Excited bound states & their role in dark matter production

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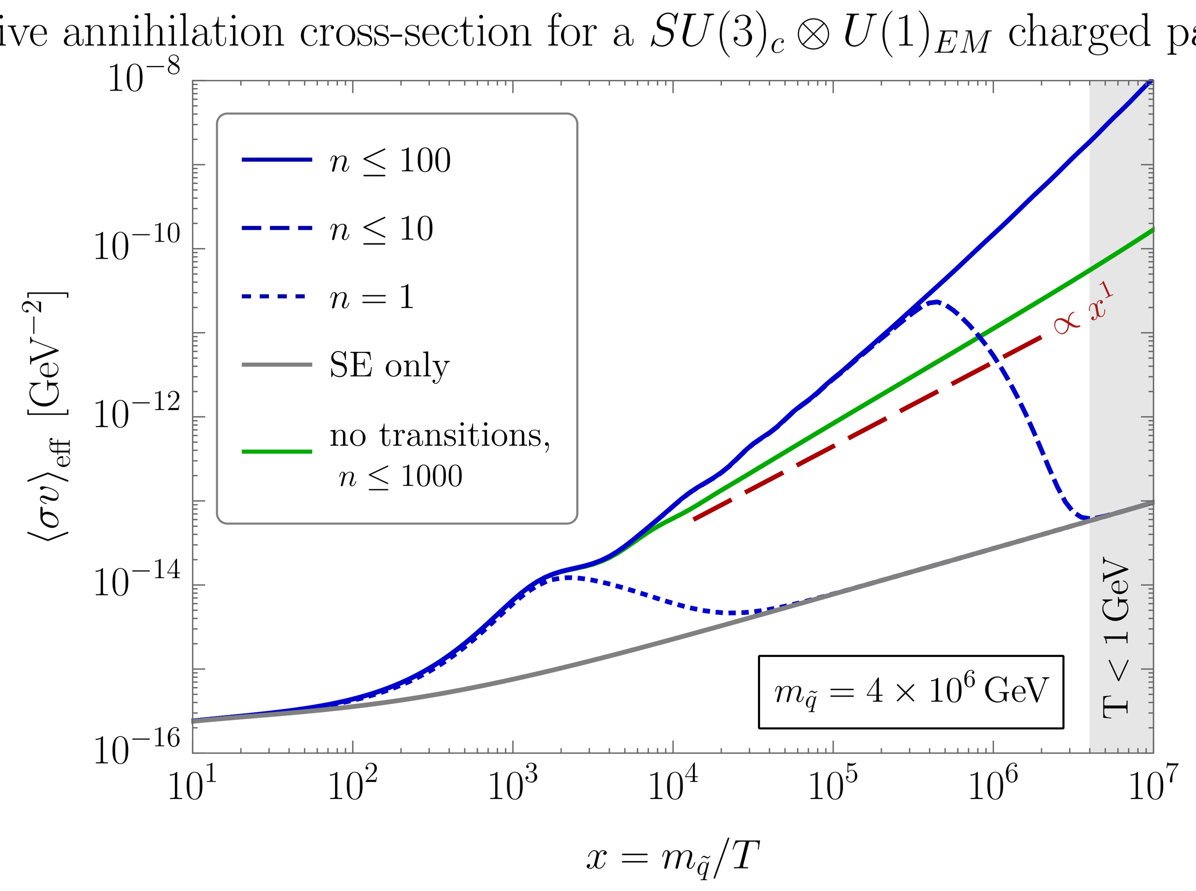
Motivation

- Bound state formation (**BSF**) can has an influence on DM freeze-out [1].
- Analytic expressions for general BSF cross-sections have only recently been published [2] and were unknown for bound-to-bound transition rates.
- From state of the art computations [1] the influence of high excitations can not be predicted. Results for $n = 1, 2$ indicate strong effects of bound states in non-Abelian theories.

The effective cross-section

Effective annihilation cross-section for a $SU(3)_c \otimes U(1)_{EM}$ charged particle.



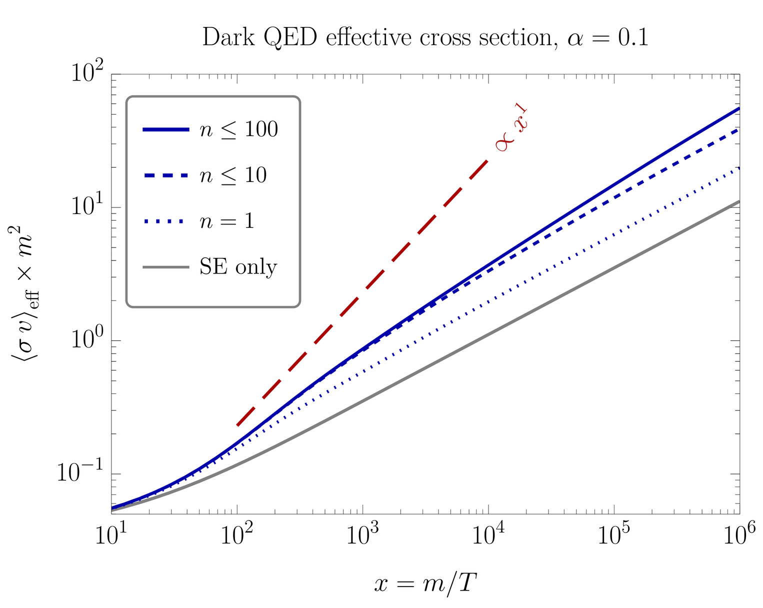
For the combined abundance of particles and anti-particles, $Y_{\tilde q}$, we can write a single Boltzmann equation in inverse temperature including all bound states (assuming chemical equilibrium and a steady state for bound states)

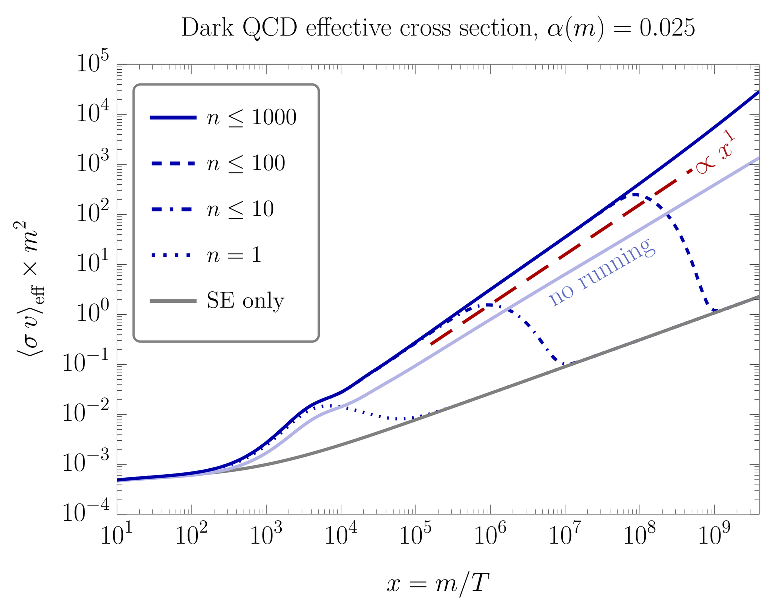
$$\frac{\mathrm{d}Y_{\tilde q}}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x} \times \frac{1}{2}\langle \sigma v\rangle_{\rm eff}\left(Y_{\tilde q}^2 - Y_{\tilde q}^{(\rm eq)^2}\right), \tag{1}$$

where $x \equiv m/T$ and the **effective cross section** includes direct annihilation as well as annihilation via BSF:

$$\langle \sigma v\rangle_{\rm eff} \equiv \langle \sigma v\rangle_{\rm annh} + \sum_{i=(n,l)} R_i \langle \sigma_{\rm BSF}\, v\rangle_i\,. \tag{2}$$

A given bound state $\mathcal{B}$ can undergo *decay* into light particles ($\mathcal{B}_{n,l=0} \to$ gauge bosons), *transition* to another bound state ($\mathcal{B}_{n,l} \to \mathcal{B}_{n',l\pm1}$) or *ionization*. The likelihood with which a BSF process into a state $i = (n,l)$ leads to a depletion of the abundance of $\tilde q$ is captured by R_i. In the absence of transitions, this is just a branching ratio $R_i = \Gamma_i^{dec} / \Gamma_i^{tot}$. This limit is automatically realized for pure non-Abelian gauge interactions forming gauge-singlet bound states, *c.f.* Eq.(4) and shown below.





Theory & Model Setup

Rapid multiple exchange of soft gauge bosons between heavy particles can be resummed into a non-local Coulomb potential in potential non-relativistic effective theory (pNREFT). This allows us to obtain the BSF, decay or transition rates for any initial or final state couplings $\alpha_{\rm in,fn}$ appearing in the potential.

We investigate a **t-channel mediator model** including a heavy scalar mediator $\tilde q$, charged under $SU(3)_c \otimes U(1)_{EM}$, and a lighter gauge sterile DM candidate χ,

$$\mathcal{L} \supset \tilde q^\dagger \left(\frac{\vec p^2}{2m} - V(r)\right)\tilde q + \lambda_\chi \tilde q \bar q \chi + h.c. \tag{3}$$

The mediator decay rate $\Gamma_{\tilde q \to q\chi} \sim \lambda_\chi^2 \lesssim 10^{-12}$ depletes the mediator prior to the QCD-phase transition, thereby setting the DM relic abundance. Equation (1) turns into 2 coupled equations for $\tilde q$ and χ. We solve the complete coupled system, although freeze-in of χ is sub-dominant for our purposes.

As $\alpha_s \gg \alpha_{EM}$, the Abelian interaction only affects bound-to-bound transitions, which are impossible in non-Abelian theories due to the repulsive adjoint effective coupling $\alpha_{\rm eff}^{[\mathbf{8}]}$:

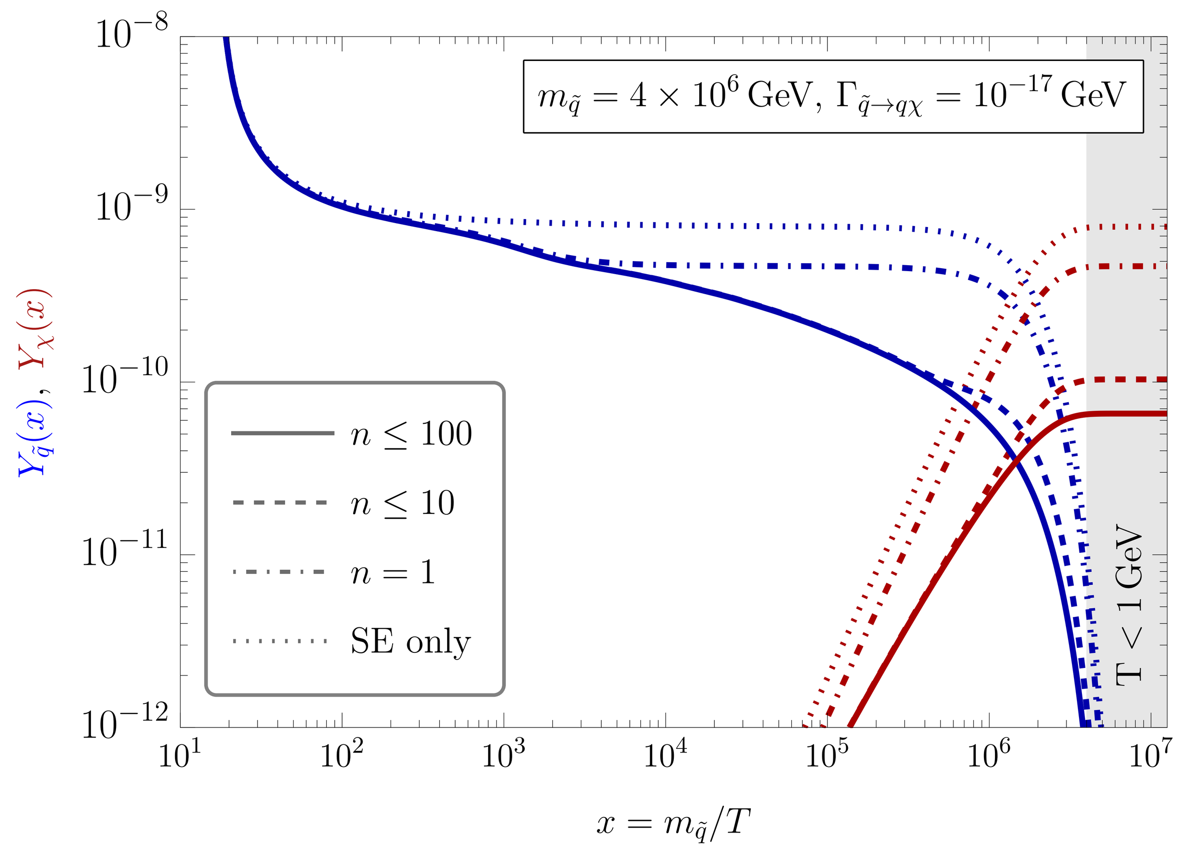
$$\mathcal{B}_{nl}^{[\mathbf{1}]} \not\to \mathcal{B}_{n'l'}^{[\mathbf{8}]} + g \quad \text{because} \quad \alpha_{\rm eff}^{[\mathbf{8}]} = \frac{-1}{2N_c}\alpha_s < 0. \tag{4}$$

Abundance Evolution

The relic abundance of DM is well measured $\Omega_{\rm DM}h^2 = 0.120$ and, in the freeze-out picture, is set by the remaining abundance today $Y^{\rm today} \approx Y(x \to \infty)$. In our model the DM abundance (red) is set by the mediator abundance (blue) through the mediator decay

$$\Omega_{\rm DM}h^2 \propto m_\chi Y_\chi^{\rm today} \sim m_\chi Y_{\tilde q}(H \sim \Gamma_{\tilde q \to q\chi})\,. \tag{5}$$

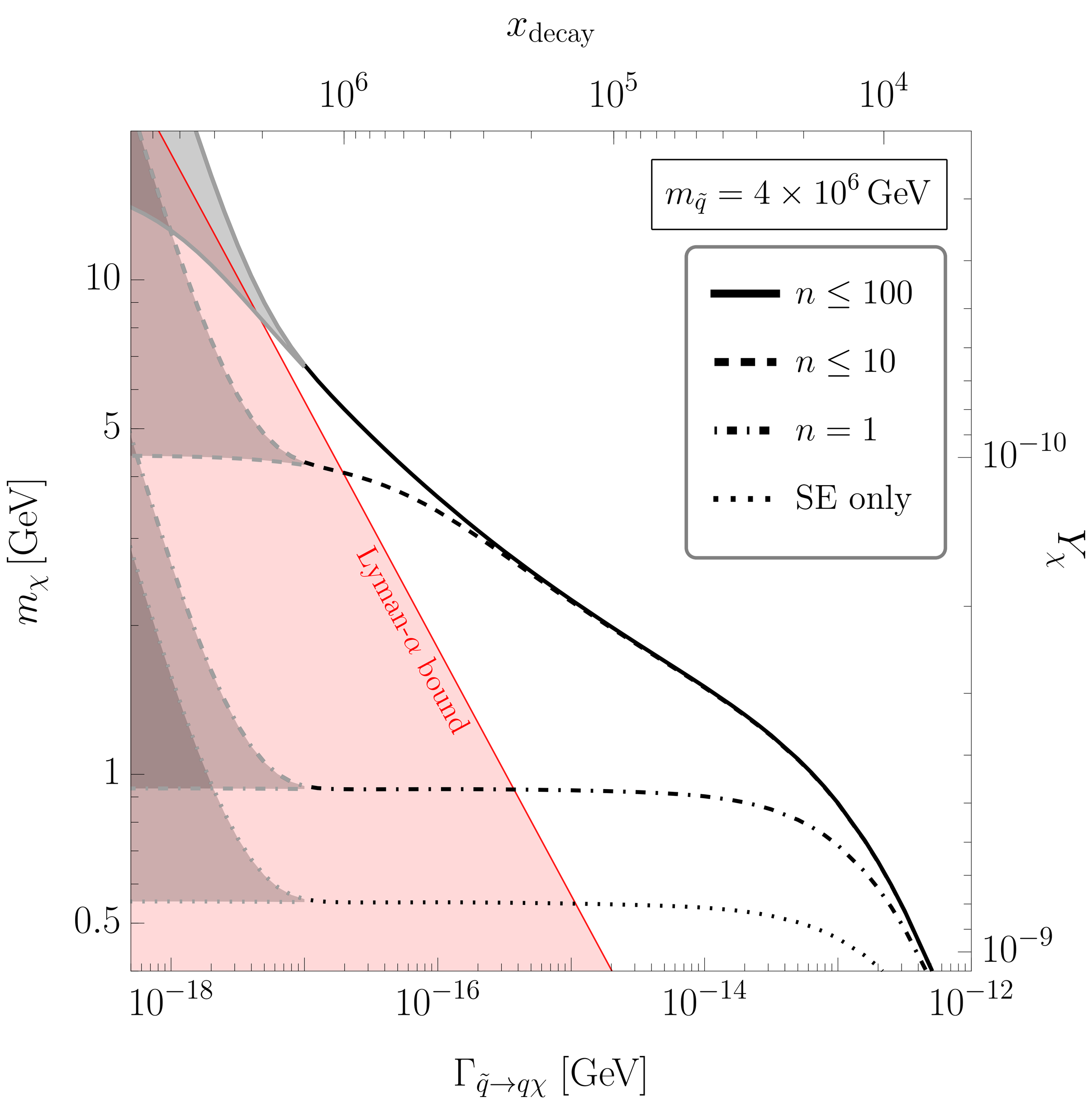
We include Sommerfeld enhancement (dotted) and also LO dipole interactions in BSF processes for all l. High n are needed for reliable results at large x.



Freeze-out occurs once $H(x) \gg \Gamma_{\tilde q\tilde q \to gg}$, which requires $\langle \sigma v\rangle_{\rm eff}(x) \propto x^\gamma$, $\gamma < 1$. We find that this is violated upon including $n \gg 1$ bound states, thus $Y_{\tilde q}$ depletes continuously up to its decay to χ around $x = x_{decay}$ and the coloured particle **does not freeze-out**.

Results for coloured t-channel models

Scanning $\Gamma_{\tilde q \to q\chi}$ yields the parameter space impact of excited states on our model.



→ Excited bound states and transitions between them are essential, even for $n \gg 1$.
→ Predicted DM masses are increased by up to an order of magnitude by BSF of $\tilde q$.
→ Including excited states relaxes Lyman-α constraints on the model.

Conclusions

- We developed a numerical method to include excited states with transitions to $n \le 100$.
- BSF and bound-to-bound transitions each strongly enhance $\langle \sigma v\rangle_{\rm eff}$.
- Under $SU(3)$, $\langle \sigma v\rangle_{\rm eff}$ converges to a supercritical power-law, preventing freeze-out. Thus, highly excited states are dominant at small T down to the Landau pole.
- No freeze-out occurs for particles charged at least under an $SU(N_c)$, $N_c \ge 3$.

References

based on: Garny et. al., 2023, arXiv: 2208.01336.
[1] M. Garny et al., *Phys. Rev. D* **2022**, *105*, 055004.
[2] S. Biondini et al., *JHEP* **2023**, *07*, 006.