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# Testing Inflation with Cosmological Observations

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Francisco Villaescusa Navarro, Licia Verde

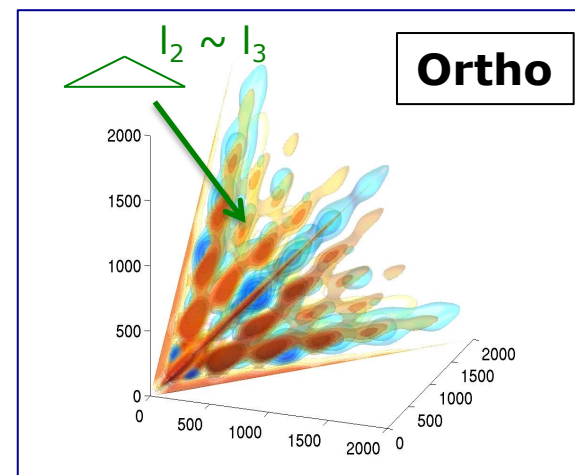
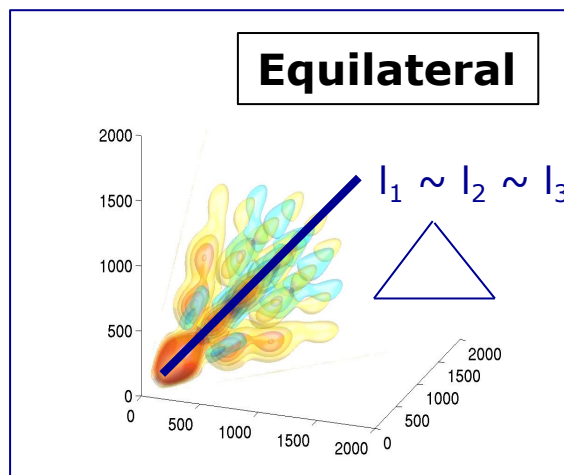
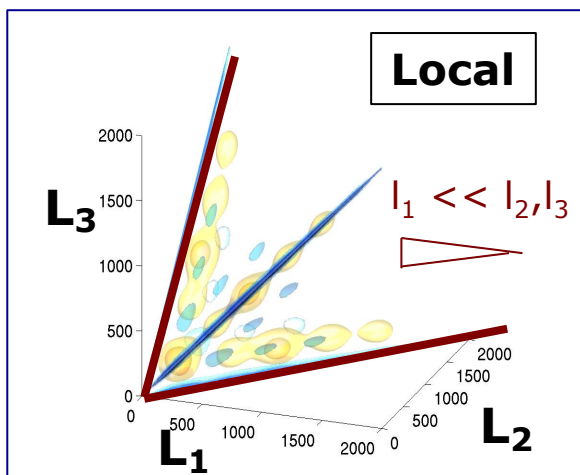
ArXiv:

Jung et al. 2022 (2206.01624)

Coulton et al. 2022 (2206.01619, 2206.15450)

# Beyond power spectra: non-Gaussianity

**Primordial non-Gaussianity.** Measures interactions. Many inflationary scenarios (notably, multi-field Inflation) predict small, model-dependent deviations from Gaussianity. **Additional information in 3-point (bispectrum) and 4-point (trispectrum) functions.**



- Multi-field
- Curvaton
- Ekpyrotic/cyclic

- Non-canonical kinetic terms (K-inflation, DBI)
- Higher derivative terms (Ghost Inflation)
- EFT

- Variants of non canonical
- Kinetic terms and higher derivatives
- EFT

# Planck LEO constraints

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Shape and method	$f_{\text{NL}}(\text{KSW})$	
	Independent	ISW-lensing subtracted
SMICA ( $T$ )		
Local . . . . .	6.7 $\pm$ 5.6	<b>-0.5 <math>\pm</math> 5.6</b>
Equilateral . . . . .	4.0 $\pm$ 67	<b>4.7 <math>\pm</math> 67</b>
Orthogonal . . . . .	-38 $\pm$ 37	<b>-15 <math>\pm</math> 37</b>
SMICA ( $T+E$ )		
Local . . . . .	4.1 $\pm$ 5.1	<b>-0.9 <math>\pm</math> 5.1</b>
Equilateral . . . . .	-25 $\pm$ 47	<b>-26 <math>\pm</math> 47</b>
Orthogonal . . . . .	-47 $\pm$ 24	<b>-38 <math>\pm</math> 24</b>

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# Future goals

- It is generally accepted that the next sensitivity target should be  $f_{\text{NL}} \sim 1$
- Local shape:  $f_{\text{NL}} > 1$  would rule out single-field inflation.  $f_{\text{NL}} < 1$  would rule out a large class of multi-field models (“spectator fields”)
- Equilateral, Orthogonal: the  $f_{\text{NL}} \sim 1$  threshold allows discriminating between the single-field slow-roll and non-slow-roll regimes.

	$f_{\text{NL}}^{\text{loc}} \lesssim 1$	$f_{\text{NL}}^{\text{loc}} \gtrsim 1$
$f_{\text{NL}}^{\text{eq, orth}} \lesssim 1$	Single-field slow-roll	Multi-field
$f_{\text{NL}}^{\text{eq, orth}} \gtrsim 1$	Single-field non-slow-roll	Multi-field

(from Alvarez et al., arXiv:1412.4671)

- This talk is mostly focused on LEO shapes, but keep in mind that there are many interesting additional shapes (e.g. oscillatory features)



# LEO shapes: forecasts

	LiteCOrE-120	COrE+	Planck 2015	LiteBird	ideal
T local	3.7	3.4	5.7	9.4	2.7
T equil.	59	56	70	92	46
T ortho.	25	25	33	58	20
E local	4.5	3.9	32	11	2.4
E equil.	46	43	141	76	31
E ortho.	21	19	72	42	13
T+E local	2.2	<b>1.9</b>	<b>5.0</b>	5.6	1.4
T+E equil.	22	<b>20</b>	<b>43</b>	40	15
T+E ortho.	10	<b>9.1</b>	<b>21</b>	23	6.7

- A cosmic variance dominated E-mode reconstruction up to  $l_{\max} \sim 3000$  (PRISM, CMBpol) allows an improvement in  $f_{\text{NL}}$  error bars by a factor  $\sim 2$  for all shapes .

# PNG from LSS

- Future LSS galaxy clustering data have the potential to improve over CMB bounds: 3D LSS density field vs 2D CMB anisotropies field => **more modes**
- **Power spectrum.** Biased tracers of the dark matter field. Specific NG scale-dependent feature in the bias. For local shape:

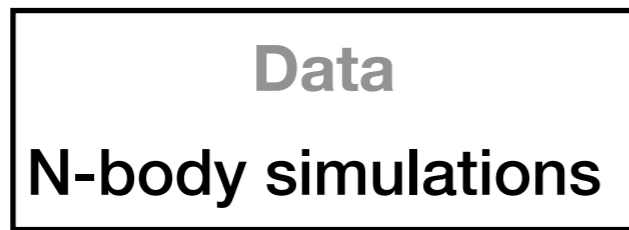
$$\Delta b(k) = 2(b - 1) f_{\text{NL}} \delta_c \frac{3\Omega_m}{2a g(a) r_H^2 k^2}$$

- **Bispectrum.** Major challenge: if we want to increase S/N we need to include Fourier modes that are in the non-linear regime of structure formation.  
**Late time NG ~ 1000 x primordial NG !**

Needs **very precise** modeling of non-linearities

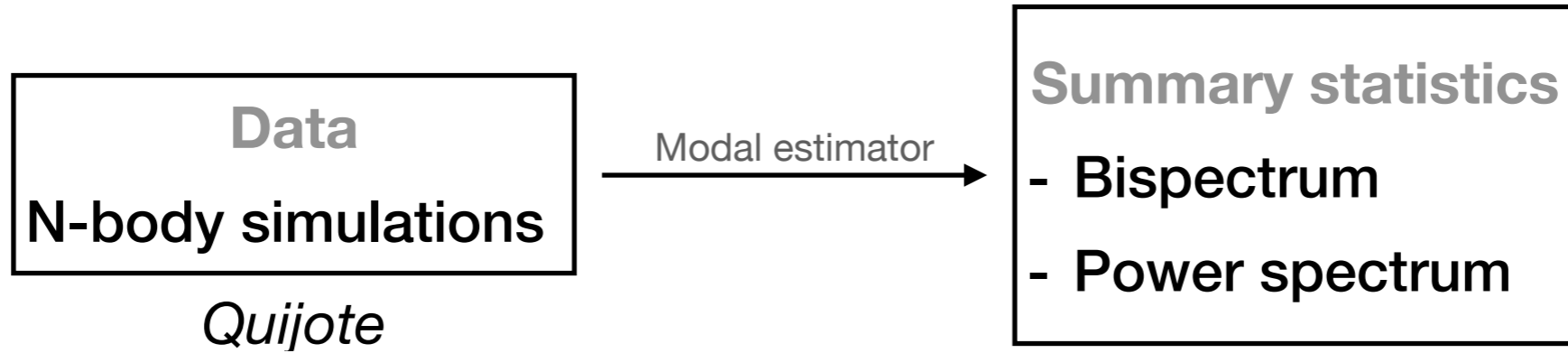
- Resort on analytical perturbative approach. Hard to go beyond  $k \sim 0.2 \text{ h/Mpc}$
- **Resort on numerical, simulation-based approaches (implicit likelihood inference)**

# Pipeline

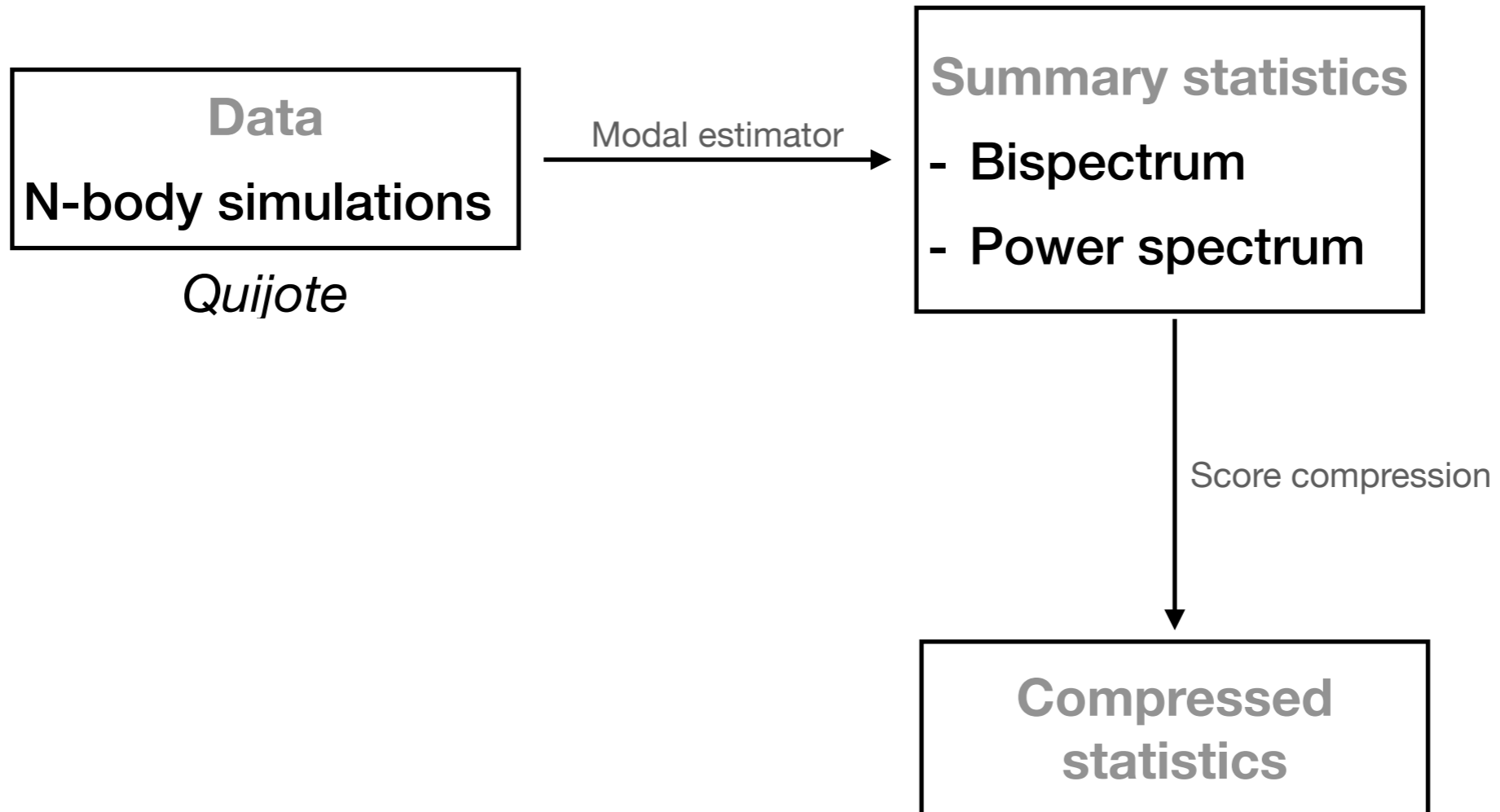


*Quijote*

# Pipeline

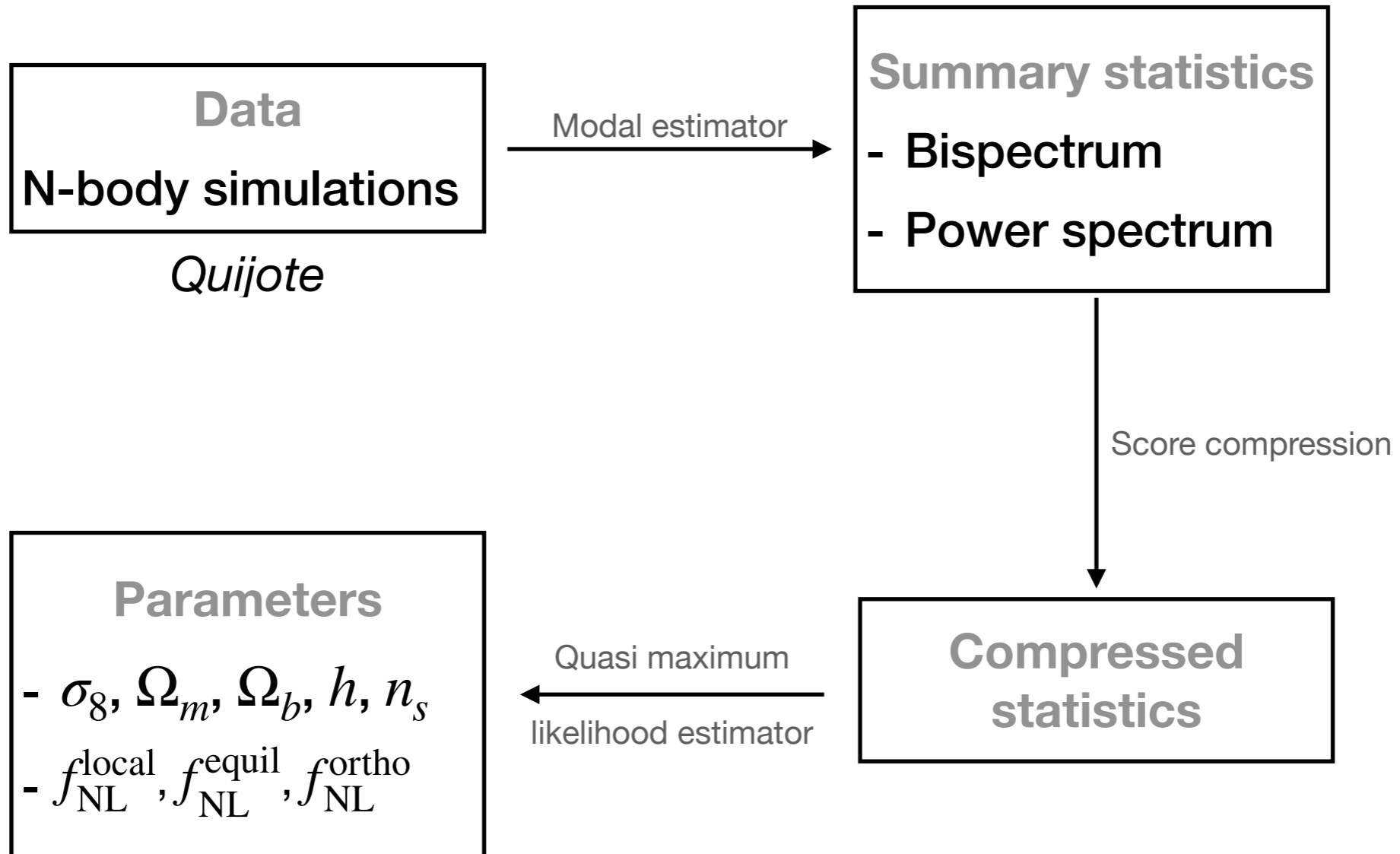


# Pipeline





# Pipeline



# Modal bispectrum estimator

Data  $\Rightarrow$  Summary statistics

- **Weighted expansion of the bispectrum:**

$$w(k_1, k_2, k_3) \mathbf{B}(k_1, k_2, k_3) = \sum_n^N \beta_n \mathbf{Q}_n(k_1, k_2, k_3)$$

- **Separable modal basis:**  $\mathbf{Q}_n(k_1, k_2, k_3) = q_r(k_1)q_s(k_2)q_t(k_3) + \text{perms}$ , with  $n \equiv \{r, s, t\}$
- **Method originally developed for CMB**
  - *Fergusson, Liguori & Shellard* (0912.3411)
  - Planck NG (1905.05697)
- **Later implemented in the LSS context**
  - *Schmittfull, Regan & Shellard* (1207.5678)
  - *Hung, Fergusson & Shellard* (1902.01830)
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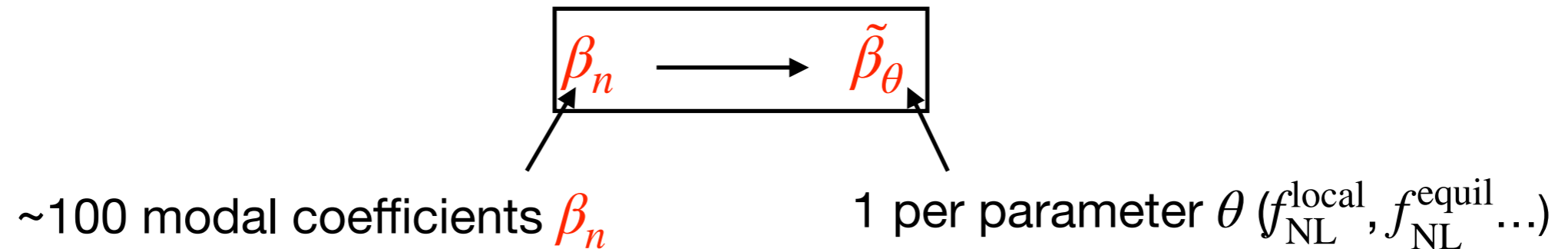
$\Rightarrow$  **Only ~100 well chosen modes** to describe the bispectrum up to  $k_{\max} = 0.5h/\text{Mpc}$

# Optimal compression

Summary statistics  $\Rightarrow$  Compressed statistics

- **Compression to the score function:**

1712.00012 (*J. Alsing, B. Wandelt*)

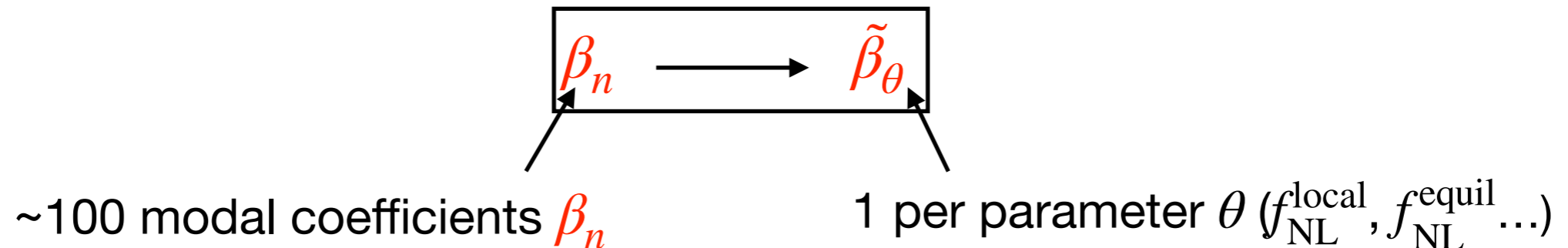


# Optimal compression

Summary statistics  $\Rightarrow$  Compressed statistics

- **Compression to the score function:**

1712.00012 (J. Alsing, B. Wandelt)



- **Assuming a Gaussian likelihood for the summary statistics:**

compression equivalent to MOPED, astro-ph/9911102 (A. Heavens, R. Jimenez & O. Lahav)

$$\tilde{\beta}_\theta = \frac{\partial \beta_i}{\partial \theta} C_{ij}^{-1} (\beta_j - \bar{\beta}_j)$$

Derivative

$$\frac{\partial \beta}{\partial \theta} = \frac{\beta(\theta_* + \Delta\theta) - \beta(\theta_* - \Delta\theta)}{2\Delta\theta}$$

Inverse covariance of  $\beta$

Mean of  $\beta$

Data to compress

$\Rightarrow$  Computed from simulations where  $\theta$  is a step  $\Delta\theta$  away from fiducial  $\theta_*$

$\Rightarrow$  Computed from simulations at fiducial cosmology  $\theta_*$



# N-body simulations

## Data

- **Quijote simulations** ( $f_{\text{NL}} = 0$ )

*<https://quijote-simulations.readthedocs.io> (F. Villaescusa-Navarro)*

Large suite (~44000) of N-body simulations with  $512^3$  particles and a size of 1 Gpc/h

⇒ 8000 simulations at fiducial cosmology to estimate **covariances**

⇒ Sets of 500 simulations for **numerical derivatives** ( $\sigma_8, \Omega_m, \Omega_b, h, n_s$ )

- **Non-Gaussian Quijote-like** ( $f_{\text{NL}} = \pm 100$ )

⇒ Sets of 500 simulations for three primordial shapes: local, equilateral and orthogonal

⇒ **Numerical derivatives** ( $f_{\text{NL}}^{\text{local}}, f_{\text{NL}}^{\text{equil}}, f_{\text{NL}}^{\text{ortho}}$ )

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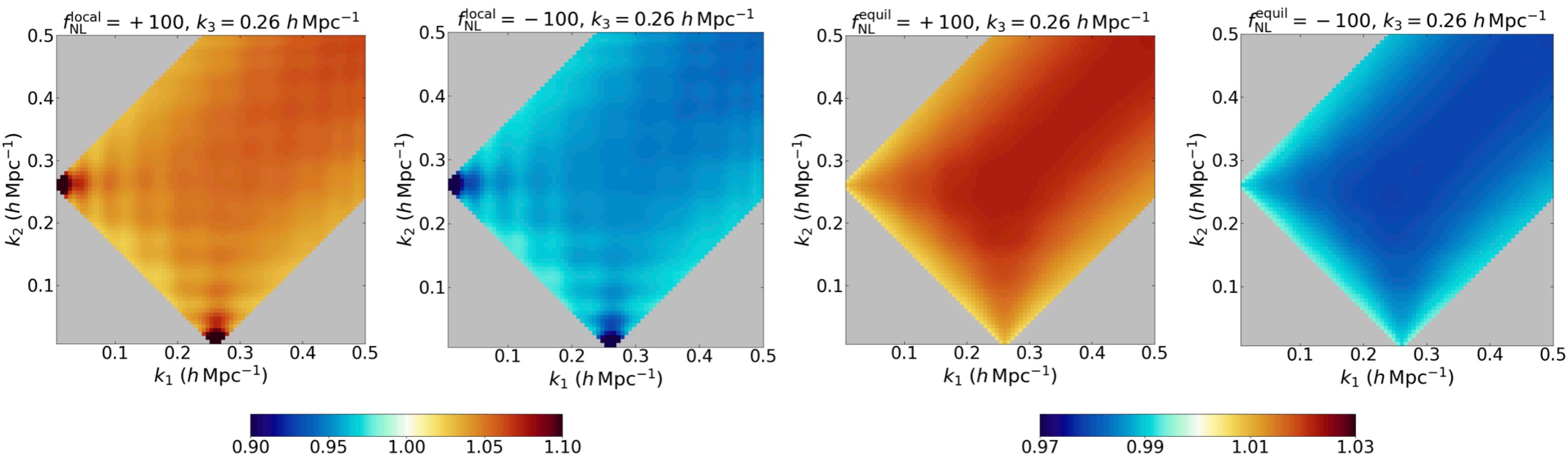
⇒ 8000 simulations at fiducial cosmology to estimate **covariances**

⇒ Sets of 500 simulations for **numerical derivatives** ( $\sigma_8$ ,  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ )

- **Non-Gaussian Quijote-like** ( $f_{\text{NL}} = \pm 100$ )

⇒ Sets of 500 simulations for three primordial shapes: local, equilateral and orthogonal

⇒ **Numerical derivatives** ( $f_{\text{NL}}^{\text{local}}$ ,  $f_{\text{NL}}^{\text{equil}}$ ,  $f_{\text{NL}}^{\text{ortho}}$ )



**Bispectrum: ratio non-Gaussian/Gaussian**

# Constraints

Compressed statistics  $\Rightarrow$  Parameters

- Pseudo maximum-likelihood estimator:

$$\hat{\theta} = \theta_* + F^{-1} \tilde{\beta}$$

where the Fisher information is  $F = \text{Cov}[\tilde{\beta}, \tilde{\beta}]$ .

# Constraints

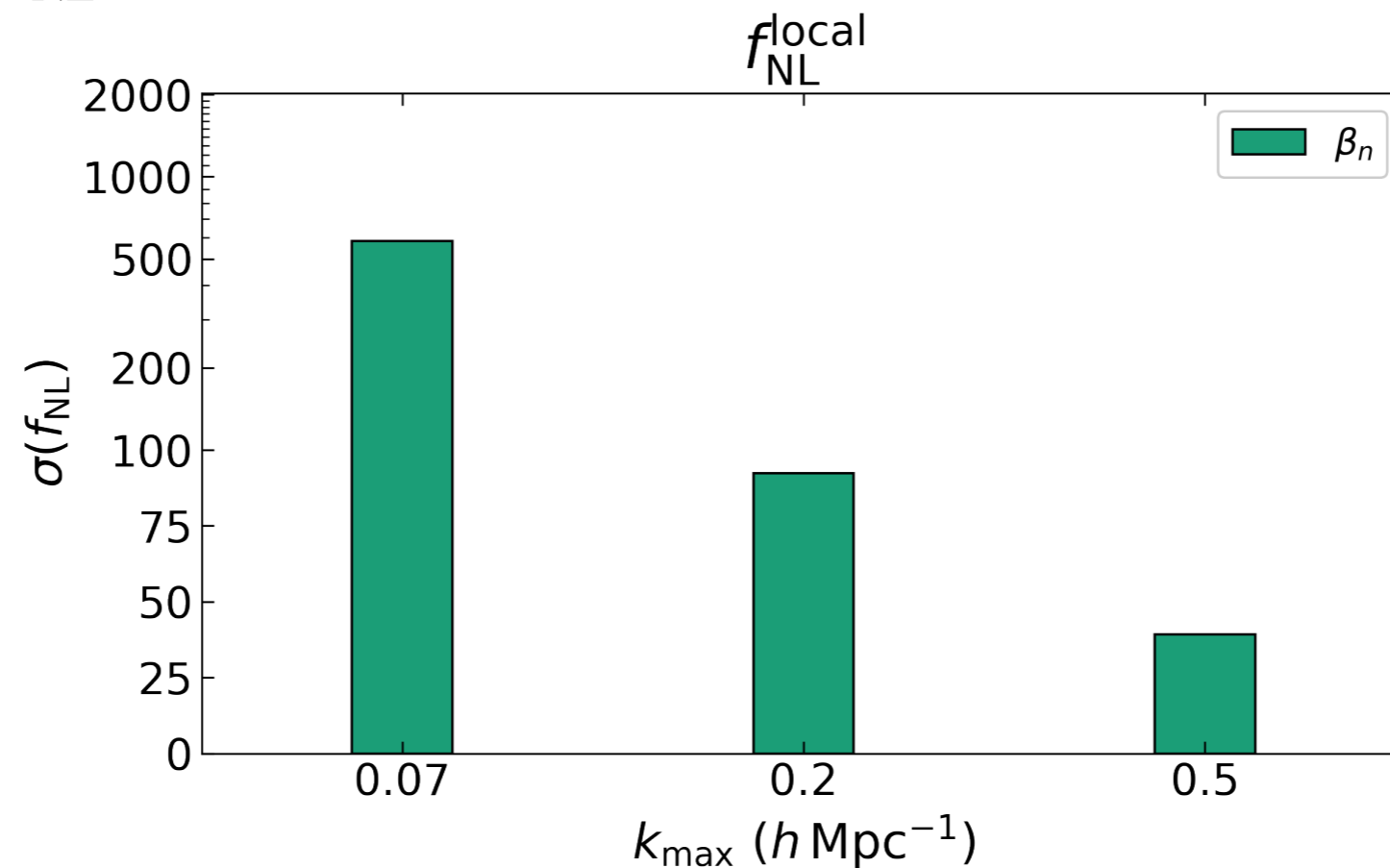
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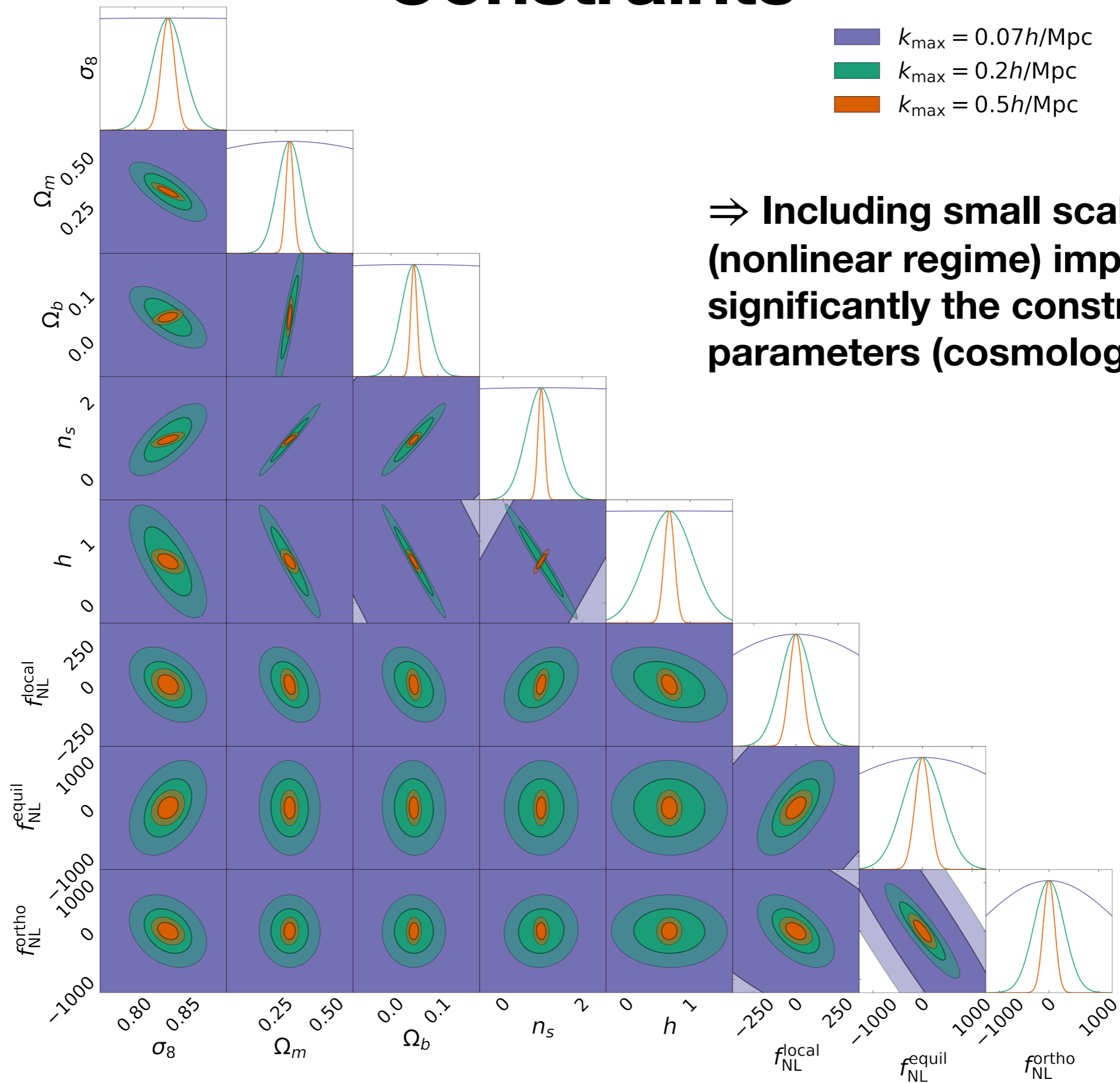
$$\hat{\theta} = \theta_* + F^{-1} \tilde{\beta}$$

where the Fisher information is  $F = \text{Cov}[\tilde{\beta}, \tilde{\beta}]$ .

- Constraints on  $f_{\text{NL}}^{\text{local}}$ :



# Constraints

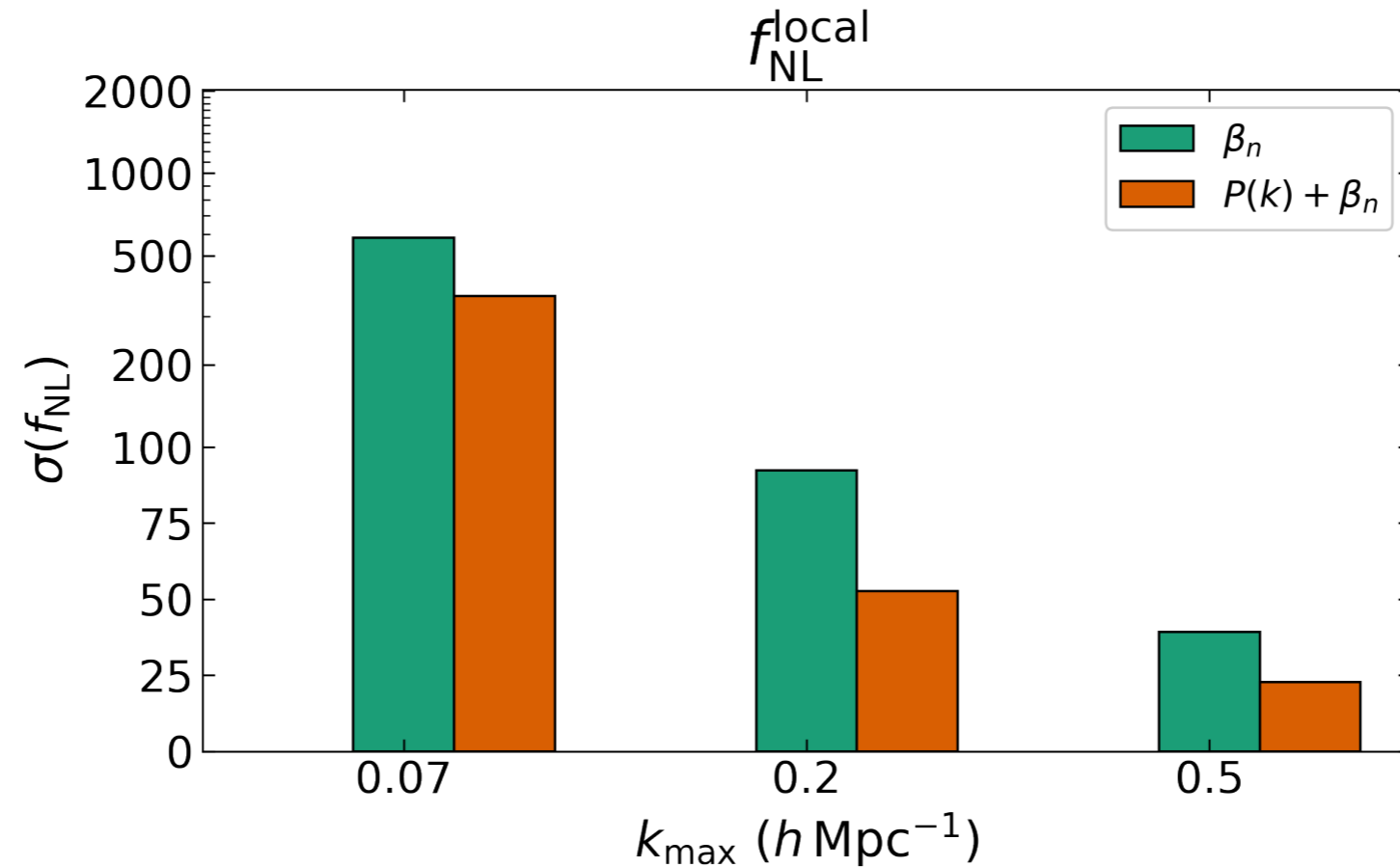


**⇒ Including small scales (nonlinear regime) improves significantly the constraints on all parameters (cosmological + PNG)**



# Constraints

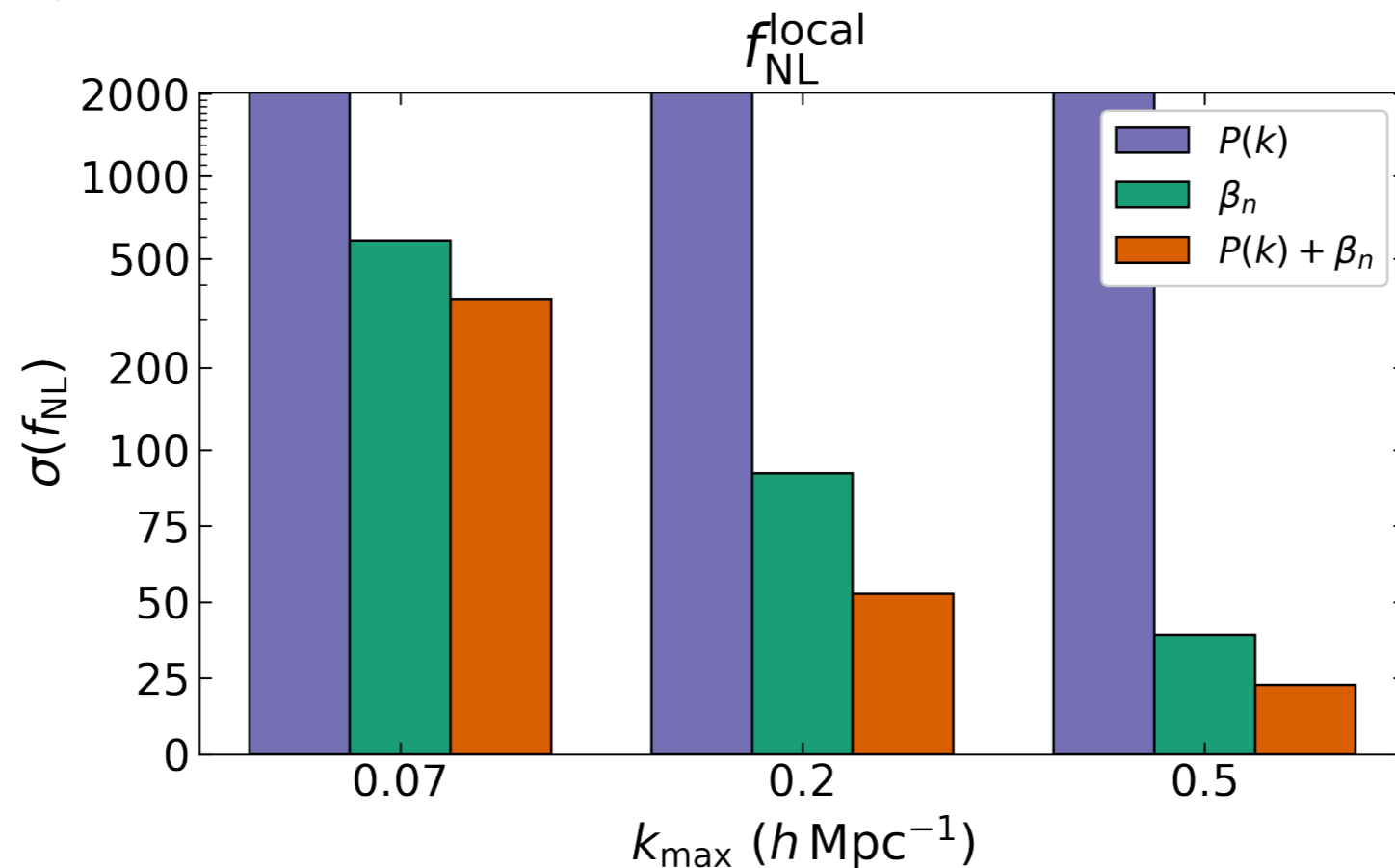
- Constraints on  $f_{\text{NL}}^{\text{local}}$ :



- Nonlinear regime ( $k_{\text{max}} = 0.5 h/\text{Mpc}$  here) improves the constraints significantly
- Combining power spectrum and bispectrum also improves the constraints by a factor  $\sim 2$

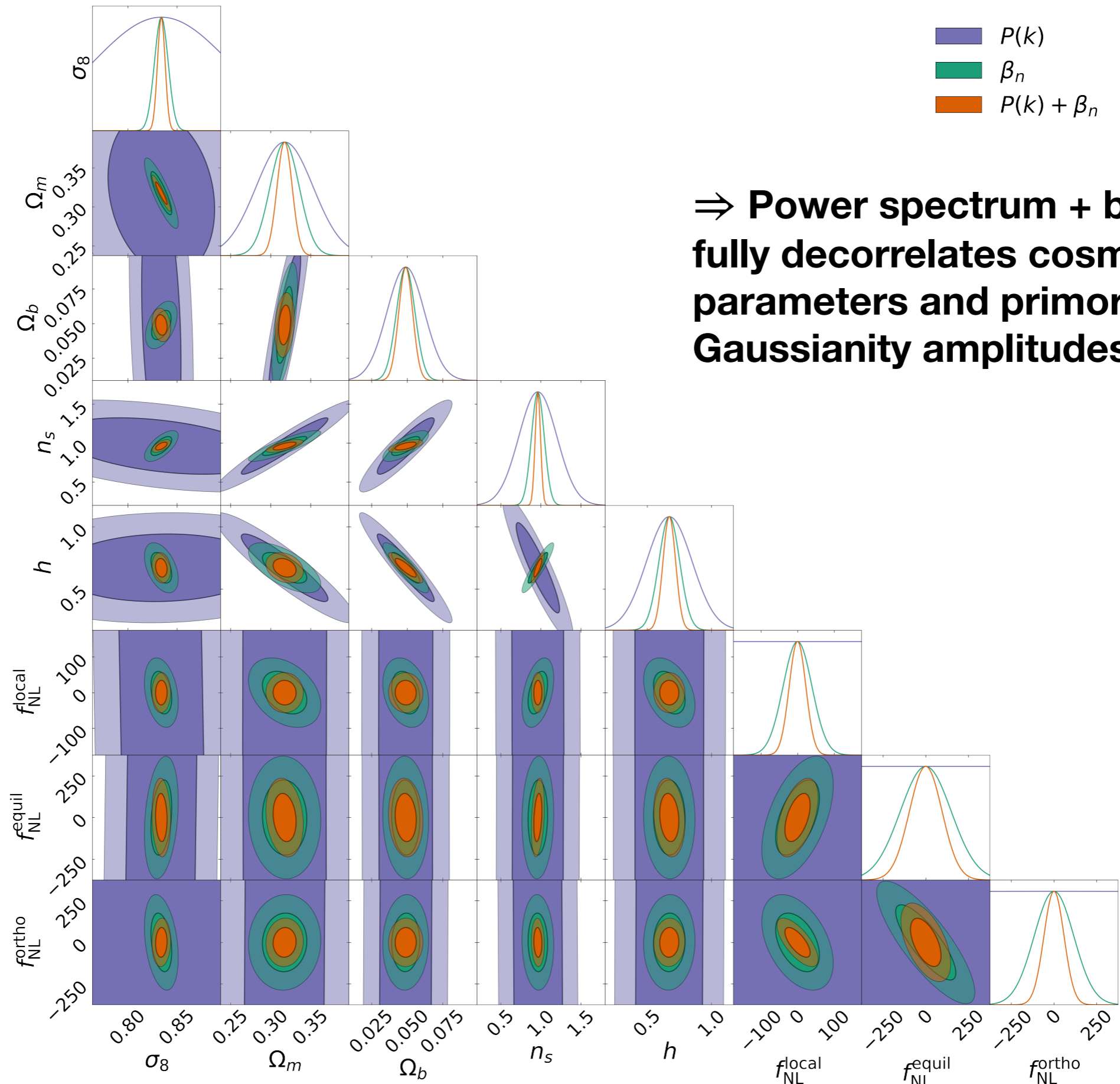
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- Constraints on  $f_{\text{NL}}^{\text{local}}$ :



- Nonlinear regime ( $k_{\text{max}} = 0.5 h/\text{Mpc}$  here) improves the constraints significantly
- Combining power spectrum and bispectrum also improves the constraints by a factor  $\sim 2$ , even if the power spectrum constraining power is very small

# Constraints



# Summary of the talk

## Conclusions

- Nonlinear scales  $\rightarrow$  (a lot of) extra information about cosmology and primordial NG
- Combining power spectrum + bispectrum, constraints on primordial NG comes for free
- Pipeline (first version) combining forward simulations, modal bispectrum estimator and data compression

## Future

- Extending the pipeline to halos and galaxies
- Find better summary statistics to study primordial NG