Quantum information measures and constraints in holography

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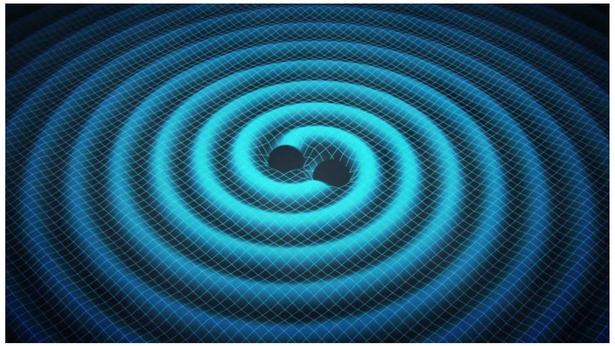
Gravity

General Relativity provides the best available theoretical description of gravity and spacetime

Its predictions have been successfully tested, including the recent measurements of gravitational waves and observation of black holes



public.virgo-gw.eu

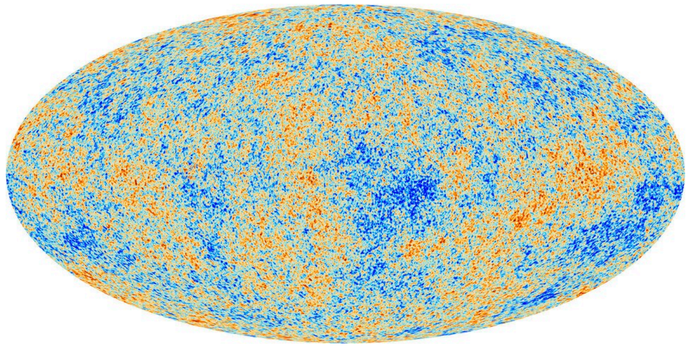
eventhorizontgelescope.org

Quantum Gravity?

A complete description needs to take into account quantum mechanical effects

These become essential in a number of situations including the physics of Black Holes and the description of early phases of our Universe

nature.com



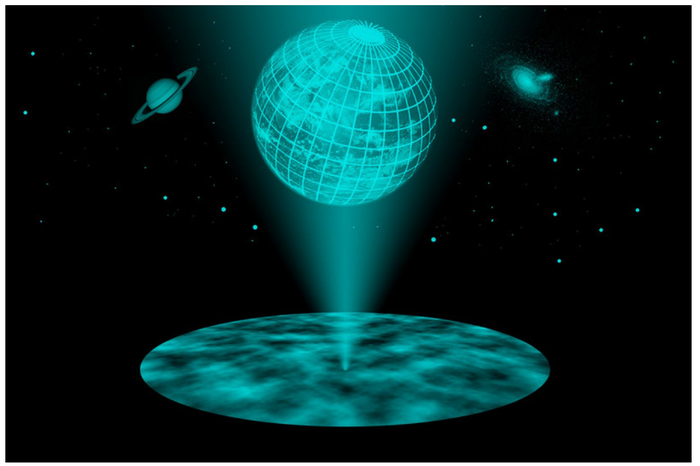
esa.int

From the theoretical point of view this remains a major open problem

Holographic Principle

Advances in the study of black hole thermodynamics and string theory have led to propose that gravity and spacetime emerge from the collective behaviour of non-gravitational degrees of freedom in lower dimensions

Gravity and spacetime as we experience them can be regarded as an effective description resulting from the physics of a lower dimensional theory, as in an hologram



The AdS/CFT correspondence gives a concrete realization of the holographic principle and provides a unique framework to study this idea

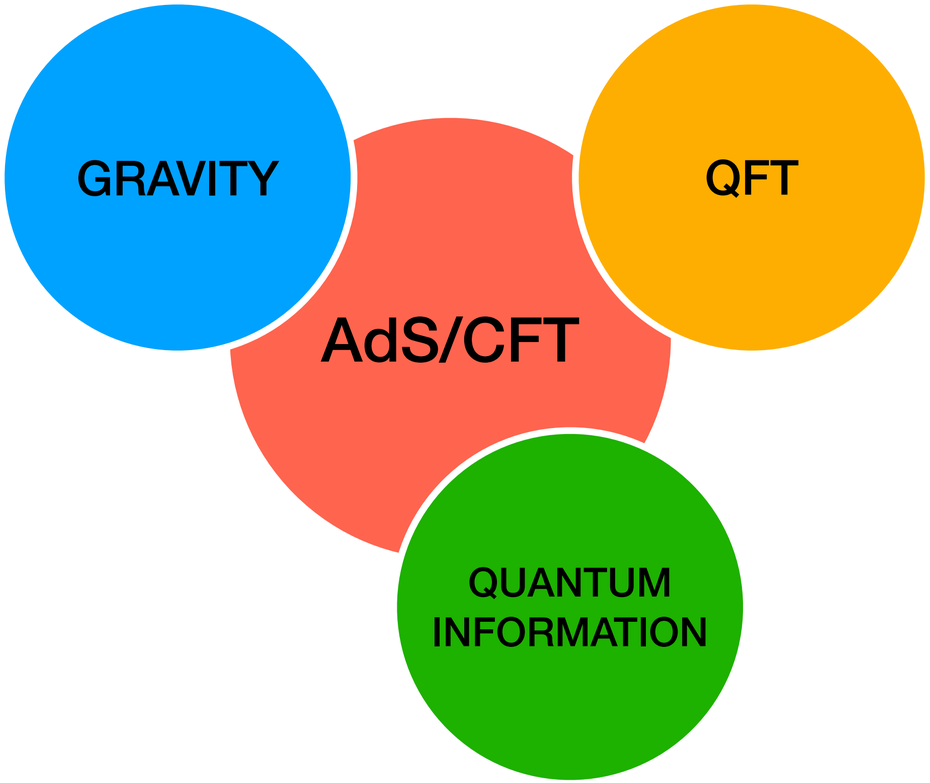
Quantum Information

Quantum information theory has emerged as a powerful language and organizing principle to structure our thinking about many deep questions in QFT and gravity

Study of entanglement bolstered by the prospect of using it a quantum resource and to perform quantum computations has led to a fruitful across-disciplines dialogue involving quantum information theory, QFT and gravity

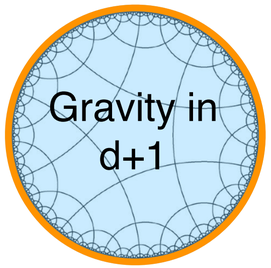
Entanglement entropy and other quantum information measures have evolved from being a probe to study and characterize new phases of matter to represent an indispensable theoretical tool for understanding fundamental properties of field theory and gravity

Introduction


GRAVITY
QFT
AdS/CFT
QUANTUM
INFORMATION

AdS/CFT Correspondence

Conjectured mathematical equivalence between two different theories: quantum field theories in d spacetime dimension and quantum gravity theories in d+1 spacetime dimensions



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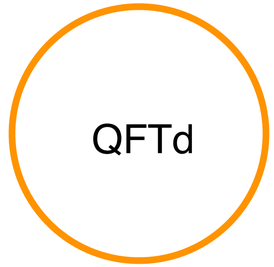
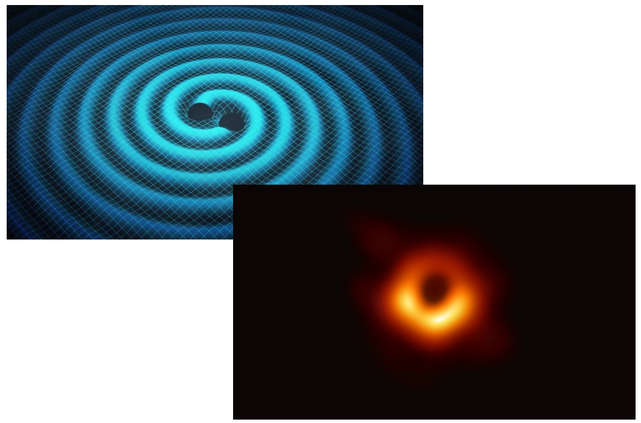
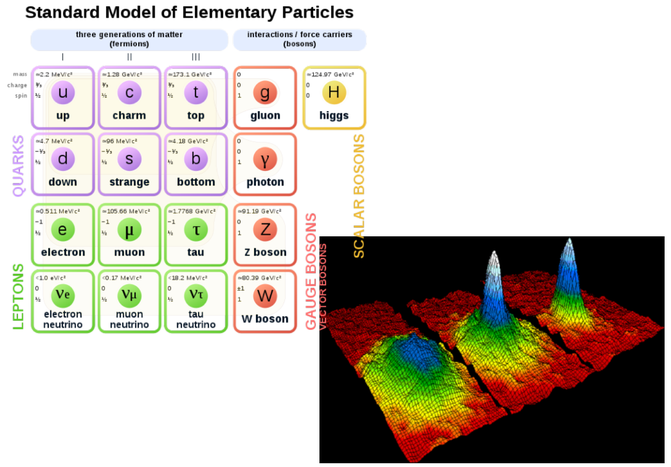


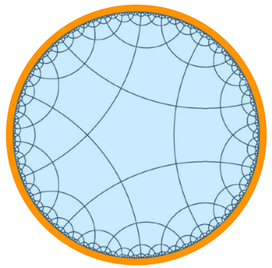




AdS/CFT Correspondence

Conjectured mathematical equivalence between two different theories: quantum field theories in d spacetime dimension and quantum gravity theories in d+1 spacetime dimensions

BULK



Gravity in Anti-DeSitter space-time
in d+1 dimensions

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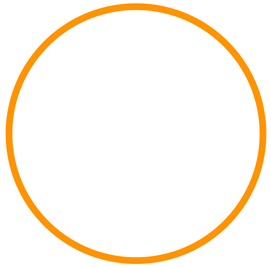
BOUNDARY



Conformal Field Theory
in d dimensions

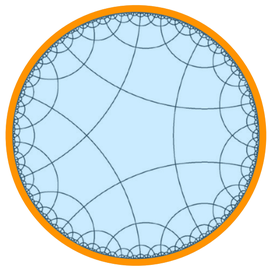
Originally formulated in string theory, but understood to be more general

Concrete realization of the holographic principle

Weak-Strong Coupling

Allows to recast difficult problems on one side in simpler terms using the dual description:
non-perturbative strongly coupled regime of QFT described in terms of classical gravity

compute here…



(semi)-classical
gravity in AdS

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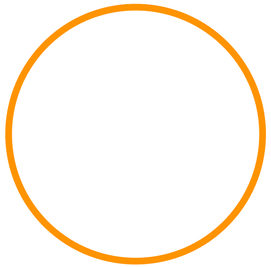
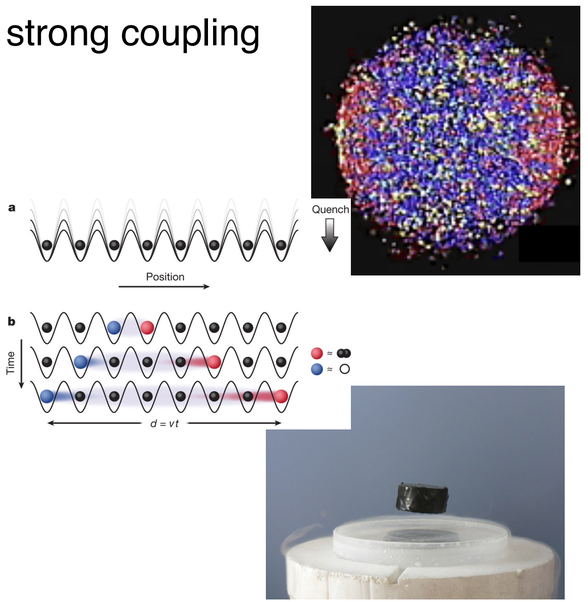


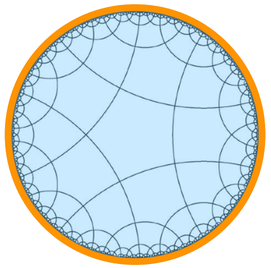
…to answer questions here

QFT at
strong coupling



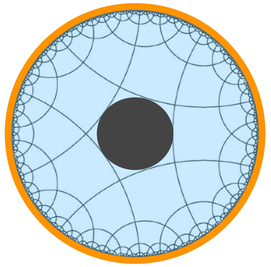
Holographic Dictionary

There exists a “dictionary” relating quantities on the two sides of the duality



Empty AdS = CFTs in its vacuum state

$|0\rangle$



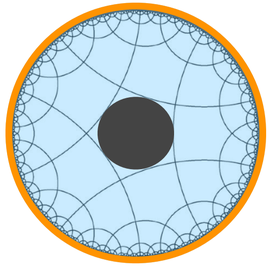
Black holes in AdS = CFTs at finite temperature

$\rho = e^{-\beta H}$

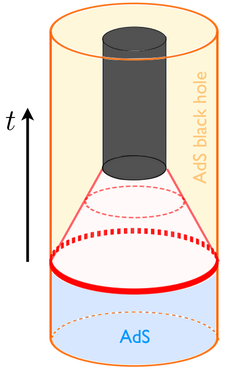
Φ Fields in AdS

Operators $\mathcal{O}$

Holographic Predictions



Falling into a black hole = dissipation process in a thermal plasma



Black holes creation in AdS = thermalization/equilibration process

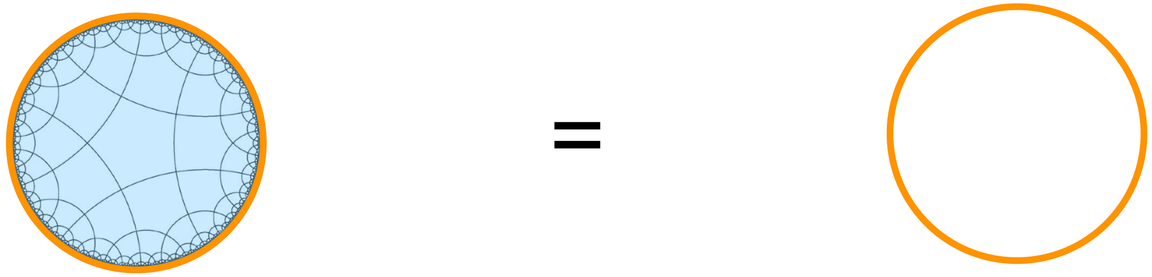
Gravitational predictions:

- Hydrodynamics has very low viscosity $\frac{\eta}{s} = \frac{1}{4\pi}$ [Kovtun,Son,Starinets 01]
- Thermalization proceeds maximally fast [Balasubramanian,Bernamonti,deBoer,Craps et al.11]

Ever growing dictionary, new entries as we understand better the duality

Emergent Gravity

A further reaching implication of the AdS/CFT correspondence linked to its holographic character: a complete microscopic description of gravity in AdS should be contained in the dual field theory



How gravity and spacetime geometry are encoded in the boundary CFT degrees of freedom?

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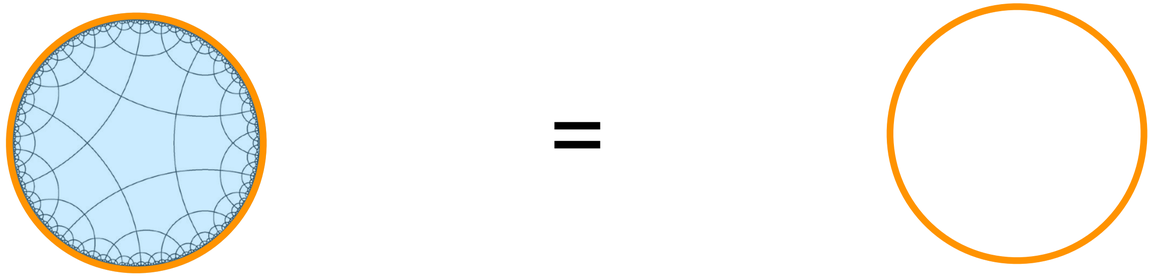
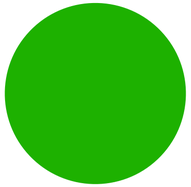


How gravity and spacetime geometry are encoded in the boundary CFT degrees of freedom?



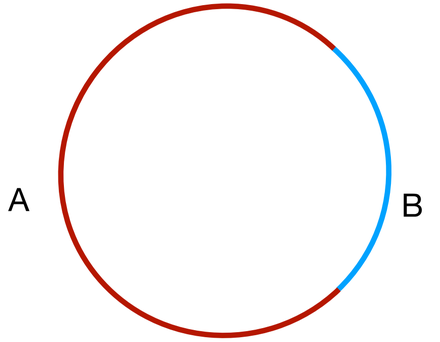
Quantum Information: promising framework to try and answer this question and gain a deeper understanding of holography

Entanglement

Key role played by the notion of entanglement and its measure in terms of entanglement entropy:

QFT in a state $\rho = |\psi\rangle\langle\psi|$

Entanglement entropy of a spatial subsystem A with its complement



$$S(A) = -\mathrm{Tr}_A\, \rho_A \log \rho_A \qquad \rho_A = \mathrm{Tr}_{\bar{A}}\, \rho \qquad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

Measures the amount of entanglement between d.o.f. in A and the rest (for a pure state)

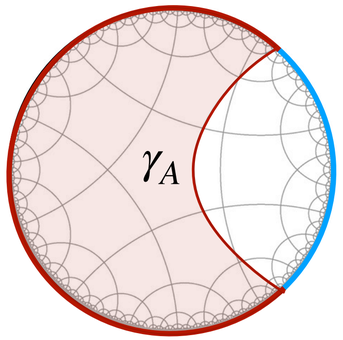
$S(A) \sim$ number of Bell pairs that entangle A and B

Encodes interesting information about the QFT (state, number of degrees of freedom…)

Holographic Entanglement Entropy

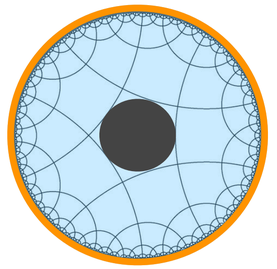
Entanglement entropy for a region A in CFT = Minimal area surface extending in AdS

[Ryu,Takayanagi 06]



$$S(A) = \frac{\mathrm{Min}(\mathrm{Area}_{\gamma_A})}{4G_N}$$

Microscopic generalization of the black hole entropy formula



$$S_{\mathrm{BH}} = \frac{\mathrm{Area(horizon)}}{4G_N}$$

[Bekenstein,Hawking 73]

→ Direct connection between bulk geometry and quantum correlation structure of states in the dual field theory on the boundary

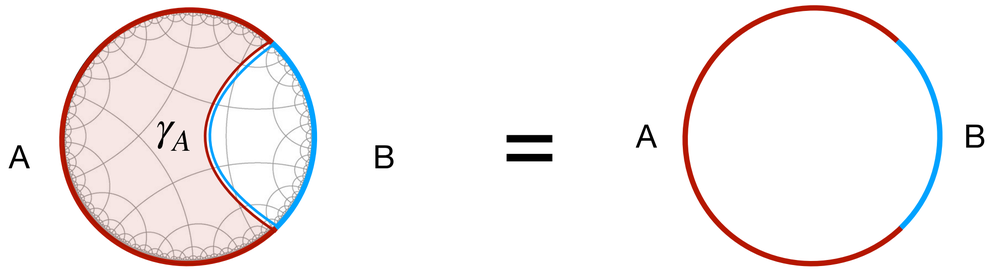
Quantum Information Constraints

First law of entanglement entropy: for an infinitesimal variation of the state $\rho_A + \delta\rho_A$

$\delta S(A) = \delta\langle H(A)\rangle$ quantum version of the 1st law of thermodynamics

Completely general property following in a simple way from the definition of $S(A)$

Assuming the holographic entropy formula

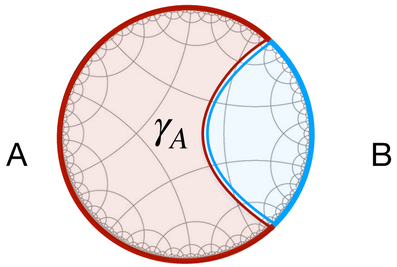


$$S(A) = \frac{\mathrm{Min}(\mathrm{Area}_{\gamma_A})}{4G_N} \qquad S(A) = -\mathrm{Tr}\rho_A \log \rho_A$$

→ The first law of entanglement on the boundary implies linearized Einstein's equations in the AdS bulk! [Faulkner,Guica,Hartman,Myers,VanRaamsdonk 14]

Geometry from Entanglement

Concrete connection between spacetime geometry and entanglement



Holographic Entanglement Entropy

$$S(A) = \frac{\mathrm{Min}(\mathrm{Area}_{\gamma_A})}{4G_N}$$

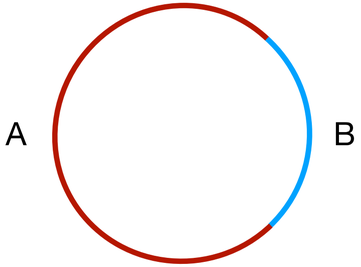
Einstein's Equations

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G_N T_{\mu\nu}$$

$=$

$\longleftrightarrow$

$\longleftrightarrow$



Entanglement Entropy

$$S(A) = -\mathrm{Tr}\rho_A \log \rho_A$$

First Law of
Entanglement Entropy

$$\delta S(A) = \delta\langle H(A)\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

Can spacetime and gravity be described in quantum information theoretical terms and as emerging from the underlying quantum correlations structure?

Take-Home Message

AdS/CFT correspondence gives a concrete realization of the holographic principle and offers a framework to explore the idea of emergent gravity and spacetime in a controlled setting

Gravity geometrizes quantum information measures and concepts

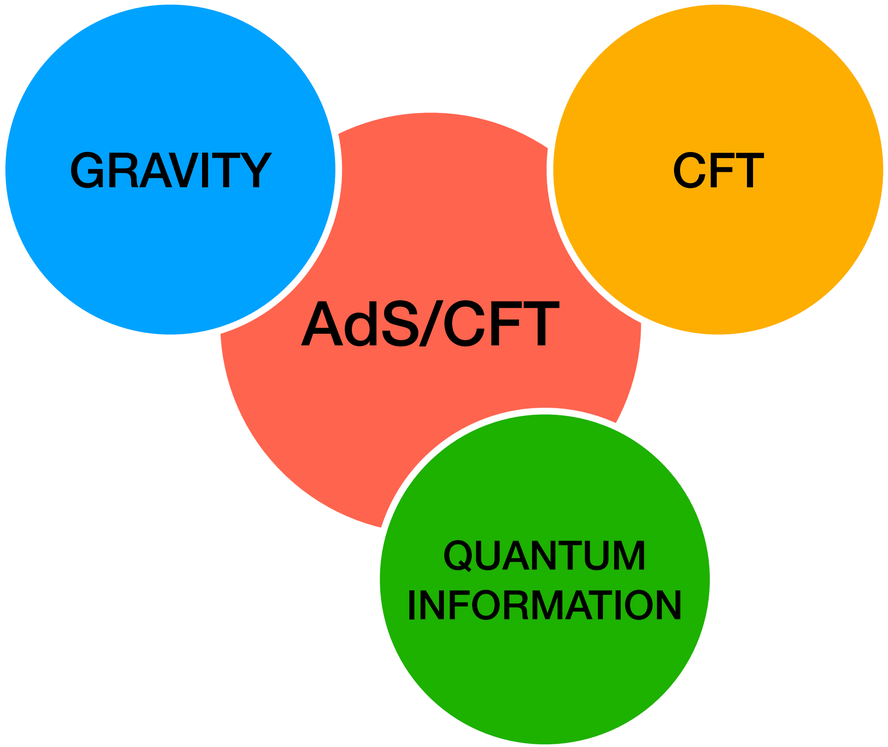
Thinking in terms of entanglement and quantum correlations gives a novel perspective on quantum field theory and gravity

Promising avenue to explore in order to gain a deeper understanding of holography

My Fellini Project

How holographic spacetime emerges from the underlying non-gravitational degrees of freedom?

- Explore further the interplay between quantum information and holography
- Understand the structure of quantum correlations in states of CFT with a gravity dual
- Characterize CFTs that admit holographic dual and how they encode gravitational structures
- Draw lessons beyond the AdS/CFT context?



- Quantum information measures: complementary information to the one given by entanglement entropy

- Quantum information constraints: have a general character and can act as a guiding principle to get new insight both for holography, and for gravity and QFT

- New instances of the holographic duality: help refine our understanding of the duality and extract new lessons

Quantum Circuit Complexity

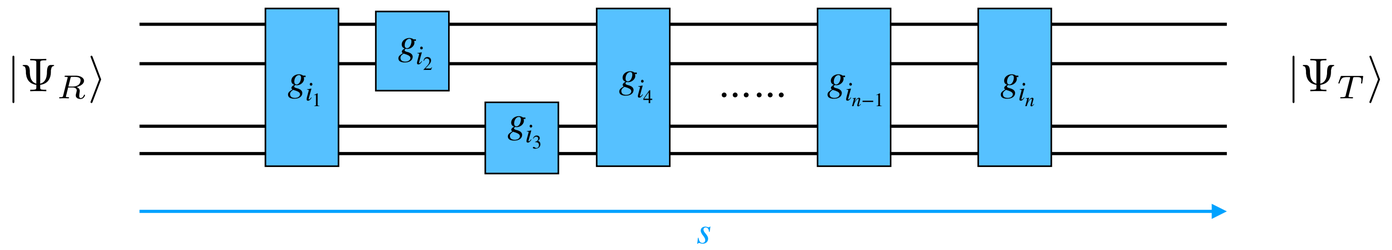
A. Bernamonti, FG, J. Hernandez, R. Myers, S. Ruan and J. Simón
PRL 2019 & J. Phys. A: Math. Theor. 2020

A. Bernamonti, F. Bigazzi, D. Billo, L. Faggi and FG JHEP 2021

Quantum Circuit Complexity

How difficult is it to prepare a particular state?

Given a simple reference state, how hard it is to prepare a target state acting with a unitary constructed from a set of generators of elementary gates



$$|\Psi_T\rangle = U_T|\Psi_R\rangle = g_{i_n} \dots g_{i_1}|\Psi_R\rangle$$

Complexity quantifies the cost of the optimal circuit generating U_T

Nielsen's geometric approach

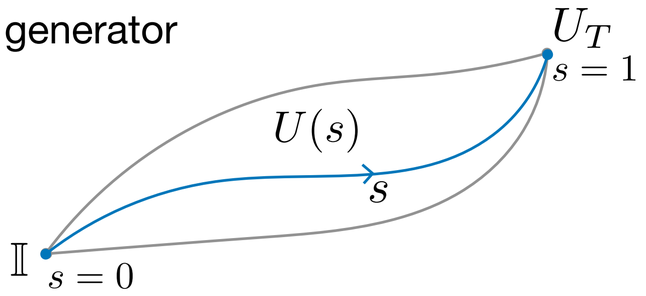
Continuous representation of unitary transformations

[Nielsen et al.]

$$U(\sigma) = \overleftarrow{\mathcal{P}} \exp\left[-i \int_0^\sigma ds H(s)\right] \qquad H(s) = \sum Y^I(s)\mathcal{O}_I$$

gate generator

control fct.



Associates a cost to each trajectory in the space of unitaries

$$\mathcal{C}\left(|\Psi_T\rangle\right) \equiv \text{Min} \int_0^1 ds\, F(U, Y^I)$$

cost fct.

Complexity: globally cost-minimizing trajectory in the space of unitaries.

Classical mechanic problem with the cost function F playing the role of the Lagrangian

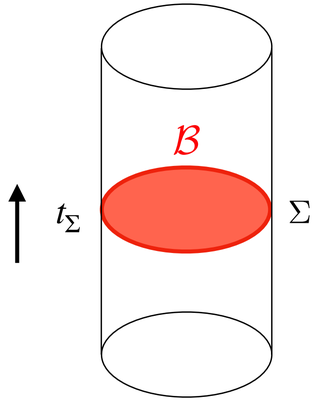
This approach has been used to define complexity in QFT in a number of cases

Holographic Complexity Proposal

Different gravitational observables in AdS have been proposed as the holographic dual of the complexity of a target state $|\Psi_T\rangle$ on a boundary Cauchy slice Σ

Maximal-Volume bulk hypersurface $\mathcal{B}$ anchored at the boundary on Σ

$$\mathcal{C}_{\rm V} = \max_{\partial\mathcal{B}=\Sigma} \frac{V(\mathcal{B})}{G_N \ell_{\rm bulk}}$$

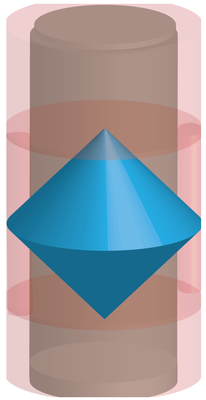


Complexity=Volume

[Susskind,Stanford 15]

Gravitational Action evaluated on the Wheeler-DeWitt patch associated to Σ

$$\mathcal{C}_A(\Sigma) = \frac{I_{\rm WDW}}{\pi}$$



Σ

Complexity=Action

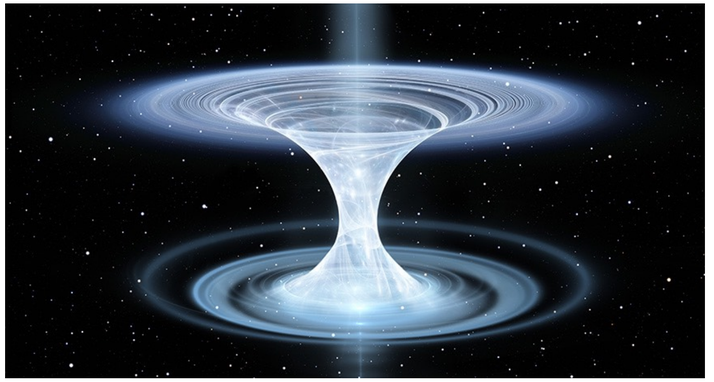
WDW patch: domain of causal dependence of a bulk slice anchored to Σ

[Brown,Roberts,Susskind,Swingle,Zhao 16]

Why Complexity?

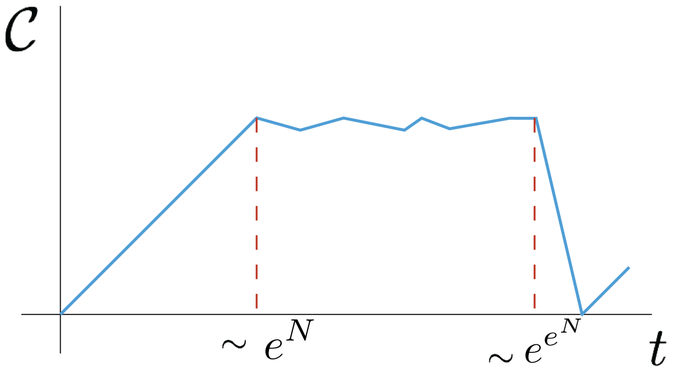
Main evidence comes from the study of AdS Black holes

Black holes grows forever: even though from the outside they appear to stay constant in size, their interior geometry keeps growing (wormhole linear growth)

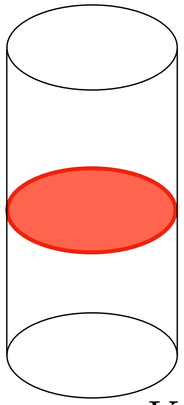


Standard observables (e.g. correlators) in holographic theories thermalize in $t \sim \log N$

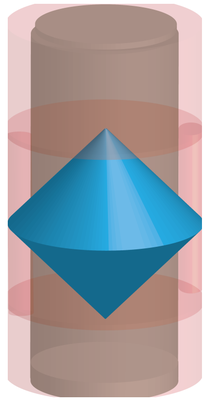
Holographic complexity captures internal black hole growth and grows linearly for times $t \sim e^N$ as expected for complexity



Holographic Complexity?



$$\mathcal{C}_{\rm V} = \max_{\partial\mathcal{B}=\Sigma} \frac{V(\mathcal{B})}{G_N \ell_{\rm bulk}}$$



$$\mathcal{C}_A(\Sigma) = \frac{I_{\rm WDW}}{\pi}$$

?

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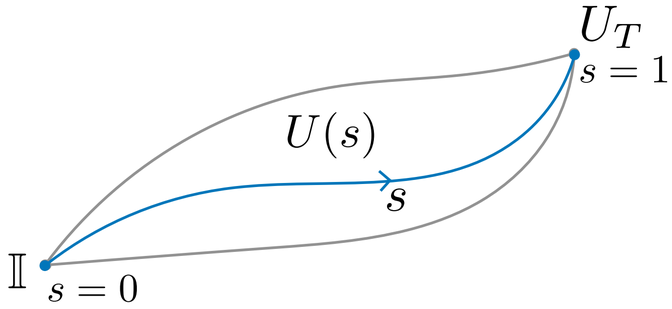


$$\mathcal{C}\left(|\Psi_T\rangle\right) \equiv \text{Min} \int_0^1 ds\, F(U, Y^I)$$

Still have a very partial understanding:

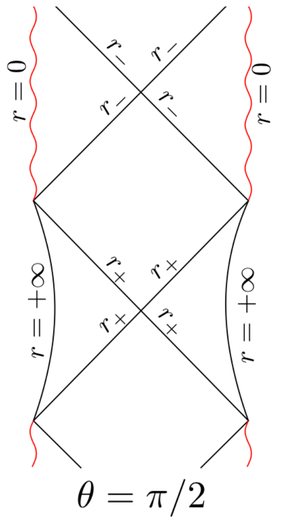
- Understanding of holographic observables far from being exhaustive
- General QFT results are still lacking (mostly free QFT computations)
- Incomplete map between the two sides

Complexity of States with Rotation

Can we gain a deeper understanding of the holographic proposals?

States with rotation: rich structure and interesting limits where to test different measures

Holographically these corresponds to spinning black holes in AdS



Kerr-AdS_4

- Better understanding of the gravitational observables associated with complexity: resolved some ambiguities in the definitions, clarified previous results obtained in the literature
- Critical rotation velocity, high T limit: qualitative agreement between gravitational and field theory complexity measures

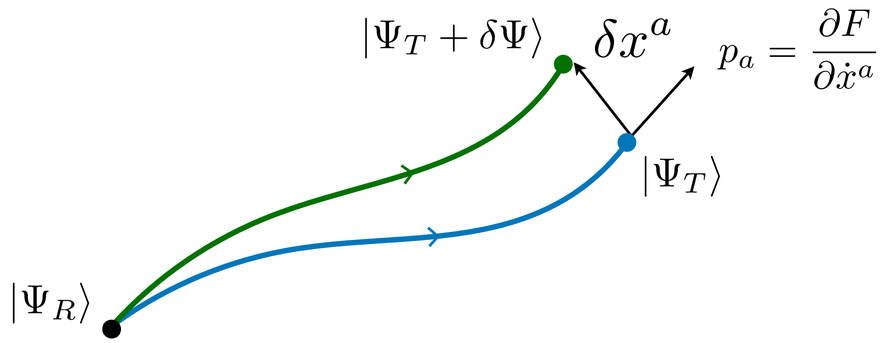
[Bernamonti, Bigazzi, Billo, Faggi, FG JHEP ’21]

First Law of Complexity

Can we build a connection between the field theory and holographic directions?

How complexity varies under a small change of the target state

$$\delta\mathcal{C} \equiv \mathcal{C}(|\Psi_T + \delta\Psi\rangle) - \mathcal{C}(|\Psi_T\rangle)$$



Using Nielsen's approach: general form of this variation

$$\delta C = p_a \delta x^a \Big|_{s=1} + \frac{1}{2}\delta p_a \delta x^a \Big|_{s=1}$$

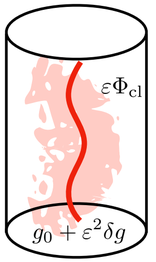
Only contributions from the endpoints

[Bernamonti, FG, Hernandez, Myers, Ruan, Simón PRL'19 & JPA'20]

First Law of Complexity

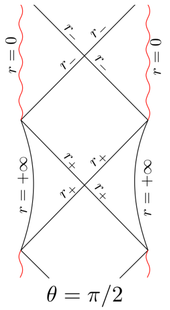
Vacuum perturbation: - coherent scalar states

[Bernamonti, FG, Hernandez, Myers, Ruan, Simón PRL'19 & JPA'20]



- non-vanishing rotation

[Bernamonti, Bigazzi, Billo, Faggi, FG JHEP'21]



- Holographic complexity variation is second order in the amplitude of the perturbation: coherent and rotating states orthogonal to vacuum AdS direction
- Restrictions on field theory costs compatible with holography
- Complexity=action result localized on the boundary of the WDW

Generalized Second Laws

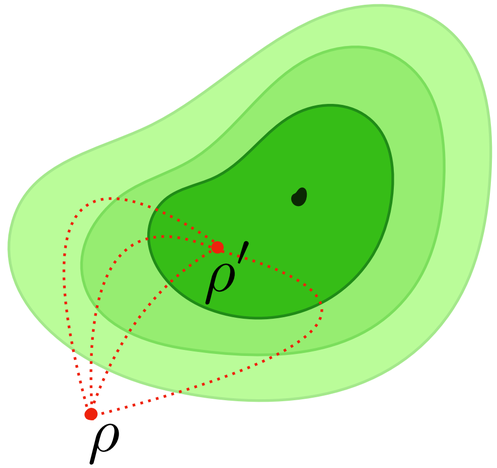
A. Bernamonti, FG, I. Reyes and R. Myers — IN PROGRESS

A. Bernamonti, FG, R. Myers, J. Oppenheim JHEP 2018

Second Law

Second law of thermodynamics: the entropy of a closed system can never decrease

$S(0) \leq S(t)$: necessary condition for any transition between states $\rho \longrightarrow \rho'$

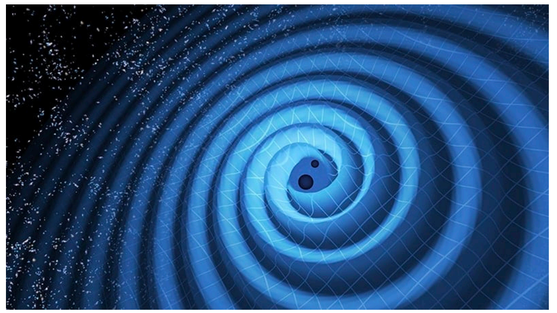


Sharp geometric interpretation in gravity:

$$S_{\rm BH} = \frac{\rm Area(horizon)}{4G_N}$$

Area theorem

Restrictions on the dynamics without solving it!



Generalized Second Laws

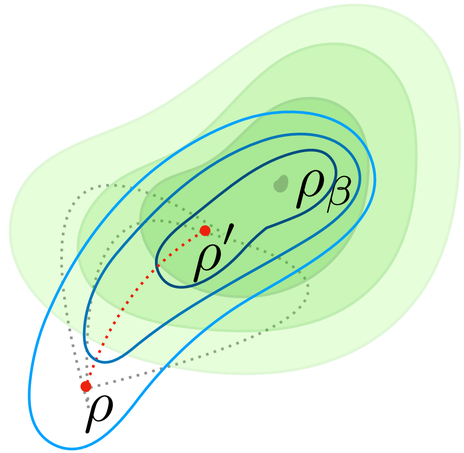
Additional laws?

Quantum thermodynamics: additional constraints on equilibration processes for quantum systems and macroscopic systems with long range correlations [Horodecki, Oppenheim 11] [Brandao, Horodecki, Ng, Oppenheim, Wehner 13] ...

Expressed in terms of quantities giving a notion of distance between states and their monotonicity property under transitions

$$D(\rho\,||\,\rho_\beta) \leq D(\rho'\,||\,\rho_\beta) \quad \text{necessary condition for} \quad \rho \longrightarrow \rho'$$

reference state



New constraints for quantum field theory and black holes, beyond the second law?

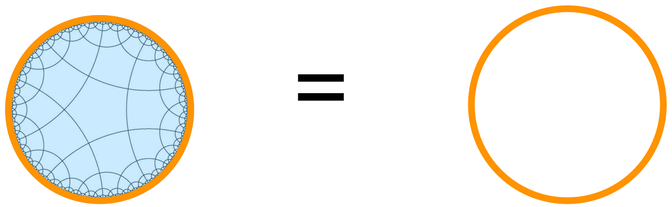
Rényi Divergences

Monotonicity of Rényi Divergences under transitions

$$D_\alpha(\rho||\rho_\beta) \equiv \frac{\mathrm{sgn}(\alpha)}{\alpha-1}\log \mathrm{tr}(\rho^\alpha \rho_\beta^{1-\alpha})$$

$$D_\alpha(\rho||\rho_\beta) \geq D_\alpha(\rho'||\rho_\beta) \qquad \text{necessary for } \rho \longrightarrow \rho'$$

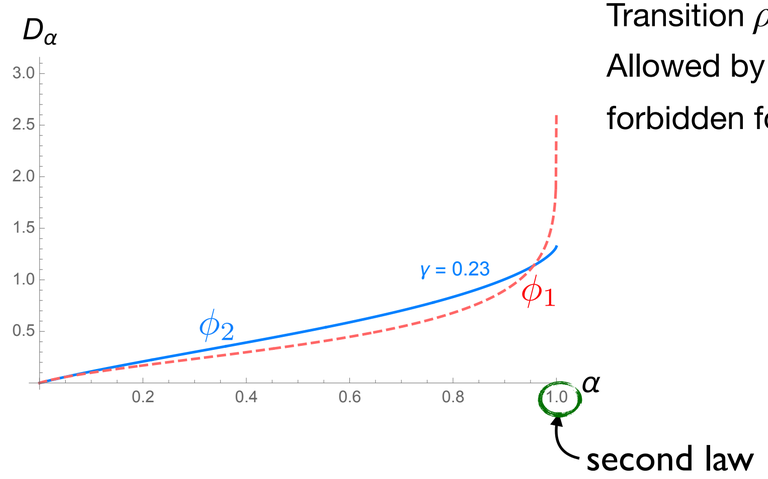
In a holograpic setup:



AdS black holes
with scalar excitations ϕ_i

=

Class of CFT
excited states



Transition $\rho_1 \to \rho_2$?
Allowed by the second law but
forbidden for small values of α

Non-trivial conditions in QFT and gravity!

[Bernamonti, FG, Myers, Oppenheim 18]

Rényi Entropies

Rényi Entropies also give a family of monotonicity constraints

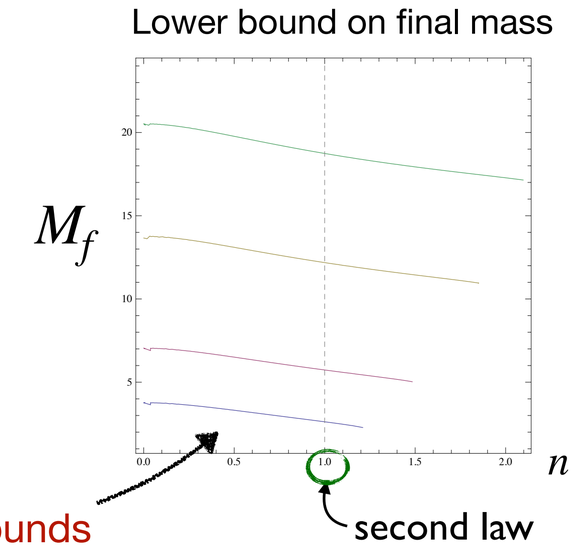
$$S_n(\rho) = \frac{1}{1-n} \log \mathrm{Tr}\rho^n \qquad S(\rho) \leq S(\rho') \qquad \text{necessary for } \rho \longrightarrow \rho'$$

Can be computed for black holes and used to study bounds on processes like black hole mergers —just like for the usual second law

Example:

AdS4-BH mergers

$$M_i + M_i \to M_f$$



[Bernamonti, FG, Myers, Reyes IN PROGRESS]

Perspectives

- Pin down bounds in processes that are informative for numerical general relativity simulations, e.g. Kerr-AdS4 super-radiant instability [Green,Holland,Ishibashi,Wald 15] [Chesler 21]

- Geometric interpretation of the bounds analog to the area theorem?

- Extension to black holes in flat spacetime

Entanglement Islands and Double Holography

A. Bernamonti, R. Emparan, A. Frassino and FG — IN PROGRESS

A. Bernamonti, D. Ge, FG and D. Neuenfeld — IN PROGRESS

Entanglement Islands

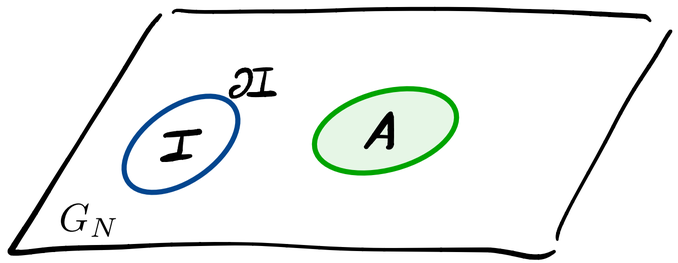
Developments in the general context described above and in the study of the entanglement entropy in gravitational systems led to the proposal that:

in a theory with dynamical gravity the entanglement entropy between regions is computed by the Island Formula

$$S_{\text{ISLAND}}(A) = \text{Min}\left\{\text{ext}_I\left[\frac{\text{Area}(\partial I)}{4G_N} + S(A \cup I)\right]\right\}$$

geometric contribution
Bekenstein-Hawking-like

QFT entanglement
entropy



Allows for contributions from additional regions of spacetime: islands

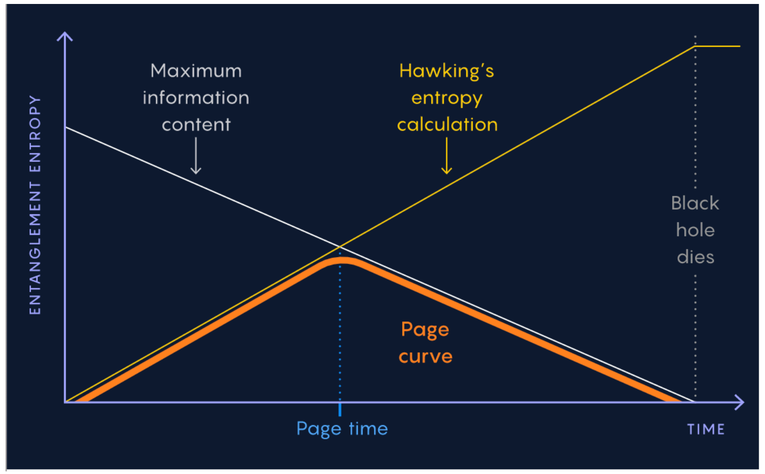
[Almheiri,Engelhardt,Marolf,Maxfield 19]

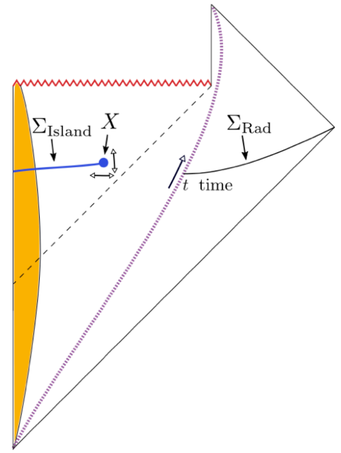
[Penington 19]

[Almheiri,Mahajan,Maldacena,Zhao 19]

Black Hole Information Paradox

Island formula yields within semi-classical gravity an evolution of the entanglement entropy for a black hole evaporation process compatible with unitarity





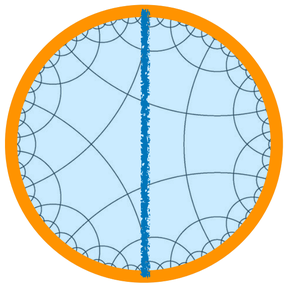
Part of the black hole interior geometry associated with the exterior “radiation” region

Holographic meaning of the island? What information exactly encodes?

Double Holography

Systems with a double holographic description allow for three complementary points of view:

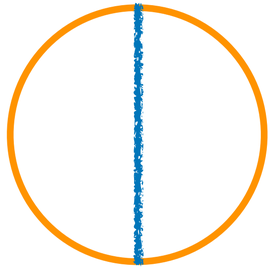
BULK



AdS(d+1) + brane

=

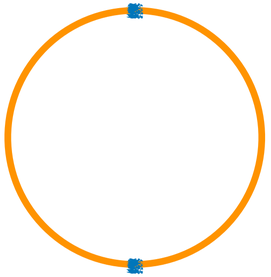
BRANE-WORLD



AdS d + CFT d

=

BOUNDARY



CFT d + defects

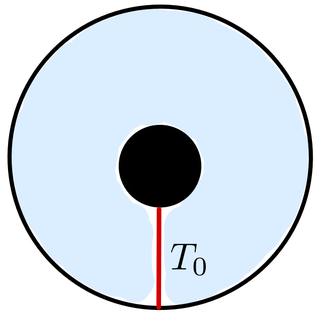
Presence of an entanglement island in the brane-world picture related to that of conformal defects on the boundary / brane in the bulk

New quantum information perspective on earlier realizations of the AdS/defect-CFT correspondence

Dynamical Double Holography

Ongoing collaborations were we are building dynamical realizations of this kind of doubly holographic models and studying the information one can obtain using the island formula

BULK



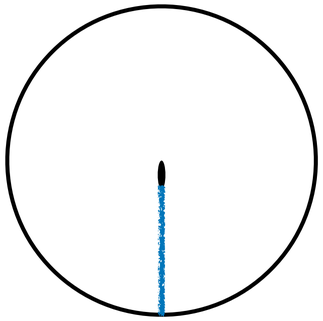
BTZ BH + brane =
Accelerated AdS3 BH

$$ds^2 = -(r^2 - r_h^2)dt^2 + \frac{dr^2}{r^2 - r_h^2} + r^2 d\phi^2$$

$$S_b = -T_0 \int d^2x\sqrt{-h}$$

=

BRANE-WORLD



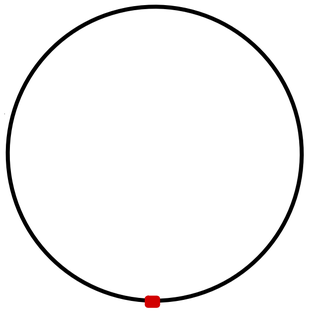
AdS2 BH + 2d CFT bath

$$ds_b^2 = -\left(\frac{\rho^2}{\ell_b^2} - r_h^2\right) dt^2 + \frac{d\rho^2}{\frac{\rho^2}{\ell_b^2} - r_h^2}$$

$$\frac{\ell_b^2}{L^2} = k^2 + 1 = \frac{1}{1 - 16\pi^2 G_N^2 L^2 T_0^2}$$

=

BOUNDARY



2d CFT with a defect

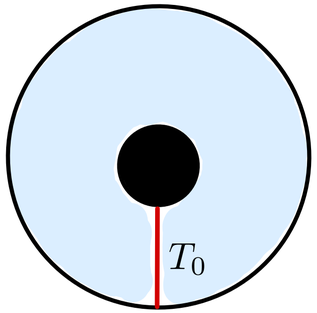
$$c_{def} = 2\log g = \frac{c}{3}\,\mathrm{arcsinh}\, k$$

[A. Bernamonti, R. Emparan, A. Frassino and FG — IN PROGRESS]

Evaporating the brane

Reducing the tension T_0 parameter:

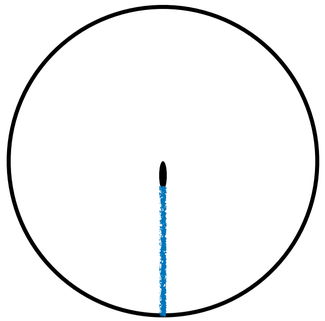
BULK



BTZ BH decelerates: moves to the center of AdS3

=

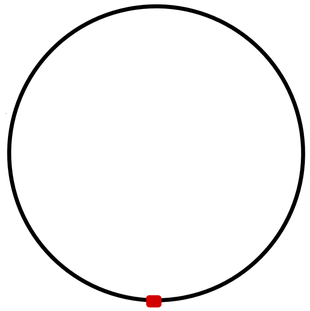
BRANE-WORLD



AdS2 black hole shrinks, brane-world disappears into the AdS3 spacetime

=

BOUNDARY



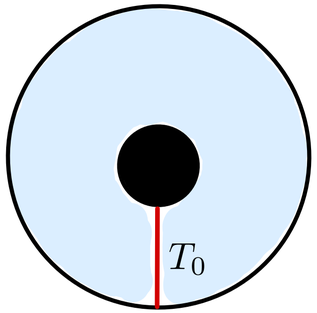
Degrees of freedom associated with the defect are decreased: defect disappear

[A. Bernamonti, R. Emparan, A. Frassino and FG — IN PROGRESS]

Evaporating the brane

Reducing the tension T_0 parameter:

BULK



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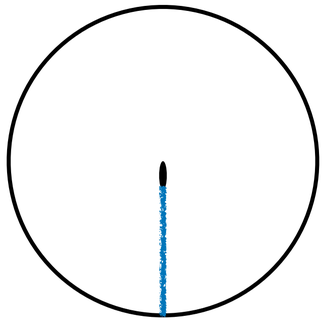
BRANE-WORLD



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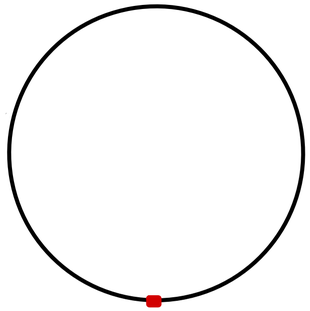
=

BOUNDARY



Degrees of freedom associated with the defect are decreased: defect disappear

Make it dynamical: concrete realization of a AdS2 BH evaporation process where to study the island formula and the information about the AdS/defect-CFT it encodes

[A. Bernamonti, R. Emparan, A. Frassino and FG — IN PROGRESS]

Conclusions

Quantum Information & AdS/CFT

Holographic principle offers a framework to explore the idea of emergent gravity and spacetime in a controlled setting

Interplay with quantum information has led to a fruitful dialogue stimulating progress in our understanding of holography, gravity and field theory

Many interesting directions and lots to explore!

Thank you!