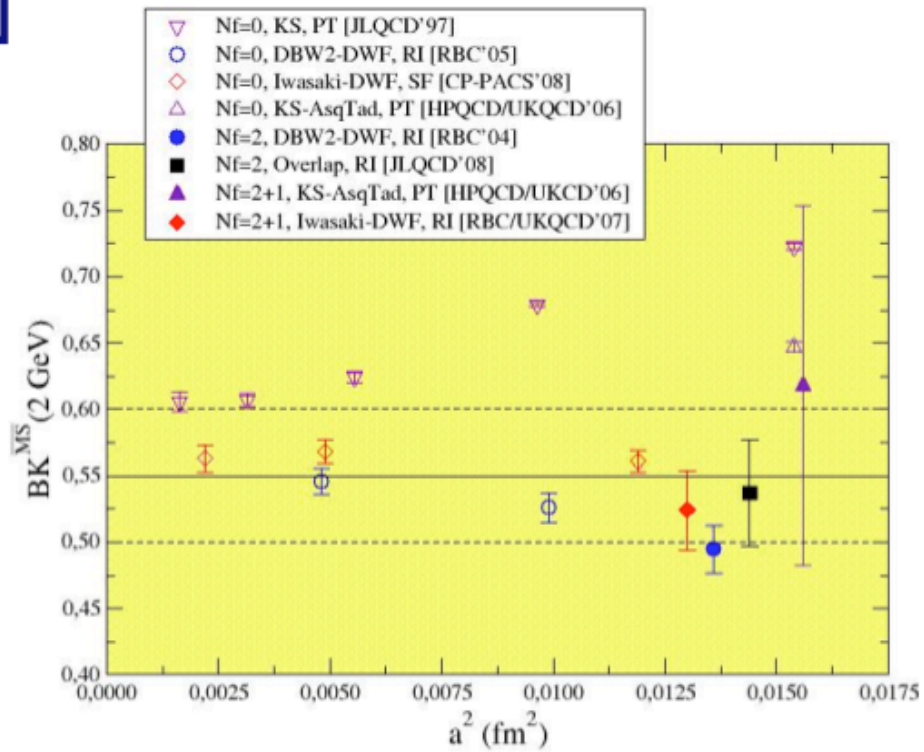
$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

$\hat{B}_K = 0.90 \pm 0.03 \pm 0.15$
S.Sharpe@Latt'96 — 17%

$\hat{B}_K = 0.86 \pm 0.05 \pm 0.14$
L.Lellouch@Latt'00 — 17%

$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$
C.Dawson@Latt'05 — 11%

$\hat{B}_K = 0.723 \pm 0.037$
L.Lellouch@Latt'08 — 5%

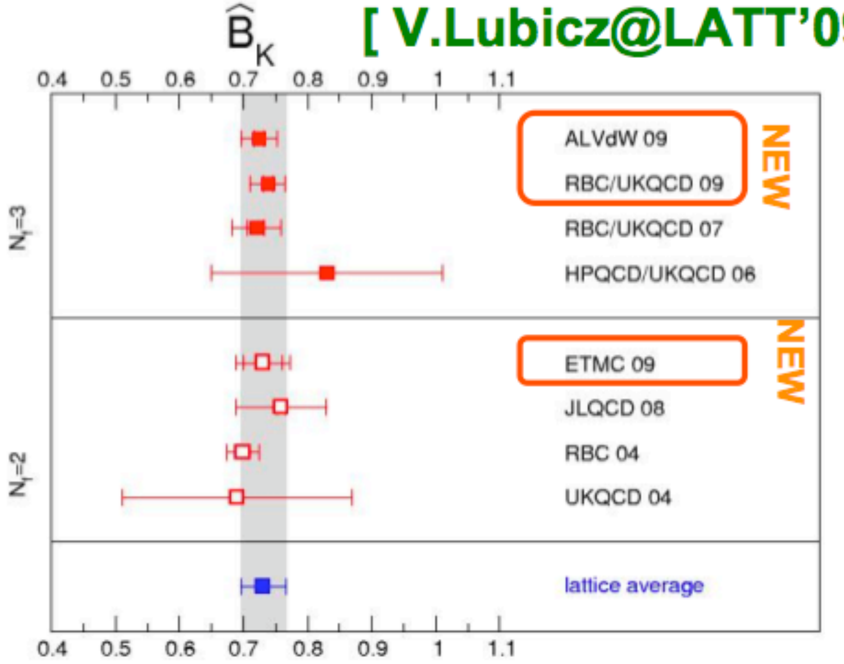


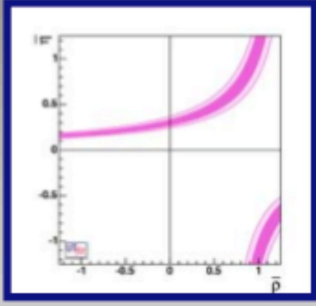
[VL, C.Tarantino 0807.4605]

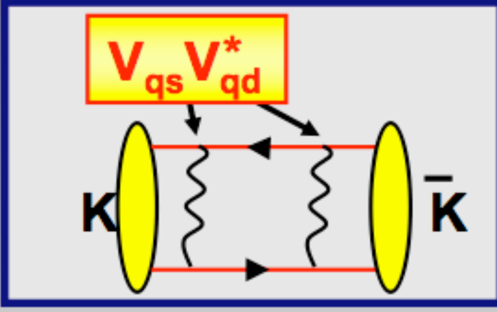
All unquenched calculations until last year at fixed (and rather large) lattice spacing

$K^0-\overline{K}^0$ mixing: B_K

[V.Lubicz@LATT'09]







$\hat{B}_K = 0.731\,(7)\,(35)$ **5%**

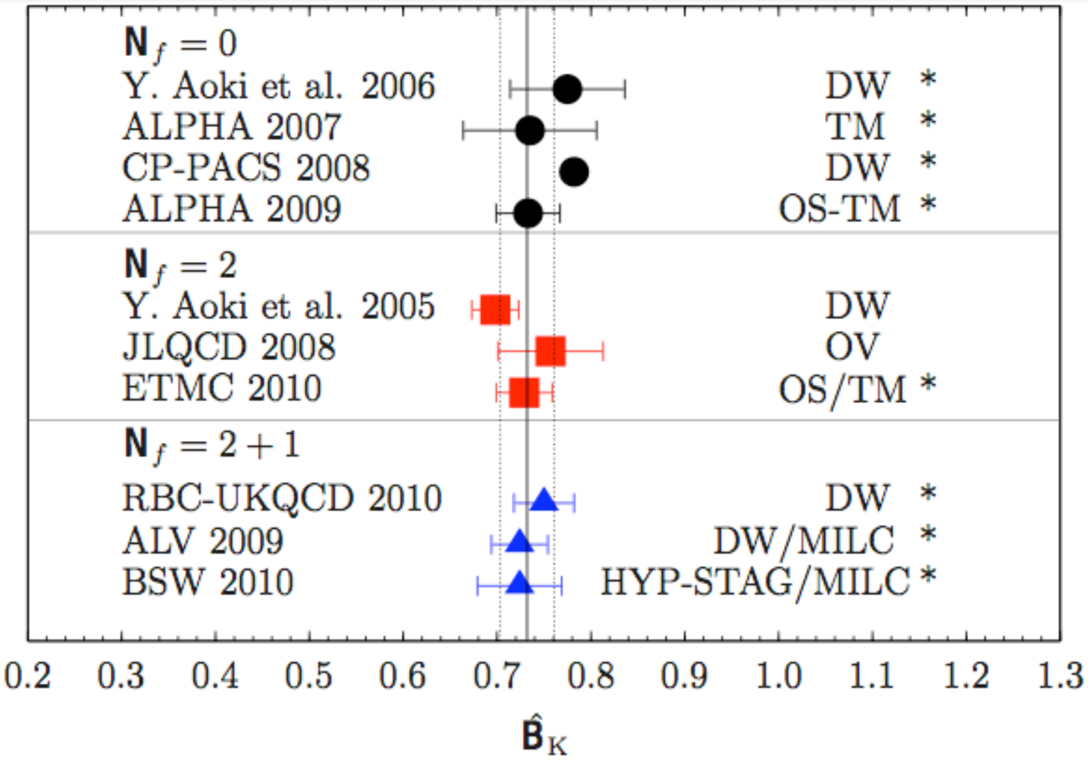
From the UT fit, assuming the validity of the Standard Model

$\hat{B}_K = 0.81\,(8)$ UTfit

(with $K\varepsilon \approx 0.92$, A.Buras, D.Guadagnoli, 0805.3887)

Predicted error with 6 TFlops 5%

Error in 2006: 11%

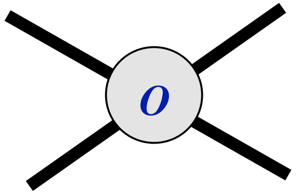


- $*$ $\longrightarrow$ result already in the **CL**.
- Average: $\hat{B}_K^{(N_f=2)}$(ETMC) $= 0.729(30)$; $\hat{B}_K^{(N_f=2+1)} = 0.732(06)(28)$
- No dependence on the strange quark (with the present precision)!
- Difference of less than $\sim 2\,\sigma$ with the most precise quenched result.

B^0 - B^0 mixing

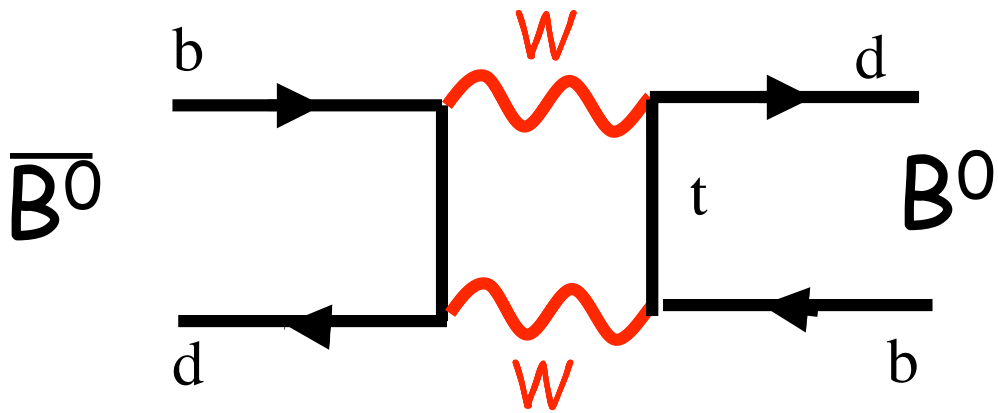
$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

$$\mathcal{H}_{eff}^{\Delta B=2} =$$



$$\propto \quad (\,\overline{\mathbf{d}}\,\gamma_\mu\,(\mathbf{1} - \gamma_5)\,\mathbf{b}\,)^2$$

ΔB=2 Transitions



Hadronic matrix element

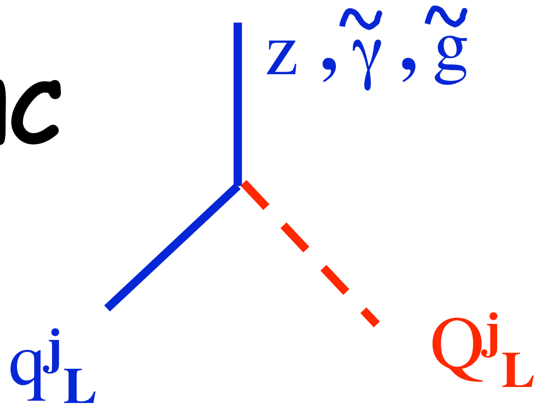
CKM

$$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16\,\pi^2}\, A^2 \lambda^6\, F_{tt}\left(\frac{m_t^2}{M_W^2}\right) \langle\, O \,\rangle$$

In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case We may either

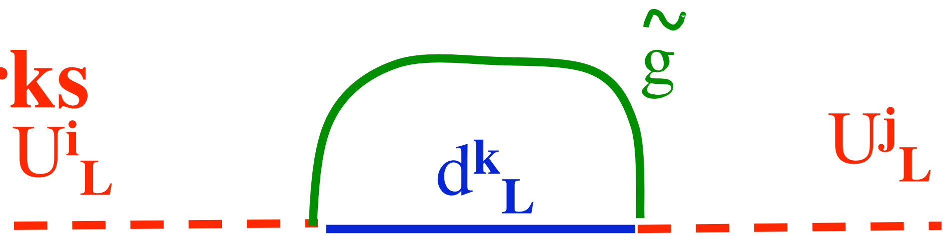
Diagonalize the SMM

FCNC

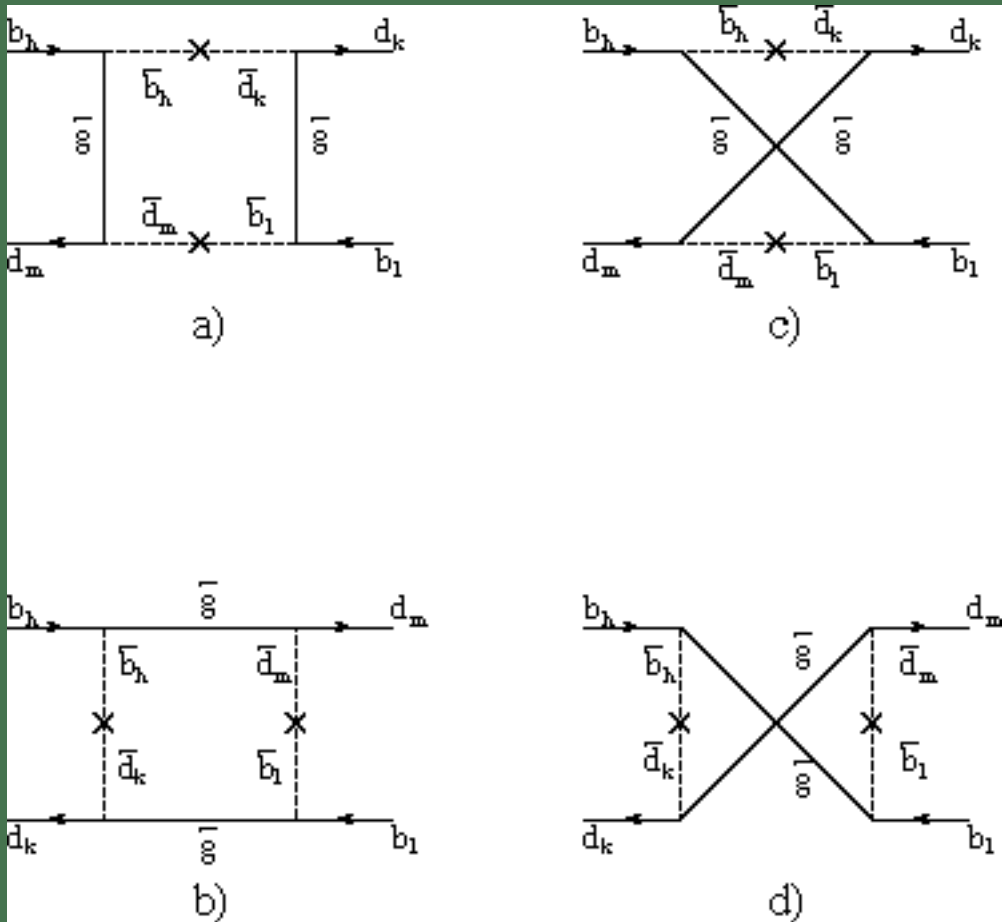


or Rotate by the same matrices the SUSY partners of the u- and d- like quarks

$(Q^j_L)' = U^{ij}_L \, Q^j_L$



In the latter case the Squark Mass Matrix is not diagonal



$$(\tilde{m}^2_Q)_{ij} = \tilde{m}^2_{average}\, 1_{ij} + \Delta\tilde{m}_{ij}^2 \qquad \delta_{ij} = \Delta\tilde{m}_{ij}^2 / \tilde{m}^2_{average}$$

New local four-fermion operators are generated

$Q_1 = (\bar{b}_L^A \gamma_\mu d_L^A)(\bar{b}_L^B \gamma_\mu d_L^B)$ SM

$Q_2 = (\bar{b}_R^A d_L^A)(\bar{b}_R^B d_L^B)$

$Q_3 = (\bar{b}_R^A d_L^B)(\bar{b}_R^B d_L^A)$

$Q_4 = (\bar{b}_R^A d_L^A)(\bar{b}_L^B d_R^B)$

$Q_5 = (\bar{b}_R^A d_L^B)(\bar{b}_L^B d_R^A)$

+ those obtained by L $\leftrightarrow$ R

Similarly for the s quark e.g.

$(\bar{s}_R^A d_L^A)(s_R^B d_L^B)$

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_1(\mu) ,$$

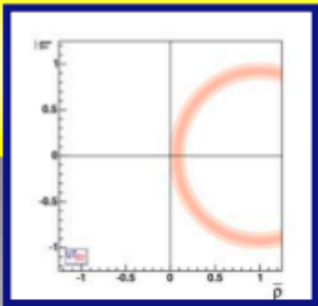
$$\langle \bar{K}^0 | O_2(\mu) | K^0 \rangle = -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_2(\mu) ,$$

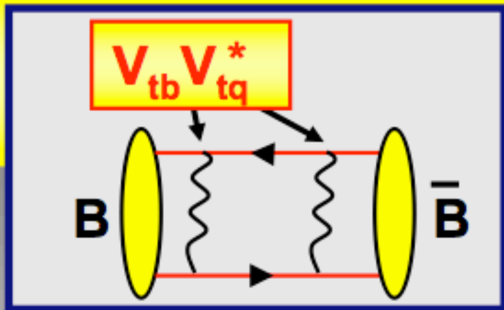
$$\langle \bar{K}^0 | O_3(\mu) | K^0 \rangle = \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_3(\mu) ,$$

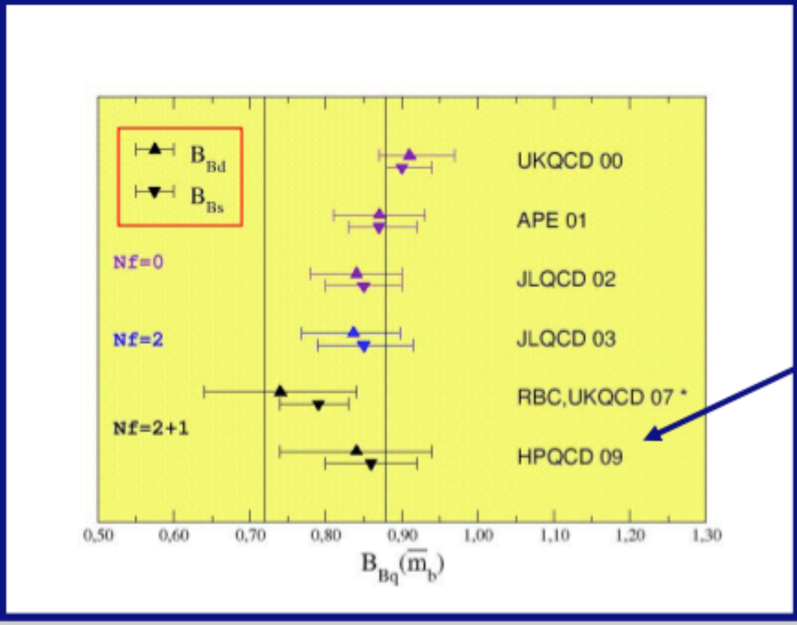
$$\langle \bar{K}^0 | O_4(\mu) | K^0 \rangle = 2 \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_4(\mu) ,$$

$$\langle \bar{K}^0 | O_5(\mu) | K^0 \rangle = \frac{2}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_5(\mu) ,$$

B-$\bar{B}$ mixing: $B_{Bd/s}$







Only one modern calculation HPQCD [0902.1815]

$$\hat{B}_{Bd} = 1.26 \pm 0.11$$

$$\hat{B}_{Bs} = 1.33 \pm 0.06$$

Combining with fB and fBs:

Error in 2006: 13%	Error in 2006: 5%
$f_{Bs}\sqrt{\hat{B}_{Bs}} = 275 \pm 13$ MeV (5%)	$\xi = 1.243 \pm 0.028$ (2%)
Predicted error with 6 TFlops: 4%	Predicted error with 6 TFlops: 3%

exps vs predictions

$f_{Bs}\sqrt{B_{Bs}} = 270 \pm 30$ MeV
(275 ± 13 MeV new)
lattice

$f_{Bs}\sqrt{B_{Bs}}=265 \pm 4$ MeV
UTA 2% ERROR !!

$\xi = 1.25 \pm 0.06$ UTA

$\xi = 1.21 \pm 0.04$
lattice

$B_K = 0.75 \pm 0.07$

$B_K = 0.75 \pm 0.07$

V. Lubicz and
C. Tarantino
0807.4605

SPECTACULAR AGREEMENT
(EVEN WITH QUENCHED
LATTICE QCD)

CONCLUSIONS I

For many quantities (quark masses, decay constants, form factors, moments of structure functions, etc.) Lattice QCD is entering the stage of precision calculations, with errors at the level of a few percent and full control of unquenching, discretization, chiral extrapolation and finite volume effects.

ma per seguir virtute e canoscenza

SISSA

CONCLUSIONS II

For non-leptonic decays (particle widths) theoretical and numerical progresses have been made, substantial improvement in the calculation of DI=3/2 amplitudes

It remains open the problem of the decays above the elastic threshold

e.g. $B \rightarrow \pi\pi$

CONCLUSIONS III

da una lettera al Presidente del 22/10/2009

Rimane invece incerto, e per noi preoccupante, il futuro delle macchine dedicate, di cui abbiamo più volte discusso. Credo sia venuto il momento di prendere delle decisioni e di far seguire a queste delle azioni tempestive, pena la perdita di competitività in un settore dove la fis teorica italiana ha avuto da sempre un ru da protagonista.