REFLECTED WAVES AND QUANTUM GRAVITY

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GIANNI'S FEST

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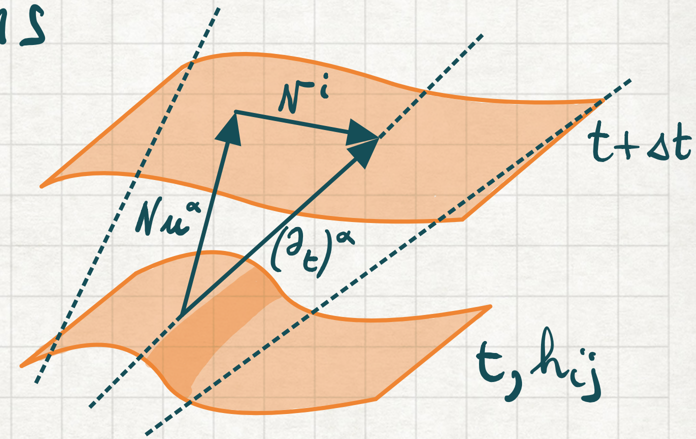
- DESPITE HIS FORMATION AS A THEORETICAL PARTICLE PHYSICIST, GIANNI UNDERSTOOD THE RELEVANCE OF COSMOLOGY AND DEDICATED MOST OF THE LAST 30 yrs. OF RESEARCH TO IT.

- THIS TALK, IN PARTICULAR, FOLLOWS OUR RESEARCH PATH TOWARD THE INVESTIGATION OF, POSSIBLE, OBSERVABLE, QUANTUM GRAVITATIONAL EFFECTS IN THE CONTEXT OF "CANONICAL QUANTUM GRAVITY" AND ADOPTING AN APPROACH, GIANNI AND HIS COLLABORATORS HAVE REFINED THROUGH THE YEARS ...

CANONICAL QUANTUM GRAVITY

* GR INVARIANCE W.R.T. DIFFEOMORPHISMS LEADS TO A SERIE OF CONSTRAINTS FOR THE SPATIAL PART OF THE METRIC h_{ij}



- AT THE QUANTUM LEVEL Wheeler-DeWitt EQ.

$$\hat{\mathcal{H}}_\alpha \Psi(\{h_{ij}\}) = 0 \quad , \quad \{h_{ij}\} \in \text{SUPERSPACE}$$

↳ APPLICATION TO COSMOLOGY ⇝ FRW (MINISUPERSPACE) + HOMOGENEOUS MATTER (INFLATON)

$$\hat{H} = \hat{H}_M + \hat{H}_G \qquad \hat{H}\Psi(a,\phi) = 0$$

!! **PROBLEMS**: ORDERING AMBIGUITIES, ABSENCE OF TIME PROBABILISTIC INTERPRETATION, CLASSICAL LIMIT

BORN-OPPENHEIMER DECOMPOSITION

- CONSISTS IN FACTORIZING $\underbrace{\Psi(a,\phi)}_{} = \overset{\text{NUCLEUS}}{\underset{\text{slow part}}{\psi(a)}}\ \overset{\text{ELECTRONS}}{\underset{\text{fast part}}{\chi(a,\phi)}}$

↳ GRAVITY EQ. + MATTER EQ.
(NUCLEUS) (ELECTRONS)

QUANTUM GRAVITATIONAL EFFECTS (non-adiabatic)

$$\begin{cases} \left[\hat{H}_G + \langle \hat{H}_M \rangle + \langle \chi | \hat{q} | \chi \rangle \right] \psi(a) = 0 \\ \frac{1}{M_p^2} \frac{\partial_a \psi}{\psi} \partial_a |\chi\rangle + \hat{H}_M |\chi\rangle + \left[\hat{Q} - \langle \chi | \hat{Q} | \chi \rangle\right] |\chi\rangle = 0 \end{cases}$$

$$\rightarrow i\partial_t |\chi\rangle + \hat{Q}_t |\chi\rangle$$

✓ Ordering ambiguities are irrelevant for large "a"

✓ TIME is introduced by gravity probability flux $\sim \dot{a}$

✓ Classical limit can be recovered by WKB, coherent-states

— PROBABILISTIC INTERPRETATION (MEASUREMENT) IS PROBLEMATIC

MUKHANOV-SASAKI EQ.

- ADDING THE PERTURBATIONS ALLEVIATES THE PROBLEM
- HANDLE TO CHECK QG WITH INFLATIONARY OBSERVABLES

$$\text{WdW} \rightarrow \left(\hat{H}_G + \hat{H}_M + \sum_k \hat{H}_k\right) \Psi\left(a, \phi, \{v_k\}\right) = 0$$

- BO decomposition : $\Psi = \psi_g(a)\, \chi_m(a,\phi) \prod_k \chi_k(a, v_k)$

- $\mathcal{P}_R = \mathcal{P}_R^{(0)} \left[1 + \alpha(\eta) \frac{H^2}{M_p^2} \left(\frac{k_0}{k}\right)^3 \right]$

COSMOMC

A COMPARISON WITH PLANCK DATA INDICATES $k_0 \sim$ galaxy radius

- LARGE SCALES DEVIATIONS
- $\alpha(\eta)$ depends on background
- k^{-3} is common to diverse approaches.
- k_0 is not FIXED BY THE THEORY

MOTIVATIONS

- NEXT GEN OF EXPERIMENTS MAY BE ABLE TO DETECT SMALL FEATURES OF PRIMORDIAL ORIGIN (NEW ENERGY SCALE)
- TYPICALLY OSCILLATORY
- ALLEVIATE TENSIONS AND ANOMALIES IN THE CURRENT DATA
- HELP TO DISCRIMINATE AMONG DIFFERENT INFLATIONARY SCENARIOS
- UNVEIL THE PATH TO QUANTUM GRAVITY

GRAVITATIONAL SECTOR

$$\partial_a^2 \tilde{\Psi}(a) + 2\tilde{M}_p^2 \left(\frac{p^2}{2a^2 \tilde{M}_p^2} + a^4 \Lambda \right) \tilde{\Psi}(a) \cong 0$$

- 2nd order ODE, $a \gg \lambda^{-1/6} \tilde{M}_p^{-1}$

$y = \tilde{M}_p^3 a^3$

$\lambda = \Lambda / \tilde{M}_p^4$

PLANE WAVE SOLUTIONS

$$\tilde{\Psi}(a) \sim A_1 e^{-i\sqrt{2\lambda}\, y} + A_2 e^{i\sqrt{2\lambda}\, y}$$

"OUT-GOING" "IN-GOING"

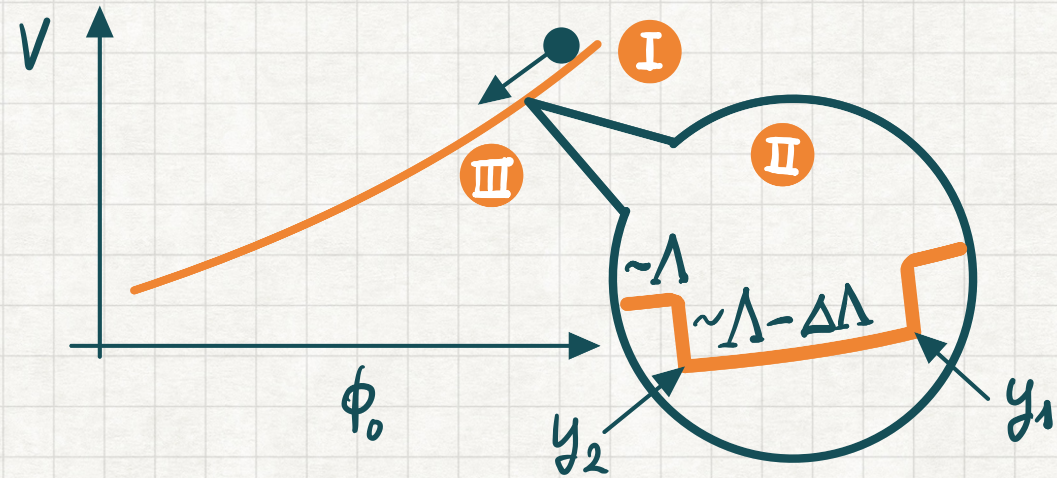
* $A_1, A_2 \rightsquigarrow$ INITIAL CONDITIONS (Vilenkin, Hartle-Hawking)

- let $A_2 \to 0 \Rightarrow \frac{1}{\tilde{M}_p^2} \frac{\partial_a \tilde{\Psi}}{\tilde{\Psi}} \partial_a \tilde{x}_K = -i\, a^2 \sqrt{\Lambda / 3M_p^2}\, \partial_a \tilde{x}_K$

CLASSICAL VELOCITY

$$= -i\, a'_{cl} \partial_a \tilde{x}_K = -i \frac{d}{d\eta} \tilde{x}_K$$

INFLATON POTENTIAL



$V \sim \Lambda$ y OUTSIDE $[y_1, y_2]$

$V \sim \Lambda \pm \Delta\Lambda$ $y \in [y_1, y_2]$

- QM POTENTIAL WELL FOR "GR wave-function"

$$\tilde{\Psi}_{I} = e^{-iqy} + r e^{iqy}, \quad \tilde{\Psi}_{II} = a e^{ipy} + b e^{-ipy}, \quad \tilde{\Psi}_{III} = t e^{-iqy}$$

I $y < y_1$ II $y_1 < y < y_2$ III $y > y_2$

* RESONANT TRANSMISSION $p \Delta y = n\pi \Rightarrow r = 0$

$$\tilde{\Psi} = e^{-iqy}\left[\theta(y_1 - y) + \theta(y - y_2)\right] + b e^{-ipy}\left(1 + \varepsilon e^{2ip(y - y_1)}\right)\theta(y - y_1)\,\theta(y_2 - y)$$

$$|\varepsilon| = \left|\frac{a}{b}\right| \sim \Delta\Lambda / \Lambda$$

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \,, \quad \frac{z''}{z} \sim 2a^2H^2(1+f(\epsilon_i))$$

- FEATURES IN THE POTENTIAL "AFFECT" MODES AFTER "HORIZON EXIT" (in the semiclassical picture), $k/aH \ll 1$

BUNCH DAVIES VACUUM

$$v_k \sim e^{-ik\eta} \rightsquigarrow v_k \sim \alpha e^{ik\eta} + \beta e^{-ik\eta}$$

GIVING RISE TO "semiclassical" OSCILLATIONS

- PRESENCE OF A "SUDDEN JUMP" IN THE POTENTIAL GENERATES A REFLECTED WAVE AS WELL

- "RESONANT TRANSMISSION" LESS GENERAL BUT EASIER TO DEFINE INITIAL CONDITIONS FOR v_k IN ABSENCE OF A REFLECTED WAVE.

MS EQUATION

$-i\, d/d\eta\, |\tilde{x}_K\rangle$ in (I) and (III)

$$\frac{1}{\tilde{M}_p^2}\frac{\partial_a\tilde{\varphi}}{\tilde{\varphi}}\partial_a|\tilde{x}_K\rangle + \left(\hat{H}_K - \langle\hat{H}_K\rangle\right)|\tilde{x}_K\rangle \approx O\left(H^2/M_p^2\right)$$

$$-i\,\frac{1+\varepsilon}{1-\varepsilon}\,\frac{1-\varepsilon e^{2ip(y-y_1)}}{1+\varepsilon e^{2ip(y-y_1)}}\,\frac{d}{d\eta}|\tilde{x}_K\rangle \quad \text{in (II)}$$

where $\frac{d}{d\eta}\langle\tilde{x}_K|\tilde{x}_K\rangle = 0$, on "REPHASING" $\tilde{x}_K = e^{i\varphi}x_K$,

$$\frac{d\varphi}{d\eta} = \frac{1-\varepsilon}{1+\varepsilon}\,\frac{1+\varepsilon e^{2ip(y-y_1)}}{1-\varepsilon e^{2ip(y-y_1)}}\langle\hat{H}_K\rangle = \frac{\langle\hat{H}_K\rangle}{m}$$

$$i\frac{d}{d\eta}|x_K\rangle \equiv \hat{H}_K|x_K\rangle$$

NON-HERMITEAN TDHO

where $\hat{H}_K = \frac{1}{2m}\hat{\pi}_v^2 + \frac{m\tilde{\omega}^2}{2}\hat{v}^2$

$\tilde{\omega}^2 = \omega^2/m^2$

PINNEY EQUATION

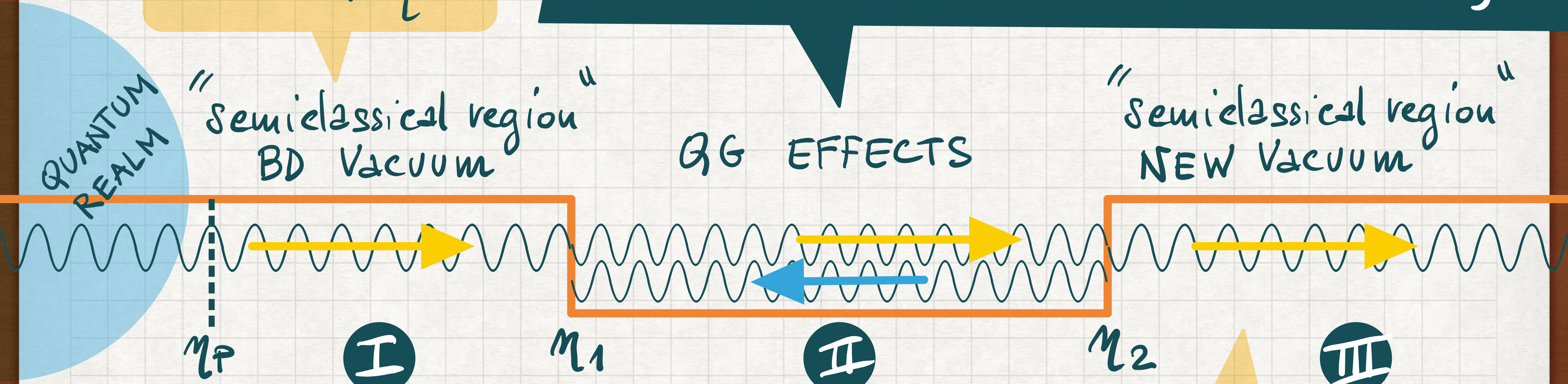
TDHO ⇝ INVARIANT OPERATORS $\quad i\frac{d}{d\eta}\hat{I} + [\hat{I},\hat{\mathcal{H}}]=0$

INV. VACUUM

$$|\chi_{k,0}\rangle = \frac{1}{(\pi\rho^2)^{1/4}} \exp\left[i\frac{m}{2}\left(\frac{\rho'}{\rho}+\frac{i}{m\rho^2}\right)v^2 - \frac{i}{2}\int_{\eta_i}^{\eta}\frac{d\bar{\eta}}{m\rho^2}\right]$$

$$\rho_{ds}^{(BD)} = \sqrt{\frac{1+k^2\eta^2}{k^3\eta^2}}$$

PINNEY EQ: $\quad \rho''+\frac{m'}{m}\rho'+\tilde{\omega}^2\rho=\frac{1}{m^2\rho^3}$



$$\rho_{ds}^{2(NEW)} \sim \frac{1+k^2\rho_0^4+\rho_0^2\rho_0'^2+\left(1-k^2\rho_0^4+\rho_0^2\rho_0'^2\right)\cos 2k\eta_2+2k\rho_0^3\rho_0'\,\mathrm{sen}\,2k\eta_2}{2\eta^2k^4\rho_0^2}$$

OBSERVABLE EFFECTS

$$\Delta_S^2 = \lim_{-k\eta\to 0} \frac{k^3}{2\pi^2} \langle \tilde{x}_k | \hat{v}^2 | \tilde{x}_k \rangle = \lim_{-k\eta\to 0} \frac{k^3}{2\pi^2} \operatorname{Re} \frac{g^2}{2}$$

- CMB : $\frac{k}{a_0} \in [10^{-4}, 10^{-1}]\ \mathrm{Mpc}^{-1}$, $H_*/M_p \sim 10^{-4} r^{1/2} < 2.5 \cdot 10^{-5}$ (95% C.L.)

$$e^{2ip(y-y_1)} \cong \exp\left[-i \underbrace{\frac{4M_p^2}{\bar{k}^3 H^2}}_{\alpha_p} \left(\frac{1}{\eta^3} - \frac{1}{\eta_1^3}\right)\right] , \quad \bar{k}^{-3} = \int_V d^3x$$

- in what follows we set

$\bar{k} = 10^3 \longrightarrow \alpha_p = 4 \cdot 10^3$, $\eta_1 = -10$, $\eta_2 \sim -7.3$ (single oscillation)

$\varepsilon = 10^{-1}$ (large)

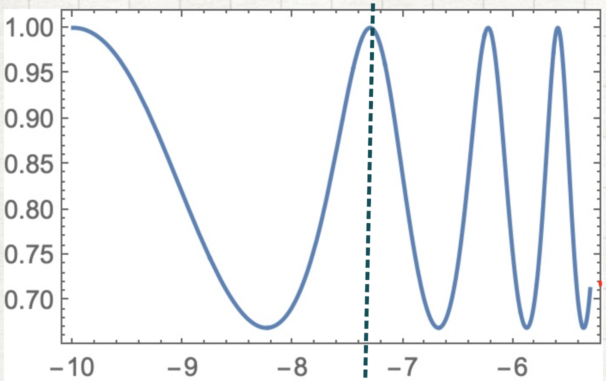
$k \in 2[10^{-1}, 10^2] \longrightarrow -k\eta_2 \gtrsim 1$, $\eta_1 \gg \eta_p$

ANALYTICAL/NUMERICAL

modified MS Eq $$v_k'' + \frac{m'}{m} v_k' + \frac{k^2 - 2/\eta^2}{m^2} v_k = 0$$

$$v_k \sim \exp\left[\pm i \int_{\eta_1}^{\eta_2} \frac{k}{m}\, d\eta\right] \sim \exp\left[\pm i k (r + i s)\right], \quad s > 0, \quad r \sim (\eta_2 - \eta_1) - O(\varepsilon)$$

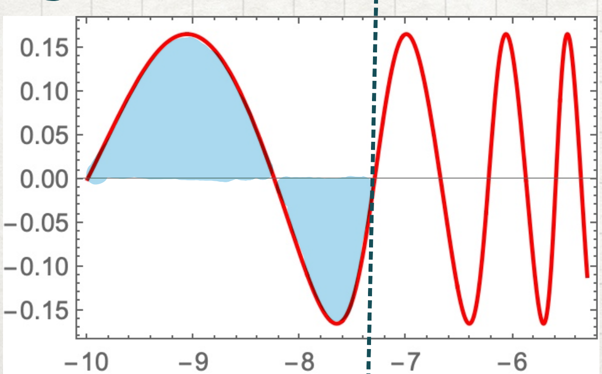
Re m^{-1} η_2

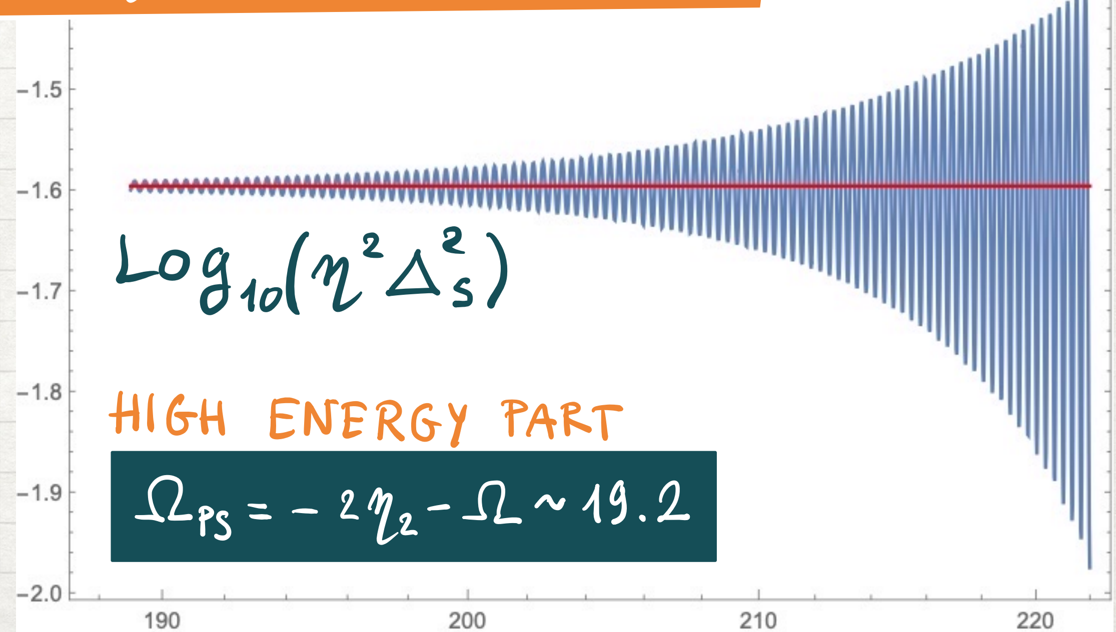


(*) for k LARGE THE "NEGATIVE" SOL. DOMINATES

$$\rho^2_{FIT} \sim \frac{1 + k^2\eta_2^2}{k^3 \eta_2^2}\left[1 + e^{a + ic + (b + i\Omega)k}\right]$$

Im m^{-1}





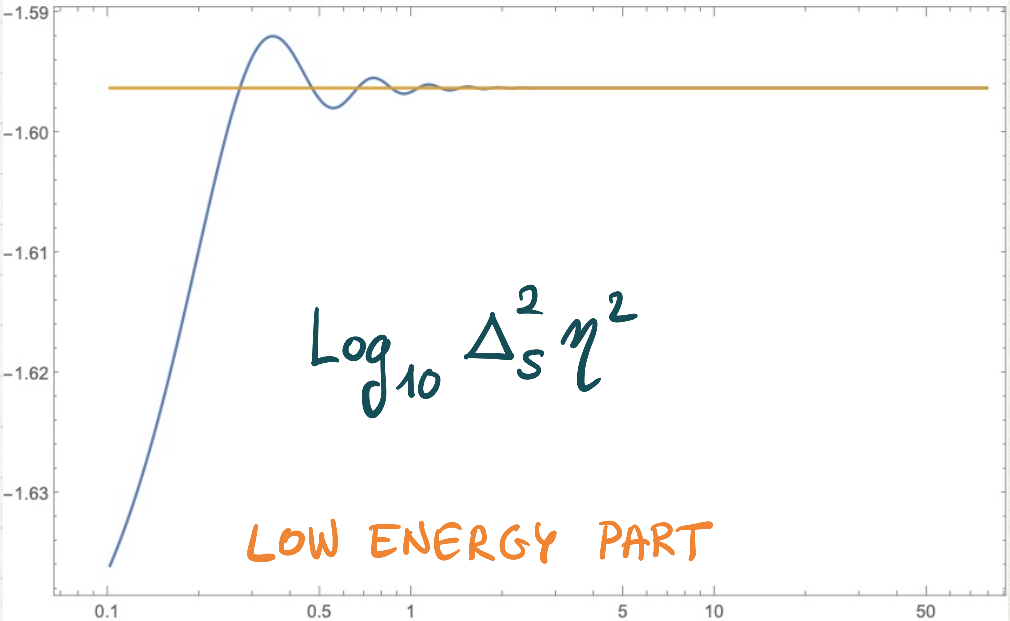
$$\Omega_{PS} = -2\eta_2 - \Omega \sim 19.2$$

- $\Omega_{PS} \sim 4 \cdot 10^4$ Mpc

$\omega \sim 300$ Mpc in the interval $2\,[10^{-2}, 10^{-1}]$ Mpc^{-1} alleviates

Some tensions and anomalies in PRESENT DATA

- THE "LARGE" K OSCILLATIONS MAY BE NON-OBSERVABLE IN CMB BUT HAVE OTHER RELEVANT EFFECTS (SGWB)
- OSCILLATIONS MAY FIT CMB DATA BETTER
- AMPLITUDE INCREASE SINCE $-k\eta \to 1$ (EXPECTED)
- DEPEND ON THE POSITION OF THE FEATURE



- $\Omega_{PS} \sim -2\eta_2$

CONCLUSIONS

- FEATURES IN THE INFLATON POTENTIAL MAY BE OBSERVABLE IN NEXT. GEN SURVEYS (CMB, LSS, ...)

- QUANTUM GRAVITATIONAL EFFECTS MAY PLAY AN IMPORTANT ROLE BUT USUALLY OVERLOOKED

- THEY ARE POTENTIALLY LARGE BUT WASHED OUT BY COSMIC HISTORY (WE LEAVE THIS TO FUTURE STUDIES)

- NEW NON-PERTURBATIVE APPROACH TO DEAL WITH QG EFFECTS ASSOCIATED TO THE BO DECOMPOSITION OF THE WdW EQUATION