Aldo Ianni, ISAPP 2023

Problem

Determine the solar neutrino capture rate in units of SNU for the following process:

$$\nu_e + {}^{205}_{81}Tl \rightarrow e^- + {}^{205}_{82}Pb \quad (1)$$

Assume that ^{205}Pb goes to the first excited state with $J^\pi=1/2^-$ with energy $\Delta E=0.002$ MeV.
Assume that the ground state of ^{205}Tl has a 10% component in the configuration:
$p[82](3s_{1/2})^{-1}$ and $n[126](2f_{5/2})^{-2}$ with $J^\pi=1/2^+$.

Solution

The capture rate per target particle is written:

$$R_{cap} = \phi_\nu \cdot \frac{\Gamma}{c} = \phi_\nu \cdot \frac{2\pi}{\hbar c} \cdot M_{205}^2 \cdot \frac{dn}{dE_e} \quad (2)$$

where E_e is the electron energy, dn/dE_e the density of states within E_e, E_e+dE_e, ϕ_ν the neutrino flux, M_{205} the matrix element for the capture transition.

The 1st excited state in ^{205}Pb has a configuration equal to: p[82] and $n[126](2f_{5/2})^{-2}(3p_{1/2})^{-1}$.
So the capture implies that one neutron in $3p_{1/2}$ goes to one proton in $3s_{1/2}$.

In (2) we have assumed $\sigma=\Gamma/c$ as the cross-section per unit interaction volume.
It turns out that $[R_{cap}]=s^{-1}\,cm^{-3}$

For (1) energy conservation gives:

$$E_e = E_\nu - \Delta M - \Delta E + m_e c^2 \quad (3)$$

where $\Delta M = M(^{205}Pb)-M(^{205}Tl) = 0.051$ MeV (using the atomic mass table in a.m.u and 931.5 MeV = 1 a.m.u.).

Because (1) is sensitive to pp solar neutrinos which are dominant, we know that on average $E_\nu = 0.266$ MeV. Therefore, on average $E_e = 0.72$ MeV.

It turns out that:

$$\frac{dn}{dE_e} = \frac{4\pi p_e^2 dp_e V}{(2\pi\hbar)^3 dE_e} = \frac{\sqrt{E_e^2 - m_e^2c^4}\, E_e}{2\pi(\hbar c)^3} V \quad (4).$$ We use V = 1.

Using the electron average energy: $\frac{dn}{dE_e} \approx \frac{0.018\, MeV^2}{(\hbar c)^3}$ per unit interaction volume.

Therefore,

$$R_{cap} \approx 6.5 \cdot 10^{10}\ cm^{-2} s^{-1}\ \frac{2\pi}{\hbar c}\ M_{205}^2\ \frac{0.018\ MeV^2}{(\hbar c)^3} \quad (5)$$

Taking into account that $[M^2_{205}]$ = MeV^2 cm^3 and $\hbar c = 197.327 \cdot 10^{-13}\ MeV\ cm$, the dimensional analysis of (5) gives $[R_{cap}]$ = s^{-1} cm^{-3} as expected.

We need to determine M^2_{205}.

We notice that in the beta decay: ${}^{206}_{81}Tl \rightarrow {}^{206}_{82}Pb + e^- + \bar{\nu}_e$ with log f_t = 5.2 the transition from the initial to the final state goes from: p[82]$(3s_{1/2})^{-1}$ and n[126]$(3p_{1/2})^{-1}$ to p[82] and n[126] $(3p_{1/2})^{-2}$.

Therefore, in both the beta decay and the capture process one neutron in $3p_{1/2}$ goes to one proton in $3s_{1/2}$. This implies that the two processes have the same transition matrix. From log f_t we obtain that M^2_{206} = 4.4x10^{-88} MeV^2cm^3 (see beta decay theory for the relationship between f_t and the transition matrix).

In conclusion:

$R_{cap} \approx 6.5 \cdot 10^{10}\ cm^{-2} s^{-1}\ \frac{2\pi}{\hbar c}\ 0.1\ M_{206}^2\ \frac{0.018\ MeV^2}{(\hbar c)^3} \approx 2.1 \cdot 10^{-36}$ capture/sec/target/unit volume of interaction = 2 SNU.

NOTE: We notice that the 90% of ^{205}Tl ground state is in p[82]$(3s_{1/2})^{-1}$ and n[126]$(3p_{1/2})^{-2}$ with $J^{\pi}=1/2^{+}$. From this configuration the neutrino capture discussed above implies one neutron from $2f_{5/2}$ to go to one proton in $3s_{1/2}$ and at the same time one proton from $2f_{5/2}$ to go to one neutron in $3p_{1/2}$. This is a two-particle process and it has a very low probability. Therefore, the capture involves the 10% configuration of the ^{205}Tl ground state.