NEUTRINOS AND MATTER

POWER SPECTRUM

The Power Spectrum of a perturbation field is defined as the expectation value of the Fourier coefficients evaluated at any two locations in Fourier space separated by a vector $\boldsymbol{k}$

$$\langle \delta(\boldsymbol{k}, z)\delta(\boldsymbol{k}', z)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k} + \boldsymbol{k}')P(\boldsymbol{k})$$

The power spectrum can be defined for the individual components (CDM, neutrinos etc.) as well as for a combination of them. Relevant to these lectures is the power spectrum of the matter, which accounts for $\delta_{CDM} + \delta_b + \delta_\nu$, and that of the cold matter only $\delta_C = \delta_{CDM} + \delta_b$.

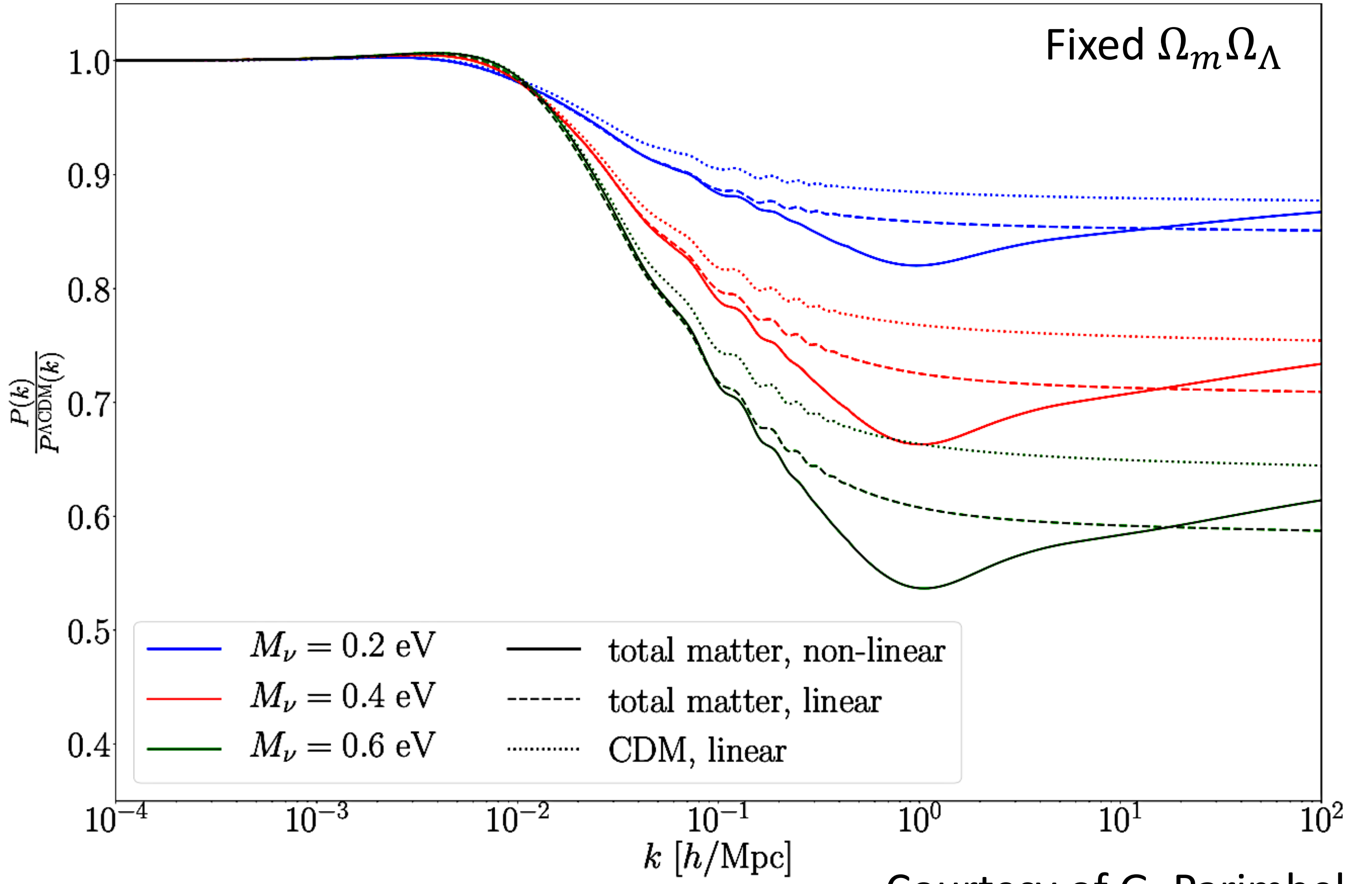
In Matteo Viel's lecture we have seen that the, in presence of massive neutrinos, on scales smaller than that of the horizon the amplitude of the power spectrum is reduced by the combined effect of the un-clustered neutrino component and of the slower growth rate of matter perturbation. In the limit of small f_ν and for large k values:

$$\frac{P(k)^{f_\nu}}{P(k)^{f_\nu=0}} \simeq -8\, f_\nu \qquad \frac{P_c(k)^{f_\nu}}{P_c(k)^{f_\nu\ =0}} \approx -6 f_\nu$$

Neutrinos and matter

Power Spectrum

On scales larger than that of the horizon at the non-relativistic transition, the amplitude of the power spectrum is the same for the massive and massless neutrino cases. On smaller scales the amplitude is damped, reaching a plateau on small scales. The amplitude of the damping depends on the sum of the neutrino masses.



Courtesy of G. Parimbelli

Modeling 2-point clustering statistics

GALAXY POWER SPECTRUM MODEL

The matter power spectrum (and its Fourier counterpart: the 2-point correlation function) is a fundamental cosmological tool.

For two reasons

- It encodes most of the cosmology-relevant information (including the footprint of the neutrino mass). Primordial cosmological fields, like the matter density fluctuation field, are known to be (very close to) Gaussian (e.g. the CMB temperature fluctuation). Gaussian fields are fully characterized by their 2-point statistics.
- It can be inferred from various types of observations. Like the spatial distribution of galaxies (mass tracers) in large spectroscopic surveys. Or the observed, systematic shape distortions of distant galaxies induced by the gravitational lensing effect of the intervening mass distribution.

Let us focus on the observed 3D galaxy power spectrum.

Since in the presence of massive neutrinos galaxies trace the density field of CDM+baryons (Castorina+ 2014) we will model the power spectrum of these two components only.

POWER SPECTRUM

What we observe are the angular positions and the redshifts of a discrete set of luminous tracers of the underlying mass density field. Whose fluctuations, especially on small scales, have evolved nonlinearly. To model the observed galaxy power spectrum we need to account for the mapping mass → galaxies, to the nonlinear evolution of density fluctuations and to other effects.

More specifically the power spectrum model must include:

1. The effect of nonlinear evolution of density fluctuations.
2. The effect of peculiar motions in the mapping between redshift and distances.
3. The impact of assuming a fiducial metric in the mapping between redshift and distances.
4. The possibility that galaxies are a biased tracer of the underlying mass field.
5. The discrete sampling of a continuous density field by a finite number of objects.

POWER SPECTRUM

Let us consider a state-of-the art galaxy power spectrum model (Blanchard+ 2020) that has been used to forecast the performances of the Euclid mission.

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

And use it as a guideline to introduce all the previous effects.

LINEAR MODEL

The starting point is the linear model for the mass power spectrum which, assuming statistical isotropy in galaxy clustering, is a function of the wavenumber modulus $k = |\boldsymbol{k}|$.

Perturbation in the curvature ζ are generated during the inflation with a power spectrum

$$\boxed{\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}}$$

Amplitude of primordial perturbations

scalar spectral index

The power spectrum of fluctuations in component i at the epoch z is obtained by applying the transfer function $\mathcal{T}_i(k, z)$ that accounts for the physical processes that modulate the evolution of the perturbations after they enter the horizon:

$$\boxed{P_i(k, z) = 2\pi^2 \mathcal{T}_i^2(k, z) \mathcal{P}_\zeta(k) k}$$

LINEAR MODEL

In the late-time ΛCDM universe, when the large-scale structures have formed, the growth of perturbation is scale independent and the transfer function can be split into a scale-dependent part $T_i(k)$ and a time-dependent part $D(z)$, the growth factor. Focusing on the matter case:

$$\sigma_8^2 = \frac{1}{2\pi^2}\int \mathrm{d}k\, P_{\delta\delta}(k, z=0)\,|W_{\mathrm{TH}}(kR_8)|^2\, k^2$$

RMS density fluctuations in spheres of R_8=8 h^{-1} Mpc

$$P_{\delta\delta}(k,z) = \left(\frac{\sigma_8}{\sigma_N}\right)^2\left[\frac{D(z)}{D(z=0)}\right]^2 T_{\mathrm{m}}^2(k)k^{n_s}$$

Normalization constant

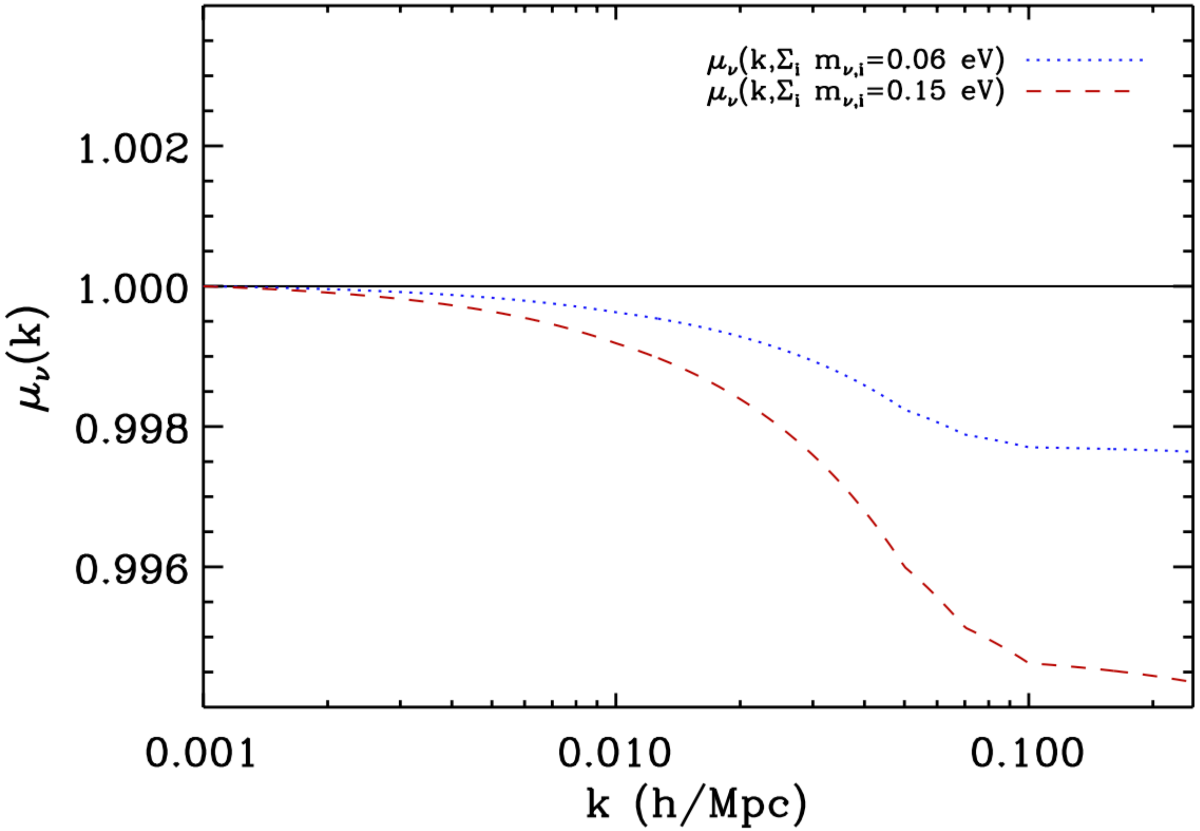
$$\sigma_N^2 = \frac{1}{2\pi^2}\int \mathrm{d}k\, T_{\mathrm{m}}^2(k)|W_{\mathrm{TH}}(kR_8)|^2 k^{n_s+2}$$

The growth factor depends on the epoch and on the neutrino mass. And so is the growth rate $\boxed{f(a) = \frac{\mathrm{d}\ln D(a)}{\mathrm{d}\ln a}}$. In the ΛCDM, massless neutrinos case: $f(z) = [\Omega_{\mathrm{m}}(z)]^\gamma$; $\gamma \approx 0.55$.

Linear model

The presence of massive neutrinos introduces a scale dependence in both $D = D(z,k)$ and f :

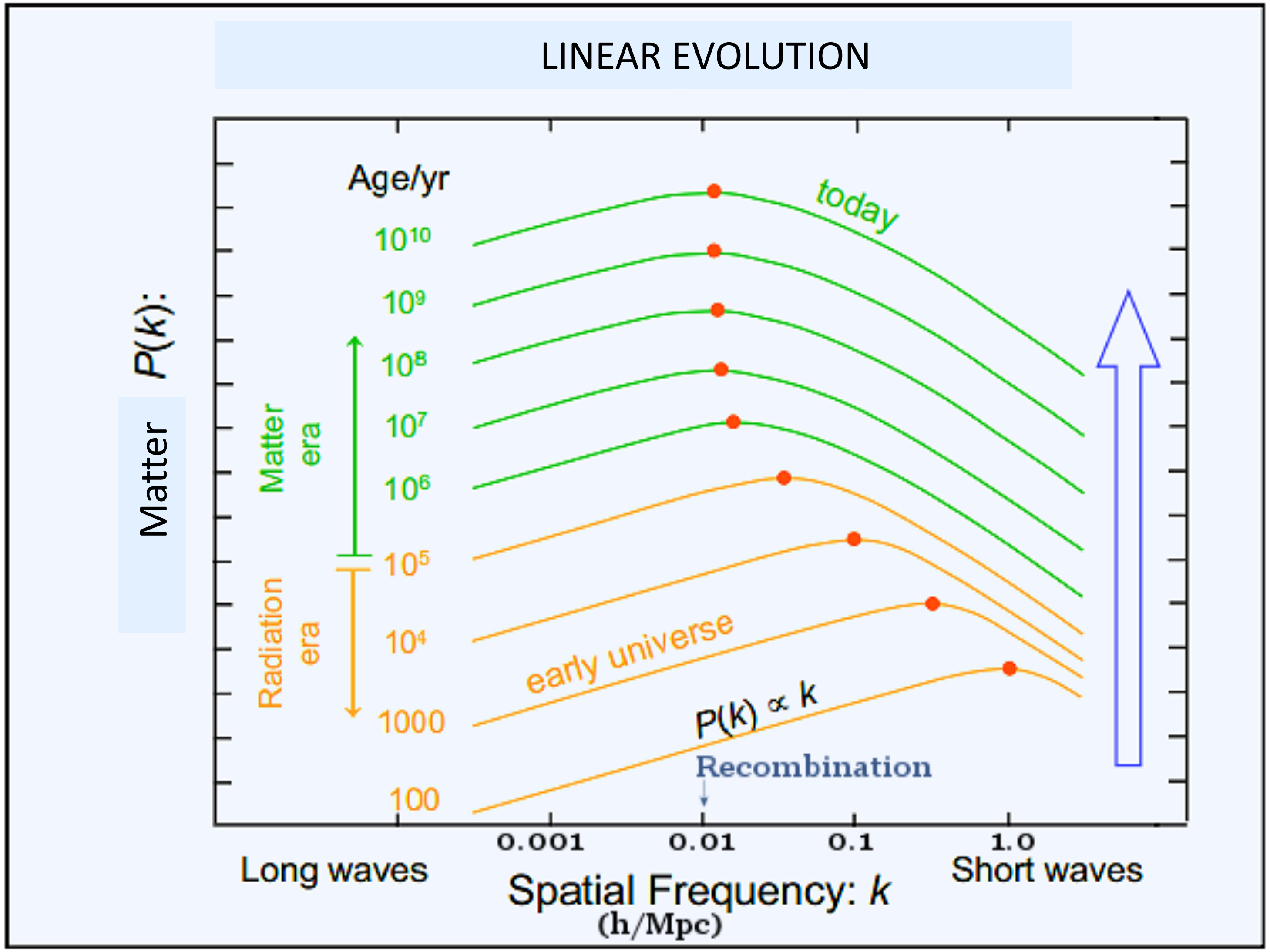
$$f(z,k;f_\nu,\Omega_{\mathrm{DE},0},\gamma) \approx \mu_\nu(k,f_\nu,\Omega_{\mathrm{DE},0})\Omega_m^\gamma(z) \quad \mu_\nu(k,f_\nu,\Omega_{\mathrm{DE},0}) \equiv 1 - A(k)\Omega_{\mathrm{DE},0}f_\nu + B(k)f_\nu^2 - C(k)f_\nu^3$$



Blanchard+ 2020

Fitting functions to numerical power spectra obtained from Boltzmann solvers like CAMB (Lewis+ 2000)

Linear model

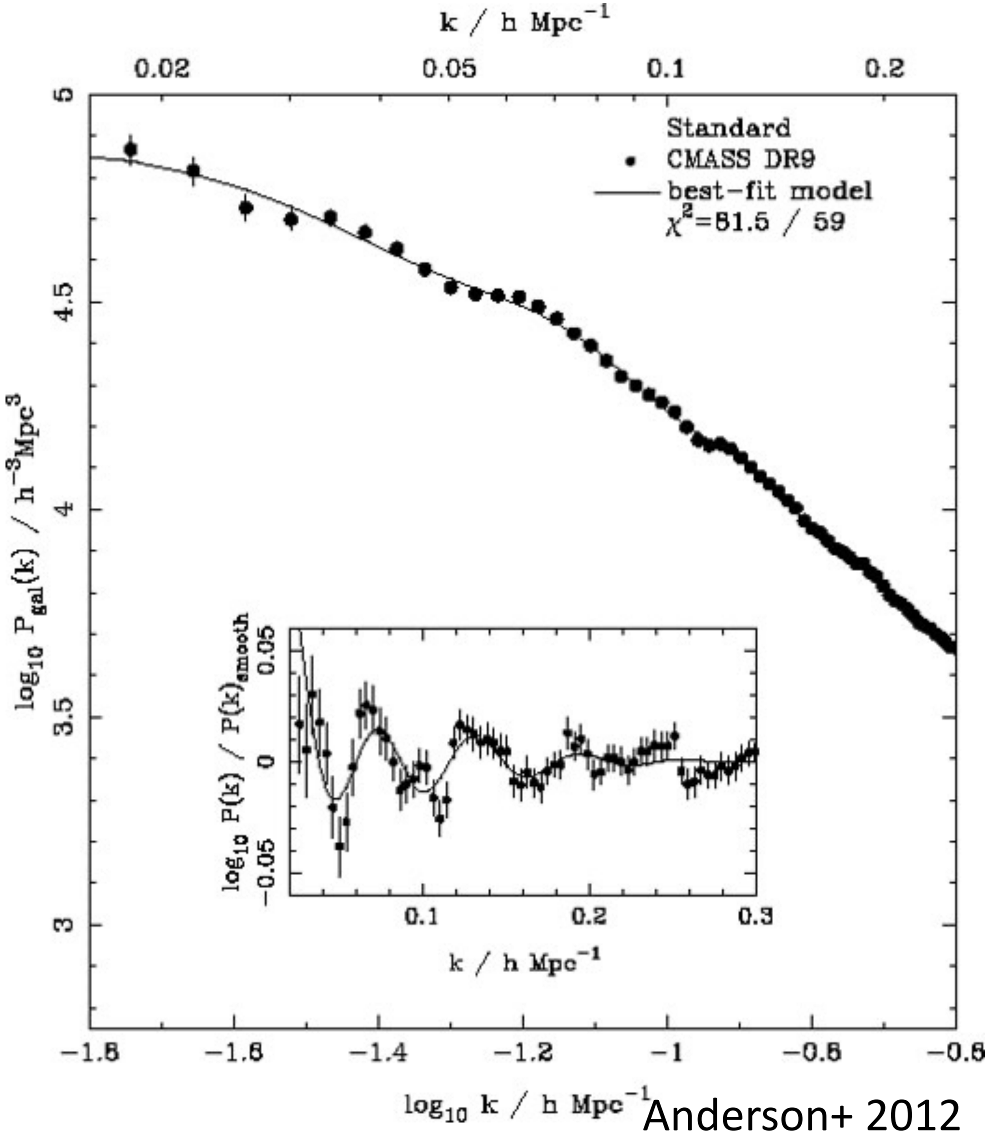


BAOs

Propagation of sounds waves in the photon-baryon plasma before recombination define a characteristic sound horizon scale (in comoving coordinates) :

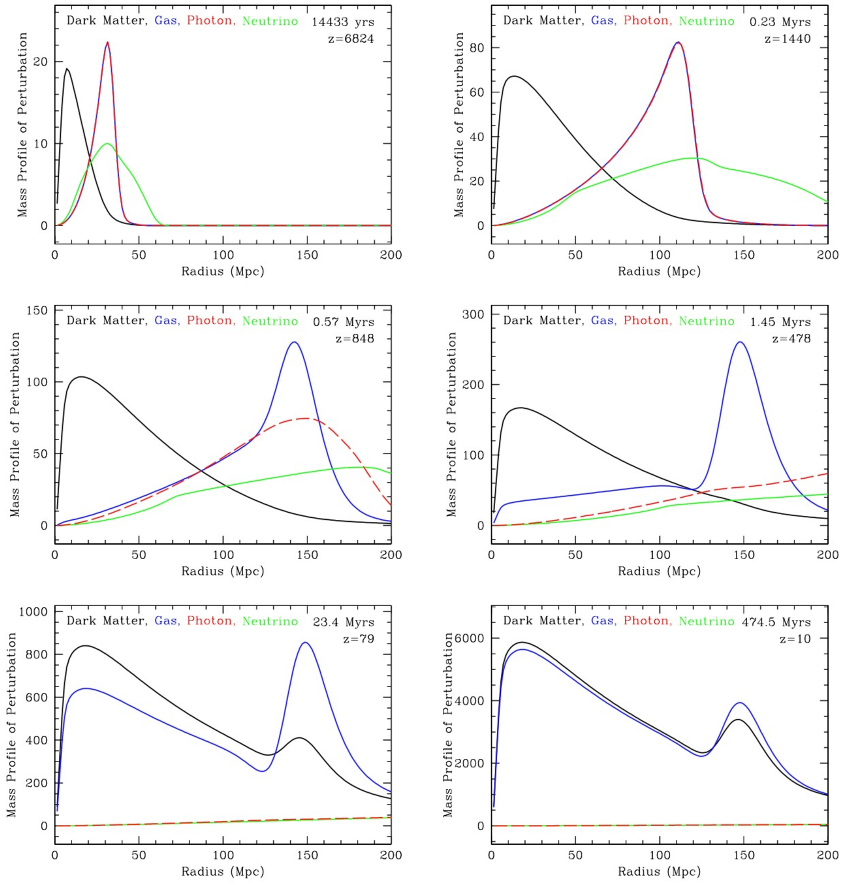
$$r_s = \int_{t_i}^{t_{dec}} \frac{c_s(t)dt'}{a(t')} = \int_{z_{dec}}^{z_i} \frac{c_s(z)dz}{H(z)}$$

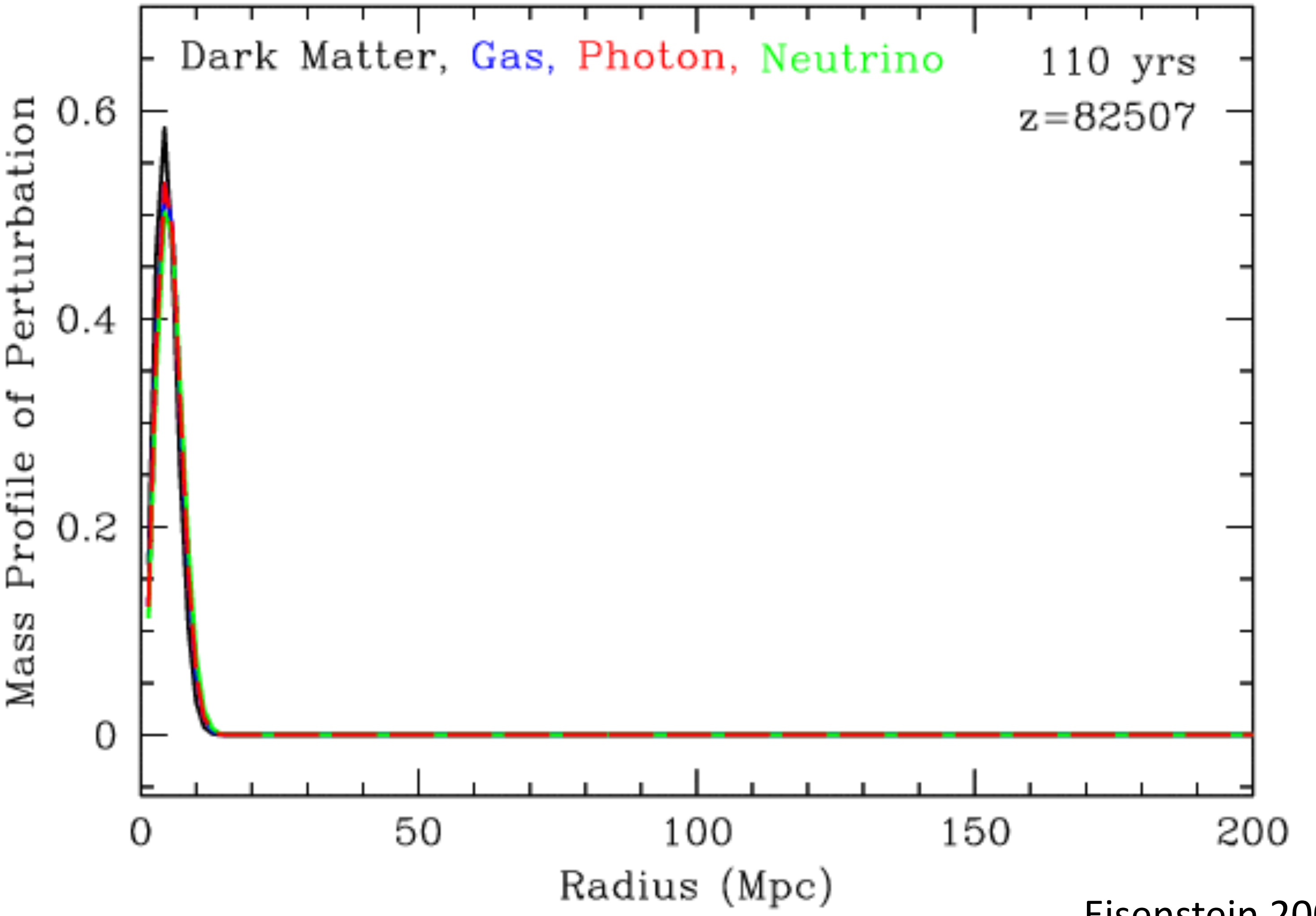
r_s is quite large ($\sim 150\ Mpc$) since $c_s \sim c/\sqrt{3}$. This sets characteristic scale in the baryonic component that, thanks to the gravitational coupling to CDM, is imprinted in the matter density fluctuation field after decoupling and can be observed in the spatial distribution of galaxies in the low-redshift Universe.



Anderson+ 2012

BAOs

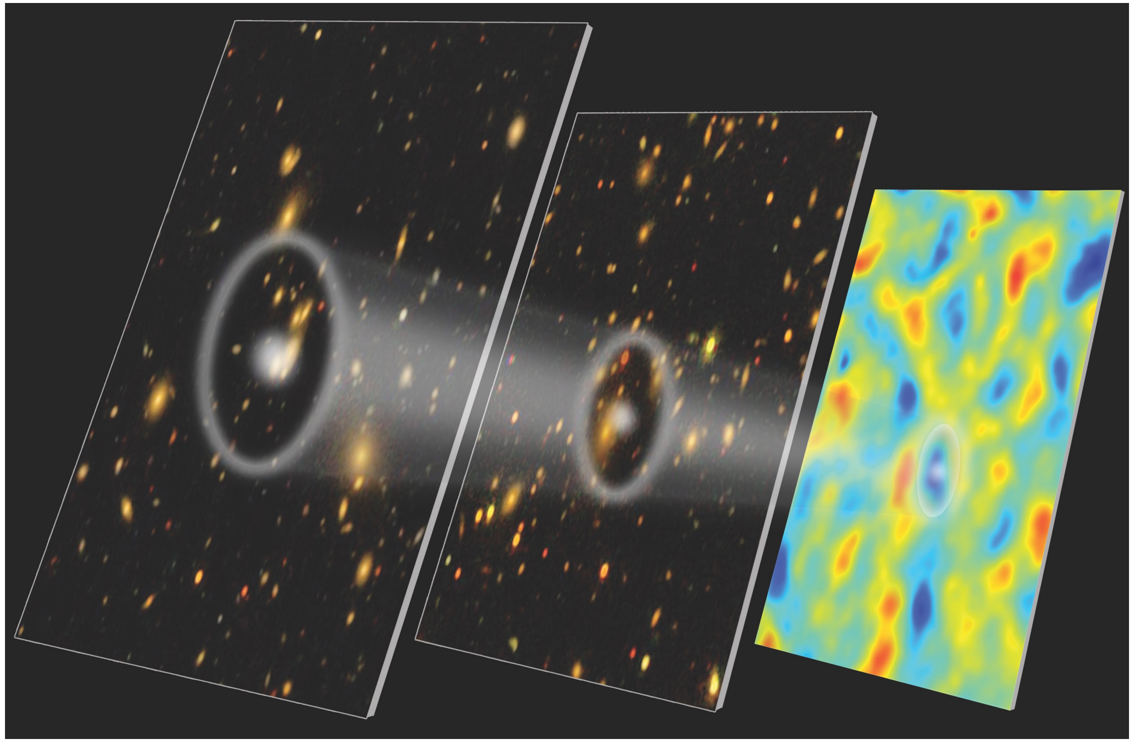




Eisenstein 2007

BAOs

BAOs provide a natural, dynamically stable standard ruler that can be used to trace the expansion history of the Universe.



Along the radial direction:

$$H(z)r_s = c\Delta z$$

Along the transverse direction:

$$r_s = \Delta\theta(1+z)D_A(z)$$

where $(1+z)D_A(z) = D_c(z) = \int_0^R \frac{dr}{\sqrt{1-kr^2}} = \int_0^{\tilde{z}} \frac{dz}{H(z)}$

Averaging among all directions: $D_V(z) = \left(D_A^2(z)\frac{cz}{H(z)}\right)^{1/3}$

It's easier to appreciate (and to measure) the BAO scale in configuration space rather than in Fourier space. So let us introduce the 2-point correlation function, 2PCF.

2-point correlation function

The 2PCF is defined as the expectation value of the fluctuation field measured in two spatial locations separated by a distance ***r***, averaged over all pairs with that separation.

$$\boxed{\xi(r) = \frac{\langle[\rho(\boldsymbol{x}) - \langle\rho\rangle][\rho(\boldsymbol{x}+\boldsymbol{r}) - \langle\rho\rangle]\rangle}{\langle\rho\rangle^2} = \langle\delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r})\rangle}$$

Having assumed statistical isotropy, 2PCF is a function of the modulus $|\boldsymbol{r}| = r$.

The 2PCF is the Fourier transform of the power spectrum $P(\boldsymbol{k})$. In an infinite volume:

$$\boxed{\xi(\boldsymbol{r}) = \frac{1}{(2\pi)^3}\int P(k)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{k}.}$$

And vice-versa:

$$P(k) = \int \xi(\boldsymbol{r})\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}$$

2-POINT CORRELATION FUNCTION

One can define the 2PCF for a discrete set of mass tracers of the underlying mass density field $\rho(\boldsymbol{x}) = \sum_i m_i \delta^{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{x}_i)$.

The probability of a tracer with mass m in a random volume element δV is: $\delta P = m^{-1}\rho(\boldsymbol{x})\delta V$

And the joint probability of a tracer in δV_1 and another in δV_2 separated by a distance r is

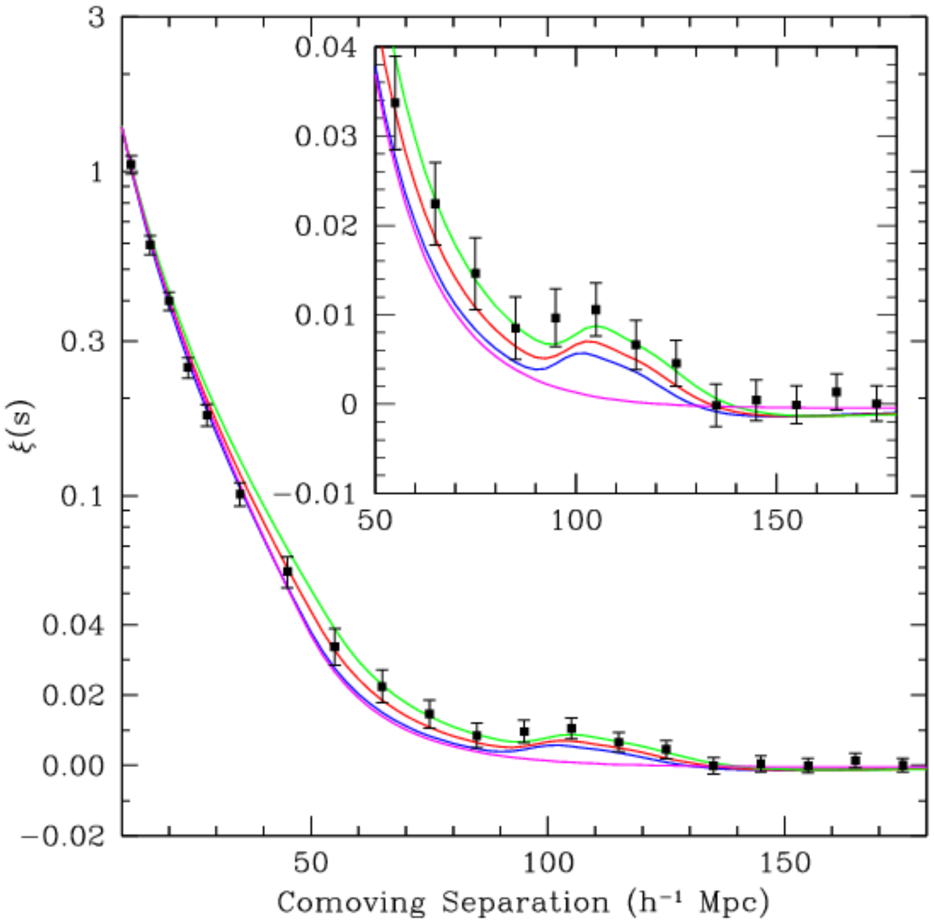
$$\boxed{\delta^2 P_2 = \frac{\langle \rho(\boldsymbol{x})\rho(\boldsymbol{x}+\boldsymbol{r})\rangle}{m^2}\delta V_1 \delta V_2 = n_V^2 \frac{\langle \rho(\boldsymbol{x})\rho(\boldsymbol{x}+\boldsymbol{r})\rangle}{\langle\rho\rangle^2}\delta V_1 \delta V_2 = n_V^2[1+\xi(r)]\delta V_1 \delta V_2}$$

which defines the 2PCF of a set of discrete tracers. It quantifies the excess (or the lack of) probability, with respect to the random case, to find a mass tracer at a distance r from a mass tracer randomly chosen in the sample. The 2PCF can be negative but larger than -1.
This definition is valid also in the case of anisotropic clustering, in which case we have $\xi(\boldsymbol{r})$.
It is a practical definition since it shows that the 2PCF can be evaluated by simply counting pairs with different separations.

2-point correlation function

The BAO phenomenon generates an excess of galaxy pair counts in correspondence with the sound horizon scale which appears as a single bump (instead of than a series of peaks) in their 2-point correlation function.
The BAO feature has been first detected in the luminous red galaxy sample of the SDSS redshift survey in 2005.

The presence of massive neutrinos modifies the amplitude and location of the BAO peak..
Its impact on the mass neutrino analyses, is maximized in combination with the CMB since it reduces the degeneracies between neutrino masses and other cosmological parameters.



Eisenstein 2005

Power Spectrum

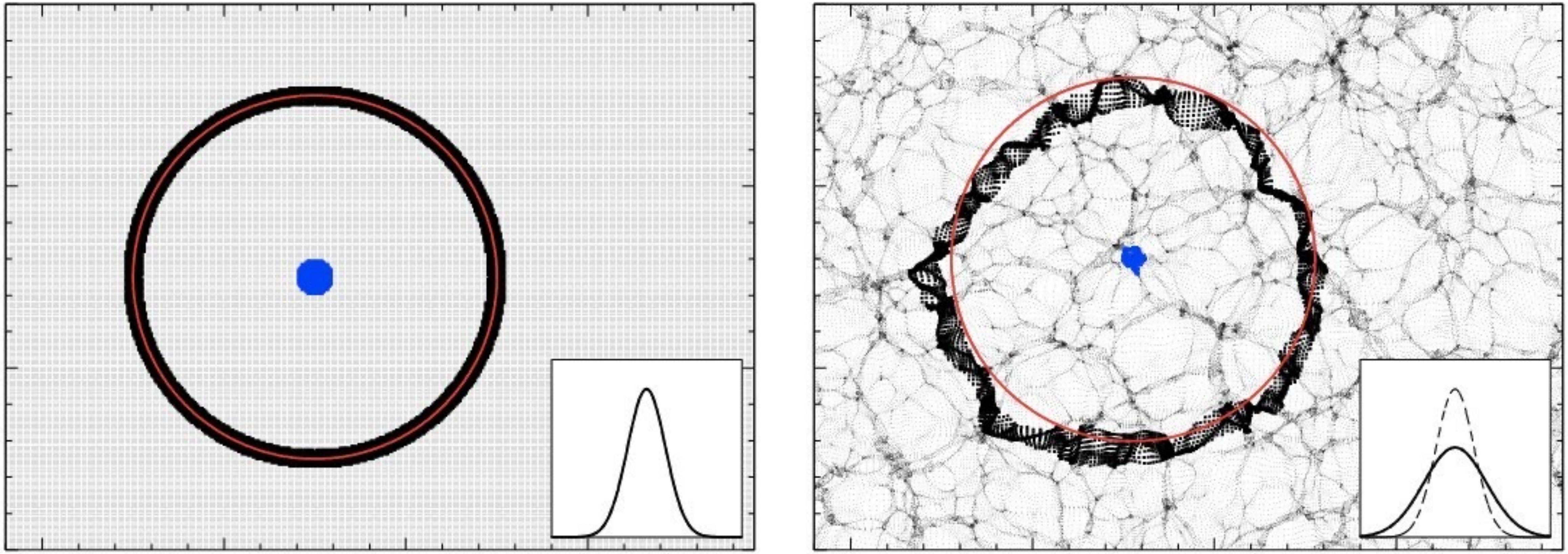
Let us go back to the power spectrum and focus on the power spectrum term

$$P_{\mathrm{obs}}(k_{\mathrm{ref}}, \mu_{\mathrm{ref}}; z) = \frac{1}{q_\perp^2 q_\parallel} \left\{ \frac{\left[b\sigma_8(z) + f\sigma_8(z)\mu^2\right]^2}{1 + [f(z)k\mu\sigma_{\mathrm{p}}(z)]^2} \right\} \frac{P_{\mathrm{dw}}(k, \mu; z)}{\sigma_8^2(z)} F_z(k, \mu; z) + P_{\mathrm{s}}(z)$$

The power spectrum in the model equation accounts for the nonlinear evolution (smearing) of the BAO peak:

BAOs

Padmanabhan+ 2012



POWER SPECTRUM

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

To include the smearing effect in the model

$$P_{\rm dw}(k,\mu;z) = P_{\delta\delta}(k;z)\,{\rm e}^{-g_\mu k^2} + P_{\rm nw}(k,\mu;z)\left(1-{\rm e}^{-g_\mu k^2}\right)$$

Wiggle suppression in the matter power spectrum

Power restoration in the broad band component

$$g_\mu(k,\mu,z) = \sigma_{\rm v}^2(z)\left\{1-\mu^2+\mu^2\left[1+f(z)\right]^2\right\}$$

is the anisotropic damping factor that depends on the cosine angle between $\boldsymbol{k}$ and the line-of-sight direction $\hat{r}$

$$\mu = \cos\vartheta = \boldsymbol{k}\cdot\hat{\boldsymbol{r}}/k$$

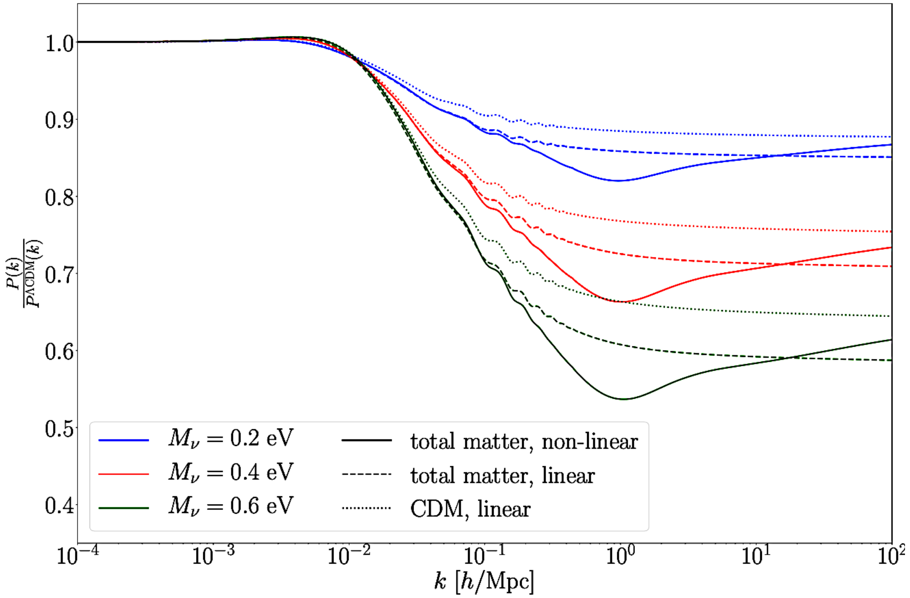
$\sigma_{\rm v}^2(z)$ is the parameter that controls the strength of the smearing and represents the galaxy-galaxy nonlinear velocity dispersion.

Power Spectrum

$$P_{\rm obs}(k_{\rm ref}, \mu_{\rm ref}; z) = \frac{1}{q_\perp^2 q_\parallel} \left\{ \frac{\left[b\sigma_8(z) + f\sigma_8(z)\mu^2\right]^2}{1 + [f(z)k\mu\sigma_{\rm p}(z)]^2} \right\} \frac{P_{\rm dw}(k, \mu; z)}{\sigma_8^2(z)} F_z(k, \mu; z) + P_{\rm s}(z)\,,$$

$$P_{\rm dw}(k, \mu; z) = P_{\delta\delta}(k; z)\,{\rm e}^{-g_\mu k^2} + P_{\rm nw}(k, \mu; z)\left(1 - {\rm e}^{-g_\mu k^2}\right)$$

For many applications, like the BAO analysis, it is sufficient to model the two spectra using linear theory. However, to analyze smaller scales where the dynamical effect of massive neutrinos is significant (see M. Viel lectures) nonlinear models are required.



POWER SPECTRUM

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

In the (Eulerian) Standard Perturbation Theory one expands the overdensity and the velocity divergence fields in power law series: $\delta_{\mathbf{k}}(t) = \delta_{\mathbf{k}}^{(1)}(t) + \delta_{\mathbf{k}}^{(2)}(t) + \ldots$
and at each order, the solution to the evolution equations can be written as (Bernardeau+ 2002)

$$\delta_{\mathbf{k}}^{(n)}(t) = \int \frac{\mathrm{d}^3\mathbf{q_1}}{(2\pi)^3}\cdots\int\frac{\mathrm{d}^3\mathbf{q_n}}{(2\pi)^3}\,\delta_D(\mathbf{k}-\mathbf{q_{1\ldots n}})\,F_n(\mathbf{q_1},\ldots,\mathbf{q_n})\,\delta_{\mathbf{q_1}}^{(1)}(t)\ldots\delta_{\mathbf{q}_n}^{(1)}(t)$$

where the kernels $F_n(\boldsymbol{q_1},\ldots\ldots,\boldsymbol{q_n})$ describe the coupling between the Fourier modes induced by nonlinear evolution. The resulting power spectrum is

$$P^{\rm SPT}(k) = P_{11}(k) + P_{13}(k) + P_{22}(k) + \ldots$$

where P_{11} is the linear power spectrum.

POWER SPECTRUM

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

The perturbative approach breaks down on small scales when the density contrast becomes of the order of unity. To describe the dynamics on those scales and how it feeds back to the larger ones the EFT of LSS theory has been introduced whose goal is to derive a correct theory of the effective cosmological fluid (i.e. Carrasco+ 2014). The EFT approach has been successfully applied to the clustering analyses of the largest and most recent datastes.

Alternatively, there are N-body simulations. These can predict accurately the matter power spectra, also in the presence of massive neutrinos, up to very small scales. As these are computationally demanding, they are often used in combination with faster methods based on interpolation techniques (emulators) or on the use of fitting functions to simulations (i.e. HALOFIT).

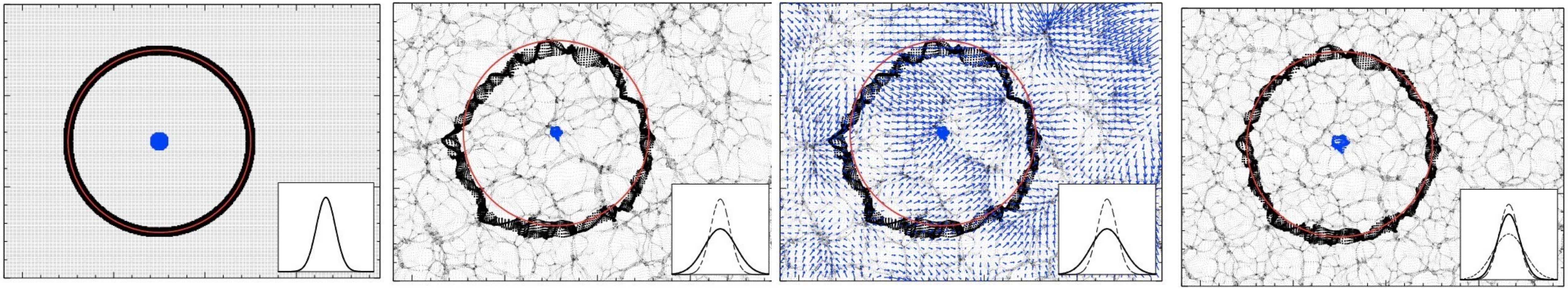
Reconstruction

Alternative one an restore the BAO peak in the linear regime by reversing the nonlinear evolution of galaxy clustering.

Various reconstruction techniques have been proposed. The most popular one is based on the Zel'dovich approximation and exploits the displacement vector field $\boldsymbol{\Psi}$ to place objects at their initial positions. Fopr this has to solve: $\boldsymbol{\nabla} \cdot \boldsymbol{\Psi} + f\boldsymbol{\nabla} \cdot \Psi_s \hat{\mathbf{s}} = \delta_m$ with $\Psi_s = \boldsymbol{\Psi} \cdot \hat{s}$

accounts for the fact that galaxies are observed at their "redshift space positions" $\boldsymbol{s}$

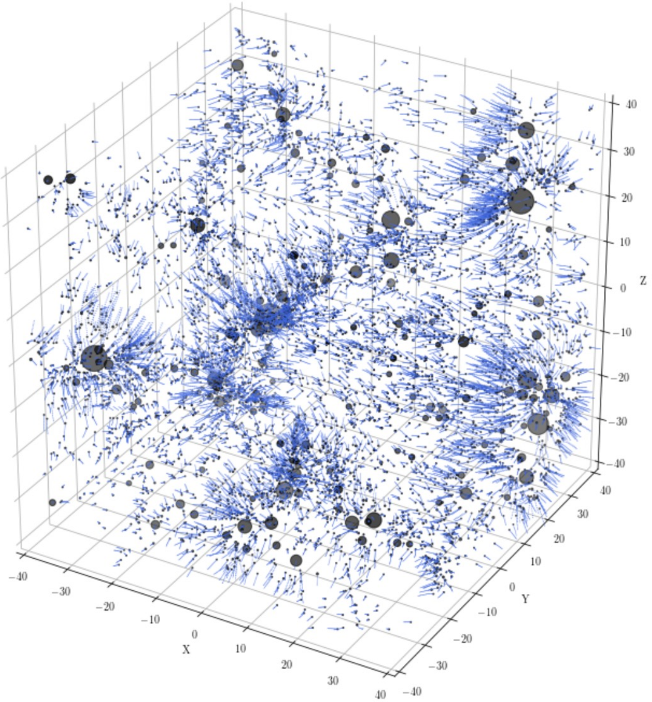
Assuming that the displacement field is irrotational, $\boldsymbol{\Psi} = \boldsymbol{\nabla}\phi$, then we need to solve a Poisson-like equation.



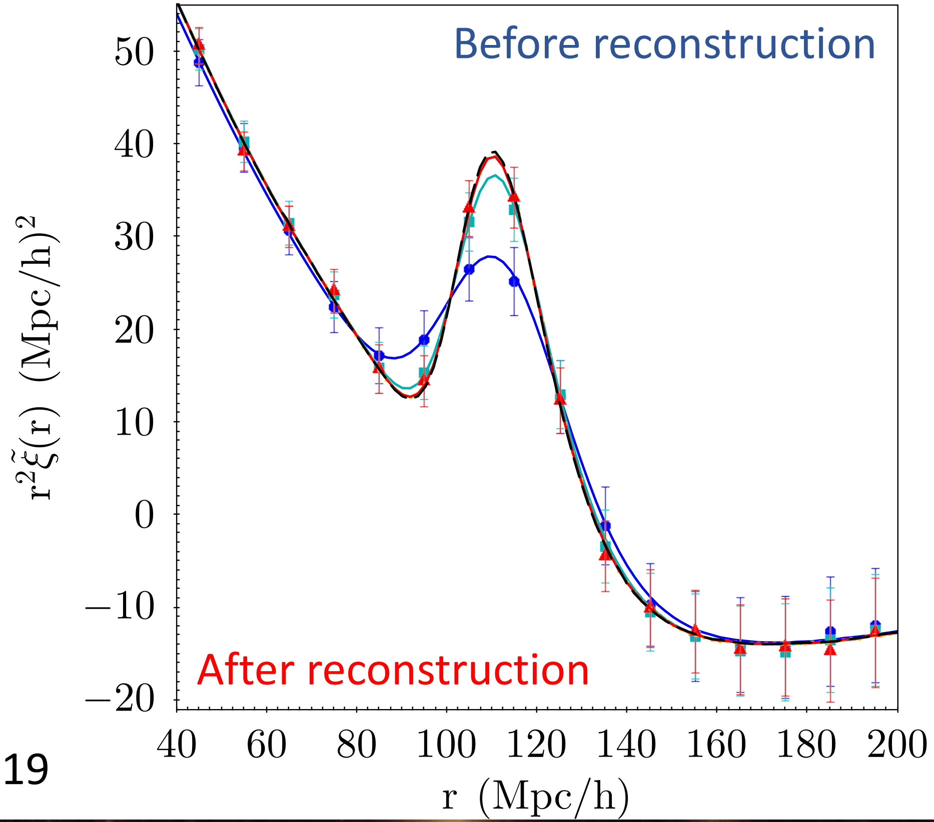
Reconstruction

More sophisticated reconstruction schemes can be used to better model the nonlinear evolution.

One of them is based on the cosmological least action principle (Peebles 1989). It exploits initial isotropy and homogeneity to solve a mixed boundary value problem and find galaxy orbits by minimizing the classical action of the system.



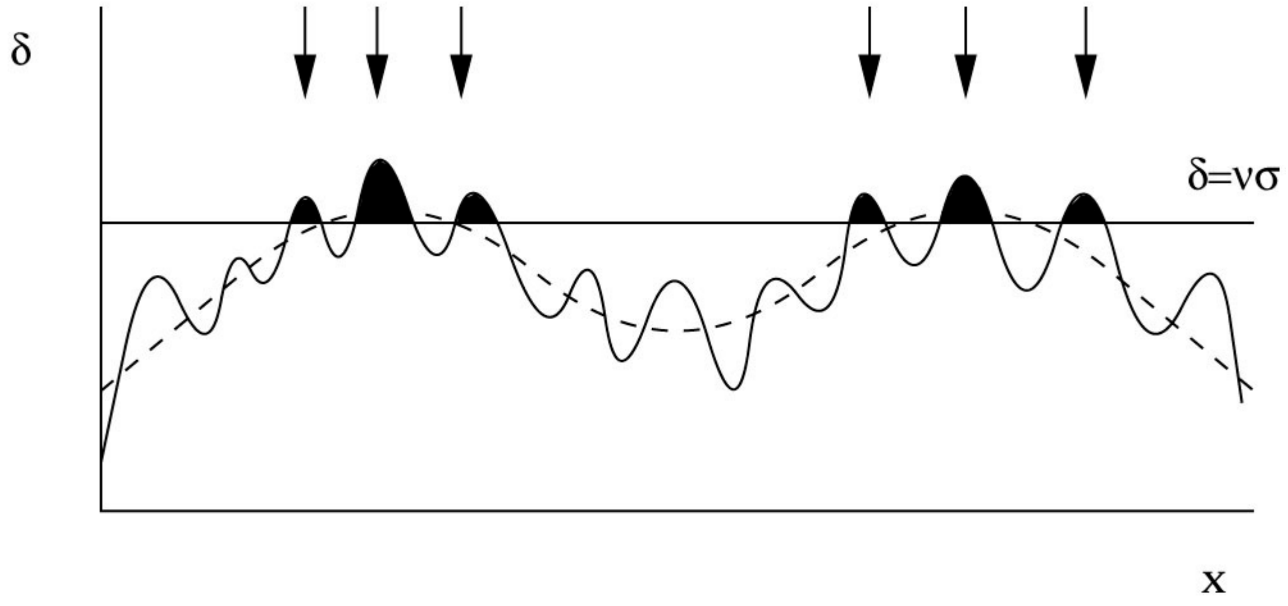
Sarpa+ 2019



GALAXY BIAS

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

Galaxies (and clusters) may not be unbiased tracers of the underlying mass density field. Rare objects that form at the highest peaks of a Gaussian density field (Kaiser 1984) clearly are not.



Galaxy bias

Galaxy bias quantifies the mapping between the mass density field and the density of a set of tracers measured over some suitable scale. The most general biasing scheme is non-local, non-linear, scale-dependent, and non-deterministic.

The halo occupation distribution prescription provides a useful receipt to describe the galaxy bias in the framework of the halo model.

The perturbative approach is an alternative way to address the biasing problem.
This approach assumes that the bias is deterministic and that the galaxy density contrast can be expanded in a perturbative series of power-law functions of the local mass density contrast

$\delta_g(\boldsymbol{x}) = \sum_{n=1} \frac{b_n}{n!}\delta(\boldsymbol{x})^n$ at the 2nd order $\delta_g(\boldsymbol{x}) = b_1\delta(x) + \frac{b_2}{2}\delta(x)^2 + \gamma_2 \mathcal{G}_2(\Phi|x)$

Linear bias. Valid on large-scale. It depends on the tracer type and on the redshift.	Nonlinear bias. To account for the complex physics of galaxy formation and evolution	Describes the relation between galaxy density and gravity potential.	Galileon operator function of the velocity divergence

PECULIAR VELOCITIES

$$P_{\rm obs}(k_{\rm ref},\mu_{\rm ref};z) = \frac{1}{q_\perp^2 q_\parallel}\left\{\frac{\left[b\sigma_8(z)+f\sigma_8(z)\mu^2\right]^2}{1+[f(z)k\mu\sigma_{\rm p}(z)]^2}\right\}\frac{P_{\rm dw}(k,\mu;z)}{\sigma_8^2(z)}F_z(k,\mu;z)+P_{\rm s}(z)\,,$$

In redshift surveys we use redshift as a distance proxy.
Measured redshifts, however, receives two contributions: the cosmological redshift of the Hubble's recession velocity and the classical Doppler shift generated by the galaxy peculiar velocities, i.e. departures from the Hubble flow. For a nearby object:

$$z_{obs} = H_0 D_p + \boldsymbol{v}_p \cdot \hat{\boldsymbol{r}}$$

As a result, the observed redshift is an unbiased distance proxy only for very distant objects.
In the liner regime peculiar velocities are highly correlated in a spatial sense since they are related to mass density fluctuation field through the continuity equation:

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = aHf\,.$$