The role of geometry projection and of regularization in Asymptotic Safety: lessons from 'CREH'

Speaker: **Maria Conti**
Università degli Studi dell'Insubria

Supervisor: **S.L. Cacciatori**
Università degli Studi dell'Insubria

Collaborators: **A.M. Bonanno**
Università degli studi di Catania

INFN Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA

Index

Why Asymptotic Safety?

GR (EH action) → perturbatively non renormalizable **QFT**

→ treat it as an **EFT**: renormalization includes ∞ counterterms when > Planck scale

Is there a way to renormalize gravity non perturbatively? [1]

AS = EFT + constraint → 'Our world is located within the UV critical hypersurface of a suitable renormalization group (RG) **fixed point**' [2,3]

[1] **S. Weinberg**, in '*General Relativity: An Einstein centenary survey*', ed. **S.W. Hawking** and **W. Israel**, 790-831, Cambridge University Press (1979)
[2] **A. Bonanno** and **F. Saueressig**, '*Asymptotically safe cosmology – a status report*' , 254-264, Comptes Rendus Physique 18 (2017)
[3] **R. Percacci**, '*A short introduction to Asymptotic Safety*', part of Time and Matter: Proceedings, 3rd International Conference, TAM2010, Budva, Montenegro, 4-8 October, 123-142 (2010)

Why Asymptotic Safety?

Renormalization Group	Mathematical framework oriented to find a fixed point in the parameter space
Fixed Point	Identifies Hamiltonian invariant under Kadanoff/scale transformations
Critical exponents	**Physics!**

Perturbative renormalizability	**Non perturbative renormalizability**
Gaussian FP	non-Gaussian 'Reuter' FP: **AS**

Background independence

RG framework choice = 'cutting away field configurations'

⬇

'**k** scale of RG must be almost physical' [4]

Γ_k = action describing effective action at scale **k**

⬇

Problem: **k** must be proper momentum (i.e. related to a specific metric).

Here the metric is **dynamical**!

[4] **M. Reuter** and **H. Weyer**, '*The role of background independence for asymptotic safety in Quantum Einstein Gravity*', Phys. Rev. D 79 (2009)

Background independence

Background field technique

to be quantized

1) fix arbitrary **background metric** $\bar{g}_{\mu\nu}$ + fluctuation $h_{\mu\nu}$

2) at the end adjust $\bar{g}_{\mu\nu}$ s.t. $\bar{h}_{\mu\nu} := \langle h_{\mu\nu} \rangle = 0$

Background independence

chosen by the system!

k is proper

k is related to the surviving metric: the background

The problem of truncation

Perspectives on previous results

Gravitational effective action: all terms compatible with **symmetry under diffeomorphisms**!

$$\Gamma[g_{\mu\nu}] = \int d^d x \sqrt{g} \left\{ q_0 + q_1 R + q_2 R^2 + \cdots \right\}$$

Right now, we cannot say for sure that we found the Reuter fixed point (as not as with 3D Wilson Fisher model [5,6]). We must show it does not depend on the truncation!

Many results have been obtained, but always working with truncations! [7,8]

[5] **T. Morris**, '*On Truncations of the Exact Renormalization Group*', Phys. Lett. B, Vol. 334, Issues 3–4 (1994)
[6] **T. Morris**, '*Derivative Expansion of the Exact Renormalization Group*', Phys. Lett. B, Vol. 329 (1994)
[7] **P.F. Machado** and **F. Saueressig**, '*On the renormalization group flow of f(R)-gravity*', Phys. Rev. D, Vol. 77 (2008)
[8] **K. Falls** *et. al.* , '*Asymptotic safety of quantum gravity beyond Ricci scalars*', Phys. Rev. D, Vol. 97, Issue 8 (2018)

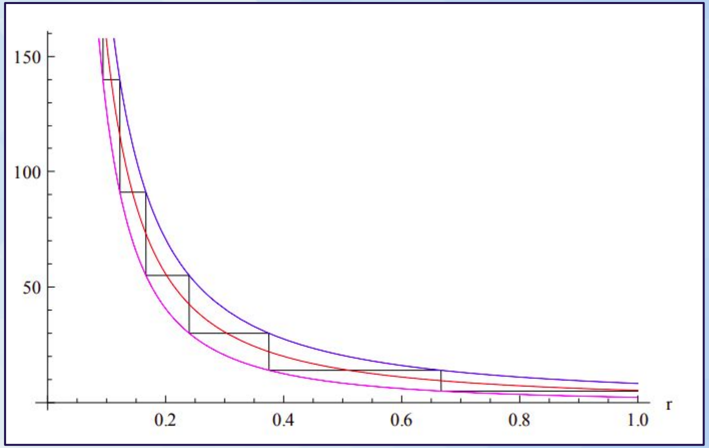
The problem of truncation

Current status

Working with general **f(R)** has been ineffective up until now...[7,8,9]

Divergent ODE for **f(R)**, where traces cannot be evaluated [9]

- » numerical approximations
- » still there is no global solution!



[9] **M. Demmel** *et al.* , JHEP, Vol. 06 (2014), Fig. 2

[7] **P.F. Machado** and **F. Saueressig**, '*On the renormalization group flow of f(R)-gravity*', Phys. Rev. D, Vol. 77 (2008)
[8] **K. Falls** *et. al.* , '*Asymptotic safety of quantum gravity beyond Ricci scalars*', Phys. Rev. D, Vol. 97, Issue 8 (2018)
[9] **M. Demmel** *et al.* , '*RG flows of Quantum Einstein Gravity on maximally symmetric spaces*', JHEP, Vol. 06 (2014)

The problem of the choice of geometry

Much of the work has been done projecting onto **flat geometry** (but also spherical geometry)

But **IF** and **HOW** is the choice of the geometry affecting results?

(Difference from the 'background independence' problem)

And what about the role of the **regulator**?

$$k\partial_k \Gamma_k = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\left(k\partial_k \rho_k(s)\right)\mathrm{Tr}e^{-s\Gamma_k^{(2)}}$$

regulator

Motivation

Our research

1) What is the role of the space we project onto when determining the **universal properties** of our theory?

2) How is the regulator concretely affecting these properties?

⬇

Main motivation

Determine a regulator for which the impact of the geometry is minimized!

The role of a simpler model: ‘CREH’

Our research

We need a tool to study the effects of switching **geometries** and **regulators**

‘CREH’

Conformally Reduced Einstein’s Gravity

All the metrics involved are **conformal factors** of a reference metric.

$$g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}$$

$$\bar{g}_{\mu\nu} = \chi_B^2 \hat{g}_{\mu\nu}$$

$$\dots$$

Each metric is represented by a single **scalar** function!

The role of a simpler model: 'CREH'

Einstein-Hilbert action: universe of pure curvature + cosmological constant

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,[R(g) - 2\Lambda]$$

⬇

$$S = \underbrace{-\frac{3}{4\pi G}}_{Z}\int d^4x\sqrt{\hat{g}}\left[\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{12}R(\hat{g})\phi^2 - \frac{1}{6}\Lambda\phi^4\right]$$

Z

extremely similar to a scalar theory!

The role of a simpler model: 'CREH'

Our research

Previous results [10] → **Spherical** geometry S^4

Regularizators family [10,11,12]

$$\rho_k(s,n) = \frac{\Gamma(n, snZk^2) - \Gamma(n, snZ\Lambda^2)}{\Gamma(n)}$$

spectrally adjusted! $\frac{1}{2} Z_k (\partial_\mu \phi)^2 \Rightarrow Z_k \bar{\Box}$

$$k\partial_k \rho_k(s,n) = -\frac{2}{\Gamma(n)} (sZ_k n \chi_B^2 \bar{k}^2) e^{-sZ_k n \chi_B^2 \bar{k}^2}$$

⟵ cutoff at $\bar{k}^2$

[10] **A. Bonanno** and **F. Guarnieri**, '*Universality and symmetry breaking in conformally reduced quantum gravity*', Phys. Rev. D, vol. 86, Issue 10, (2012)

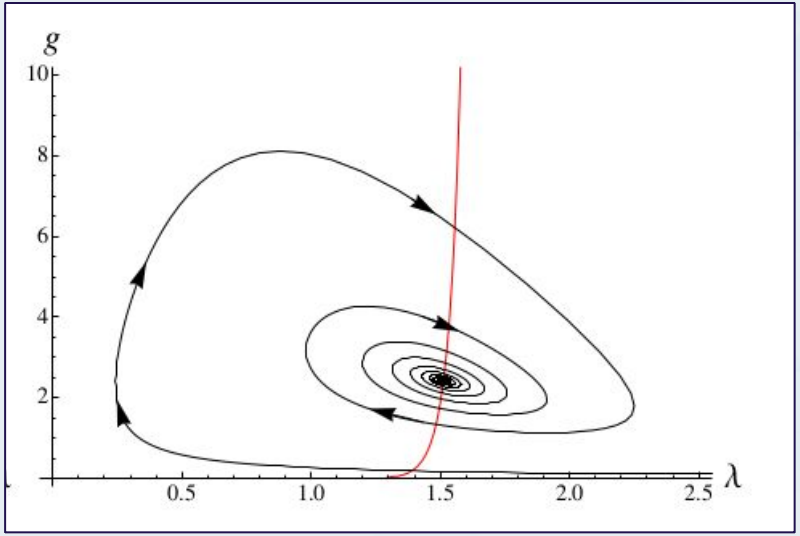
[11] **A. Bonanno** *et al.* , '*On Exact Proper Time Wilsonian RG Flows*', Eur. Phys. J. C, Vol. 80, Issue 3 (2020)

[12] **A. Bonanno** *et al.* , '*Structural aspects of FRG in quantum tunnelling computations*', Annals Phys., Vol. 445 (2022)

The role of a simpler model: ‘CREH’

Our research

Reuter UV fixed point!



[10] **A. Bonanno** and **F. Guarnieri**, Phys. Rev. D, vol. 86, Issue 10, (2012), Fig. 2

Conformal factor

Due to its instability, in QG its fluctuations dominate the path integral [1,10,13]

renormalizability of conformal sector

=

renormalizability of whole theory!

[1] **S. Weinberg**, in ‘*General Relativity: An Einstein centenary survey*’, ed. **S.W. Hawking** and **W. Israel**, 790-831, Cambridge University Press (1979)
[10] **A. Bonanno** and **F. Guarnieri**, ‘*Universality and symmetry breaking in conformally reduced quantum gravity*’, Phys. Rev. D, vol. 86, Issue 10, (2012)
[13] **M. Reuter** , ‘*Nonperturbative Evolution Equation for Quantum Gravity*’, Phys. Rev. D, Vol. 57, (1998)

Our results in the CREH+R^2 model

Our research

Starting point

Einstein-Hilbert action + R^2 : what happens at bigger orders in curvature?

$$S = \int d^4x \sqrt{g} \left[\left(-\frac{1}{16\pi G} \right) (R(g) - 2\Lambda) + \beta R^2 \right]$$

1) Spherical geometry
2) Usual regulator family

→ **Today's results**

Our results in the CREH+R² model

Our research

Reduced number of degrees of freedom: we are able to compute traces **exactly**

$$\partial_t \Gamma_k = -\frac{1}{2}\int_0^\infty \frac{ds}{s}(\partial_t \rho_k(s,n))\mathrm{Tr}e^{-s\Gamma_k^{(2)}} \quad + \quad \phi = \phi_0 + \tilde{\phi}$$

l.h.s

$$\partial_t \Gamma_k = \int d^d x \sqrt{\hat{g}}\left[-\frac{1}{2}(\partial_t \tilde{Z}_k[\phi_0])\tilde{\phi}\hat{\Box}\tilde{\phi} + (\partial_t U_k[\phi_0]) + 36(\partial_t \beta_k[\phi_0])\frac{(\hat{\Box}\tilde{\phi})^2}{\phi_0^2}\right]$$

$$\partial_t U_k[\phi_0] = \frac{1}{12}(\partial_t Z_k)\hat{R}\phi_0^2 - \frac{1}{6}(\partial_t Z_k \Lambda_k)\phi_0^4 + (\partial_t \beta_k)\hat{R}^2.$$

Our results in the CREH+R² model

Our research

r.h.s ➡ $\Gamma_k^{(2)} = \mathcal{X} + \mathcal{Y}$ **BCH expansion!**

$$\mathcal{X} = -Z_0\hat{\Box} + U_0^{(2)} + 36c_1\hat{\Box}^2$$

$$\mathcal{Y} = -\frac{Z_0'}{2}[(\hat{\Box}\tilde{\phi}) + \hat{\Box}(\tilde{\phi}\,\cdot) + \tilde{\phi}\hat{\Box}] + \tilde{\phi}U_0^{(3)} + 72c_2[(\hat{\Box}\tilde{\phi})\hat{\Box} + \hat{\Box}((\hat{\Box}\tilde{\phi})\cdot) + \hat{\Box}(\tilde{\phi}\hat{\Box})] +$$
$$-\frac{Z_0''}{2}[\tilde{\phi}(\hat{\Box}\tilde{\phi}) + \frac{1}{2}\hat{\Box}(\tilde{\phi}^2\cdot) + \frac{1}{2}\tilde{\phi}^2\hat{\Box}] + \frac{\tilde{\phi}^2}{2}U_0^{(4)} + 36c_3[\hat{\Box}(\tilde{\phi}^2\hat{\Box}) + 2\hat{\Box}(\tilde{\phi}(\hat{\Box}\tilde{\phi})\cdot) + 2\tilde{\phi}(\hat{\Box}\tilde{\phi})\hat{\Box} + (\hat{\Box}\tilde{\phi})^2]$$

$$c_1 = 2\frac{\beta_0}{\phi_0^2},$$
$$c_2 = \frac{\beta_0'}{\phi_0^2} - 2\frac{\beta_0}{\phi_0^3},$$
$$c_3 = 6\frac{\beta_0}{\phi_0^4} - 4\frac{\beta_0'}{\phi_0^3} + \frac{\beta_0''}{\phi_0^2}.$$

Our results in the CREH+R² model

Our research

spherical geometry

$\hat{R} = \text{const.}$ + $\tilde{\phi} = 0$ → $\mathcal{Y} = 0$

$$r.h.s. = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\left(-\frac{2}{\Gamma(n)}(sZ_k\chi_B^2\bar{k}^2)^n e^{-sZ_k\chi_B^2\bar{k}^2}\right)\mathrm{Tr}\, e^{-s\left(-Z_k\chi_B^2\bar{\Box}+\frac{1}{6}Z_k\hat{R}-2Z_k\Lambda_k\chi_B^2+72\frac{\beta_k}{\chi_B^2}\bar{\Box}^2\chi_B^4\right)}$$

$$= -\frac{1}{2}\int_0^\infty \frac{ds}{s}\left(-\frac{2}{\Gamma(n)}(sZ_k\chi_B^2k^2)^n e^{-sZ_k\chi_B^2k^2}\right)e^{-sZ_k\left(\frac{1}{6}\hat{R}-2\Lambda_k\chi_B^2\right)}\,\mathrm{Tr}\,\underbrace{e^{-s\left(-Z_k\bar{\Box}\chi_B^2+72\beta_k\bar{\Box}^2\chi_B^2\right)}}_{W(-\bar{\Box})}$$

Heat kernel expansion

Our results in the CREH+R^2 model

Our research

Finite n

1) Beta functions **without divergencies**
2) $\forall$ n there is a **single fixed point** located at $(g^* < 0, \lambda^* > 0, b^* < 0)$
3) The structure of critical exponents depends on n: slight **regulator dependence**

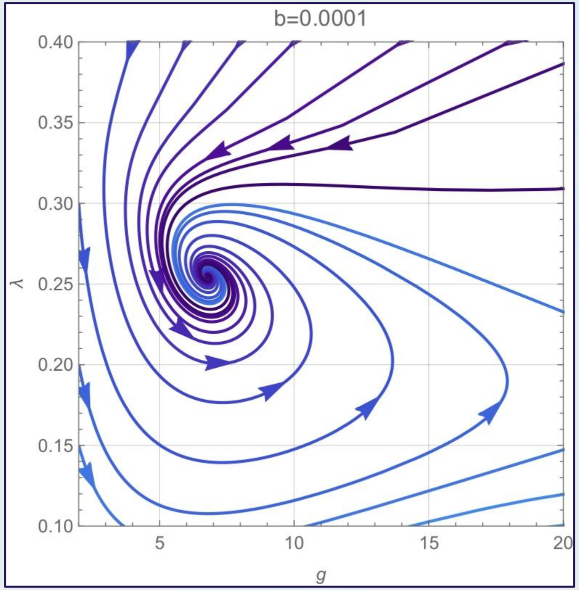
n	z^*	g^*	λ^*	b^*	$g^*\lambda^*$	λ_1	λ_2	λ_3
3	0.00984	−24.25919	0.39148	−0.00061	−9.49699	12.928	$-3.32257 + i\,1.08259$	$-3.32257 - i\,1.08259$
5	0.00966	−24.70644	0.31413	−0.00056	−7.76122	12.54881	$-3.47640 + i\,0.97049$	$-3.47640 + i\,0.97049$
7	0.00825	−28.92136	0.26102	−0.00051	−7.54914	12.61331	−4.14312	−3.51035
9	0.00686	−34.80991	0.22289	−0.00047	−7.75899	12.78006	−5.47819	−2.95752
20	0.00278	−85.98612	0.12301	−0.00030	−10.57733	13.84397	−10.32259	−2.45510

Our results in the CREH+R^2 model

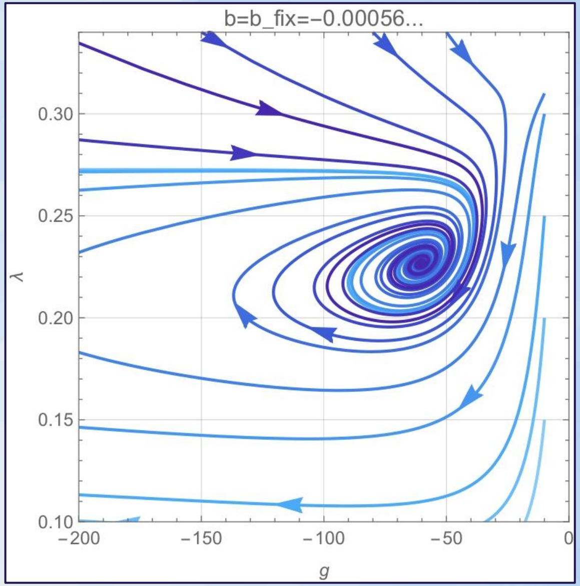
Our research

Trajectories on slices with fixed value of b

$b \gtrsim 0$



$b = b^*$



Our results in the CREH+R^2 model

Our research

n = + ∞

n	Beta functions β_z, β_λ and β_b
$+\infty$	$\partial_t z = -2z + \frac{e^{2\lambda} z \left(\sqrt{2\pi}\sqrt{z} e^{\frac{z}{288b}} \operatorname{erfc}\left(\frac{\sqrt{z}}{12\sqrt{2}\sqrt{b}}\right) - 24\sqrt{b}\right)}{27648\pi^2 b^{3/2}}$ $\partial_t \lambda = \frac{24\sqrt{b}\left(e^{2\lambda}(\lambda-3) - 2304\pi^2 b\lambda\right) - \sqrt{2\pi}(\lambda-3)\sqrt{z}\operatorname{erfc}\left(\frac{\sqrt{z}}{12\sqrt{2}\sqrt{b}}\right) e^{\frac{z}{288b}+2\lambda}}{27648\pi^2 b^{3/2}}$ $\partial_t b = \frac{e^{2\lambda} z \left(24\sqrt{b} - \sqrt{2\pi}\sqrt{z} e^{\frac{z}{288b}} \operatorname{erfc}\left(\frac{\sqrt{z}}{12\sqrt{2}\sqrt{b}}\right)\right)}{3981312\pi^2 b^{3/2}}$

There is no common fixed point!

Dependence on regulator! Small n seems to capture more info

Our results in the CREH+R^2 model

Notice that previous results in flat space (R^2 truncation) do not show any fixed point! [14]
At least for $g^* > 0$

↓

Work in progress in the CREH+R^2 model to study $g^* < 0$

↓

Evidence for dependence on geometry projection?

[14] **B. Knorr** , '*Lessons from conformally reduced quantum gravity*', "Class. Quant. Grav., Vol. 38, Issue 6 (2021)

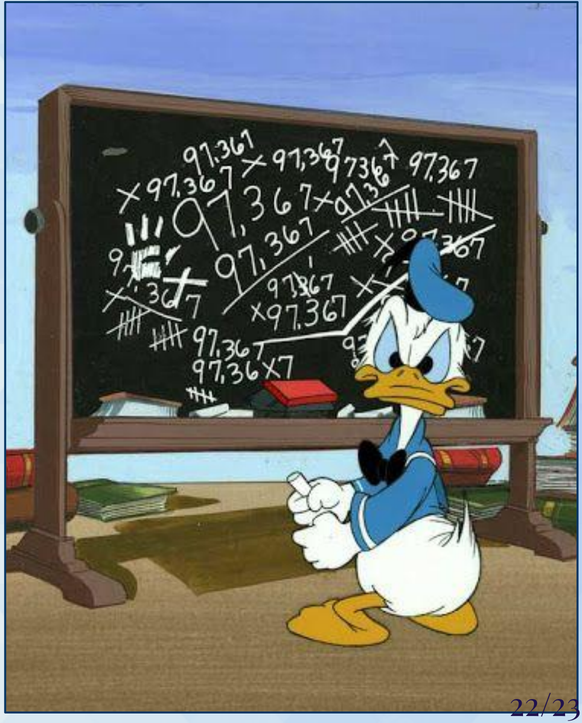
Beyond our first step

Starting point

- **CREH** + **R²**, projection onto **S⁴** sphere

And then?

- **CREH** + **R²**, projection onto **R⁴** flat space ?
- **CREH** + **R³**, projection onto **S⁴** sphere ?
- Can we write the equations for **Rⁿ** on the **S⁴** sphere ?
- Can we write the equations for **f(R)** on the **S⁴** sphere ?
- Etc.



Thanks!