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A couple of back-of-the-envelope calculations (“Fermi problems”)

Use information in the lecture slides to estimate the following.

- **#1:** Estimate the number of solar neutrino interactions occurring in a typical human body over a human lifetime. Consider neutrino-electron elastic scattering only. Assume the human body is made of water and that it has a mass of 80 kg. Assume the solar neutrino flux (^{8}B) is $\phi \sim 2 \times 10^6$ cm^{-2}s^{-1}.

- **#2:** Estimate the number of $\nu_e - {}^{40}$Ar charged-current neutrino interactions in a 40-kton liquid-argon detector for a core collapse at the center of the Milky Way. Estimate also the number for a core collapse in M31. (Do not cheat by reading event rates directly off any plot!)

- **#1 SOLUTION**

  Interaction rate is given by $R_{\text{int}} = \sigma\phi N_t$, where σ is the cross section, ϕ is the flux and N_t is the number of targets. For this back-of-the-envelope estimate we will take the σ and ϕ as constant, although a more accurate calculation would fold both σ and ϕ as a function of energy.

  The number of targets is $N_t \sim \frac{MtN_A}{A}$, where M is the mass, N_A is Avogadro's number, A is the molar mass and t is targets per atom or molecule. Plugging in $M = 80$ kg, $t = 10$ electrons per water molecule, $A = 18$ g/mol for water, and $N_A = 6 \times 10^{23}$, and then plugging in (eyeballing from the plots in lecture) $\sigma \sim 5 \times 10^{-44}$ cm^2 (electron scattering cross-section above a few MeV) and $\phi \sim 2 \times 10^6$ cm^{-2}s^{-1} (flux above a few MeV, mostly from ^{8}B neutrinos), and then multiplying by 80 years[1], we get about 7 solar neutrino interactions per lifetime.

- **#2 SOLUTION**

  Here we employ the same relation $R_{\text{int}} = \sigma\phi N_t$. For the cross section we take $\sigma \sim 2 \times 10^{-41}$ cm^2, and for $N_t = 1$ we plug in $M = 4 \times 10^{10}$ g, $A = 40$ g/mol for argon. To find the total fluence (flux integrated over time) at the center of the Milky Way ($\sim$ 8.5 kpc), we assume the neutron star binding energy is about $E_b \sim 2.5 \times 10^{53}$ ergs, and that this energy goes entirely into neutrinos, equally distributed among flavors. If each neutrino has about $E_\nu \sim 15$ MeV, then the fluence per flavor is $F = \frac{E_b}{6 \cdot 4\pi d^2 E_\nu} \sim 2 \times 10^{11}$ cm^{-2} at $d = 8.5$ kpc.

  Plugging this in, we get an expected number of neutrino events of 2600. (Note that a high-energy tail in the spectrum will increase this, as cross section goes as E_ν^2.) The distance to M31 is about 770 kpc, so one expects $2600 \times (\frac{8.5}{770})^2 = 0.31$ neutrino events.

[1] Useful fact: one year is close to $\pi \times 10^7$ seconds.