



Overview of the Muon $g-2$ Experiment Results

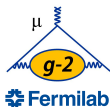
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on behalf of the Muon $g-2$ Collaboration



UNIVERSITÀ DI PISA



Dark Matter Studies in Accelerator Physics
3rd DMNet international symposium

26-28 September 2023
Palazzo Moroni, Padua, Italy



Introduction: the muon anomaly

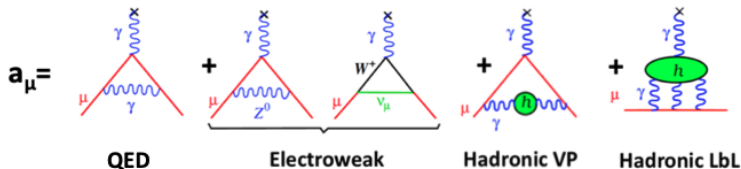
- **Muon:** elementary particle with spin-1/2 and magnetic moment proportional to spin through the **g-factor**:

$$\vec{\mu} = g \frac{q}{2m_{\mu}} \vec{S}$$

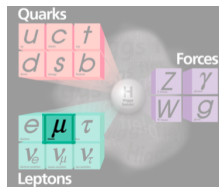
- At first order (Dirac theory for $s = 1/2$ particles) $g = 2$ but with higher order corrections $g > 2$:

$$\underbrace{g_{\mu} = 2 (1 + a_{\mu})}_{\text{Dirac}} \Rightarrow \boxed{a_{\mu} = \frac{g - 2}{2}} \quad \text{muon anomaly}$$

→ Theoretically calculated using the Standard Model (SM) :

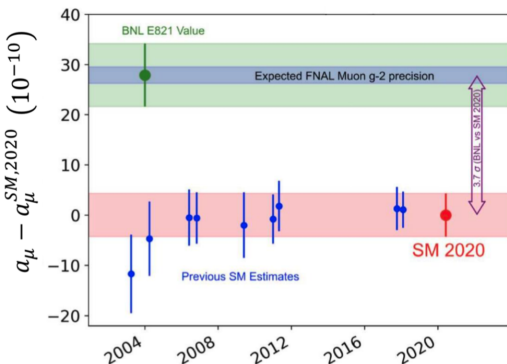


→ Comparison to measurement allows for a **precise test of the SM and to look for new physics**



Experimental measurement vs. SM calculation in 2020

- Long-standing $> 3\sigma$ discrepancy



- **E821 (BNL) experimental value:**

$$a_{\mu}^{E821,BNL} = 116592080(63) \times 10^{-11}$$

[Phys. Rev. D, 73 (2006) 072003]

- **SM value** re-evaluated in 2020 by Muon $g-2$ Theory Initiative:

$$a_{\mu}^{SM,2020} = 116591810(43) \times 10^{-11}$$

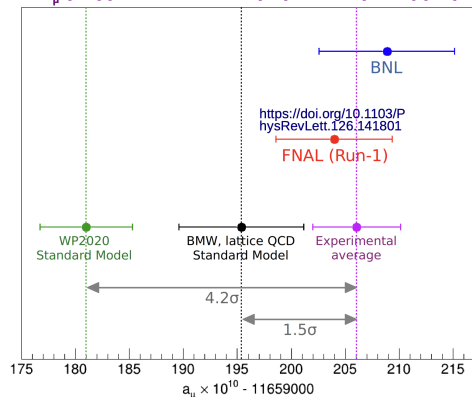
[Phys. Rept. 887, 1 (2020)]

- In the meantime: **FNAL Exp.** was built and is collecting data since 2018 aiming to **improve uncertainty** with 140 ppb goal

Experimental measurement vs. SM calculation (2021)

- In April 2021 were published:

$$a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11} \text{ (0.35 ppm)}$$



- a new measurement from **FNAL Muon g - 2 Exp. Run-1 data** that confirmed result from BNL:

$$a_\mu(\text{FNAL}) = 116592040(54) \cdot 10^{-11} \text{ (460 ppb)}$$

$$a_\mu(\text{BNL}) = 116592089(63) \cdot 10^{-11} \text{ (540 ppb)}$$

$$a_\mu(\text{Exp}) = 116592061(41) \cdot 10^{-11} \text{ (350 ppb)}$$

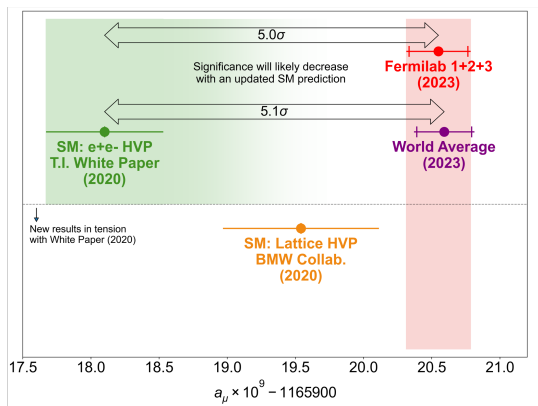
[Phys. Rev. Lett. **126**, no.14, 141801 (2021)]

- a new theoretical calculation $a_\mu(\text{BMW, HVP - LO})$ based on Lattice QCD in tension with $a_\mu(\text{WP, HVP - LO})$ calculation based on e^+e^- data [Nature **593** (2021) 51-55]

- In this talk:
review of the FNAL Run-1 measurement and...

New Experimental Measurement vs. SM calculation

- Adding the **new FNAL Muon g-2 result from Run-2/3 data**:



- **Dark Matter (DM)** may be responsible for the potential discrepancy between experimental measurement and SM prediction
- In this talk: overview of the **first-ever direct DM search with muons in a storage ring** using the data collected by the Muon g-2 experiment

Experimental technique

1. Inject polarized muons into a magnetic storage ring
2. Muons circulate around the ring at the cyclotron frequency:

$$\vec{\omega}_C = \frac{q}{\gamma m_\mu} \vec{B}$$

3. Muon spin precession frequency (Larmor) is given by:

$$\vec{\omega}_S = \frac{q}{\gamma m_\mu} \vec{B} (1 + \gamma a_\mu)$$

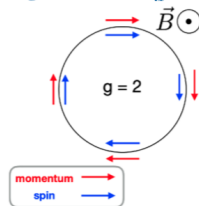
4. Muon anomaly is related to **anomalous precession frequency**:

$$\vec{\omega}_a \cong \vec{\omega}_S - \vec{\omega}_C \cong a_\mu \frac{q}{m_\mu} \vec{B}$$

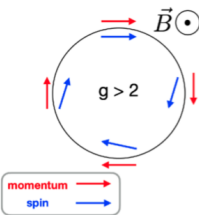
5. Measure B and ω_a to extract the anomaly



$$\text{If } g = 2 \Rightarrow \vec{\omega}_a = 0$$

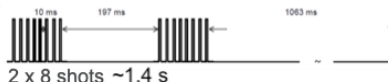


$$g \neq 2 \Rightarrow \vec{\omega}_a \cong a_\mu \frac{e}{m} \vec{B}$$



Production of the muon beam

- **Recycler Ring:** 8 GeV protons from Booster are divided in 4 bunches
- **Target Station:** p -bunches are collided with target and π^+ with 3.1 GeV/c ($\pm 10\%$) are collected
- **Beam Transport and Delivery Ring:** magnetic lenses select μ^+ from $\pi^+ \rightarrow \mu^+ \nu_\mu$ then μ^+ are separated from p and π^+ in circular ring
- **Muon Campus:** polarized μ^+ are ready to be injected into the storage ring

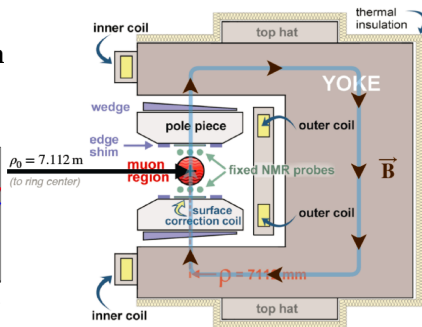
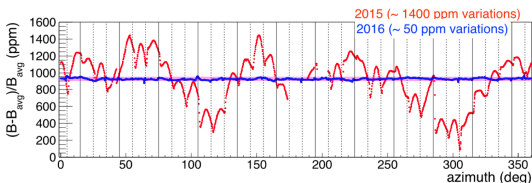
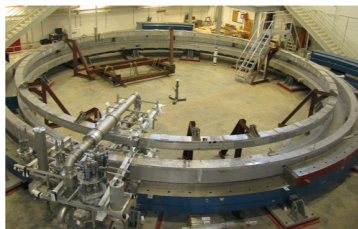


The storage ring journey: from BNL to FNAL in Summer 2013



Storage ring magnet

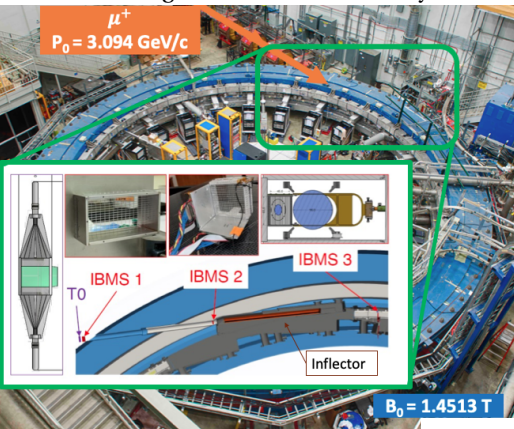
- Three superconducting coils provide 1.45 T vertical magnetic field
- Vacuum chambers surrounded by a cryosystem and C-shaped **yokes** to allow the decay positrons to reach the detectors.
- Achieved 50 ppm on field uniformity thanks to low-carbon steel **poles**, **edge shims**, **steel wedges**, **surface correction coil**



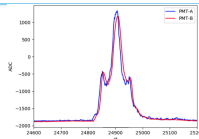
final field ~ 3 times more uniform than at BNL

Injection of the muons into the ring

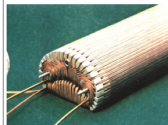
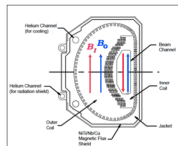
- Beam enters the ring through a 2.2 m-long 10 cm hole in the iron yoke



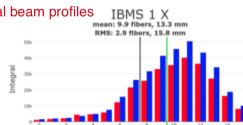
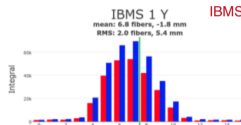
- **T0 Counter** (thin scintillator read out by PMTs) to measure **beam time profile**



- **Inflector magnet** provides nearly field free region for muons to enter the storage region

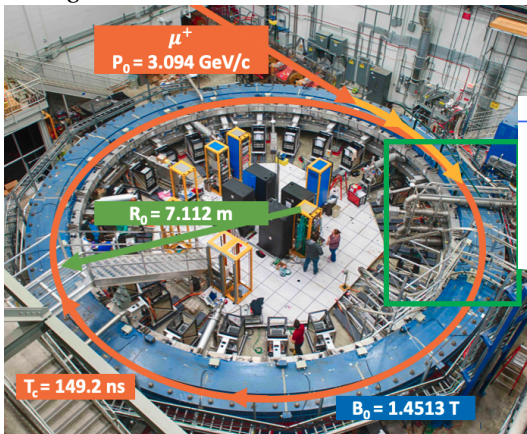


- **Inflector Beam Monitoring System** (scintillator fiber grids) to measure **beam spatial profile**



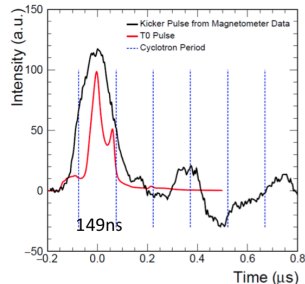
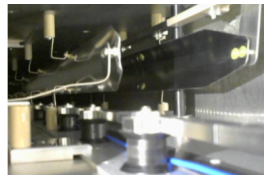
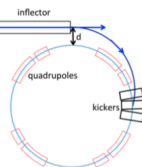
Muon storage

- Injected beam is 77 mm off from storage region center

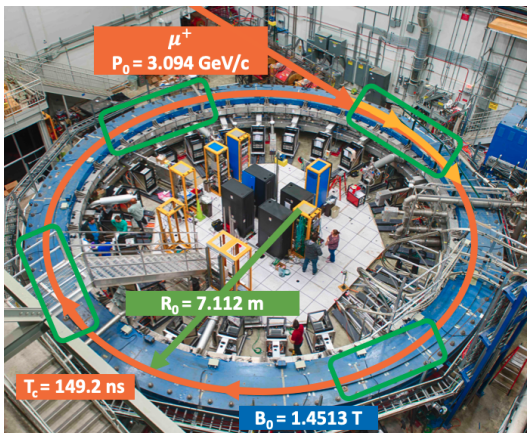


Kicker Magnets

- 3 pulsed magnets deflect beam $\sim 10 \text{ mrad}$ onto the closed storage orbit in less than 150 ns

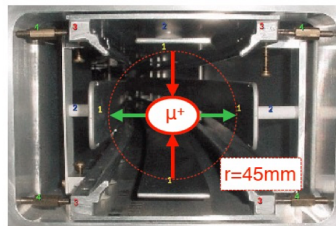


Vertical focusing



Electrostatic Quadrupoles

- 4 sets of quads provide vertical beam focusing

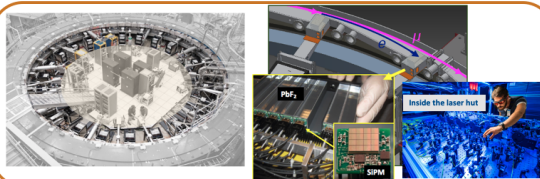


- E -field component cancels out (at first order) when muons at *magic momentum*:

$$\vec{\omega}_a \cong -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

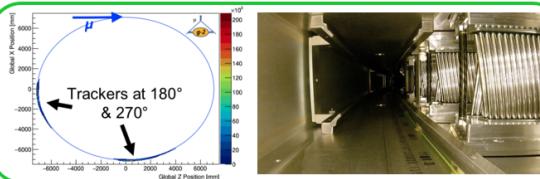
~ 0 if $\gamma = 29.3$ i.e., $p_\mu = 3.094 \text{ GeV}/c$

Detectors and field probes



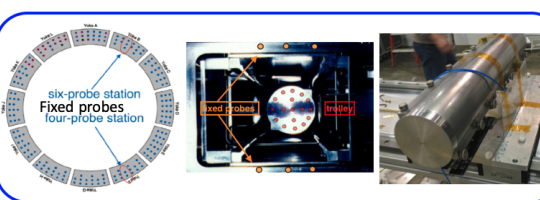
24 Calos around the ring

- Each made of 6×9 PbF₂ crystals read out by large-area SiPMs
- 1296 channels individually calibrated by 405nm-laser system



2 in-vacuum straw trackers

- Each with 8 modules consisting of 128 gas filled straws



2 types of field probes

- 378 fixed NMR probes above and below storage region
 - measure B-field 24/7
- Trolley with 17-probe NMR
 - 2D profile of B over the entire azimuth when beam is OFF

Final formula

Muon anomaly is determined with:

$$a_\mu = \underbrace{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}}_{\text{ratio of frequencies } (R_\mu) \text{ measured by us}} \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}_{\text{fundamental factors (combined uncertainty 25 ppb)}}$$

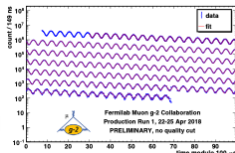
ratio of frequencies (R_μ)
measured by us

fundamental factors
(combined uncertainty 25 ppb):

ω_a : muon anomalous precession frequency

Extract from decay positron time spectra

$$N(t) = N_0 e^{-t/\tau_\mu} [1 + A \cos(\omega_a t + \phi)]$$



$\tilde{\omega}'_p(\mathbf{T}_r)$: magnetic field B in terms of (shielded) proton precession frequency (proton NMR $\hbar\omega_p = 2\mu_p B$) and weighted by the muon distribution

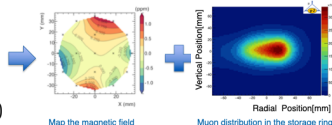
(shielded = measured in spherical water sample at $T_r = 34.7^\circ\text{C}$)

$\mu'_p(T_r)/\mu_e(H)$ from [Metrologia **13**, 179 (1977)]

$\mu_e(H)/\mu_e$ from [Rev. Mod. Phys. **88** 035009 (2016)]

m_μ/m_e from [Phys. Rev. Lett. **82**, 711 (1999)]

$g_e/2$ from [Phys. Rev. A **83** 052122 (2011)]



Master formula for Muon g-2 analysis

$$R_\mu = \frac{\overbrace{f_{clock} \cdot \omega_a^{meas}}^{\omega_a} \cdot \overbrace{(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}^{\text{beam dynamics corrections}}}{\underbrace{f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi)}_{\tilde{\omega}'_p(T_r)} \cdot \underbrace{(1 + B_k + B_q)}_{\text{field corrections}})}$$

f_{clock} : blinded clock
 ω_a^{meas} : measured precession frequency

f_{calib} : absolute magnetic field calibration
 $\omega'_p(x, y, \phi)$: field maps
 $M(x, y, \phi)$: muon beam distribution

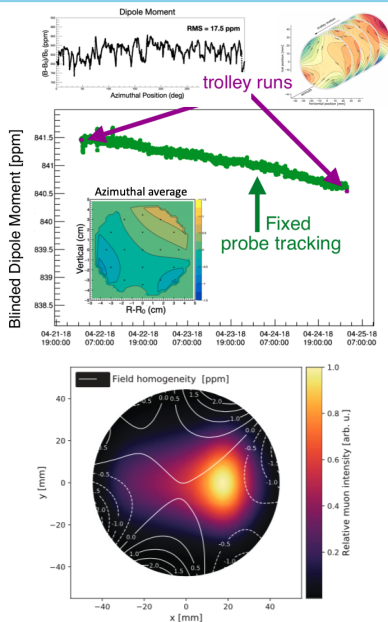
C_e : electric field correction
 C_p : pitch correction
 C_{ml} : muon loss correction
 C_{pa} : phase-acceptance correction
 C_{dd} : differential-decay correction

B_k : transient field from eddy current in kicker
 B_q : transient field from quad vibration

Measuring the magnetic field seen by the muons

$$R_{\mu} = \frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}{f_{calib} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)}$$

- ω_p' is proportional to the magnetic field and it is mapped every 3 days using 17 NMR probes on a trolley
- During data taking fixed NMR probes located above and below the storage region monitor the field
- Fixed probes to interpolate the field between trolley runs
- Field maps are weighted by beam distribution (extrapolated from the decay e^+ trajectory measured by the trackers and simulations)

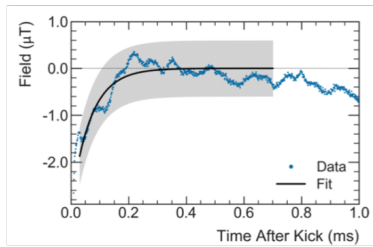
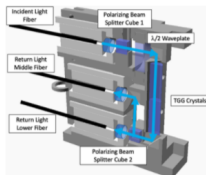


Magnetic field corrections

$$R\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}{f_{calib} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

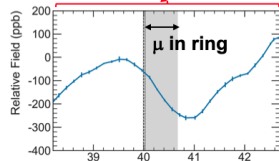
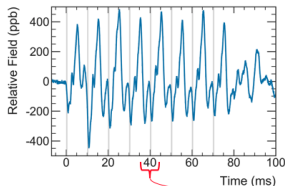
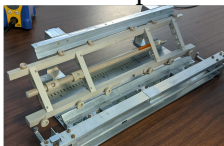
Kicker transient field

- due to eddy currents produced by kicker pulses
- measured using Faraday magnetometers



Quads transient field

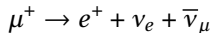
- due to mechanical vibrations from pulsing the quads
- mapped using special NMR probes



Measuring ω_a

$$R_\mu = \left(\frac{f_{\text{clock}} \cdot \omega_a^{\text{meas}} \cdot (1 + C_e + C_p + C_{mi} + C_{pa} + C_{dd})}{f_{\text{calib}} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

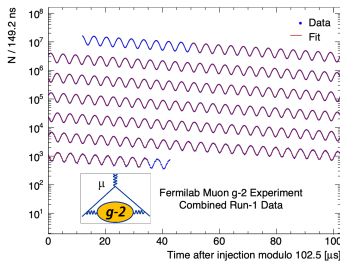
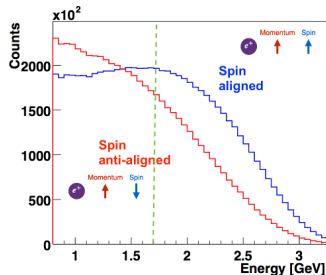
- Polarized muon decay:



- High energy e^+ are preferentially emitted in direction of μ^+ spin (parity violation of the weak decay)
- Energy spectrum modulates at the ω_a frequency
- Counting the number of e^+ with $E_{e^+} > E_{\text{threshold}}$ as a function of time (wobble plot) leads to ω_a :

$$N(t) = \underbrace{N_0}_{\text{normalization}} e^{-t/\tau} \left[1 + \underbrace{A \cos(\omega_a t + \varphi)}_{\text{g-2 asymmetry}} \right]$$

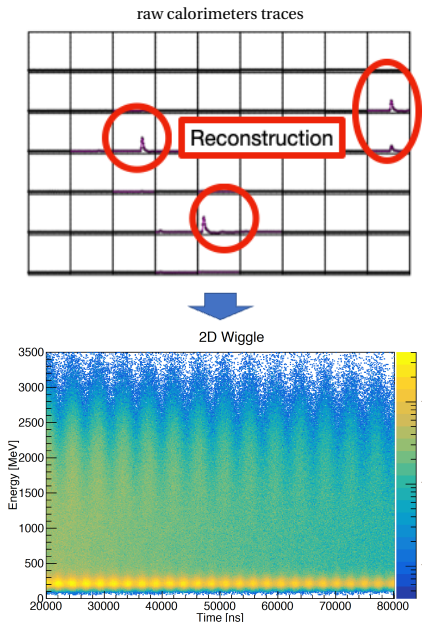
muon lab-frame lifetime
g-2 phase



E_{e^+} and t are measured by the calorimeters with a blinding factor applied to the digitization rate

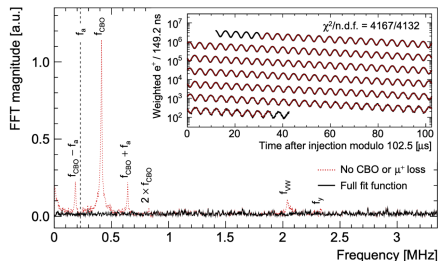
Wiggle plot

- Calorimeters data is reconstructed into energies and times
 - Two independent reconstruction routines
- Different analysis techniques used to reduce systematic errors :
 - **Threshold (T) Method**
 - only energy threshold applied to select positrons
 - **Asymmetry-Weighted (A) Method:**
 - positrons divided into energy bins and weighted by $g-2$ asymmetry
 - **Ratio (R) Method**
 - exponential decay due to muon lifetime is removed before fitting
 - **Integrated Charge (Q) Method:**
 - sum of raw calorimeter traces (unique method independent of reconstruction)



Fitting procedure

- Fit \rightarrow Residuals \rightarrow Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_y, \omega_{VW} \text{ vertical oscillations}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue = fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

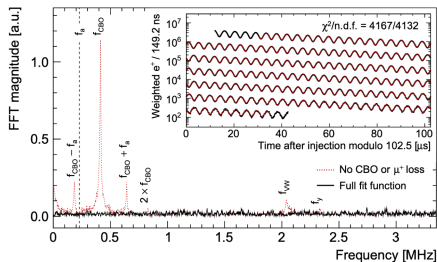
$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_0 t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

ω_y, ω_{VW} vertical oscillations

Red = free parameters

Blue = fixed parameters

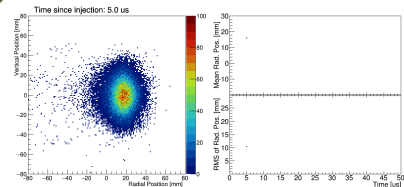
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t)} - 1$$

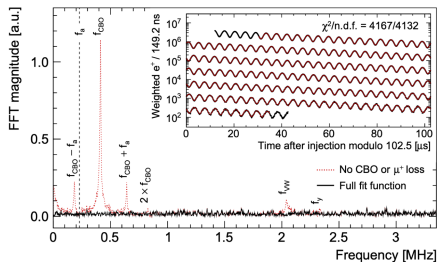
$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_0 t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

ω_y, ω_{VW} vertical oscillations

Red = free parameters

Blue = fixed parameters

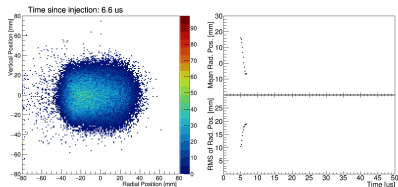
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t)} - 1$$

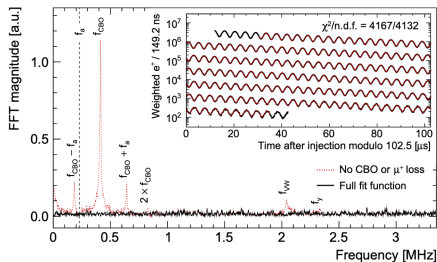
$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_B e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_B t + \phi_B \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_Y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

ω_Y, ω_{VW} vertical oscillations

$$N_Y(t) = 1 + A_Y \cos(\omega_Y t + \phi_Y) e^{-\frac{t}{\tau_Y}}$$

Red = free parameters

Blue = fixed parameters

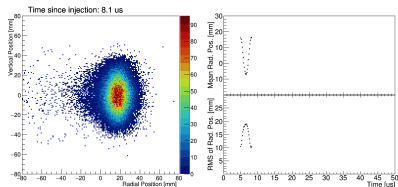
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_Y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

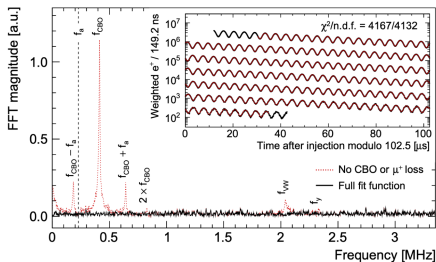
$$\omega_{VW}(t) = \omega_c - 2\omega_Y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_B e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_B t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_Y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_Y(t) = 1 + A_Y \cos(\omega_Y(t) + \phi_Y) e^{-\frac{t}{\tau_Y}}$$

ω_Y, ω_{VW} vertical oscillations

Red = free parameters

Blue = fixed parameters

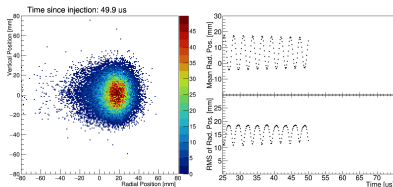
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_Y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

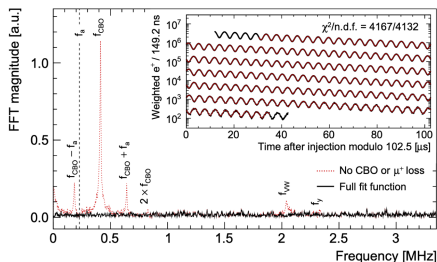
$$\omega_{VW}(t) = \omega_c - 2\omega_Y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit \rightarrow Residuals \rightarrow Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



Additional term to account for muons that hit the collimators and are lost:

$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_B t + \phi) \cdot \phi_{BO}(t)) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_y, \omega_{VW} \text{ vertical oscillations}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

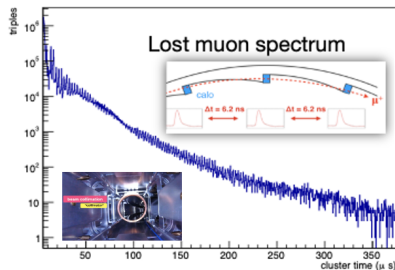
Blue = fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

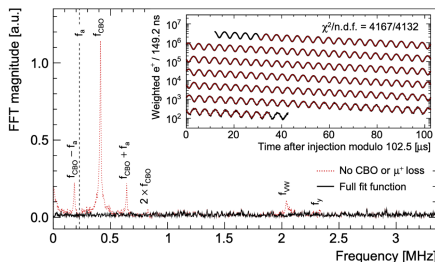
$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_Y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_Y, \omega_{VW} \text{ vertical oscillation}$$

$$N_Y(t) = 1 + A_Y \cos(\omega_Y(t) + \phi_Y) e^{-\frac{t}{\tau_Y}}$$

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_Y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_Y(t)$$

Red = free parameters
Blue = fixed parameters

+ beam dynamics corrections:

$$\omega_a = \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa}) + C_{dd} \text{ (in Run-2/3)}$$

Diagram showing corrections: Electric Field (green box), Pitch (blue box), Muon Loss (purple box), Phase Acceptance (red box). Arrows point from these boxes to the corresponding terms in the equation above.

Correction for Effect on Spin Precession

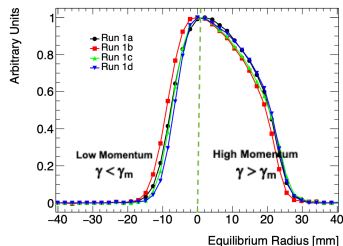
$$R_\mu = \left(\frac{f_{\text{clock}} \cdot \omega_a^{\text{meas}} \cdot (1 + \boxed{C_e} + \boxed{C_p} + C_{ml} + C_{pa} + C_{dd})}{f_{\text{calib}} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Electric Field

- due to momentum spread around p_{magic}

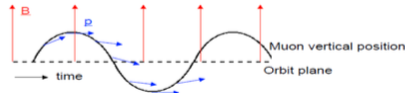
$$\vec{\omega}_a \cong -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- measured using momentum distribution provided by the calorimeters in terms of equilibrium radius

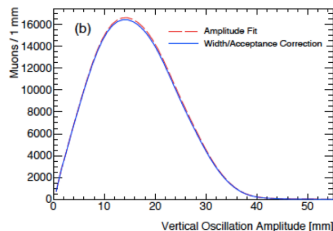


Pitch

- due to vertical beam oscillation



- measured using the beam vertical amplitude from the trackers, calorimeter data, and simulations



Corrections for Phase-Changing Effects

Muon losses

- cause a phase shift because muon-phase and muon loss rate are momentum-dependent
- measured using data-driven technique

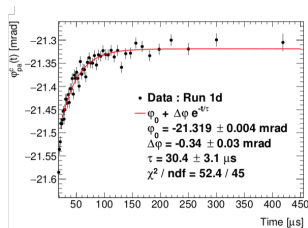
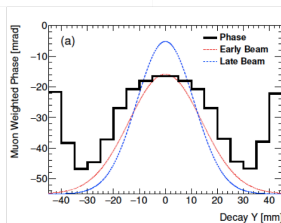
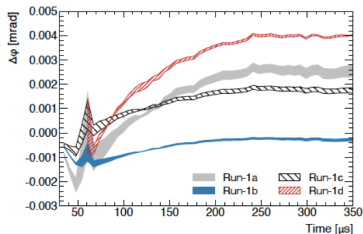
Differential Decay

- correction to account for high momentum muons having a longer lifetime

Phase acceptance

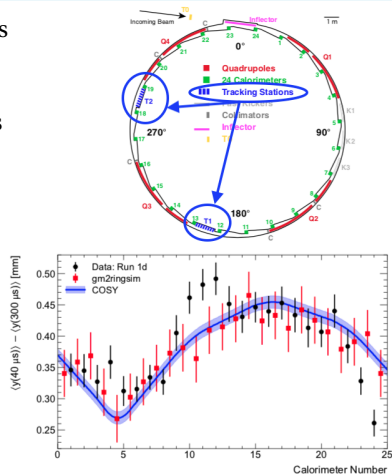
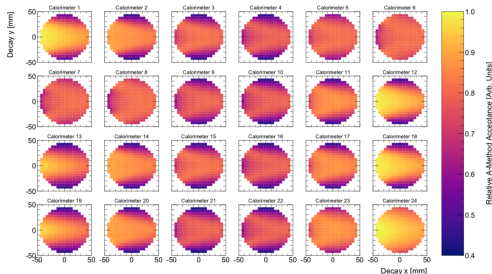
- phase changes due to early to late variations of the beam
- measured using tracker data and simulations

$$R_\mu = \frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}{f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)}$$



Simulations for phase-acceptance

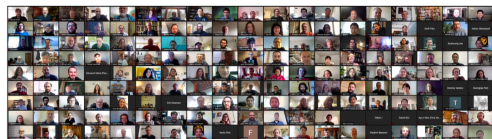
- Time-dependence of beam spatial distributions are measured by trackers in two locations
- Two independent **simulations** are used to extrapolate beam profile from tracker locations around the ring
 - based on **COSY-INFINITY** and **GEANT-4**
 - cross-checked against data
- The beam profiles in the ring are then folded with calorimeter acceptance maps produced with the **GEANT-4** based simulation



$$R_\mu = \left(\frac{f_{\text{clock}} \cdot \omega_a^{\text{meas}} \cdot (1 + C_e + C_p + C_{mi} + C_{pa} + C_{dd})}{f_{\text{calib}} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Clock frequency (f_{clock}):

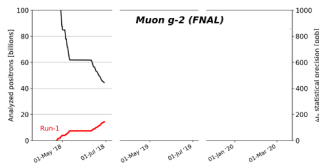
- frequency that our DAQ clock ticks
- **stable** at ppt level
- hardware-blinded to have $(40 - \varepsilon)$ MHz
 - ε kept **secret** from all collaborators
- **revealed** only when physics analysis is completed:
 - Run-1 result unblinded on Feb 25, 2021 during a virtual meeting
 - Run-2/3 result unblinded on Jul 24, 2023 during the collaboration meeting



First production run

Statistics:

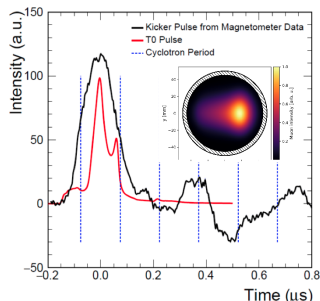
- March 26 – July 7 2018 : **Run1**
- $1.2 \times$ BNL after data quality selection



Main challenges:

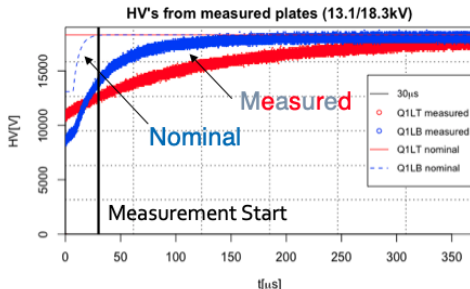
- Non-ideal kick

- low amplitude and ringing
- beam not centered in storage region



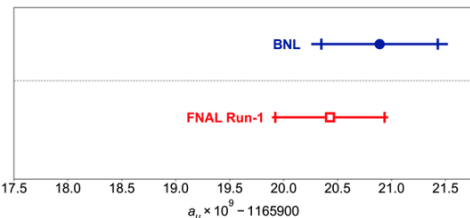
- 2 of 32 HV Quad resistors were damaged

- slow recovery time, enhanced C_{pa}



- Temperature variations larger than 1°C

Run-1 Result



- Run-1 result uncertainty is **statistics dominated**
- Major systematic uncertainties: **PA and field transients**
- **Next:** reduce as much as possible the experimental uncertainty on $g-2$!

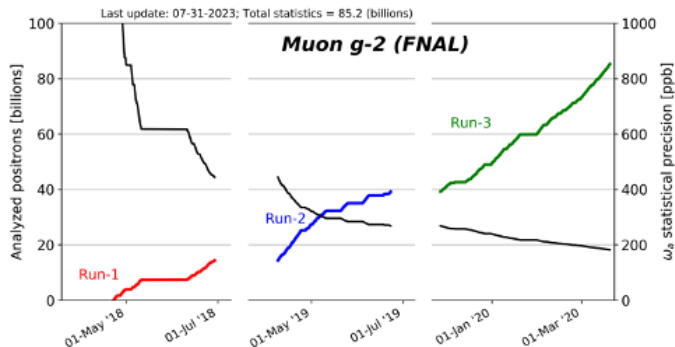
- First **FNAL** $g-2$ result :

$$a_\mu = 116592040(54) \times 10^{-11} \text{ (462 ppb)}$$

- Good agreement with **BNL** $g-2$

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a (statistical)	-	434
ω_a (systematic)	-	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p^l(x, y, \phi) \times M(x, y, \phi) \rangle$	-	56
B_k	-27	37
B_q	-17	92
$\mu_p^l(34.7^\circ)/\mu_e$	-	10
m_μ/m_e	-	22
$g_e/2$	-	0
Total systematic	-	157
Total fundamental factors	-	25
Totals	544	462

Run-2 and Run-3 Statistics Improvement



Statistics:

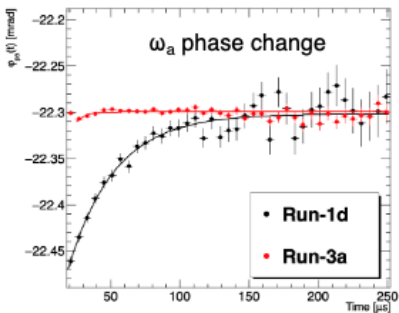
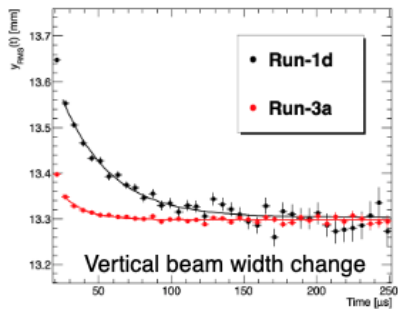
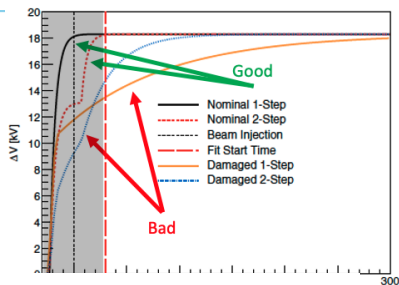
- ~ 4.7 more data in Run-2/3 than Run-1

Dataset	Stat. Unc.
Run-1	434 ppb
Run-2/3	201 ppb
Run-1+Run-2/3	185 ppb

Run-2 and Run-3 Hardware Improvements

• Before Run-2:

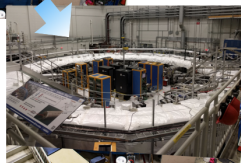
- Replaced bad quads HV resistors
- Less beam motion and reduced C_{pa}



Run-2 and Run-3 Hardware Improvements

● Before Run-2:

- > Replaced bad quads HV resistors
- > Magnet covered with a thermal blanket



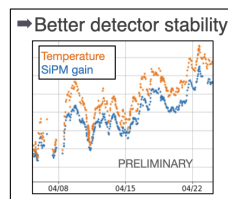
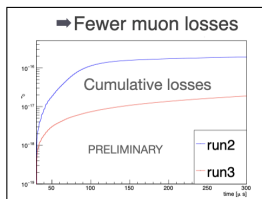
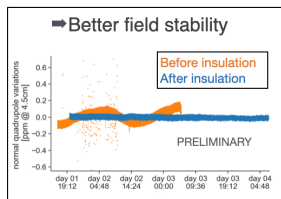
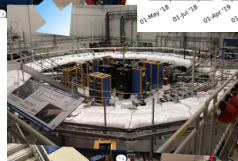
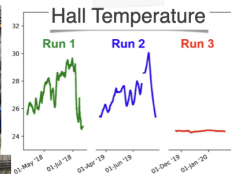
Run-2 and Run-3 Hardware Improvements

● Before Run-2:

- Replaced bad quads HV resistors
- Magnet covered with a thermal blanket

● Before Run-3:

- Hall temperature control improved



Run-2 and Run-3 Hardware Improvements

● Before Run-2:

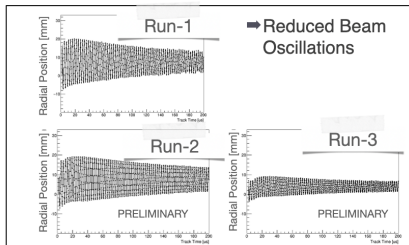
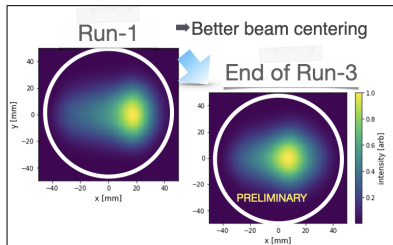
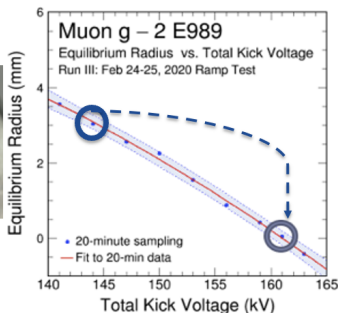
- Replaced bad quads HV resistors
- Magnet covered with a thermal blanket

● Before Run-3:

- Hall temperature control improved

● During Run-2 and Run-3:

- Replaced kicker cables \Rightarrow kickers at HV design value



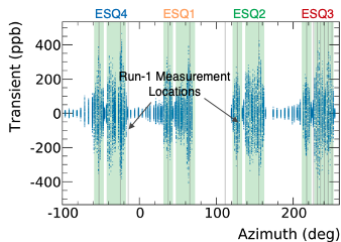
Run-2 and Run-3 Measurement Improvements

- Improved **quadrupole field transient** (B_q) by measuring both time and space

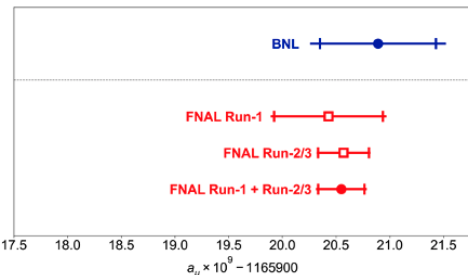
$$\delta_{B_q} \sim 92 \text{ ppb} \rightarrow \sim 20 \text{ ppb}$$

- Improved **kicker field transient** (B_k) by performing cross-check of the measurement also with a new magnetometer

$$\delta_{B_k} \sim 37 \text{ ppb} \rightarrow \sim 13 \text{ ppb}$$



Run-2/3 Result



- Both Run-1 and Run-2/3 results uncertainties are **statistics dominated**
- Run-2/3 systematic uncertainty of 70 ppb is lower than our TDR goal of 100 ppb!
- Combination assuming systematics 100% correlated with final uncertainty of 203 ppb

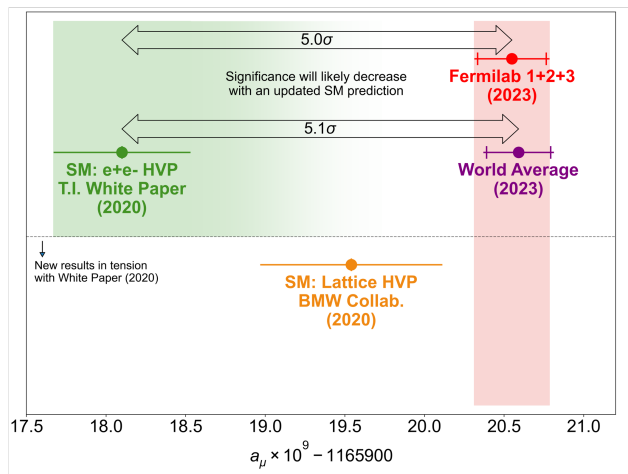
- First **New FNAL $g - 2$** result :

$$a_\mu = 116592057(25) \times 10^{-11} \text{ (215 ppb)}$$

- Good agreement with **FNAL Run-1 BNL $g - 2$**

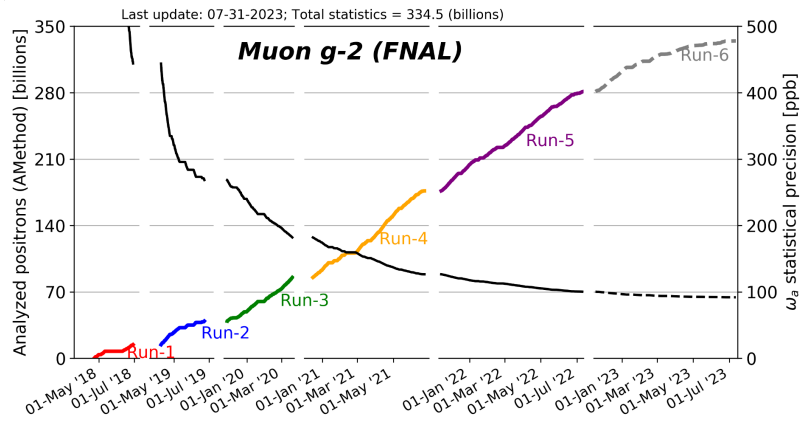
Quantity	Correction [ppb]	Uncertainty [ppb]
ω_α^m (statistical)	-	201
ω_α^m (systematic)	-	25
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{mi}	0	3
$f_{\text{calib}}(\omega'_p(\vec{r}) \times M(\vec{r}))$	-	46
B_h	-21	13
B_q	-21	20
$\mu'_p(34.7^\circ)/\mu_e$	-	11
m_μ/m_e	-	22
$g_e/2$	-	0
Total systematic	-	70
Total external parameters	-	25
Totals	622	215

Experimental measurement vs. SM calculation



- 5.1 σ discrepancy between 2023 World average and WP (2020)
- BMW result (*i.e.*, changing in WP (2020) result the HVP term from dispersion with lattice-QCD calculation) falls in between WP (2020) and the experiment

What's next?



- Completed Run-6 before summer: there is more data still to analyze!
- Not only Muon $g-2$ measurement: there are other analysis: EDM, CPT/LV and **Dark Matter** searches.

Dark Matter Searches

Muon $g-2$ experiment enables the **direct search for two ultralight DM candidates** that primarily interact with muons:

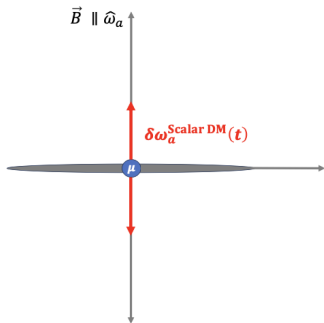
- **Scalar DM (signature: Parallel perturbation)**

- **Scalar DM perturbs the muon's mass:**

$$\begin{aligned}\omega_a(t) &= a \frac{q}{m(t)} B, \text{ where} \\ & (m(t) \rightarrow m_0(1 + A'_{\text{DM}} \cos m_{\text{DM}} t)) \\ &= a \frac{q}{m_0(1 + A'_{\text{DM}} \cos m_{\text{DM}} t)} B \\ &\approx a \frac{q}{m_0} B (1 - A'_{\text{DM}} \cos m_{\text{DM}} t)\end{aligned}$$

$$\therefore \delta\omega_a^{\text{Scalar DM}}(t) = A_{\text{DM}} \cos(m_{\text{DM}} t + \varphi_{\text{DM}})$$

- **Therefore, it causes modulation of ω_a at m_{DM}**

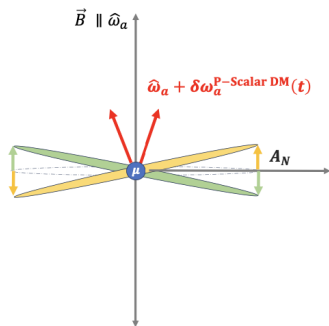


Slide courtesy of Byungchul Yu (University of Mississippi)

Dark Matter Searches

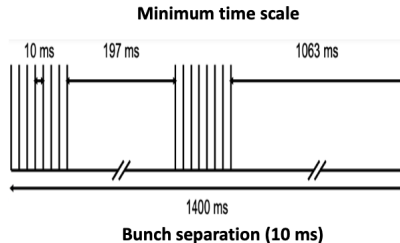
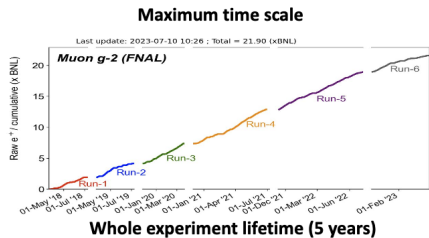
Muon $g-2$ experiment enables the **direct search for two ultralight DM candidates** that primarily interact with muons:

- **Scalar DM (signature: Parallel perturbation)**
 - **PseudoScalar DM (signature: Perpendicular perturbation)**
- Pseudoscalar DM works as an anomalous magnetic field that interacts only with the muon spin, making the muon's spin precession plane swing.
 - Up-down number asymmetry refers to a difference in the number of decay positrons accepted at the crystals in a calorimeter's upper and lower parts.
 - The decay positron is preferentially emitted to the spin direction, leading to the up-down number asymmetry.
 - Therefore, it causes the modulation of A_N (Amplitude of up-down number asymmetry) at m_{DM}



Slide courtesy of Byungchul Yu (University of Mississippi)

Estimation of Dark Matter mass sensitivity

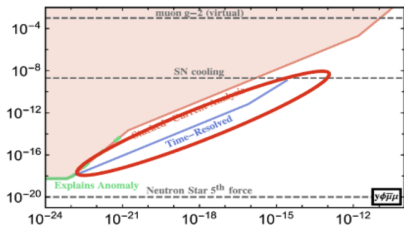


Targeted Dark Matter mass range with its corresponding frequency

Slide courtesy of Byungchul Yu (University of Mississippi)

Dark Matter Model sensitivity

Scalar DM model sensitivity plot

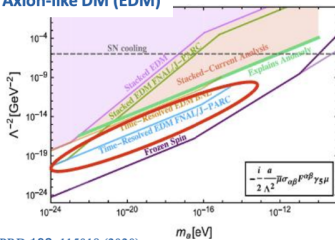


Ryan Janish, Harikrishnan Ramani n_ϕ [eV]
PRD 102, 115018 (2020)

Projected sensitivity of “direct” DM search using the Muon $g - 2$ data is beyond the astronomical bounds.

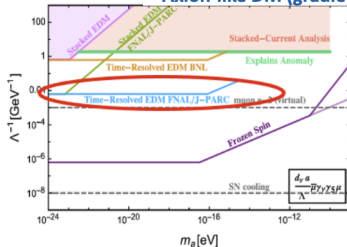
Pseudoscalar DM model projected sensitivity plots

Axion-like DM (EDM)



Ryan Janish, Harikrishnan Ramani PRD 102, 115018 (2020)

Axion-like DM (gradient)



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Summary and Conclusions

- FNAL $g - 2$ Experiment goal is to measure a_μ with a **precision of 140 ppb** (4×BNL precision)
- The result from the analysis of the Run-1/2/3 data **confirmed** result from BNL experiment
- **Run-2 and Run-3 data** measurement achieved a factor 2 uncertainty reduction both in statistics and systematics!
- With **Run-4, Run-5 and Run-6** we expect to achieve the uncertainty goal and other analysis including **dark matter searches** are been developed.

Thanks!

