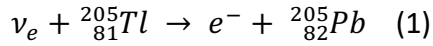


### Problem

Determine the solar neutrino capture rate in units of SNU for the following process:



Assume that  ${}^{205}\text{Pb}$  goes to the first excited state with  $J^\pi=1/2^-$  with energy  $\Delta E=0.002$  MeV. Assume that the ground state of  ${}^{205}\text{Tl}$  has a 10% component in the configuration:  $p[82](3s_{1/2})^{-1}$  and  $n[126](2f_{5/2})^{-2}$  with  $J^\pi=1/2^+$ .

### Solution

The capture rate per target particle is written:

$$R_{cap} = \phi_\nu \cdot \frac{\Gamma}{c} = \phi_\nu \cdot \frac{2\pi}{\hbar c} \cdot M_{205}^2 \cdot \frac{dn}{dE_e} \quad (2)$$

where  $E_e$  is the electron energy,  $dn/dE_e$  the density of states within  $E_e$ ,  $E_e+dE_e$ ,  $\phi_\nu$  the neutrino flux,  $M_{205}$  the matrix element for the capture transition.

The 1<sup>st</sup> excited state in  ${}^{205}\text{Pb}$  has a configuration equal to:  $p[82]$  and  $n[126](2f_{5/2})^{-2}(3p_{1/2})^{-1}$ . So the capture implies that one neutron in  $3p_{1/2}$  goes to one proton in  $3s_{1/2}$ .

In (2) we have assumed  $\sigma=\Gamma/c$  as the cross-section per unit interaction volume. It turns out that  $[R_{cap}]=s^{-1} \text{ cm}^{-3}$

For (1) energy conservation gives:

$$E_e = E_\nu - \Delta M - \Delta E + m_e c^2 \quad (3)$$

where  $\Delta M = M({}^{205}\text{Pb}) - M({}^{205}\text{Tl}) = 0.051$  MeV (using the atomic mass table in a.m.u and  $931.5$  MeV = 1 a.m.u.).

Because (1) is sensitive to pp solar neutrinos which are dominant, we know that on average  $E_\nu = 0.266$  MeV. Therefore, on average  $E_e = 0.72$  MeV.

It turns out that:

$$\frac{dn}{dE_e} = \frac{4\pi p_e^2 dp_e V}{(2\pi\hbar)^3 dE_e} = \frac{\sqrt{E_e^2 - m_e^2 c^4} E_e}{2\pi(\hbar c)^3} V \quad (4). \text{ We use } V = 1.$$

Using the electron average energy:  $\frac{dn}{dE_e} \approx \frac{0.018 \text{ MeV}^2}{(\hbar c)^3}$  per unit interaction volume.

Therefore,

$$R_{cap} \approx 6.5 \cdot 10^{10} \text{ cm}^{-2} \text{ s}^{-1} \frac{2\pi}{\hbar c} M_{205}^2 \frac{0.018 \text{ MeV}^2}{(\hbar c)^3} \quad (5)$$

Taking into account that  $[M_{205}^2] = \text{MeV}^2 \text{ cm}^3$  and  $\hbar c = 197.327 \cdot 10^{-13} \text{ MeV cm}$ , the dimensional analysis of (5) gives  $[R_{cap}] = \text{s}^{-1} \text{ cm}^{-3}$  as expected.

We need to determine  $M_{205}^2$ .

We notice that in the beta decay:  ${}^{206}_{81}\text{Tl} \rightarrow {}^{206}_{82}\text{Pb} + e^- + \bar{\nu}_e$  with  $\log f_t = 5.2$  the transition from the initial to the final state goes from:  $p[82](3s_{1/2})^{-1}$  and  $n[126](3p_{1/2})^{-1}$  to  $p[82]$  and  $n[126](3p_{1/2})^{-2}$ .

Therefore, in both the beta decay and the capture process one neutron in  $3p_{1/2}$  goes to one proton in  $3s_{1/2}$ . This implies that the two processes have the same transition matrix. From  $\log f_t$  we obtain that  $M_{206}^2 = 4.4 \times 10^{-88} \text{ MeV}^2 \text{ cm}^3$  (see beta decay theory for the relationship between  $f_t$  and the transition matrix).

In conclusion:

$$R_{cap} \approx 6.5 \cdot 10^{10} \text{ cm}^{-2} \text{ s}^{-1} \frac{2\pi}{\hbar c} 0.1 M_{206}^2 \frac{0.018 \text{ MeV}^2}{(\hbar c)^3} \approx 2.1 \cdot 10^{-36} \text{ capture/sec/target/unit}$$

volume of interaction = 2 SNU.

NOTE: We notice that the 90% of  ${}^{205}\text{Tl}$  ground state is in  $p[82](3s_{1/2})^{-1}$  and  $n[126](3p_{1/2})^{-2}$  with  $J^\pi=1/2^+$ . From this configuration the neutrino capture discussed above implies one neutron from  $2f_{5/2}$  to go to one proton in  $3s_{1/2}$  and at the same time one proton from  $2f_{5/2}$  to go to one neutron in  $3p_{1/2}$ . This is a two-particle process and it has a very low probability. Therefore, the capture involves the 10% configuration of the  ${}^{205}\text{Tl}$  ground state.