

JUNO+TAO MC SENSITIVITY
TALK 5: ONE-SIDED SENSITIVITY, $\sin^2 2\theta_{13}$

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PREVIOUS WORK

Summary

- JUNO+TAO sensitivity estimation with MC with full systematics
- Follow the analysis definition in the TechNote doc-7489
- Stat only analysis: doc8889
- Stat+syst analysis: doc8916
- Uncertainties: doc8921
- One sided intervals: doc8958

See also

- Excellent summary on statistics by Marco: doc8937



OVERVIEW

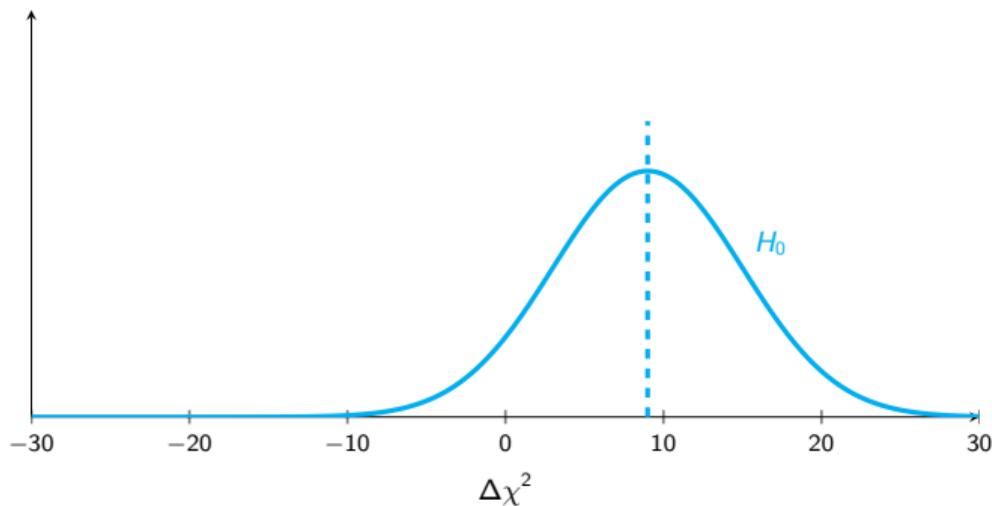
Setup

- JUNO+TAO with constrained antineutrino spectrum: 100% at 20 keV or equivalent.
(unconstrained fits are unstable and fail often).
- Statistic: Pearson's χ^2 , unbiased Pearson's χ^2 , log-Poisson, combined Neyman-Pearson
- Fit parameters: Δm_{31}^2 , Δm_{21}^2 , $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{13}$, spectral parameters.
- Minimization: MINUIT, no scan.
- 10'000 MC experiments \times NMO \times statistic
- Explicit random seed for each job. Goal: all fits to converge.

NMO



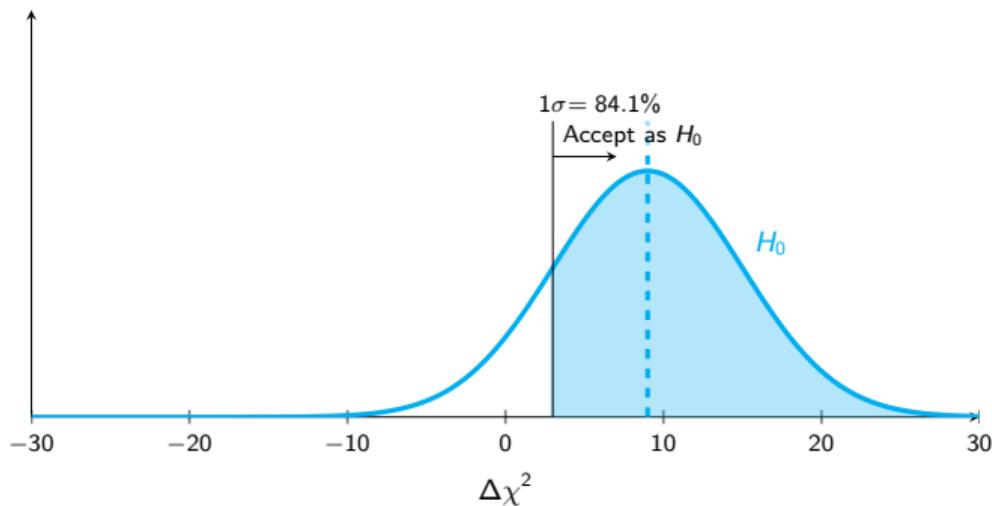
DEFINITION: GENERAL



- $\Delta\chi^2 = \min \chi_{H_1}^2 - \min \chi_{H_0}^2$
- H_0 — true hypothesis,
 H_1 — false hypothesis.



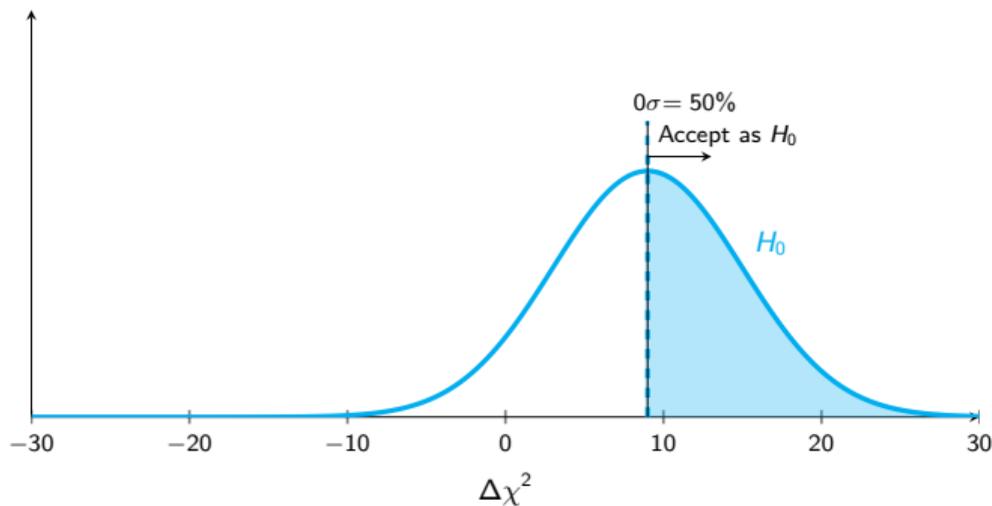
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- Sensitivity: probability to accept H_0
as the correct one.



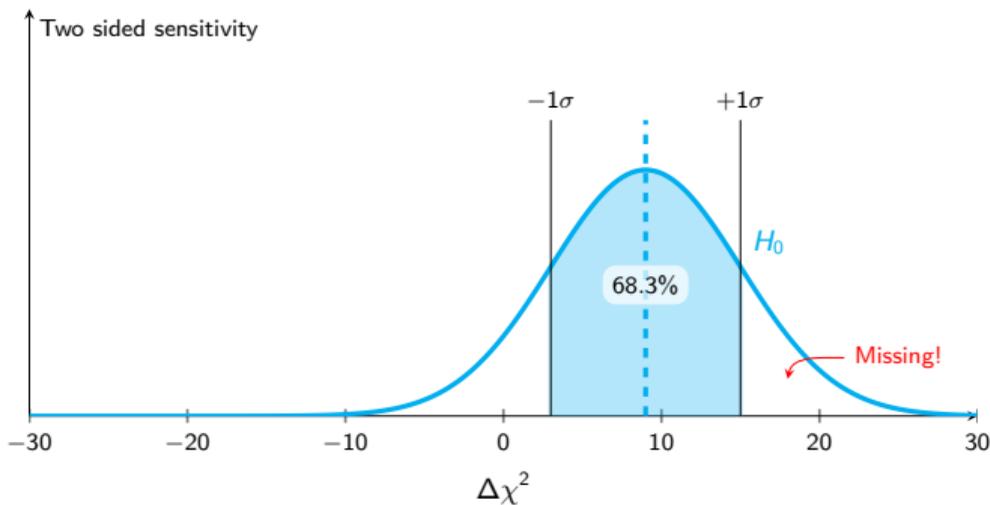
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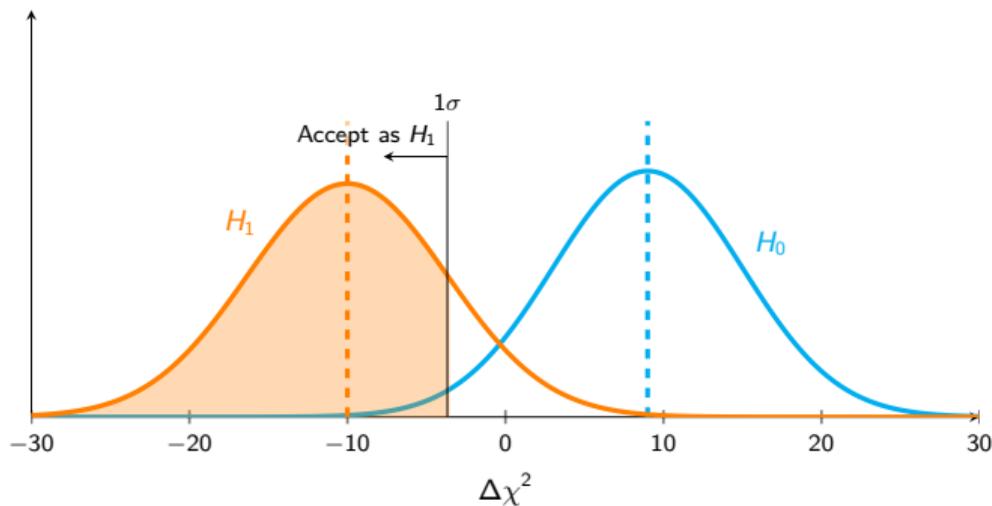
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Conversion

$$A_{\text{one side}} = \frac{1 + A_{\text{two sides}}}{2}$$



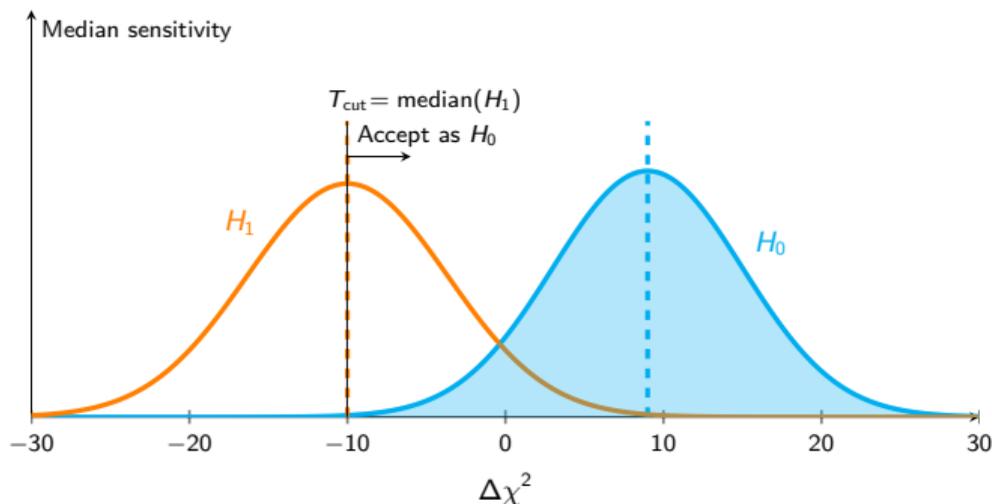
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- Error of the II kind:
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DEFINITION: SENSITIVITIES



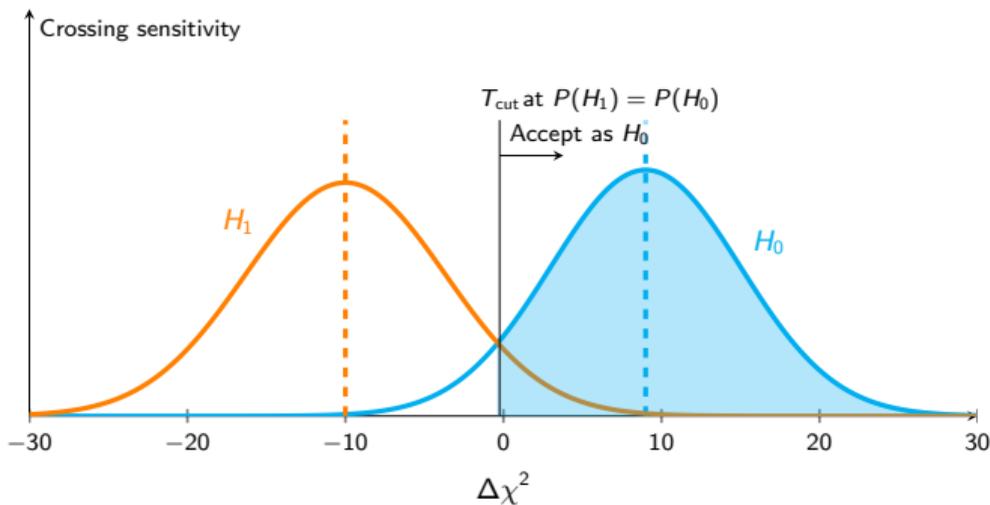
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Setups

- Median sensitivity: accept H_1 in 50% cases



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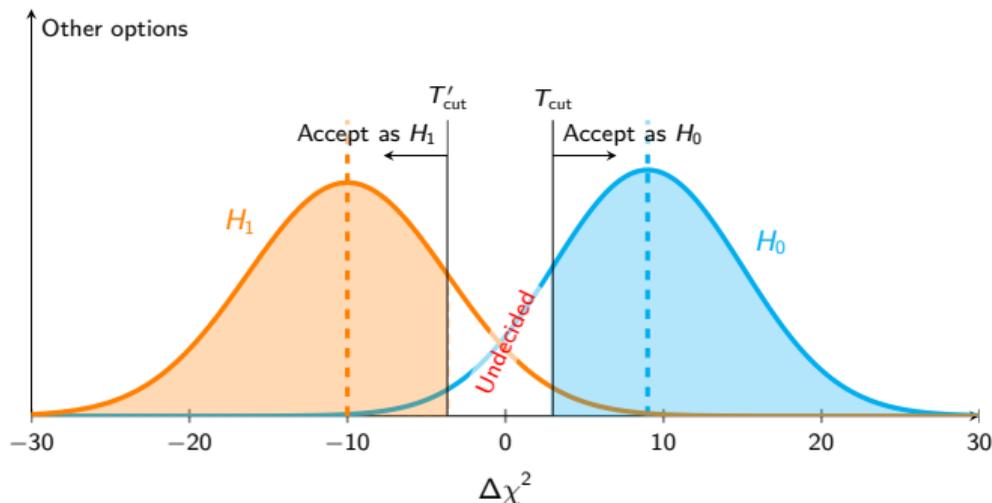
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- Crossing sensitivity: equal sensitivity
regardless of H_0/H_1



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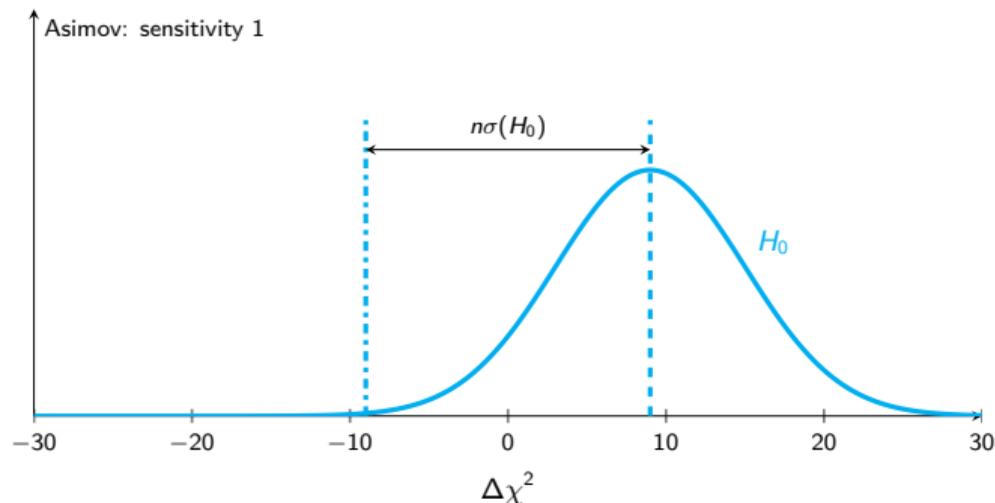
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DEFINITION: ASIMOV SENSITIVITIES



One sided formulas

$$S_1 = \int_{-\Delta\chi_{H_0}^2}^{\infty} \mathcal{N}(\Delta\chi_{H_0}^2, 2\sqrt{\Delta\chi_{H_0}^2})$$

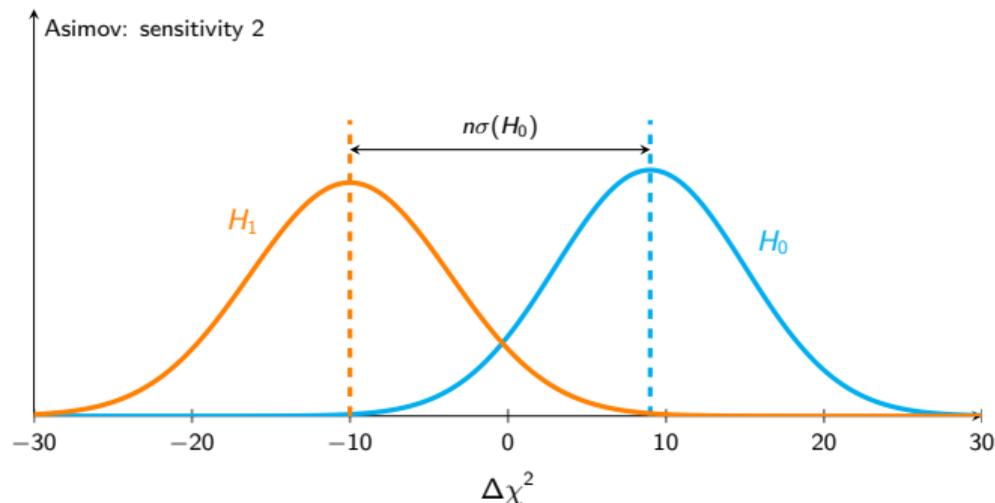
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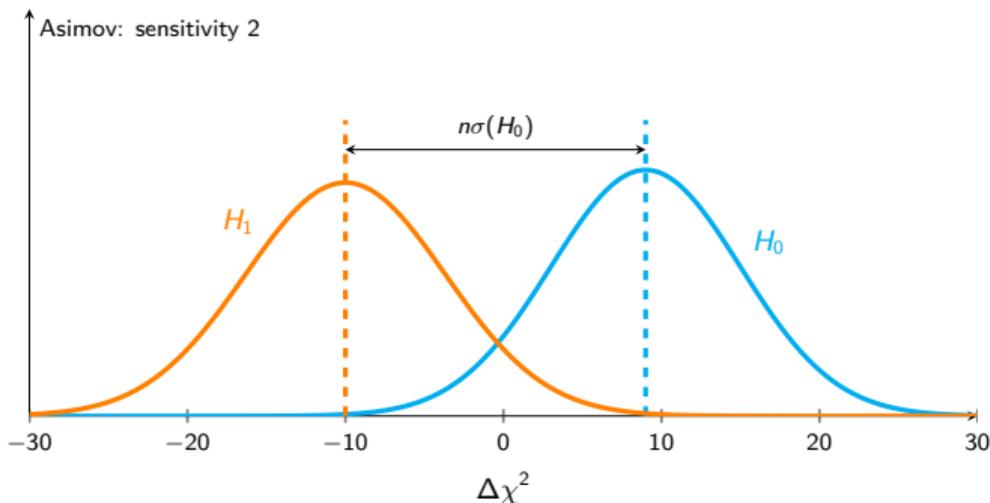
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One sided formulas

$$S_1 = \int_{-\Delta\chi_{H_0}^2}^{\infty} \mathcal{N}(\Delta\chi_{H_0}^2, 2\sqrt{\Delta\chi_{H_0}^2}) \quad S_2 = \int_{\Delta\chi_{H_1}^2}^{\infty} \mathcal{N}(\Delta\chi_{H_0}^2, 2\sqrt{\Delta\chi_{H_0}^2})$$



DEFINITION: ASIMOV SENSITIVITIES



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Setups

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Two sided formulas: not to be used

$$S_1 = \sqrt{\Delta\chi_{H_0}^2} \quad S_2 = \frac{\Delta\chi_{H_0}^2 + \Delta\chi_{H_1}^2}{2\sqrt{\Delta\chi_{H_0}^2}}$$

MC WITH SYSTEMATICS



MC outline

- 2-step MC:
 1. Randomize nuisance parameters:
 - ✗ Antineutrino spectrum is fixed, not randomized¹
 - ✓ Uncorrelated or fully correlated parameters
 - ✓ Parameters with partial correlations: fission fractions
 1. Predict
 2. Randomize prediction with covariance matrix, including:
 - Statistical uncertainties
 - Bin-to-bin uncertainties
- Fit with full systematics
 - ✓ Including spectral parameters



FLUCTUATIONS AND NUISANCE TERMS

| | | Asimov | | MC | |
|--------------|-----------------------------|---------|---------|-----------------|-----------------------|
| | | stat | all | stat | all |
| Data | | IBD | IBD+Bkg | IBD | IBD+Bkg |
| Fluctuations | Free oscillation parameters | central | central | central | central |
| | Spectral parameters | central | central | central | central |
| | $\sin^2 2\theta_{13}$ | central | central | central | Gaussian |
| | Systematic parameters | central | central | central | Gaussian |
| | Bin-to-bin | Asimov | Asimov | Poisson stat | Gaussian stat+syst |
| Nuisance | $\sin^2 2\theta_{13}$ | yes | yes | yes | yes |
| | Spectral parameters | no | no | yes | yes |
| | Systematic parameters | no | yes | no | yes |
| | Bin-to-bin covariance | no | yes | no | yes |



Changes since the technote doc-7489 (v9)

- Reason: stabilize the minimization process
- Cross section threshold: 1st order \rightarrow [exact](#). Impact on the analysis: negligible.
- LSNL: smoothen common input curves. Impact on the sensitivity $\Delta^2\chi^2 \sim 0.01$.



MINOR NOTES

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- Reason: stabilize the minimization process
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Fluctuations

- Antineutrino spectrum parameters: **no fluctuation**, nuisance 100% for 20 keV equivalent.
- NOTE: SNF rate uncertainty is 30%, thus in a few samples out of 10'000 SNF may be **subtracted** from the reactor spectrum. TODO: correct method is to use logarithmic parameter or log-normal distribution.



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Fluctuations

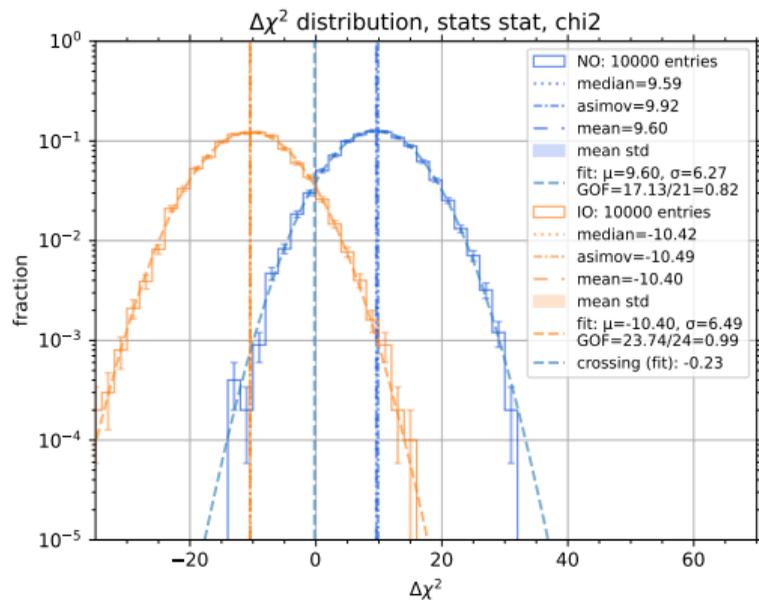
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Minimization

- $\sim 10/10'000$ samples fail with a jump to non-physical region. Will be fixed with another starting point.



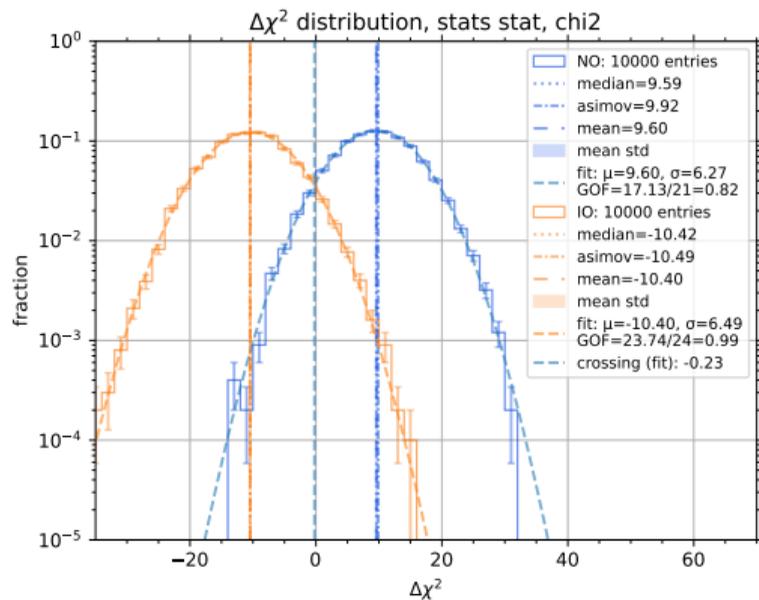
TEST DISTRIBUTIONS: STAT ONLY MODE



- $\Delta\chi^2 = \min \chi_{IO}^2 - \min \chi_{NO}^2$
- Median sensitivity:
 - ▶ accepted fraction (true NMO)
 - ▶ 50% rejected (opposite NMO)
- Crossing sensitivity:
 - accepted frac. (true NO) = accepted frac. (true IO)

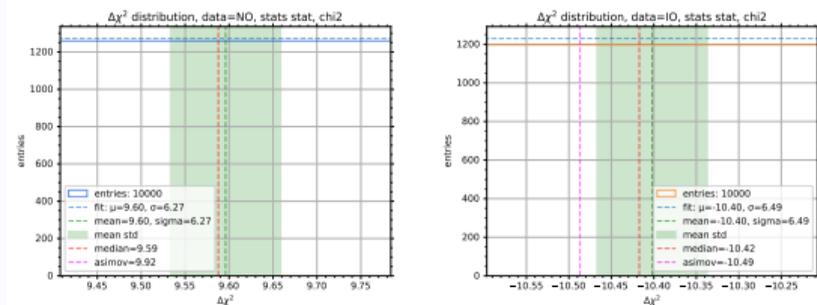


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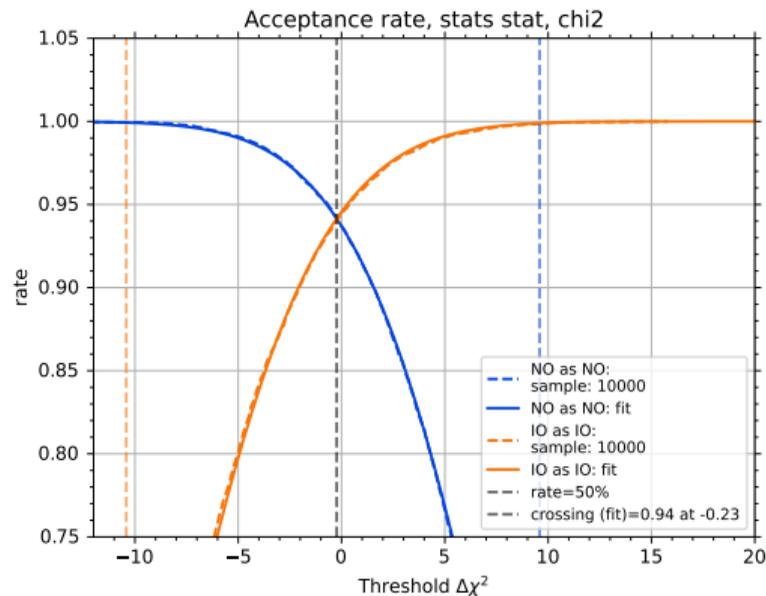
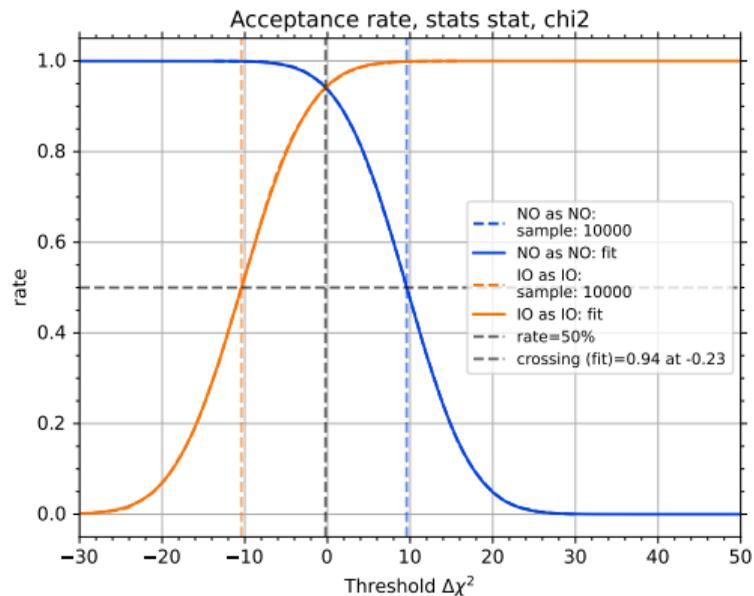
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Mean, median, Asimov:





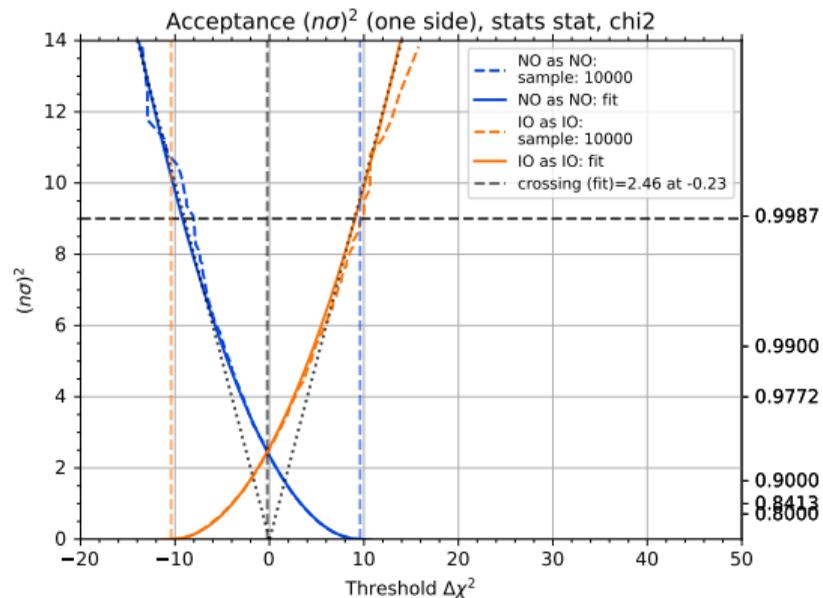
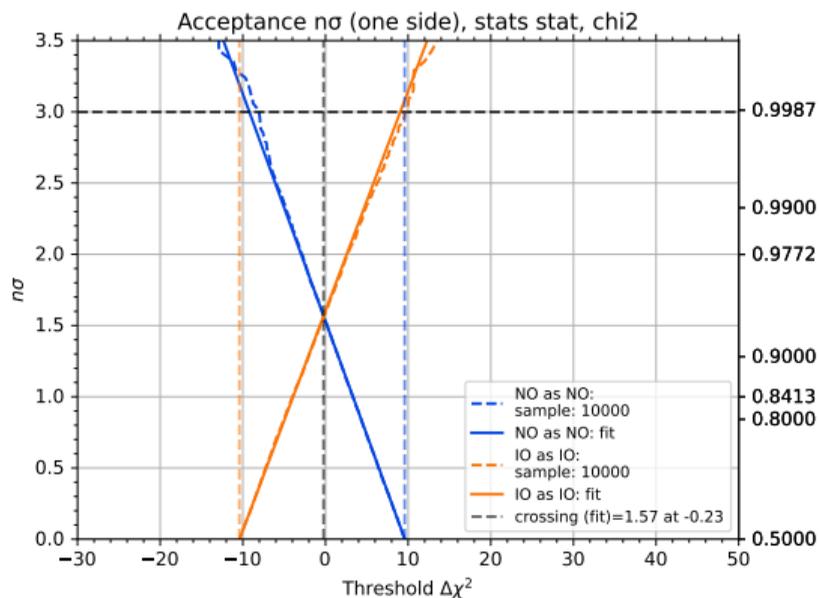
ACCEPTANCE RATE



- Solid: analytic, from the fit
- Dashed: based on MC data



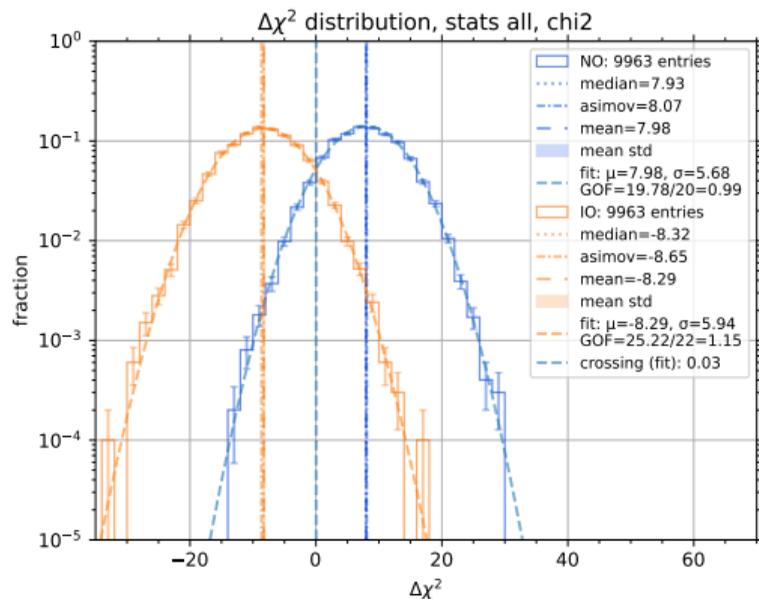
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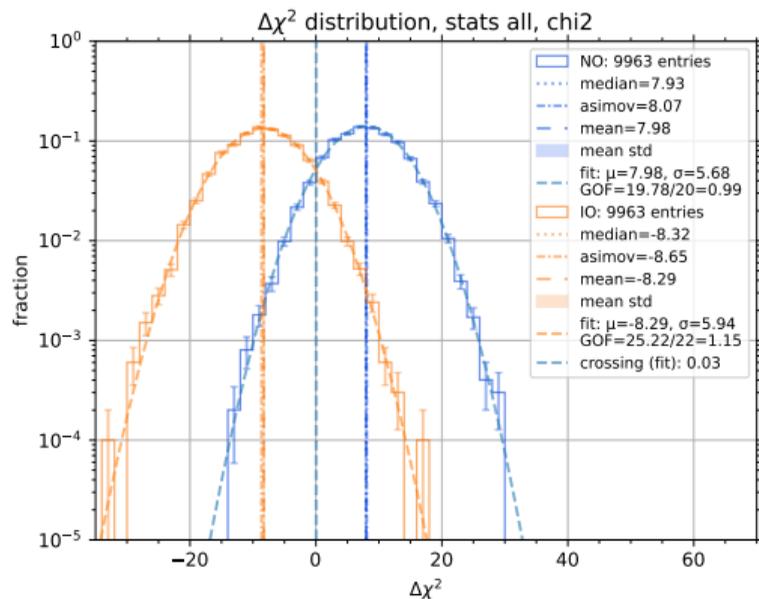
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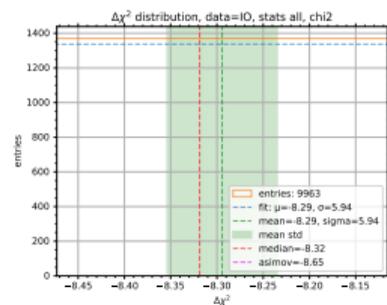
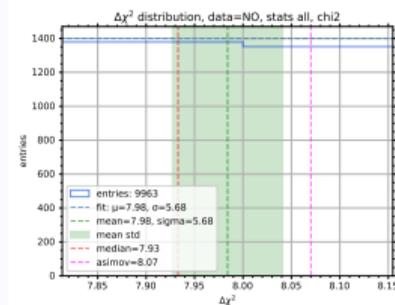


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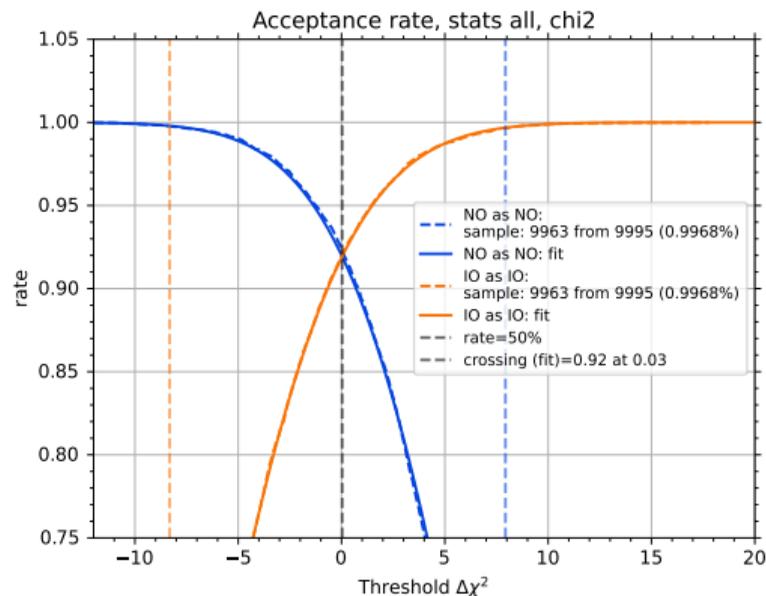
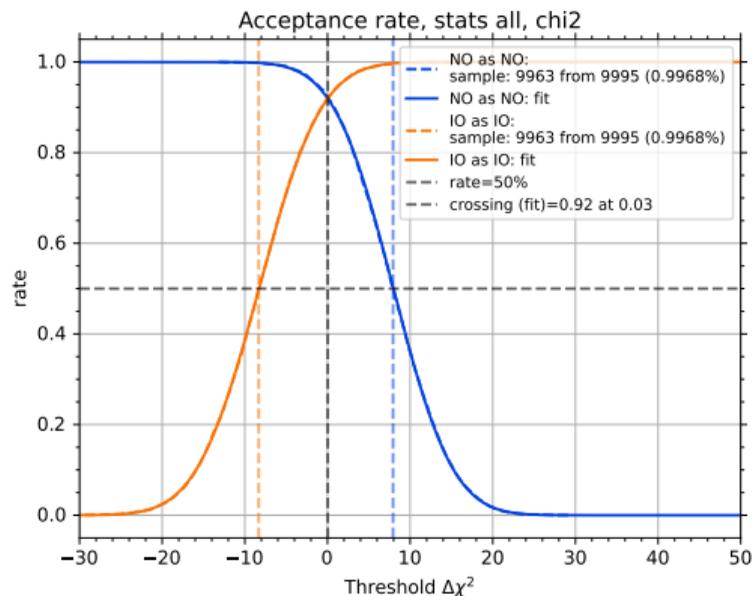
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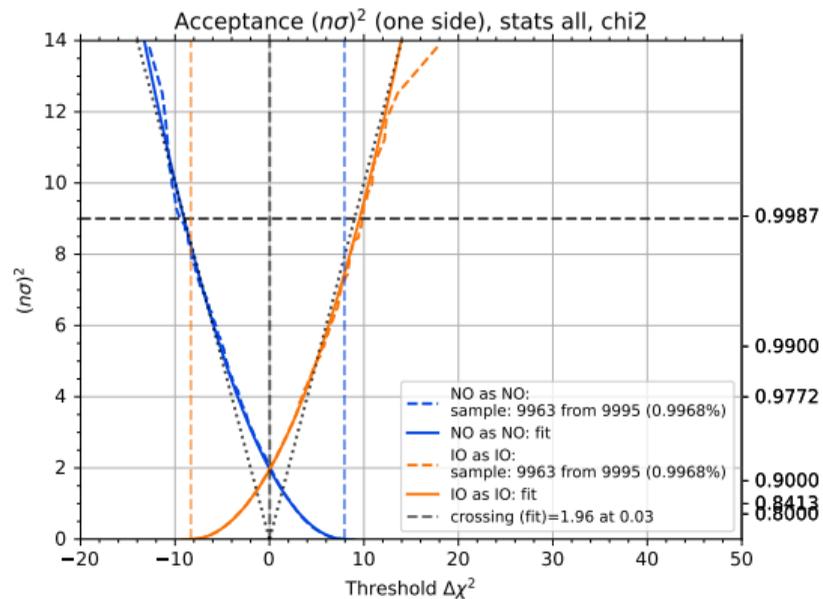
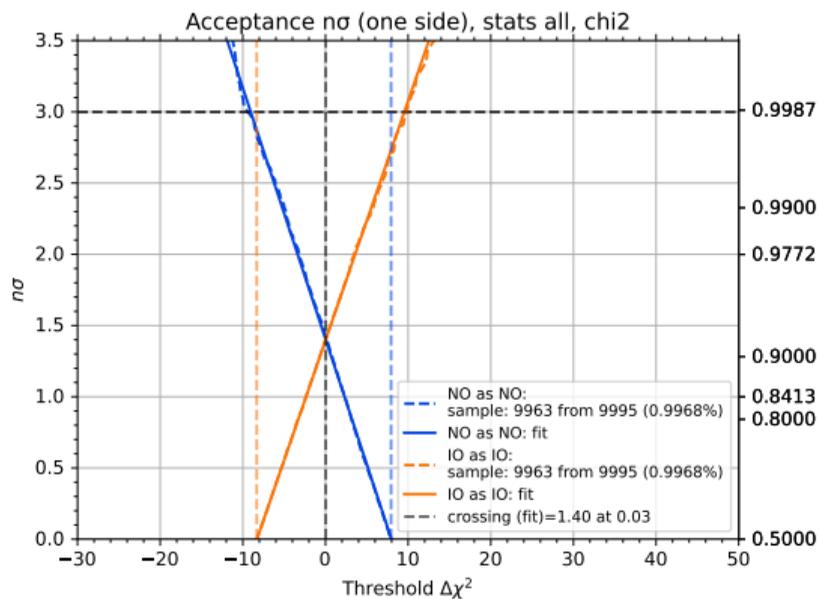
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SENSITIVITY

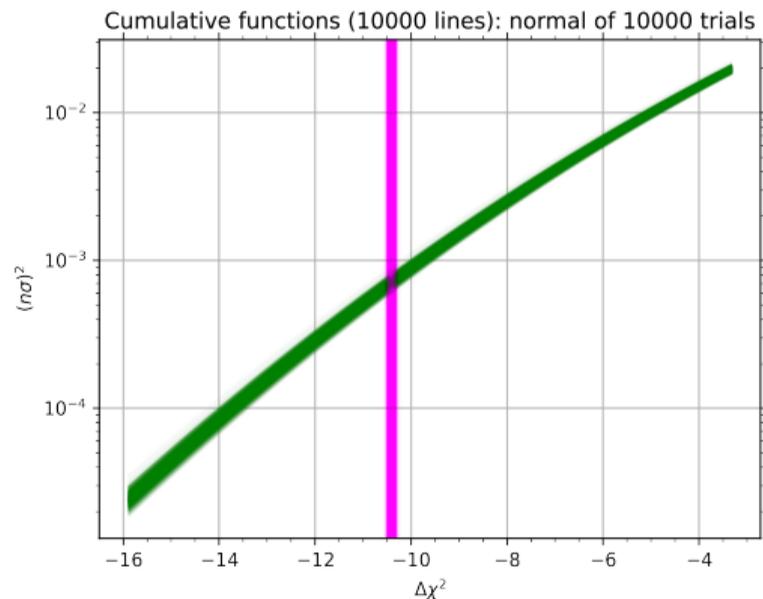


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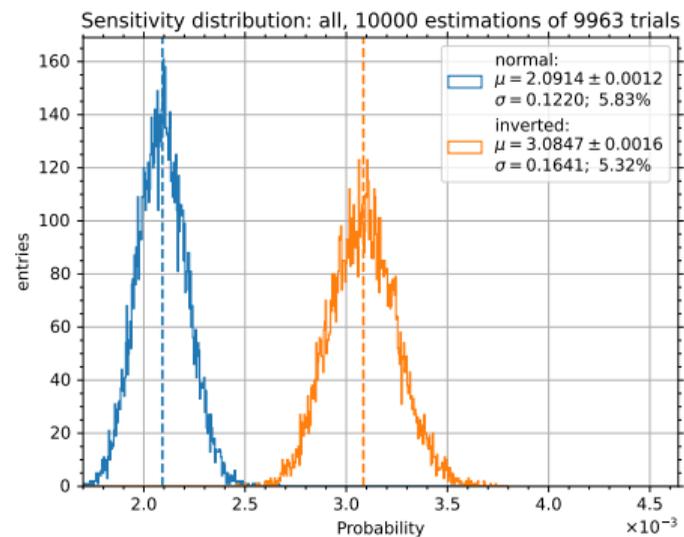
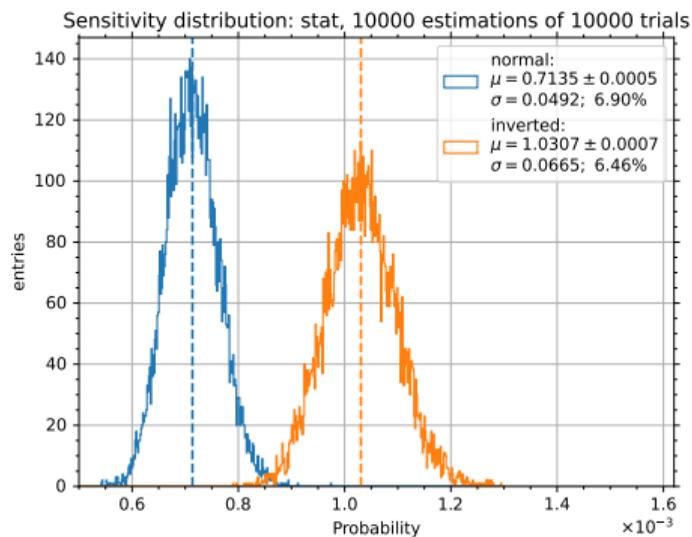
SENSITIVITY UNCERTAINTY: METHOD

1. Take mean, sigma, N from $\Delta\chi^2$ distributions
(assume mean=median for simplicity)
2. Build Asimov Gaussians for IO and NO
3. Fit with Gaussian: estimate covariance matrix
for mean/sigma
4. Build 10'000 samples for mean/sigma and
mean (opposite IO) for each NMO ▶
5. Estimate sensitivity for each Gaussian and opposite
NMO mean
6. Plot distributions



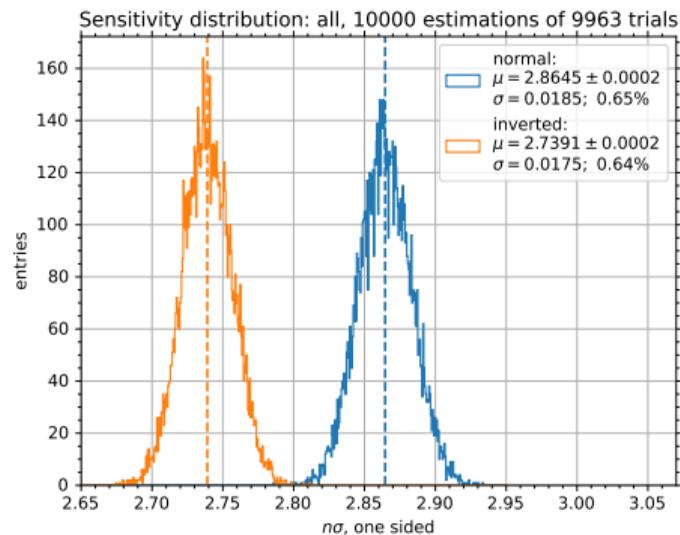
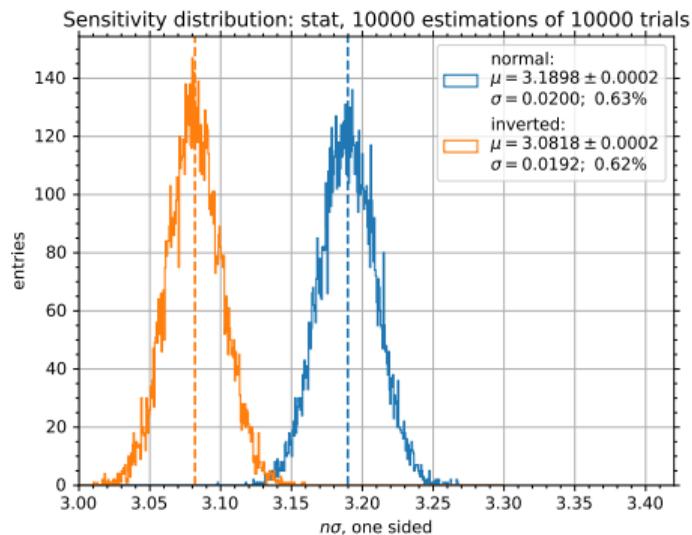


SENSITIVITY UNCERTAINTY: RESULTS



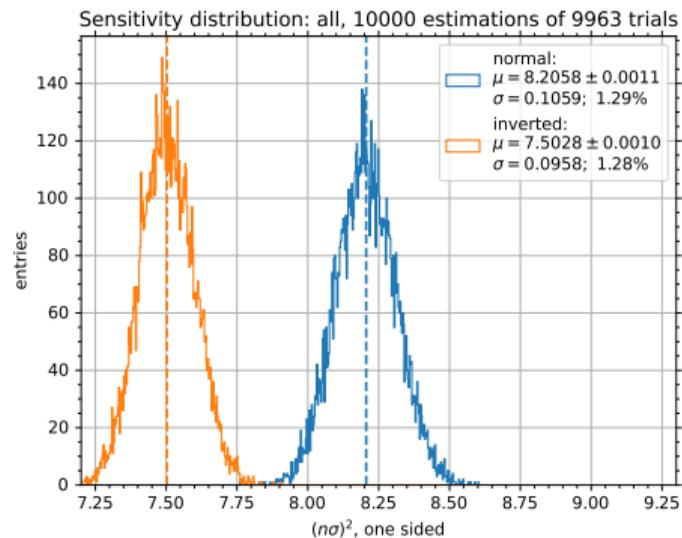
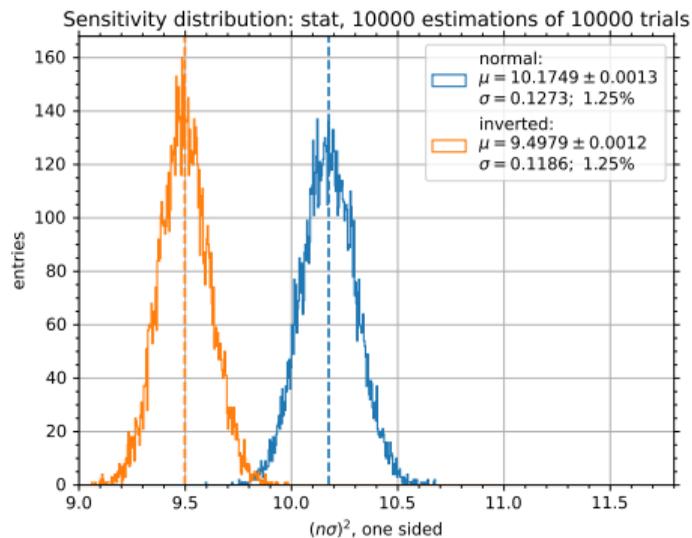


SENSITIVITY UNCERTAINTY: RESULTS





SENSITIVITY UNCERTAINTY: RESULTS





SENSITIVITY: RESULTS

Stat only

| | NO | IO |
|---|-----------------------|----------------------|
| $\Delta\chi^2$, MC, mean | 9.59597 | 10.402 |
| $\Delta\chi^2$, MC, median | 9.58779 | 10.4171 |
| $\Delta\chi^2$, Asimov | 9.91528 | 10.4869 |
| $\sigma(\Delta\chi^2)$, MC | 6.27065 | 6.48632 |
| $2\sqrt{\Delta\chi^2}$, Asimov | 6.29771 | 6.4767 |
| $n\sigma$, Asimov, two-sided, equal | 3.14885 | 3.23835 |
| $n\sigma$, Asimov, two-sided | 3.23962 | 3.15009 |
| P. accept, Asimov, two-sided, equal | 0.998361 | 0.998798 |
| P. accept, Asimov, two-sided | 0.998803 | 0.998368 |
| P. accept, Asimov, one-sided | 0.999402 | 0.999184 |
| P. accept, MC (fit), one-sided | 0.999293 | 0.998972 |
| P. accept, MC (sample), one-sided | 0.999465 | 0.998606 |
| P. reject, Asimov, two-sided, equal, $\times 10^{-3}$ | 1.63912 | 1.20223 |
| P. reject, Asimov, two-sided, $\times 10^{-3}$ | 1.19689 | 1.6322 |
| P. reject, Asimov, one-sided, $\times 10^{-3}$ | 0.598443 | 0.816099 |
| P. reject, MC (fit), one-sided, $\times 10^{-3}$ | 0.707193 ± 0.0492 | 1.02809 ± 0.0665 |
| P. reject, MC (sample), one-sided, $\times 10^{-3}$ | 0.534828 | 1.39398 |
| $n\sigma$, Asimov, one-sided | 3.23962 | 3.15009 |
| $n\sigma$, MC (fit), one-sided | 3.1917 ± 0.0200 | 3.08199 ± 0.0192 |
| $n\sigma$, MC (sample), one-sided | 3.27153 | 2.9902 |
| $(n\sigma)^2$, Asimov, one-sided | 10.4951 | 9.92307 |
| $(n\sigma)^2$, MC (fit), one-sided | 10.1869 ± 0.1273 | 9.49869 ± 0.1186 |
| $(n\sigma)^2$, MC (sample), one-sided | 10.7029 | 8.94129 |



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| P. accept, MC (fit), one-sided | 0.999293 | 0.998972 |
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| P. reject, MC (sample), one-sided, $\times 10^{-3}$ | 0.534828 | 1.39398 |
| $n\sigma$, Asimov, one-sided | 3.23962 | 3.15009 |
| $n\sigma$, MC (fit), one-sided | 3.1917 ± 0.0200 | 3.08199 ± 0.0192 |
| $n\sigma$, MC (sample), one-sided | 3.27153 | 2.9902 |
| $(n\sigma)^2$, Asimov, one-sided | 10.4951 | 9.92307 |
| $(n\sigma)^2$, MC (fit), one-sided | 10.1869 ± 0.1273 | 9.49869 ± 0.1186 |
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Full syst

| | NO | IO |
|---|----------------------|----------------------|
| $\Delta\chi^2$, MC, mean | 7.98405 | 8.29437 |
| $\Delta\chi^2$, MC, median | 7.933 | 8.31866 |
| $\Delta\chi^2$, Asimov | 8.07005 | 8.65149 |
| $\sigma(\Delta\chi^2)$, MC | 5.67823 | 5.9438 |
| $2\sqrt{\Delta\chi^2}$, Asimov | 5.68157 | 5.88268 |
| $n\sigma$, Asimov, two-sided, equal | 2.84078 | 2.94134 |
| $n\sigma$, Asimov, two-sided | 2.94312 | 2.8425 |
| P. accept, Asimov, two-sided, equal | 0.9955 | 0.996732 |
| P. accept, Asimov, two-sided | 0.996751 | 0.995524 |
| P. accept, Asimov, one-sided | 0.998375 | 0.997762 |
| P. accept, MC (fit), one-sided | 0.997956 | 0.996836 |
| P. accept, MC (sample), one-sided | 0.997797 | 0.996492 |
| P. reject, Asimov, two-sided, equal, $\times 10^{-3}$ | 4.50028 | 3.26795 |
| P. reject, Asimov, two-sided, $\times 10^{-3}$ | 3.24922 | 4.47609 |
| P. reject, Asimov, one-sided, $\times 10^{-3}$ | 1.62461 | 2.23804 |
| P. reject, MC (fit), one-sided, $\times 10^{-3}$ | 2.04435 ± 0.1220 | 3.16412 ± 0.1641 |
| P. reject, MC (sample), one-sided, $\times 10^{-3}$ | 2.20324 | 3.50834 |
| $n\sigma$, Asimov, one-sided | 2.94312 | 2.8425 |
| $n\sigma$, MC (fit), one-sided | 2.87124 ± 0.0185 | 2.73027 ± 0.0175 |
| $n\sigma$, MC (sample), one-sided | 2.84749 | 2.69605 |
| $(n\sigma)^2$, Asimov, one-sided | 8.66196 | 8.07982 |
| $(n\sigma)^2$, MC (fit), one-sided | 8.244 ± 0.1059 | 7.45438 ± 0.0958 |
| $(n\sigma)^2$, MC (sample), one-sided | 8.10823 | 7.2687 |

Oscillation parameters



FIT FUNCTIONS

| Function | Value at model=data | Bias | | Reasons |
|----------------------------------|------------------------|--------|-----|---|
| | | Asimov | MC | |
| log-Poisson | not 0 | no | no | |
| log-Poisson ratio | 0 | no | no | |
| Neyman's χ^2 | 0 | no | yes | fluctuations in covariance matrix |
| Pearson's χ^2 | 0 | no | yes | interplay between fluctuations and running covariance matrix |
| Pearson's χ^2 iterative | 0 | no | no? | covariance matrix is fixed during minimization requires few iterations |
| Combined Neyman-Pearson χ^2 | 0 | no | no | ad-hoc, compensates bias for the normalization |
| Pearson's $\chi^2 + \ln \det V$ | not 0 | yes | no | defined by the statistics |

Notes

- $w/2$ — half width at same height covering $\sim 68.3\%$ of the plot
- σ — width of the distribution of the best fit values



FIT FUNCTIONS

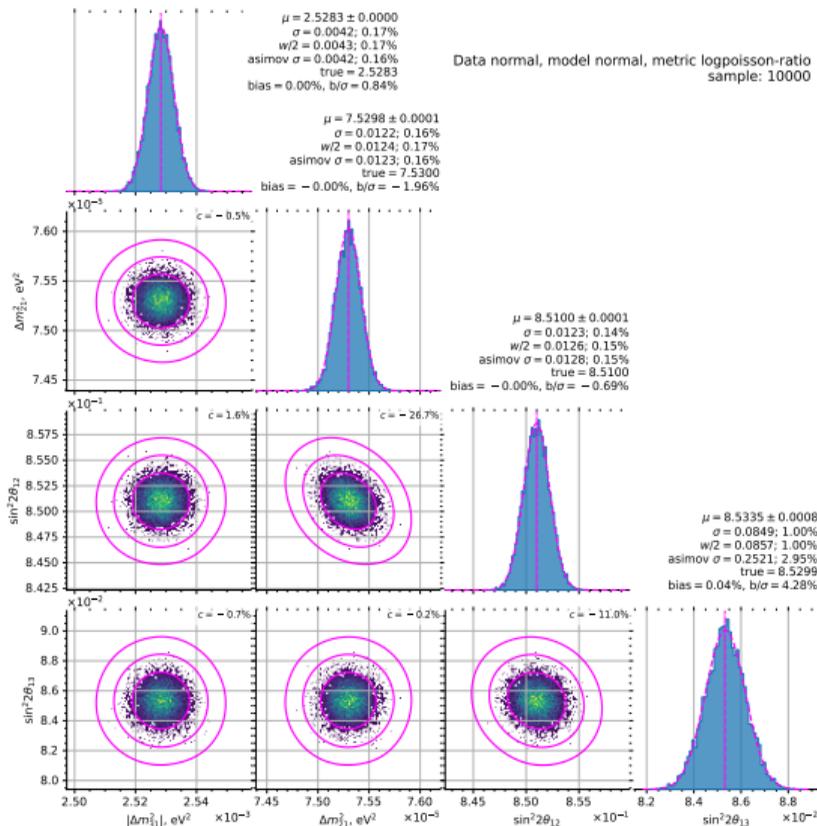
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Notes

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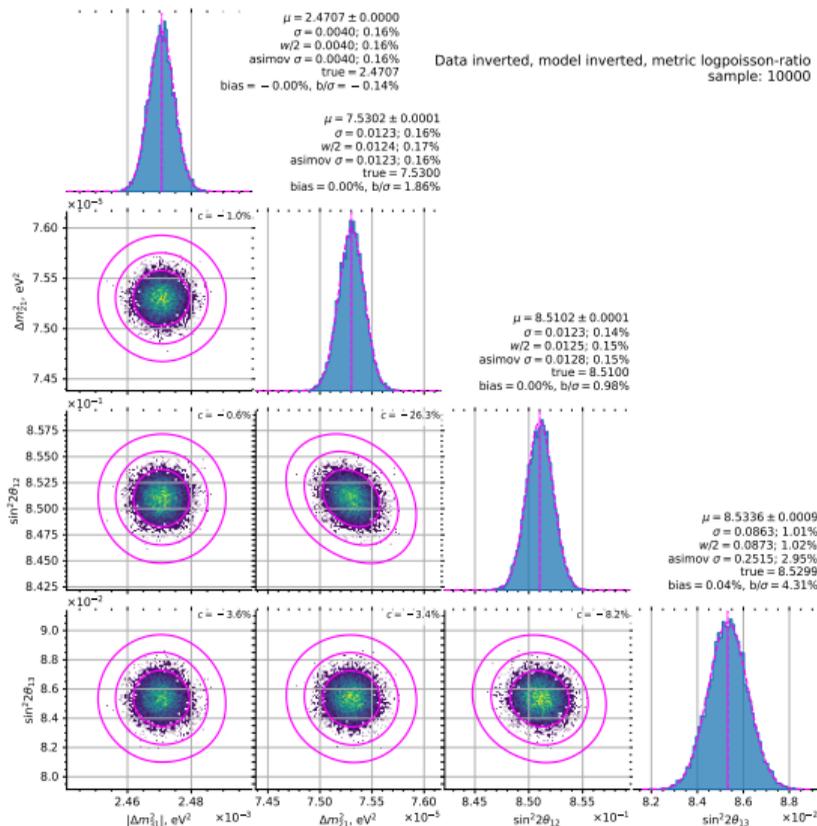
OSC PARS: STAT ONLY MODE



- Contours: kernel density estimation
- ✓ Normal behaviour
- ✓ Δm_{31}^2 , Δm_{21}^2 and $\sin^2 2\theta_{12}$ consistent with Asimov
- ✓ $\sin^2 2\theta_{13}$ width is expected
- Stat only:
 - ✓ Poisson: no bias
 - ▶ largest correlation 30% between $\sin^2 2\theta_{12}$ and Δm_{21}^2



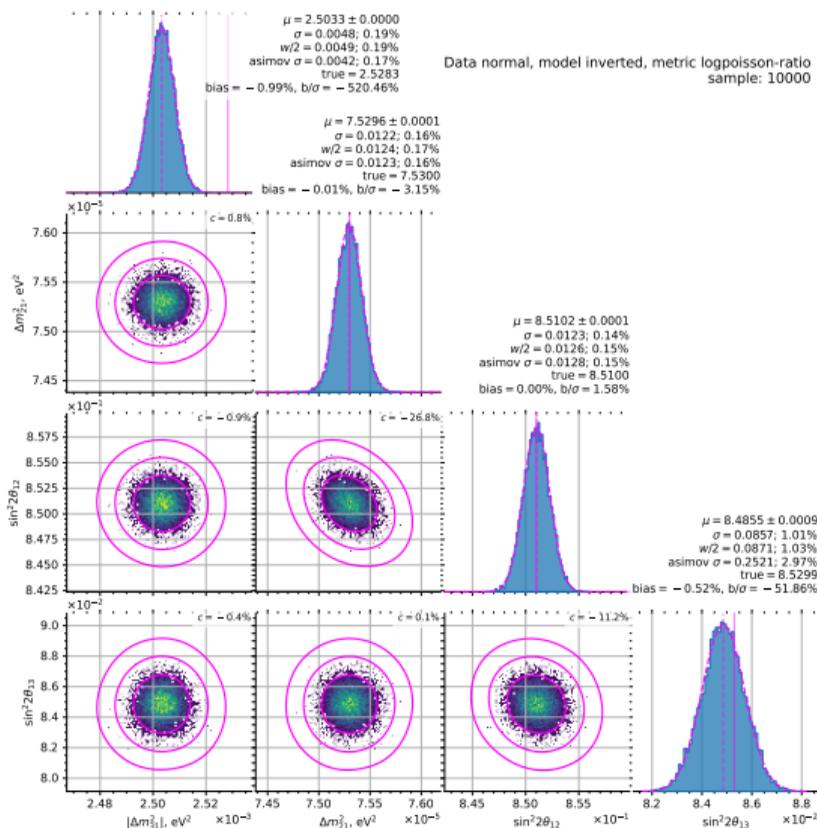
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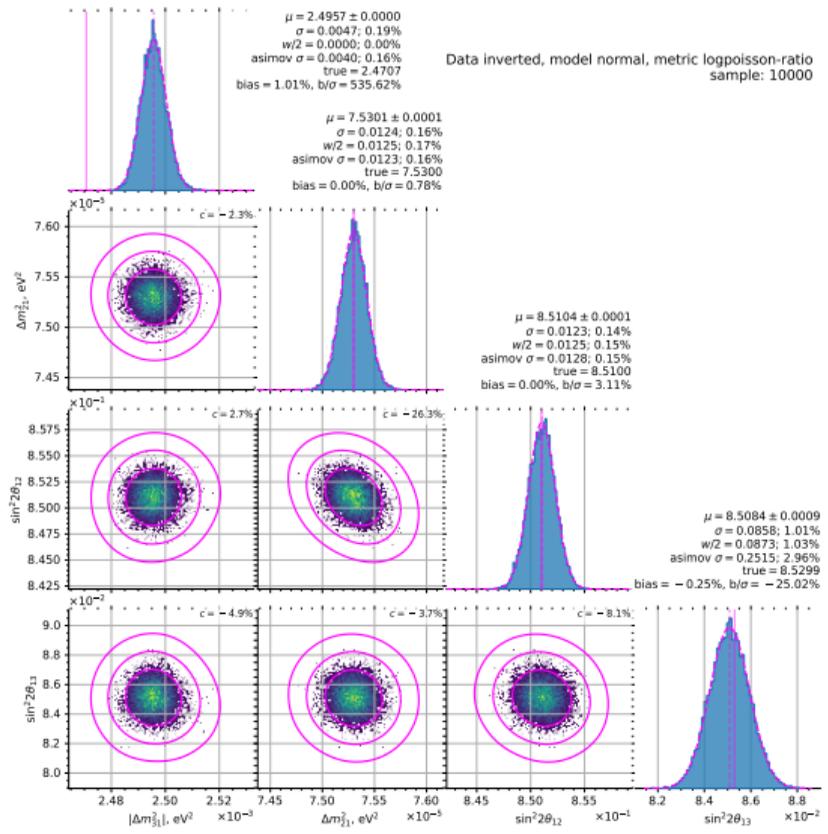
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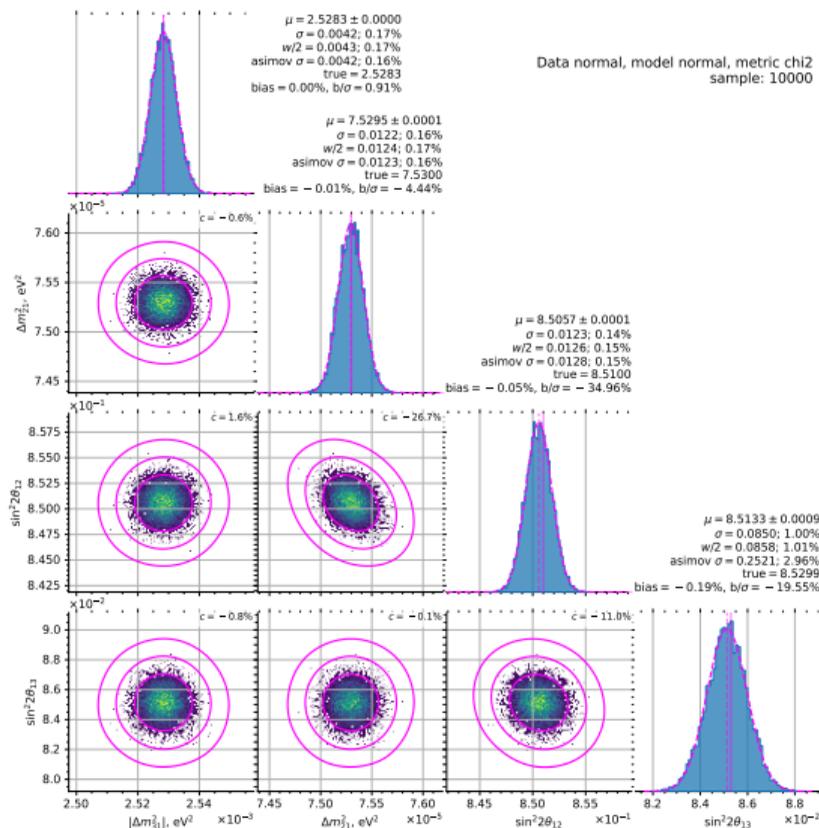
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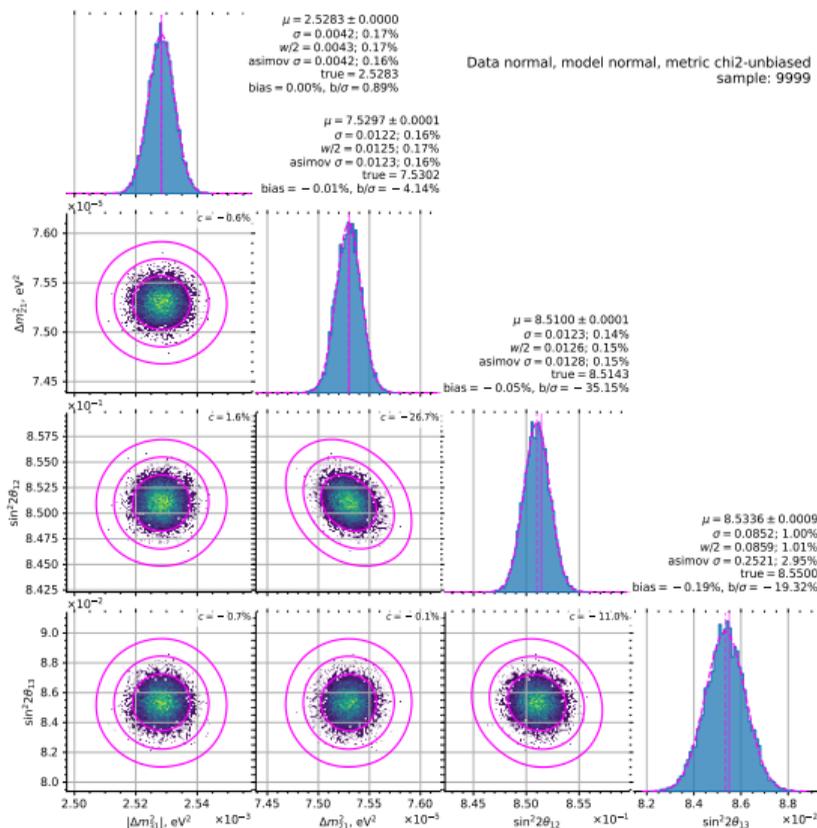
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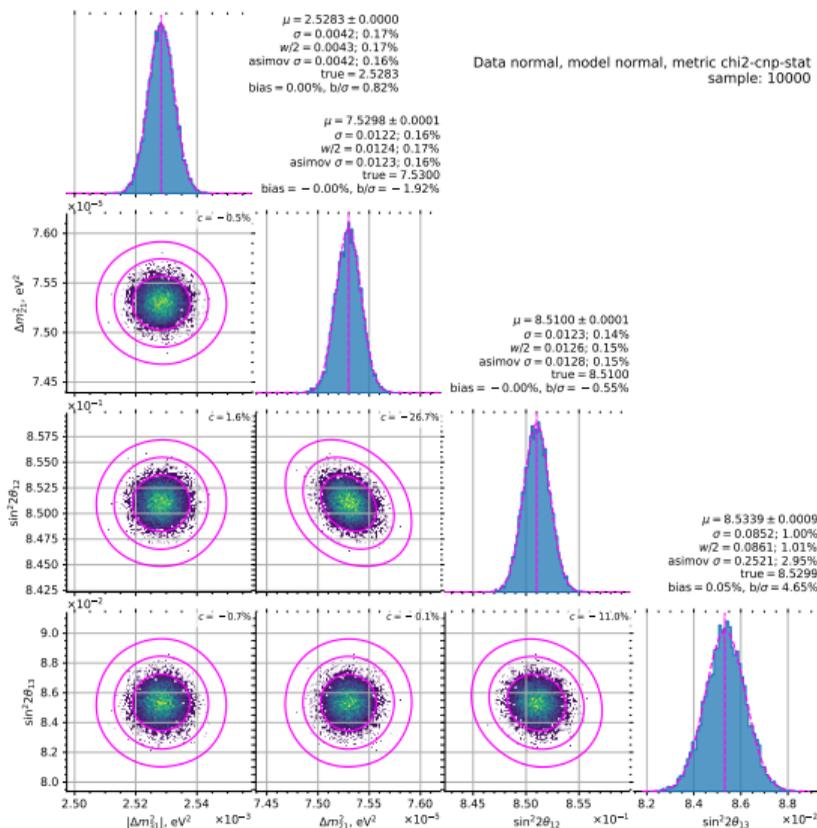
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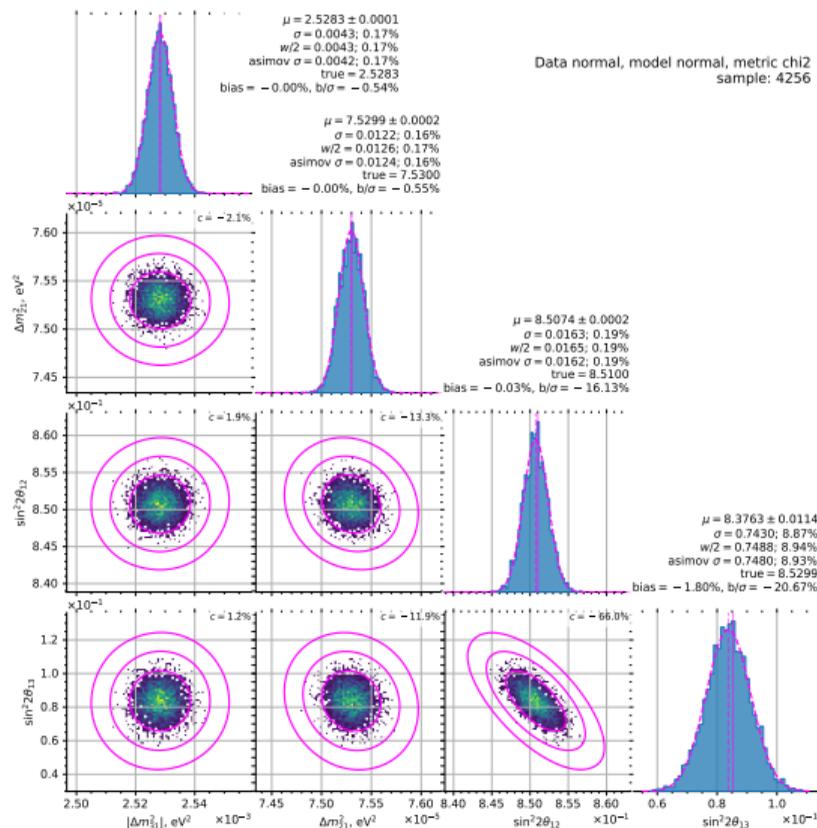
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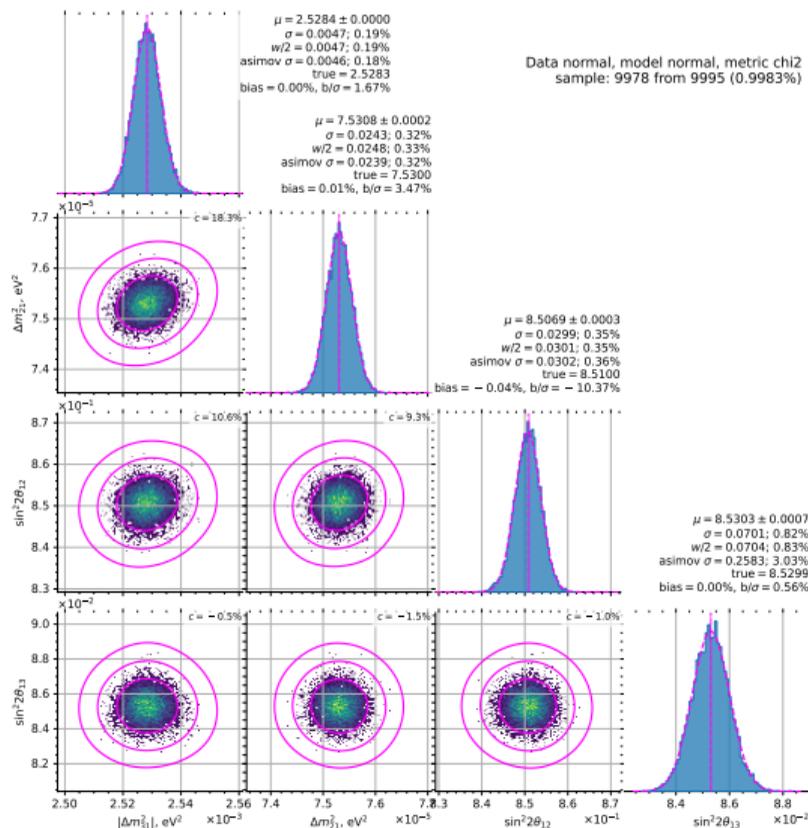
OSC PARS: STAT ONLY MODE, FREE $\sin^2 2\theta_{13}$



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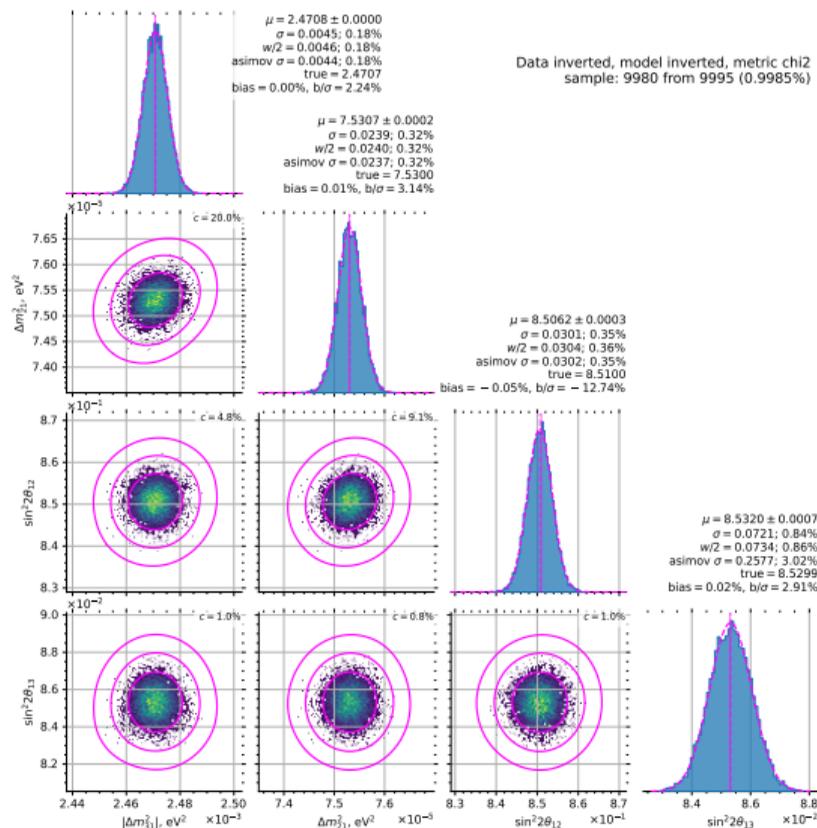
OSC PARS: STAT+SYST MODE



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- Stat+syst
 - ✗ Pearson's χ^2 introduces bias within 10% of σ



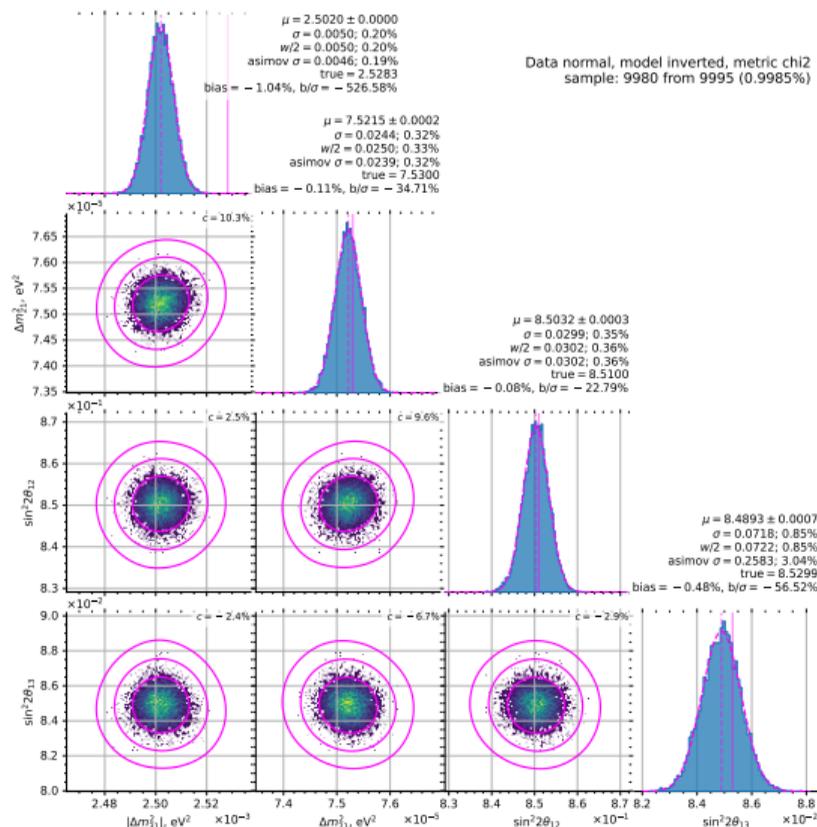
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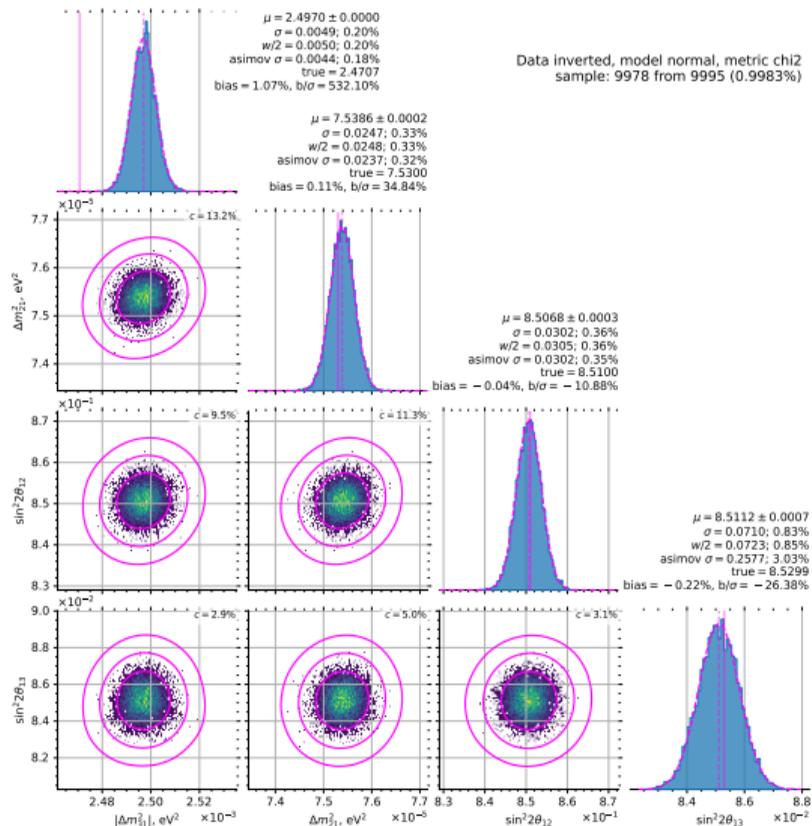
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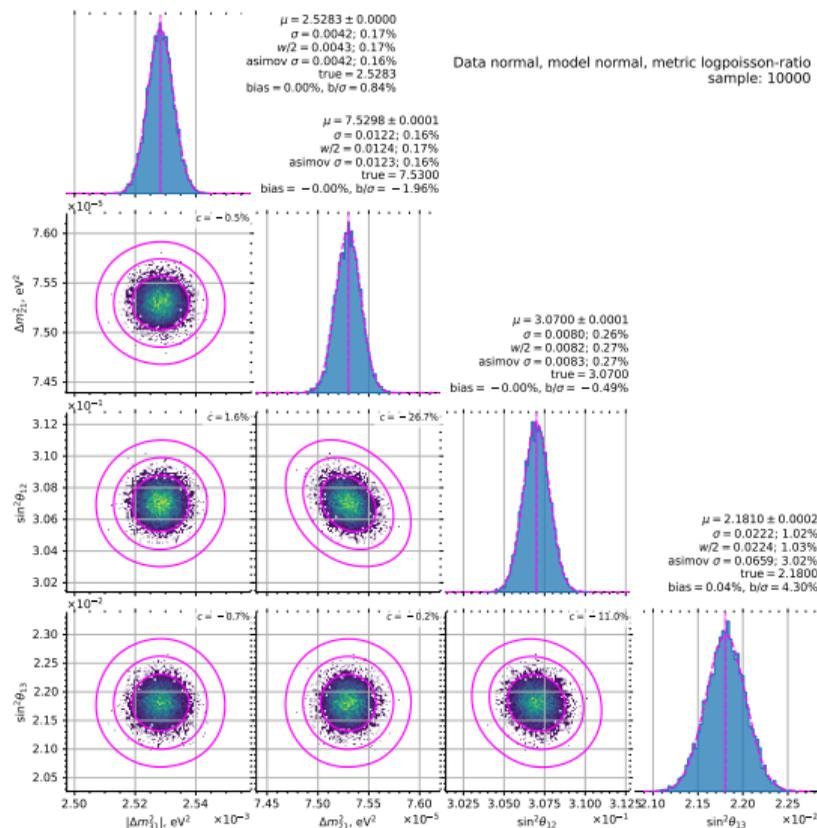
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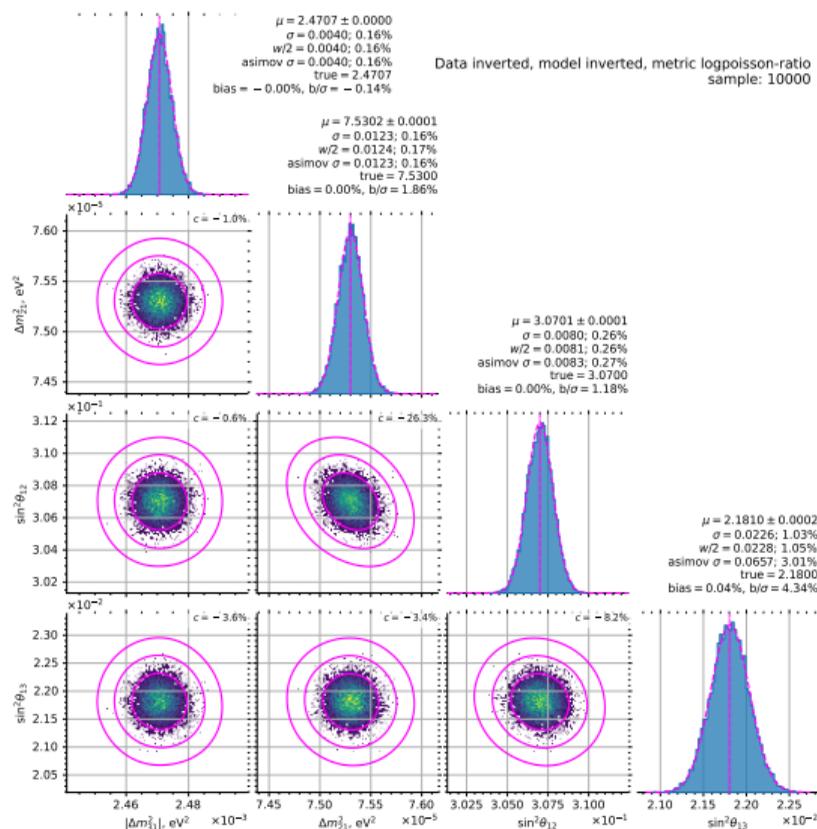
OSC PARS, SINGLE ANGLE: STAT ONLY MODE



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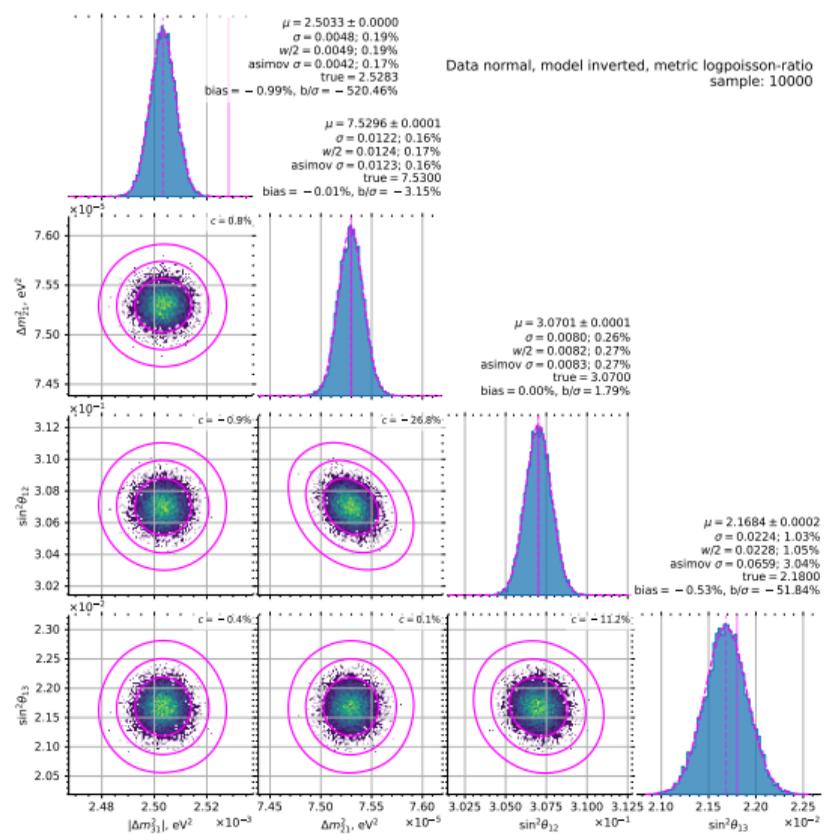
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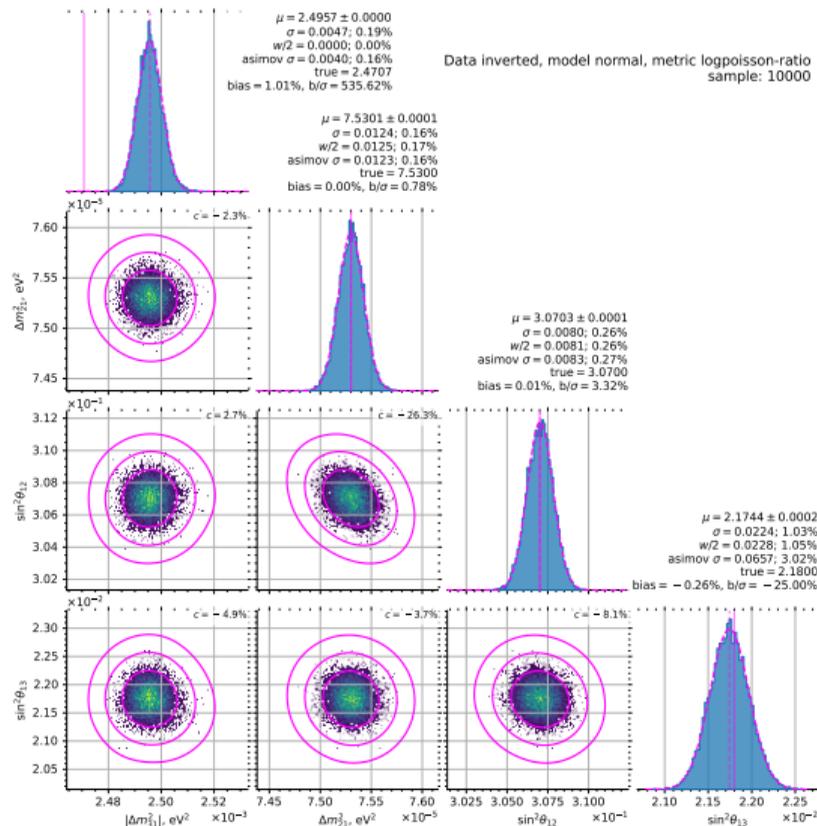
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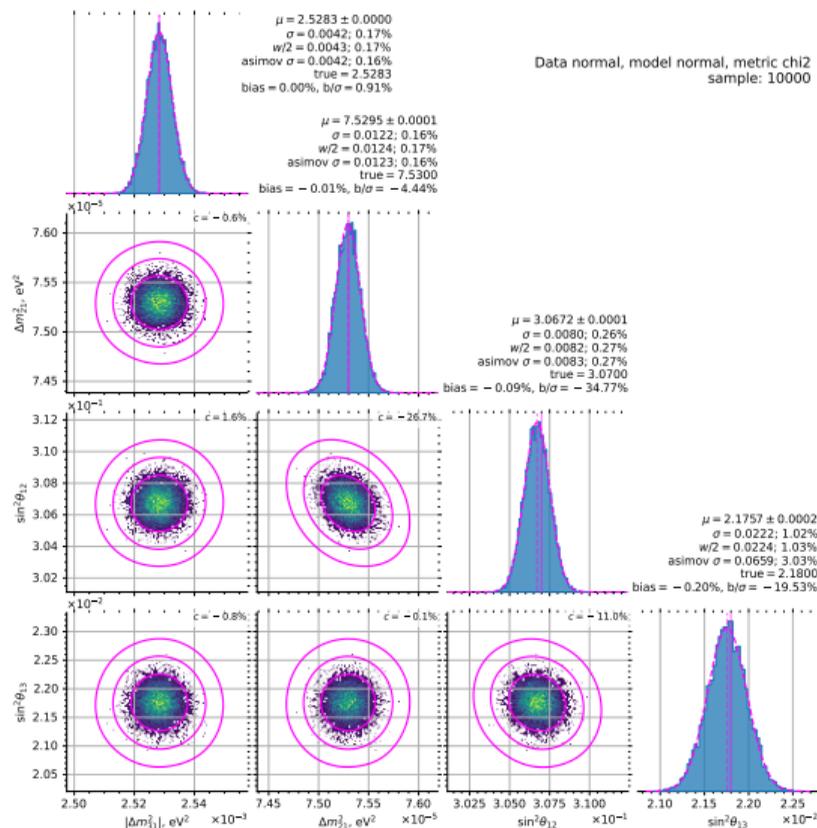
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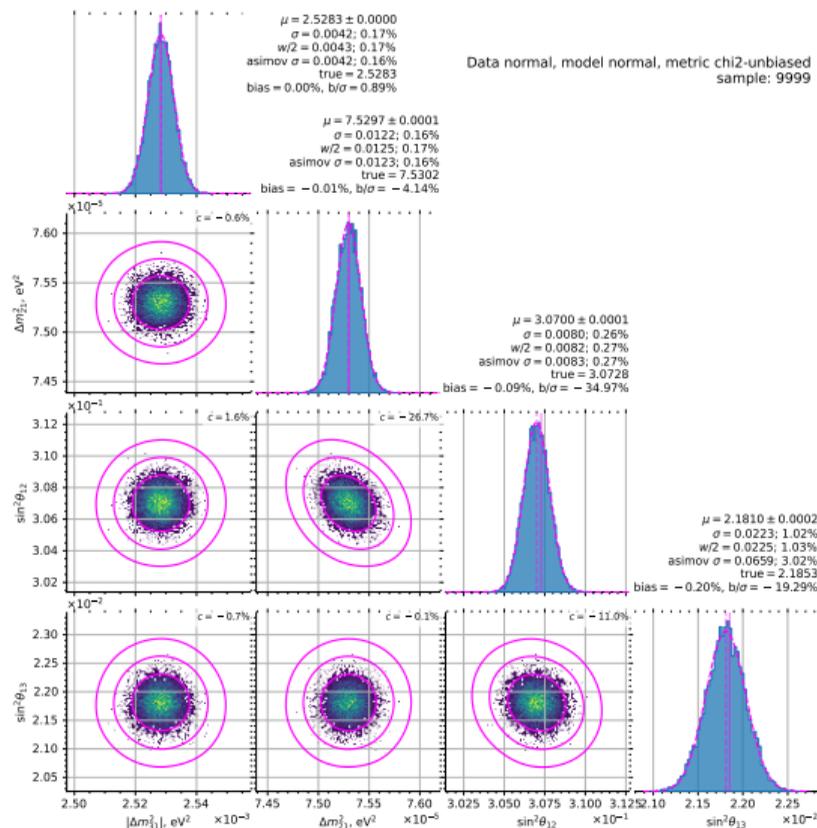
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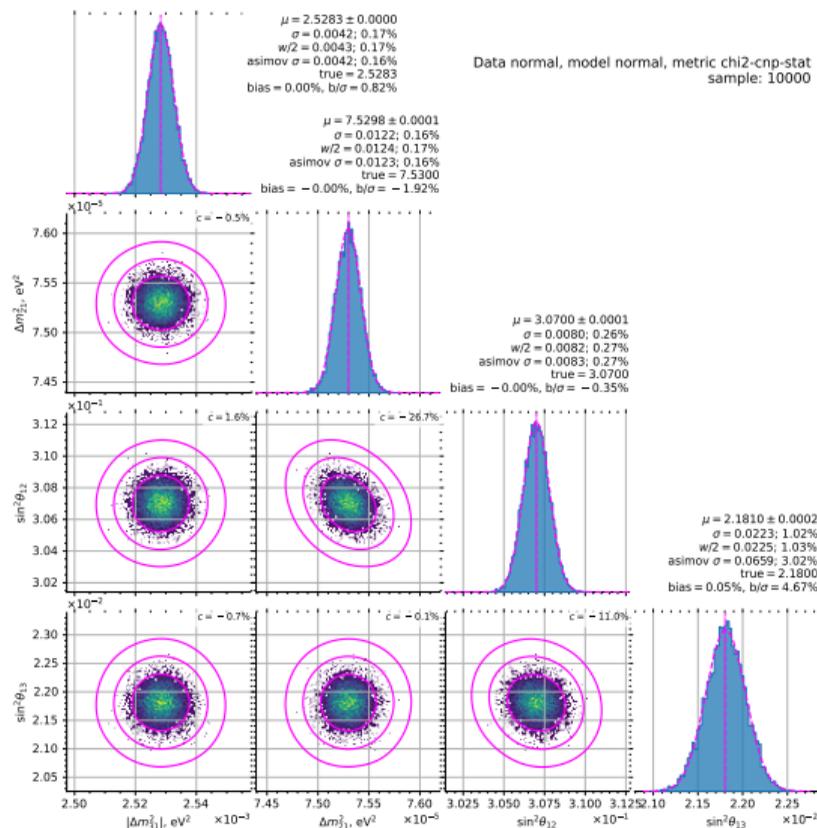
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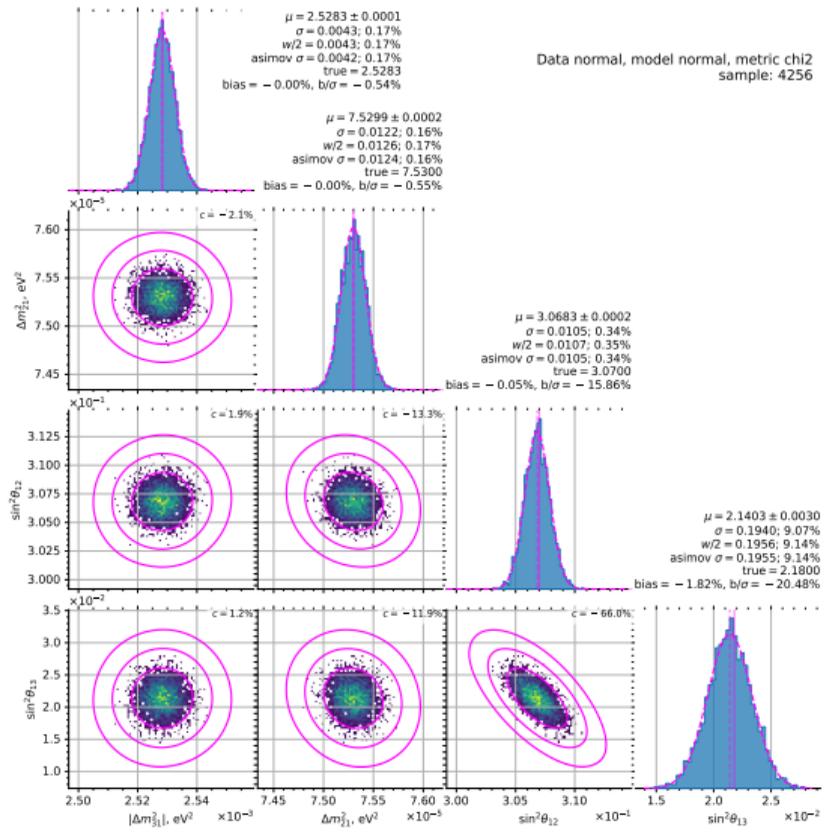
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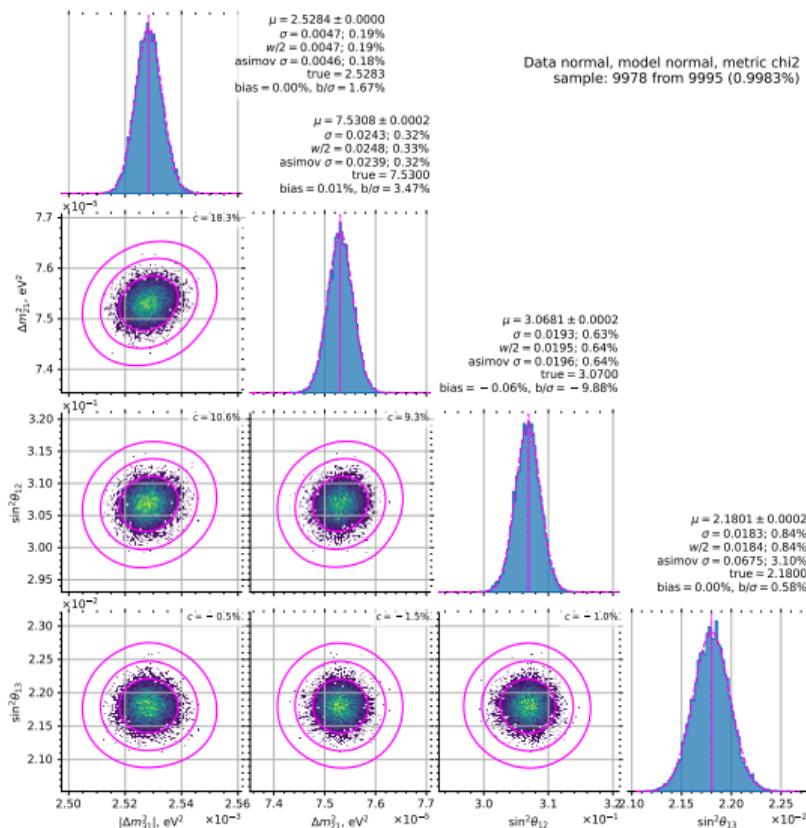
OSC PARS, SINGLE ANGLE: STAT ONLY MODE, FREE $\sin^2 \theta_{13}$



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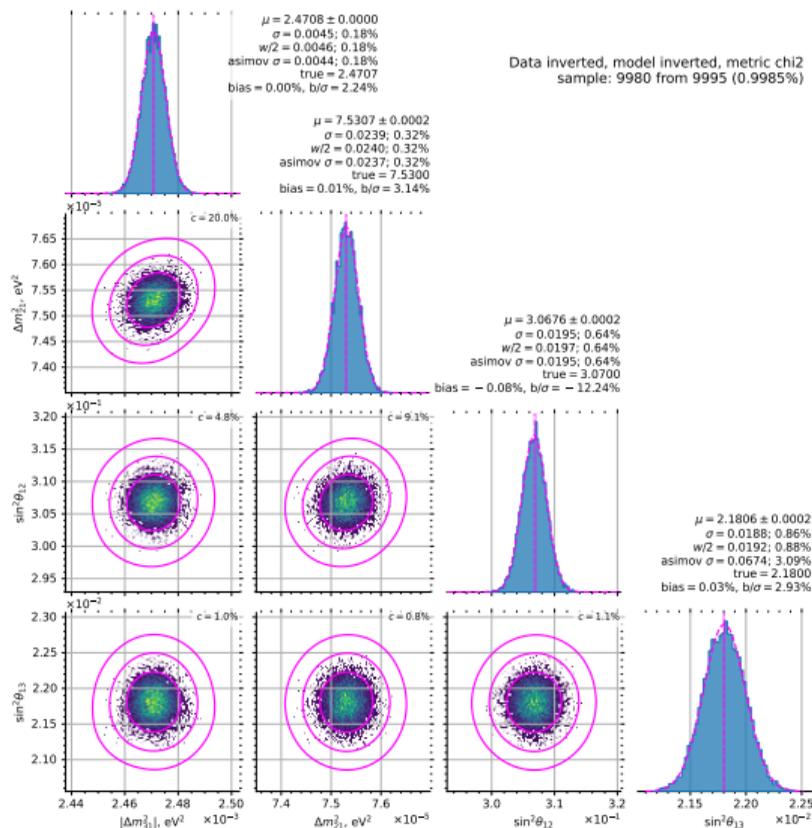
OSC PARS, SINGLE ANGLE: STAT+SYST MODE



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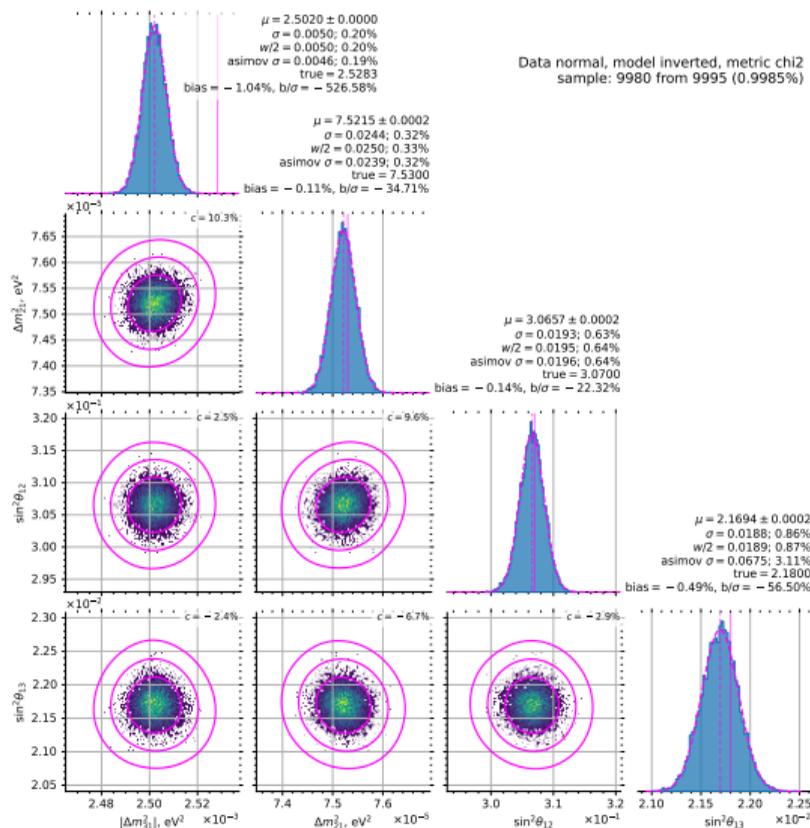
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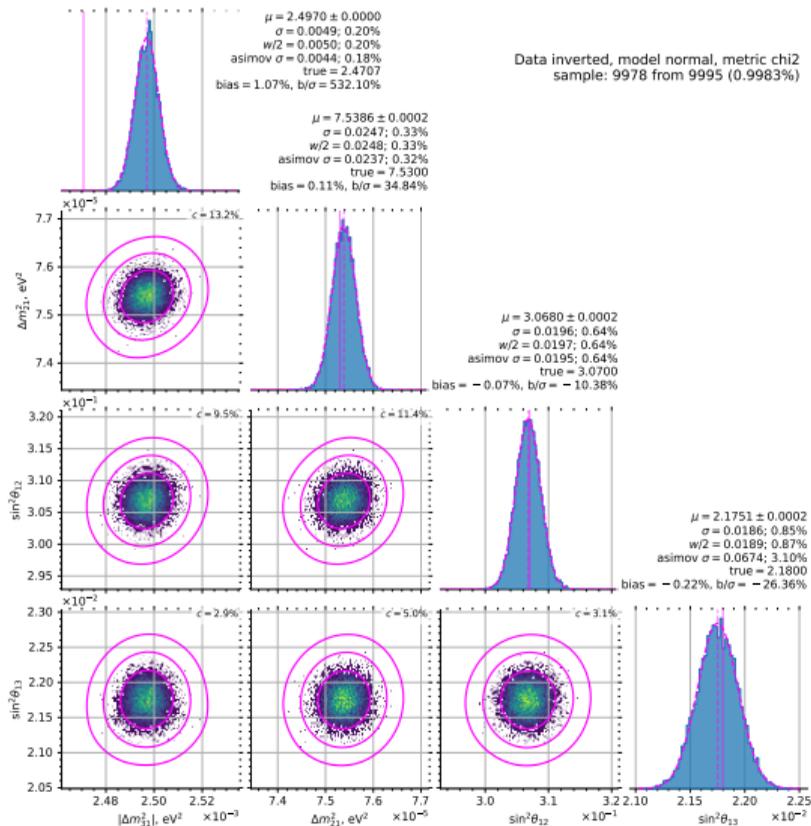
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THE PROBLEM

Inputs

- $\sin^2 2\theta_{13}$ uncertainty from unconstrained fit: 9%
- $\sin^2 2\theta_{13}$ nuisance term: 3%

Outputs

- Uncertainty from profiling the Asimov: $\sim 3\%$
- Width of the MC distribution: $\sim 1\%$



TOY MODEL

Summary

- Histogram μ_0
- Deficit $k \sim \sin^2 2\theta_{13}$
- Data x with fluctuations Δ

$$\mu(k) = (1 - k)\mu_0. \quad (1)$$

$$x = \mu(k_0) + \Delta, \quad (2)$$



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$$\mu(k) = (1 - k)\mu_0. \quad (1)$$

$$x = \mu(k_0) + \Delta, \quad (2)$$

Statistic

$$\chi^2 = (x - \mu(k))^T V^{-1} (x - \mu(k)) + \frac{(k - k_e)^2}{\sigma_{ke}^2}, \quad (3)$$

- External constraint on k around k_e with σ_{ke}^2
- Diagonal stat. covariance matrix V



TOY MODEL: SOLUTION

Solution

- From $\frac{d\chi^2}{dk} = 0$:

$$k = k_0 \frac{1 + \frac{k_e (1 - k_0)}{k_0 M_0 \sigma_{ke}^2} - \frac{\Delta_S}{k_0 M_0}}{1 + \frac{1 - k_0}{M_0 \sigma_{ke}^2}}. \quad (4)$$

- M_0 — sum of all bins of μ_0
- Δ_S — sum of all bins of Δ



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Frequentist uncertainty: confidence interval of an average experiment

$$(\sigma_k^F)^2 = \frac{1}{\frac{1}{2} \frac{d^2\chi^2}{dk^2}} = \frac{1}{\frac{M_0}{1 - k_0} + \frac{1}{\sigma_{ke}^2}} \quad (5)$$



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Frequentist uncertainty: confidence interval of an average experiment

$$\left(\sigma_k^F\right)^2 = \frac{1}{\frac{1}{2} \frac{d^2\chi^2}{dk^2}} = \frac{1}{\frac{1}{\sigma_{ks}^2} + \frac{1}{\sigma_{ke}^2}} \quad \sigma_{ks}^2 = \frac{1 - k_0}{M_0} \quad (5)$$

- σ_{ks} — statistical uncertainty, no external constraints



TOY MODEL: WIDTH OF THE DISTRIBUTION

Width of the distribution of k

$$\sigma_k^W = \sqrt{\langle (k - \langle k \rangle)^2 \rangle} \quad (6)$$



TOY MODEL: WIDTH OF THE DISTRIBUTION

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$$(\sigma_k^W)^2 = \frac{\langle \Delta_S^2 \rangle}{M_0^2} \frac{1}{\left(1 + \frac{1 - k_0}{M_0 \sigma_{ke}^2}\right)^2}. \quad (6)$$

$$\langle \Delta_S^2 \rangle = (1 - k_0) M_0 \quad (7)$$



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$$\langle \Delta_S^2 \rangle = (1 - k_0) M_0 \quad (7)$$

$$\sigma_k^W = \frac{\sigma_{ks}}{1 + \left(\frac{\sigma_{ks}}{\sigma_{ke}}\right)^2}. \quad (8)$$



TOY MODEL: COMPARISON

Frequentist confidence interval

$$\sigma_k^F = \frac{1}{\sqrt{\frac{1}{\sigma_{ks}^2} + \frac{1}{\sigma_{ke}^2}}} \quad (9)$$

Width of the distribution

$$\sigma_k^W = \frac{\sigma_{ks}}{1 + \left(\frac{\sigma_{ks}}{\sigma_{ke}}\right)^2} \quad (10)$$



TOY MODEL: COMPARISON

Frequentist confidence interval

- An interval of k , for which the best fit statistic (data given k) falls into 68.3% band.

$$\sigma_k^F = \frac{1}{\sqrt{\frac{1}{\sigma_{ks}^2} + \frac{1}{\sigma_{ke}^2}}} \quad (9)$$

Width of the distribution

- The width of the distribution of k :
 - ▶ averaged over statistical fluctuations of data
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$$\delta_k^F = \frac{1}{\sqrt{\frac{1}{\delta_{ks}^2} + \frac{1}{\delta_{ke}^2}}} = \frac{1}{\sqrt{\frac{1}{9^2} + \frac{1}{3^2}}} \% = \frac{9}{\sqrt{10}} \% = 2.84\% \quad (9)$$

Width of the distribution

- The width of the distribution of k :
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$$\delta_k^W = \frac{\delta_{ks}}{1 + \left(\frac{\delta_{ks}}{\delta_{ke}}\right)^2} = \frac{9\%}{1 + \left(\frac{9\%}{3\%}\right)^2} = 0.9\% \quad (10)$$



CONCLUSION AND ROADMAP

Conclusion

- ✓ Obtained JUNO's MC sensitivity to NMO and oscillation pars
 - ✓ Both stat-only and stat+syst options demonstrate Gaussian behavior
 - ✓ Compare one-sided uncertainties
 - ✗ In these studies: MC sensitivities are worse than Asimov estimates
 - ✓ Uncertainty due to finite sample is estimated
- ✓ Studied sensitivity to the oscillation parameters
 - ✓ For Δm_{31}^2 , Δm_{21}^2 and $\sin^2 2\theta_{12}$ the results are consistent with Asimov
 - ✓ For $\sin^2 2\theta_{13}$ the uncertainty is smaller on MC: understood
 - ✓ Bias due to Pearson's χ^2 is small for stat+syst mode

TODO

- Update the TechNote
- 100k samples for full systematics: ongoing, ~2k left

Open questions

- ✗ Minor: biased unbiased χ^2

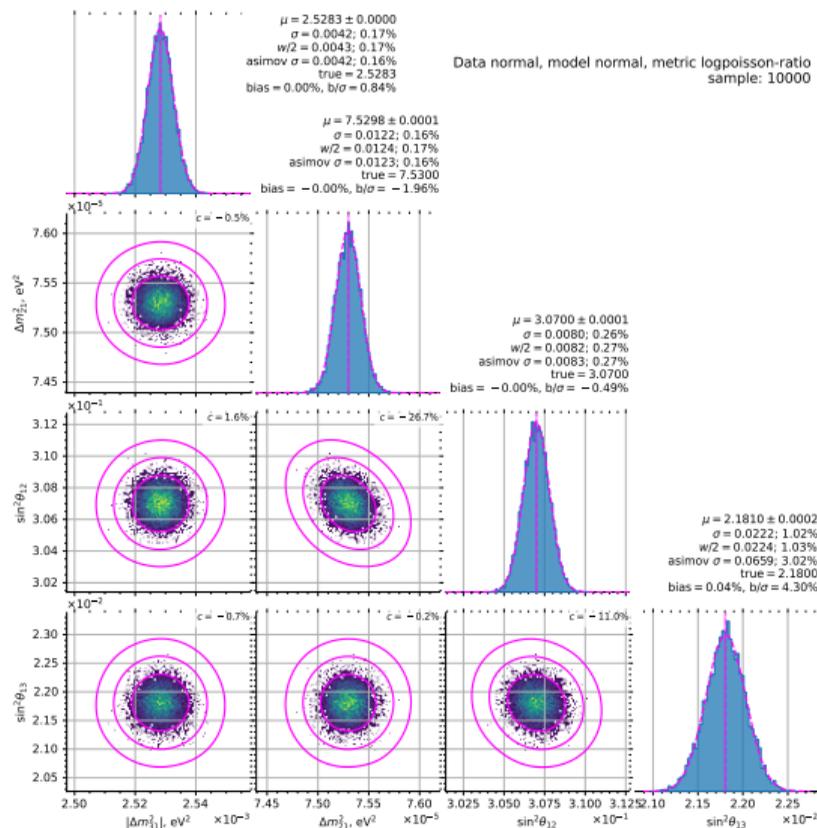
Thank you for your attention!

Spare slides:

5 OSCILLATION PARAMETERS

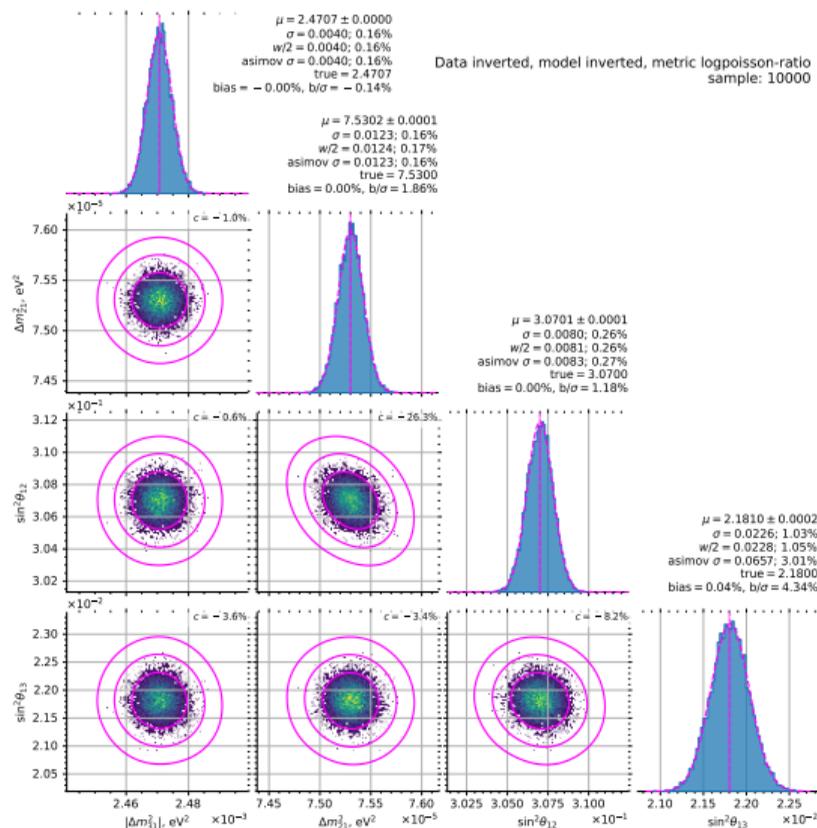
6 CALCULATION TIME

OSC PARS, SINGLE ANGLE: STAT ONLY MODE



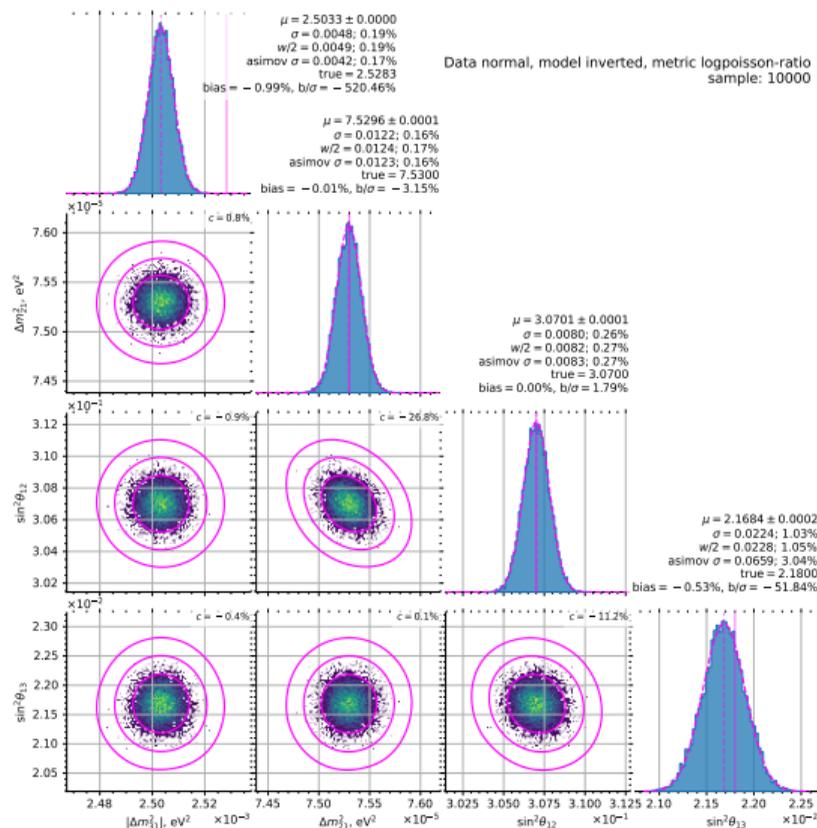
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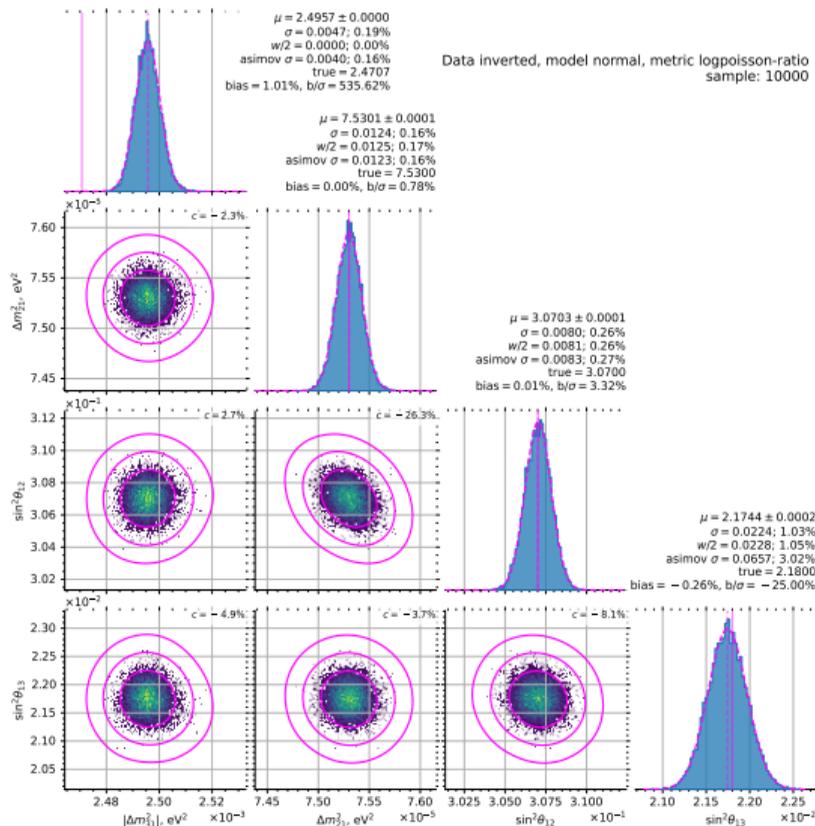
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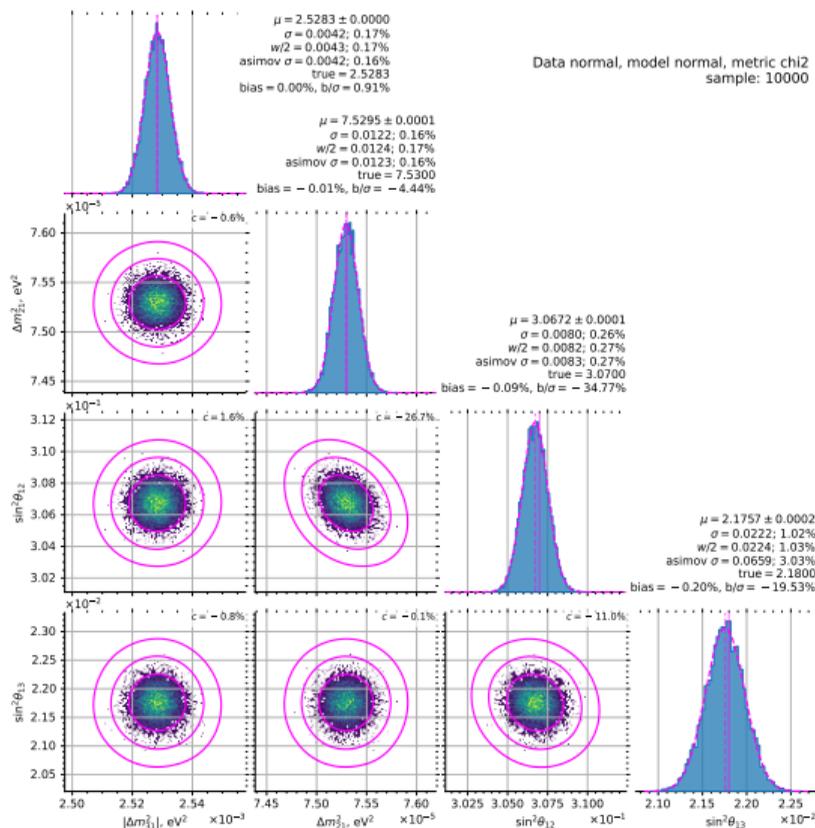
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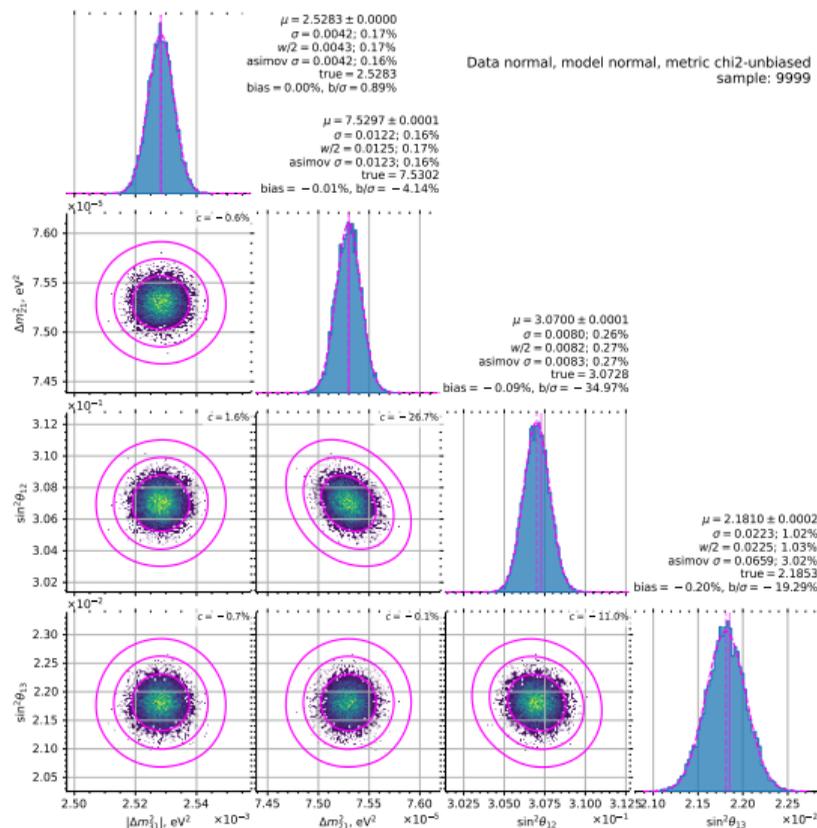
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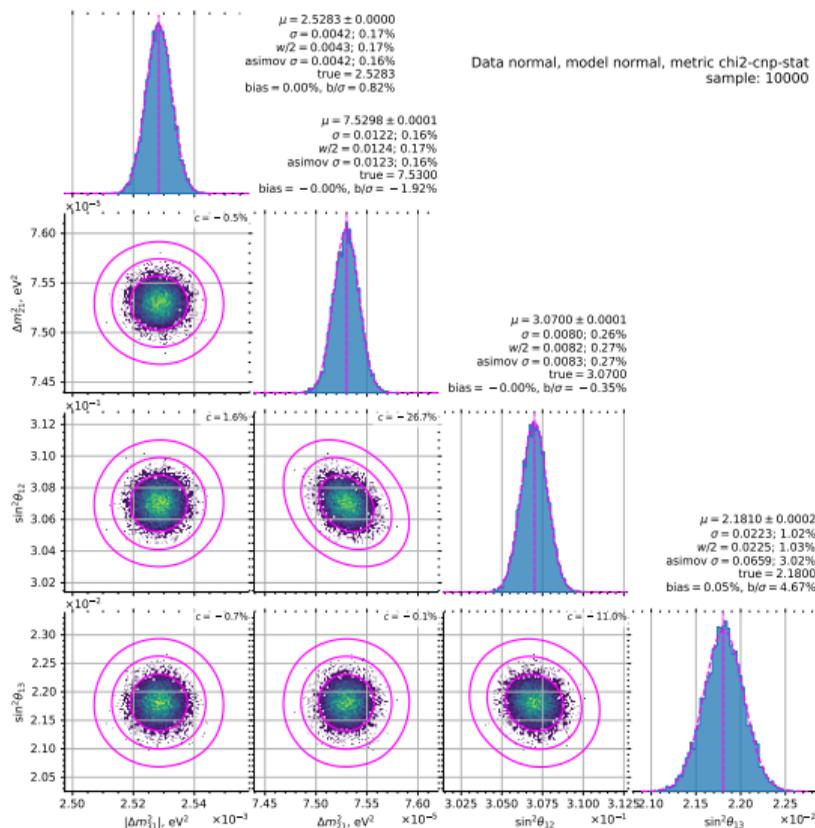
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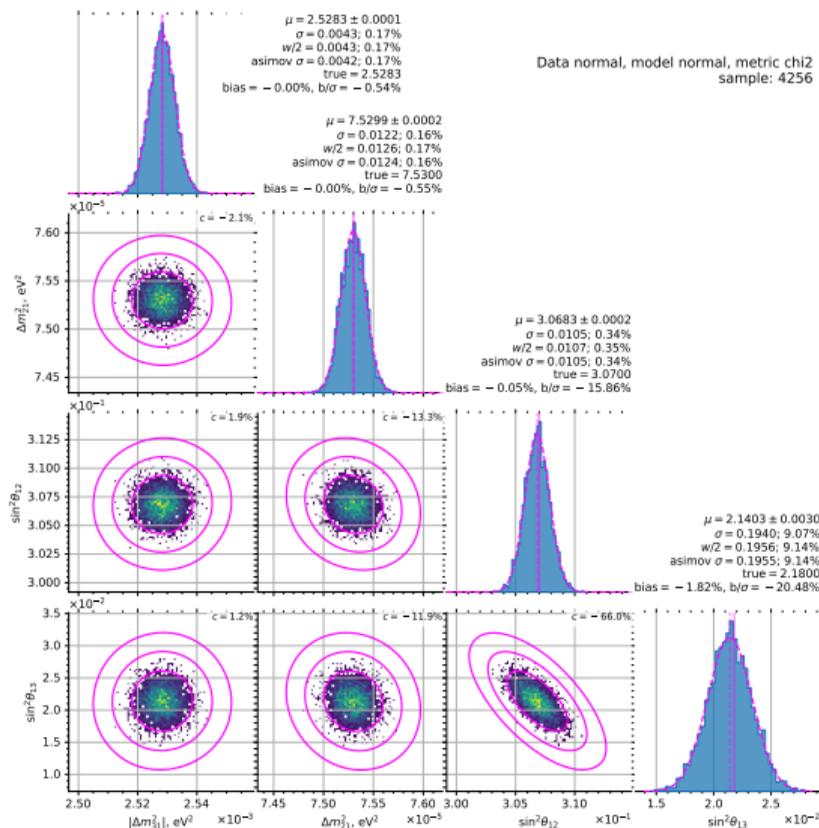
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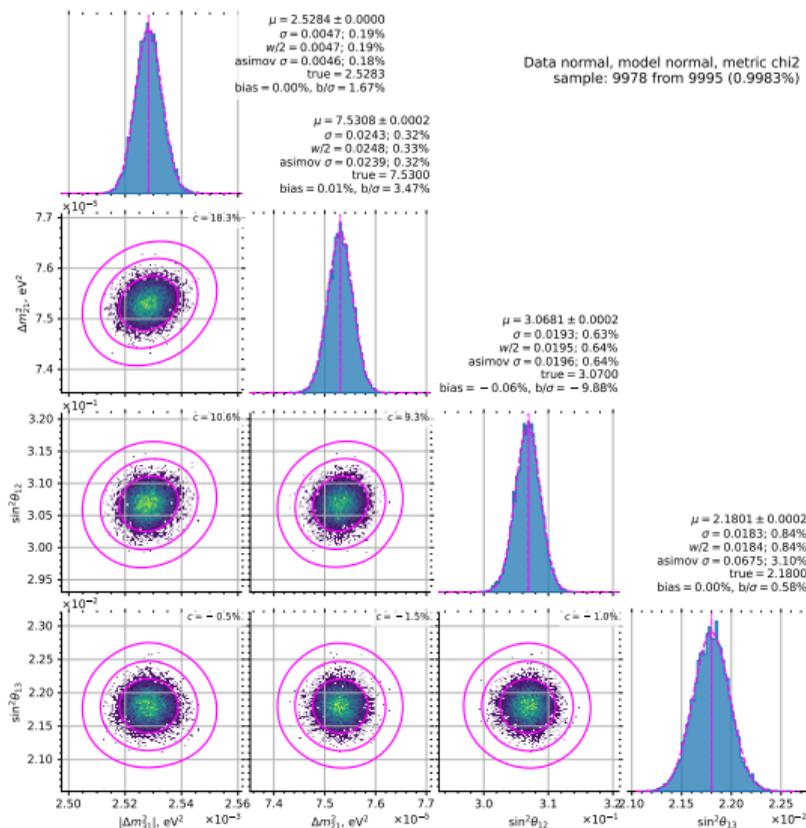
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OSC PARS, SINGLE ANGLE: STAT ONLY MODE, FREE $\sin^2 2\theta_{13}$



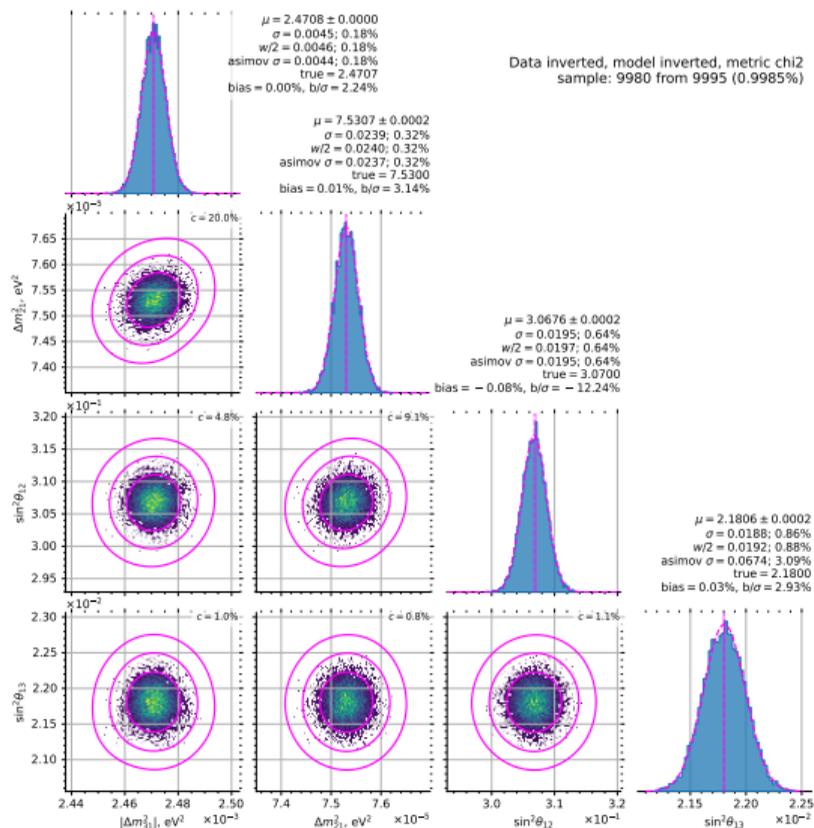
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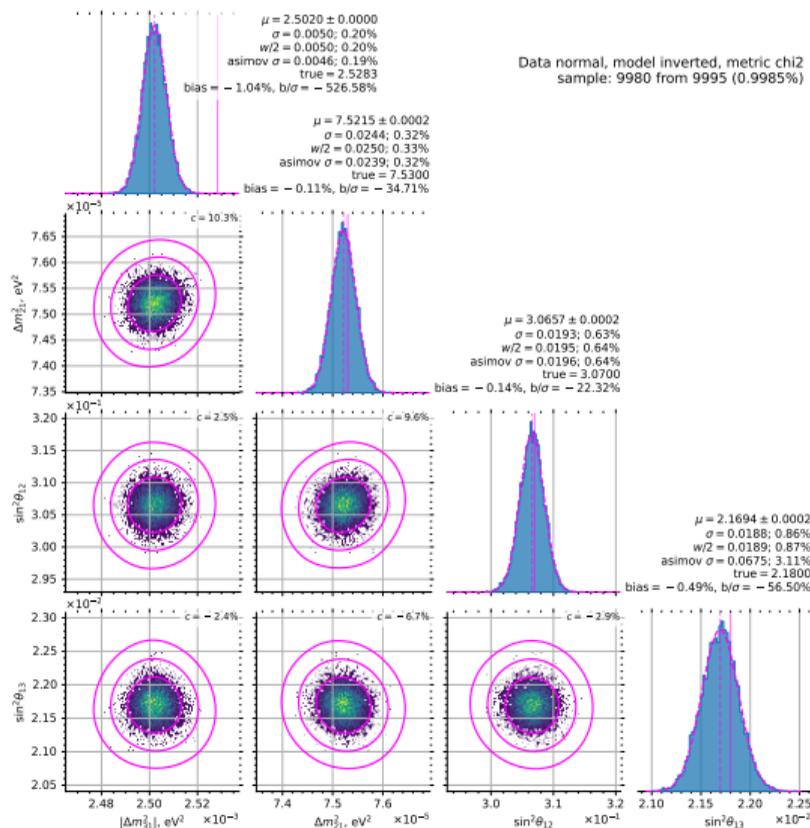
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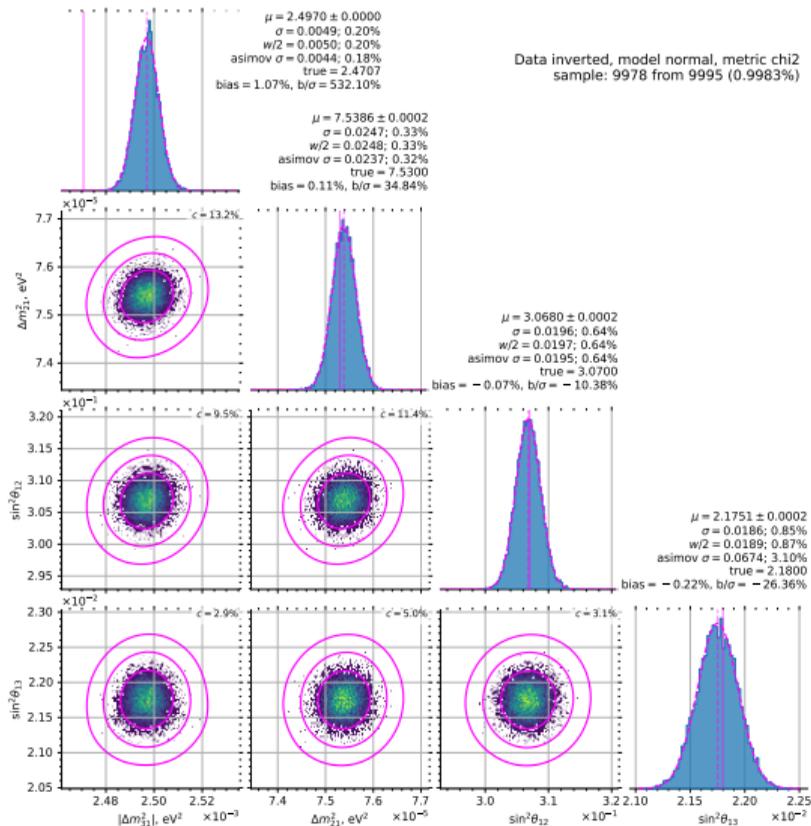
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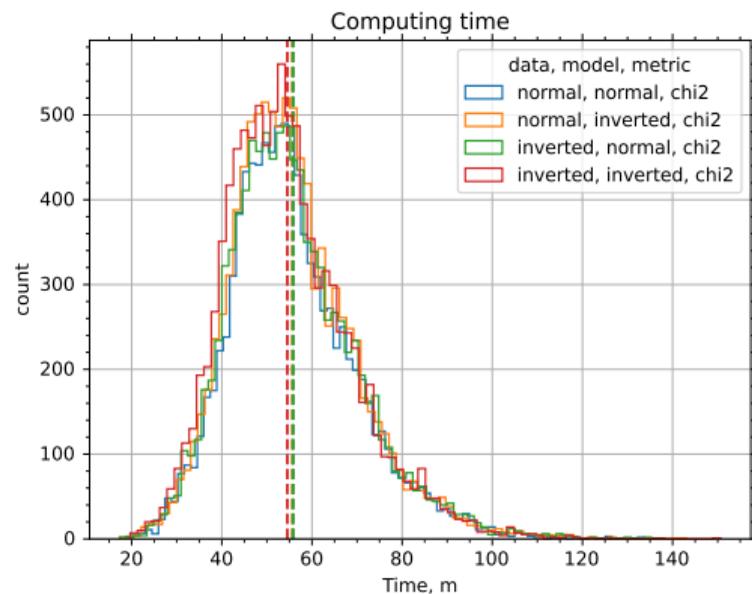
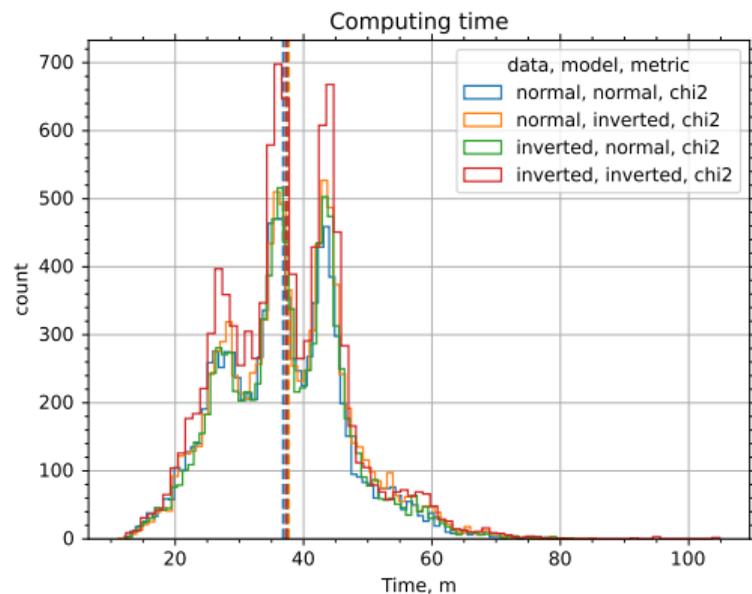
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CALCULATION TIME



- Statistics 100k samples is reachable