Gravitational multipoles beyond Einstein gravity

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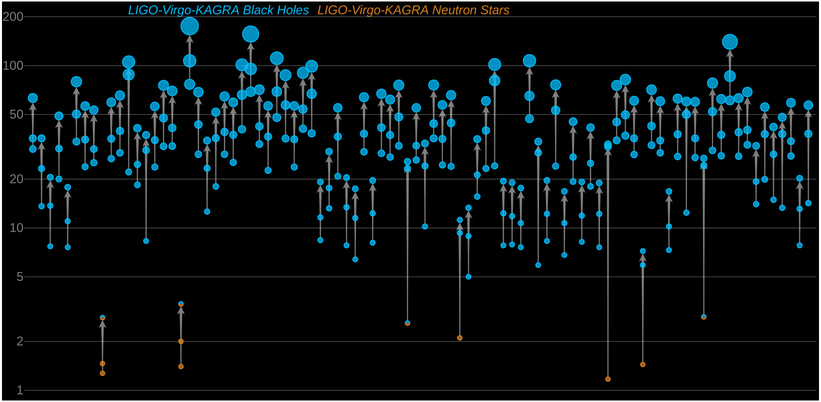
Based on 2208.01044, 22xx.xxxx

GR and beyond

What are the physics beyond classical gravity?



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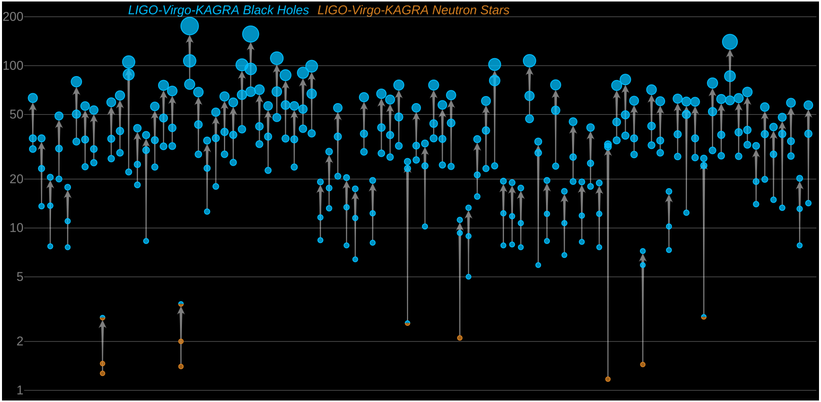

- Testing beyond GR requires predictions comparable to experimental data
- Not so easy for EFT as well as top-down approaches

Multipoles as the first step beyond

We can ease our burden by first learning about

Gravitational multipoles can be computed just from an asymptotic expansion of the metric.

- ⧫ EFT perspective $\rightarrow$ Constrain scale of new physics

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- Top-down approaches $\rightarrow$ Can we get arbitrarily close to the BH values and what does it impose

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Gravitational multipoles can be computed just from an asymptotic expansion of the metric.

- EFT perspective → Constrain scale of new physics
- Top-down approaches → Can we get arbitrarily close to the BH values and what does it impose

Important *now*

*Because LISA and other GW experiments are coming online **soon***

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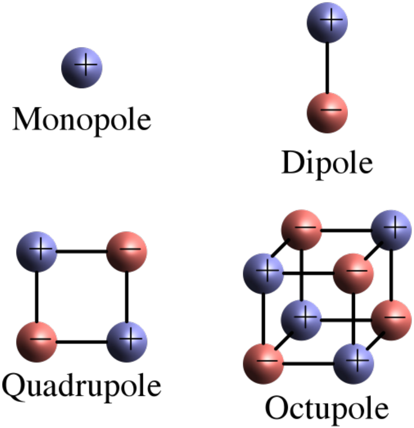
EM multipoles

$$V(r,\theta,\phi) = \frac{1}{4\,\pi}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{Q_{\ell\,m}}{r^{\ell+1}}\sqrt{\frac{4\,\pi}{2\,\ell+1}}Y_{\ell\,m}(\theta,\phi)$$

C_{00} - monopole

$C_{1,-1}$, $C_{1,0}$, $C_{1,1}$ - dipole

EM multipoles



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This expansion can be used at any radius r.

Gravitational multipoles

Non-linearities + tensors = complications
⇒ So we go to asymptotically flat infinity
and expand the metric [Thorne]

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{\ell \geq 1}^{\infty} \frac{2}{r^{\ell+1}} \left(\tilde{M}_\ell P_\ell + \sum_{\ell' < \ell} c_{\ell\ell'}^{(tt)} P_{\ell'} \right),$$

$$g_{t\phi} = -2r \sin^2 \theta \left[\sum_{\ell \geq 1}^{\infty} \frac{1}{r^{\ell+1}} \left(\frac{\tilde{S}_\ell}{\ell} P'_\ell + \sum_{\ell' < \ell} c_{\ell\ell'}^{(t\phi)} P'_{\ell'} \right) \right],$$

$$M_\ell = \sum_{k=0}^{\ell} \binom{\ell}{k} \tilde{M}_k \left(-\frac{\tilde{M}_1}{\tilde{M}_0} \right)^{\ell-k}, \quad S_\ell = \sum_{k=0}^{\ell} \binom{\ell}{k} \tilde{S}_k \left(-\frac{\tilde{M}_1}{\tilde{M}_0} \right)^{\ell-k}.$$

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Example: Kerr

$$M_{2n} = M(-a^2)^n, \quad M_{2n+1} = 0, \quad S_{2n} = 0, \quad S_{2n+1} = M\, a(-a^2)^n.$$

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Example: Kerr and Kerr-Newman

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Gravitational multipoles in string theory and their ratios

- Richer structure: $M_{2n+1} \neq 0$ and $S_{2n} \neq 0$, dependence on electric and magnetic charges, $J = S_1 \neq a\,M$.
- There is a Kerr limit, so we can embed Kerr in a generalised BH solution (STU BH)
- Look at ratios of multipoles that are naively ill-defined for Kerr and take the limit [Bena]

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Multipole ratios examples

$$\frac{M_{n+1}M_{n+2}}{M_n M_{n+3}} \xrightarrow[\text{Kerr lim}]{} 1 - \frac{4}{3+(-1)^n(2\,n+1)},$$

$$\frac{M_2 S_n}{M_{n+1}S_1} \xrightarrow[\text{Kerr lim}]{} 1\,.$$

Deviations of ratios

String theory prediction?

- Small deviations: $M_n = M_n^{(Kerr)} + \epsilon\, m_n$, $S_n = S_n^{(Kerr)} + \epsilon\, s_n$
- Ratios determine:

$$S_{2n} = -n\, M(-a^2)^n \epsilon, \quad M_{2n+1} = n\, M\, a(-a^2)^n \epsilon$$

$$M_{2n} - M_{2n}^{(Kerr)} = -n^2 M(-a^2)^n \Big(\frac{2n-3}{4n}\Big)\epsilon^2$$

$$S_{2n+1} - S_{2n+1}^{(Kerr)} = -n^2 M\, a(-a^2)^n \Big(\frac{2n+1}{4n}\Big)\epsilon^2$$

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String theory restrictions?

- Demand ratios of BH-like solutions to match BH ones in the BH limit
- Puts constraints on moduli space of allowed solutions in for example string theory

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Definitions

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R + \ell^4\mathcal{L}_{(6)} + \ell^6\mathcal{L}_{(8)} + \dots\right\},$$

where

$$\mathcal{L}_{(6)} = \lambda_{\rm ev} R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm odd} R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu},$$

$$\mathcal{L}_{(8)} = \epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\mathcal{C}\tilde{\mathcal{C}},$$

$$\mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma},$$

$$\tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R_{\alpha\beta}{}^{\rho\sigma}.$$

LO type IIB string theory on a torus: $\epsilon_1 = \epsilon_2$, $\epsilon_3 = 0$

Complex function: $Z_n = M_n + iS_n$

Kerr: $Z_n^{(0)} = M(ia)^n$

$$M_{2n} = M(-a^2)^n, \quad M_{2n+1} = 0, \quad S_{2n} = 0, \quad S_{2n+1} = M\,a(-a^2)^n.$$

6-Derivative corrections

$$Z_n = Z_n^{(0)} \left[1 + \left(\hat{\lambda}_{\text{ev}} + i\hat{\lambda}_{\text{odd}}\right) f_n\left(\chi\right)\right], \quad \chi = \frac{a}{M}.$$

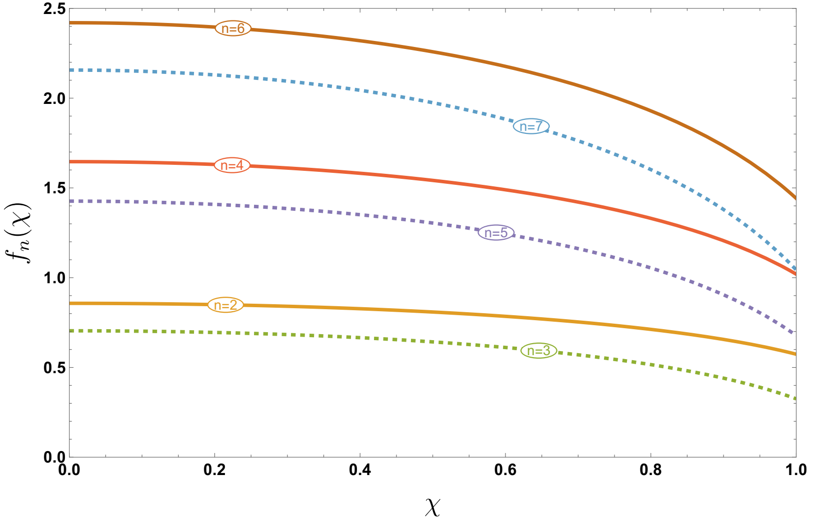
$$f_2(\chi) = -\frac{4}{7\,\chi^6}\left(8 - 4\chi^6 + 15\chi^4 - 20\chi^2 - 8(1-\chi^2)^{5/2}\right),$$

$$f_3(\chi) = \frac{4}{7\,\chi^8}\Big[4\sqrt{1-\chi^2}\,(8\chi^6 - 10\chi^4 + \chi^2 + 16) - 16 + 4\chi^8 - 13\chi^6 + 10\chi^4 - 8\chi^2 + 15\chi\arcsin(\chi)\Big]$$

$$f_4(\chi) = \frac{8}{49\,\chi^8}\Big[\sqrt{1-\chi^2}\,(56\chi^6 - 108\chi^4 + 48\chi^2 - 311) + 28\chi^8 - 103\chi^6 + 136\chi^4 + 24\chi^2 + 416 - \frac{840\,(1+2\chi^2)}{\chi}\arcsin(\chi)\Big],$$

Regular everywhere!

6-Derivative corrections



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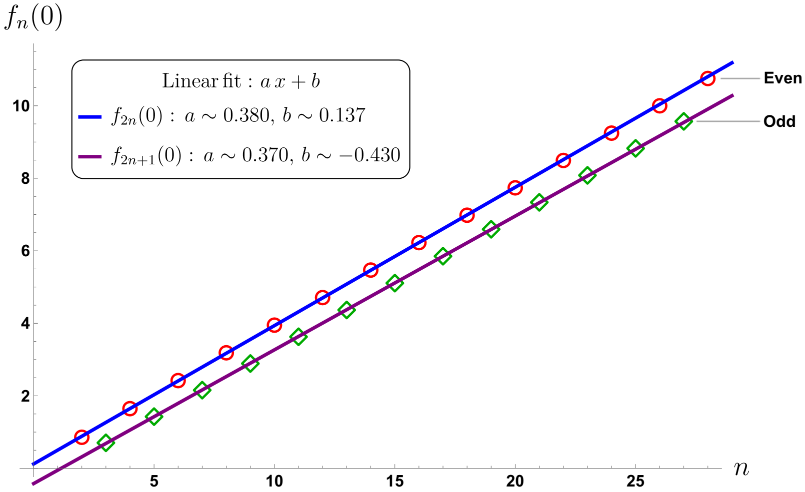

8-Derivative corrections

$$Z_n = Z_n^{(0)} \left[1 + (\hat{\epsilon}_1 + \hat{\epsilon}_2)\, g_n\,(\chi) + (\hat{\epsilon}_1 - \hat{\epsilon}_2 + i\,\hat{\epsilon}_3)\, h_n\,(\chi)\right] .$$

$$\begin{aligned} h_2(\chi) &= -\frac{8}{25\,\chi^{10}}\big(64 - 80\chi^{10} + 660\chi^8 - 1545\chi^6 + 1500\chi^4 - 600\chi^2 \\ &\quad - 8(1-\chi^2)^{5/2}(8 + 35\chi^4 - 55\chi^2)\big)\,, \\ h_3(\chi) &= \frac{8}{25\,\chi^{10}}\big(192 + 80\chi^{10} - 620\chi^8 + 1165\chi^6 - 620\chi^4 \\ &\quad - 200\chi^2 + 8(1-\chi^2)^{5/2}(35\chi^4 - 35\chi^2 - 24)\big)\,. \end{aligned}$$

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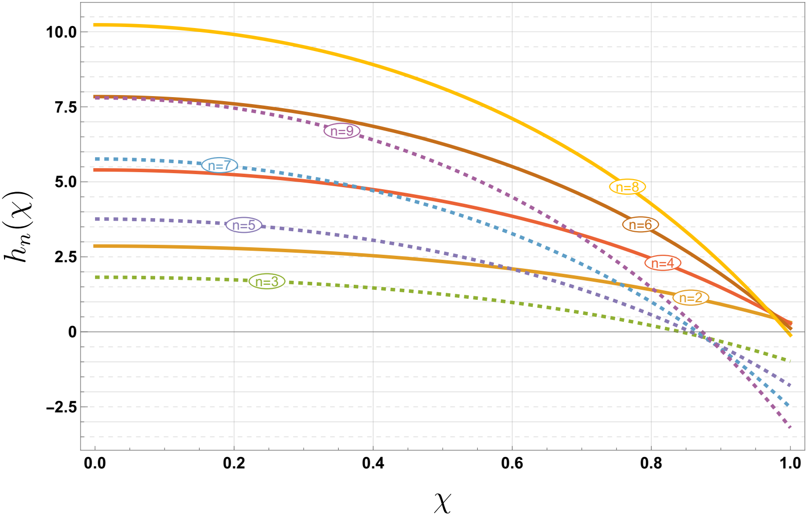

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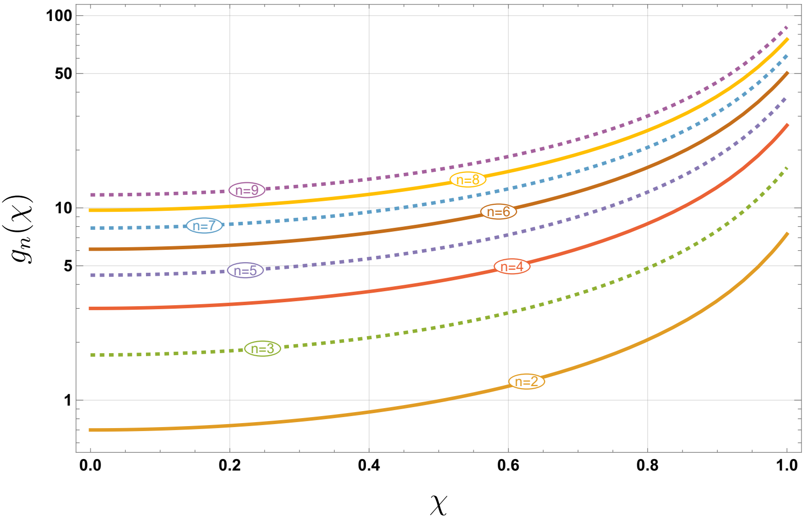

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Dimensionless multipoles deviation with LISA

Extreme-mass-ratio inspirals (EMRIs)

- Stellar mass ($\sim 10\, M_\odot$) compact objects (CO) inspiralling into massive BHs ($\sim 10^5 - 10^7\, M_\odot$)
- Inspiral takes very long time $\rightarrow$ orbit depends strongly on background geometry
- CO acts like a probing needle of space-time structure outside massive BH $\rightarrow$ measure lowest multipoles well
- Up to $\Delta(M_2/M^3) \sim 10^{-4}$ [Barack]

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However...

due to the huge mass $\rightarrow$ bad constraint on scale of new physics

$$\ell \lesssim 0.1(\lambda_{\text{ev}}\chi^2)^{-1/4} M \sim 10^4\,\text{km}$$

Similar mass binaries with ground experiments

$$M_{2,(i)} = -\kappa_{(i)} M_{(i)}^3 \chi_{(i)}^2, \qquad \kappa_{(i)} = 1 + \delta\kappa_{(i)},$$
$$\delta\kappa^{(\pm)} = (1/2)(\delta\kappa_{(1)} \pm \delta\kappa_{(2)}).$$

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Bounds on $\delta\kappa^{(\pm)} \Rightarrow$ constraint on ℓ

- Current observations: $\delta\kappa^{(s)} \lesssim 6.66 \Rightarrow \ell \lesssim 1 - 10\,\text{km}$ [Ligo]
- Future Einstein Telescope: $\delta\kappa^{(s)} \lesssim 10^{-2} \Rightarrow \ell \lesssim 0.1 - 1\,\text{km}$ [Arun]

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Best current bound - compatible with future bounds from tidal Love numbers and QNMs.

Multipole ratios and string theory

For $\hat{\epsilon}_1 = \hat{\epsilon}_2$, $\hat{\epsilon}_3 = 0$ (only $g_n(\chi)$ terms) we recover leading higher-curvature correction of type IIB string theory on a torus,

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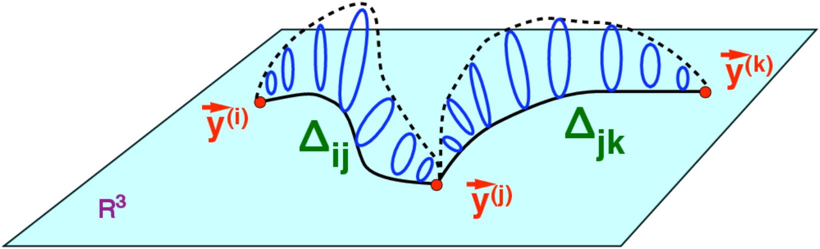
Multi-centred bubbling geometries



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Future Directions

Multipoles are an imprint of structure near the gravitating object. Computing them and modelling their effects on observables can put constraints on and teach us about viable GR extensions.

- More general theories with matter fields.
- STU BH higher derivative corrections and ratios matching.
- Do it for other constructions in string theory - like microstate geometries?
- To that end, extend definition to $D > 4$.
- Ultimately, we need modelling of signatures of GR modifications - so go and do more! Tidal Love numbers, backreaction of linear perturbations, mergers of fuzzballs, etc.