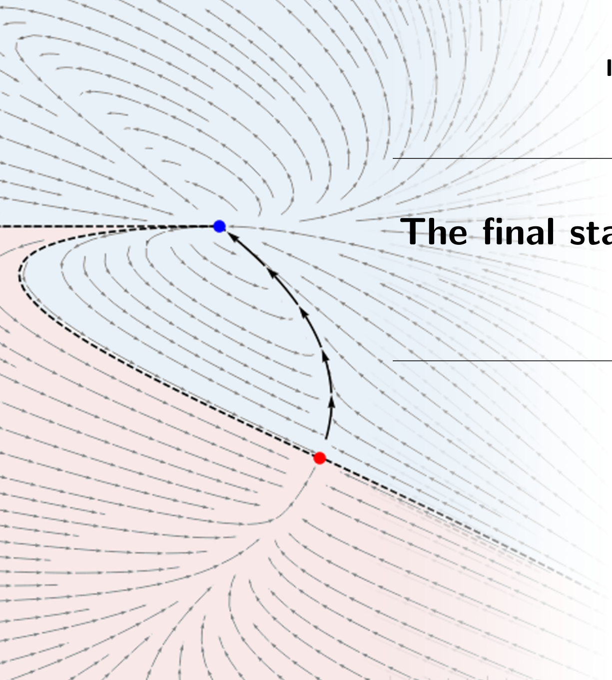


IV FLAG meeting: "The Quantum and Gravity"

Trento, 7th September 2022

The final stages of black hole evaporation in quadratic gravity

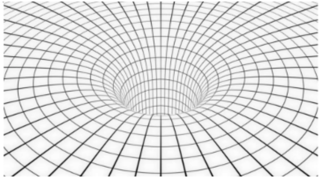
Samuele Marco Silveravalle

in collaboration with Alfio Bonanno

UNIVERSITÀ DI TRENTO

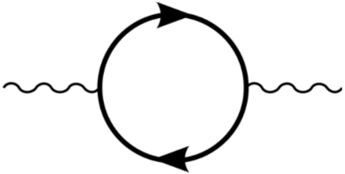
TIFPA Trento Institute for Fundamental Physics and Applications

Why black hole evaporation? - Semiclassical gravity



Classical curved spacetime

$+$



Quantum Field Theory

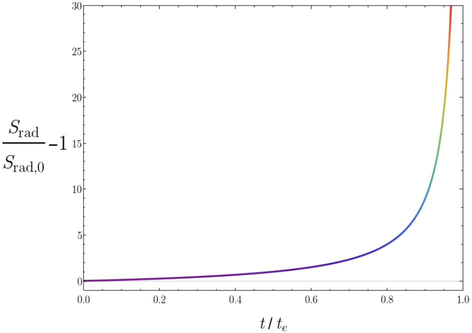
$\Downarrow$

Black hole evaporation

Fundamental assumption: $E_{Quantum\ Gravity} \gg E_{Standard\ Model}$

Why black hole evaporation? - Information paradox

Final stages of evaporation



Initial state: pure

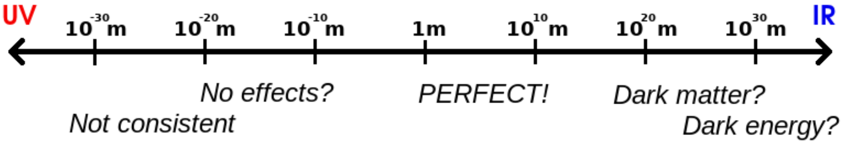
Final state: thermal radiation

$\Longrightarrow$ Information paradox

(Divergent entropy of radiation)

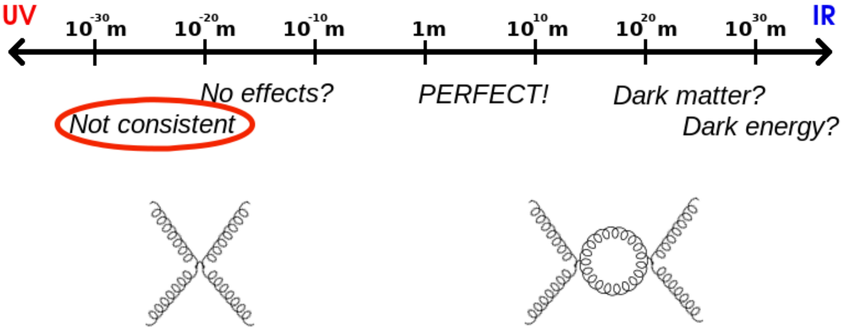
Solution: quantum corrections for gravity?

Why quadratic gravity? - The Quantum and Gravity



Coupling constant $[G] = E^{-2} \implies$ one-loop corrections $G^2 E^2_{\mathrm{cutoff}} \to \infty$

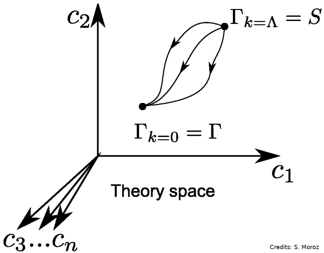
Why quadratic gravity? - The Quantum and Gravity



Coupling constant $[G] = E^{-2}$ $\Longrightarrow$ one-loop corrections $G^2 E^2_{\text{cutoff}} \to \infty$

Why quadratic gravity? - Wilsonian approach

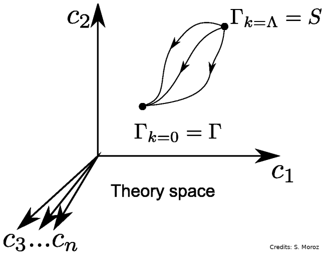
Non-renormalizable theory $\Longrightarrow$ effective field theory at low energies



$$\mathcal{S}_{eff} = \int d^4x\sqrt{-g}\left[c_1 + c_2 R + c_3 R^2 + c_4 R^{\mu\nu}R_{\mu\nu} + c_5 R^3 + \dots\right] \quad (1)$$

Why quadratic gravity? - Wilsonian approach

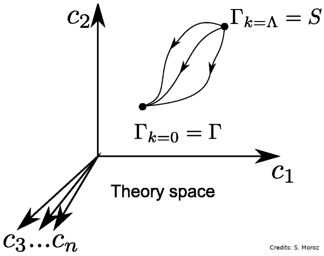
Non-renormalizable theory $\implies$ effective field theory at low energies



$$\mathcal{S}_{eff} = \int d^4x \sqrt{-g} \left[\underbrace{c_1 + c_2 R}_{GR} + c_3 R^2 + c_4 R^{\mu\nu} R_{\mu\nu} + c_5 R^3 + \dots \right] \tag{1}$$

Why quadratic gravity? - Wilsonian approach

Non-renormalizable theory $\implies$ effective field theory at low energies



$$\mathcal{S}_{eff} = \int d^4x\sqrt{-g}\left[c_1 + \underbrace{c_2 R + c_3 R^2 + c_4 R^{\mu\nu}R_{\mu\nu}}_{Quadratic\ \ Gravity} + c_5 R^3 + \ldots\right] \quad (1)$$

Quadratic gravity: a classical model for quantum corrections

$$\mathcal{S}_{QG} = \int d^4x\sqrt{-g}\left[\gamma\, R + \beta\, R^2 - \alpha\, C^{\mu\nu\rho\sigma}\, C_{\mu\nu\rho\sigma}\right] \begin{cases} S = 2,\ m = 0 \\ S = 0,\ m_0^2 = \gamma/6\beta \\ S = 2,\ m_2^2 = \gamma/2\alpha \end{cases} \tag{2}$$

PRO: renormalizable, general, IR limit of fundamental theories
K. Stelle (1977), B. Zwiebach (1985)

CON: negative energy states $\implies$ non-unitary theory

Effective theory: classical solutions as first quantum corrections

Quadratic gravity: a classical model for quantum corrections

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Effective theory: classical solutions as first quantum corrections

A no-(scalar) hair theorem

$C_{\mu\nu\rho\sigma}$ is traceless $\implies$ trace of vacuum e.o.m. is $\left(\Box - m_0^2\right) R = 0$

$$\left\{\begin{array}{l}\text{staticity}\\ \text{asymptotic flatness}\\ \text{presence of event horizon}\end{array}\right. \implies R = 0 \text{ in all spacetime}$$

R^2 term is irrelevant $\implies$ $C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$ term is crucial (ghosts!)

Symmetries and weak field limit

Staticity, spherical symmetry:

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \tag{3}$$

Asymptotic flatness (isolated objects):
K. Stelle (1978), A. Bonanno and S.S. (2019)

$$\begin{aligned} h(r) &\sim 1 - \frac{2M}{r} + \quad 2 S_2^- \frac{\mathrm{e}^{-m_2 r}}{r} \\ f(r) &\sim 1 - \frac{2M}{r} + S_2^- \frac{\mathrm{e}^{-m_2 r}}{r} (1 + m_2 r) \end{aligned} \tag{4}$$

Symmetries and weak field limit

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Internal boundaries

Series expansion around fixed radius r_0:
A. Perkins et al. (2015)

$$
\begin{aligned}
h(r) &= (r-r_0)^t \left[\sum_{n=0}^{N} h_{t+n/\Delta} (r-r_0)^{\frac{n}{\Delta}} + O\left((r-r_0)^{\frac{N+1}{\Delta}}\right)\right] \\
f(r) &= (r-r_0)^s \left[\sum_{n=0}^{N} f_{s+n/\Delta} (r-r_0)^{\frac{n}{\Delta}} + O\left((r-r_0)^{\frac{N+1}{\Delta}}\right)\right]
\end{aligned}
\tag{5}
$$

Generic solution: $t=0,\ s=0,\ \Delta=1$

Black holes: $t=1,\ s=1,\ \Delta=1$

Wormholes: $t=0,\ s=1,\ \Delta=2$

Internal boundaries

Series expansion around fixed radius r_0:
A. Perkins et al. (2015)

$$
\begin{aligned}
h(r) &= (r - r_0)\left[\sum_{n=0}^{N} h_{1+n}(r - r_0)^n + O\left((r - r_0)^{N+1}\right)\right] \\
f(r) &= (r - r_0)\left[\sum_{n=0}^{N} f_{1+n}(r - r_0)^n + O\left((r - r_0)^{N+1}\right)\right]
\end{aligned}
\tag{5}
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Black holes: $t = 1,\ s = 1,\ \Delta = 1$

Wormholes: $t = 0,\ s = 1,\ \Delta = 2$

Behaviour close to the origin

Series expansion around origin:
A. Perkins et al. (2015)

$$h(r) = r^t \left[\sum_{n=0}^{N} h_{t+n} r^n + O\left(r^{N+1}\right) \right]$$
$$f(r) = r^s \left[\sum_{n=0}^{N} f_{s+n} r^n + O\left(r^{N+1}\right) \right] \implies \begin{aligned} t &= \lim_{r\to 0} \chi_h = \lim_{r\to 0} \frac{\mathrm{d}\log(h(r))}{\mathrm{d}\log(r)} \\ s &= \lim_{r\to 0} \chi_f = \lim_{r\to 0} \frac{\mathrm{d}\log(f(r))}{\mathrm{d}\log(r)} \end{aligned} \quad (6)$$

Regular solutions (stars): $t = 0$, $s = 0$

Divergent metric: $t = -1$, $s = -1$

Vanishing metric: $t = 2$, $s = -2$

Behaviour close to the origin

Series expansion around origin:
A. Perkins et al. (2015)

$$h(r) = r^t \left[\sum_{n=0}^{N} h_{t+n} r^n + O\left(r^{N+1}\right) \right] \qquad t = \lim_{r\to 0} \chi_h = \lim_{r\to 0} \frac{\mathrm{d}\log(h(r))}{\mathrm{d}\log(r)}$$
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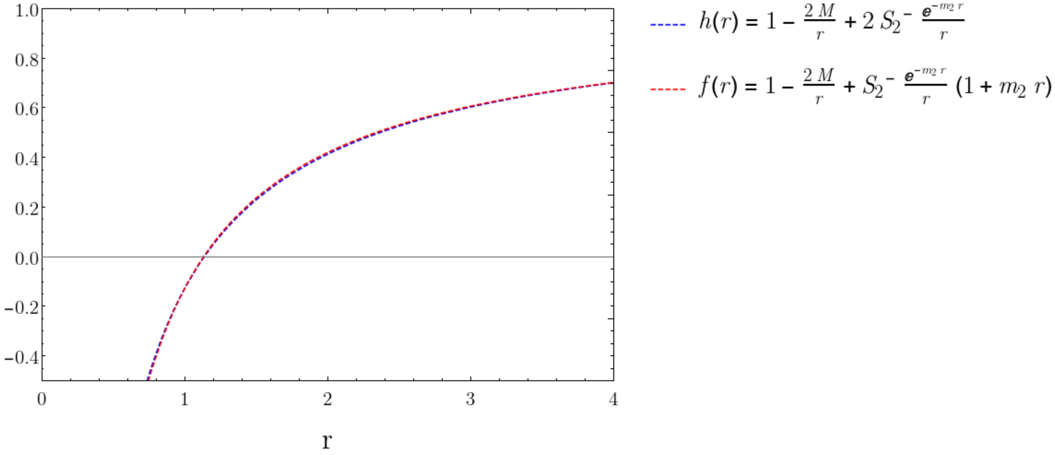
Regular solutions (stars): $t = 0$, $s = 0$

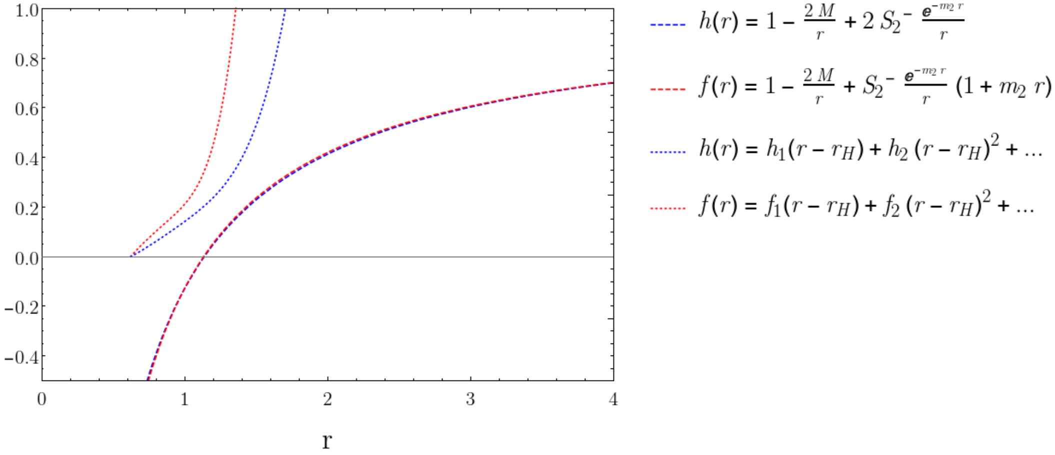
Divergent metric: $t = -1$, $s = -1$

Vanishing metric: $t = 2$, $s = -2$ $\implies$ Not present in GR

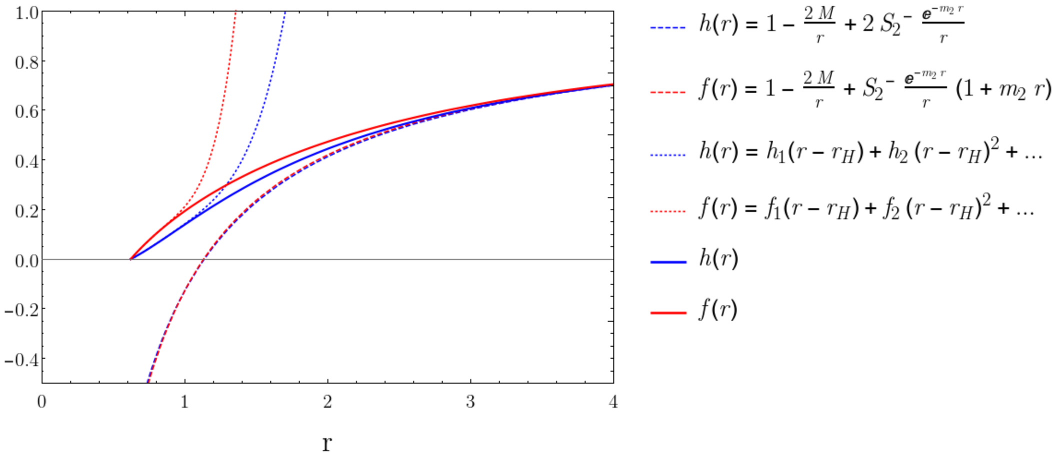
Numerical methods: shooting method



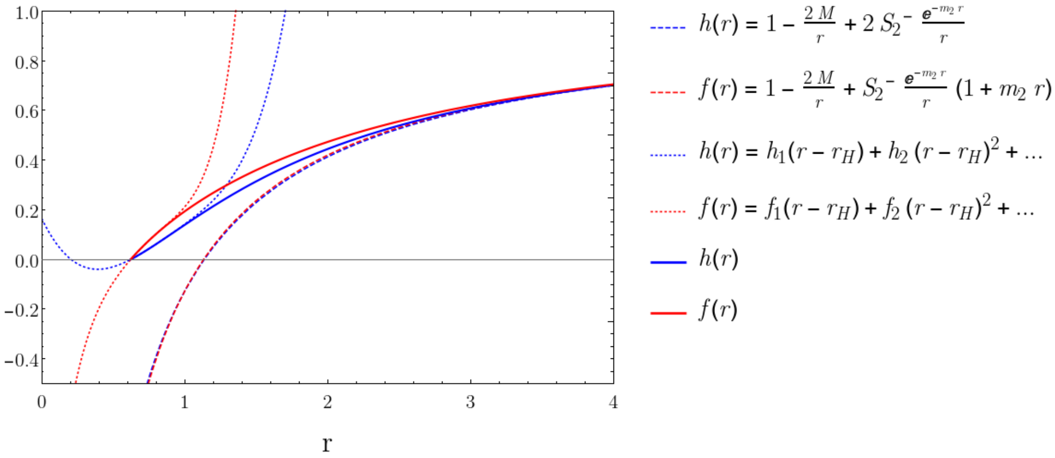
Numerical methods: shooting method



Numerical methods: shooting method



Numerical methods: shooting method



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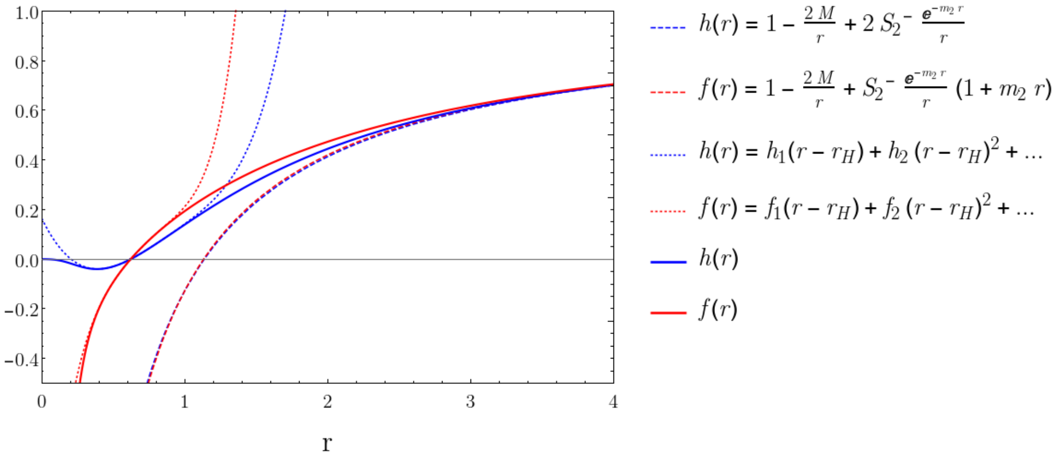

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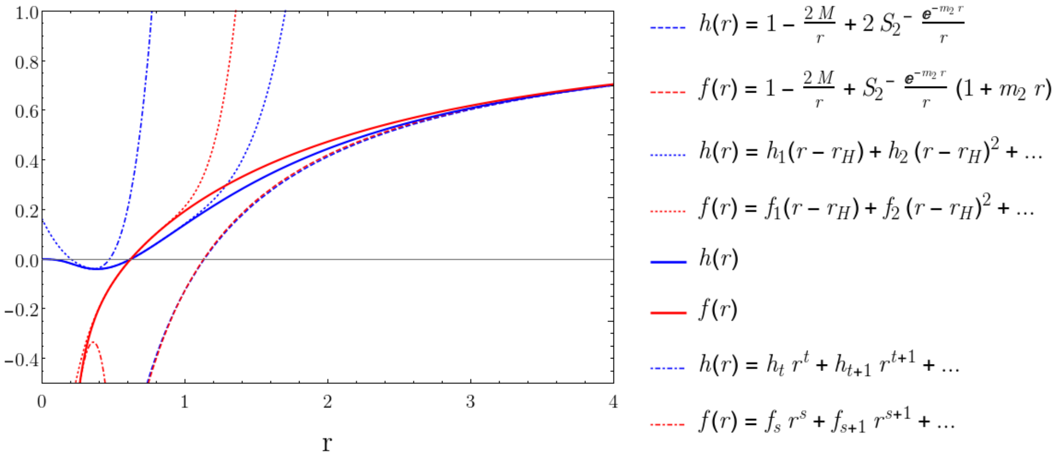
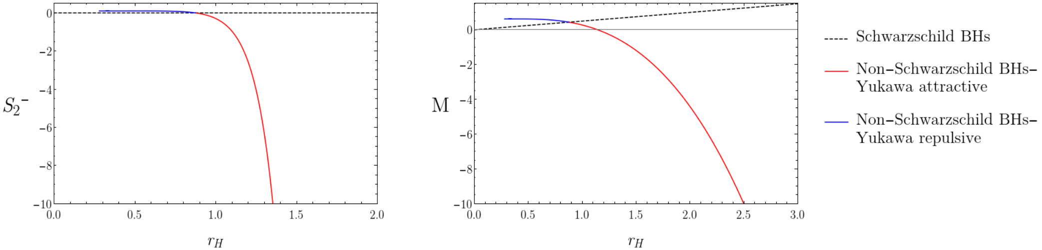


Large distances behaviour: gravitational field

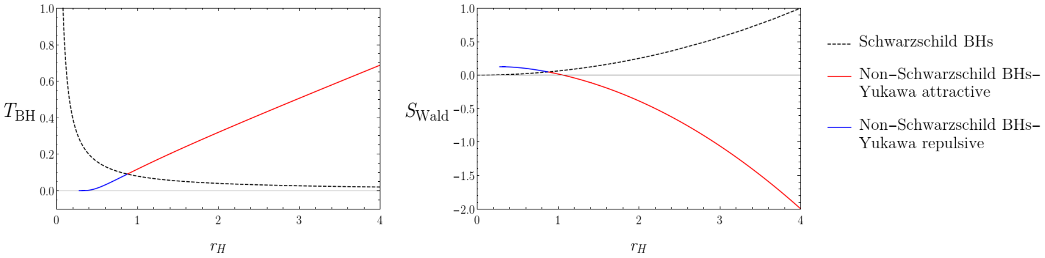
$S_2^- > 0 \implies$ Yukawa repulsive, $S_2^- < 0 \implies$ Yukawa attractive



Attractive in the "ghost" sector $\implies$ repulsive gravitational field

Near-horizon behaviour: thermodynamical properties

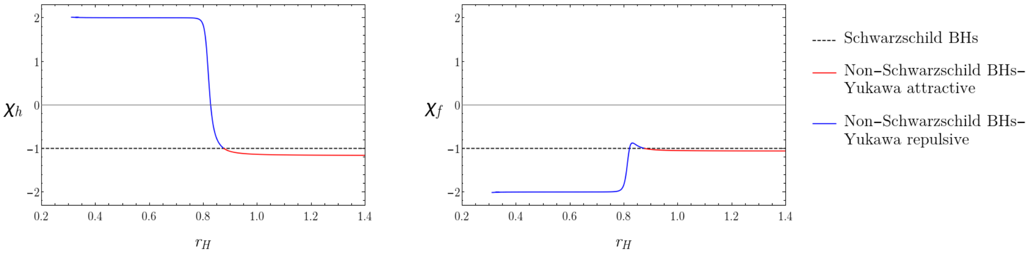
Black hole temperature: $T_{BH} = \frac{\kappa}{2\pi}$ Black hole entropy: $\delta S_{Wald} = \frac{1}{T_{BH}} \delta M$



Negative entropy: $S \propto -\langle \log(p) \rangle < 0 \implies$ Non-unitarity: $p > 1$

Near-origin behaviour: characterization of the singularity

Yukawa attractive/repulsive: are they that different?



Yukawa repulsive $\Longrightarrow$ vanishing, Yukawa attractive $\Longrightarrow$ divergent

Instability of Schwarzschild black holes

Transverse and traceless perturbation of Schwarzschild metric in General Relativity

$$\Delta_L \, h_{\mu\nu} = \Box h_{\mu\nu} + 2R_{\mu\rho\nu\sigma} h^{\rho\sigma} = 0 \tag{7}$$

Transverse and traceless perturbation of Schwarzschild metric in quadratic gravity
K. Stelle et al. (2017)

$$\left(\Delta_L - m_2^2\right) \Delta_L h_{\mu\nu} = 0 \tag{8}$$

Instability of Schwarzschild black holes

$a)$ $\Delta_L h_{\mu\nu} = 0 \implies$ Schwarzschild in Schwarzschild

$b)$ $\left(\Delta_L - m_2^2\right) h_{\mu\nu} = 0 \implies$ Schwarschild in non-Schwarzschild

Equation $b)$ is satisfied only at $r_H \sim 0.876 \implies$ analytical check

$r_H \sim 0.876$ is also the minimum radius stable Schwarzschild solutions!

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Black hole phase transition: a thermodynamical argument

Yukawa repulsive have

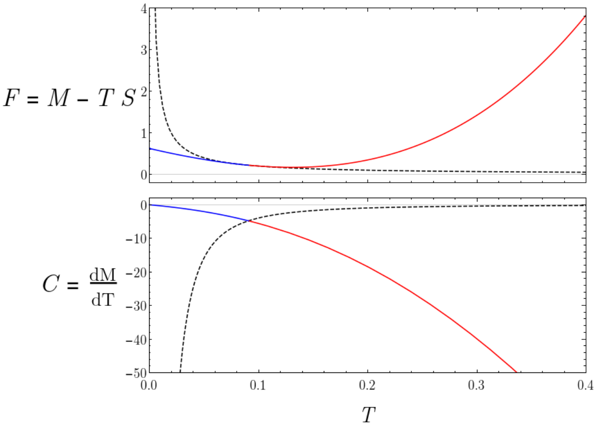
- free energy smaller than
- specific heat greater than

stable Schwarzschild black holes

$\Downarrow$

more thermodynamically stable

K. Stelle et al. (2017)

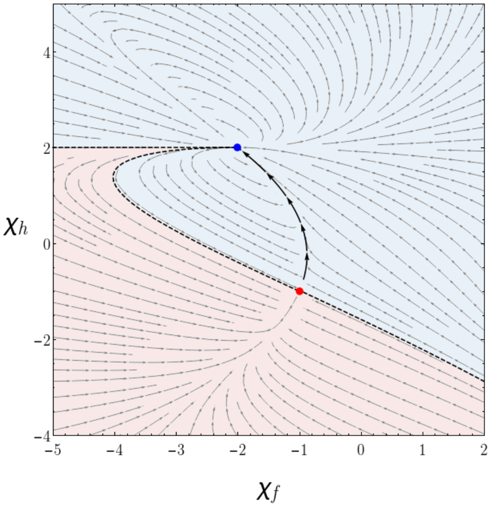


Black hole phase transition: a dynamical argument

Equations for χ_h and χ_f:

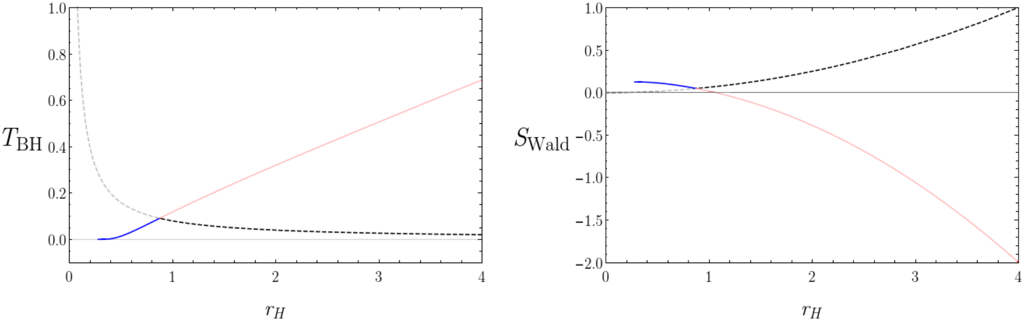
$$\chi_h' = \frac{1}{2}\left(\chi_h\chi_f + 4\chi_f + \chi_h^2 + 2\chi_h + 4\right)$$

$$\chi_f' = \frac{1}{2(\chi_h - 2)}\left(2\chi_f^2\chi_h - \chi_f^2 + \chi_s\chi_h^2 + \right.$$
$$\left. + 8\chi_f - \chi_h^3 + 3\chi_h^2 + 8\right)$$



Black hole phase transition: consequences

Schwarzschild $\Longrightarrow$ Yukawa repulsive phase transition



No black hole explosion?

Black hole evaporation: time evolution

Time dependent equations of motion in adiabatic approximation

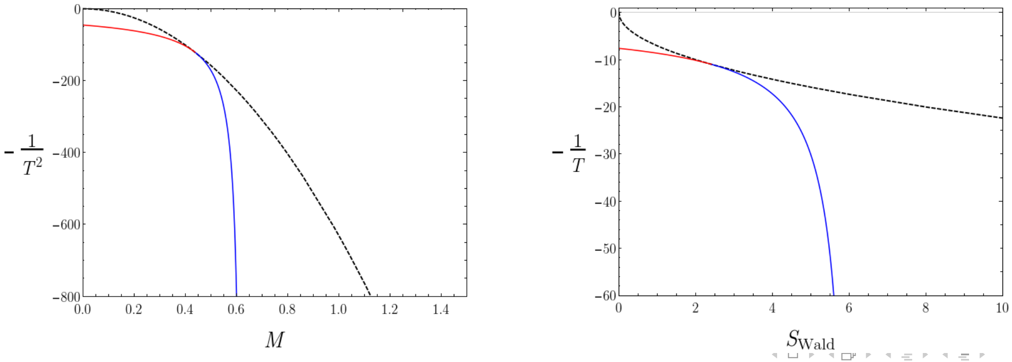
$$\mathcal{H}_{tr} = \frac{1}{2}\langle T_{tr}\rangle \tag{9}$$

Exact calculations at large distances or close to the horizon

$$\frac{\mathrm{d}M}{\mathrm{d}t} = T\frac{\mathrm{d}S_{Wald}}{\mathrm{d}t} = -\sum_{l,m}\int \frac{\mathrm{d}\omega}{2\pi}|B_{lm\omega}|^2 \frac{\omega}{\mathrm{e}^{\omega/T}-1} \sim -\alpha T^2 \tag{10}$$

Black hole evaporation: time evolution

$$t = -\frac{1}{\alpha}\int_{M_0}^{M}\frac{\mathrm{d}M}{T^2} = -\frac{1}{\alpha}\int_{S_0}^{S}\frac{\mathrm{d}S_{Wald}}{T} \tag{11}$$



Black hole evaporation: no way out?

$$\left\{ \begin{array}{l} \text{Thermodynamical stability} \\ \text{Dynamical stability} \end{array} \right. \Longrightarrow \text{Schwarzschild evaporates in Yukawa repulsive}$$

Energy flux $\Longrightarrow$ Yukawa repulsive evaporates in Scharzschild/Yukawa attractive

Black hole evaporation: no way out?

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$$\text{Energy flux} \implies \text{Yukawa repulsive evaporates in Scharzschild/Yukawa attractive}$$

Black hole evaporation: ghost Hawking radiation

Quadratic gravity predicts ghost particles!

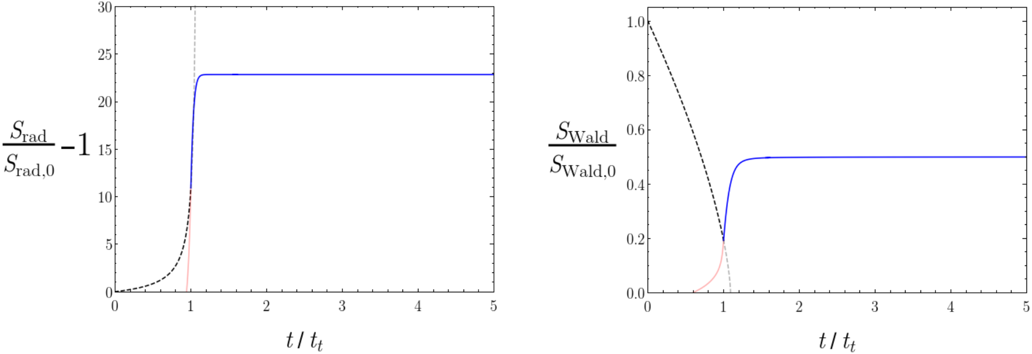
Toy-model: ghost scalar field

$$\frac{\mathrm{d}M}{\mathrm{d}t} = T\frac{\mathrm{d}S_{Wald}}{\mathrm{d}t} = +\sum_{l,m}\int \frac{\mathrm{d}\omega}{2\pi}|B_{lm\omega}|^2 \frac{\omega}{\mathrm{e}^{\omega/T}-1} = +\alpha T^2 \tag{12}$$

Fundamental assumption: ghosts dominate radiation after the phase transition

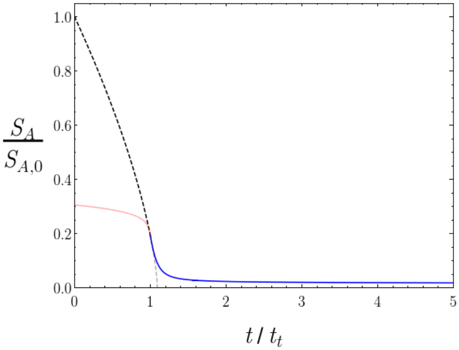
Black hole evaporation: time evolution with ghosts

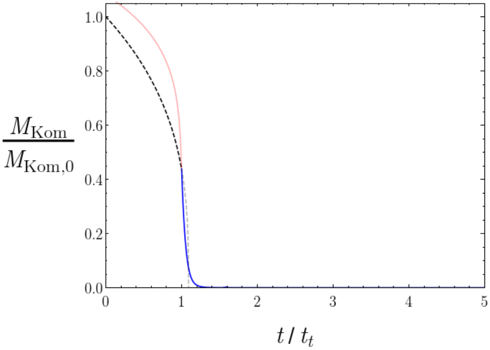
Infinite time of evaporation, no diverging entropies $\implies$ no information paradox?



Black hole evaporation: time evolution with ghosts

Vanishing quasi-local quantities $\implies$ pure ghost radiation?





Conclusions

Simple and conservative approach:

Information paradox: semiclassical gravity breaks down at high energies

$\Longrightarrow$ inclusion of first order quantum corrections to gravity

Many strong (but sensible) assumptions:

- classical solutions of quadratic gravity as first-order quantum corrections
- Schwarzschild $\Longrightarrow$ Yukawa repulsive phase transition
- ghosts dominated Hawking radiation

Conclusions

Consequences:

- infinite time of evaporation
- finite entropies
- finite temperatures

Simple semiclassical assumptions $\Longrightarrow$ simple remnants?

Conclusions

Consequences:

- infinite time of evaporation
- finite entropies
- finite temperatures

Thank you, and see you soon!

Simple semiclassical assumptions $\Longrightarrow$ simple remnants?

Black hole evaporation: ghost Hawking radiation

Grey body factor for spin 2 particles $\Longrightarrow$ Teukolsky equation

$$\left(\left(D-3\epsilon+\bar{\epsilon}-4\rho-\bar{\rho}\right)\left(\Delta+\mu-4\gamma\right)-\left(\delta+\bar{\pi}-\bar{\alpha}-3\beta-4\tau\right)\left(\bar{\delta}+\pi-4\alpha\right)-3\psi_2\right)\tilde{\psi}_0=T \tag{13}$$

T is a said to be a source term given that $T=T\left(R_{\mu\nu}\right)$

- General Relativity: vacuum $\Longrightarrow$ $T=0$
- quadratic gravity: vacuum $\Longrightarrow$ $T\neq 0$

Black hole phase transition: a geometrical argument

Riemannian Penrose Inequality:

$$R_{\mu\nu}u^{\mu}u^{\nu} \geq 0 \implies M \geq \sqrt{\frac{A}{16\pi}}$$

$$M < \sqrt{\frac{A}{16\pi}} \implies R_{\mu\nu}u^{\mu}u^{\nu} < 0$$

Locally repulsive gravity!