Novel loop-diagrammatic approach to QCD θ parameter and application to the left-right model

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JHEP 03 (2023) 150 [arXiv:2301.13405]

Strong CP Problem

the QCD θ angle $|\bar{\theta}| < 10^{-10}$ $\ll$ the phase in the CKM matrix $\delta_{\rm CKM} = \mathcal{O}(1)$

$$\bar{\theta} \equiv \theta_G - \arg\det[\mathcal{M}_u \mathcal{M}_d]$$

bare — the chiral (ABJ) anomaly

▶ mass phase contribution

where
$$\mathcal{L}_{\rm SM} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$

$\mathcal{M}_{u/d}$: mass matrix for quarks

$\left(\tilde{G}^{\hat{a}\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^{\hat{a}}_{\rho\sigma}\right)$

S. L. Adler, Phys. Rev. **177** (1969) 2426-38
J. S. Bell, R. Jackiw, Nuovo. Cim. A **60** (1969) 47-61

TWO promising solutions

Axion

- dynamical solution
- predict a dark matter candidate
- **conflict with the quantum gravity**

R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. **38** (1977)

Left-Right model

The parity symmetry forbids $G\tilde{G}$!!

$\bar{\theta}\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$: P- and $T(CP)$-odd operator

parity(P) symmetric model

▼ spontaneous breaking

small quark mass phases ← arise at loop level

▶ bottleneck!

▼ Fujikawa method

$\mathcal{L}_{\rm SMEFT} \ni \bar{\theta}\frac{\alpha_s}{8\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$ small! ← unevaluated

conventional researches
- arise at loop level
- ▶ bottleneck!
- unevaluated
- ▶ solve or not?
- ▶ examination?

M. A. B. Beg, H. S. Tsao, Phys. Rev. Lett. **41** (1978) 278

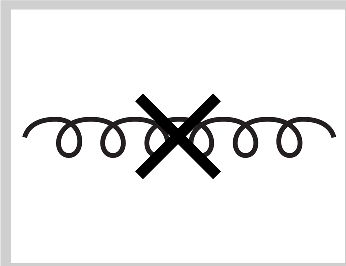
Aim of This Study

Are mass phases unique contributions to $\bar{\theta}$?

Do we need to modify the conventional method to derive $\bar{\theta}$, which is Fujikawa method + loop masses?

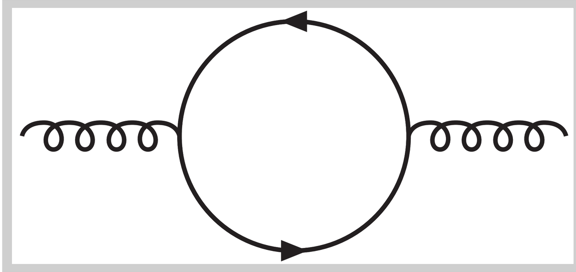
Diagrammatic Approach

$\bar{\theta} =$



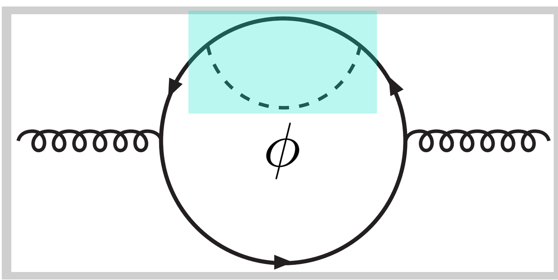
θ_G

$+$



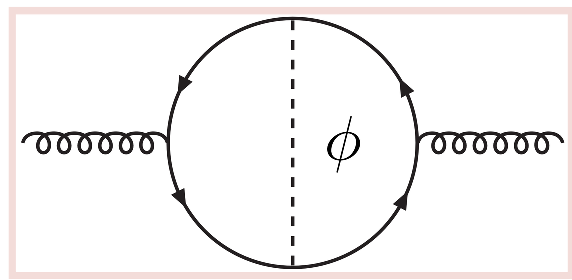
$-m_{\rm CP}/m$

$+$



$-\frac{\delta m^{(1)}_{\rm CP}}{m} + \frac{m_{\rm CP}}{m}\frac{\delta m^{(1)}}{m}$

$+$



New!

(assumption: a scalar ϕ which interacts with ψ)

These loop graphs are perturbatively **uncalculable** because

total derivative
$$G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} = \partial^\mu \epsilon_{\mu\nu\rho\sigma}\left(A^{\hat{a}}_\nu G^{\hat{a}}_{\rho\sigma} - \frac{g_s}{3} f^{\hat{a}\hat{b}\hat{c}} A^{\hat{a}}_\nu A^{\hat{b}}_\rho A^{\hat{a}}_\sigma\right)$$

$$\sum p^\mu = 0$$

strategy

- building a gluon effective theory described by not gauge field $A^{\hat{a}}_\mu$ but the field-strength $G^{\hat{a}}_{\mu\nu}$
- fixing a background field-strength $G^a_{\mu\nu}$
- temporarily breaking the translation symmetry

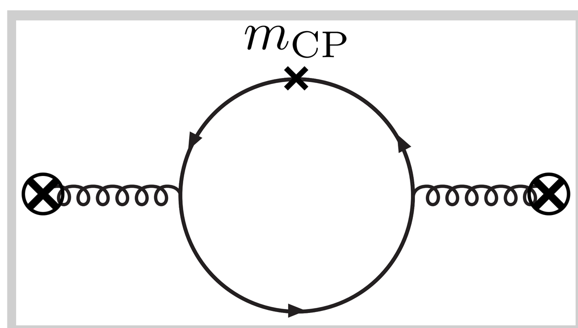
Fock-Schwinger gauge: $(x^\mu - x^\mu_0)A^{\hat{a}}_\mu(x) = 0$

V. A. Novikov, *et al.*, Fortsch. Phys. **32** (1984) 585

- fixing a background $A^{\hat{a}}_\mu(x) = \frac{1}{2}(x^\nu - x^\nu_0)G^{\hat{a}}_{\nu\mu}(x_0) + \cdots$
- breaks the translation symmetry, but it revives in the result of gauge invariant quantities

S. N. Nikolaev, *et al.*, Nucl. Phys. **B** 213 (1983) 285-304

$$\mathcal{L} = \bar{\psi}i\not{D}\psi - m\bar{\psi}\psi - m_{\rm CP}\bar{\psi}i\gamma_5\psi - \frac{1}{4}G^{\hat{a}}_{\mu\nu}G^{\hat{a}\mu\nu} + \theta_G\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$



$= -\frac{m_{\rm CP}}{m}$ **consistent with Fujikawa method!**

Conventional Method

$\not{P}, \not{T}$ & 1 flavor QCD (example) — $T(CP)$-odd, mass phase

$$\mathcal{L}_{\not{P},\not{T}} \ni \theta_G\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - \left(m + \delta m^{(1)}\right)\bar{\psi}\psi - \left(m_{\rm CP} + \delta m^{(1)}_{\rm CP}\right)\bar{\psi}i\gamma_5\psi$$

loop masses + mass diagonalize (chiral rotation)

—Fujikawa method—

$$\mathcal{L}_{\not{P},\not{T}} \ni \bar{\theta}^{\rm loop}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - M^{(1)}\bar{\psi}_M\psi_M$$

$\left(M^{(1)} = (m + \delta m^{(1)}) + \frac{(m_{\rm CP} + \delta m^{(1)}_{\rm CP})^2}{m + \delta m^{(1)}}\right)$

conventional radiative corrections to $\bar{\theta}$

$$\bar{\theta}^{\rm loop} \simeq \theta_G - \frac{m_{\rm CP}}{m} - \frac{\delta m^{(1)}_{\rm CP}}{m} + \frac{m_{\rm CP}}{m}\frac{\delta m^{(1)}}{m}$$

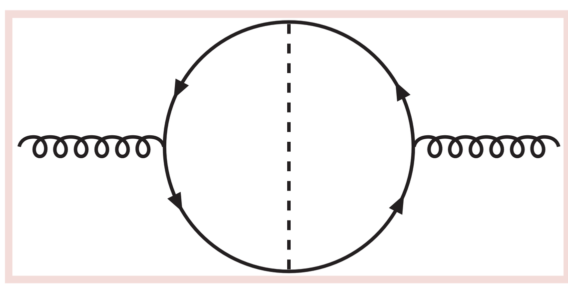
Conclusion

- We showed the **NEW type contribution** to $\bar{\theta}$ diagrammatically.

$$\bar{\theta} = \theta_G + \arg\det[\mathcal{M}_u\mathcal{M}_d]$$

bare — Fujikawa + tree mass

$$+ \arg\det[\delta\mathcal{M}_u\delta\mathcal{M}_d] +$$



Fujikawa + loop mass

New!

- We **formulated the calculation method** for this type diagram using Fock-Schwinger gauge.