Plan

Matteo Viel

- Lecture 1: Cosmological effects of neutrinos in linear perturbation theory
- Lecture 2: Non-linear regime
- Lecture 3: Neutrinos in Intergalactic space
- Lecture 4: New ways of probing neutrino masses

Regimes

$\delta \ll 1$ **Linear theory**: use public codes like CAMB/CLASS or your own solver
analytical neutrino calculations also valid in some regimes

$\delta \leq 1$ **Perturbation theories**: Standard Perturbation theory or other methods, Effective field theory of the LSS, …..

$\text{any } \delta$ **Non-linear regime: A)** analytical models for non-linear power like the halo model (or some log-normal approximation for the density field)
B) N-body simulations

N-body sims in the context of neutrino cosmologies

1980: Bond et al. 1980 – linear theory (also Russian school with Zeldovich)
1983: Bond et al. – Evolution of Boltzmann-Einstein equations. Clustering properties of galaxies not reproduced if the universe is dominated by neutrinos (White et al. 1983) – numerical experiment

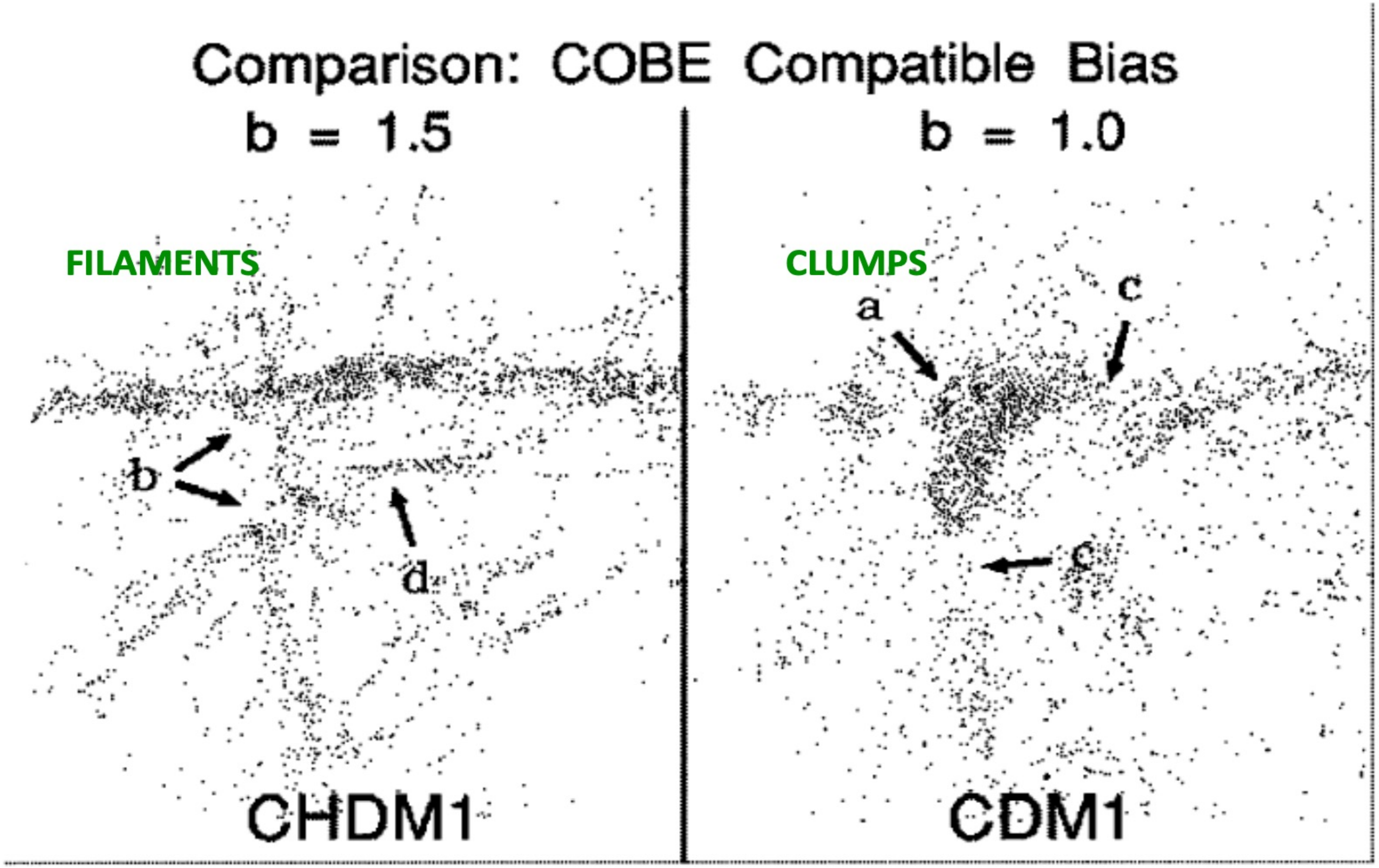
1992: Davis et al. HDM or CHDM models P3M codes with neutrino particles placed as the dark matter ones (same CDM spectrum + velocities): 32^3 particles

1993: Klypin et al. 2 x 128^3 particles at z_IC=14 with the right power spectrum

1994: Ma & Bertschinger approximate linear scheme evolved at z=13 and after that pure N-body

N-body sims in the context of neutrino cosmologies

Pure HDM not allowed. However CHDM is still viable and impacts on the cosmic web



Brodbeck+98

Numerical methods for structure formations

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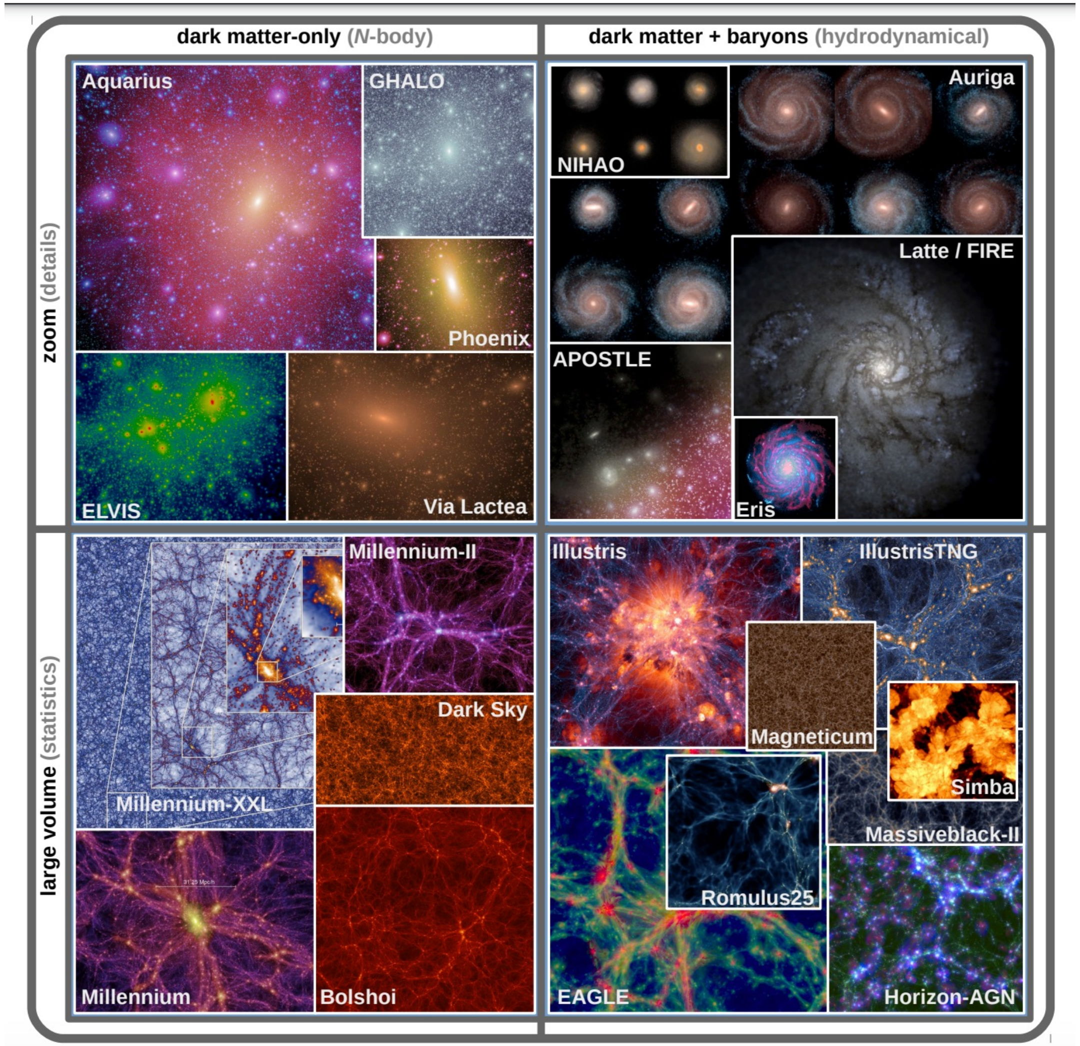
Vogelsberger+20
Nature Review



Physics implemented

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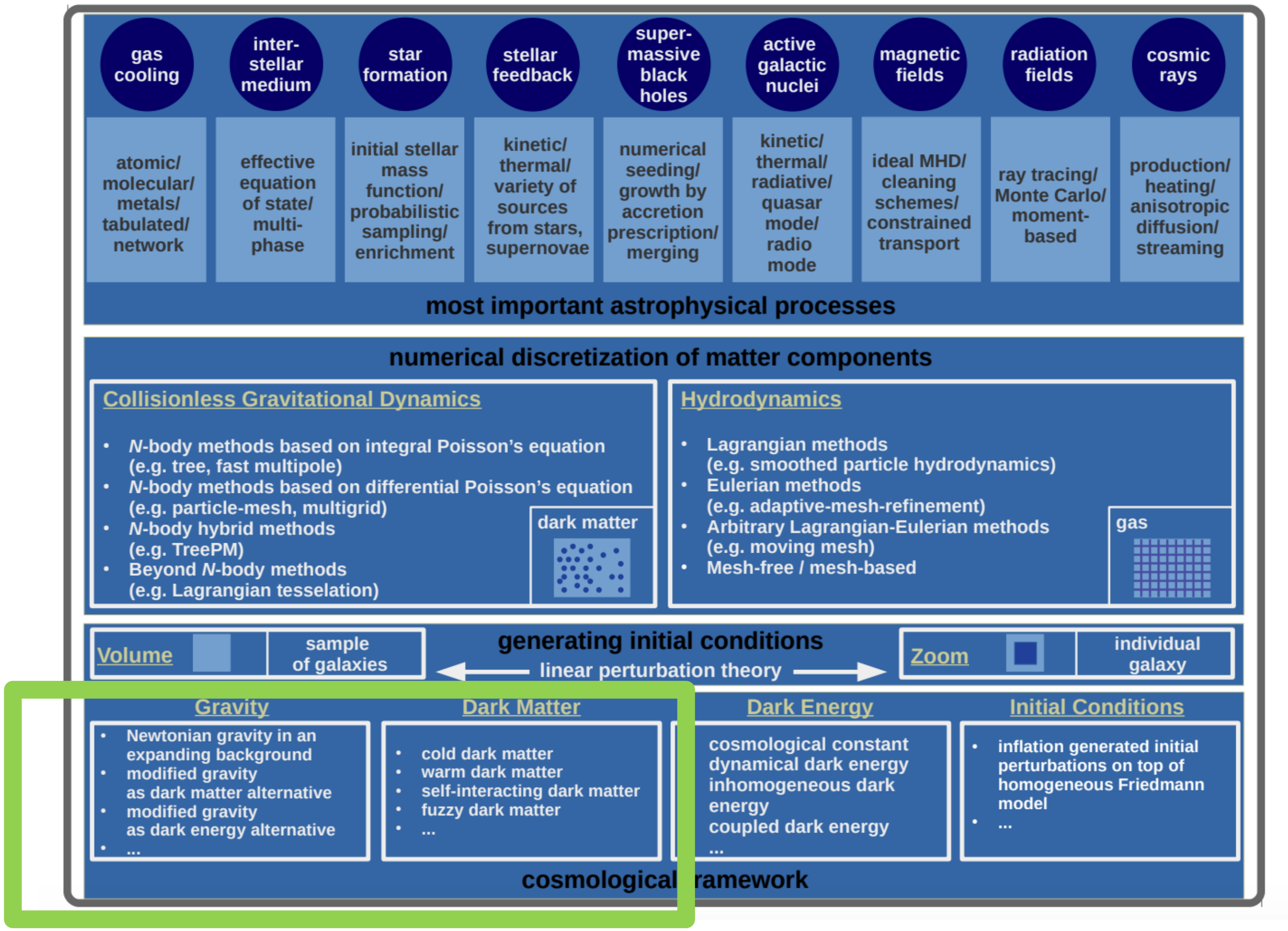


Setting Initial conditions

It is wise to start the simulations in the MD era, to set up initial conditions we will use the Results of Lecutre 1: i.e. linear theory

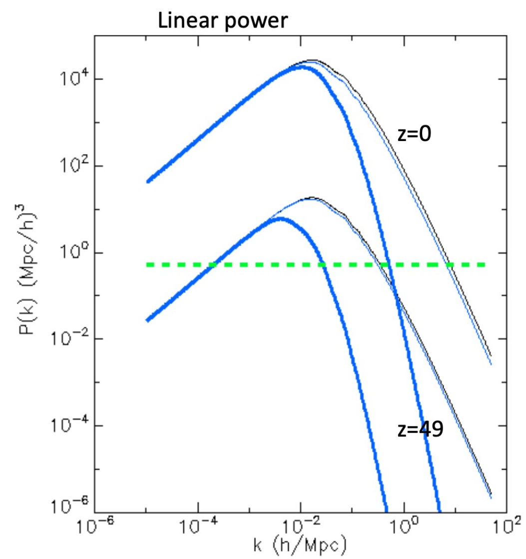
Generating initial conditions

initial positions: $\mathbf{x} = \mathbf{q} + D(t)\Psi(\mathbf{q})$ **initial velocities:** $a(t)\,\dot{\mathbf{x}} = a(t)\,\frac{\mathrm{d}D(t)}{\mathrm{d}t}\,\Psi(\mathbf{q}) = a(t)H(t)\,\frac{\mathrm{d}\ln D}{\mathrm{d}\ln a}\,D(t)\Psi(\mathbf{q})$

Comoving initial positions, $\mathbf{x}$, are assigned based on the unperturbed particle position, $\mathbf{q}$, the linear growth factor, $D(t)$, and the scale factor, a, which is related to the initial redshift, $z = 1/a - 1$. The curl-free displacement field Ψ is computed by solving the linearized continuity equation $\nabla \cdot \Psi = -\delta/D(t)$, where δ is the relative density fluctuation.

Zel'dovich approximation is used

However: note that for neutrinos
the initial FD thermal velocities are large
Note also: Poissonian shot noise



$$x(q, z) = q + D_+(z)\nabla_q \phi_i(q)$$

Black is matter power

Blue is matter power in neutrino simulations

Thick blue is neutrino power

Green dashed is shot noise level for a typical run

Zeldovich displacement of the particles

$$\phi_i(x) = -\frac{3}{2} H^2 a^3 \Phi_i(x)$$

Setting ICs: Zel'dovich approximation

Particle displacements:

$$\mathbf{d}_i(t) = \mathbf{x}_i(t) - \mathbf{q}_i$$

Density change due to displacements:

$$\rho(\mathbf{x}) = \frac{\rho_0}{\left|\frac{\partial \mathbf{x}}{\partial \mathbf{q}}\right|} = \frac{\rho_0}{\left|\delta_{ij} + \frac{\partial \mathbf{d}}{\partial \mathbf{q}}\right|}$$

For small displacements:

$$\left|\delta_{ij} + \frac{\partial \mathbf{d}}{\partial \mathbf{q}}\right| \simeq 1 + \nabla_{\mathbf{q}} \cdot \mathbf{d}$$

Resulting density contrast:

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_0}{\rho} = -\nabla_{\mathbf{q}} \cdot \mathbf{d}$$

During linear growth:

$$\delta(t) = D(t)\delta_0$$

$$\mathbf{d}(t) = D(t)\mathbf{d}_0$$

⇒

$$\dot{\mathbf{x}} = \dot{\mathbf{d}} = \dot{a}\frac{\mathrm{d}D}{\mathrm{d}a}\mathbf{d}_0 = \frac{\dot{a}}{a}\frac{a}{D}\frac{\mathrm{d}D}{\mathrm{d}a}\mathbf{d}$$

Particle velocities:

$$\dot{\mathbf{x}} = H(a) f(\Omega)\mathbf{d} \qquad f(\Omega) = \frac{\mathrm{d}\ln D}{\mathrm{d}\ln a} \simeq \Omega^{0.6}$$

Note: Particles move on straight lines in the Zeldovich approximation.

N-body sims primer

Modeling dark matter

collisionless Boltzmann equation: $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$ **Poisson's equation:** $\nabla^2\Phi = 4\pi G\int f\mathrm{d}\mathbf{v}$

The collisionless Boltzmann equation describes the evolution of the phase-space density or distribution function of dark matter, $f = f(\mathbf{r},\mathbf{v},t)$, under the influence of the collective gravitational potential, Φ, given by Poisson's equation. The collisionless Boltzmann equation states the conservation of the local phase-space density; i.e. Liouville's theorem.

Table 1: Major galaxy formation simulation codes

code name	gravity treatment[a]	hydrodynamics treatment[b]	parallelization technique[c]	code availability[d]	primary reference
ART	PM/ML	AMR	data-based	public	Kravtsov (1997)[27]
RAMSES	PM/ML	AMR	data-based	public	Teyssier (2002)[38]
GADGET-2/3	TreePM	SPH	data-based	public	Springel (2005)[39]
Arepo	TreePM	MMFV	data-based	public	Springel (2010)[40]
Enzo	PM/MG	AMR	data-based	public	Bryan et al. (2014)[41]
ChaNGa[e]	Tree/FM	SPH	task-based	public	Menon et al. (2015)[42–44]
GIZMO[f]	TreePM	MLFM/MLFV	data-based	public	Hopkins et al. (2015)[45]
HACC	TreePM/P^{3}M	CRK-SPH	data-based	private	Habib et al. (2016)[46]
PKDGRAV3	Tree/FM	–	data-based	public	Potter et al. (2017)[47]
Gasoline2	Tree	SPH	task-based	public	Wadsley et al. (2017)[48]
SWIFT	TreePM/FM	SPH	task-based	public	Schaller et al. (2018)[49]

[a] PM: particle-mesh; TreePM: tree + PM, FM: fast multipole, P^{3}M: particle-particle-particle-mesh; ML: multilevel; MG: multigrid
[b] SPH: smoothed particle hydrodynamics, CRK-SPH: conservative reproducing kernel smoothed particle hydrodynamics , AMR: adaptive-mesh-refinement, MMFV: moving-mesh finite volume, MLFM/MLFV: mesh-free finite mass / finite volume
[c] data-based: data parallelism focuses on distributing data across different nodes, which operate on the data in parallel; task-based: task parallelism focuses on distributing tasks concurrently performed
[d] private: private code; public: publicly available code (in some cases with limited functionality)
[e] gravity solver is based on PKDGRAV3
[f] based on the GADGET-3 code

N-body sims: the Particle Mesh method

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$$\ddot{\mathbf{x}}_i = -\nabla_i \, \Phi(\mathbf{x}_i)$$

$$\Phi(\mathbf{x}) = -G \sum_{j=1}^{N} \frac{m_j}{[(\mathbf{x}-\mathbf{x}_j)^2+\epsilon^2]^{1/2}}$$

Note: The naïve computation of the forces is an N^2 - task.

Direct summation is unfeasible
One can rely on approximations

Poisson's equation can be solved in real-space by a convolution of the density field with a Green's function.

$$\Phi(\mathbf{x}) = \int g(\mathbf{x}-\mathbf{x}')\, \rho(\mathbf{x})\, \mathrm{d}\mathbf{x}'$$

In Fourier-space, the convolution becomes a simple multiplication!

$$\hat{\Phi}(\mathbf{k}) = \hat{g}(\mathbf{k}) \cdot \hat{\rho}(\mathbf{k})$$

→ **Solve the potential in these steps:**

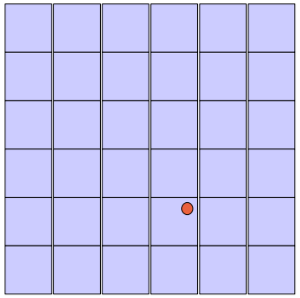
(1) FFT forward of the density field
(2) Multiplication with the Green's function
(3) FFT backwards to obtain potential

The four steps of the PM algorithm
(a) Density assignment
(b) Computation of the potential
(c) Determination of the force field
(d) Assignment of forces to particles

N-body sims: force calculation in the PM method

Finite differencing of the potential to get the force field

Approximate the force field $\mathbf{f} = -\nabla\Phi$ with finite differencing



2nd order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h}$$

4th order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{4}{3}\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h} + \frac{1}{3}\frac{\Phi_{i+2,j,k} - \Phi_{i-2,j,k}}{4h}$$

Interpolating the mesh-forces to the particle locations

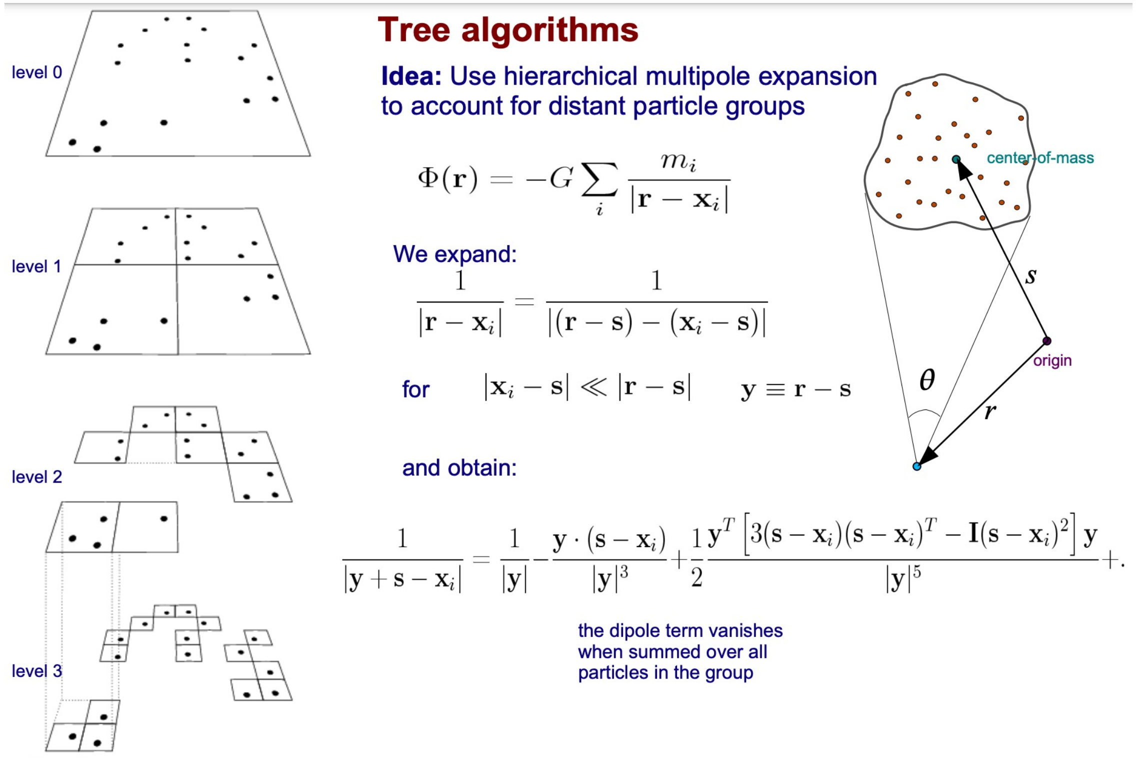
$$F(\mathbf{x}_i) = \sum_{\mathbf{m}} W(\mathbf{x}_i - \mathbf{x}_{\mathbf{m}})\, f_{\mathbf{m}}$$

The interpolation kernel needs to be the same one used for mass-assignment to ensure force anti-symmetry.

N-body sims: method

PM methods are fast and simple they are good for species which do not cluster much (neutrinos?)
However, DM will cluster more, PM is not good (limited by force resolution on the mesh).

Tree algorithm: group distant particles together and use their multipole expansion only log(N) force
Terms per particle



N-body sims: Tree-PM

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THE TREE-PM FORCE SPLIT

Periodic peculiar potential

$$\nabla^2\phi(\mathbf{x}) = 4\pi G[\rho(\mathbf{x}) - \overline{\rho}] = 4\pi G \sum_{\mathbf{n}} \sum_i m_i \left[\tilde{\delta}(\mathbf{x} - \mathbf{x}_i - \mathbf{n}L) - \frac{1}{L^3}\right]$$

Idea: Split the potential (of a single particle) in Fourier space into a long-range and a short-range part, and compute them separately with PM and TREE algorithms, respectively.

Poisson equation in Fourier space:

$$\phi_{\mathbf{k}} = -\frac{4\pi G}{\mathbf{k}^2}\rho_{\mathbf{k}} \qquad (\mathbf{k} \neq 0)$$

DM small scale: Tree
DM large scale: PM
Neutrinos (all over): PM

$$\phi_{\mathbf{k}}^{\mathrm{long}} = \phi_{\mathbf{k}} \exp(-\mathbf{k}^2 r_s^2)$$

Solve with PM-method

- CIC mass assignment
- FFT
- multiply with kernel
- FFT backwards
- Compute force with 4-point finite difference operator
- Interpolate forces to particle positions

$$\phi_{\mathbf{k}}^{\mathrm{short}} = \phi_{\mathbf{k}} \left[1 - \exp(-\mathbf{k}^2 r_s^2)\right]$$

FFT to real space $\phi(r) = -\frac{Gm}{r}\,\mathrm{erfc}\left(\frac{r}{2r_s}\right)$

Solve in real space with TREE

Key results of N-body simulations

1) Large/Medium/small scale distribution of matter
2) Halo mass function
3) Structure of dark matter haloes
4) Halo substructure

Till 2008 all the above points were tackled without considering neutrinos as a Separate fluid but implementing the matter power spectrum of a massive neutrino cosmology

Key results of N-body simulations

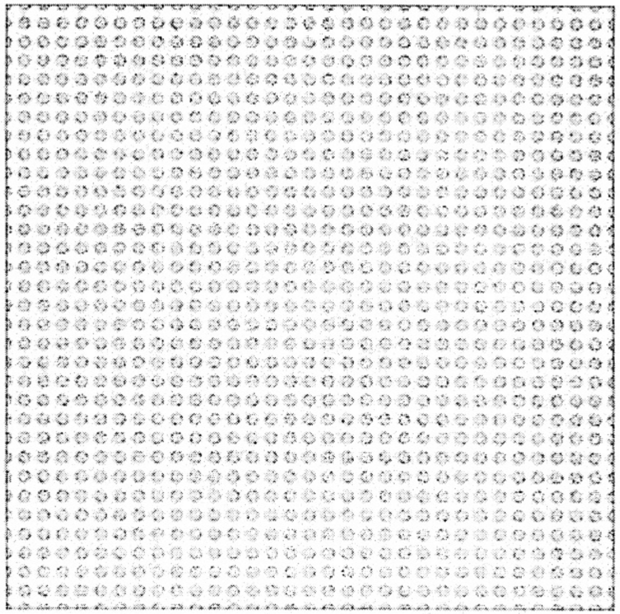
A CALCULATION OF THE FULL NEUTRINO PHASE SPACE IN COLD + HOT DARK MATTER MODELS

CHUNG-PEI MA[1] AND EDMUND BERTSCHINGER
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

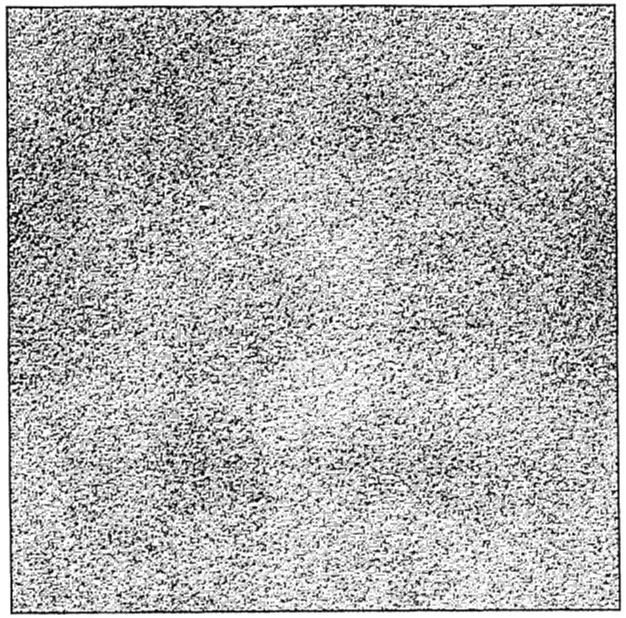
Received 1993 August 6; accepted 1993 December 30

Monte Carlo method to sample (integrate geodesic equation)
the neutrino phase space-density from $z=10^9$ to $z=0$ First they use linear theory to $z=13$, then they switch to N-body

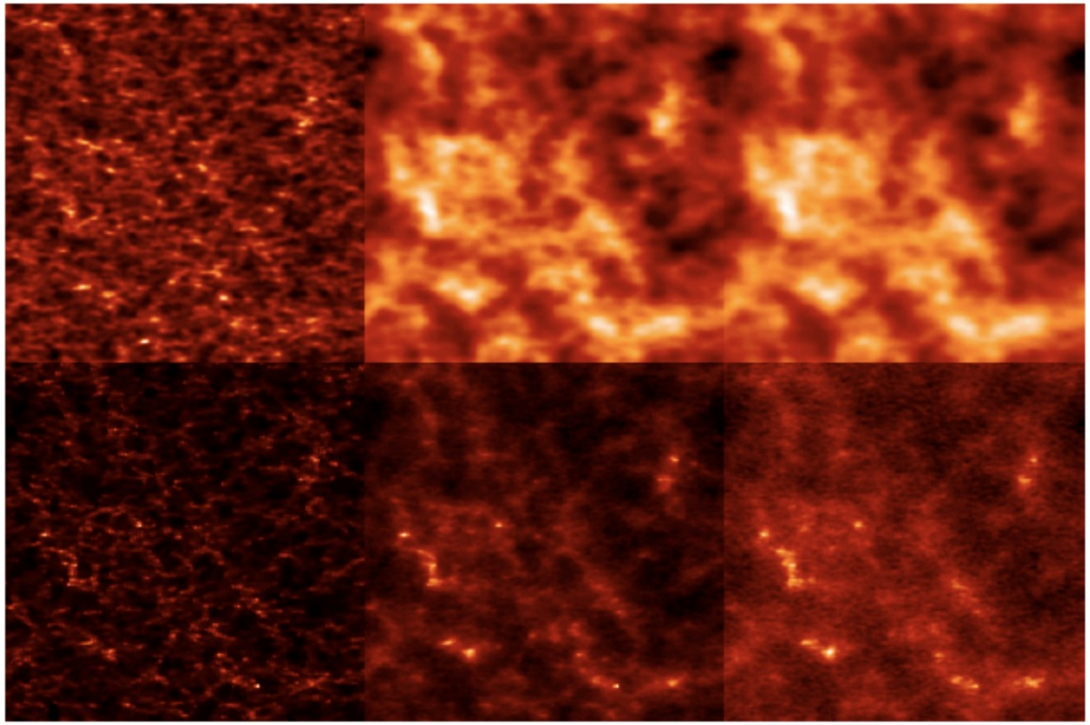
z=10,000



z=13.5



Neutrino simulation with particles

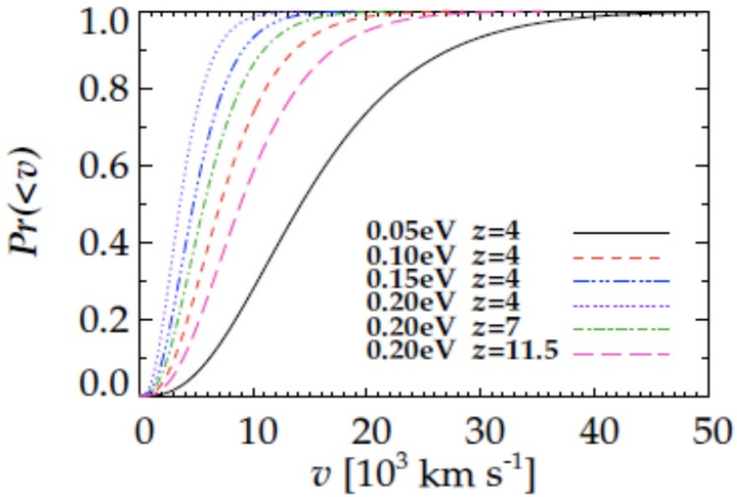


COLD DM NEUTRINOS 0.6 eV NEUTRINOS 0.3 eV

Simulation of neutrinos as an independent set of particles that interact gravitationally

Brandbyge et al 08

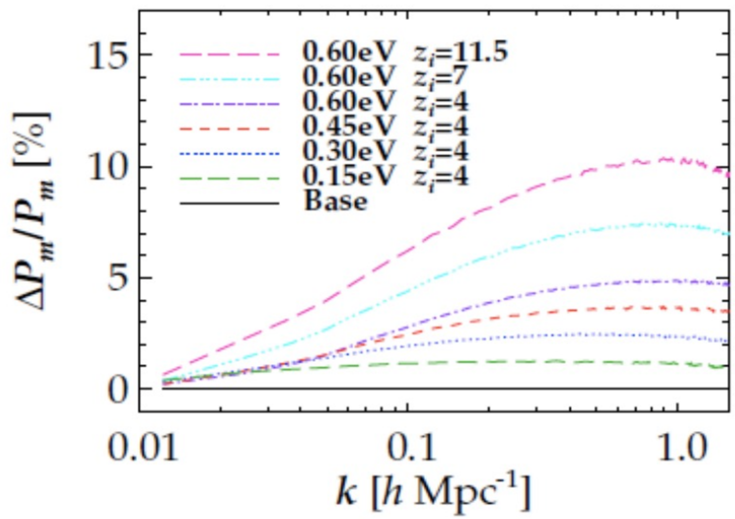
Neutrino simulations: role of thermal velocities



$$T_\nu \simeq T_\gamma (4/11)^{1/3}$$

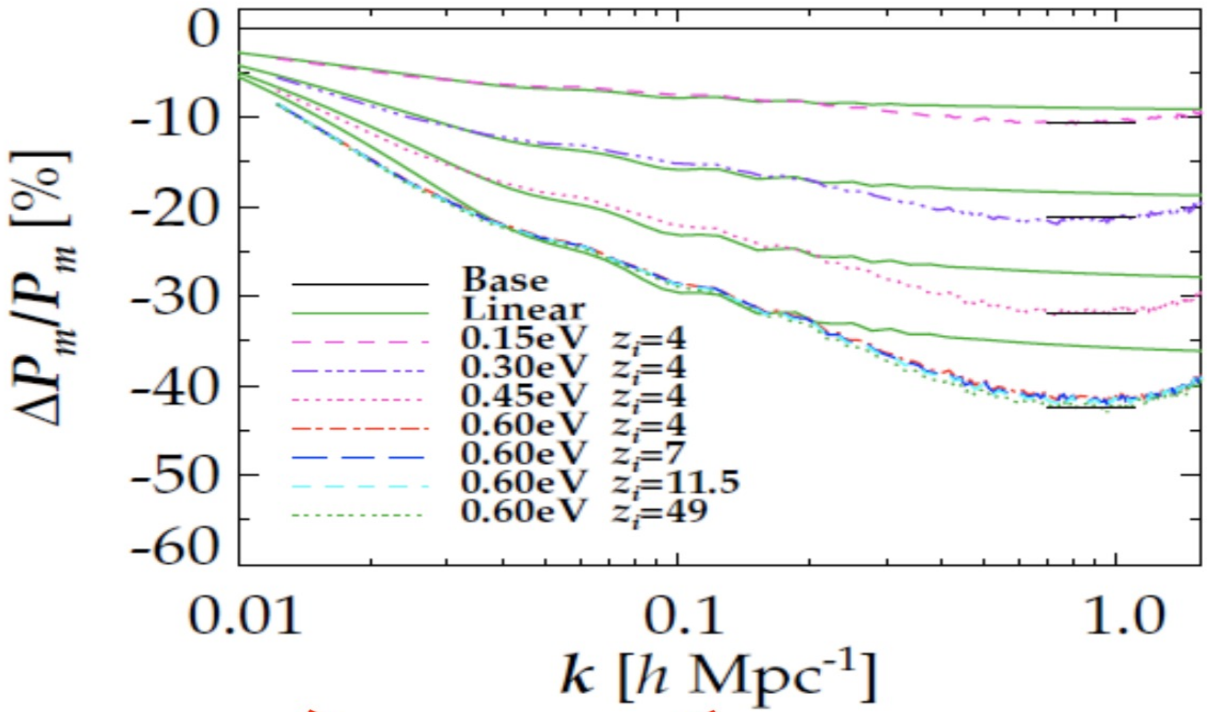
$$Pr(<p) = N \int_0^p \frac{p'^2}{e^{p'c/k_b T_\nu} + 1} dp'$$

Draw velocity from Fermi-Dirac distribution



Brandbyge et al 08a

Non-linear matter power



Brandbyge et al 08

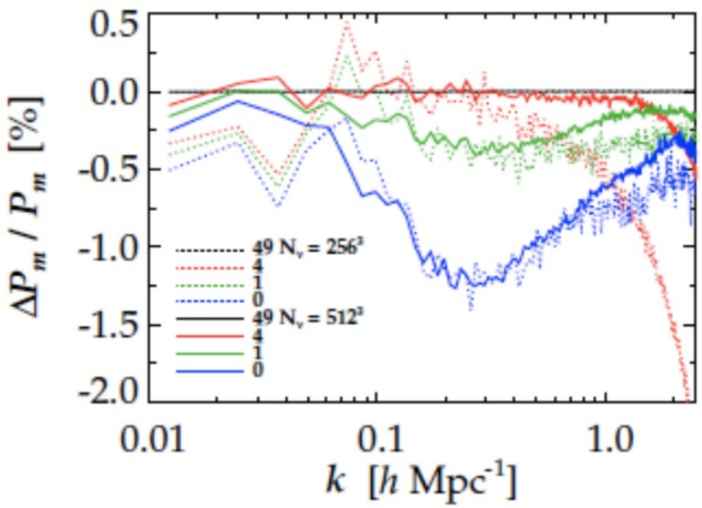
$$\left.\frac{\Delta P}{P}\right|_{\max} \sim -8\frac{\Omega_\nu}{\Omega_m} \quad \rightarrow \quad \left.\frac{\Delta P}{P}\right|_{\max} \sim -9.8\frac{\Omega_\nu}{\Omega_m}$$

Non-linear matter power

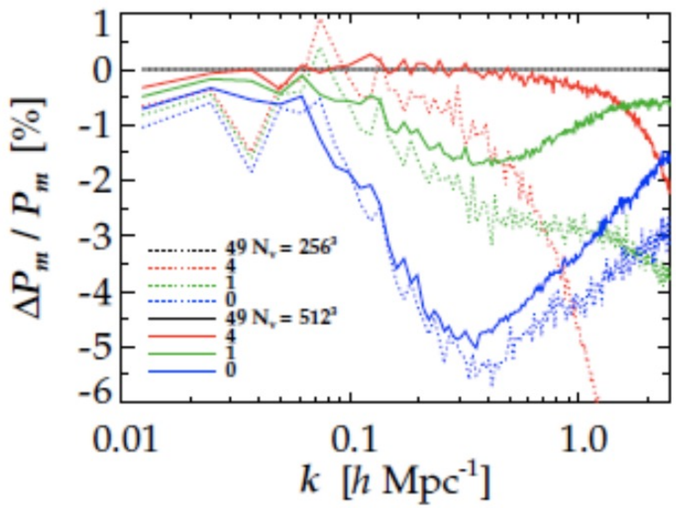
Computing the neutrino gravitational potential on the PM grid and summing up its contribution to the total matter gravitational potential

COMPARISON GRID VS PARTICLES

M ν =0.6 eV



M ν = 1.2 eV



Brandbyge et al 08b

Non-linear matter power

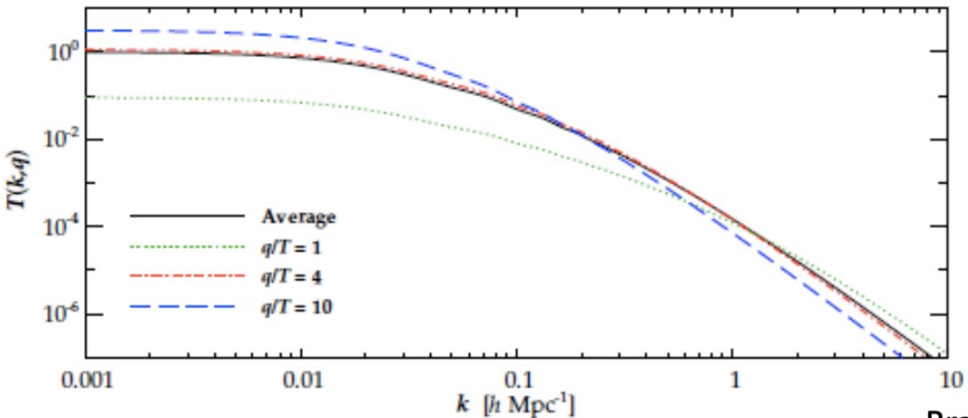
$$f = f_0 + \frac{\partial f_0}{\partial T}\delta T = f_0(1+\Psi) \qquad f_0(q) = \frac{1}{e^{q/T}+1}$$

After neutrino decoupling CBE

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = 0$$

$$\delta\rho_\nu(k) = 4\pi a^{-4}\int q^2 dq \epsilon f_0 \Psi_0 \qquad \epsilon = (q^2 + a^2 m^2)^{1/2}$$

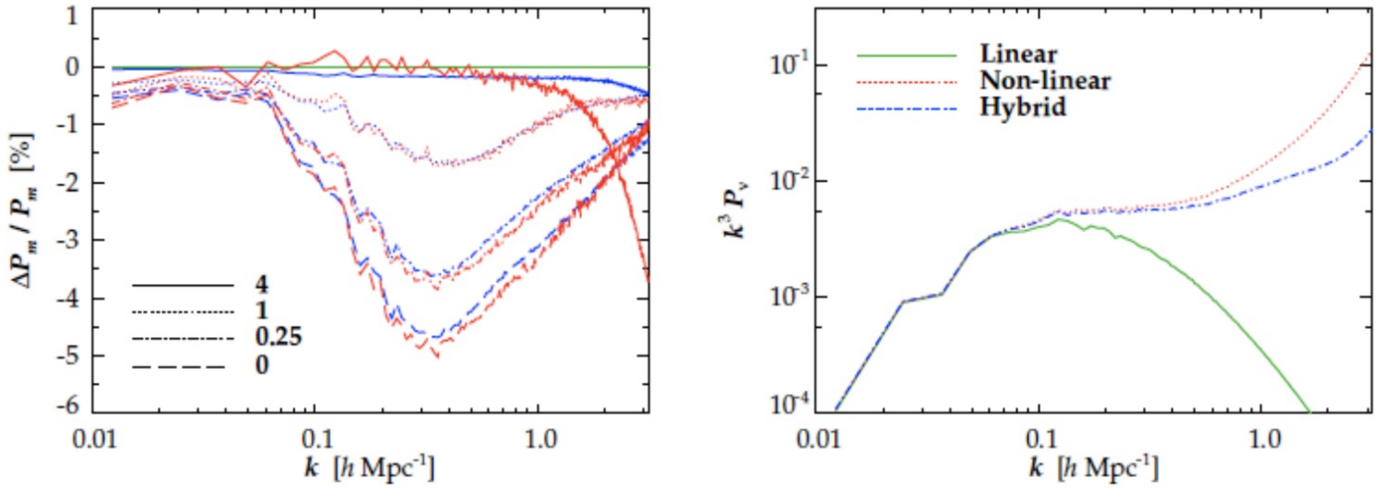
$$\dot\Psi_0 = -\frac{qk}{3\epsilon}\Psi_1 - \dot\phi\frac{d\ln f_0}{d\ln q},$$
$$\dot\Psi_1 = \frac{qk}{\epsilon}\left(\Psi_0 - \frac{2}{5}\Psi_2\right) - \frac{\epsilon k}{q}\psi\frac{d\ln f_0}{d\ln q},$$
$$\dot\Psi_l = \frac{qk}{\epsilon}\left(\frac{l}{2l-1}\Psi_{l-1} - \frac{l+1}{2l+3}\Psi_{l+1}\right),\ l \geq 2$$



$$\Psi_0(k,q,z) = T(k,q,z)\Psi_0^I(k,q)$$

Brandbyge & Hannestad 09

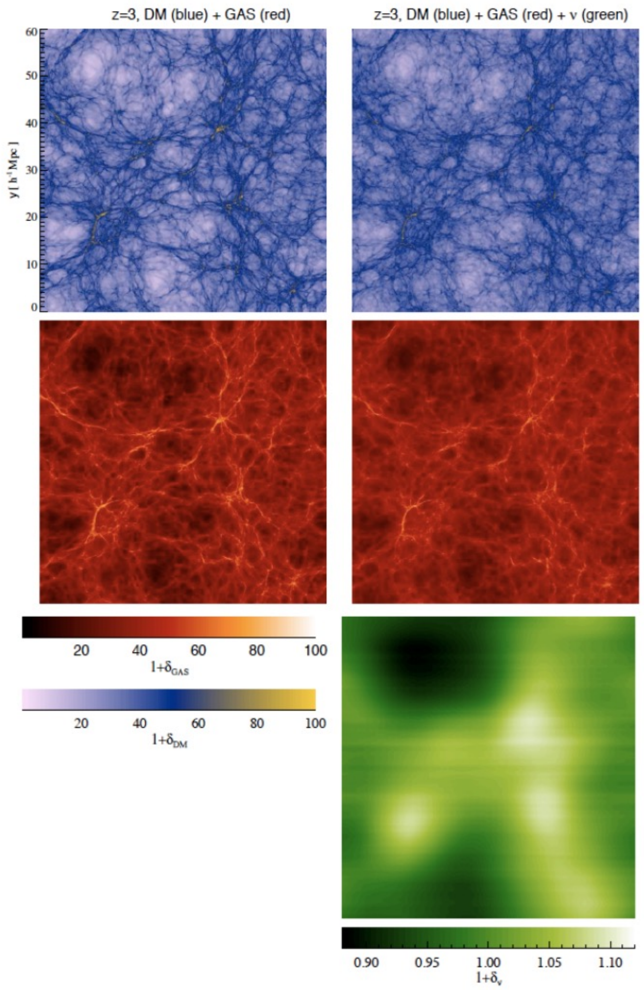
Non-linear matter power



PARTICLES: accurate non-linear sampling but prone to shot-noise errors

GRID: fast and accurate but no phase mixing (i.e. non-linear regime suppression maybe it is less than it should be)

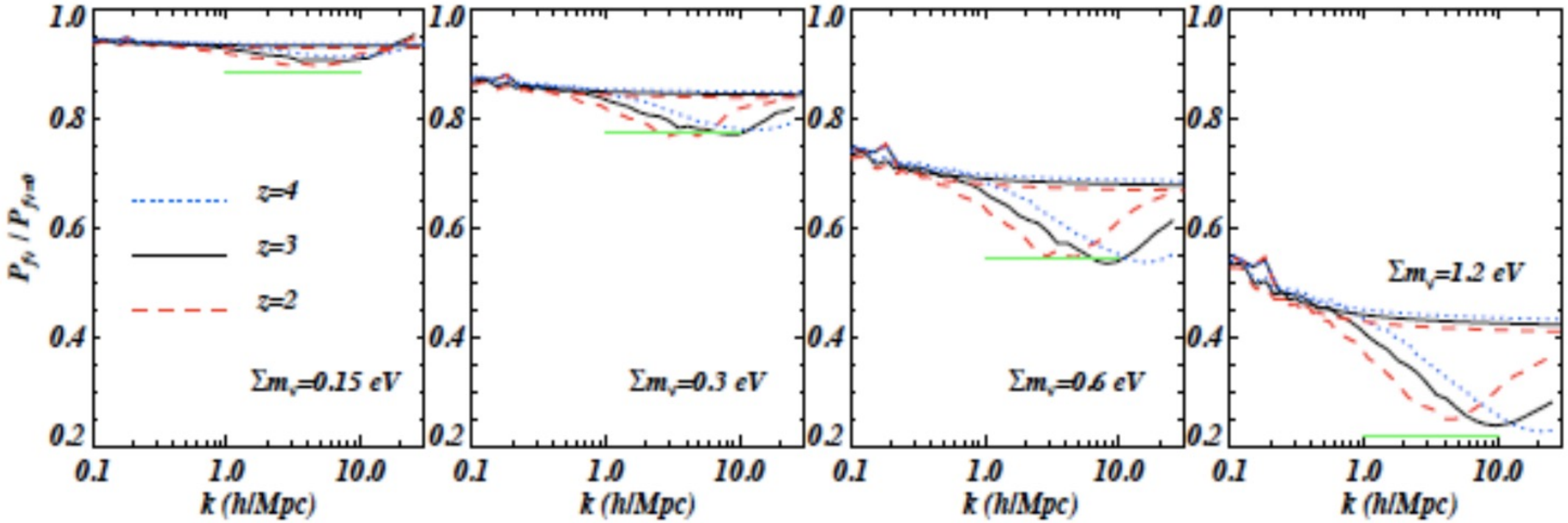
First hydro sim



TreeSPH code Gadget-III
follows DM, neutrinos, gas and star particles in a cosmological volume

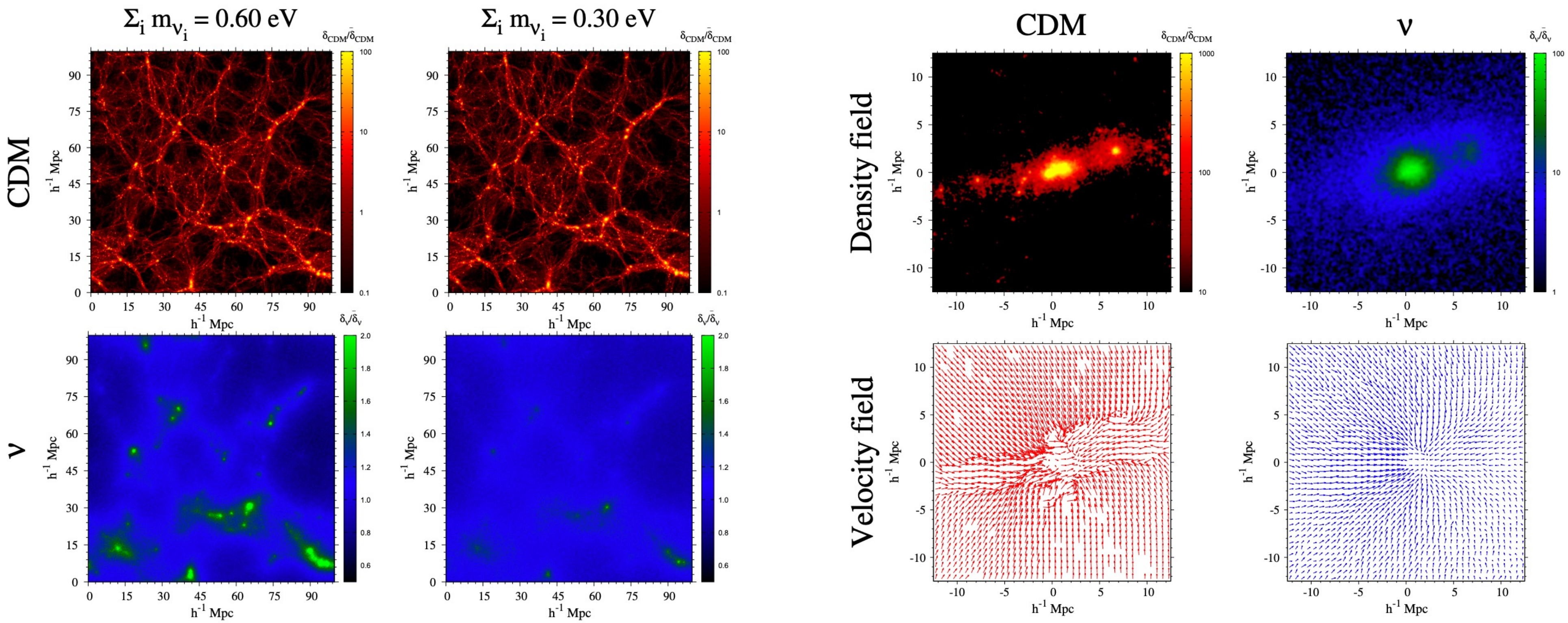
Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

Total matter power suppression

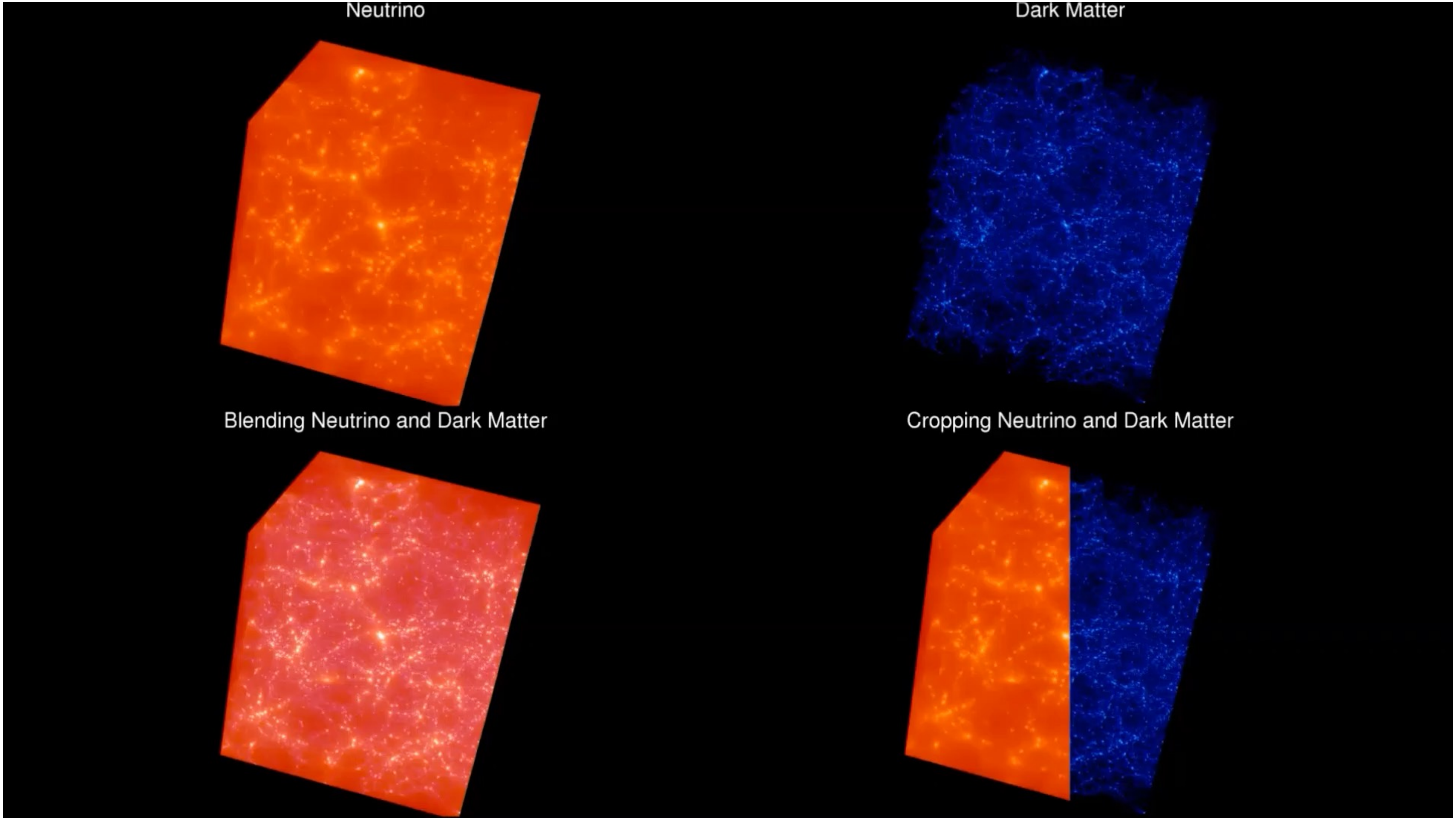


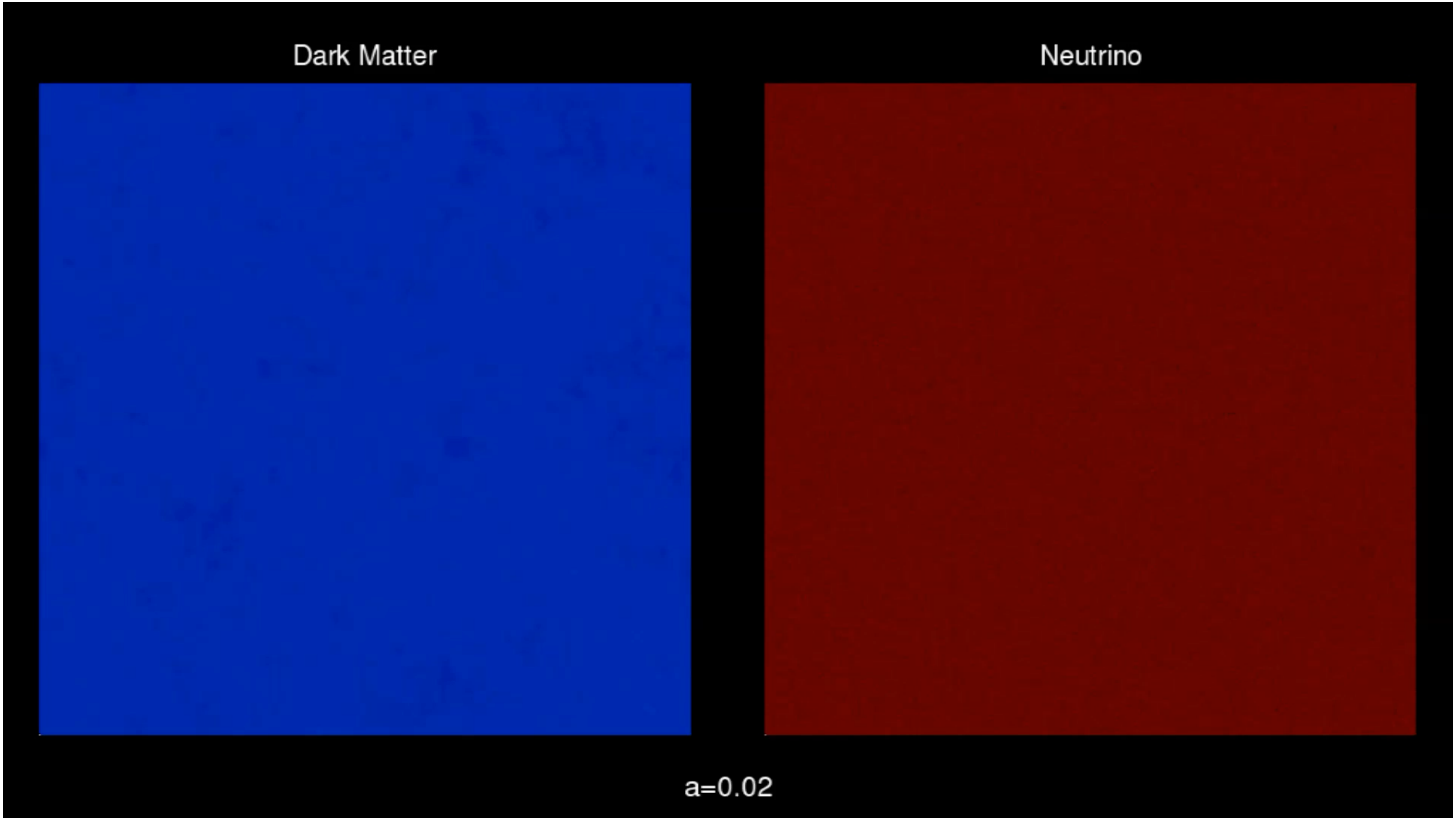
Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

Neutrino fluid clustering

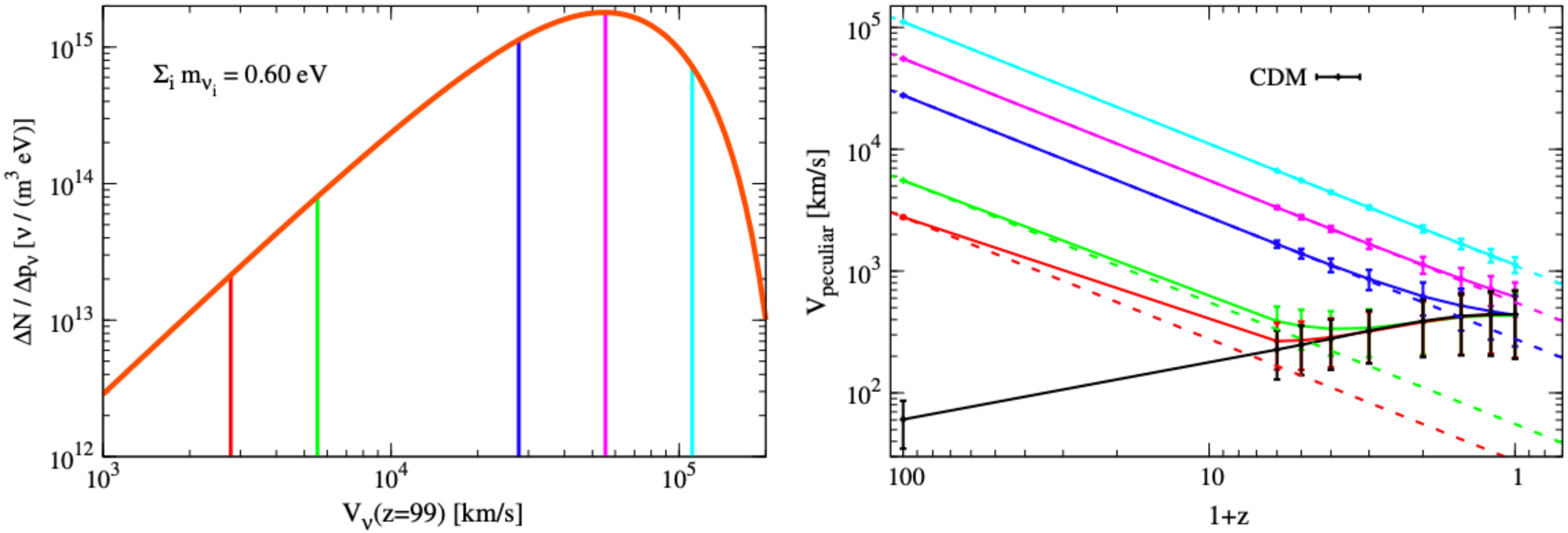


Villaescus-Navarro, Bird, Pena-Garay, Viel 2013


Neutrino
Dark Matter
Blending Neutrino and Dark Matter
Cropping Neutrino and Dark Matter


Dark Matter
Neutrino
a=0.02

Neutrino peculiar velocity evolution



Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Neutrino peculiar velocity evolution – II

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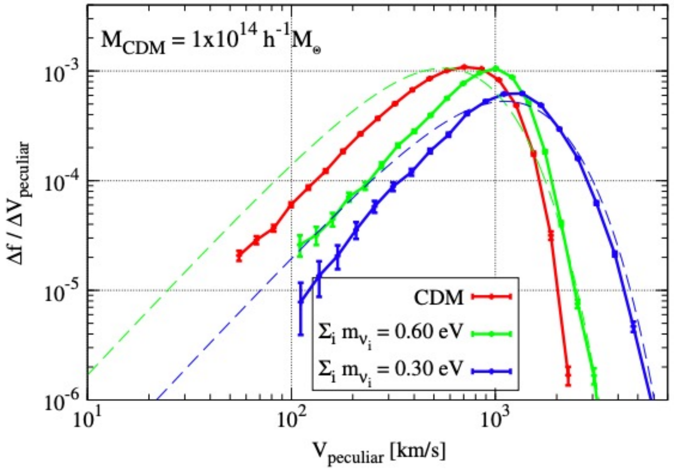
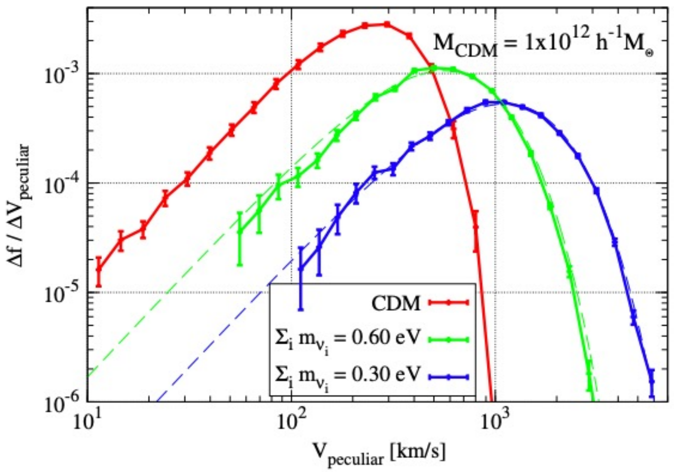


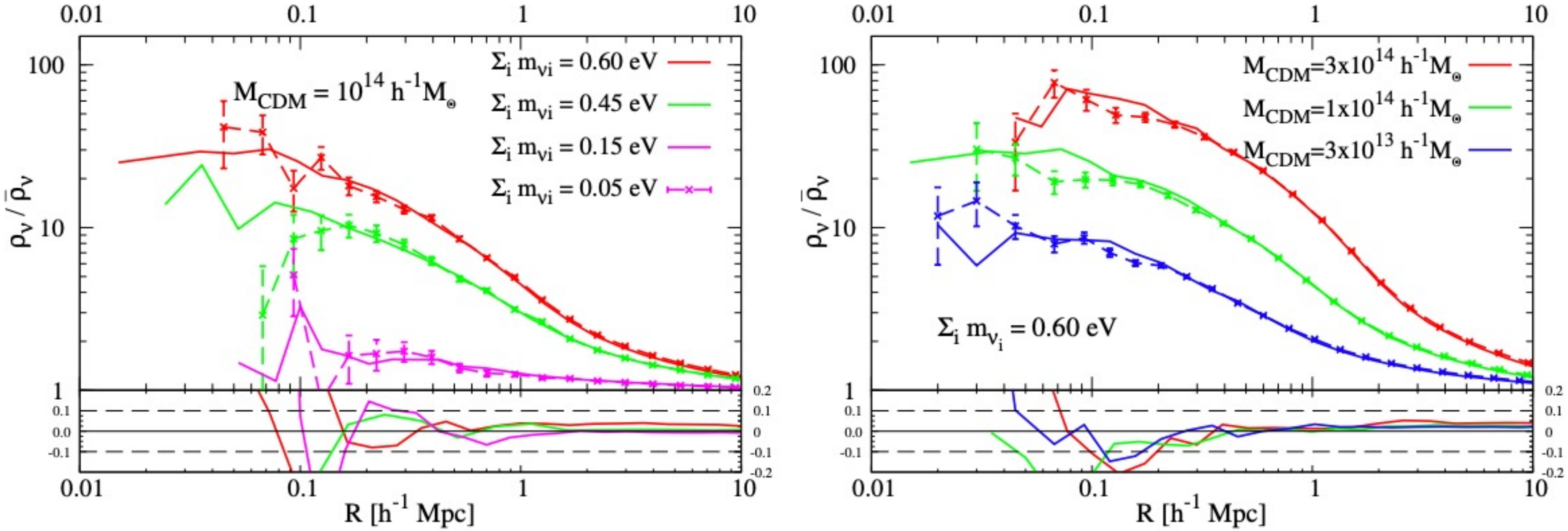


This is by selecting neutrino particles around haloes of a given mass

Neutrino slow down and cluster in a “selective” way

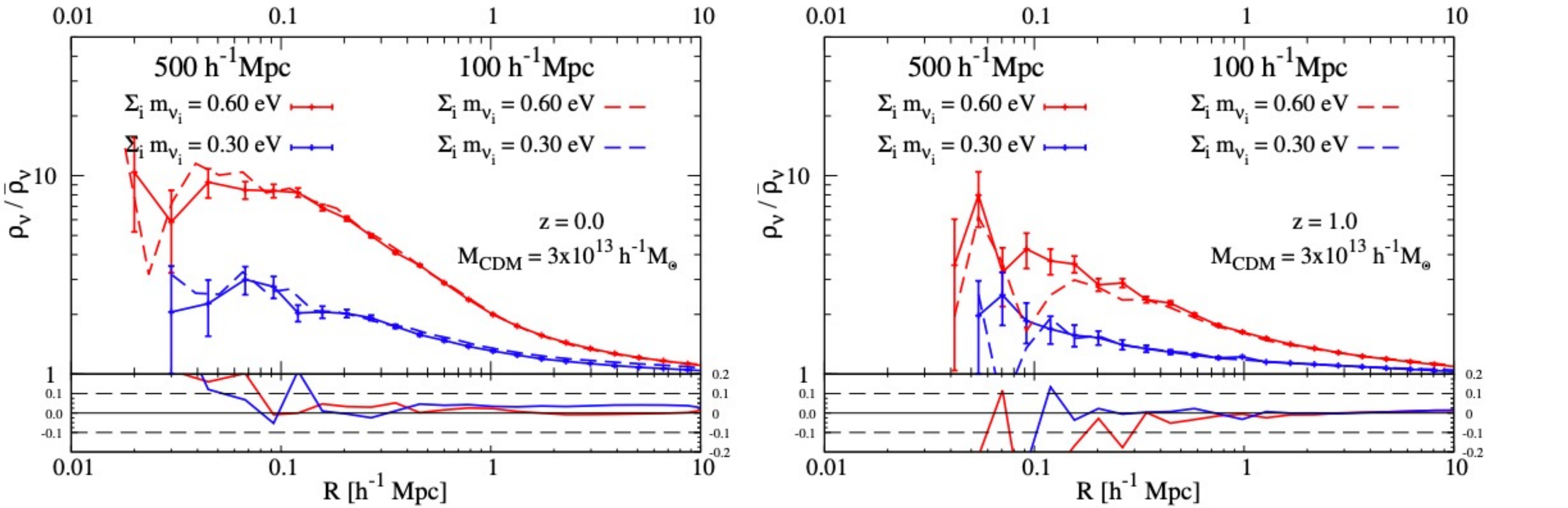
Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

The neutrino halo



Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

The neutrino halo



Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

Mass functions - I

Castorina, Sefusatti, Sheth, Villaescusa-Navarro, Viel 2013

FoF halos : b=0.2

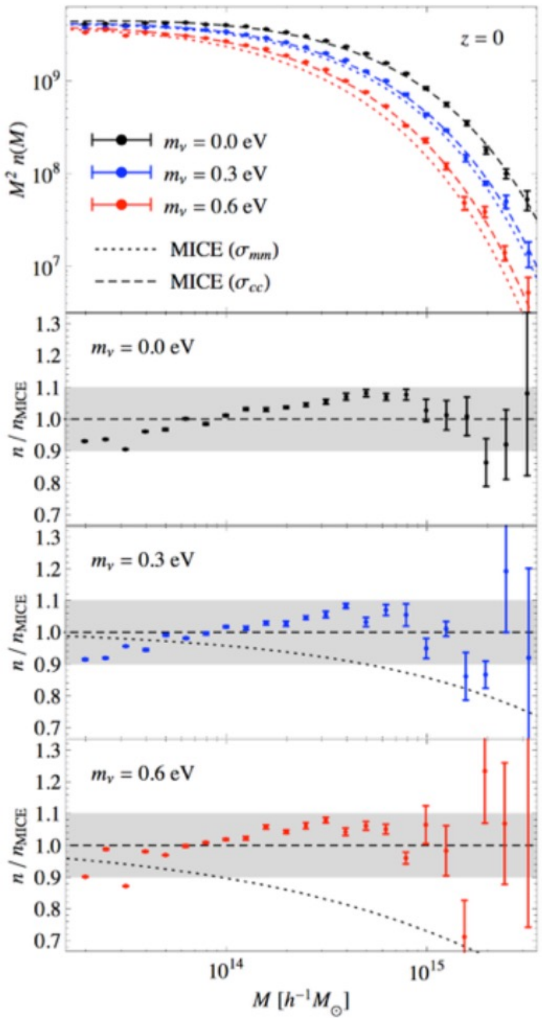
$$\frac{dn(M,z)}{dM} = \underbrace{\nu\, f(\nu)\, \frac{\rho_m}{M^2}\, \frac{d\ln\nu}{d\ln M}}$$

Matter prescription

$\rho_m \rightarrow \rho_{cdm} \quad P_m(k) \rightarrow P_m(k)$

Cold dark matter prescription

$\rho_m \rightarrow \rho_{cdm} \quad P_m(k) \rightarrow P_{cdm}(k)$



Note that

$$P_{cdm}(k,z) \geq P_m(k,z)$$

Mass functions - II

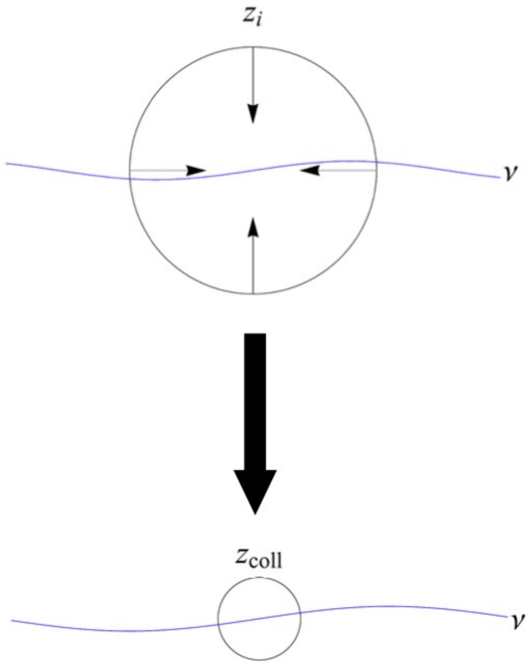
$$\delta_{cdm} > \delta_{crit} + a\,\delta_{\nu}$$

The physical picture:

the free streaming length is much larger than Lagrangian size of halos, neutrino perturbations do not play any role in the collapse.

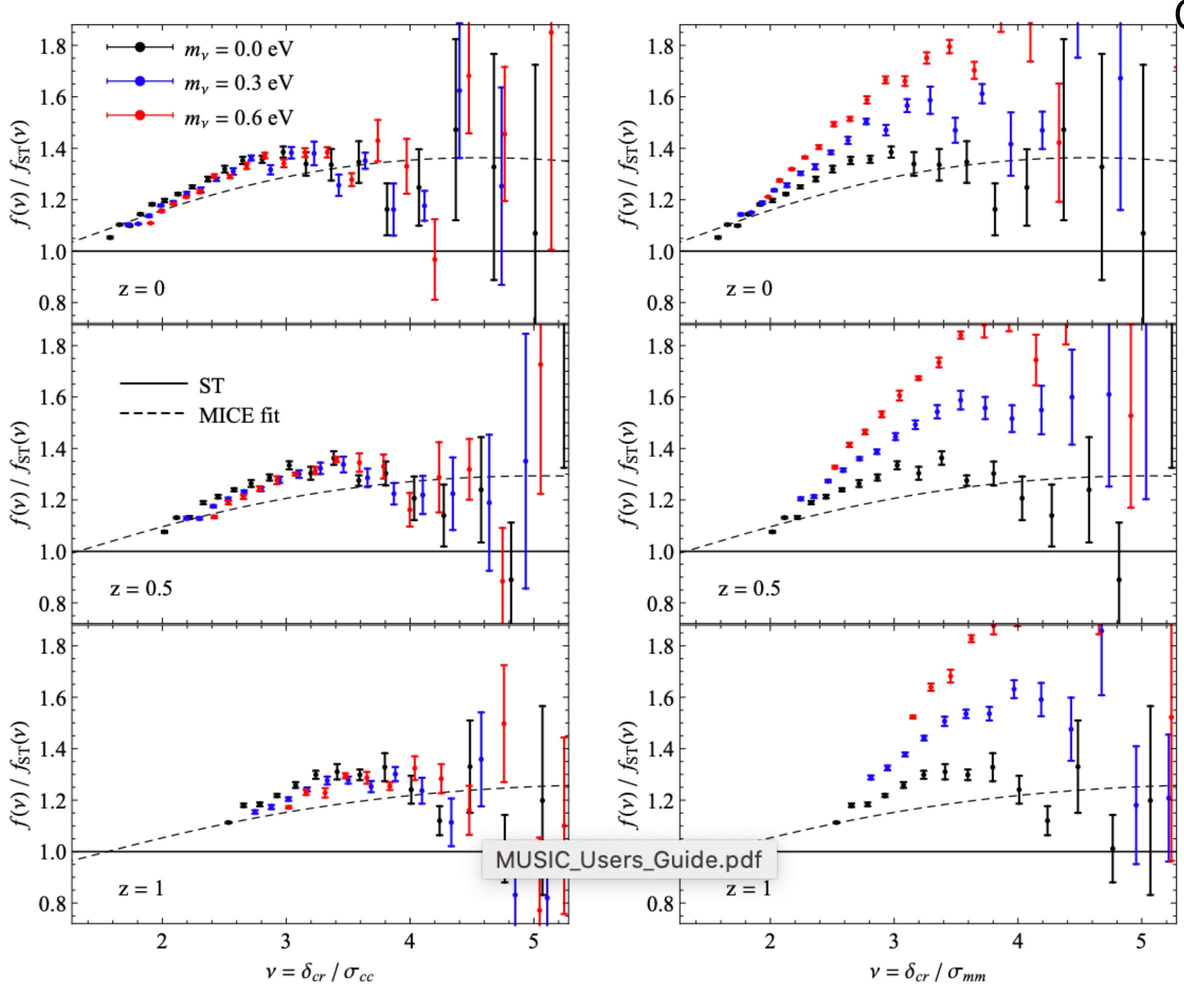
Ichiki&Takada(2012) studied the spherical collapse with massive neturinos, finding sub % effect on the collapse threshold.

They can be treaded as a background cosmology effect, like a Cosmological Constant, and we can and should use the CDM power spectrum.



Mass functions - III

Castorina+14



Halo bias?

Castorina+14

Halos and galaxies are biased tracers of the underlying mass distribution

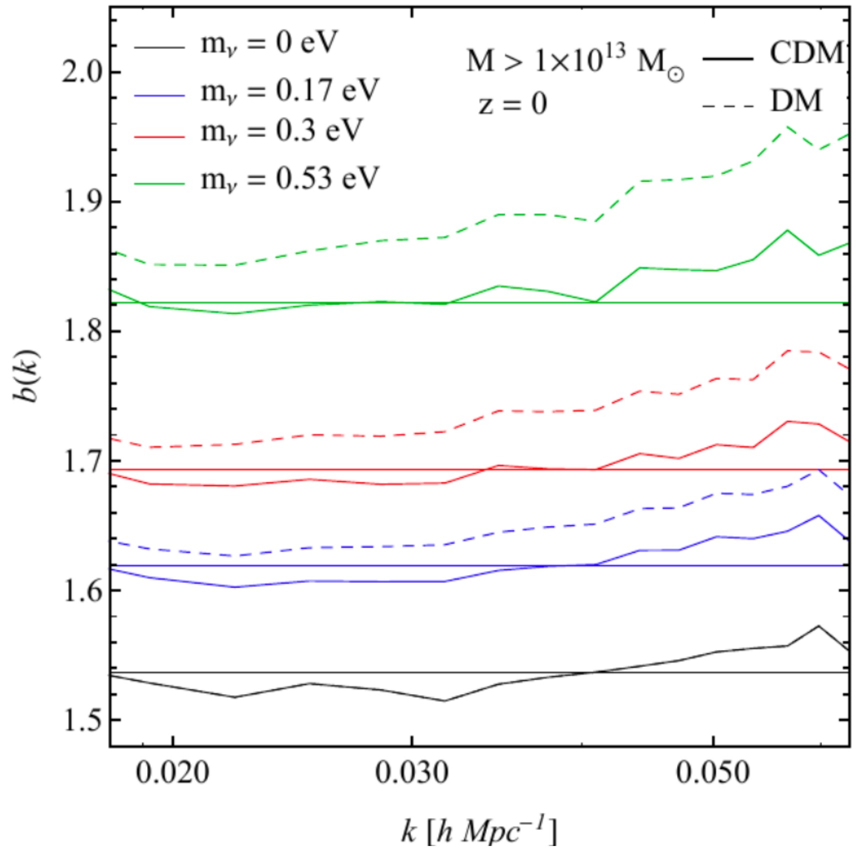
$$\delta_h(x) = b\,\delta_m(x)$$

Linear bias is expected to be scale-independent on large scales.

$$b_c^{(hh)} \equiv \sqrt{\frac{P_{hh}(k)}{P_{cc}(k)}}$$

$$b_m^{(hh)} \equiv \sqrt{\frac{P_{hh}}{P_{mm}}} = b_c^{(hh)}\sqrt{\frac{P_{cc}}{P_{mm}}}$$

Potential systematic error in galaxy clustering measurements.



Main conclusions (so far)

- N-body simulations recover linear growth at large scale
- Non-linear regime presents an overall suppression of power which is about 25% larger than in linear theory
- There is a distinctive spoon-shape feature at small scales, however, this is not due to neutrinos, but it is due to the different growth of small vs. large scales and their relative coupling → it can be used to constrain neutrinos once amplitude at larger scales is fixed
- Neutrino halo is characterized and it is cored for haloes $> 10^{13.5}$ Msun/h
- FD distribution of neutrinos is perturbed inside virialized haloes
- Momentum distribution is also perturbed over large cosmological volumes (the more massive they are the closer they are to CDM)

Villaescus-Navarro, Bird, Pena-Garay, Viel 2013

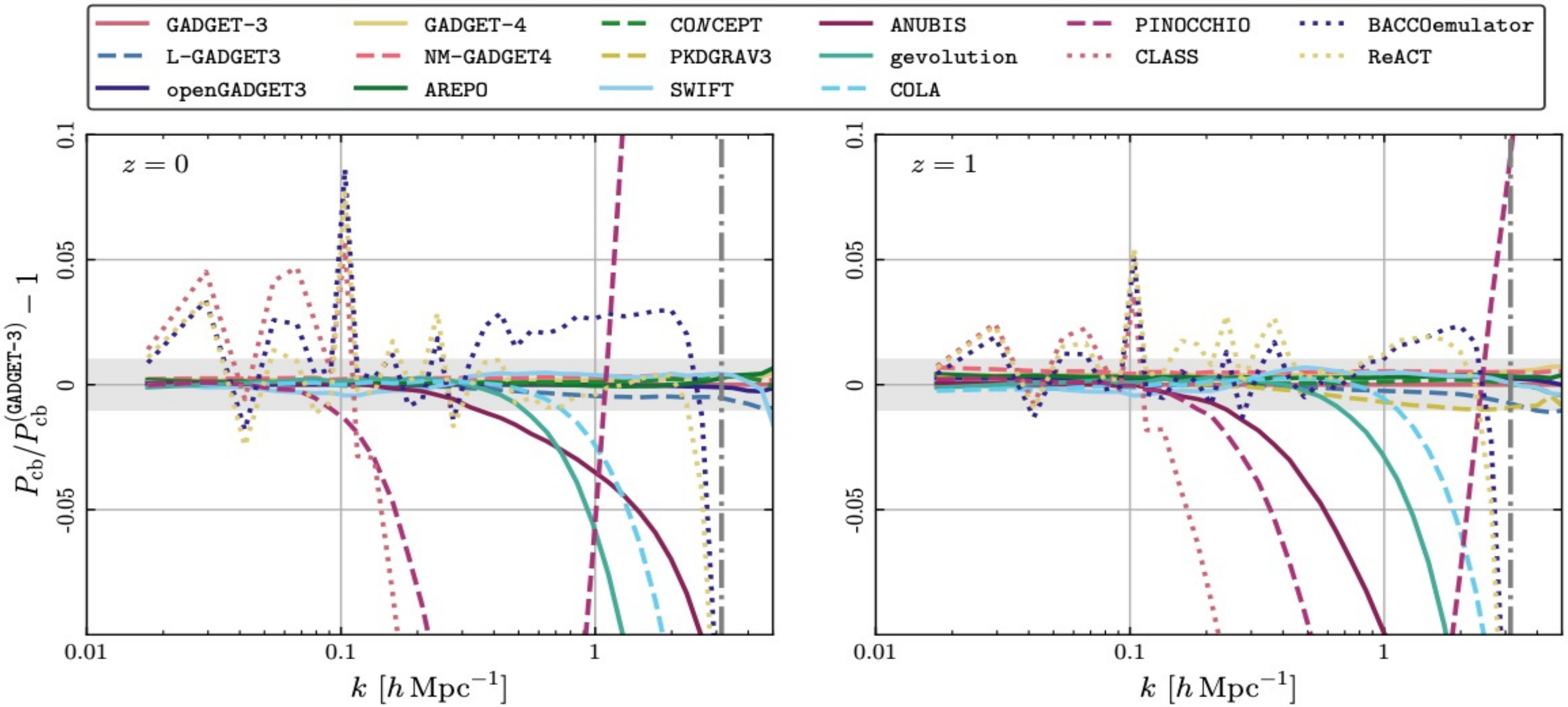
Are numerical effects under control?

Euclid*: Modelling massive neutrinos in cosmology – a code comparison

J. Adamek[1] R. E. Angulo[2,3] C. Arnold,[4] M. Baldi[5,6,7]

Code	type	neutrino method(s)
GADGET-3	N-body (Tree-PM)	particle
L-GADGET3	N-body (Tree-PM)	mesh
openGADGET3	N-body (Tree-PM)	particle
GADGET-4	N-body (Tree-PM)	particle
NM-GADGET4	N-body (Tree-PM)	Newtonian motion gauge
AREPO	N-body (moving mesh)	particle
CONCEPT	N-body (P^3M)	mesh
PKDGRAV3	N-body (Tree + FMM)	mesh
SWIFT	N-body (PM + FMM)	particle / δf
ANUBIS	N-body (PM + AMR)	particle
gevolution	N-body (uniform PM)	particle / mesh
COLA	N-body surrogate	mesh
PINOCCHIO	N-body surrogate	linear growth factor
ReACT	$P(k)$ prediction	halo-model reaction
BACCOemulator	$P(k)$ prediction	emulation
EuclidEmulator2	$P(k)$ prediction	emulation
Cosmic Emu	$P(k)$ prediction	emulation

Are numerical effects under control?



Halo model

Matteo Viel

$$\rho(\mathbf{x}) = \sum_i \mathcal{N}_i M_i u(\mathbf{x} - \mathbf{x}_i | M_i)$$

$$\begin{aligned} \langle \rho(\mathbf{x}) \rangle &= \int \mathrm{d}M\, M\, n(M) \sum_i \Delta V_i\, u(\mathbf{x} - \mathbf{x}_i | M) \\ &= \int \mathrm{d}M\, M\, n(M) \int \mathrm{d}^3\mathbf{x}' u(\mathbf{x} - \mathbf{x}' | M) \\ &= \int \mathrm{d}M\, M\, n(M) = \overline{\rho}, \end{aligned}$$

$$\langle \rho(\mathbf{x}_1) \rho(\mathbf{x}_2) \rangle = \sum_{i,j} \langle \mathcal{N}_i M_i \mathcal{N}_j M_j u(\mathbf{x}_1 - \mathbf{x}_i | M_i) u(\mathbf{x}_2 - \mathbf{x}_j | M_j) \rangle .$$

$$\xi(r) \equiv \langle \delta(\mathbf{x}_1) \delta(\mathbf{x}_2) \rangle = \xi^{1\mathrm{h}}(r) + \xi^{2\mathrm{h}}(r) \qquad (r \equiv |\mathbf{x}_1 - \mathbf{x}_2|) ,$$

where

$$\xi^{1\mathrm{h}}(r) = \frac{1}{\overline{\rho}^2} \int \mathrm{d}M\, M^2 n(M) \int \mathrm{d}^3\mathbf{y}\, u(\mathbf{y} - \mathbf{x}_1 | M) u(\mathbf{y} - \mathbf{x}_2 | M),$$

$$\begin{aligned} \xi^{2\mathrm{h}}(r) = \frac{1}{\overline{\rho}^2} & \int \mathrm{d}M_1\, M_1 n(M_1) \int \mathrm{d}M_2\, M_2 n(M_2) \int \mathrm{d}^3\mathbf{x} \int \mathrm{d}^3\mathbf{x}' \\ & \times u(\mathbf{x}_1 - \mathbf{x} | M_1) u(\mathbf{x}_2 - \mathbf{x}' | M_2) \xi_{\mathrm{hh}}(\mathbf{x} - \mathbf{x}' | M_1, M_2). \end{aligned}$$

Mo, Van den Bosch & White
"Galaxy formation and evolution"
book

Halo model

$$\rho(\mathbf{k}) = \sum_i \mathcal{N}_i M_i \tilde{u}(\mathbf{k}|M_i) e^{-i\mathbf{k}\cdot\mathbf{x}_i}.$$

Here $\tilde{u}(\mathbf{k}|M_i)$ is the Fourier transform of the density profile, which for a spherically profile truncated at the virial radius r_h is given by

$$\tilde{u}(k|M) = \frac{4\pi}{M} \int_0^{r_h} \rho(r|M) \frac{\sin kr}{kr} r^2 \, dr.$$

$$P(k) \equiv V_u \langle |\delta_{\mathbf{k}}|^2 \rangle = P^{1h}(k) + P^{2h}(k),$$

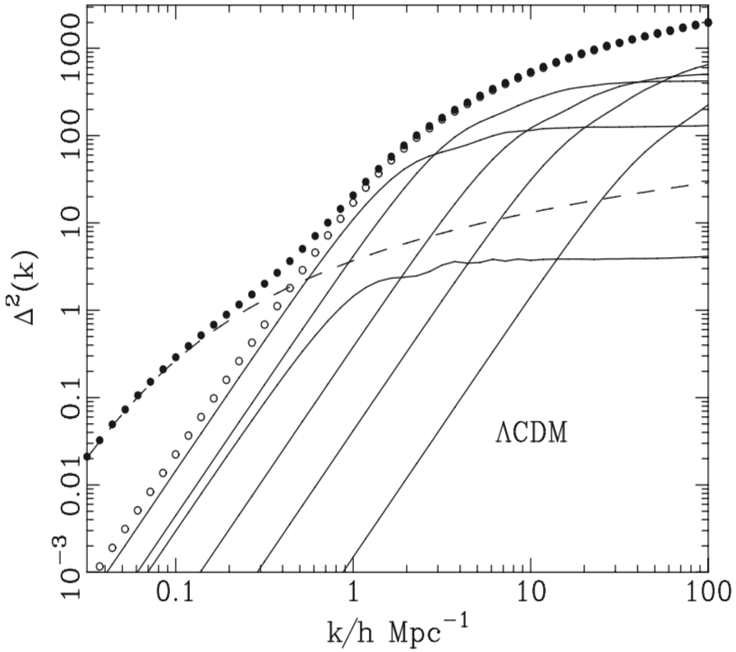
where

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM \, M^2 n(M) \, |\tilde{u}(k|M)|^2,$$

$$P^{2h}(k) = \frac{1}{\bar{\rho}^2} \int dM_1 \, M_1 n(M_1) \tilde{u}(k|M_1) \int dM_2 \, M_2 n(M_2) \tilde{u}(k|M_2) P_{hh}(k|M_1, M$$

$$P_{hh}(k|M_1, M_2) = b_h(M_1) \, b_h(M_2) \, P_{lin}(k),$$

$$P^{2h}(k) = P_{lin}(k) \left[\frac{1}{\bar{\rho}} \int dM \, M \, n(M) \, b_h(M) \, \tilde{u}(k|M) \right]^2$$



A neutrino halo model

$$P(k) = \left(\frac{\bar{\rho}_c}{\bar{\rho}}\right)^2 P_c(k) + 2\,\frac{\bar{\rho}_c\bar{\rho}_\nu}{\bar{\rho}^2}\,P_{c\nu}(k) + \left(\frac{\bar{\rho}_\nu}{\bar{\rho}}\right)^2 P_\nu(k)$$

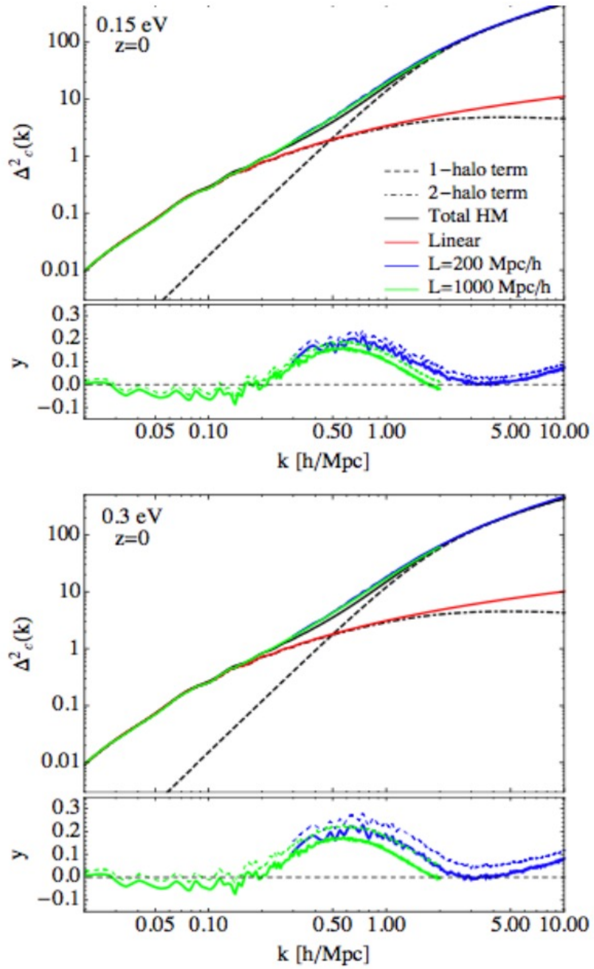
- Assumption: all matter within haloes 1h and 2h terms

- Simple modelling of non-linear power spectra (including cross-spectra)

- When used to predict ratios w.r.t. massless case it is as good as hydro/N-body to 2% level

- When used to compute actual power it suffers from limitation and it is good at the 20% level

NON LINEAR POWER SPECTRA



Massara, Villaescusa, MV (2014) – Castorina+ (2014) for bias and mass functions

Perturbation theory

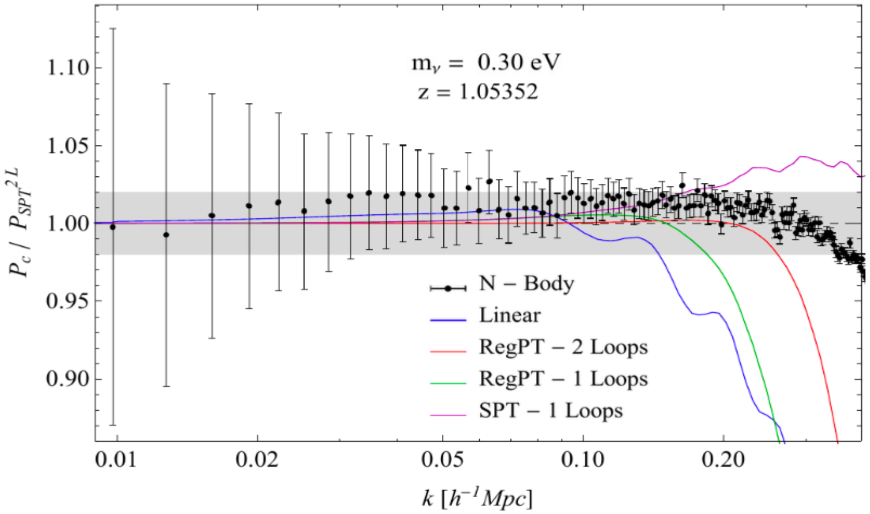
PT results (I)

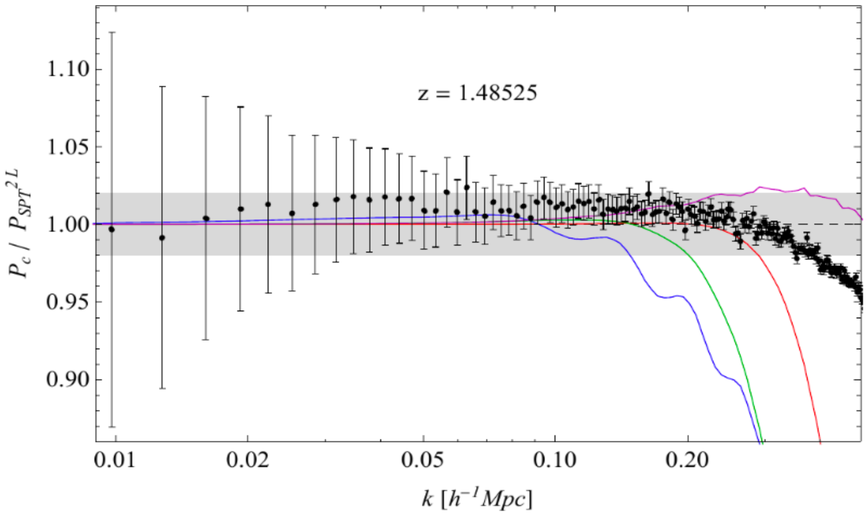
Non linear just in the CDM component.

Neglect scale dependent growth factor, see Blas+14

PT works at mildly non linear scales as in LCDM cosmology

At z=1.5 PT is accurate at a few % level up to kmax = 0.4 h/Mpc





What about Pmm?

Summary of Lecture 2

Neutrino non-linear clustering is selective: depends on halo mass, redshift, scales

This is good: quantitative predictions in a variety of environments,
mass functions, Bias of haloes/galaxis, neutrino haloes

All the effects are somewhat at the percent or sub-percent level

Precision post-Planck cosmology will address this