30 years of quantum black holes in Bologna

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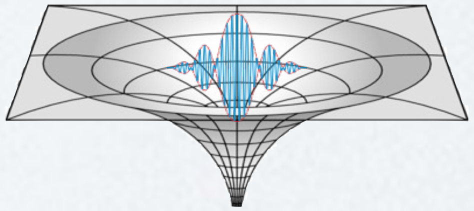
INFN

IV FLaG Meeting

Povo (TN)

06/10/2022

Theory and Phenomenology
of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA



How it all began (for me…)

- Meet Gianni:

PHYSICAL REVIEW VOLUME 160, NUMBER 5 25 AUGUST 1967

Quantum Theory of Gravity. I. The Canonical Theory*

BRYCE S. DEWITT
Institute for Advanced Study, Princeton, New Jersey
and
Department of Physics, University of North Carolina, Chapel Hill, North Carolina†
(Received 25 July 1966; revised manuscript received 9 January 1967)

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universe is studied in detail, and its static wave functions in the WKB approximation are obtained. In order to obtain nonstatic wave functions which resemble a dynamical universe evolving it is necessary to introduce a clock. The combined wave functions of universe-*cum*-clock are studied, and it is pointed out that normalizability of the wave functions requires precise commensurability between the periods of universe and clock. Wave packets exhibiting quasiclassical behavior are constructed in Sec. 8, in three different representations. Two of these make use of proper times defined by the clock and the universe respectively; the third treats universe and clock symmetrically through their mutual correlations. Attention is called to the deficiencies of the first two representations arising from the fact that, in a covariant theory, time is only a phenomenological concept. In the third representation probability flows in a closed finite circuit in configuration space, and wave packets do *not* ultimately spread in time. Use is made of

x^0 = constant are distinguished by means of a prefixed superscript (3). These conventions have the property that ${}^{(4)}R$ is non-negative in a space-time containing normal matter and satisfying Einstein's equations, and that ${}^{(3)}R$ is positive in a 3-space of positive curvature.

2. EXTRINSIC AND INTRINSIC CURVATURE. CLASSIC FORM OF THE LAGRANGIAN

The canonical theory begins with the following decomposition of the metric tensor:

$$(g_{\mu\nu})=\begin{pmatrix}-\alpha^2+\beta_k\beta^k & \beta_j\\ \beta_i & \gamma_{ij}\end{pmatrix}, \tag{2.1}$$

$$(g^{\mu\nu})=\begin{pmatrix}-\alpha^{-2} & \alpha^{-2}\beta^j\\ \alpha^{-2}\beta^i & \gamma^{ij}-\alpha^{-2}\beta^i\beta^j\end{pmatrix}, \tag{2.2}$$

$$\gamma_{ik}\gamma^{kj}=\delta_i{}^j,\quad \beta^i=\gamma^{ij}\beta_j.$$

(2.4)

$$\ldots\equiv\gamma^{ik}\gamma^{jl}K_{kl},$$

$$K\equiv\gamma^{ij}K_{ij}, \tag{2.5}$$

… denoting covariant differentiation based on the 3-metric γ_{ij}.

The quantity K_{ij}, which transforms as a symmetric tensor under spatial coordinate transformations, is known as the *second fundamental form*. It describes the curvature of the hypersurface x^0= constant as viewed from the 4-dimensional space-time in which it is em-

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…sity …nd, from the Lagrangian (2.6) a surface integral E_∞ given by

$$E_\infty=\int_\infty \alpha\gamma^{1/2}\gamma^{ij}(\gamma_{ik,j}-\gamma_{ij,k})dS^k, \tag{2.7}$$

and hence adds a corresponding quantity to the canonical energy. In an asymptotically flat world it is always possible to find an asymptotically Minkowskian reference frame in which α, β_i, and γ_{ij} take the static Schwarzschild forms

$$\alpha\underset{r\to\infty}{\to}1-\frac{M}{16\pi r},\quad \beta_i\underset{r\to\infty}{\to}0,\quad \gamma_{ij}\underset{r\to\infty}{\to}\delta_{ij}+\frac{M}{8\pi}\frac{x^ix^j}{r^3}, \tag{2.8}$$

where $r^2\equiv x^ix^i$ and M is the effective gravitational mass of the field distribution. Substitution of (2.8) into (2.7) yields

$$E_\infty=M. \tag{2.9}$$

It is to be noted that the removal of E_∞ from the Lagrangian does not correspond to a mere redefinition

How it all began (for me…)

- Gravitational collapse

PHYSICAL REVIEW D VOLUME 8, NUMBER 10 15 NOVEMBER 1973

Canonical Quantization of Relativistic Balls of Dust*

Fernando Lund†
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 21 May 1973)

The Hamiltonian form for the equations of a relativistic perfect fluid is considered and later specialized to the case of spherical symmetry and vanishing pressure. When comoving coordinates are used in the canonical formalism, one gets a reduced Hamiltonian which is independent of time. The continuous number of degrees of freedom are decoupled and the Schrödinger equation separates from a functional differential equation to a set of identical ordinary differential equations. Boundary conditions for these equations are naturally obtained by requiring that the minisuperspace be geodesically complete. The formalism remains the same whether one treats a closed nonhomogeneous universe or a collapsing star. The problem of singularities is discussed, and it is concluded that in this minisuperspace quantum formalism there is no inevitable singularity.

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where p is the pressure and ρ the density of total mass energy. Here, ϕ is a scalar field related to the four-velocity U_ν through

$$U_\nu = h^{-1}\phi_{,\nu}, \tag{9}$$

where h is the specific enthalpy

$$h = \rho_0^{-1}(\rho + p), \tag{10}$$

the field variables ϕ, μ, λ, their canonically conjugate momenta p^ϕ, π_μ, π_λ, and the Hamiltonian

$$H = \int dr\{N(\mathcal{H}^0 + \mathcal{E}) + N_1(\mathcal{H}^1 + \mathcal{P}^1)\}$$

given in terms of them by (5), (6), (14), and (15). The dynamical equations are (the dot denotes differentiation with respect to time):

$$\dot\lambda = \frac{\delta H}{\delta \pi_\lambda} = -\tfrac{1}{4}\pi_\mu N e^{-\mu-2\lambda} + N_1 e^{-2\mu}\lambda', \tag{16}$$

$$\dot\mu = \frac{\delta H}{\delta \pi_\mu} = \tfrac{1}{4} N e^{-\mu-2\lambda}(\pi_\mu - \pi_\lambda) + (N_1 e^{-2\mu})' + N_1 e^{-2\mu}\mu', \tag{17}$$

$$\dot\pi_\mu = -\frac{\delta H}{\delta\mu} = N\mathcal{H}^0 - 4(Ne^{-\mu}\lambda' e^{2\lambda})' + 4Ne^{\mu} + Np^\phi e^{-2\mu}(\phi')^2[1 + e^{-2\mu}(\phi')^2]^{-1/2} + (N_1 e^{-2\mu}\pi_\mu)' + 2N_1 e^{-2\mu}p^\phi\phi' - 2N_1\mathcal{H}^1, \tag{18}$$

$$\dot\pi_\lambda = -\frac{\delta H}{\delta\lambda} = 2N\mathcal{H}^0 - 8Ne^{-\mu+2\lambda}[2\lambda'' - 2\lambda'\mu' + 3(\lambda')^2 - e^{2\mu-2\lambda}] - 4(Ne^{-\mu+2\lambda})'' - 4(Ne^{-\mu+2\lambda}\mu')' + 12(Ne^{-\mu+2\lambda}\lambda')' - 4Ne^{\mu} - Np^\phi[1 + e^{-2\mu}(\phi')^2]^{1/2} + (N_1 e^{-2\mu}\pi_\lambda)' - N_1 e^{-2\mu}p^\phi\phi', \tag{19}$$

$$\dot\phi = \frac{\delta H}{\delta p^\phi} = N[1 + e^{-2\mu}(\phi')^2]^{1/2} + N_1 e^{-2\mu}\phi', \tag{20}$$

$$\dot p^\phi = -\frac{\delta H}{\delta\phi} = \{Np^\phi e^{-2\mu}\phi'[1 + e^{-2\mu}(\phi')^2]^{-1/2}\}' + (N_1 e^{-2\mu}p^\phi)', \tag{21}$$

and the constraints are

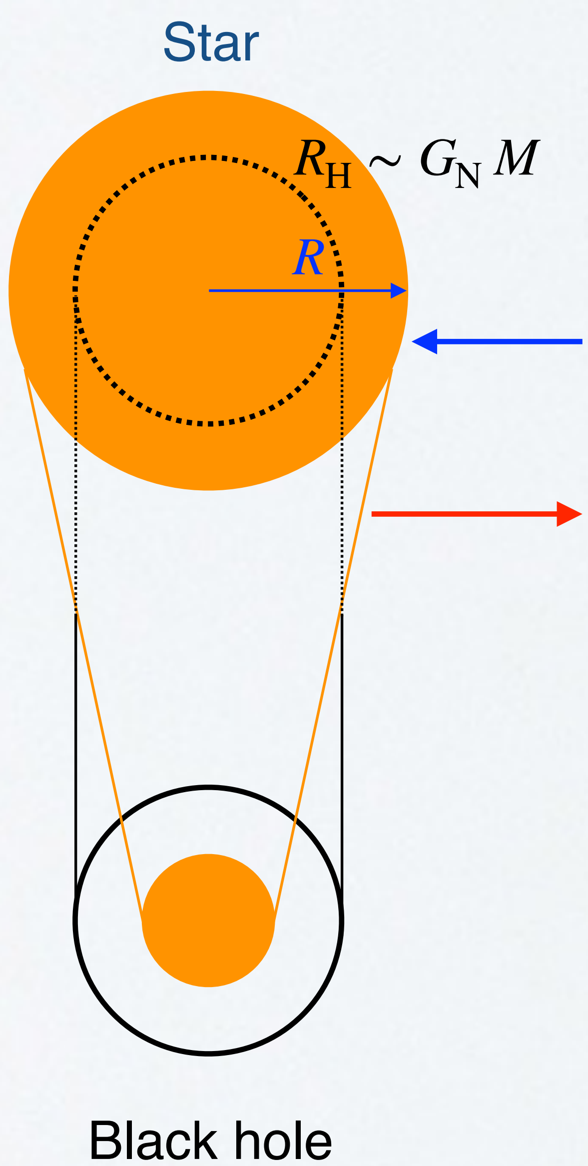
$$\mathcal{H}^0 + \mathcal{E} = 0, \tag{22}$$

$$\mathcal{H}^1 + \mathcal{P}^1 = 0. \tag{23}$$

We are now in a position to apply the ADM reduction procedure. First of all, we need to specify a time variable. A time-coordinate built from the gravitational degrees of freedom alone

Gravitational collapse and black hole evaporation

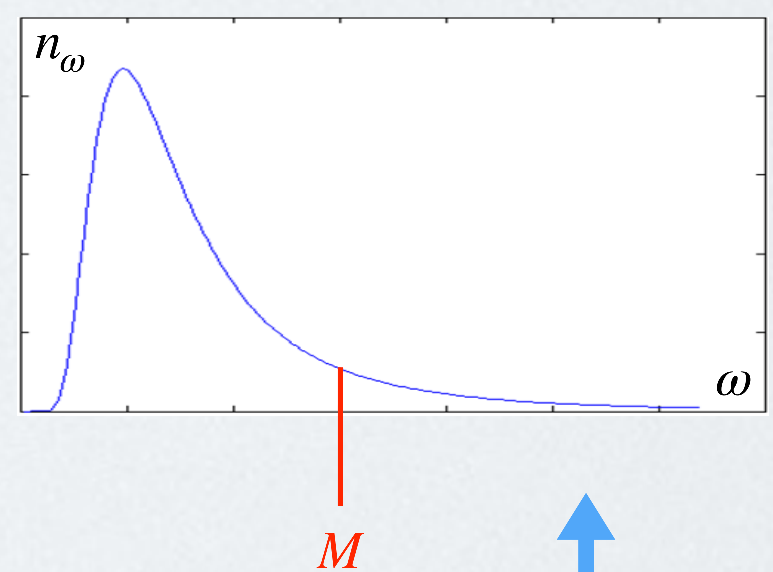
- BH form from collapse - the semiclassical (classical background) view:



- Classical star + $|\,vacuum\ \text{fluctuations}\rangle$
- Classical inner and outer geometry + $|\,vacuum\ \text{fluctuations}\rangle$

Time-dependent
(gravitational blue/redshift)

$|\,\text{Thermal excitations}\rangle$



BH temperature: $T_H \sim m_p \dfrac{m_p}{M}$

BH decay rate: $\dfrac{dM}{dt} \sim -\dfrac{m_p^2}{M^2}$

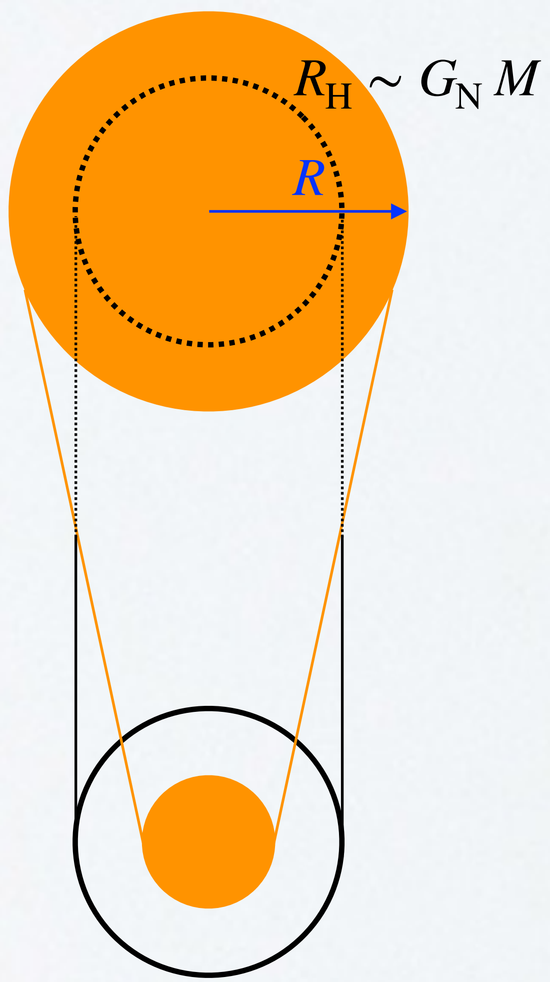
Microcanonical corrections*: $\dfrac{dM}{dt} \sim -\left(\dfrac{M}{m_p}\right)^{\alpha}$

"quantum hair"

* R.C., B. Harms, PRD 58 (1998) 044014 [gr-qc/9712017]

Gravitational collapse and black hole evaporation

- BH form from collapse - the quantum view:



- $|\text{matter}\rangle$ ~ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\text{gravity}\rangle$ ~ very large number of gravitons ($N_G \sim M_\odot^2 \sim 10^{76}$)

- $|\text{gravity}\rangle$ always entangled with $|\text{matter}\rangle$ $\Longleftrightarrow$ "quantum hair" *

Dynamics

$$|\mathbf{g}\,\phi\rangle = \sum_{ij} C_{ij} |\mathbf{g}_i\rangle |\phi_j\rangle \quad \not\Rightarrow \quad \left(\sum_{ab} c_{ab} |\mathbf{g}_a\rangle |\phi_b\rangle \right) \left(\sum_{AB} c_{AB} |\mathbf{g}_A\rangle |\phi_B\rangle \right)$$

$$\hat{H}^\mu |\mathbf{g}\,\phi\rangle = 0$$

BH interior — BH exterior

* X. Calmet, R.C., S.D.H. Hsu, F. Kuipers, PRL 128 (2022) 111301 [arXiv:2110.09386]

Quantum physics and bound states

- Quantum *vs* classical physics:

  Quantum space = not all $\phi_{\rm cl}$ may be realised by a $|\psi_{\rm cl}\rangle$!! (*e.g.* hydrogen atom, BH? Universe?)

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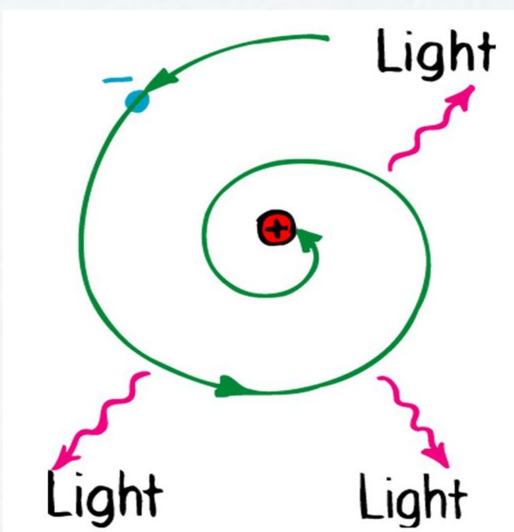
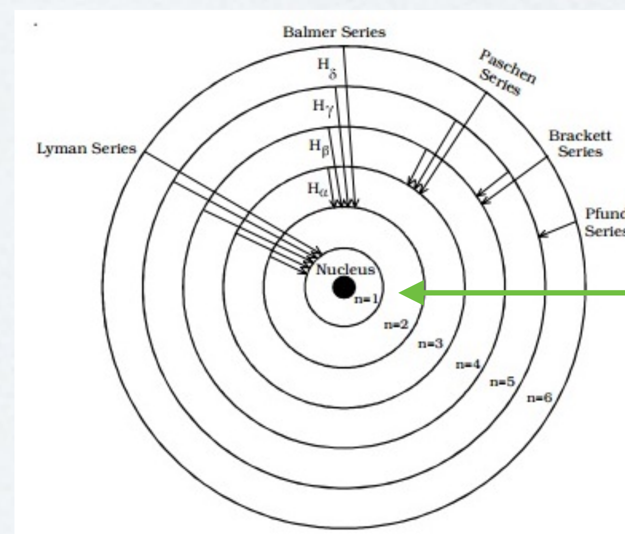
UV catastrophe of CED*



Same dynamics

QED → QM potential theory



Bound ground state

CED Hy lifetime $\tau \approx 10^{-11}\,\text{sec}$

Actual Hy lifetime $\sim \tau \approx 10^{17}\,\text{sec}$

(CED off by $\sim 10^{28}$)

* And gravity: S. Deser, EPJC 82 (2022) 424 [arXiv:2202.00786]

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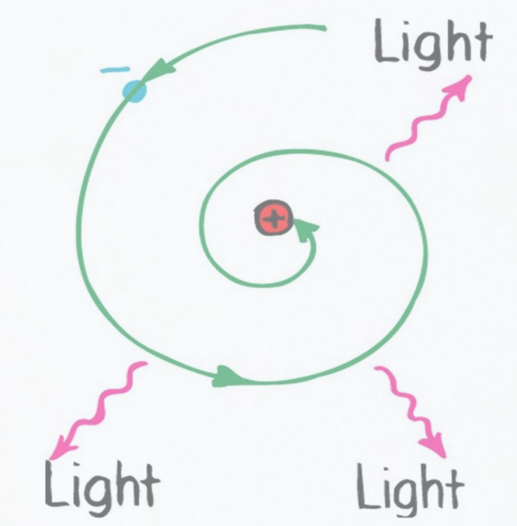

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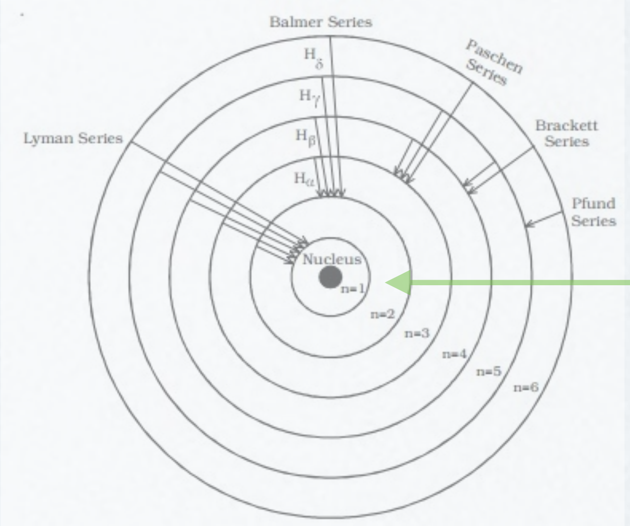

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- Classical (sector of) gravity is long-range, nonlinear, and universal (*equivalence principle*):

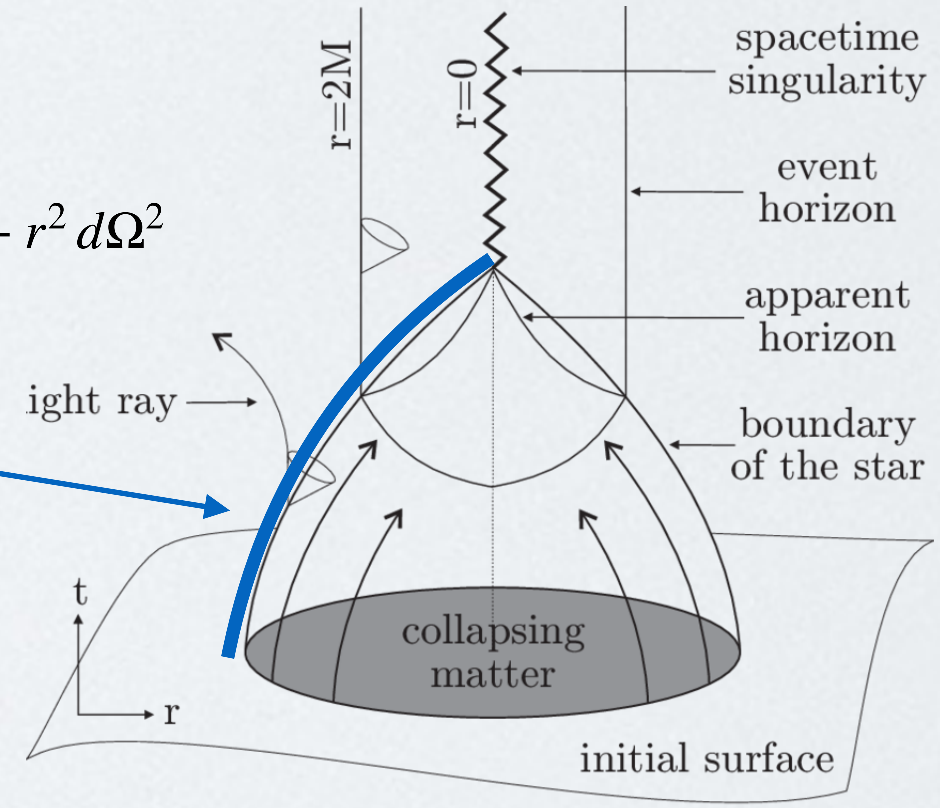
GR $\longrightarrow$ QG very complicated!

Quantum dust core

- Role of non-linearity: collapsing ball of dust *

$$ds^2 = -\left(1 - \frac{2\,G_N\,M}{r}\right) dt^2 + \left(1 - \frac{2\,G_N\,M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$$

$$\left(\frac{dR}{d\tau}\right)^2 + 1 - \frac{2\,G_N\,M}{R} \simeq \frac{E^2}{M^2}$$



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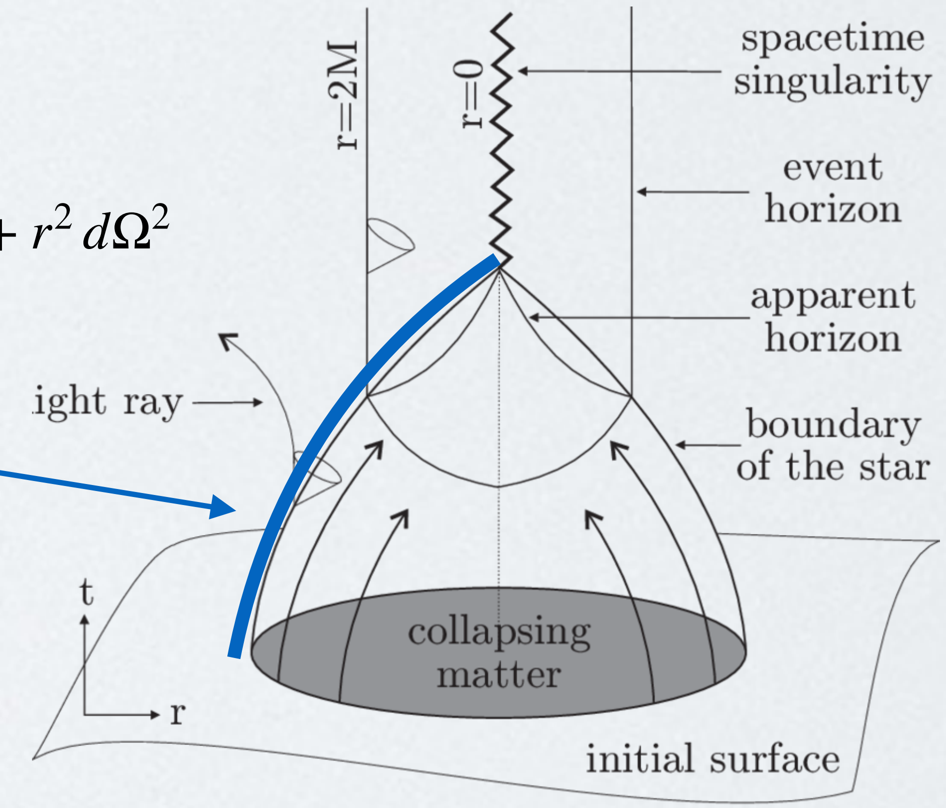

- Effective Hamiltonian:

$$H \equiv \frac{P^2}{2\,M} - \frac{G_N\,M^2}{R} = \frac{M}{2}\left(\frac{E^2}{M^2} - 1\right) \equiv \mathscr{E}$$

- Schrödinger equation: $\hat{H}\,\Psi_n = \mathscr{E}_n\Psi_n$

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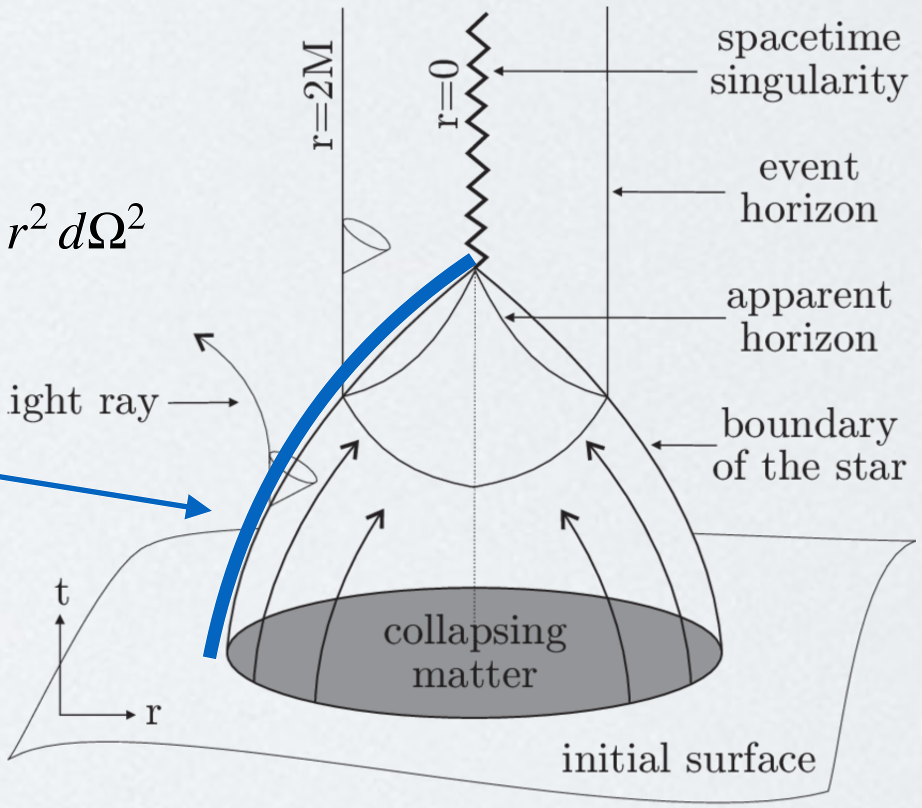

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- Schrödinger equation: $\hat{H}\,\Psi_n = \mathscr{E}_n\,\Psi_n$

- Spectrum of bound states ($n \geq 1$):

"GR" *

$$\frac{\mathscr{E}_n}{M} \simeq -\frac{G_N^2\,M^4}{2\,\hbar^2\,n^2} = -\frac{1}{2\,n^2}\left(\frac{M}{m_p}\right)^4 = \frac{1}{2}\left(\frac{E_n^2}{M^2} - 1\right)$$

$$R_n \equiv \langle \Psi_n | R | \Psi_n \rangle \simeq n^2\,\ell_p \left(\frac{m_p}{M}\right)^3$$

Newtonian spectrum

* R.C., EPJC 82 (2022) 1 [arXiv:2103.14582]

Quantum dust core

- Allowed spectrum *: $0 \leq \frac{E_n^2}{M^2} \simeq 1 - \frac{1}{n^2}\left(\frac{M}{m_{\rm p}}\right)^4$

$$n \geq N_M \simeq \frac{M^2}{m_{\rm p}^2}$$

$$\mathscr{E}_n \geq \mathscr{E}_{N_M} \simeq -\frac{M}{2}$$

$$R_n \geq R_{N_M} \simeq G_{\rm N}\, M = \ell_{\rm p}\,\frac{M}{m_{\rm p}}$$

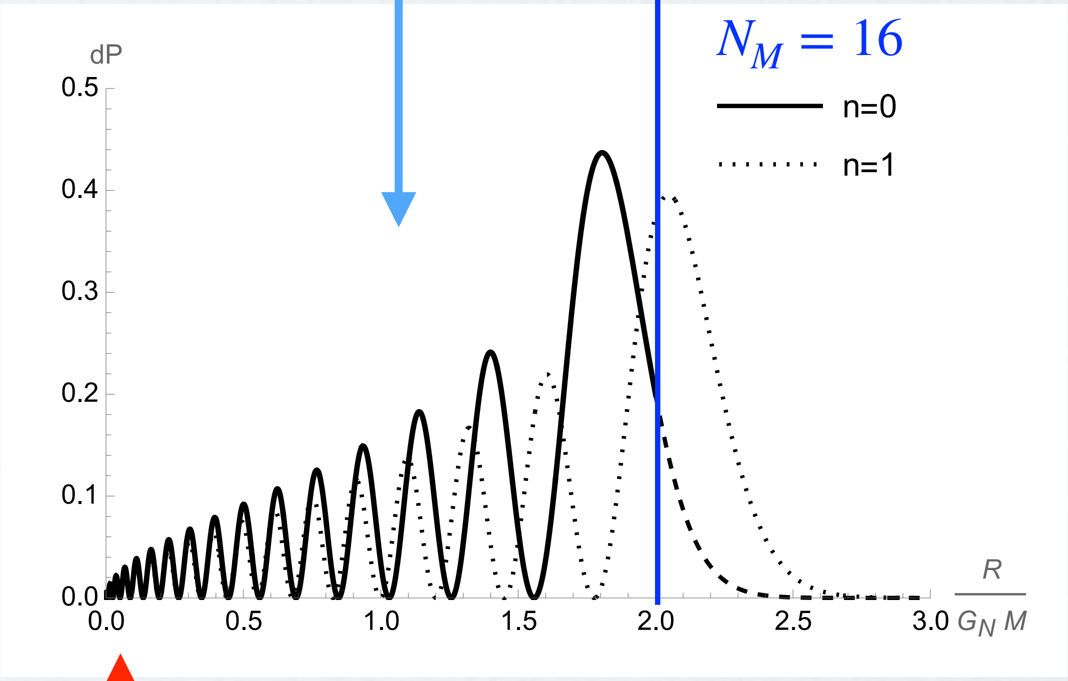
- "Energy" levels: $|\mathscr{E}_{n+1} - \mathscr{E}_n| \simeq m_{\rm p}\,\frac{m_{\rm p}}{M} \ll m_{\rm p}$

$$|E_{n+1} - E_n| \simeq m_{\rm p}$$

- Bounded compactness: $\frac{G_{\rm N}\, M}{R_n} \lesssim 1$

$(n \to N_M + n)$

$N_M = 16$



$$R_{n=1} \sim \ell_{\rm p}\left(\frac{m_{\rm p}}{M}\right)^3$$

non-linearity

↓

* *Classicalization* ~ GUP in action?

Conclusions

- Black holes as (macroscopic) quantum objects (*ground state* very far from vacuum + information entropy *)

- Singularity is not resolved (regular or fuzzy geometry)

- Exterior quantum hair (from background and loop corrections)

- No Cauchy horizon (for electrically charged black holes **)

- No Cauchy horizon for rotating black holes?

- Effective cosmological DM **

* R.C., R. Da Rocha, P. Meert, L. Tabarroni, W. Barreto, *Configurational entropy of black hole quantum cores*, arXiv:2206.10398

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*** A. Giusti, S. Buffa, L. Heisenberg, R.C., PLB 826 (2022) 136900 [arXiv:2108.05111]

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