Top-Antitop Production at Hadron Colliders

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Plan of the Talk

- General Introduction
  - Top Quark at the Tevatron
  - LHC Perspectives
- Status of the Theoretical Calculations
  - The General Framework
  - The NLO Corrections
- Two-Loop QCD Corrections: Analytic Calculation

  R. B., A. Ferroglia, T. Gehrmann, D. Maître, and C. Studerus, JHEP **0807** (2008) 129
  R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067
  R. B., A. Ferroglia, T. Gehrmann, A. von Manteuffel, and C. Studerus, JHEP **1101** (2011) 102
  R. B., A. Ferroglia, T. Gehrmann, A. von Manteuffel, and C. Studerus, in preparation
- Conclusions

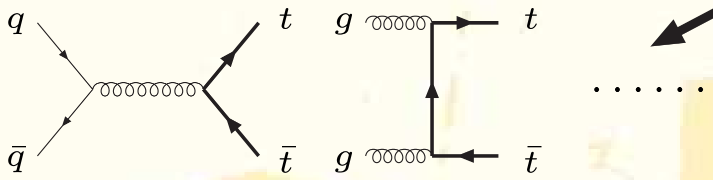
Top Quark

Top Quark

- With a mass of $m_t = 173.3 \pm 1.1$ GeV (July 2010), the TOP quark (the up-type quark of the third generation) is the heaviest elementary particle produced so far at colliders.
- Because of its mass, top quark is going to play a unique role in understanding the EW symmetry breaking $\Rightarrow$ Heavy-Quark physics crucial at the LHC.
- Two production mechanisms

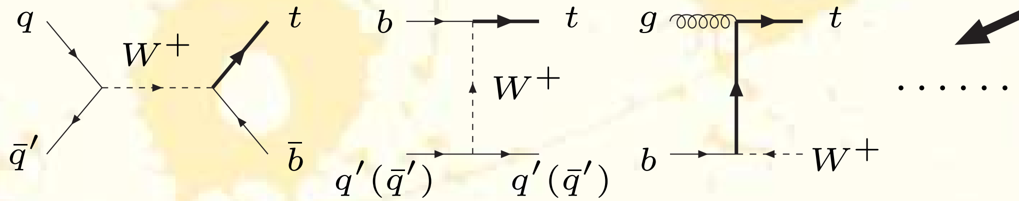
Pair Production

  - $pp(\bar{p}) \to t\bar{t}$

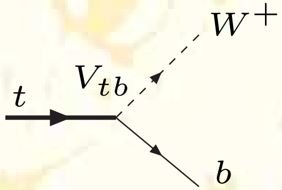


Single Top

  - $pp(\bar{p}) \to t\bar{b}, tq'(\bar{q}'), tW^-$



- Top quark does not hadronize, since it decays in about $5 \cdot 10^{-25}$s (one order of magnitude smaller than the hadronization time) $\Longrightarrow$ opportunity to study the quark as single particle
  - Spin properties
  - Interaction vertices
  - Top quark mass
- Decay products: almost exclusively $t \to W^+ b \quad (|V_{tb}| \gg |V_{td}|, |V_{ts}|)$



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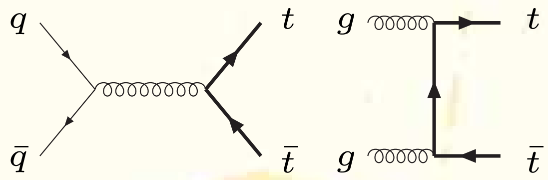

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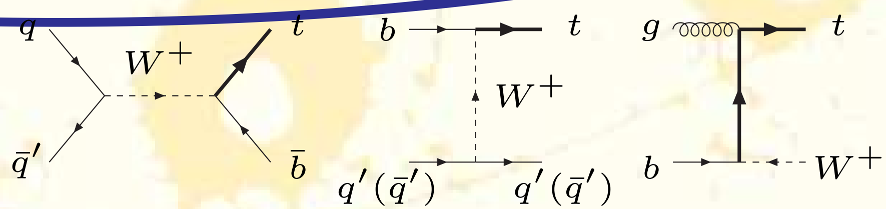

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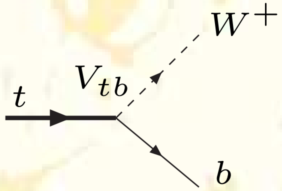

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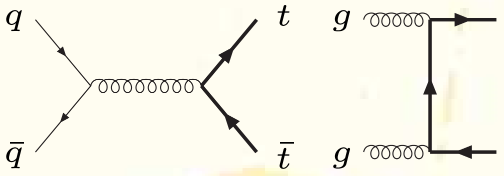

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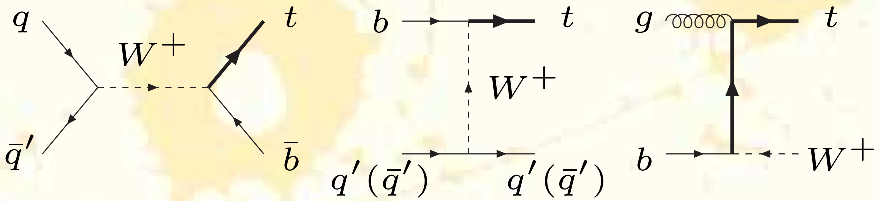

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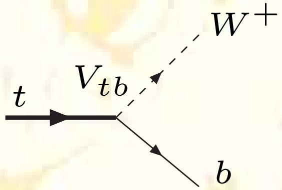
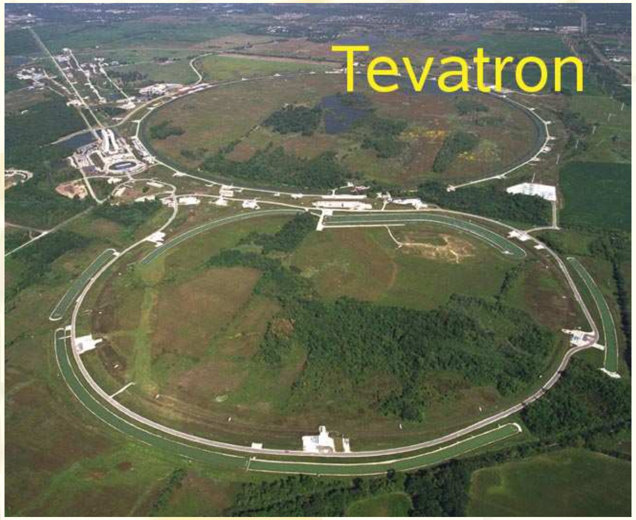


Top Quark

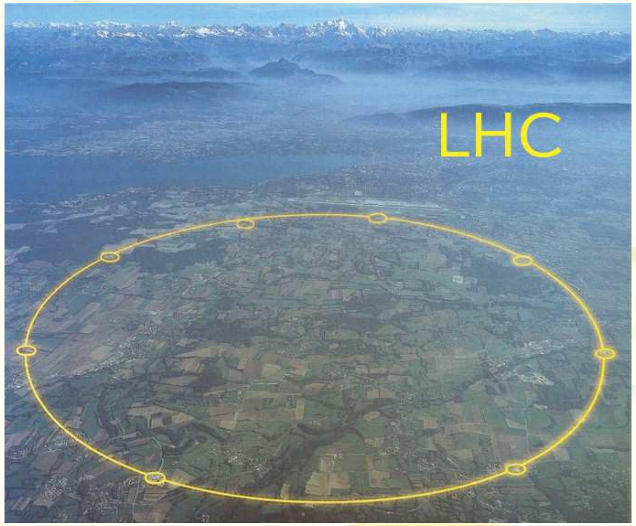
Tevatron

- Up to 2010 the Top quark could be produced and studied only at the Tevatron (discovery 1995)
- $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV
- $L \sim 9\text{fb}^{-1}$ reached in 2010 (12fb^{-1} expected by 2011)
- $\mathcal{O}(10^3)$ $t\bar{t}$ pairs produced so far
- Only recently confirmation of single-t



LHC

- Running since end 2009
- pp collisions at $\sqrt{s} = 7\,(14)$ TeV
- LHC will be a factory for heavy quarks ($\mathcal{L} \sim 100fb^{-1}/year$ at $\sqrt{s} = 14$ TeV, $t\bar{t}$ at ∼1Hz!)
- Even in the first low-luminosity phase (2 years $\sim 1\text{fb}^{-1}$ @ 7 TeV) $\sim \mathcal{O}(10^4)$ registered $t\bar{t}$ pairs



Top Quark @ Tevatron

Top Quark @ Tevatron

Events measured at Tevatron

$\sigma_{t\bar{t}} \sim$ 7pb

- $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-\bar{b} \rightarrow l\nu l\nu b\bar{b}$ — Dilepton $\sim 10\%$
- $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-\bar{b} \rightarrow l\nu q\bar{q}'b\bar{b}$ — Lep+jets $\sim 44\%$
- $p\bar{p} \rightarrow t\bar{t} \rightarrow W^+bW^-\bar{b} \rightarrow q\bar{q}'q\bar{q}'b\bar{b}$ — All jets $\sim 46\%$

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2 high-p_T lept, $\geq$ 2 jets and ME

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NO lept, $\geq$ 6 jets and low ME

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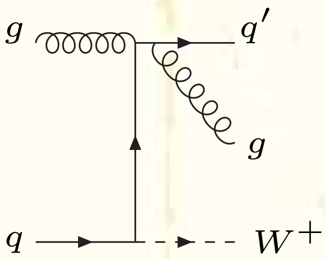
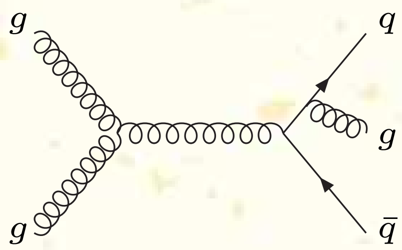
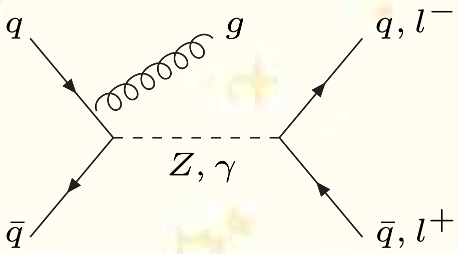
Background Processes



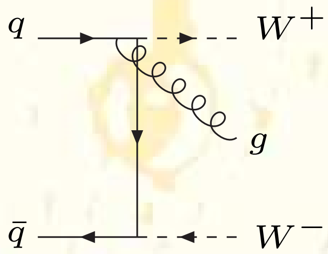
W+jets



QCD



Drell-Yan



Di-boson

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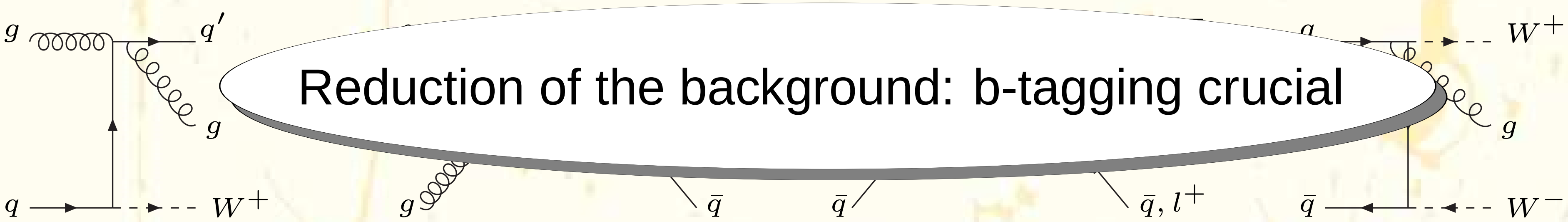


Reduction of the background: b-tagging crucial

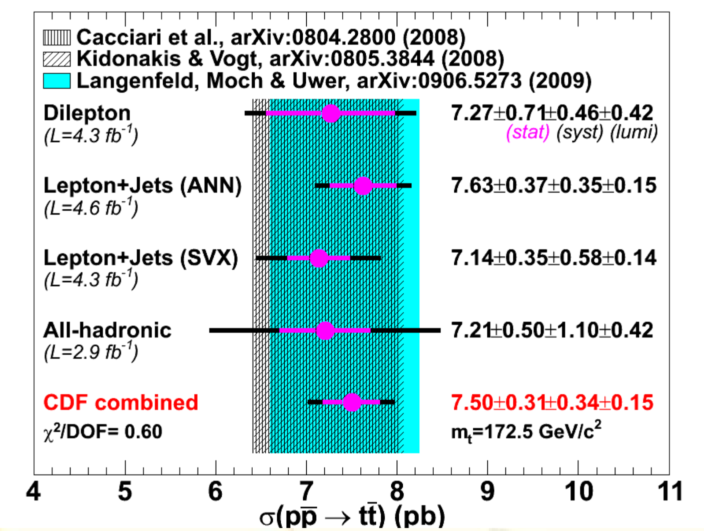
W+jets QCD Drell-Yan Di-boson

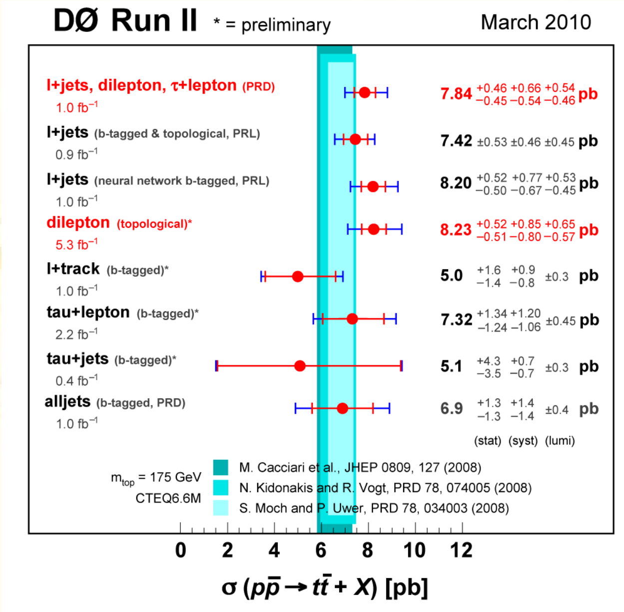
Top Quark @ Tevatron

- Total Cross Section

$$\sigma_{t\bar{t}} = \frac{N_{data} - N_{bkgr}}{\epsilon\, L}$$

  - Good test for the SM (in particular QCD)





Combination CDF-D0 ($m_t = 175$ GeV)

$$\sigma_{t\bar{t}} = 7.0 \pm 0.6\,\text{pb} \qquad (\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 9\%)$$

using 4.6 fb^{-1} of data

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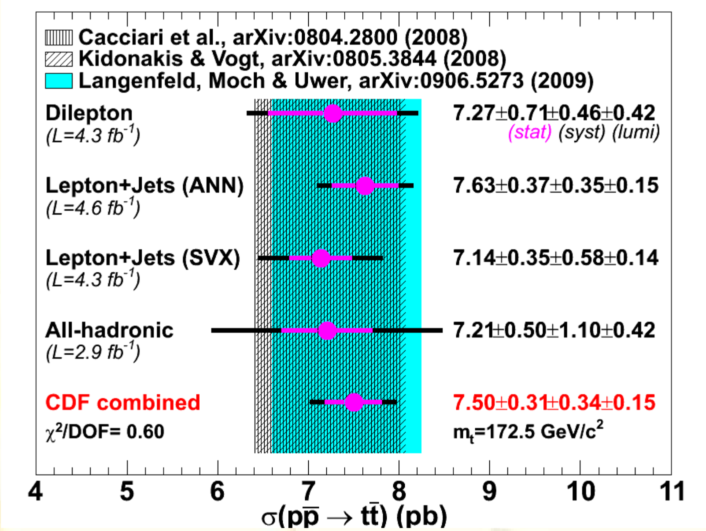

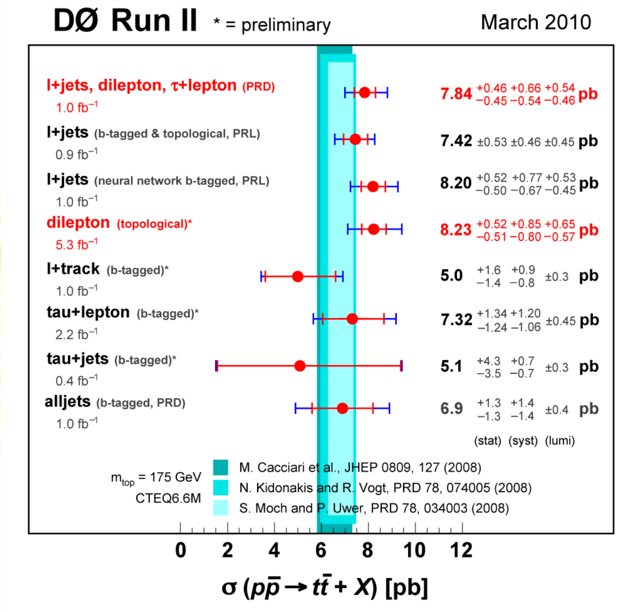

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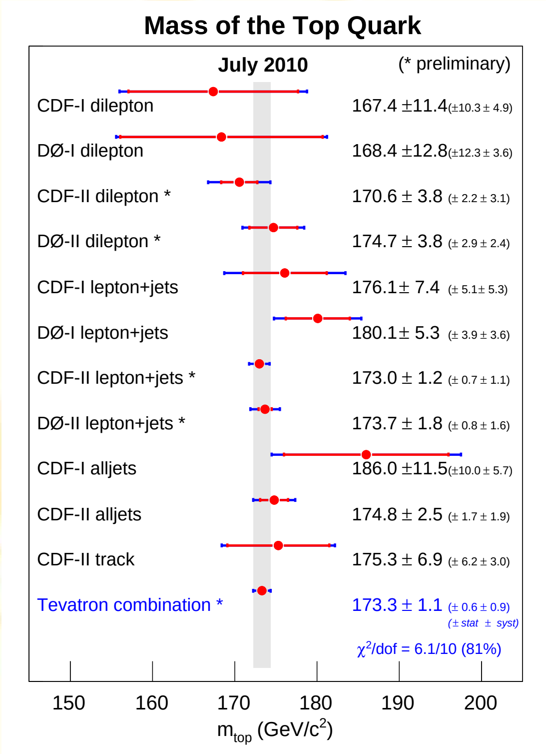
very recently $\Longrightarrow \Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 6.5\%$ (σ_Z for the luminosity)

Top Quark @ Tevatron

- Top-quark Mass
  - Fundamental parameter of the SM. A precise measurement useful to constraint Higgs mass from radiative corrections (Δr)
  - A possible extraction: $\sigma_{t\bar{t}} \Longrightarrow$ need of precise theoretical determination

$$\frac{\Delta m_t}{m_t} \sim \frac{1}{5} \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}$$



Combination CDF-D0 (July 2010)

$$m_t = 173.3 \pm 1.1 \text{ GeV } (0.63\%)$$

- In spite of the high precision is not totally clear which mass corresponds to the parameter measured at Tevatron: something near the “pole mass”?
- Top-quark pole mass is “physically” not well defined (although in pQCD it has a precise meaning) due to non-PT effects: $\mathcal{O}(\Lambda_{QCD})$ ambiguity. Probably better to move to other mass definitions (for in stance $\overline{\text{MS}}$). Transformation known at the three-loop level in QCD K. Melnikov and T. van Ritbergen ’99

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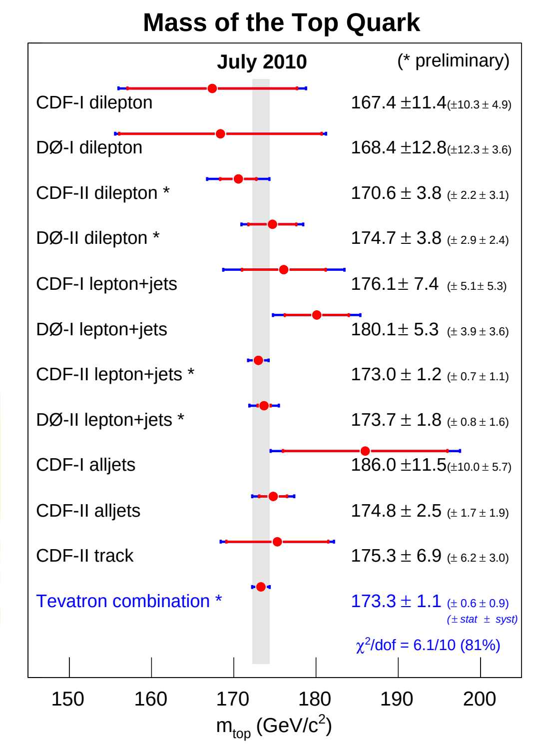


Combination CDF-D0 (July 2010)

$$\boxed{m_t = 173.3 \pm 1.1\,\text{GeV}\ (0.63\%)}$$

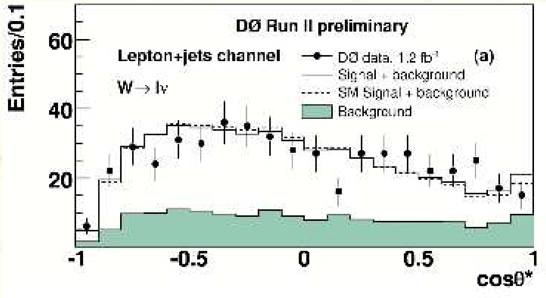
- Top-Anti Top Mass Difference @ D0 $\boxed{\Delta m_t = 3.8 \pm 3.7\,\text{GeV}}$
- Top-quark Width $\boxed{\Gamma_t < 7.6\,\text{GeV}\ (95\%\,CL)}$

Top Quark @ Tevatron

- W helicity fractions $F_i = B(t \to bW^+(\lambda_W = i))$ $(i = -1, 0, 1)$ measured fitting the distribution in θ^* (the angle between l^+ in the W^+ rest frame and W^+ direction in the t rest frame)

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{4}F_0 \sin^2\theta^* + \frac{3}{8}F_-(1-\cos\theta^*)^2 + \frac{3}{8}F_+(1+\cos\theta^*)^2$$

$$F_0 + F_+ + F_- = 1$$



$$\boxed{F_0 = 0.66 \pm 0.16 \pm 0.05 \quad F_+ = -0.03 \pm 0.06 \pm 0.03}$$

- Spin correlations measured fitting the double distribution

($\theta_1\,(\theta_2)$ is the angle between the dir of flight of $l_1\,(l_2)$ in the $t(\bar{t})$ rest frame and the $t(\bar{t})$ direction in the $t\bar{t}$ rest frame)

$$\frac{1}{N}\frac{d^2N}{d\cos\theta_1\, d\cos\theta_2} = \frac{1}{4}(1 + \kappa\cos\theta_1\cos\theta_2)$$

$$\boxed{\kappa = 0.32^{+0.55}_{-0.78}}$$

Top Quark @ Tevatron

- Forward-Backward Asymmetry

$$A_{FB}^{(lab)} = \frac{N(y_t > 0) - N(y_t < 0)}{N(y_t > 0) + N(y_t < 0)}$$

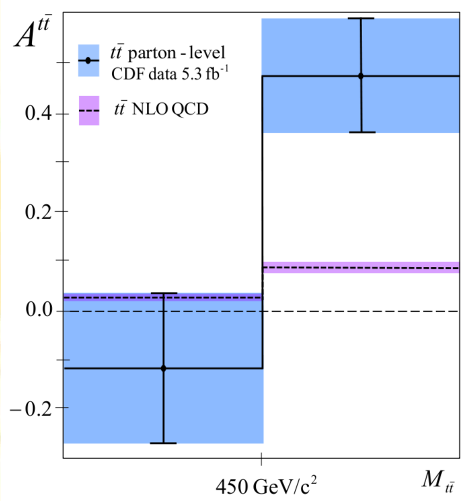
  - At LO top and anti-top quarks have identical distribution. A_{FB} starts at $\mathcal{O}(\alpha_S^3)$
  - CDF ($5.3\, fb^{-1}$)

    $A_{FB}^{(lab)} = (15.0 \pm 5\, stat \pm 2.4\, syst)\,\%$

    $A_{FB}^{(t\bar{t})} = (15.8 \pm 7.2\, stat\, \pm 1.7\, syst)\,\%$

    $A_{FB}^{(t\bar{t})}(M_{tt} < 450\text{GeV}) = (-11.6 \pm 14.6\, stat\, \pm 4.7\, syst)\,\%$

    $A_{FB}^{(t\bar{t})}(M_{tt} > 450\text{GeV}) = (47.5 \pm 10.1\, stat\, \pm 4.9\, syst)\,\%$



  - D0 ($4.3\, fb^{-1}$)

    $A_{FB}^{(t\bar{t})} = (8.0 \pm 4.0\, stat\, \pm 1\, syst)\,\%$ (uncorrected)

  - THEORY

    $A_{FB}^{(lab)} = (5.1 \pm 0.6)\,\%$ (NLO QCD+EW, J. Kühn and G. Rodrigo '98)

    $A_{FB}^{(t\bar{t})} = (7.8 \pm 0.9)\,\%$ (NLO QCD+EW, J. Kühn and G. Rodrigo '98)

    $A_{FB}^{(t\bar{t})} = (7.3^{+1.1}_{-0.7})\,\%$ (NLO + NNLL, Ahrens et al. '10)

Top Quark @ Tevatron

Tevatron searches of physics BSM in top events

- New production mechanisms via new spin-1 or spin-2 resonances: $q\bar{q} \to Z' \to t\bar{t}$ in lepton+jets and all hadronic events. Bumps in the invariant-mass distribution (excluded at 95% CL vector resonances with mass in the range 450–1500 GeV)
- Top charge measurements (recently excluded exotic top-quark with $Q_t = -4/3$)
- Anomalous couplings

$$L = -\frac{g}{\sqrt{2}}\bar{b}\left\{\gamma^{\mu}(V_L P_L + V_R P_R) + \frac{i\sigma^{\mu\nu}(p_t - p_b)_{\nu}}{M_W}(g_L P_L + g_R P_R)\right\} t W_{\mu}^{-}$$

  - From helicity fractions
  - From asymmetries in the final state
- Forward-backward asymmetry
- Non SM Top decays. Search for charged Higgs: $t \to H^+ b \to q\bar{q}' b(\tau\nu b)$
- Search for heavy $t' \to W^+ b$ in lepton+jets (recently excluded t' with $m_{t'} < 360$ GeV)

Top Quark @ Tevatron

Tevatron searches of physics BSM in top events

- New pr … $\rightarrow t\bar{t}$ in
  lepton-
  (excluc … eV)
- Top ch … $/3$)
- Anoma

  $L = -$ …

  - F
  - F
- Forwar

No Evidence
of New Physics so far

- Non SM Top decays. Search for charged Higgs: $t \rightarrow H^+ b \rightarrow q\bar{q}' b(\tau\nu b)$
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Top Quark @ LHC

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Very recently, new results became available from CMS and ATLAS collaborations, for pp collisions at $\sqrt{s} = 7$ GeV, analysing almost 3 pb^{-1}:

- CMS $\boxed{\sigma_{t\bar{t}} = 194 \pm 72(stat.) \pm 24(syst.) \pm 21(lumi.)\ \text{pb}}$ arXiv:1010.5994
  - Only di-lepton channel: e^+e^-, $\mu^+\mu^-$, $e^\pm\mu^\pm$
  - Based on 3.1 pb^{-1} of data (11 events, 2.1 ± 1.0 background)
  - Background (Drell-Yan ...) estimated from data and/or modeled with MADGRAPH
  - Selection efficiency of signal events: MADGRAPH + PYTHIA + CMS detector simulation
- ATLAS $\boxed{\sigma_{t\bar{t}} = 145 \pm 31^{+42}_{-27}\ \text{pb}}$ arXiv:1012.1792
  - Lepton+jet and di-lepton channels
  - Based on 2.9 pb^{-1} (37 events in l+j and 9 in di-l, 12.2 ± 3.9 and 2.5 ± 0.6 background)
  - Background and selection efficiency modeled with MC@NLO, ALPGEN

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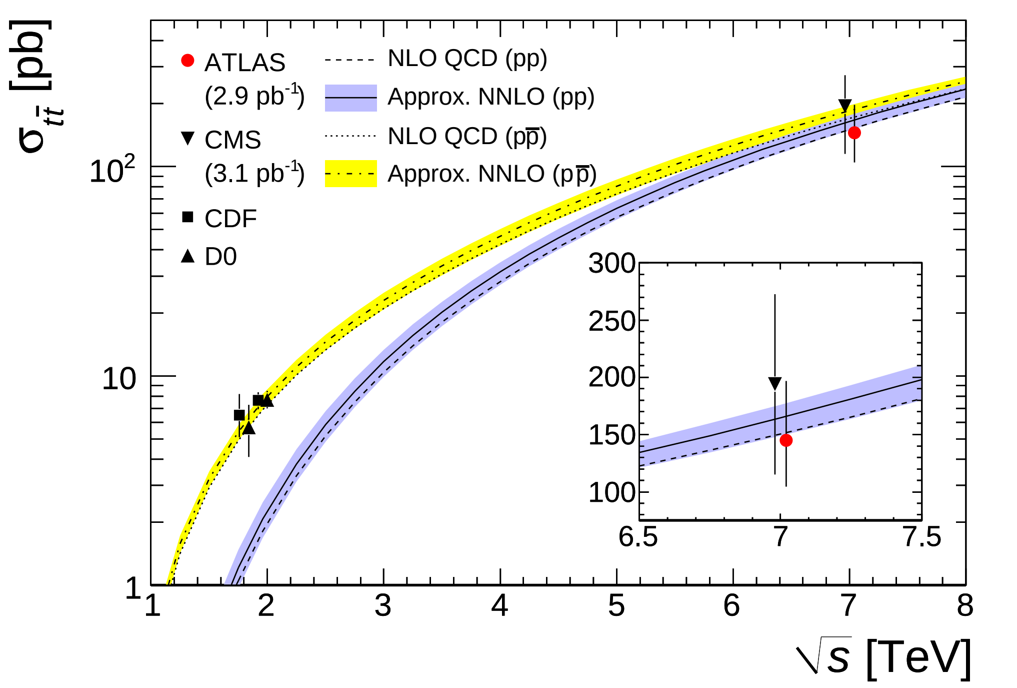

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Top Quark @ LHC: Perspectives

- Cross Section
  - With $100\,\text{pb}^{-1}$ of accumulated data an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 15\%$ is expected (dominated by statistics!)
  - After 5 years of data taking an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 5\%$ is expected
- Top Mass
  - With $1\,\text{fb}^{-1}$ Mass accuracy: $\Delta m_t \sim 1 - 3$ GeV
- Top Properties
  - W helicity fractions and spin correlations with $10\,\text{fb}^{-1}$ $\Longrightarrow$ 1-5%
  - Top-quark charge. With $1\,\text{fb}^{-1}$ we could be able to determine $Q_t = 2/3$ with an accuracy of $\sim 15\%$
- Sensitivity to new physics
  - all the above mentioned points
  - Narrow resonances: with $1\,\text{fb}^{-1}$ possible discovery of a Z' of $M_{Z'} \sim 700$ GeV with $\sigma_{pp\to Z'\to t\bar{t}} \sim 11$ pb

ATLAS CSC book

Top Quark @ LHC: Perspectives

- Cross Section
  - With $100\,\text{pb}^{-1}$ of accumulated data an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 15\%$ is expected (dominated by statistics!)
  - After 5 years of data taking an error of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim 5\%$ is expected
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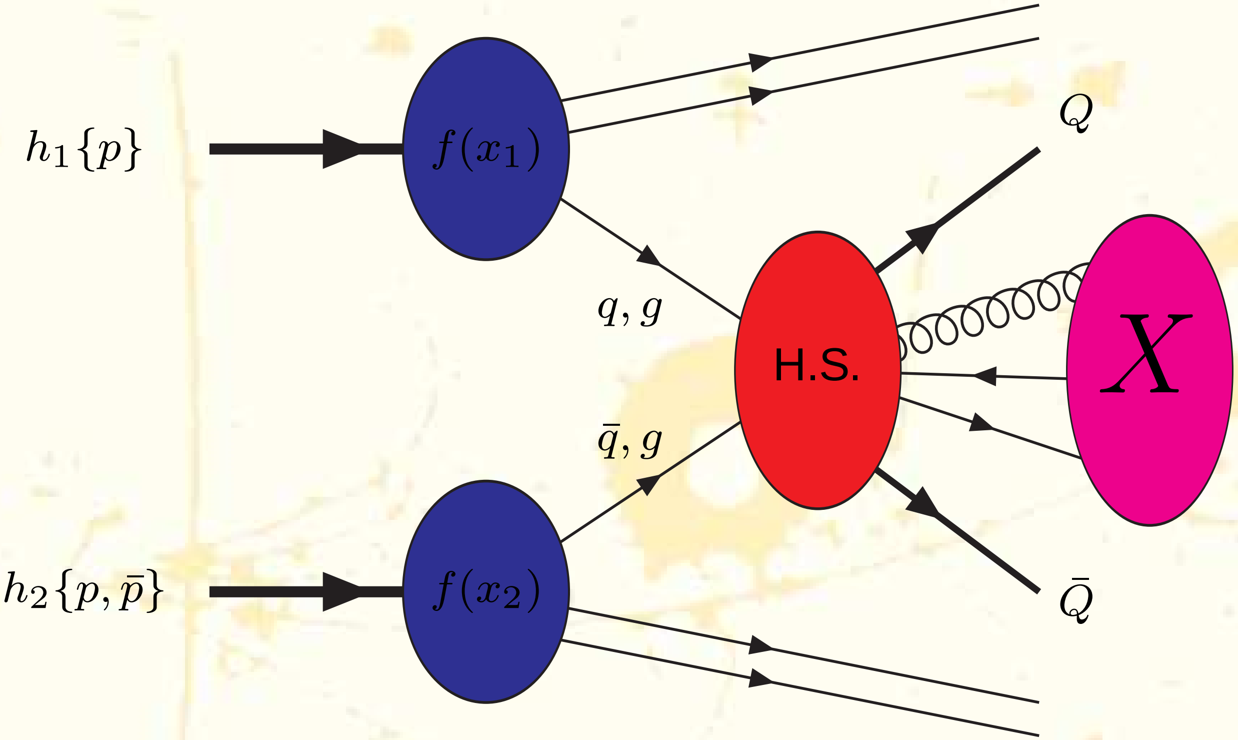
ATLAS CSC book

Theoretical Framework: QCD

Let us consider the heavy-quark production in hadron collisions $h_1 + h_2 \rightarrow Q\bar{Q} + X$

According to the FACTORIZATION THEOREM the process can be sketched as follows:



$$\sigma^{Q\bar{Q}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F) f_{h_2,j}(x_2, \mu_F)\ \hat{\sigma}^{Q\bar{Q}}_{ij}\ (\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R)$$

$$s = (p_{h_1} + p_{h_2})^2\ ,\ \hat{s} = x_1 x_2 s$$

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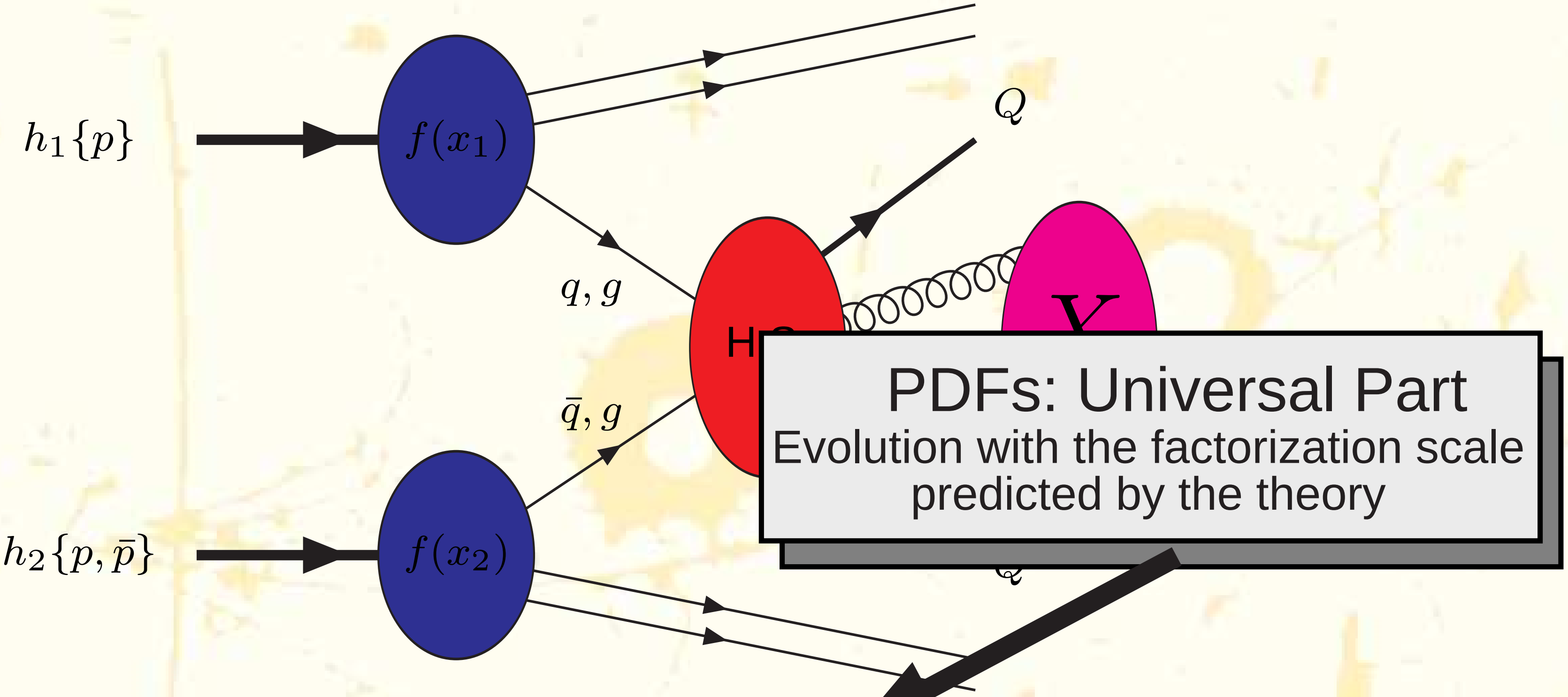

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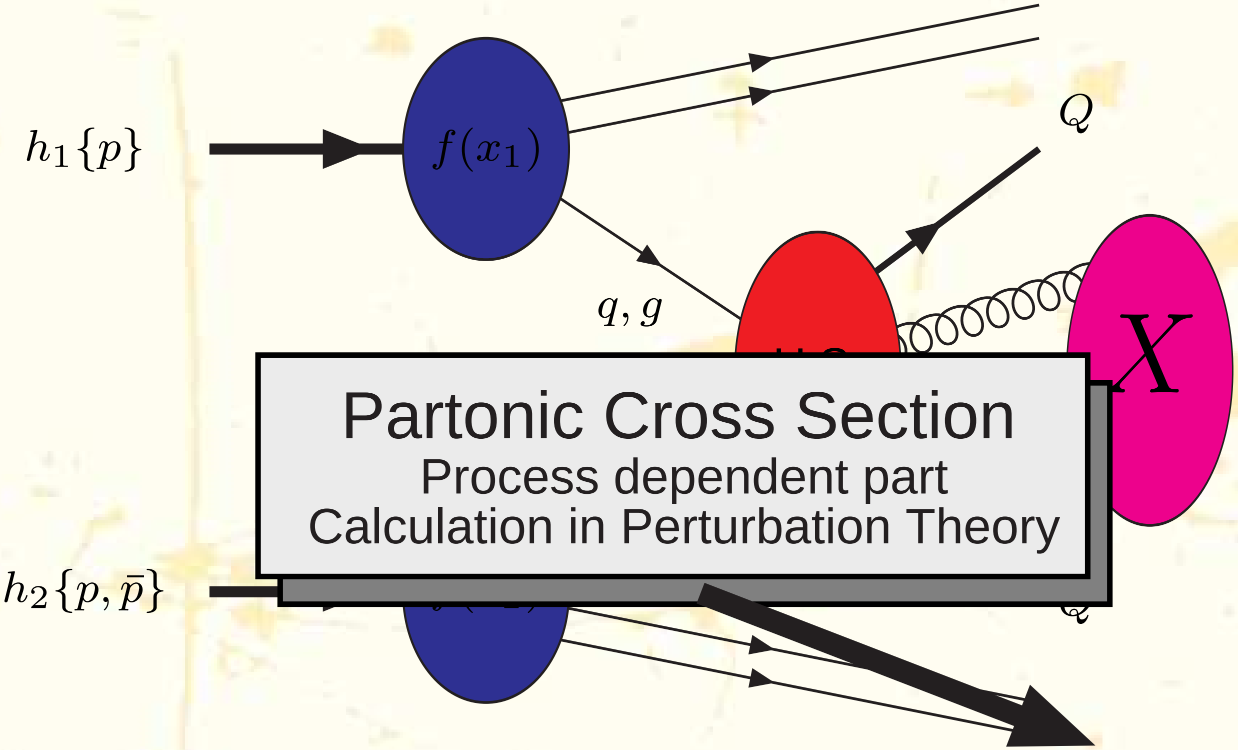

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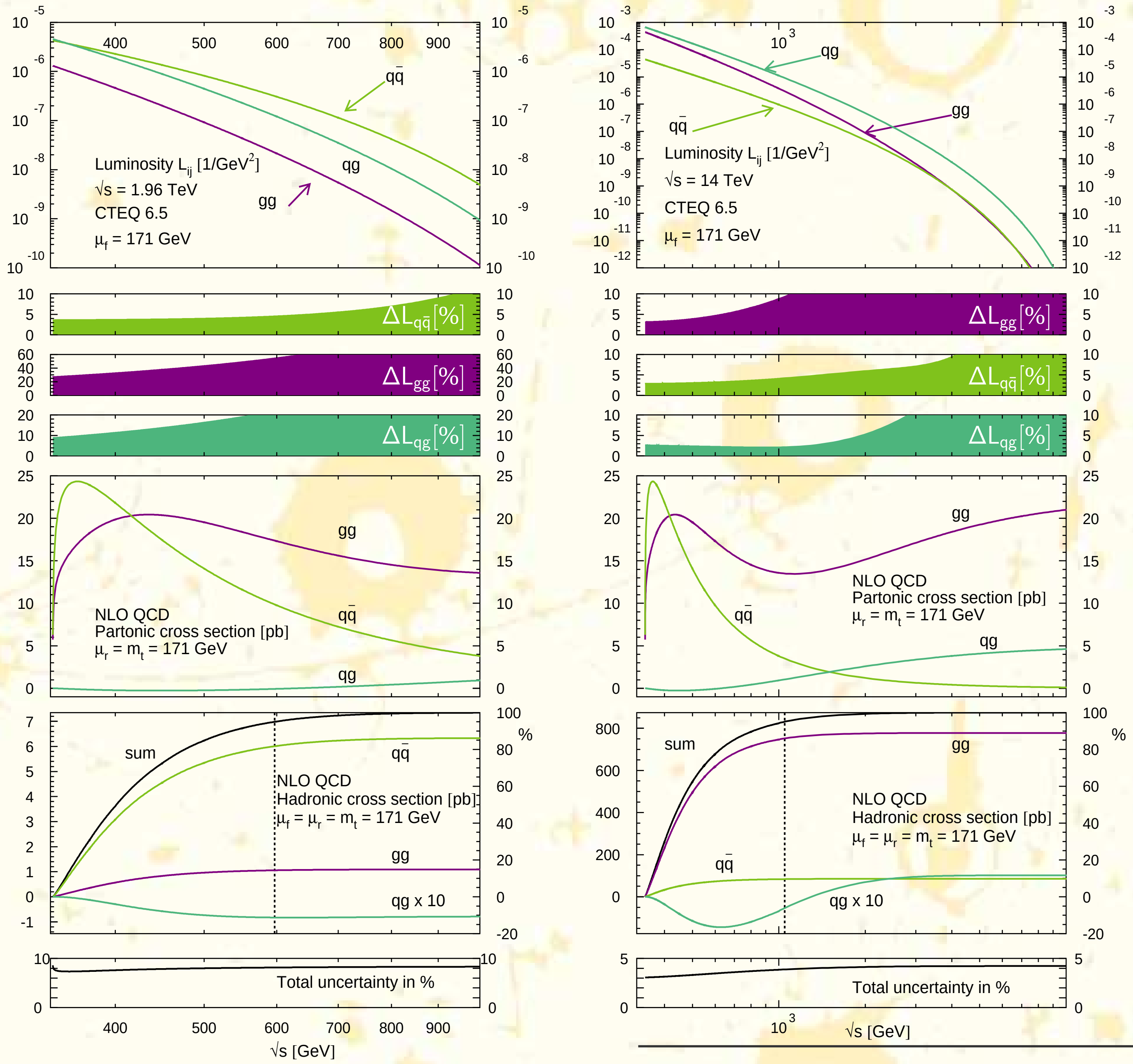
Partonic Luminosity

$$L_{ij} = \frac{1}{s_{had}} \int_{\hat{s}}^{s_{had}} \frac{dx}{x} \times f_{h_1,i}(x/s_{had}, \mu_F) \times f_{h_2,j}(\hat{s}/x, \mu_F)$$

$$\sigma = \int_{4m_t^2}^{s_{max}} ds L(s) \hat{\sigma}(s)$$

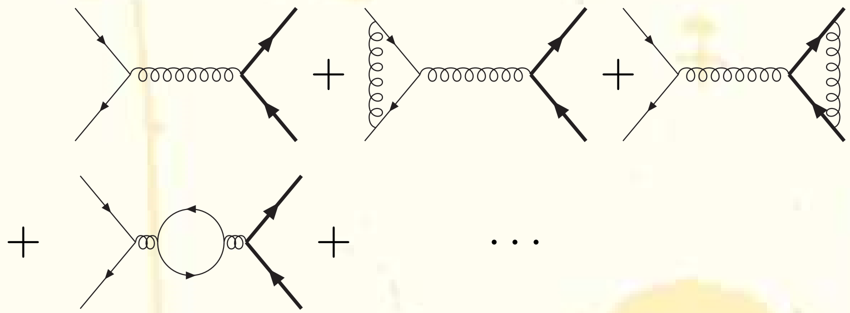


Plots from
Moch and Uwer '08

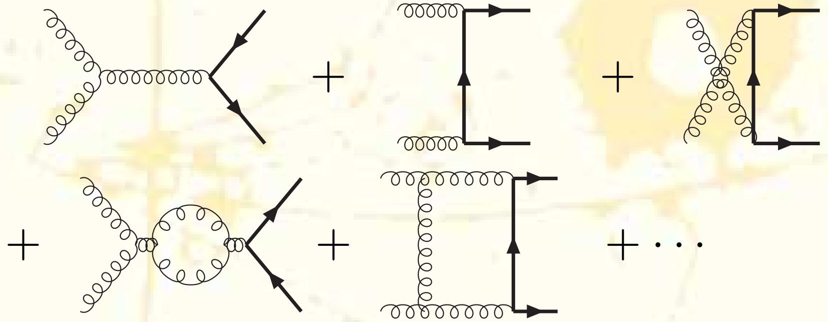
Partonic Cross Section: PT Expansion

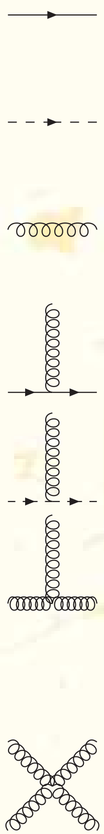
$$\hat{\sigma}_{ij}^{Q\bar{Q}} \propto \left|\mathcal{M}_{ij}^{Q\bar{Q}}\right|^2 = \left|\mathcal{M}_{ij,0}^{Q\bar{Q}} + \alpha_S\,\mathcal{M}_{ij,1}^{Q\bar{Q}} + \alpha_S^2\,\mathcal{M}_{ij,2}^{Q\bar{Q}} + \cdots\right|^2$$

$$\mathcal{M}_{q\bar{q}}^{Q\bar{Q}} =$$



$$\mathcal{M}_{g\bar{g}}^{Q\bar{Q}} =$$





$$\rightarrow \quad \frac{\delta_{ij}(-i\,\not{k}+m)}{k^2+m^2-i\epsilon}$$

$$\rightarrow \quad \frac{\delta_{ab}}{k^2-i\epsilon}$$

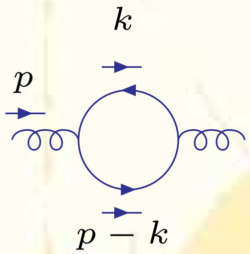
$$\rightarrow \quad \frac{\delta_{\mu\nu}\,\delta_{ab}}{k^2-i\epsilon}$$

$$\rightarrow \quad i g_S\, t^a_{ij}\,\gamma^\mu$$

$$\rightarrow \quad -i g_S\, f^{cab}\, p^\mu$$

$$\rightarrow \quad i g_S\, f^{abc}[\delta_{\mu\nu}(p_\sigma - q_\sigma) + \delta_{\nu\sigma}(q_\mu - k_\mu) + \delta_{\mu\sigma}(k_\nu - p_\nu)]$$

$$\rightarrow \quad -g_S^2[f^{gac} f^{gbd}(2\delta_{\mu\nu}\delta_{\sigma\tau} - \delta_{\mu\sigma}\delta_{\nu\tau} - \delta_{\mu\tau}\delta_{\nu\sigma}) + \cdots$$



$$\propto \frac{\alpha_S}{\pi}\int d^4k\,\frac{tr\{t^a t^b\}\, tr\{\gamma^\mu(-i\,\not{k}+m)\gamma^\nu[i(\not{p}-\not{k})+m]\}}{(k^2+m^2)[(p-k)^2+m^2]}$$

Cross Section: LO (stable top)

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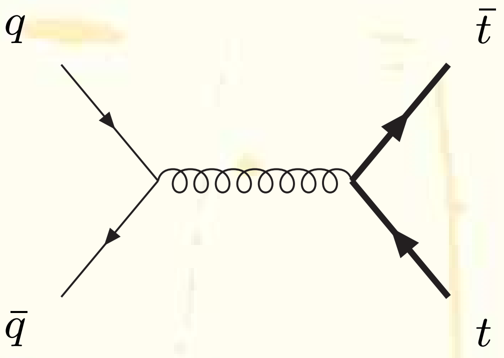
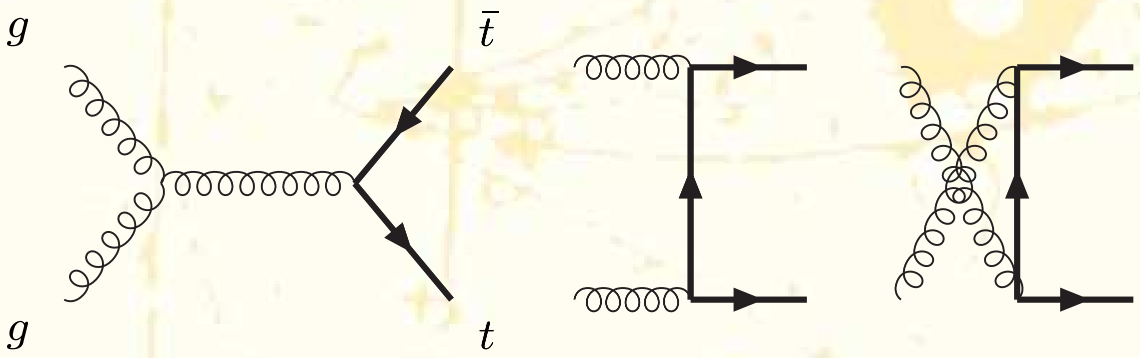
$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at Tevatron

$\sim 85\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at LHC

$\sim 90\%$

$\sigma_{t\bar{t}}^{LO}(LHC, m_t = 171\,\text{GeV}) = 583\,\text{pb} \pm 30\%$

$\sigma_{t\bar{t}}^{LO}(Tev, m_t = 171\,\text{GeV}) = 5.92\,\text{pb} \pm 44\%$

Cross Section: NLO (stable top)

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Fixed Order

- The NLO QCD corrections are quite sizable: + 25% at Tevatron and +50% at LHC. Scale variation $\pm 15\%$.

  Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91; Mangano, Nason, Ridolfi '92; Frixione et al. '95; Czakon and Mitov '08

- Mixed NLO QCD-EW corrections are small: - 1% at Tevatron and -0.5% at LHC.

  Beenakker *et al.* '94 Bernreuther, Fuecker, and Si '05-'08
  Kühn, Scharf, and Uwer '05-'06; Moretti, Nolten, and Ross '06.

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- Even if $\alpha_S \ll 1$ (perturbative region) we can have at all orders $\alpha_S^n \ln^m (1-\rho) \sim \mathcal{O}(1)$

  Resummation $\Longrightarrow$ improved perturbation theory

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All-order Soft-Gluon Resummation

- Leading-Logs (LL)

  Laenen et al. '92-'95; Berger and Contopanagos '95-'96; Catani et al. '96.

- Next-to-Leading-Logs (NLL)

  Kidonakis and Sterman '97; R. B., Catani, Mangano, and Nason '98-'03.

- Next-to-Next-to-Leading-Logs (NNLL)

  Moch and Uwer '08; Beneke et al. '09-'10; Czakon et al. '09; Kidonakis '09; Ahrens et al. '10

NLO+NLL Theoretical Prediction

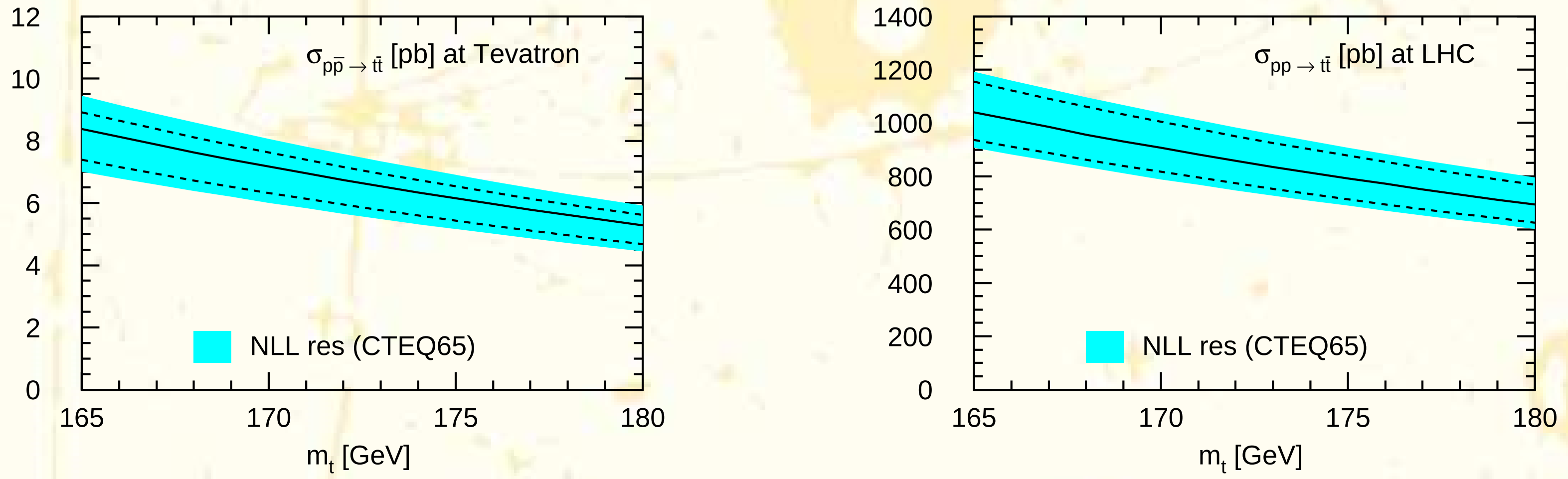
TEVATRON

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171\ {\rm GeV}, {\rm CTEQ6.5}) = 7.61\ {}^{+0.30(3.9\%)}_{-0.53(6.9\%)}\ ({\rm scales})\ {}^{+0.53(7\%)}_{-0.36(4.8\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

LHC

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171\ {\rm GeV}, {\rm CTEQ6.5}) = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ ({\rm scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ ({\rm PDFs})\ \ {\rm pb}$$

M. Cacciari, S. Frixione, M. Mangano, P. Nason, and G. Ridolfi, JHEP 0809:127,2008

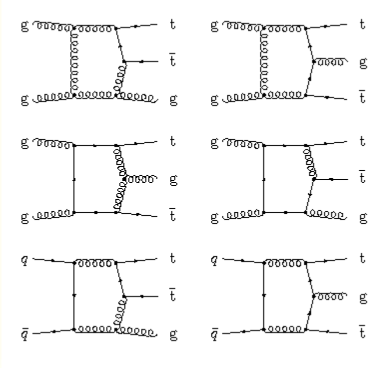


S. Moch and P. Uwer, Phys. Rev. D **78** (2008) 034003

Distributions

Distributions

- $p\bar{p} \rightarrow t\bar{t} + 1\,jet$
  - Important for a deeper understanding of the $t\bar{t}$ prod (possible structure of the top-quark)
  - Technically complex involving multi-leg NLO diagrams



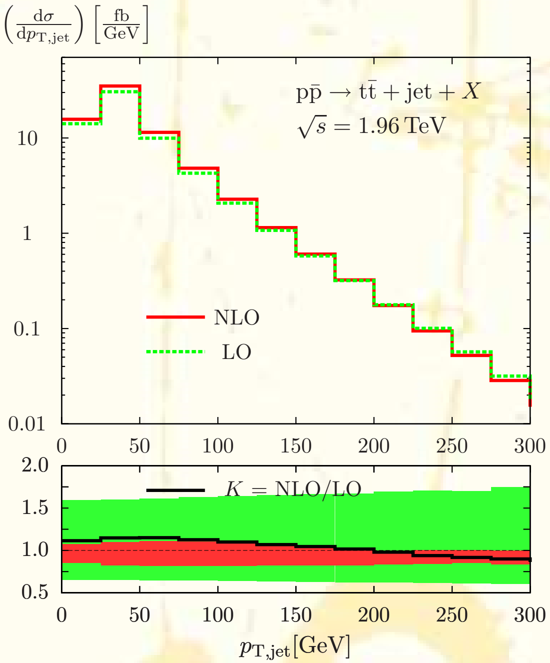
$\sigma_{t\bar{t}+j}$ (LHC) = 376.2$^{+17}_{-48}$ pb

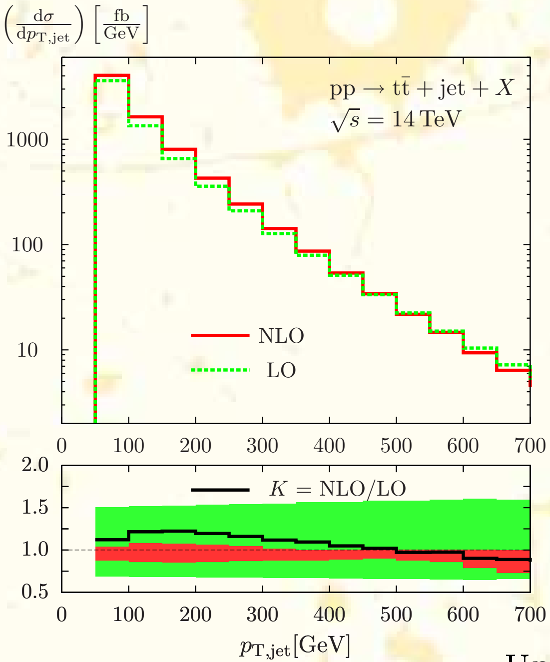
$\sigma_{t\bar{t}+j}$ (Tev) = 1.79$^{+0.16}_{-0.31}$ pb

$\sigma^{CDF}_{t\bar{t}+j}$ = 1.6 ± 0.2 (stat) $\pm\, 0.5$ (syst) pb

confirmed by G. Bevilacqua, M. Czakon, C.G. Papadopoulos, M. Worek, Phys.Rev.Lett. 104 (2010) 162002
K. Melnikov and M. Schulze, Nucl.Phys. B840 (2010) 129-159

$t\bar{t} + 2j$

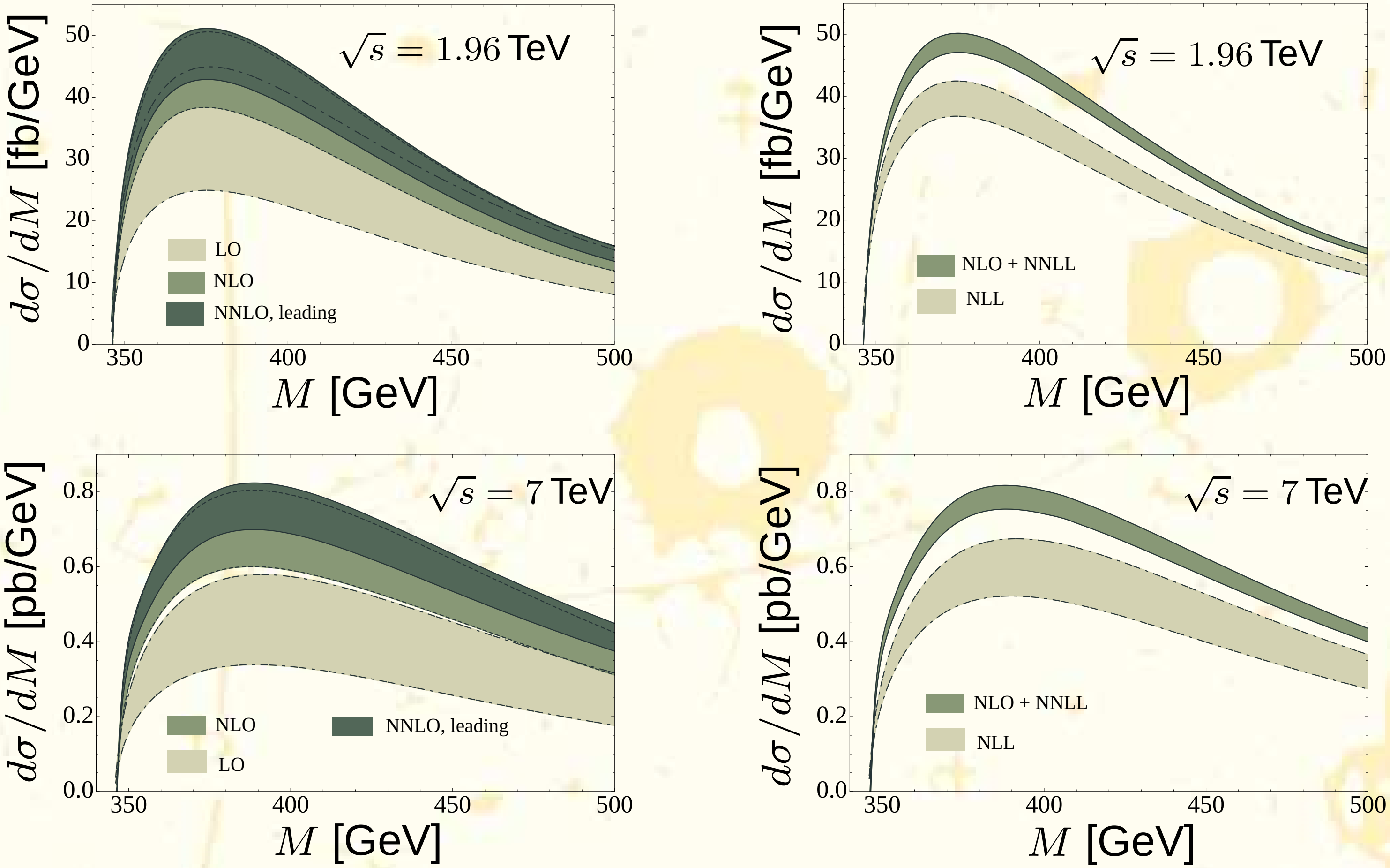




S. Dittmaier, P. Uwer and S. Weinzierl,
Eur. Phys. J. C **59** (2009) 625

Distributions

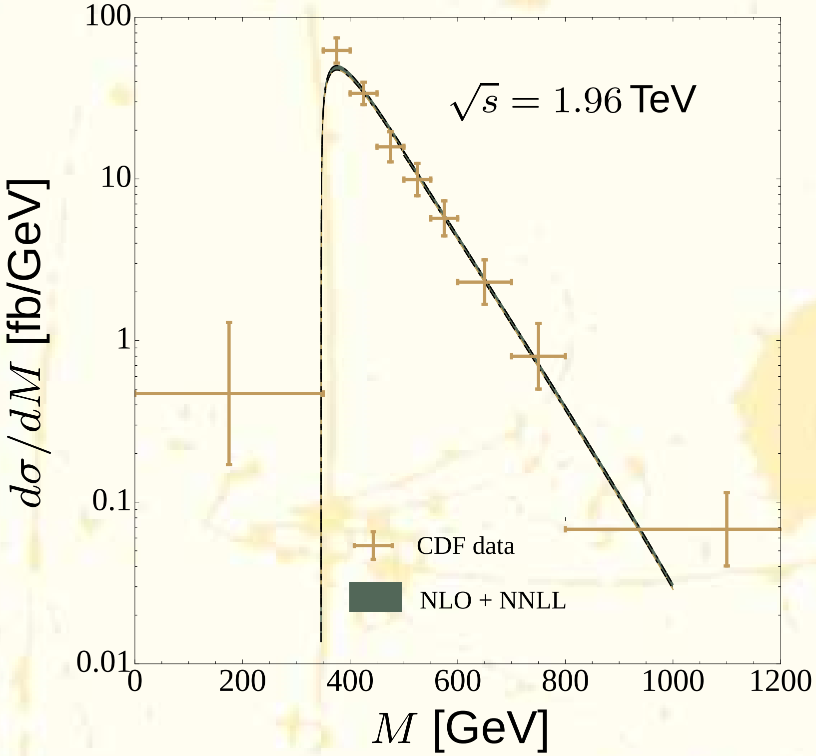
- Invariant mass distributions: fixed-order and resummed (SCET) PT

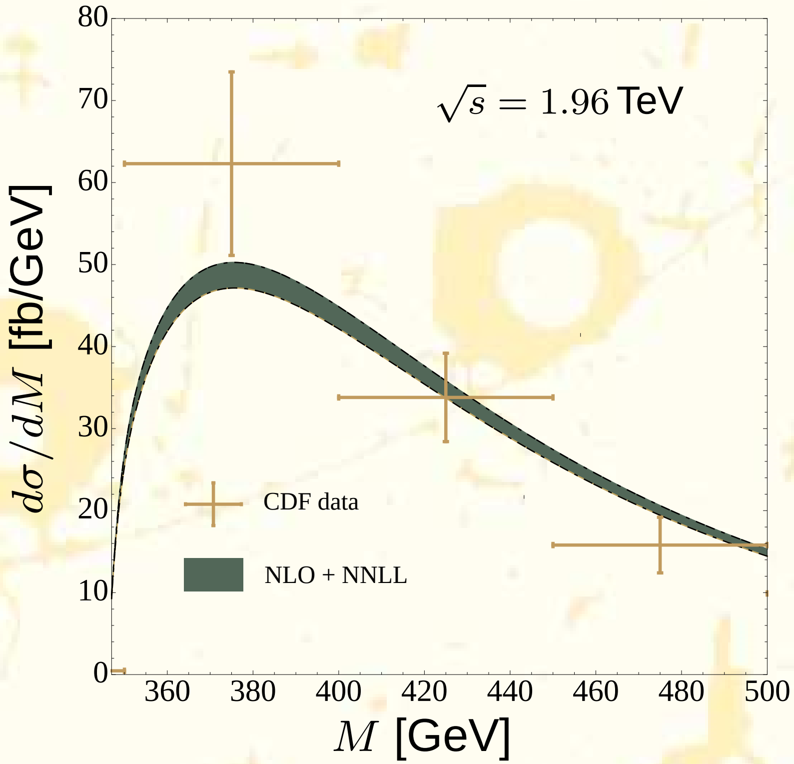


V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, and L. L. Yang, JHEP 1009 (2010) 097

Distributions

- Invariant mass distributions:
  comparison with CDF data (Phys. Rev. Lett. 102, 222003 (2009))





V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, and L. L. Yang, JHEP 1009 (2010) 097

NLO with decay Products: Fact. Correctons

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- The calculations shown so far consider a stable top (anti-top) quark. Advantage: reduction in the complexity of a NLO calculation
- In “reality” the out states are leptons and hadrons $\Longrightarrow$ experiments put cuts on leptons and hadrons. Desirable a description of the process in terms of actual out states

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- In "reality" the out states are leptons and hadrons $\Longrightarrow$ experiments put cuts on leptons and hadrons. Desirable a description of the process in terms of actual out states
- Factorizable corrections do not mix production and decay stages!

$$\lim_{\Gamma_t/m_t \to 0} \frac{1}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} = \frac{\pi}{m_t \Gamma_t} \delta(p_t^2 - m_t^2)$$

$$+\,\mathcal{O}(\Gamma_t/m_t)$$

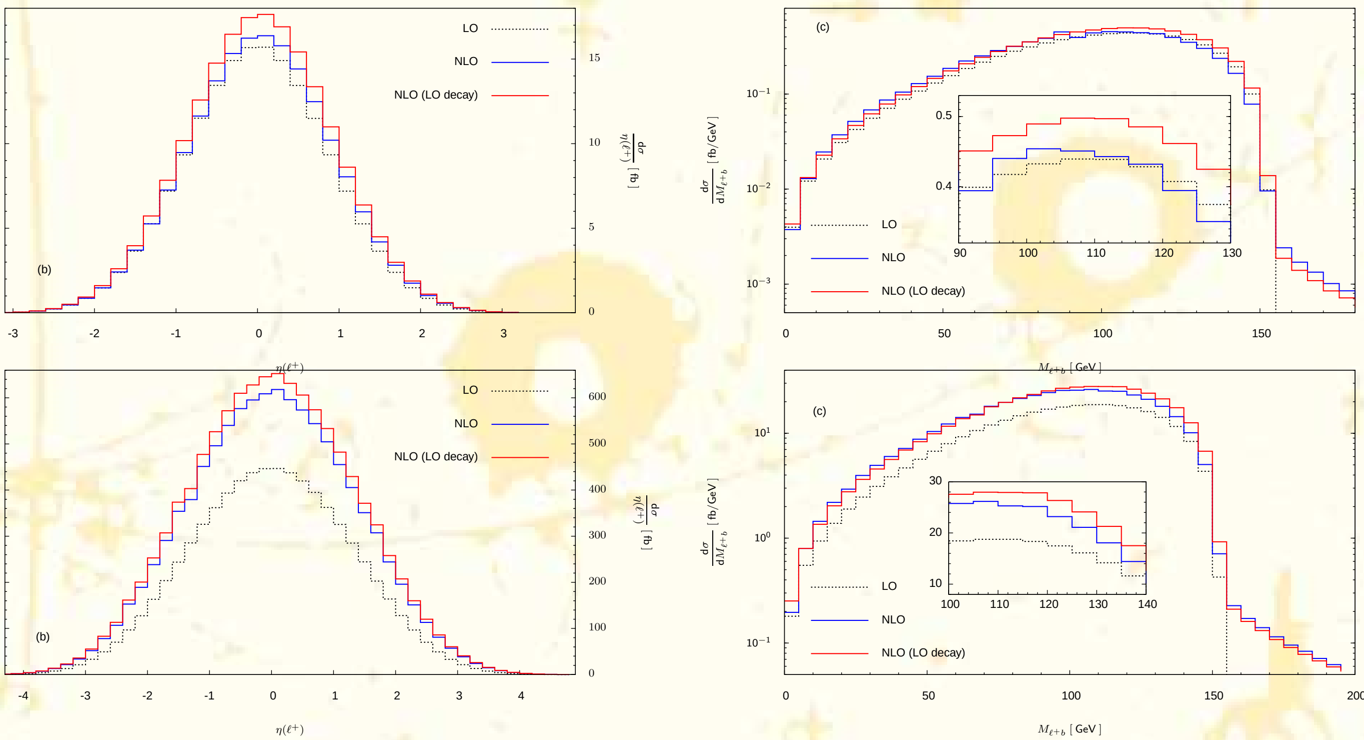
- The non-factorizable corrections do not decouple, but in sufficiently inclusive observables they become small: $\sim \mathcal{O}(\Gamma_t/m_t)$ Fadin, Khoze, Martin '94; Aeppli, van Oldenborgh, Wyler '94; Melnikov, Yakovlev '94; Beenakker, Berends, Chapovsky '99
- One can keep track of the spin of the top and anti-top and compute spin correlations

NLO with decay Products: Fact. Correctons

- NLO corrections to various kinematic distributions for Tevatron and LHC (Bernreuther and Si include also EW corrections)

Tevatron

LHC



- The study can be extended at NNLO

K.Melnikov and M. Schulze, JHEP 0908 (2009) 049

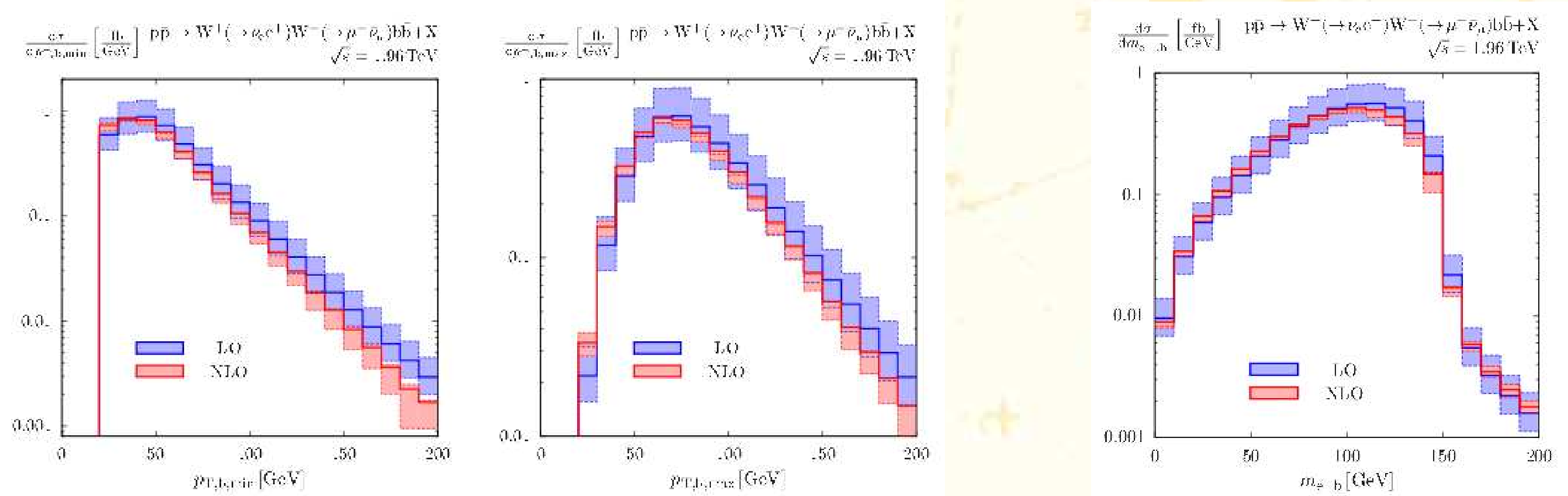
W. Bernreuther and Z. Si, Nucl.Phys. B837 (2010) 90-121

NLO with decay Products: Full Calculation

NLO with decay Products: Full Calculation

Finally, very recently two groups computed the full set of NLO corrections to $\boxed{pp \to WWbb}$

- Calculation technically challenging (~ 1500 Feynman diagrams, up to 6 external legs)
- The direct calculation confirms that for inclusive quantities the non-factorizable corrections are of $\mathcal{O}(\Gamma_t/m_t)$
- Possibility to study many distributions imposing realistic experimental cuts



(Plots S. Pozzorini's ZH talk)

A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, arXiv:1012.3975

G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos, and M. Worek, arXiv:1012.4230

Measurement Requirements for $\sigma_{t\bar{t}}$

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Experimental requirements for $\sigma_{t\bar{t}}$:

- Tevatron $\Delta\sigma/\sigma \sim 9\% \Longrightarrow$ already < $(\Delta\sigma/\sigma)_{TH}$
- LHC (14 TeV, high luminosity) $\Delta\sigma/\sigma \sim 5\% \ll (\Delta\sigma/\sigma)_{TH}$!!

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Different groups presented approximated higher-order results for $\sigma_{t\bar{t}}$

- Including scale dep at NNLO, NNLL soft-gluon contributions, Coulomb corrections

$$\sigma_{t\bar{t}}^{\rm NNLOappr}({\rm Tev}, m_t = 173 \text{ GeV}, {\rm MSTW2008}) = 7.04 \,^{+0.24}_{-0.36} \,({\rm scales})\,^{+0.14}_{-0.14} \,({\rm PDFs}) \;\; {\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NNLOappr}({\rm LHC}, m_t = 173 \text{ GeV}, {\rm MSTW2008}) = 887 \,^{+9}_{-33} \,({\rm scales})\,^{+15}_{-15} \,({\rm PDFs}) \;\; {\rm pb}$$

Kidonakis and Vogt ’08; Moch and Uwer ’08; Langenfeld, Moch, and Uwer ’09

- Integration of the Invariant mass distribution at NLO+NNLL

$$\sigma_{t\bar{t}}^{\rm NLO+NNLL}({\rm Tev}, m_t = 173.1 \text{ GeV}, {\rm MSTW2008}) = 6.48 \,^{+0.17}_{-0.21} \,({\rm scales})\,^{+0.32}_{-0.25} \,({\rm PDFs}) \;\; {\rm pb}$$

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V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak, L. L. Yang, arXiv:1006.4682

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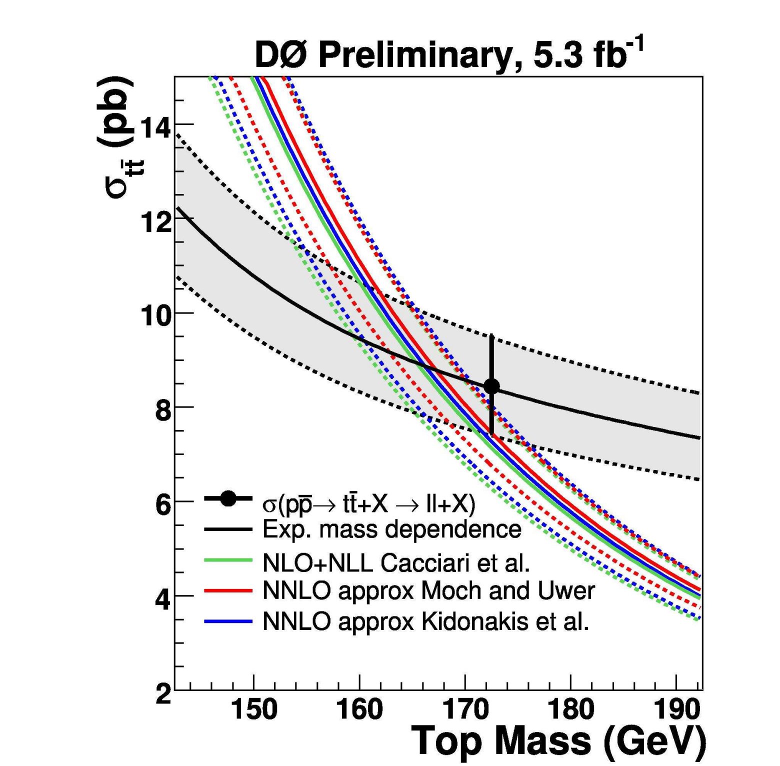

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Langenfeld, Moch, and Uwer '09

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V. Ahrens et al. '10

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- Virtual Corrections
  - two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - interference of one-loop diagrams

Körner et al. ’05-’08; Anastasiou and Aybat ’08

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- Real Corrections
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$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\mathcal{A}_2^{(2\times0)} = N_c C_F\left[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\right]$$

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  R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

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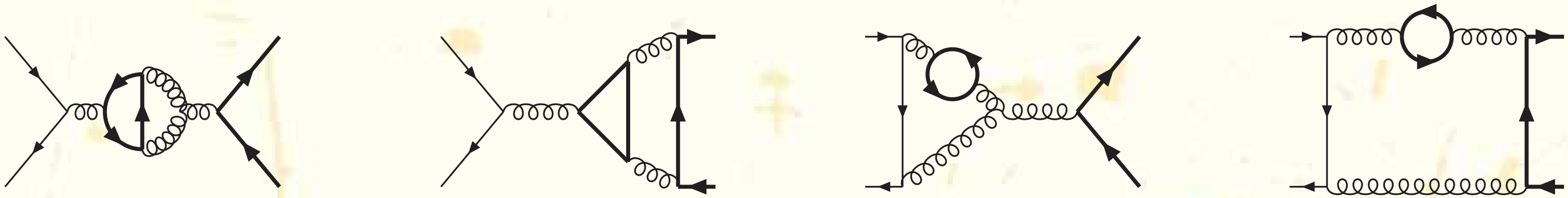
- The poles of $\mathcal{A}_2^{(2\times 0)}$ (and therefore of B and C) are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

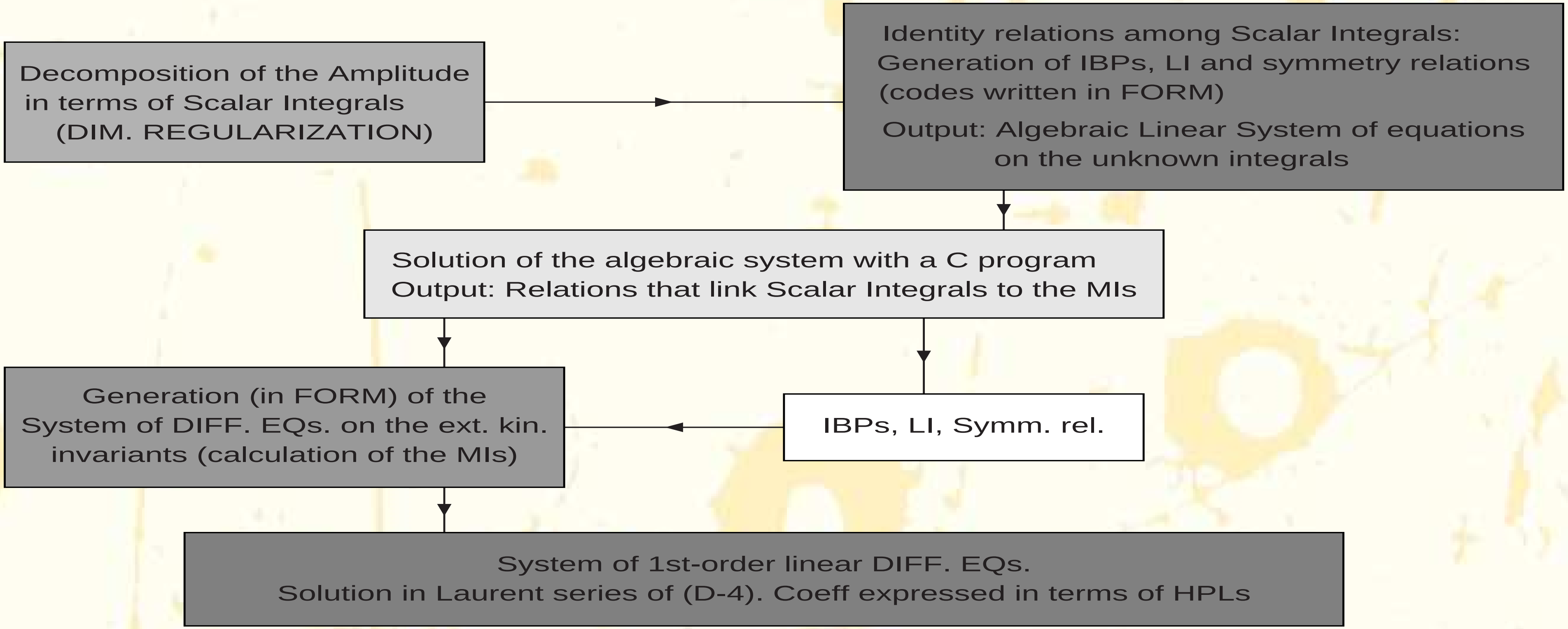
- D_i, E_i, F_i come from the corrections involving a closed (light or heavy) fermionic loop:



- A the leading-color coefficient, comes from the planar diagrams:

- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs

Laporta Algorithm and Diff. Equations



PUBLIC PROGRAMS

- AIR – Maple package
  (C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046)
- FIRE – Mathematica package (A. V. Smirnov, JHEP **0810** (2008) 107)
- REDUZE – REDUZE2 C++/GiNaC packages
  (C. Studerus, Comput. Phys. Commun. **181** (2010) 1293;
  A. von Manteuffel and C. Studerus, in preparation)

Master Integrals for N_l and N_h

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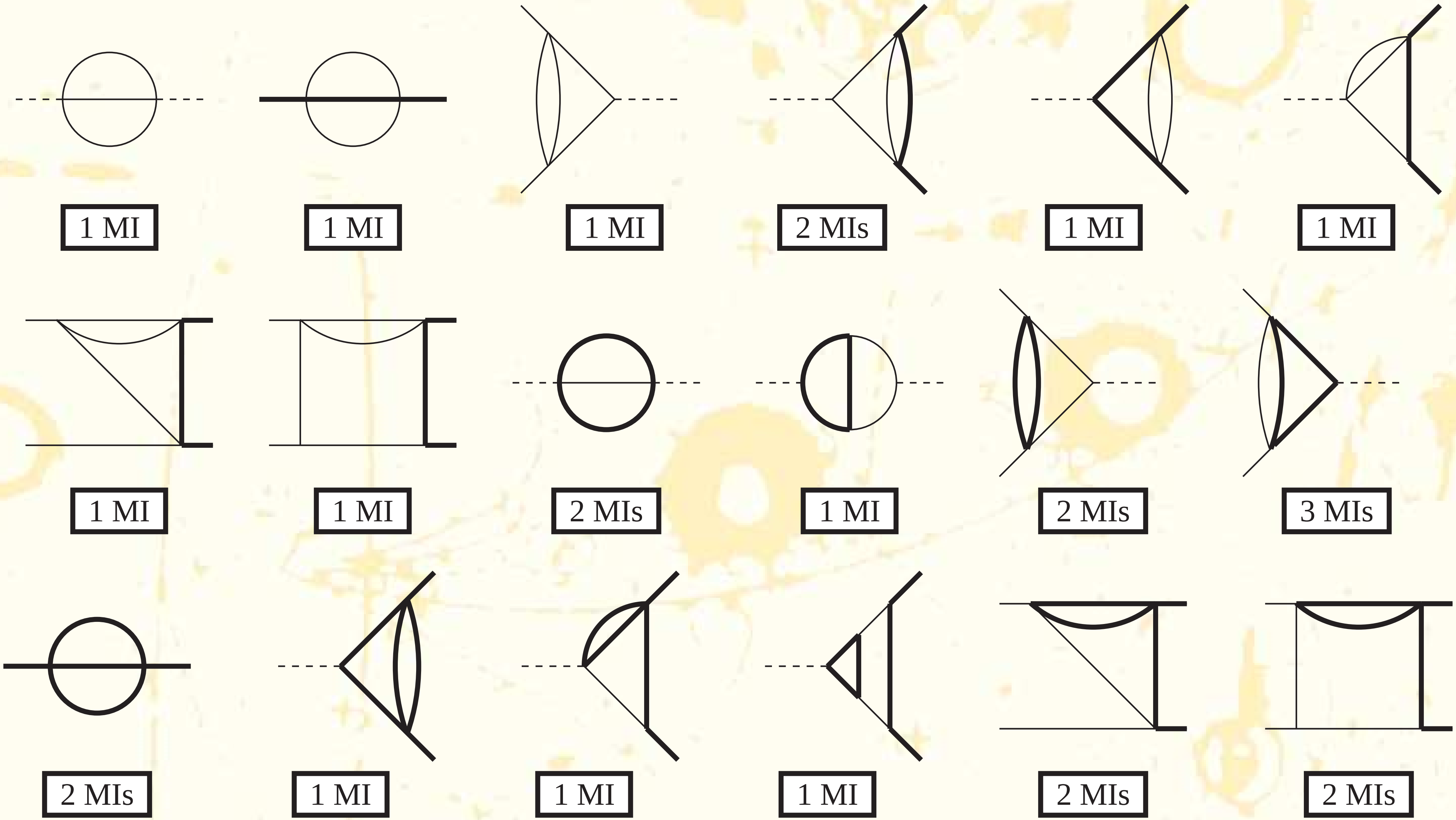

18 irreducible two-loop topologies (26 MIs)

R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.

Master Integrals for the Leading Color Coeff

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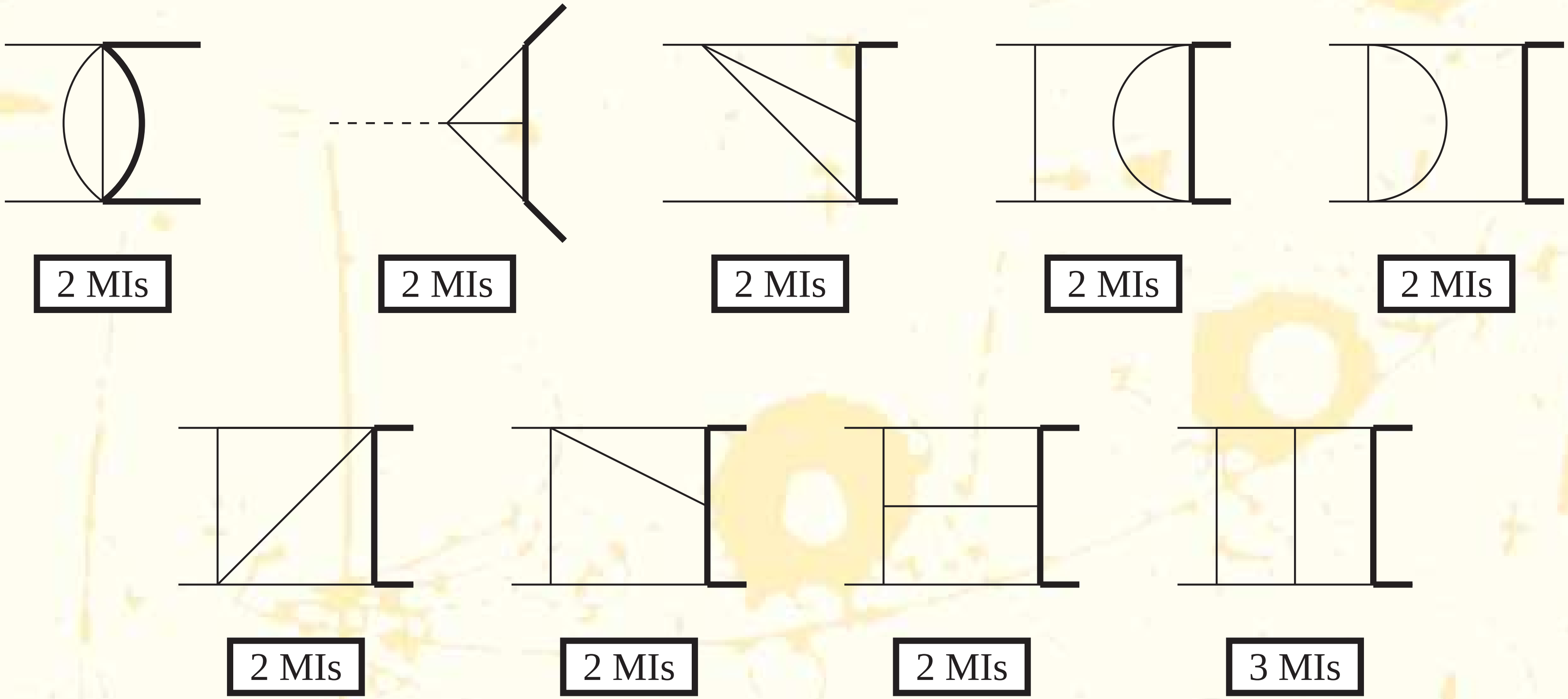

For the leading color coefficient there are 9 additional irreducible topologies (19 MIs)

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

Differential Equations for the MIs

Differential Equations for the MIs

For a given topology, when the system of identities is not reducible, we have a small number of MI's. In the case of three-point functions:

$$F_i(Q^2, p_1^2, p_2^2) = \int d^D k_1 d^D k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{D_1^{m_1} \cdots D_t^{m_t}}$$

Using all the identity-relations (IBP's, LI, Sym) we can construct the following system of first-order linear differential equations:

$$\frac{dF_i}{dQ^2} = \sum_j h_j(Q^2, m^2)\, F_j + \Omega_i$$

where $i, j = 1, ..., N_{MIs}$.

Ω_i | This term involves integrals of the class $I_{t-1,r,s}$ (sub-topologies) to be considered KNOWN

V. Kotikov, *Phys. Lett.* **B254** (1991) 158; **B259** (1991) 314; **B267** (1991) 123.
E. Remiddi, *Nuovo Cim.* **110A** (1997) 1435.

Diff. Eqs. for the Crossed Vertex Diagram

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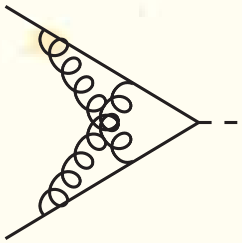
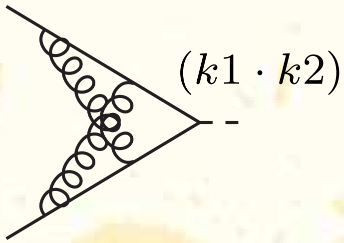
The reduction process $\rightarrow$ to 2 MI's. We choose ($a = m^2$):

$F_1(\epsilon, a, Q^2) =$ 

$F_2(\epsilon, a, Q^2) =$ 

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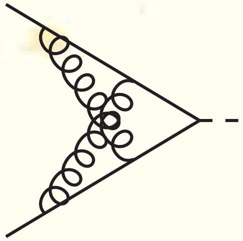
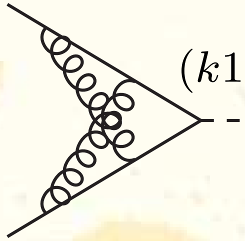
$F_1(\epsilon, a, Q^2) =$ 

$F_2(\epsilon, a, Q^2) =$ 

The system of first-order linear differential equations is:

$$\frac{dF_1}{dQ^2} = -(1+2\epsilon)\left[\frac{1}{Q^2} + \frac{(1-2\epsilon)}{(Q^2+4a)}\right] F_1 - \frac{2\epsilon}{a}\left[\frac{1}{Q^2} - \frac{(1-2\epsilon)}{(Q^2+4a)}\right] F_2 + \Omega^{(1)}$$

$$\frac{dF_2}{dQ^2} = \epsilon\, F_1 - \frac{(1-2\epsilon)}{2}\left[\frac{1}{Q^2} + \frac{1}{(Q^2+4a)}\right] F_2 + \Omega^{(2)}$$

where $\Omega^{(i)}$ are combinations of simpler MI's.

R. B., E. Remiddi, P. Mastrolia, *Nucl. Phys.* **B661** (2003) 289.

Solution as a Laurent series in ϵ

- We look for a solution expanded in Laurent series of ϵ:

$$F_1(\epsilon, a, Q^2) = \sum_{i=-2}^{0} \epsilon^i F_i^{(1)}(a, Q^2) + \mathcal{O}(\epsilon) \qquad F_2(\epsilon, a, Q^2) = \sum_{i=-2}^{0} \epsilon^i F_i^{(2)}(a, Q^2) + \mathcal{O}(\epsilon)$$

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- The homogeneous system at $\epsilon = 0$ ($D = 4$) decouples:

$$\frac{df_1(a,y)}{dy} = -\left[\frac{1}{y} + \frac{1}{(y+4a)}\right] f_1(a,y)$$

$$\frac{df_2(a,y)}{dy} = -\frac{1}{2}\left[\frac{1}{y} + \frac{1}{(y+4a)}\right] f_2(a,y)$$

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- The solution of the homogeneous system at $\epsilon = 0$:

$$f_1(a,y) = \frac{k_1}{y(y+4a)} \qquad f_2(a,y) = \frac{k_2}{\sqrt{y(y+4a)}}$$

Euler's Method (variation of constants)

By means of the Euler's method of the variation of the constants k_1 and k_2, we find, order by order in ϵ, the solution of the non-homogeneous system:

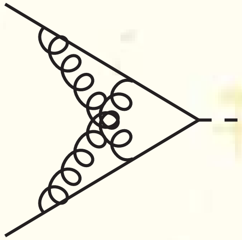
$$
\begin{aligned}
F_i^{(1)}(a,Q^2) &= \frac{1}{Q^2(Q^2+4a)}\left\{\int^{Q^2} dy\, y(y+4a)\left[-2\left(\frac{1}{y}+\frac{1}{(y+4a)}\right)F_{i-1}^{(1)}(a,y)\right.\right. \\
&\quad + \frac{4}{(y+4a)}F_{i-2}^{(1)}(a,y) - \frac{2}{a}\left(\frac{1}{y}-\frac{1}{(y+4a)}\right)F_{i-1}^{(2)}(a,y) \\
&\quad \left.\left. - \frac{4}{a(y+4a)}F_{i-2}^{(2)}(a,y) + \Omega_i^{(1)}(a,y)\right] + k_i^{(1)}\right\} \\
F_i^{(2)}(a,Q^2) &= \frac{1}{\sqrt{Q^2(Q^2+4a)}}\left\{\int^{Q^2} dy\, \sqrt{y(y+4a)}\left[F_{i-1}^{(1)}(a,y)\right.\right. \\
&\quad \left.\left. + \left(\frac{1}{y}-\frac{1}{(y+4a)}\right)F_{i-1}^{(2)}(a,y) + \Omega_i^{(2)}(a,y)\right] + k_i^{(2)}\right\}
\end{aligned}
$$

For the determination of $k_i^{(1)}$ and $k_i^{(2)}$ we have to impose initial conditions.

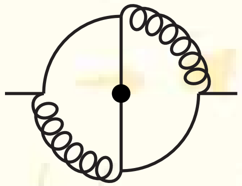
Initial conditions

The vertex topology is not singular at $Q^2 = 0$ (the only singularity is the threshold $Q^2 = -4a$). We can perform the limit directly in the integral getting:

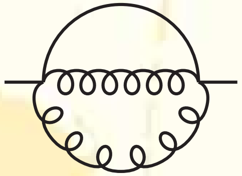
$$F_1(\epsilon, a, Q^2 = 0) = \lim_{Q^2 \to 0}$$



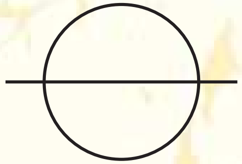
$$=$$



$$= -\frac{3\epsilon(2-3\epsilon)(1-3\epsilon)}{4a^3(1-4\epsilon)(1+2\epsilon)}$$

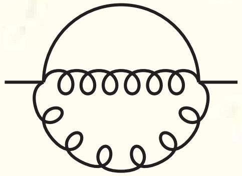


$$+ \frac{3(2-3\epsilon)(1-3\epsilon)}{64a^3\epsilon}$$

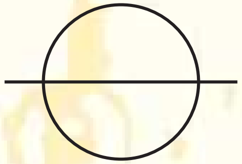


$$+ \frac{(1-\epsilon)^2(9+3\epsilon-160\epsilon^2-196\epsilon^3)}{64a^4\epsilon(1-2\epsilon)(1+2\epsilon)} T^2(\epsilon, a)$$

$$F_2(\epsilon, a, Q^2 = 0) = \frac{(2-3\epsilon)(1-3\epsilon)}{8a^2\epsilon}$$



$$+ \frac{(2-3\epsilon)(1-3\epsilon)}{32a^2\epsilon^2}$$



$$+ \frac{(1-\epsilon)^2(3-15\epsilon+16\epsilon^2)}{32a^3\epsilon^2(1-2\epsilon)} T^2(\epsilon, a)$$

Change of variable and HPL's

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It turns out to be a very convenient choice to change the variable Q^2 in x defined as follows:

$$x = \frac{\sqrt{Q^2+4a} - \sqrt{Q^2}}{\sqrt{Q^2+4a} + \sqrt{Q^2}}$$

with which

$$Q^2 = a\frac{(1-x)^2}{x} \qquad (Q^2+4a) = a\frac{(1+x)^2}{x}$$

In terms of x the solutions of the homogeneous system are:

$$f_1(a,x) \quad = \quad -\frac{k_1}{4}\left[\frac{1}{(1-x)} + \frac{1}{(1+x)} - \frac{1}{(1-x)^2} - \frac{1}{(1+x)^2}\right]$$

$$f_2(a,x) \quad = \quad \frac{k_2}{2}\left[\frac{1}{(1-x)} - \frac{1}{(1+x)}\right]$$

With this choice the basis for the calculation is constituted by HPL's of x:

$$F_i^{(1)} = \int^x dt \left\{\frac{1}{t}\,;\,\frac{1}{(1-t)}\,;\,\frac{1}{(1+t)}\right\}\left\{F_{i-1}^{(1)}(a,x), F_{i-1}^{(2)}(a,x), \Omega_i^{(1)}(a,x)\right\}$$

Harmonic Polylogarithms (HPLs)

- Weight $= 1$

$$H(0,x) = \ln x \quad H(-1,x) = \int_0^x \frac{dt}{1+t} = \ln(1+x) \quad H(1,x) = \int_0^x \frac{dt}{1-t} = -\ln(1-x)$$

- Weight > 1
  If $\vec{a} = \vec{0}$ we define $H(\vec{0},x) = \frac{1}{\omega!} \ln^\omega x$. If $\vec{a} \neq \vec{0}$:

$$H(\vec{a},x) = \int_0^x dt\, f(a_1,x)\, H(\vec{a}_{\omega-1},t) \quad \frac{d}{dx} H(\vec{a},x) = f(a_1,x)\, H(\vec{a}_{\omega-1},x)$$

- The Algebra: $\omega_{\vec{a}} \times \omega_{\vec{b}} = \omega_{\vec{a}} \times \omega_{\vec{b}}$

$$H(\vec{a},x)\, H(\vec{b},x) = \sum_{\vec{c}=\vec{a}\uplus\vec{b}} H(\vec{c},x)$$

- Integration by Parts

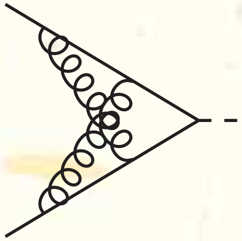
$$H(m_1,...,m_q,x) = H(m_1,x)H(m_2,...,m_q,x) - ... + (-1)^{q+1} H(m_q,...,m_1,x)$$

- Connection with Nielsen's polylog and Spence functions:

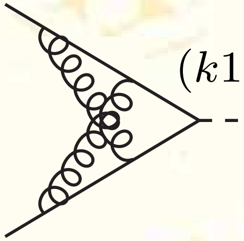
$$S_{n,p}(x) = H(\vec{0}_n,\vec{1}_p,x) \quad Li_n(x) = H(\vec{0}_{n-1},1,x)$$

A.B.Goncharov, *Math. Res. Lett.* **5** (1998), 497-516.
E. Remiddi and J. A. M. Vermaseren, *Int. J. Mod. Phys.* **A15** (2000) 725.

Solution for the MIs of the Crossed



$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-1}^{0} \epsilon^i R_i + \mathcal{O}(\epsilon)$$



$(k1 \cdot k2)$

$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} S_0 + \mathcal{O}(\epsilon)$$

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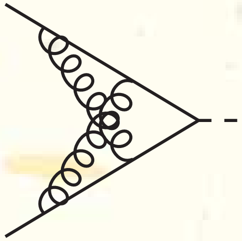

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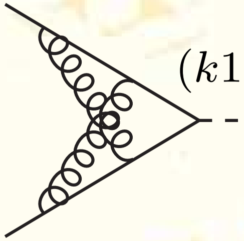
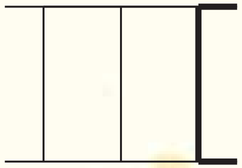


$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} S_0 + \mathcal{O}(\epsilon)$$

$$\begin{aligned}
a^2 R_{-1} &= -\frac{1}{4}\left[\frac{1}{(1-x)} - \frac{1}{(1-x)^2} + \frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right][\zeta(3)+\zeta(2)H(0,x) + 2H(0,0,0,x) \\
&\quad +2H(0,1,0,x) - 2H(0,-1,0,x)] \\
a^2 R_0 &= -\frac{1}{4}\left[\frac{1}{(1-x)} - \frac{1}{(1-x)^2} + \frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right]\left[\frac{37\zeta^2(2)}{10} + H(0,x) - 4H(-1,x) + \zeta(3)H(1,x)\right. \\
&\quad -2\zeta(2)H(0,0,x) - 4\zeta(2)H(-1,0,x) - 2\zeta(2)H(0,-1,x) - 2\zeta(2)H(0,1,x) + 4\zeta(2)H(1,0,x) \\
&\quad +12H(0,0,0,0,x) + 8H(-1,0,-1,0,x) - 8H(-1,0,0,0,x) - 8H(-1,0,1,0,x) \\
&\quad +20H(0,-1,-1,0,x) - 16H(0,-1,0,0,x) - 12H(0,-1,1,0,x) - 24H(0,0,-1,0,x) \\
&\quad -16H(0,0,1,0,x) - 12H(0,1,-1,0,x) + 8H(0,1,0,0,x) + 4H(0,1,1,0,x) \\
&\quad \left.-8H(1,0,-1,0,x) + 8H(1,0,0,0,x) + 8H(1,0,1,0,x)\right] \\
a S_0 &= \left[\frac{1}{(1+x)} - \frac{1}{(1-x)}\right]\left\{\frac{\zeta^2(2)}{10} - \zeta(3)H(0,x) + \zeta(2)(2H(1,0,x) + 3H(0,-1,x))\right. \\
&\quad \left.+\frac{1}{2}H(0,0,0,0,x) + H(0,-1,0,0,x) + H(0,0,-1,0,x) + H(0,1,0,0,x) + 2H(1,0,0,0,x)\right\}
\end{aligned}$$

Example: Box for the Leading Color Coeff



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + \mathcal{O}(\epsilon^0)$$

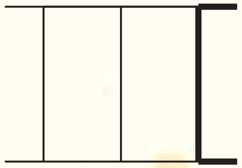
$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)},$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6G(1;x)\Big],$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 12G(-1;y)G(1;x) + 8G(-1,-1;y)\Big],$$

$$\begin{aligned}
A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[&-13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G(-1/y;x)\\
&+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/y;x)G(-1,-1;y)\\
&-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(-1/y;x)G(0,-1;y) + 6G(-y;x)G(0,-1;y)\\
&+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x)\\
&-6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y)\\
&-12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x)\\
&-6G(-1/y,0,0;x) + 12G(-1/y,0,1;x) + 6G(-1/y,1,0;x) - 12G(-1/y,1,1;x) + 6G(-y,1,0;x)\\
&-12G(-y,1,1;x)\Big]
\end{aligned}$$

Example: Box for the Leading Color Coeff



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + \mathcal{O}(\epsilon^0)$$

$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)},$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6\ldots$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + \ldots$$

$$A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G\left(-1/y;x\right)$$

$$\rho = \frac{-4m^2}{\hat{s}} \to 1$$

$$+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G\left(-1/y;x\right)G(-1,-1;y)$$

$$-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G\left(-1/y;x\right)G(0,-1;y) + 6G(-y;x)G(0,-1;y)$$

$$+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G\left(-1/y,0;x\right) + 12G(-1;y)G\left(-1/y,1;x\right)$$

$$-6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - [illegible]G(-1,0,-1;y)$$

$$-12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x)$$

$$-6G\left(-1/y,0,0;x\right) + 12G\left(-1/y,0,1;x\right) + 6G\left(-1/y,1,0;x\right) - 12G\left(-1/y,1,1;x\right) + 6G(-y,1,0;x)$$

$$-12G(-y,1,1;x)\Big]$$

1- and 2-dim GHPLs

GHPLs

- One- and two-dimensional Generalized Harmonic Polylogarithms (GHPLs) are defined as repeated integrations over set of basic functions. In the case at hand

$$f_w(x) = \frac{1}{x-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -y, -\frac{1}{y}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right\}$$

$$f_w(y) = \frac{1}{y-w}\,, \quad \text{with} \quad w \in \left\{0, 1, -1, -x, -\frac{1}{x}, 1 - \frac{1}{x} - x\right\}$$

- The weight-one GHPLs are defined as

$$G(0;x) = \ln x\,, \qquad G(w;x) = \int_0^x dt\, f_w(t)$$

- Higher weight GHPLs are defined by iterated integrations

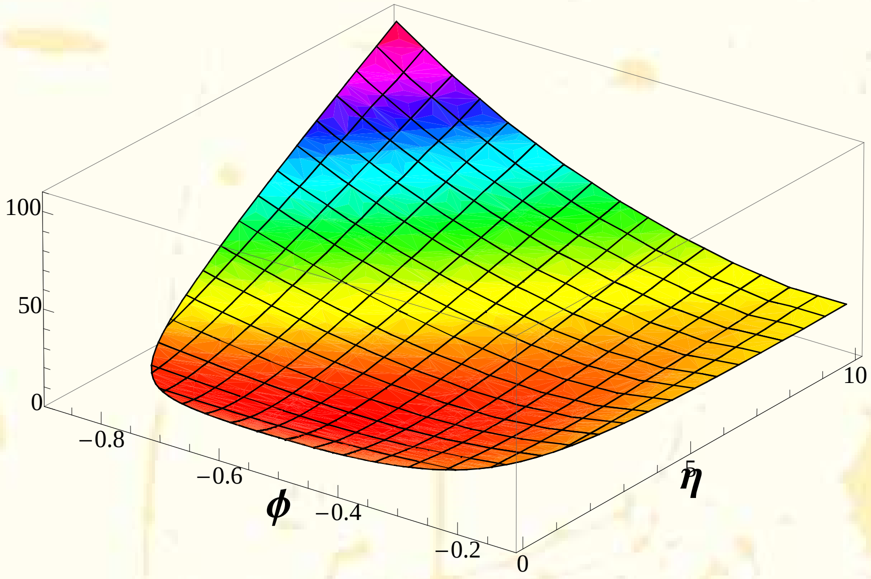
$$G(\underbrace{0,0,\cdots,0}_{n};x) = \frac{1}{n!}\ln^n x\,, \qquad G(w,\cdots;x) = \int_0^x dt\, f_w(t) G(\cdots;t)$$

- Shuffle algebra. Integration by parts identities

Remiddi and Vermaseren '99, Gehrmann and Remiddi '01-'02, Aglietti and R. B. '03, Vollinga and Weinzierl '04, R. B., A. Ferroglia, T. Gehrmann, and C. Studerus '09

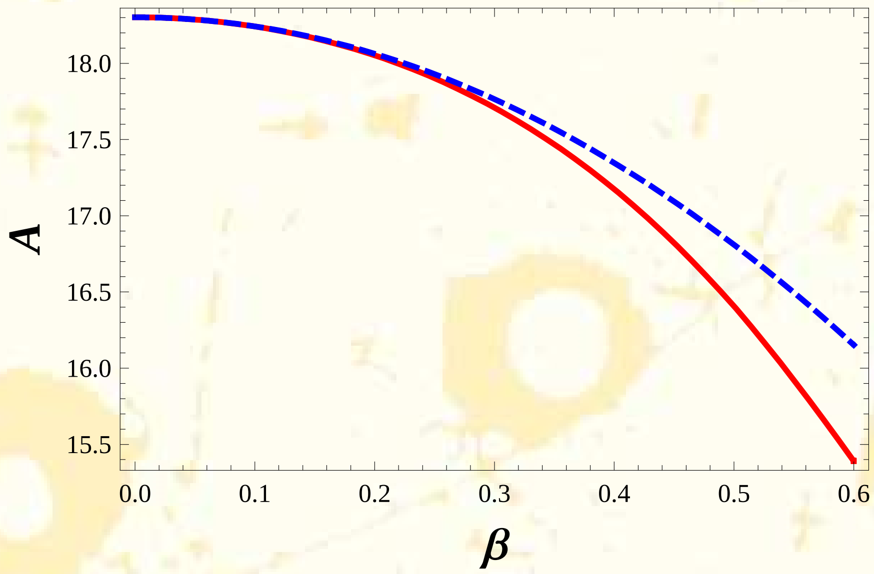
Coefficient A

Finite part of A



$$\eta = \frac{s}{4m^2} - 1\,, \quad \phi = -\frac{t - m^2}{s}$$

Threshold expansion versus exact result



$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

partonic c.m. scattering angle $= \frac{\pi}{2}$

Numerical evaluation of the GHPLs with GiNaC C++ routines (Vollinga and Weinzierl ’04).

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

Two-Loop Corrections to $gg \to t\bar{t}$

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} \quad = \quad & (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Big)\end{aligned}$$

789 two-loop diagrams contribute to 16 different color coefficients

- No numeric result for $\mathcal{A}_2^{(2\times0)}$ published yet
- The poles of $\mathcal{A}_2^{(2\times0)}$ are known analytically

Ferroglia, Neubert, Pecjak, and Li Yang '09

- The coefficient A is done. E_l–I_l can be evaluated analytically as for the $q\bar{q}$ channel

R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus '10, in preparation

Two-Loop Corrections to $gg \to t\bar{t}$

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$$\mathcal{A}_2^{(2\times 0)} = (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h$$

$$+ N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h$$

$$+ N_c N_l N_h H_{lh} + \frac{N_l}{N_c} I_l + \ldots$$

For the leading-color coefficient

NO additional MI

789 two-loop diagrams contribute to 16 different ...

- No numeric result for $\mathcal{A}_2^{(2\times 0)}$ published yet
- The poles of $\mathcal{A}_2^{(2\times 0)}$ are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09

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Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

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- For the light-fermion contrib

11 additional MIs

different color coefficients

...ed yet

...lytically

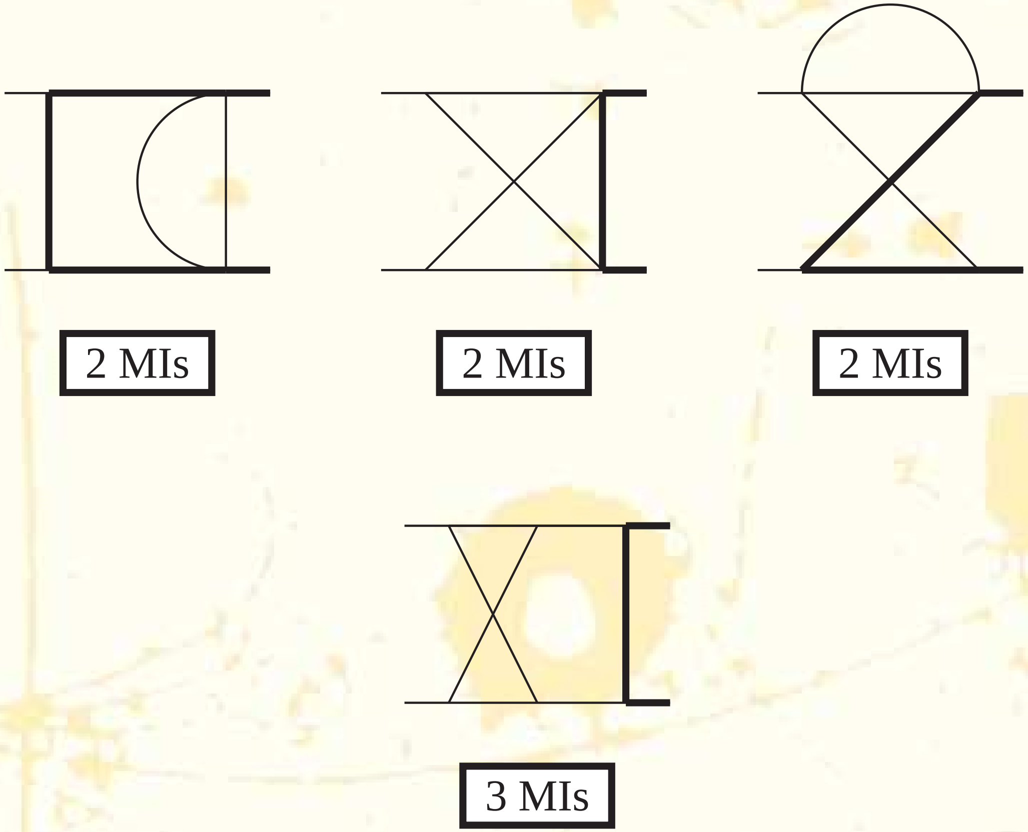
Ferroglia, Neubert, Pecjak, and Li Yang '09

- The coefficient A is done. E_l–I_l can be evaluated analytically as for the $q\bar{q}$ channel

R. B., Ferroglia, Gehrmann, von Manteuffel and Studerus '10, in preparation

Master Integrals for the N_l Coeff

Master Integrals for the N_l Coeff

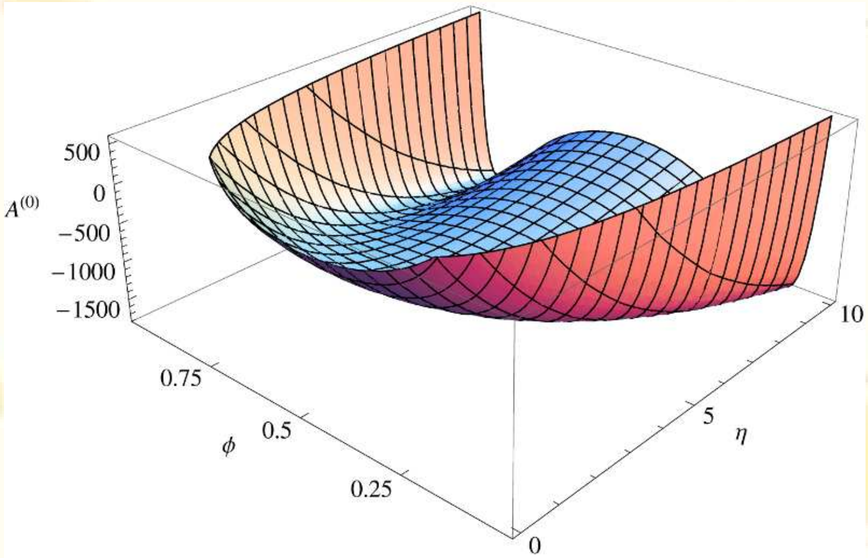


For the N_l coefficients in the gg channel there are 5 additional irreducible topologies (11 MIs)

R. B., A. Ferroglia, T. Gehrmann, A. von Manteuffel, and C. Studerus, in preparation

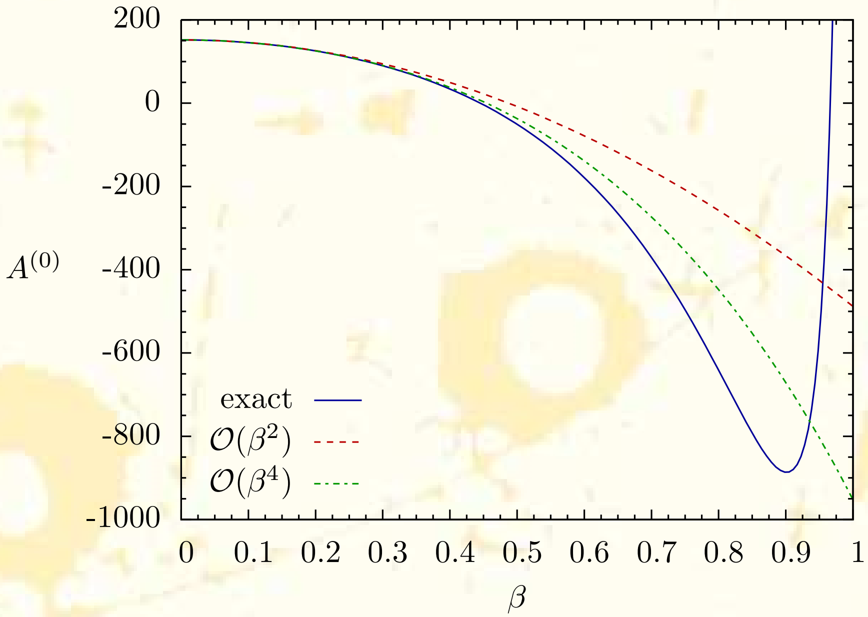
Coefficient A in gg

Finite part of A



$$\eta = \frac{s}{4m^2} - 1\,, \quad \phi = -\frac{t-m^2}{s}$$

Threshold expansion versus exact result



$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

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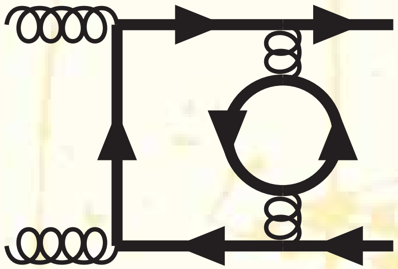
Numerical evaluation of the GHPLs with GiNaC C++ routines (Vollinga and Weinzierl ’04).

R. B., A. Ferroglia, T. Gehrmann, A. von Manteuffel, and C. Studerus, JHEP **1101** (2011) 102

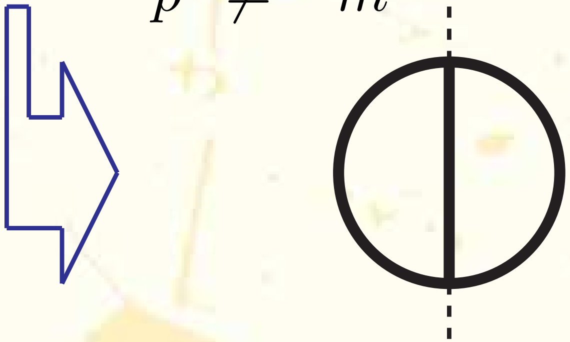
The other coefficients ...

The remaining coefficients present additional problems:

- CROSSED TOPOLOGIES
  - In particular in the coefficients B, C of the $q\bar{q}$ channel and B, C, D of the gg channel $\Longrightarrow$ many Master Integrals for each topology
- The N_h terms in the gg channel, E_h–I_h cannot be expressed in terms of GHPLs, since they have as a common subtopology the equal-mass sunrise



$p^2 \neq -m^2$



Elliptic Functions

$$K(z) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-zt^2)}}$$

S. Laporta and E. Remiddi, Nucl. Phys. B **704** (2005) 349

Conclusions

- In the last 15 years, Tevatron explored top-quark properties reaching a remarkable experimental accuracy. The top mass could be measured with $\Delta m_t/m_t = 0.63\%$ and the production cross section with $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 9\%$. Other observables could be measured only with bigger errors.

- At the LHC the situation will further improve. The production cross section of $t\bar{t}$ pairs is expected to reach the accuracy of $\Delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} = 5\%$!!

- This experimental precision requires a complete NNLO theoretical analysis.

- In this talk I briefly reviewed the analytic evaluation of the two-loop matrix elements.
  - The corrections involving a fermionic loop (light or heavy) in the $q\bar{q}$ channel are completed, together with the leading color coefficient.
  - The leading color coefficient in the gg channel is completed and light-fermion corrections can be calculated with the same technique and are at the moment under study.
  - Advantages of the analytic formulas: Beauty! Fast and precise numerical evaluation!

- The calculation of the crossed diagrams and of the diagrams with a heavy loop have still to be afforded.