

# Nuclear Energy Density Functionals

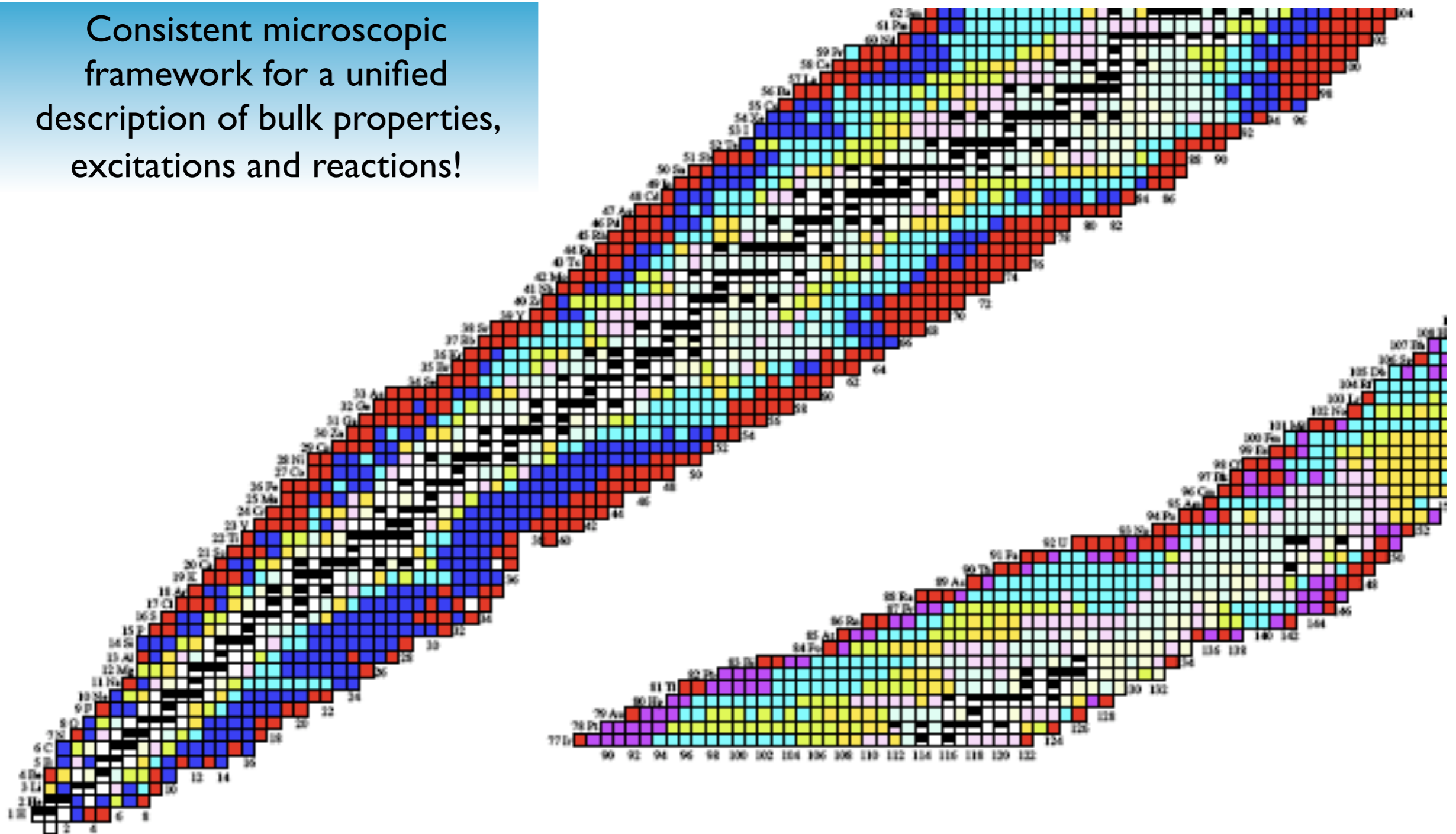
Dario Vretenar  
University of Zagreb

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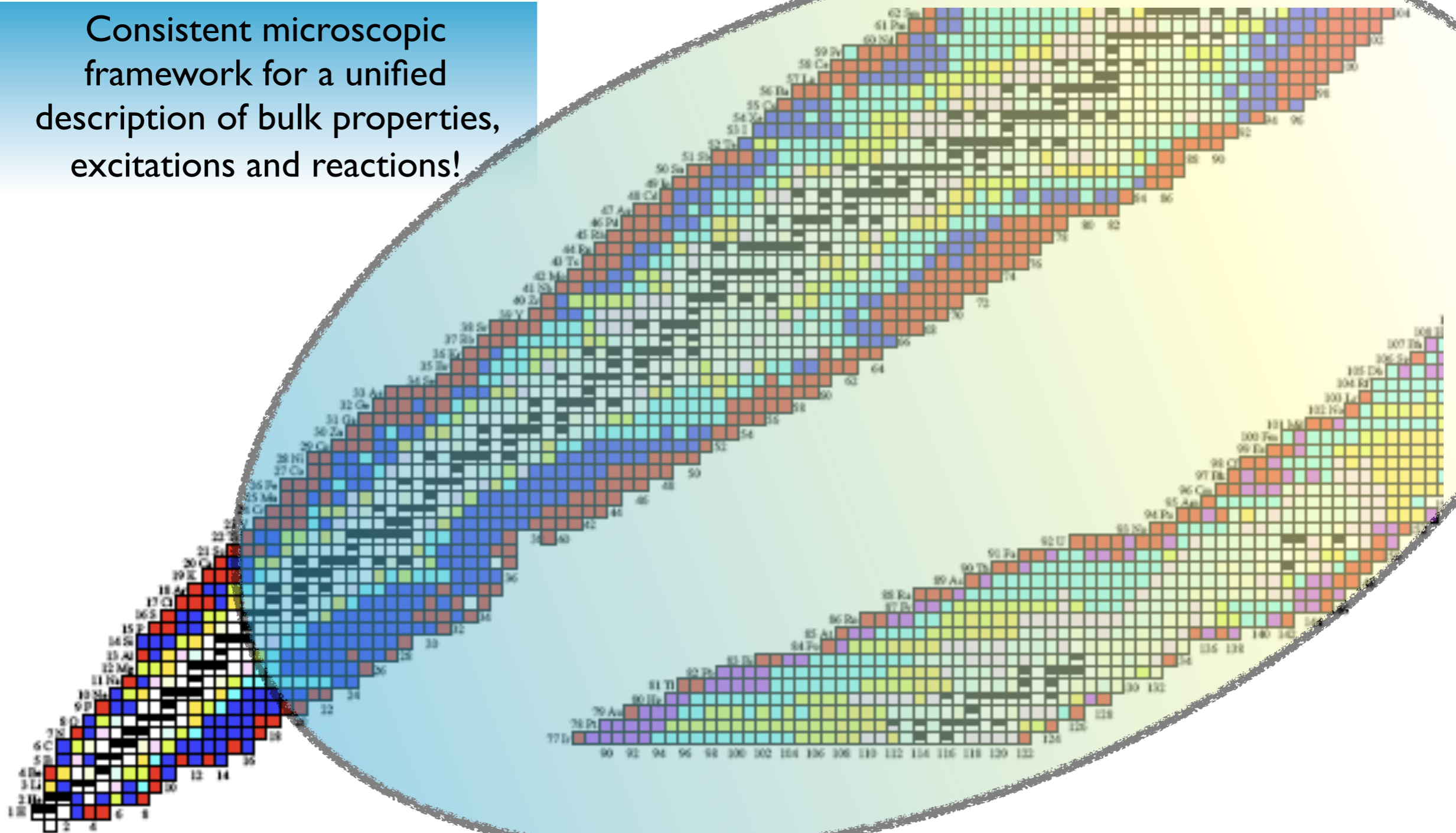
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Consistent microscopic framework for a unified description of bulk properties, excitations and reactions!



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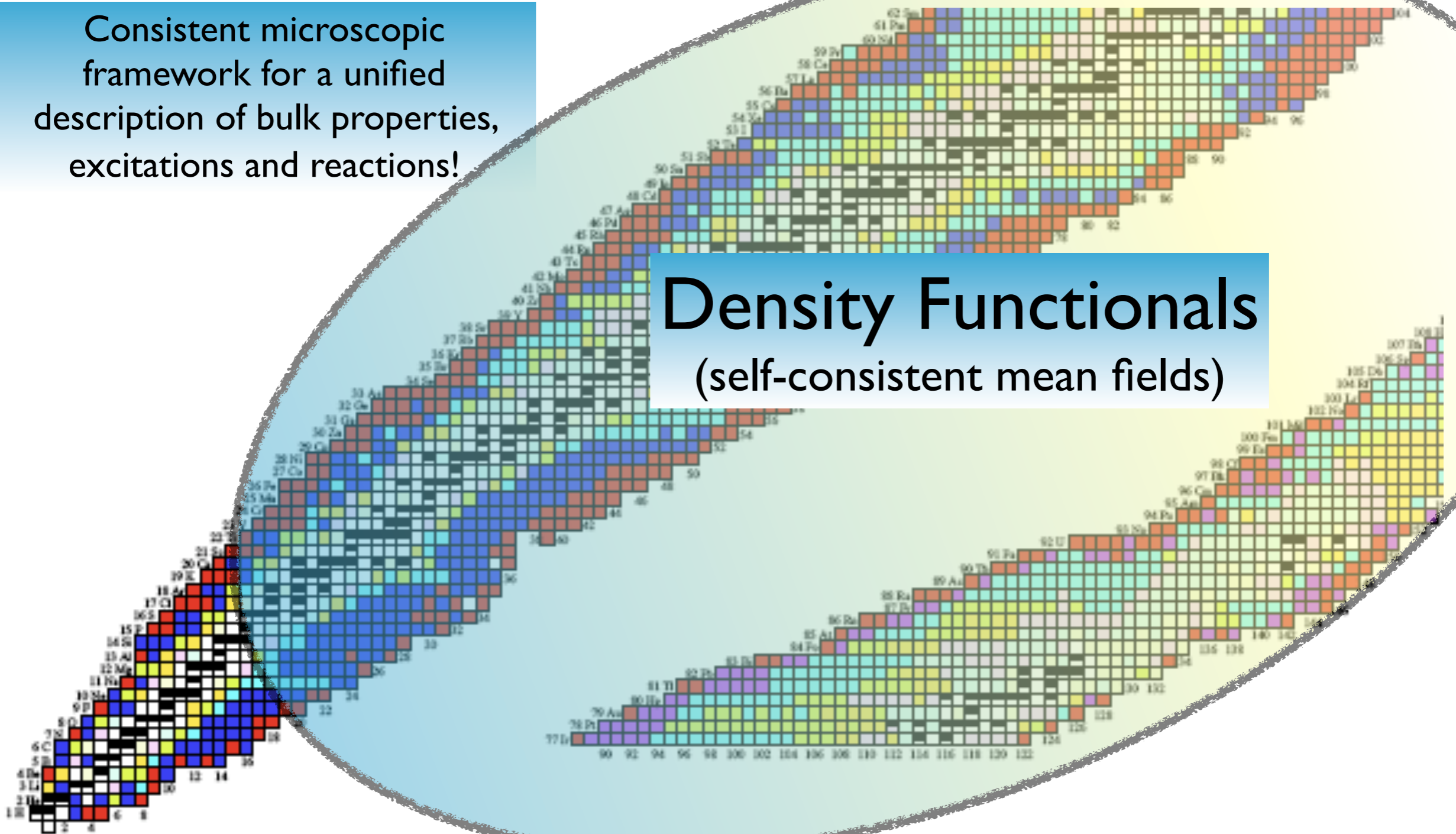
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**Density Functionals**  
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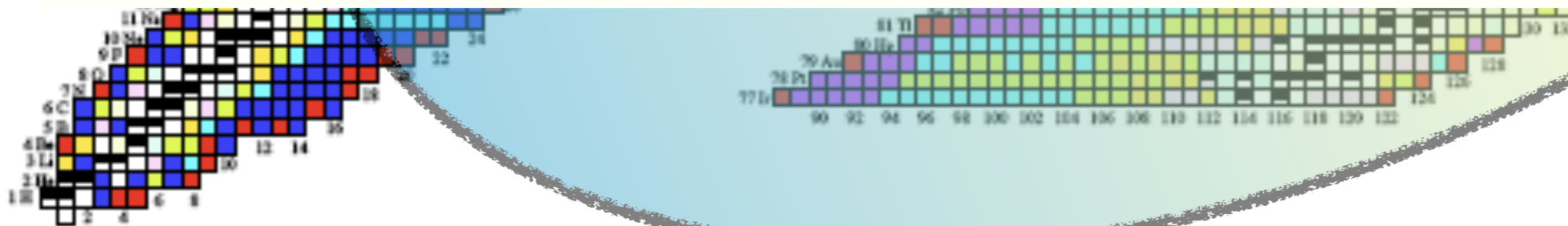


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The many-body problem is mapped onto a one body problem without explicitly involving inter-nucleon interactions!



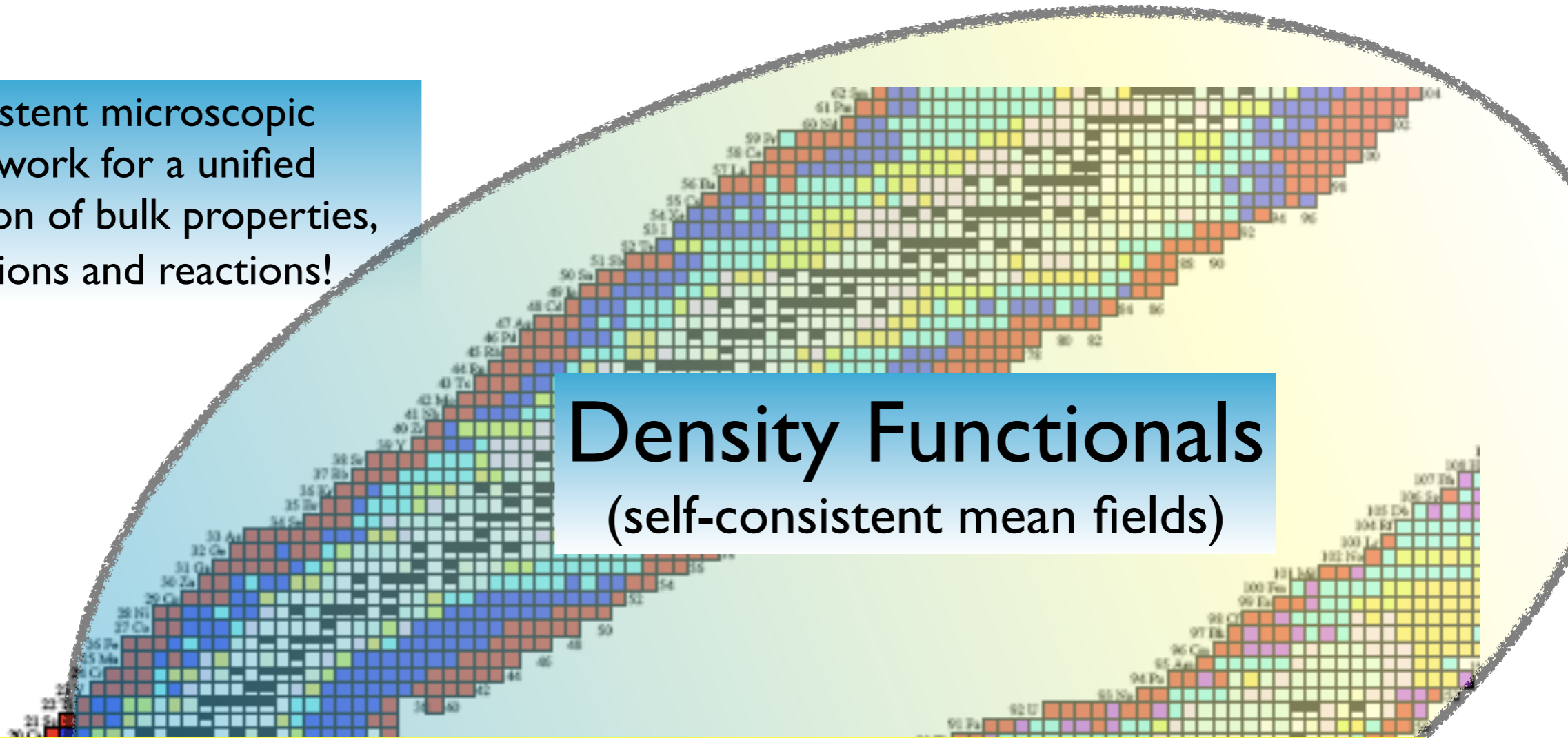
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Density Functionals  
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The exact density functional is approximated with **powers and gradients of ground-state nucleon densities and currents.**



## Local densities and currents:

T=0 density:

$$\rho_0(\mathbf{r}) = \rho_0(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma\tau)$$

T=I density:

$$\rho_1(\mathbf{r}) = \rho_1(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma\tau) \tau$$

T=0 spin density:

$$\mathbf{s}_0(\mathbf{r}) = \mathbf{s}_0(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma'\tau) \boldsymbol{\sigma}_{\sigma'\sigma}$$

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Current:

$$\mathbf{j}_T(\mathbf{r}) = \frac{i}{2} (\nabla' - \nabla) \rho_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

Spin-current tensor:

$$\mathcal{J}_T(\mathbf{r}) = \frac{i}{2} (\nabla' - \nabla) \otimes \mathbf{s}_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

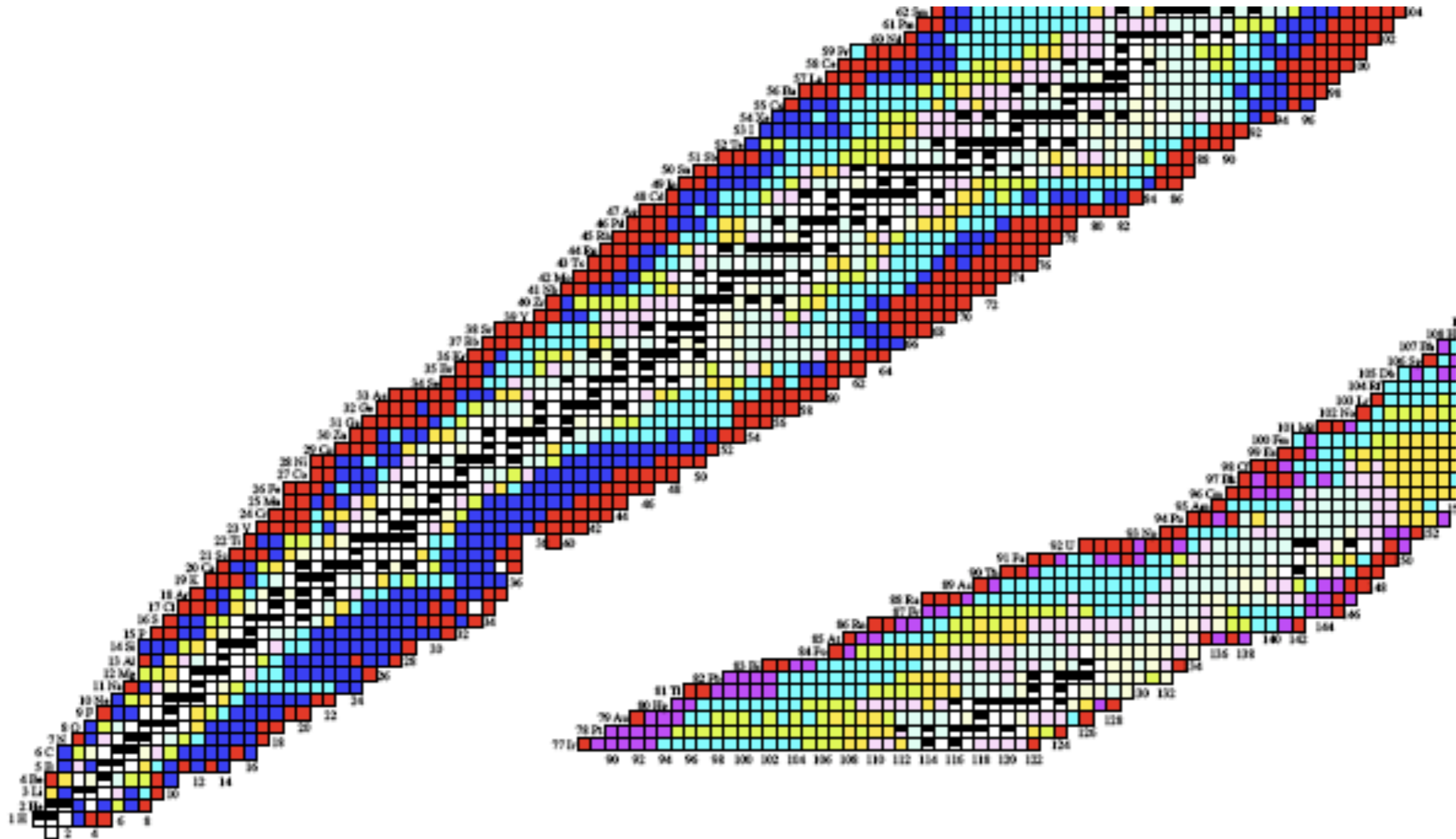
Kinetic density:

$$\tau_T(\mathbf{r}) = \nabla \cdot \nabla' \rho_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

Kinetic spin-density:

$$\mathbf{T}_T(\mathbf{r}) = \nabla \cdot \nabla' \mathbf{s}_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

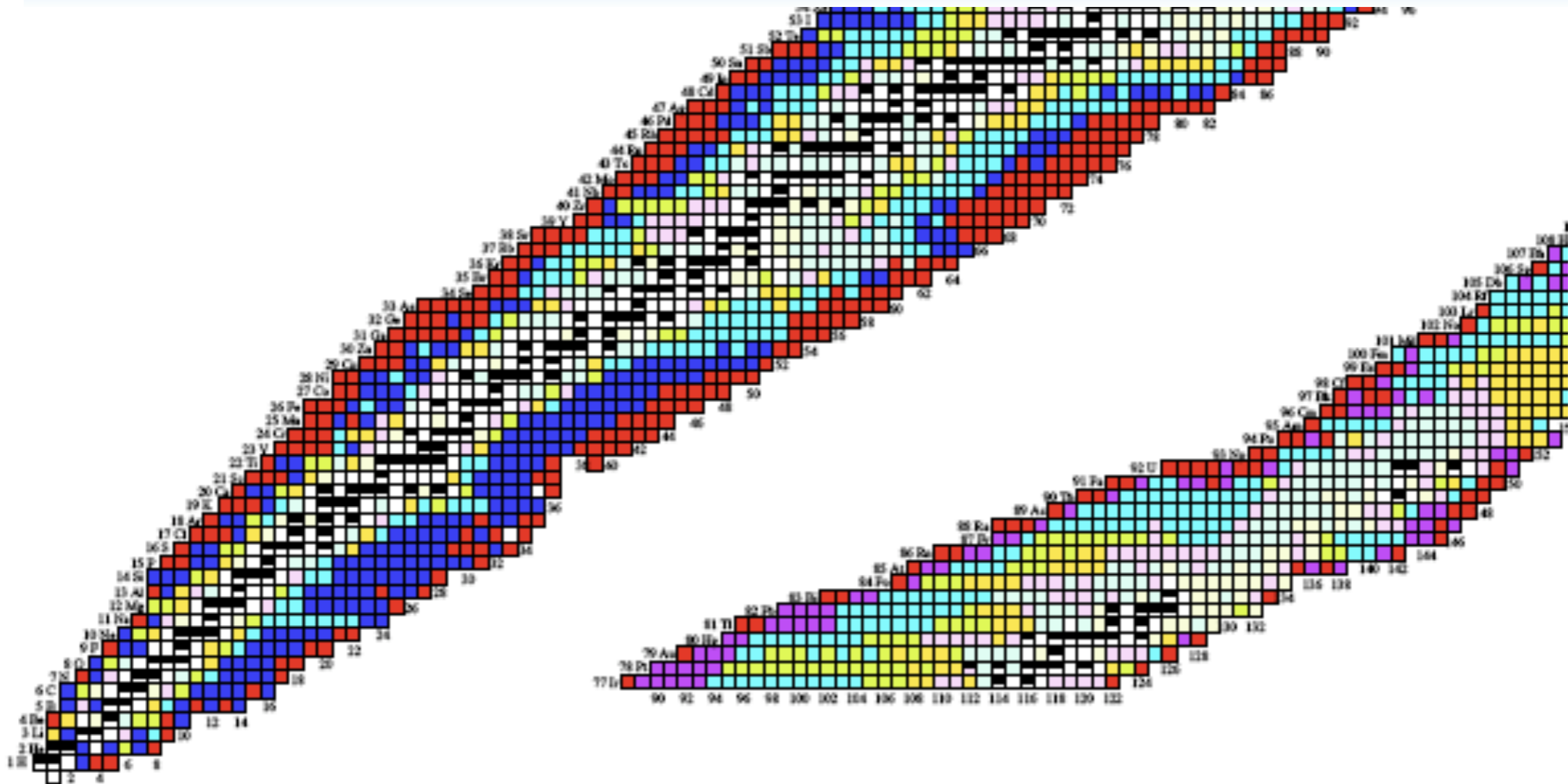
# Relativistic Energy Density Functionals





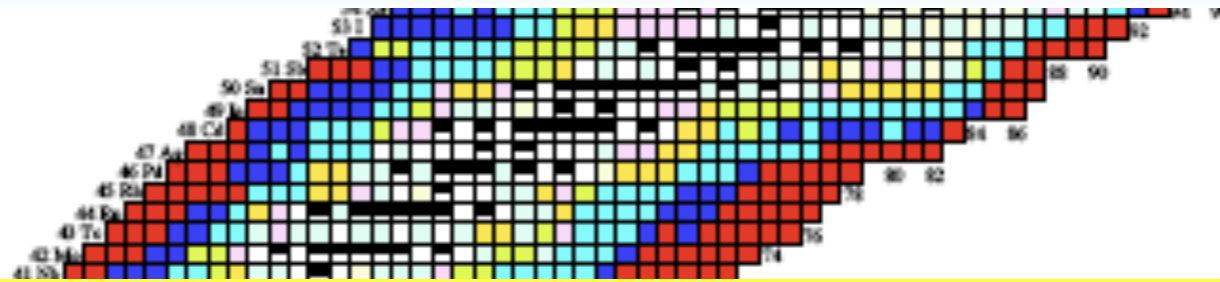
# Relativistic Energy Density Functionals

- ✓ natural inclusion of the **spin degree of freedom** (spin-orbit potential with empirical strength)

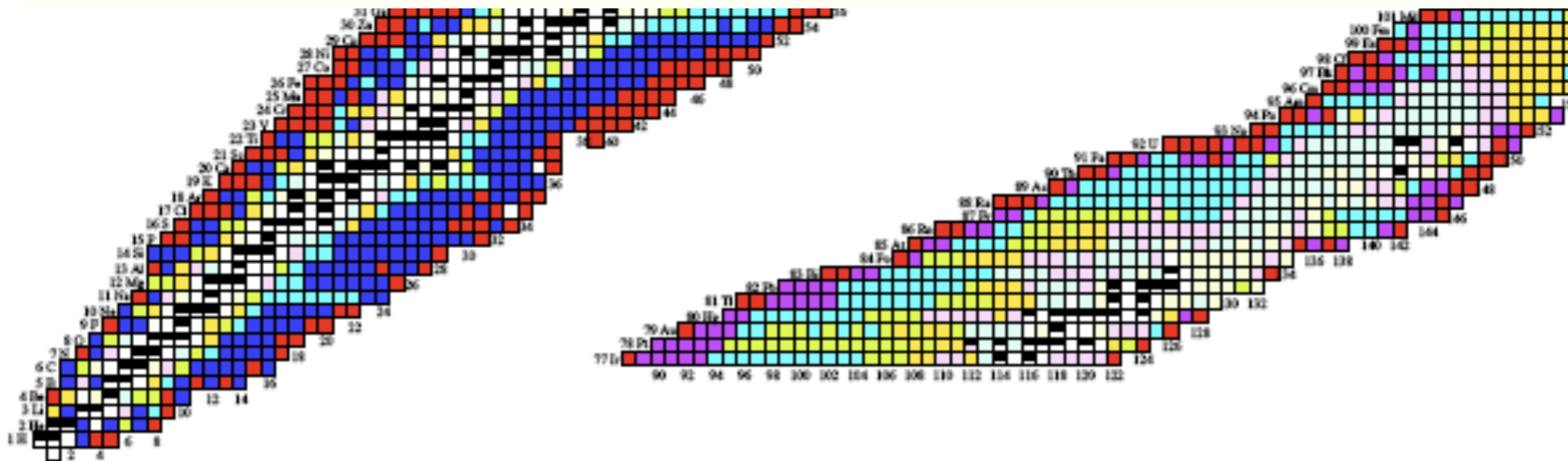


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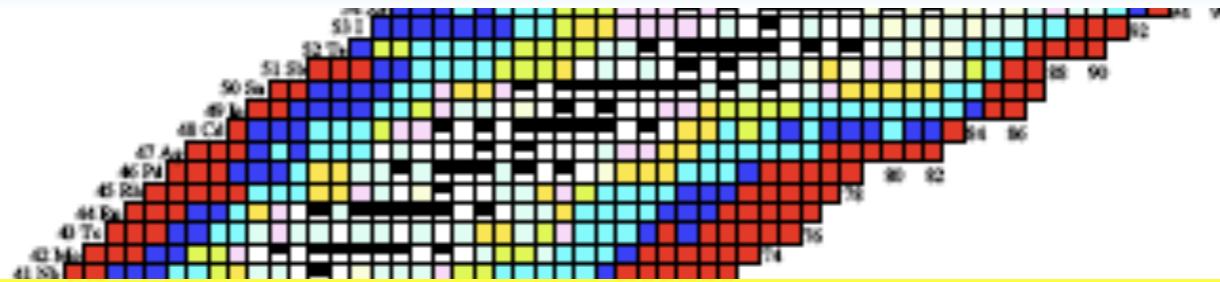


- ✓ unique parameterization of **time-odd components** (currents) of the nuclear mean-field



# Relativistic Energy Density Functionals

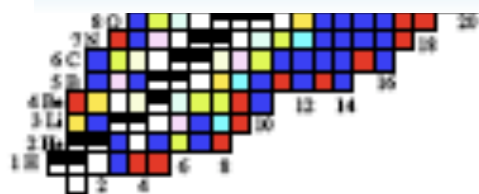
- ✓ natural inclusion of the **spin degree of freedom** (spin-orbit potential with empirical strength)



- ✓ unique parameterization of **time-odd components** (currents) of the nuclear mean-field



- ✓ the distinction between scalar and vector self-energies leads to a natural **saturation mechanism for nuclear matter**



# Advantages of the Energy Density Functional approach to nuclear structure

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- ✓ the use of ***universal density functionals*** that can be applied to all nuclei throughout the periodic chart

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Important for extrapolations to regions far from stability!





... microscopic foundation for a universal EDF framework, related to and constrained by low-energy QCD

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... correlations related to restoration of broken symmetries and fluctuations of collective coordinates

# Semi-empirical functionals

Infinite nuclear matter cannot determine the density functional on the level of accuracy that is needed for a quantitative description of structure phenomena in finite nuclei.

... start from a favorite microscopic nuclear matter EOS

... the parameters of the functional are fine-tuned to data of finite nuclei

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Nikšić, Vretenar, and Ring, Phys. Rev. C **78**, 034318 (2008)

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Nikšić, Vretenar, and Ring, Phys. Rev. C **78**, 034318 (2008)

... starts from microscopic nucleon self-energies in nuclear matter.

... parameters adjusted in self-consistent mean-field calculations of masses of **64** axially deformed nuclei in the mass regions  $A \sim 150-180$  and  $A \sim 230-250$ .

... calculated masses of finite nuclei are primarily sensitive to the three leading terms in the empirical mass formula:

$$E_B = a_v A + a_s A^{2/3} + a_4 \frac{(N - Z)^2}{4A} + \dots$$

... generate families of effective interactions characterized by different values of  $a_v$ ,  $a_s$  and  $a_4$ , and determine which parametrization minimizes the deviation from the empirical binding energies of a large set of deformed nuclei.

## DD-PCI

volume energy:

$$a_v = -16.06 \text{ MeV}$$

surface energy:

$$a_s = 17.498 \text{ MeV}$$

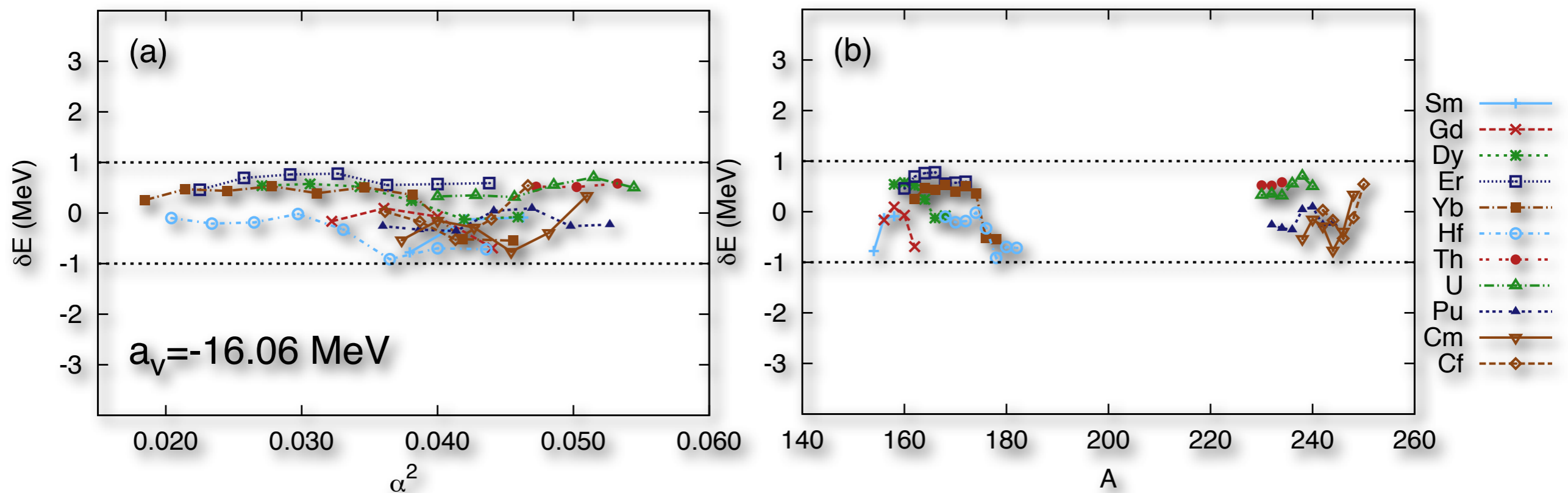
symmetry energy:

$$\langle S_2 \rangle = 27.8 \text{ MeV} \quad (a_4 = 33 \text{ MeV})$$

# Deformed nuclei

Binding energies used to adjust the parameters of the functional:

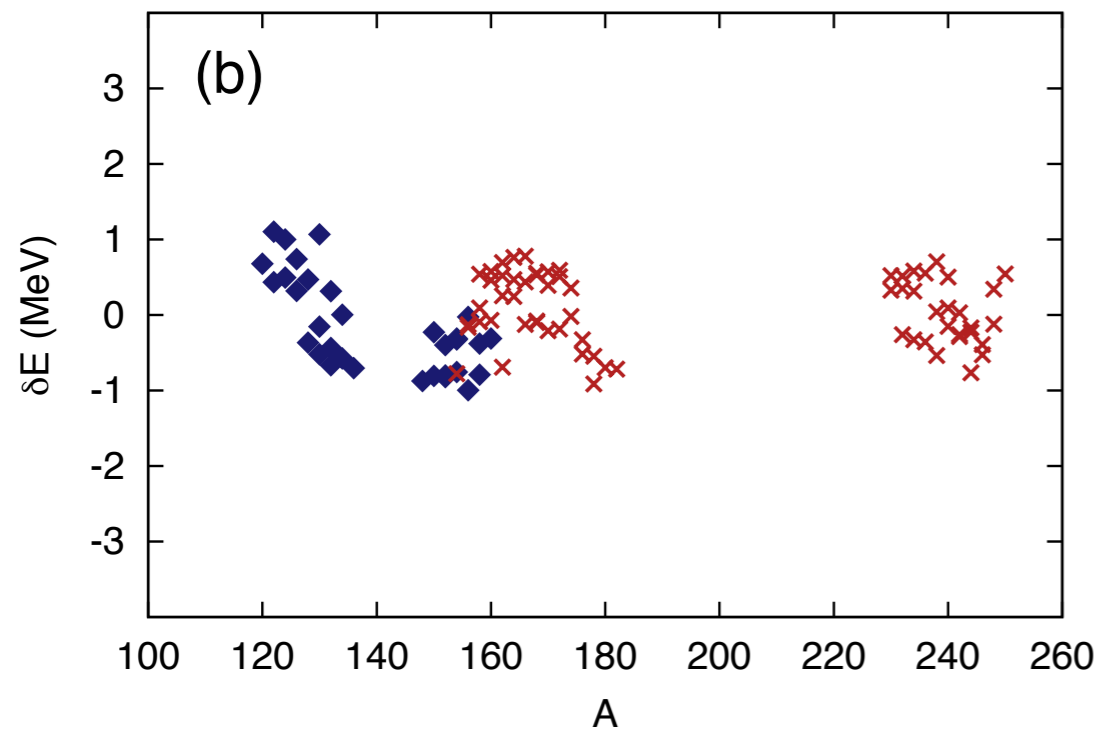
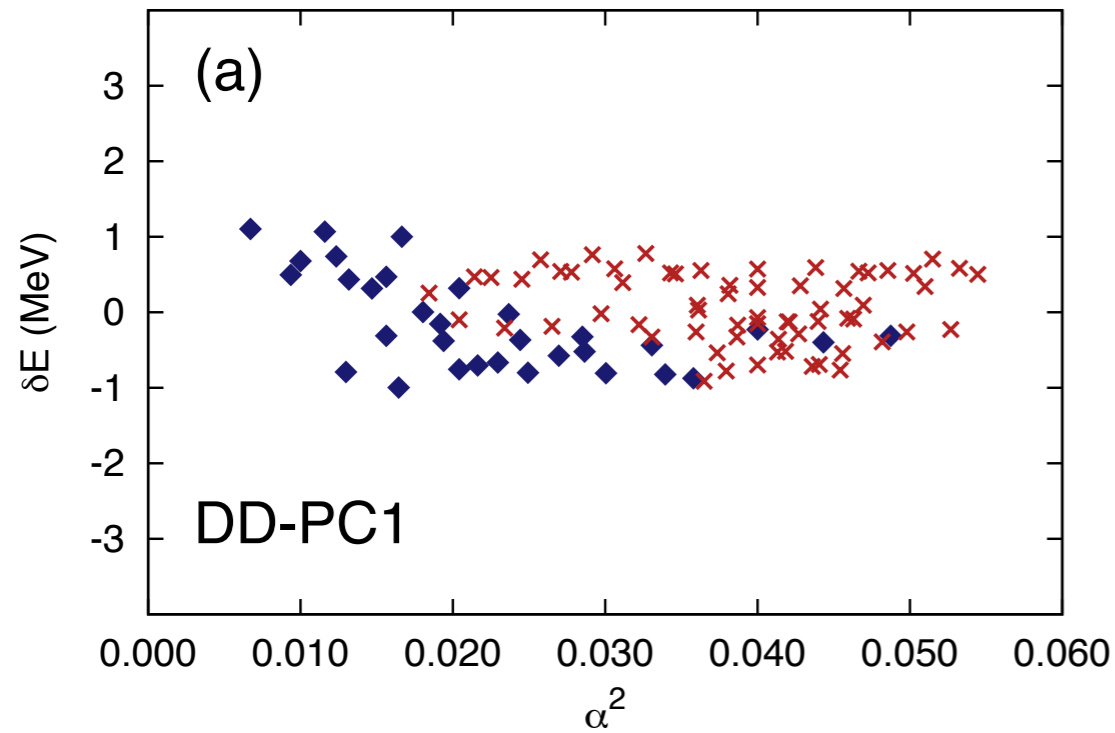
$Z$	62	64	66	68	70	72	90	92	94	96	98
$N_{min}$	92	92	92	92	92	72	140	138	138	142	144
$N_{max}$	96	98	102	104	108	110	144	148	150	152	152



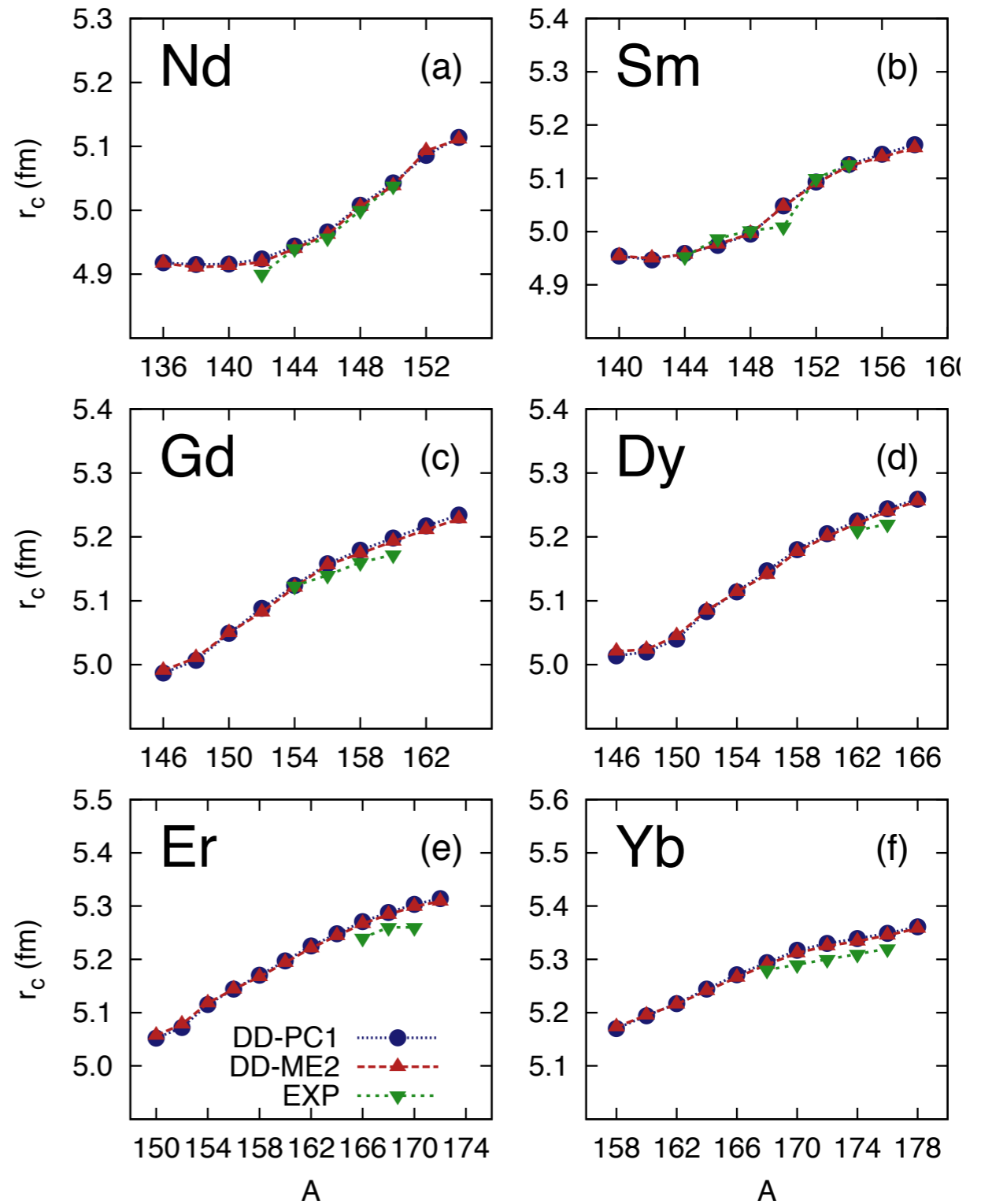
Nikšić, Vretenar, and Ring, Phys. Rev. C **78**, 034318 (2008)

# Systematic calculation of ground-state properties:

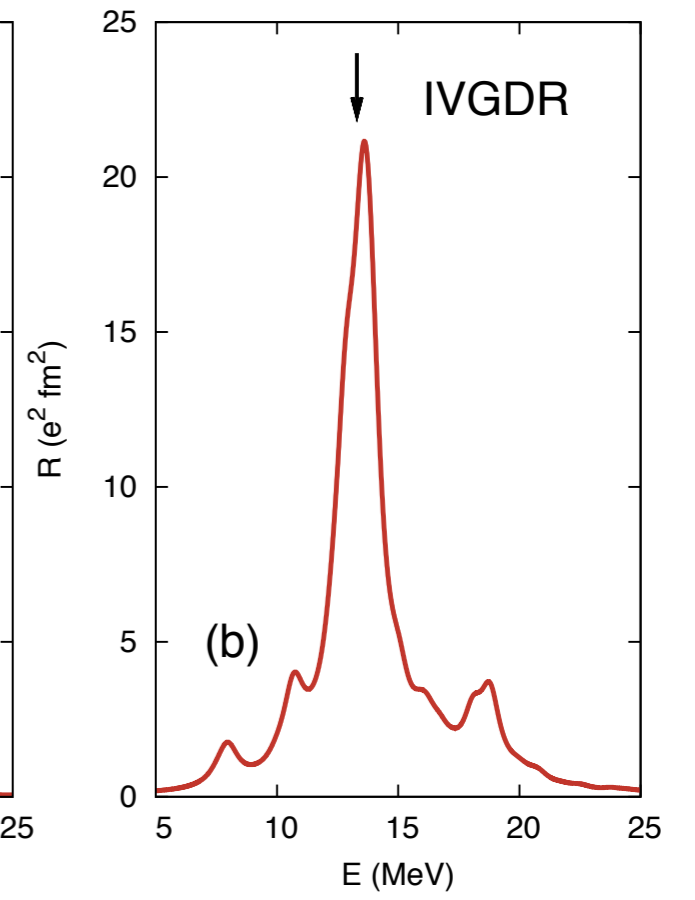
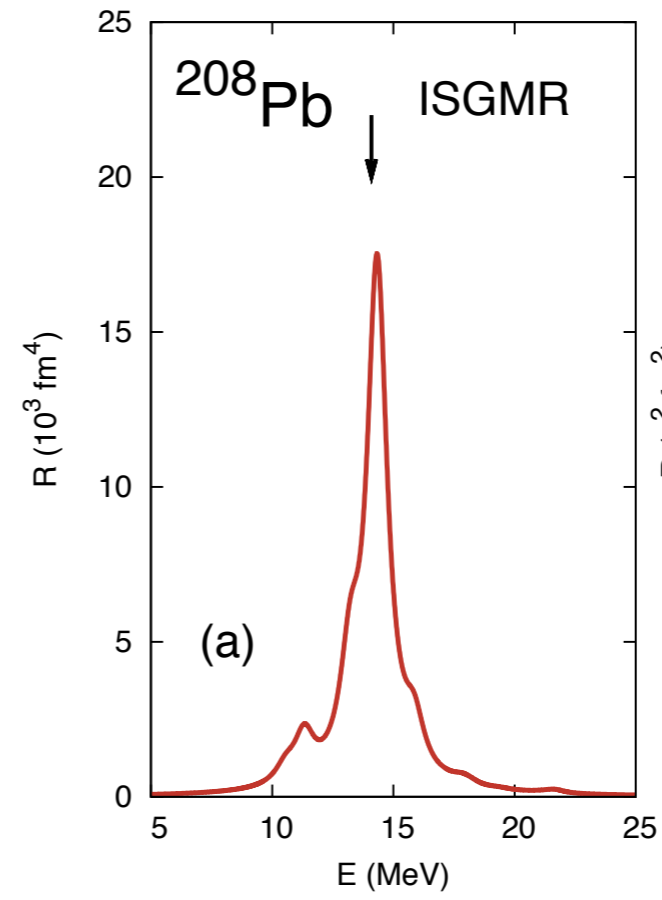
Absolute error of calculated masses:



Charge radii:

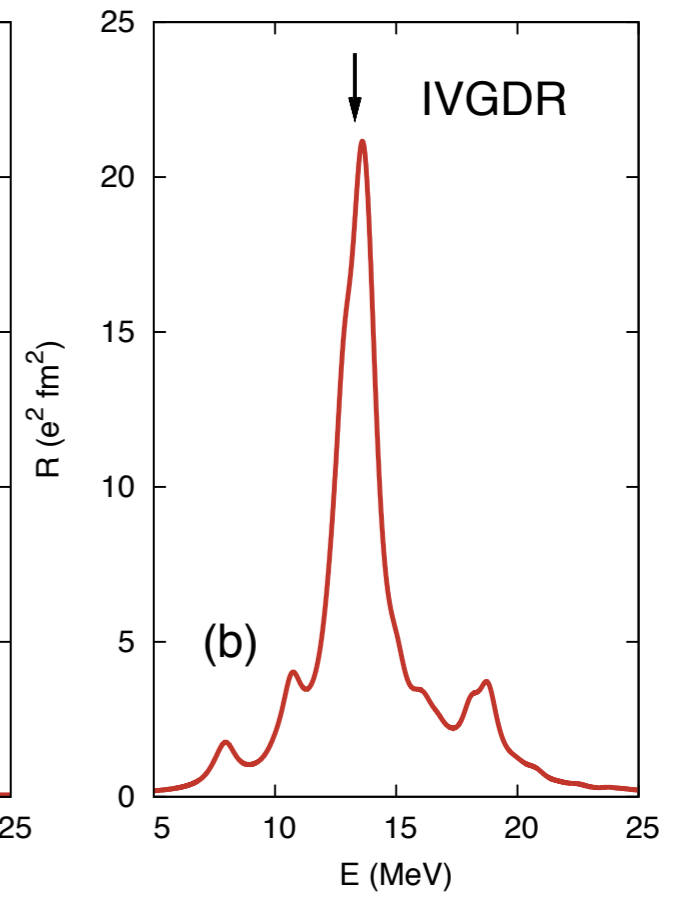
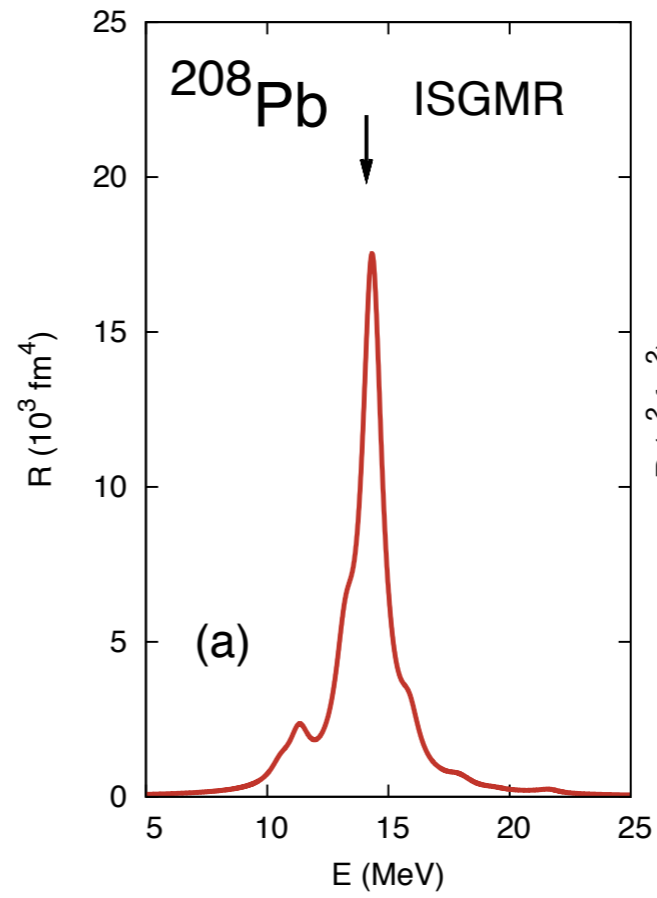


# Excitation energies of collective modes:



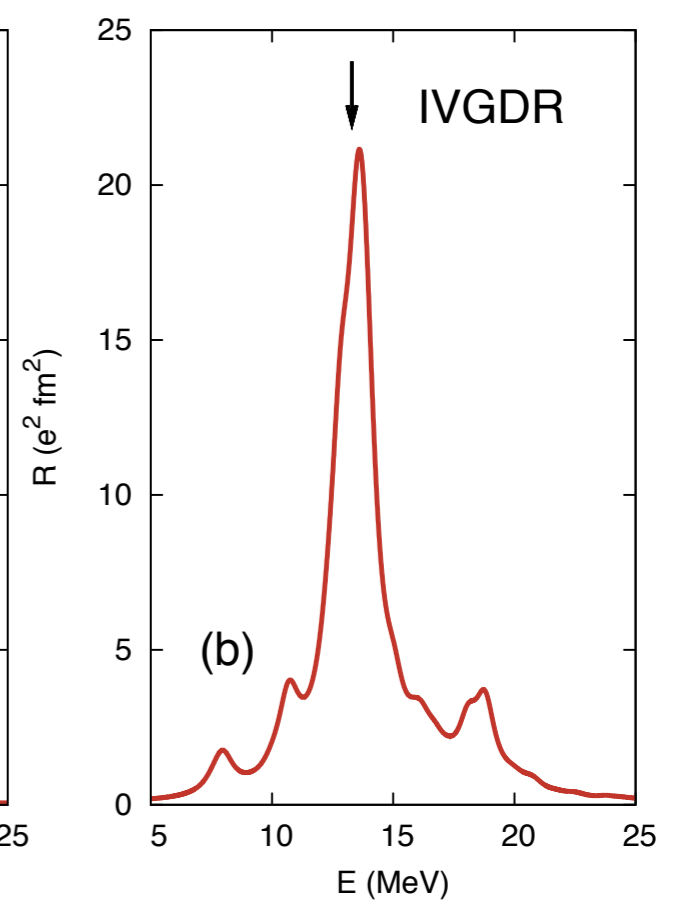
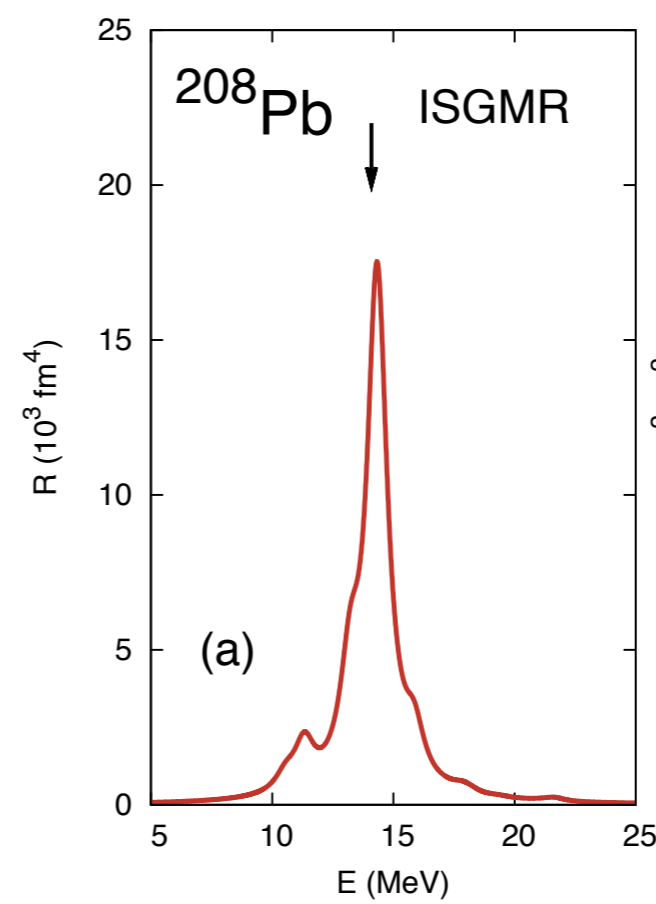
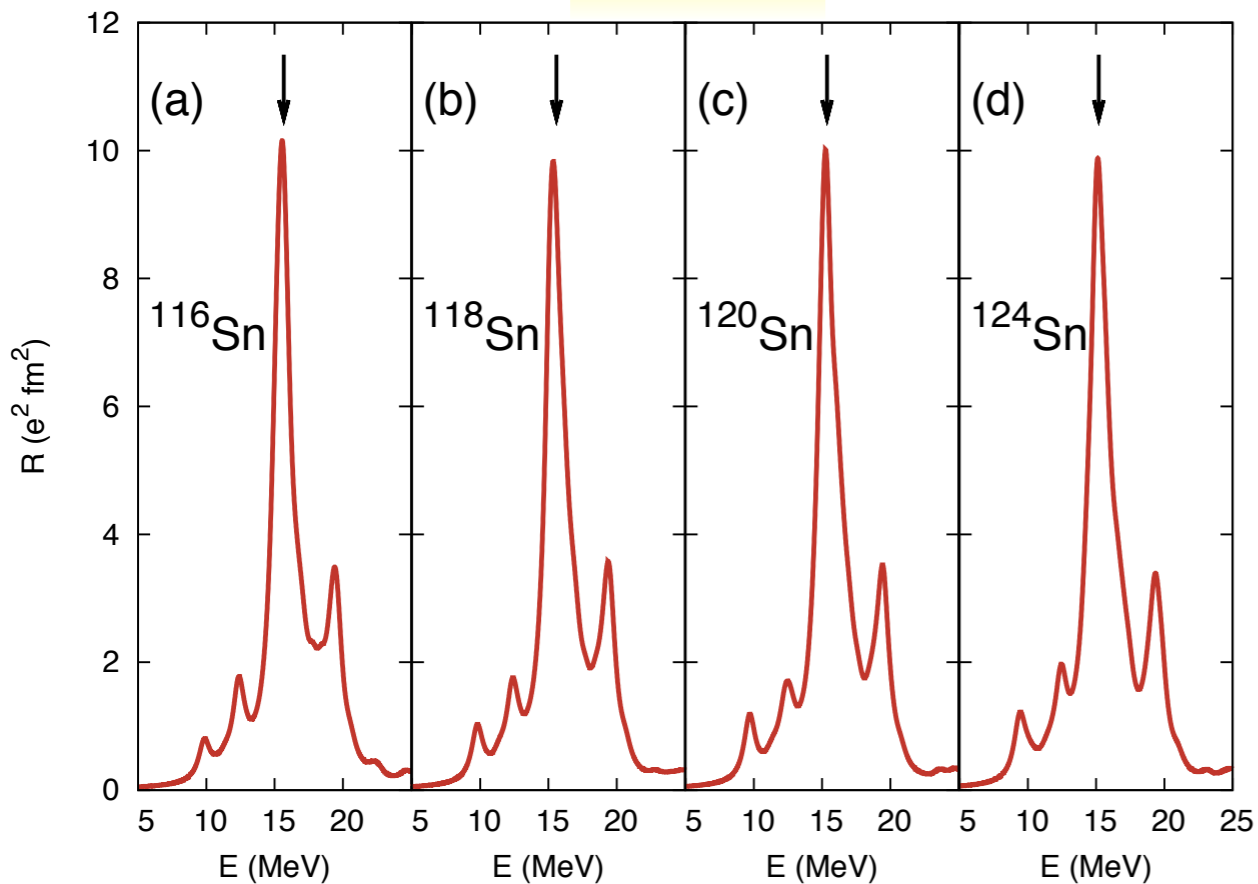
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IVGDR



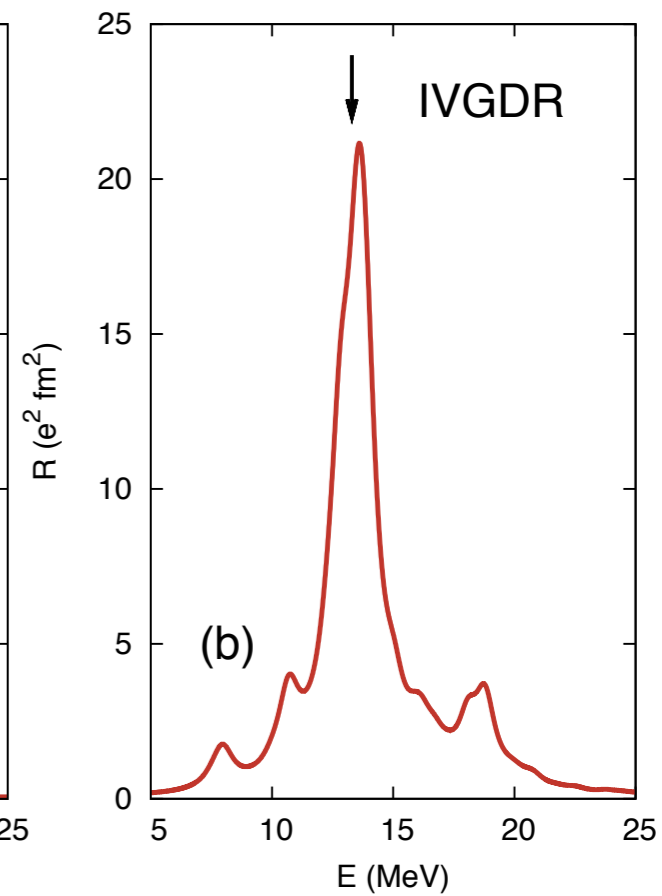
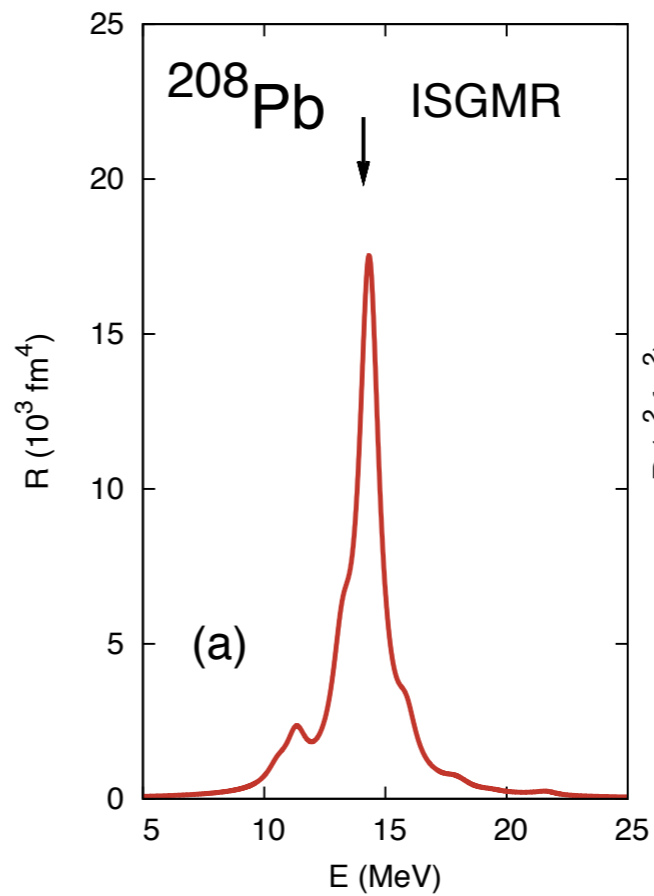
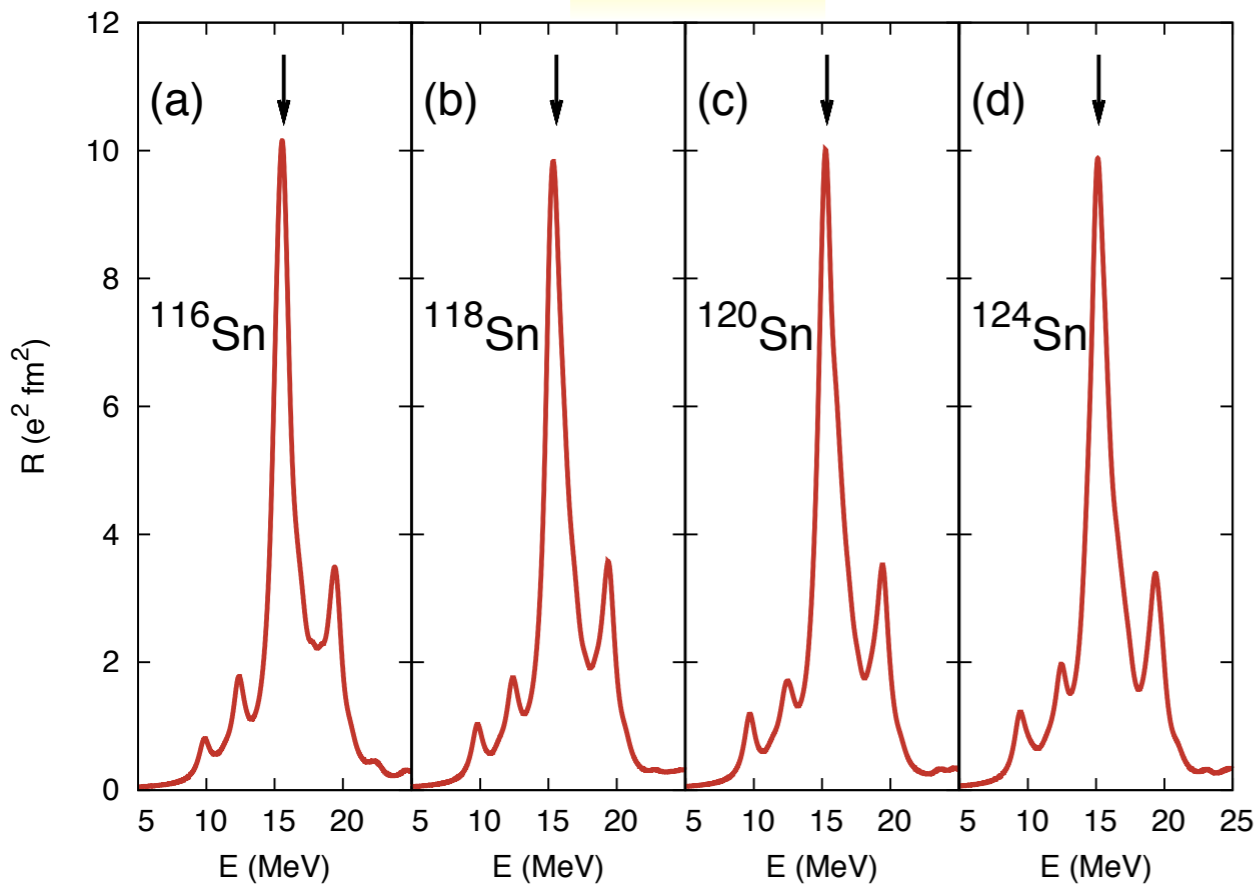
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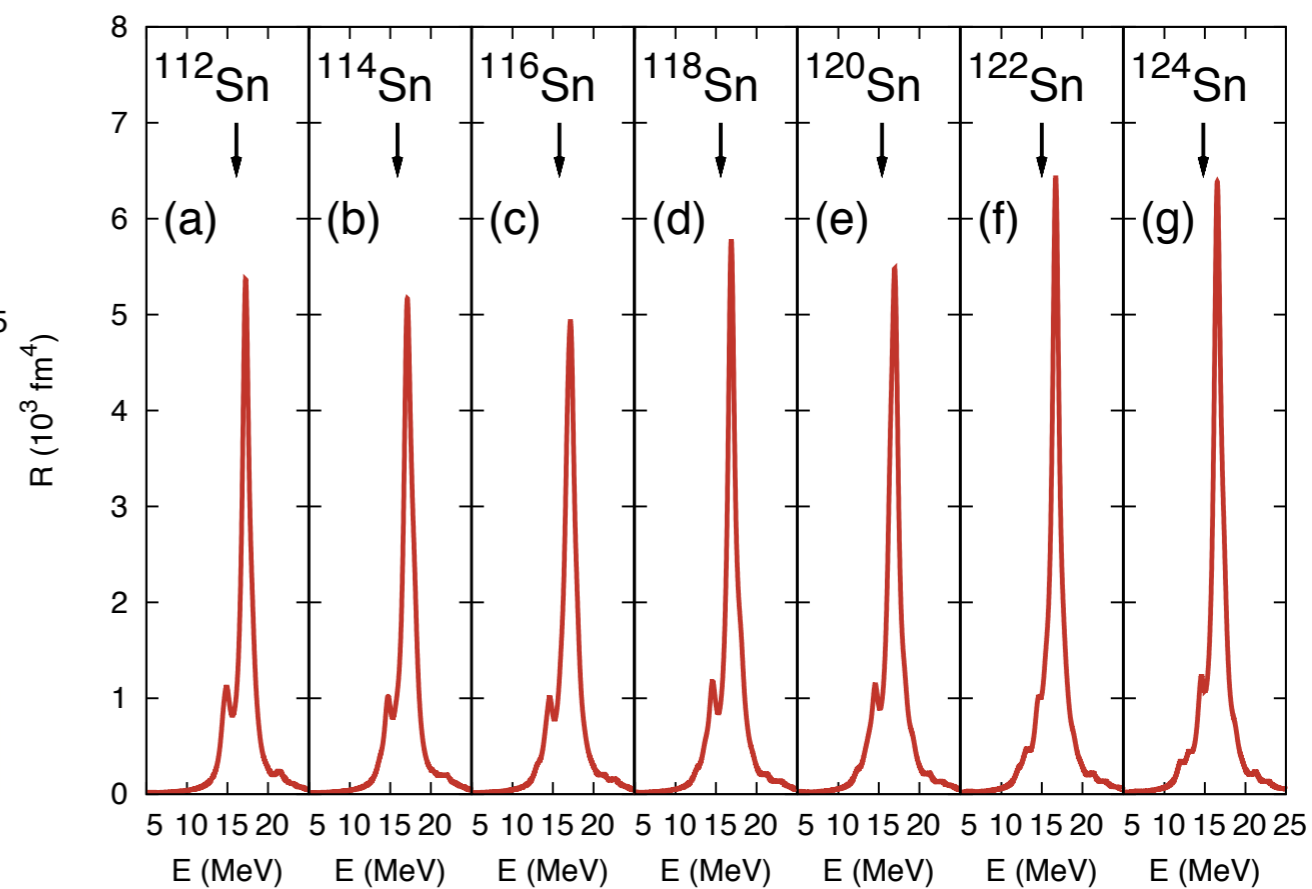
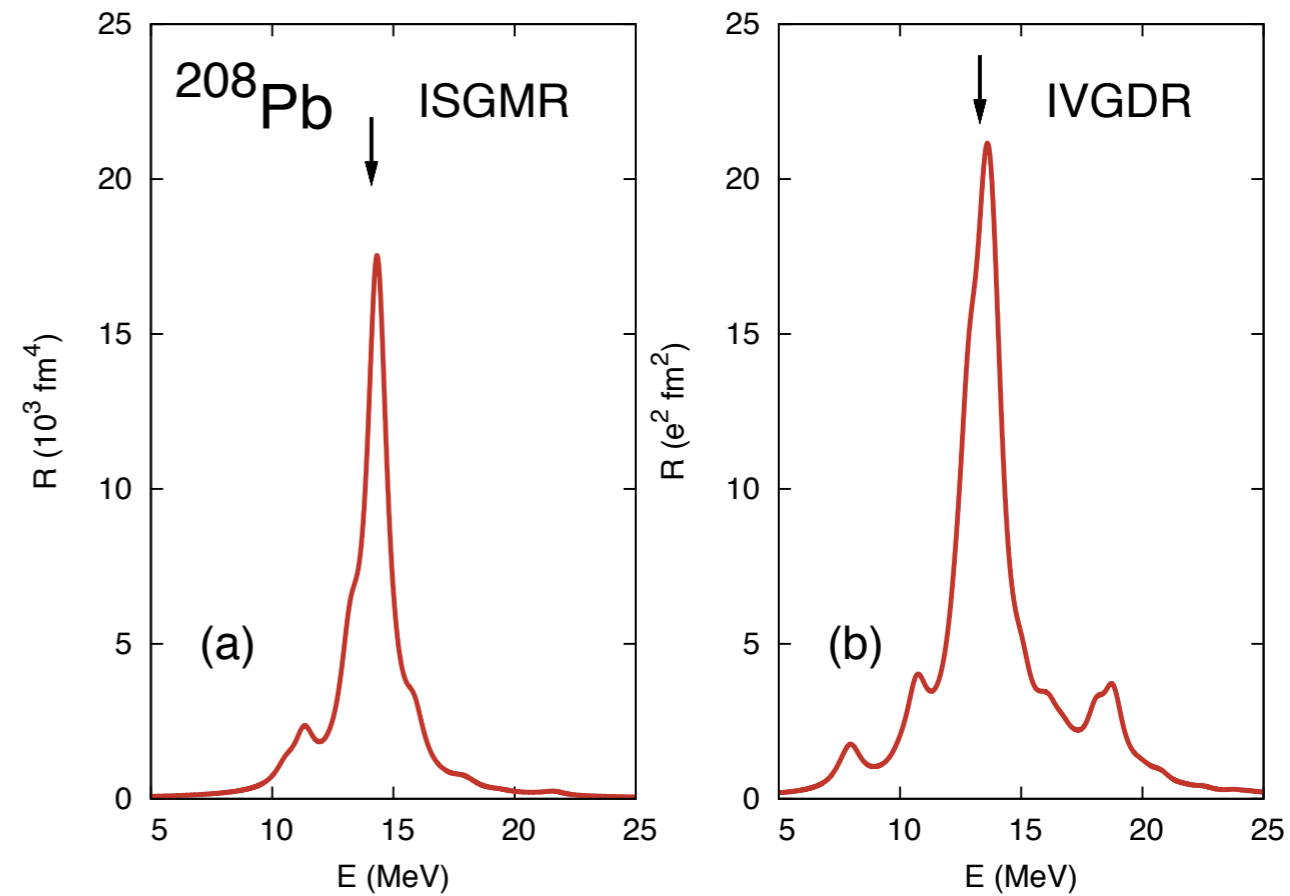
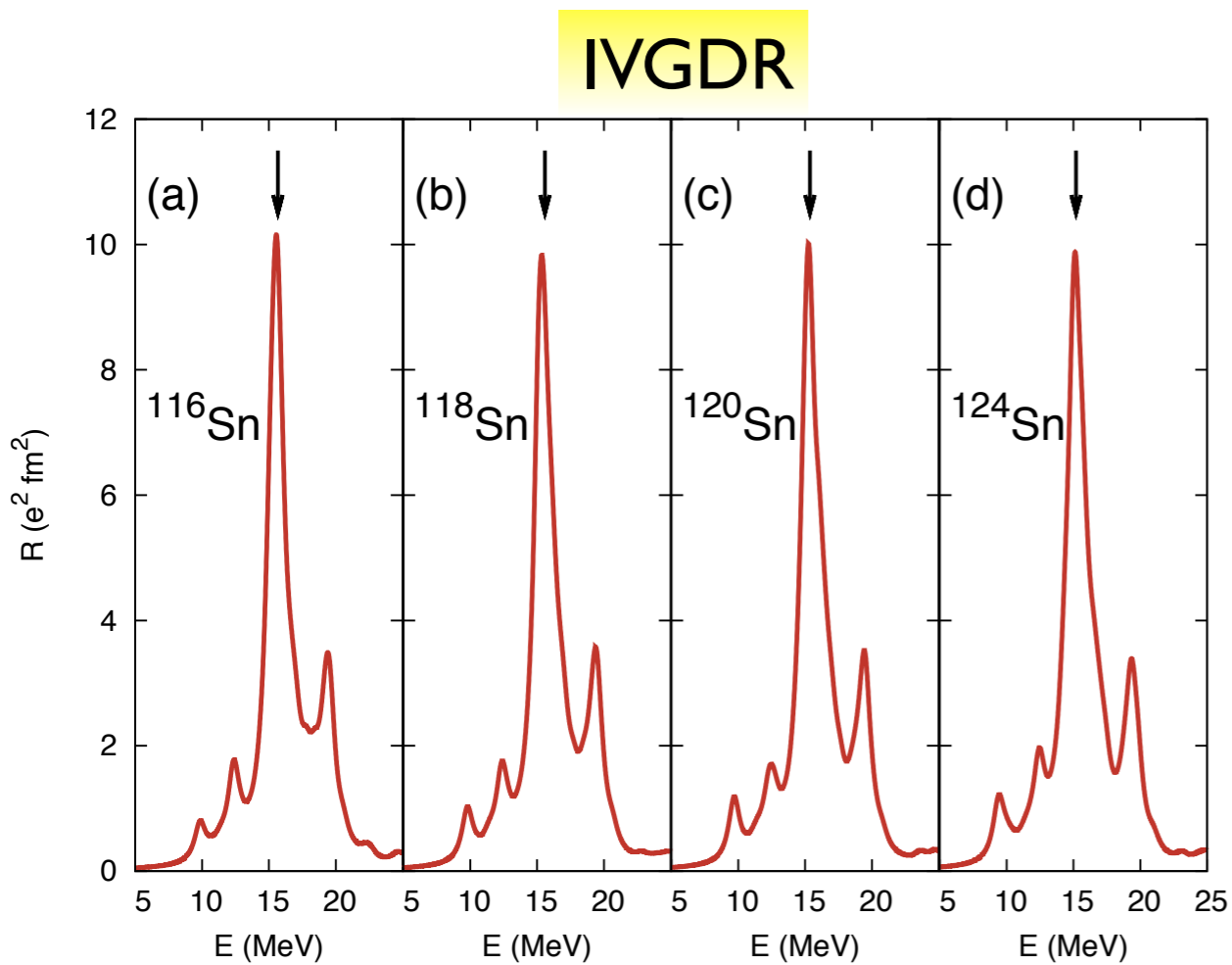
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ISGMR



# Excitation energies of collective modes:



# Nuclear Many-Body Correlations



## **short-range**

(hard repulsive core of the NN-interaction)

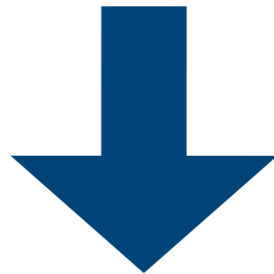
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nuclear resonance modes  
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large-amplitude soft modes:  
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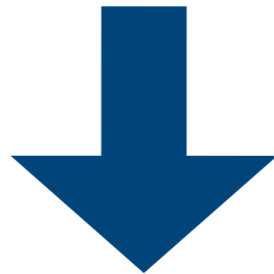
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Energy Density Functional.

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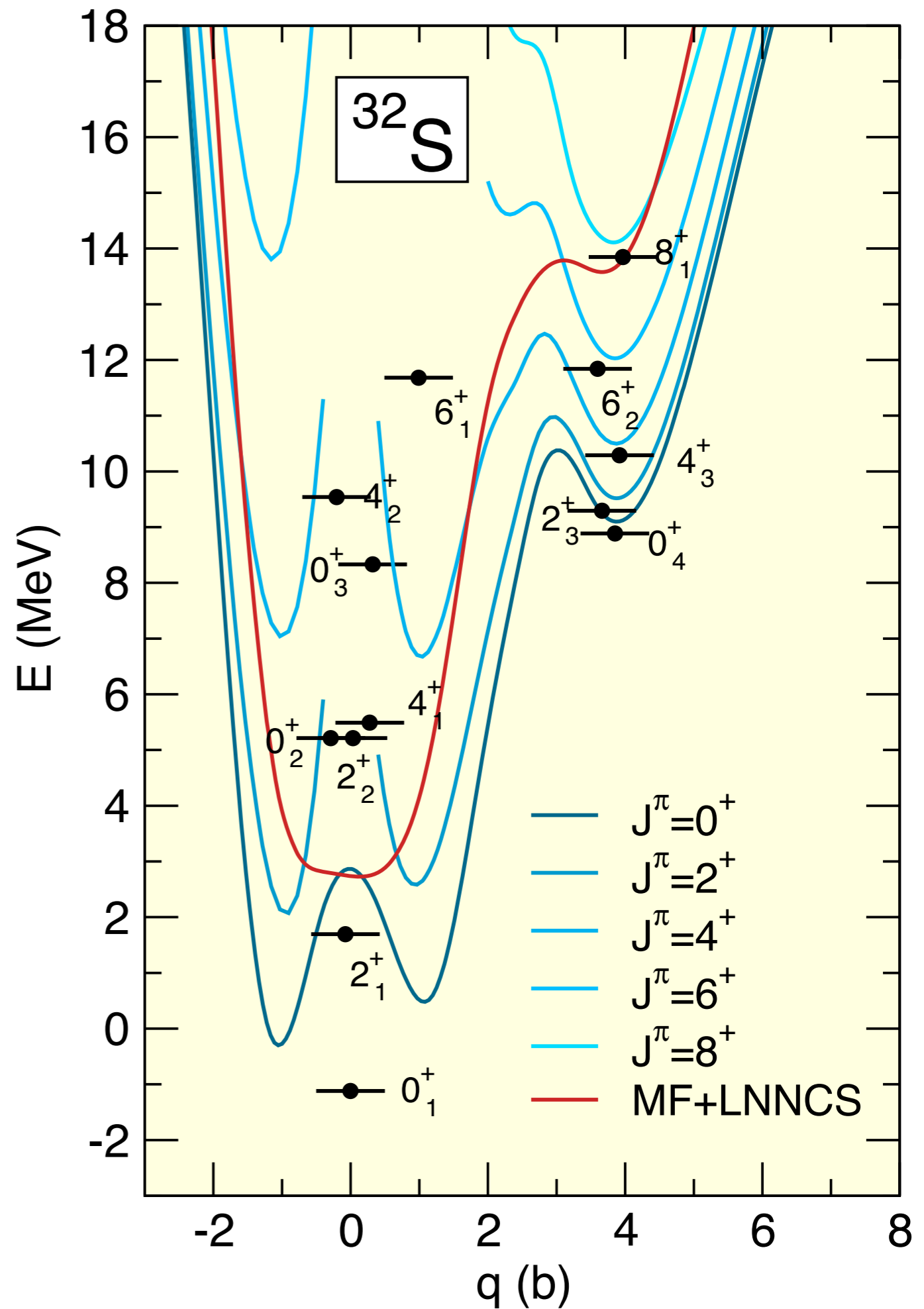
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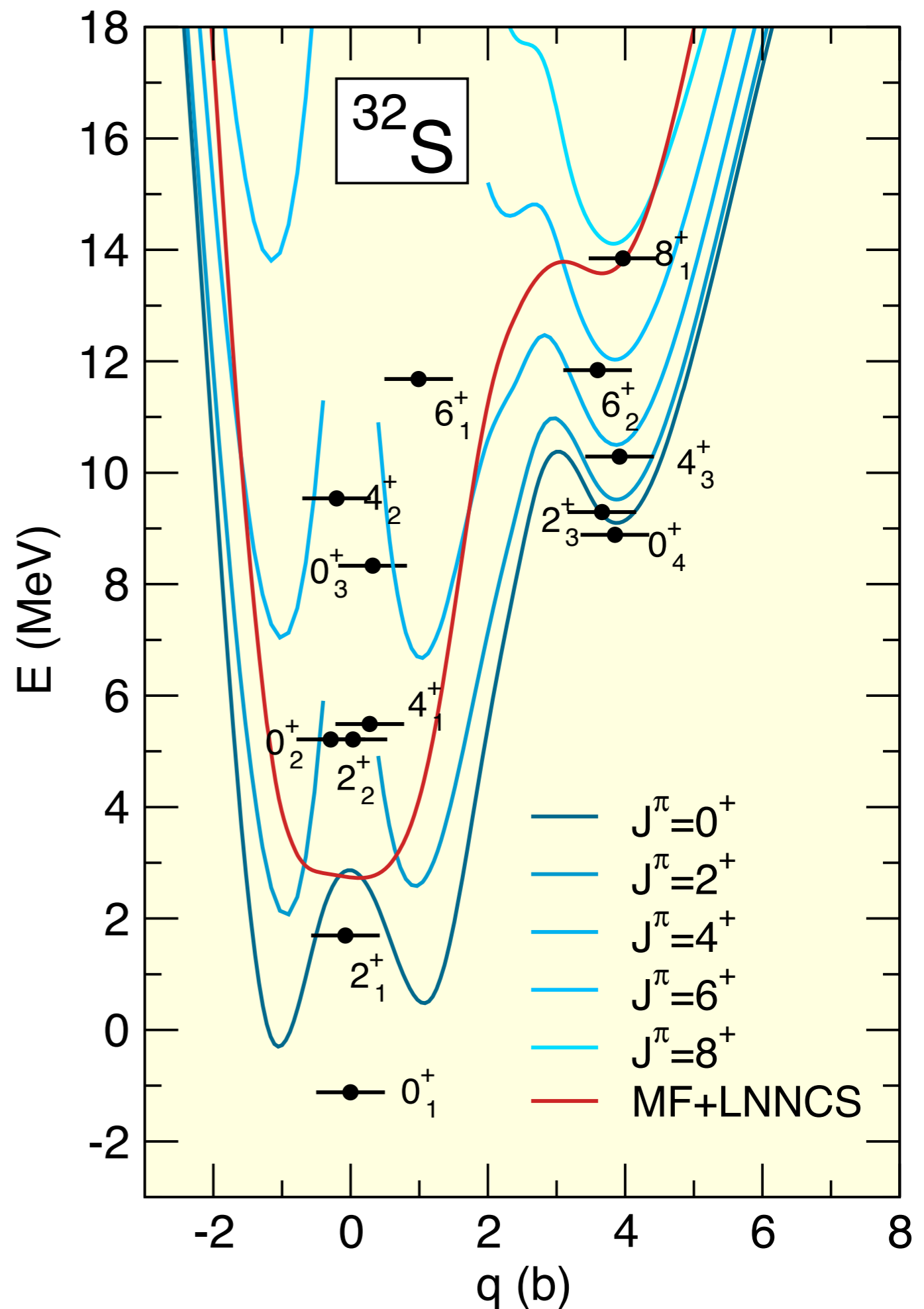
...sensitive to shell-effects and strong variations  
with nucleon number!  
Cannot be included in a simple EDF framework.

Collective correlations →  
restoration of broken  
symmetries and fluctuations  
of collective variables



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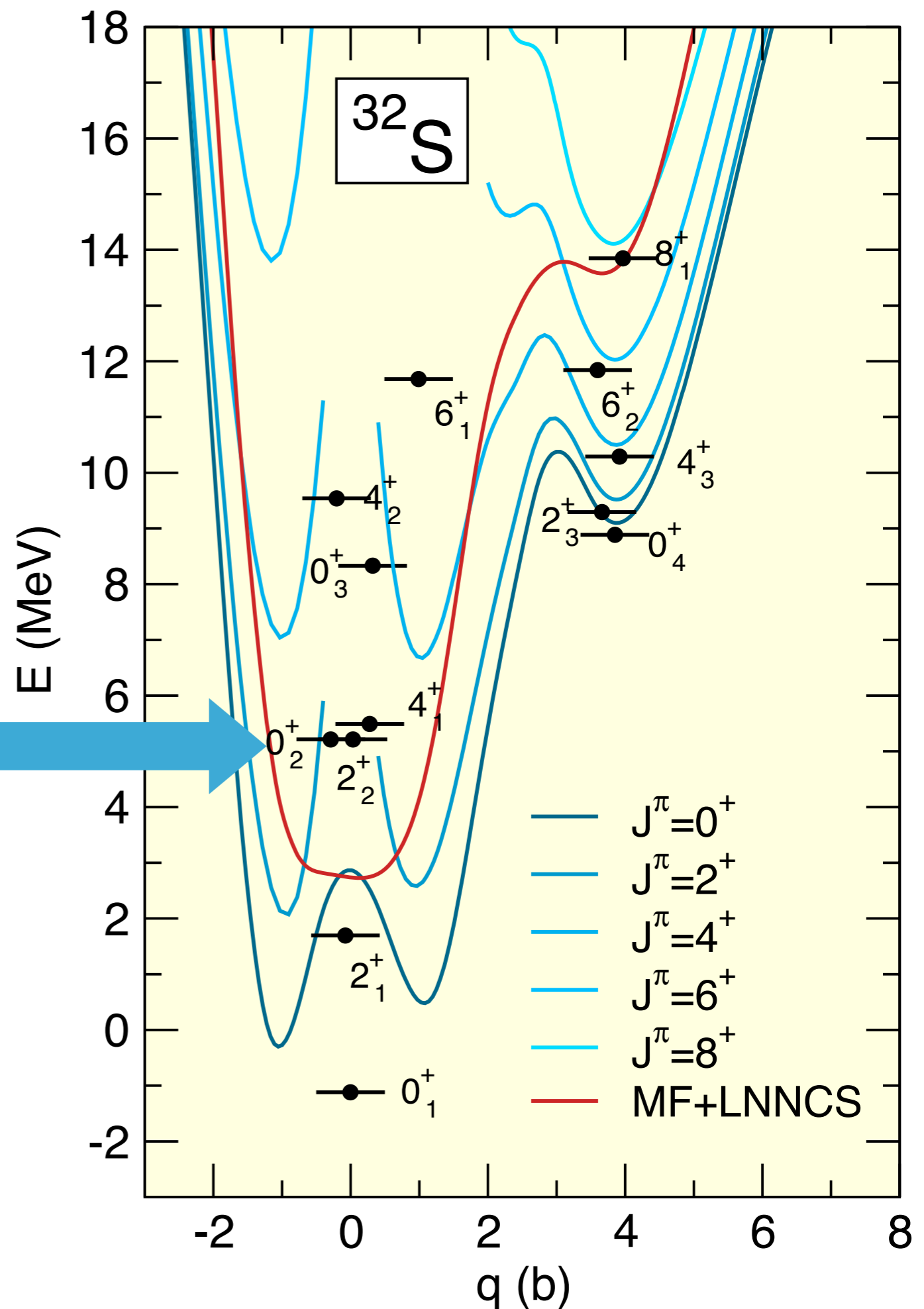
1. Mean-field calculations, with a constraint on the quadrupole moment.
2. Angular-momentum and particle-number projection.
3. Generator Coordinate Method ⇒ configuration mixing





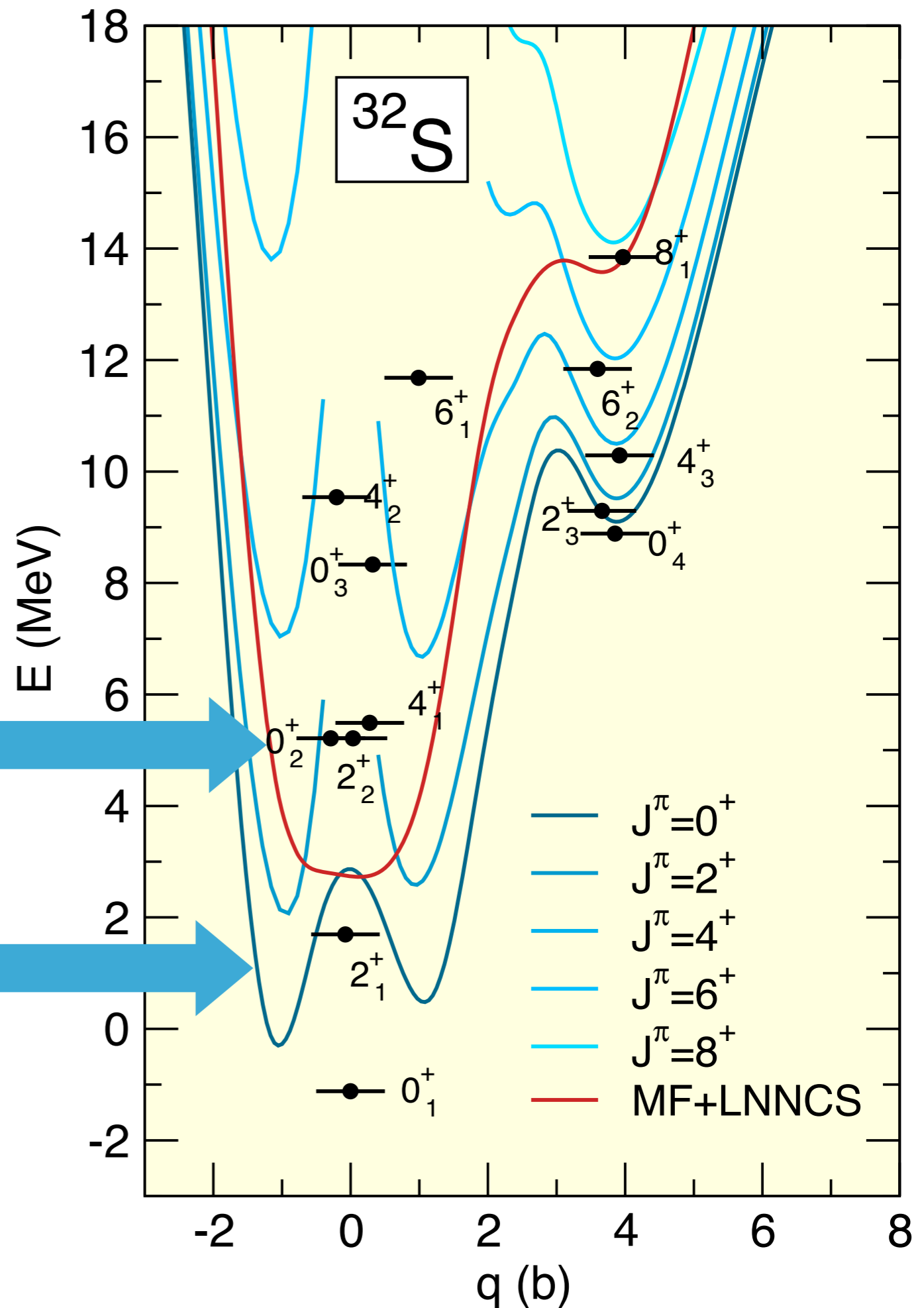
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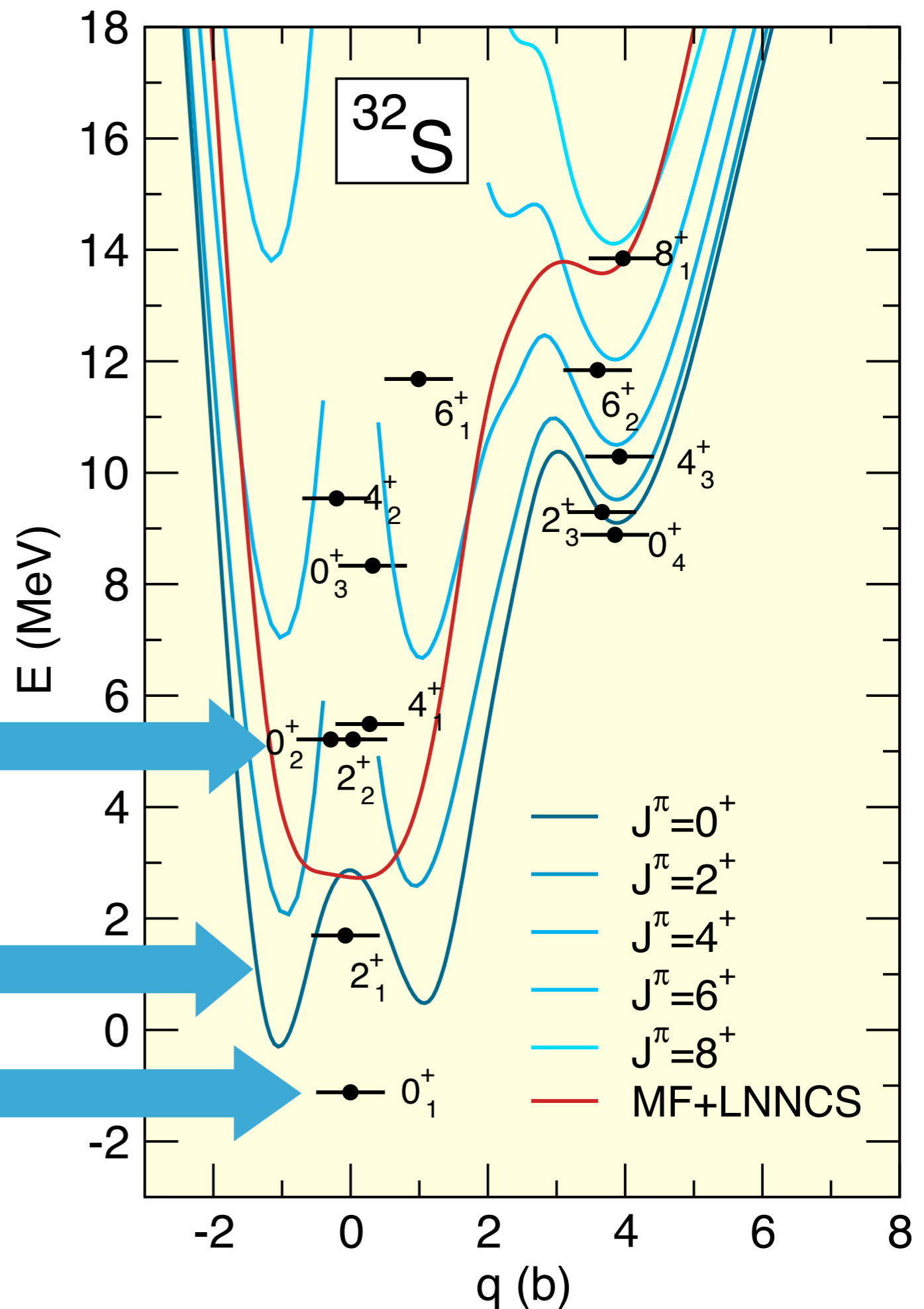
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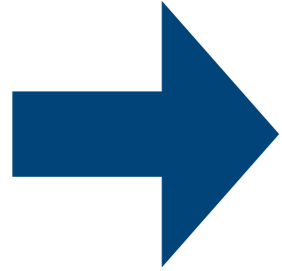


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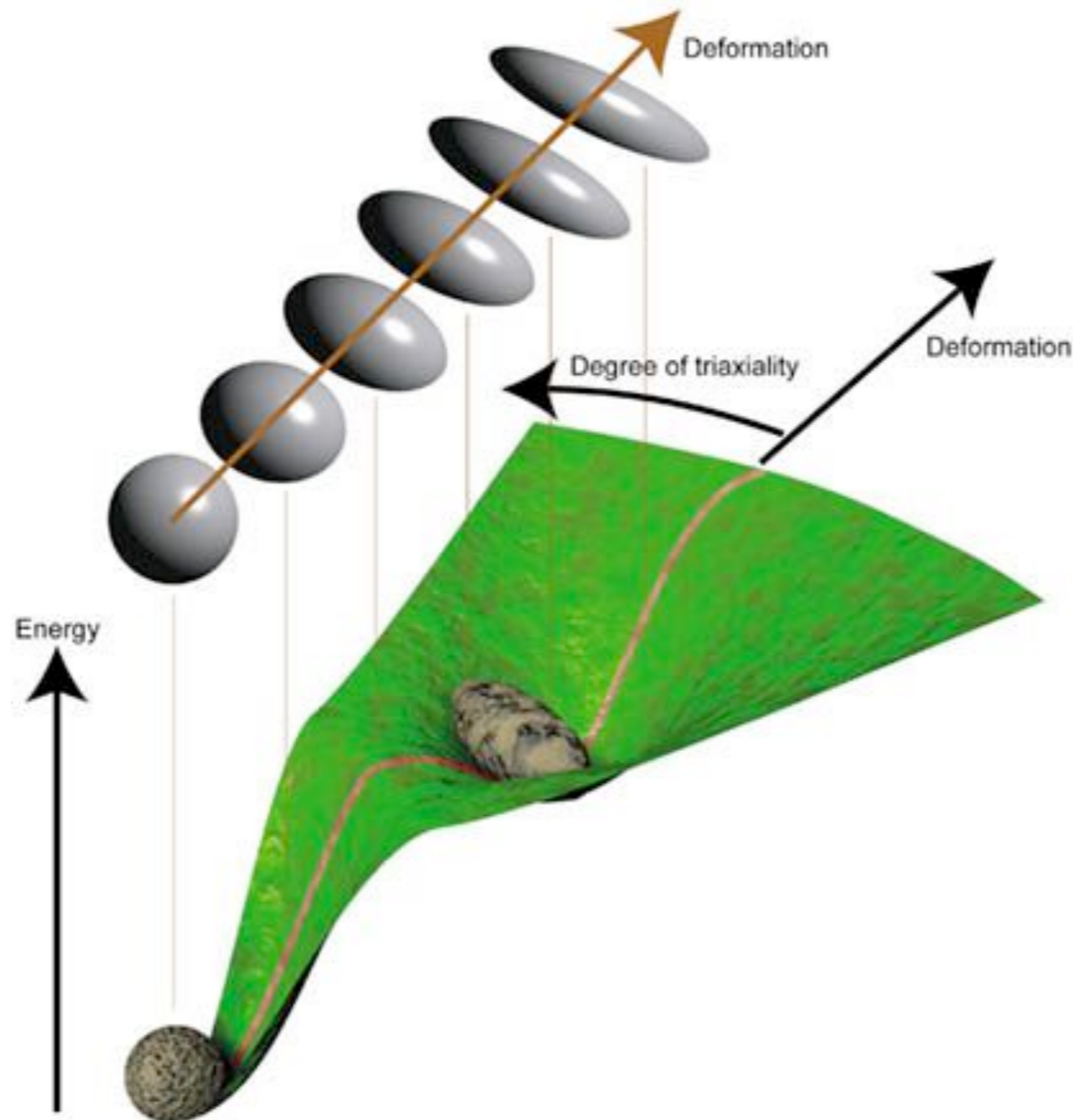
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... larger variational space for projected GCM calculations!



triaxial shapes, breaking time-reversal invariance, different deformations for proton and neutron distributions, ...



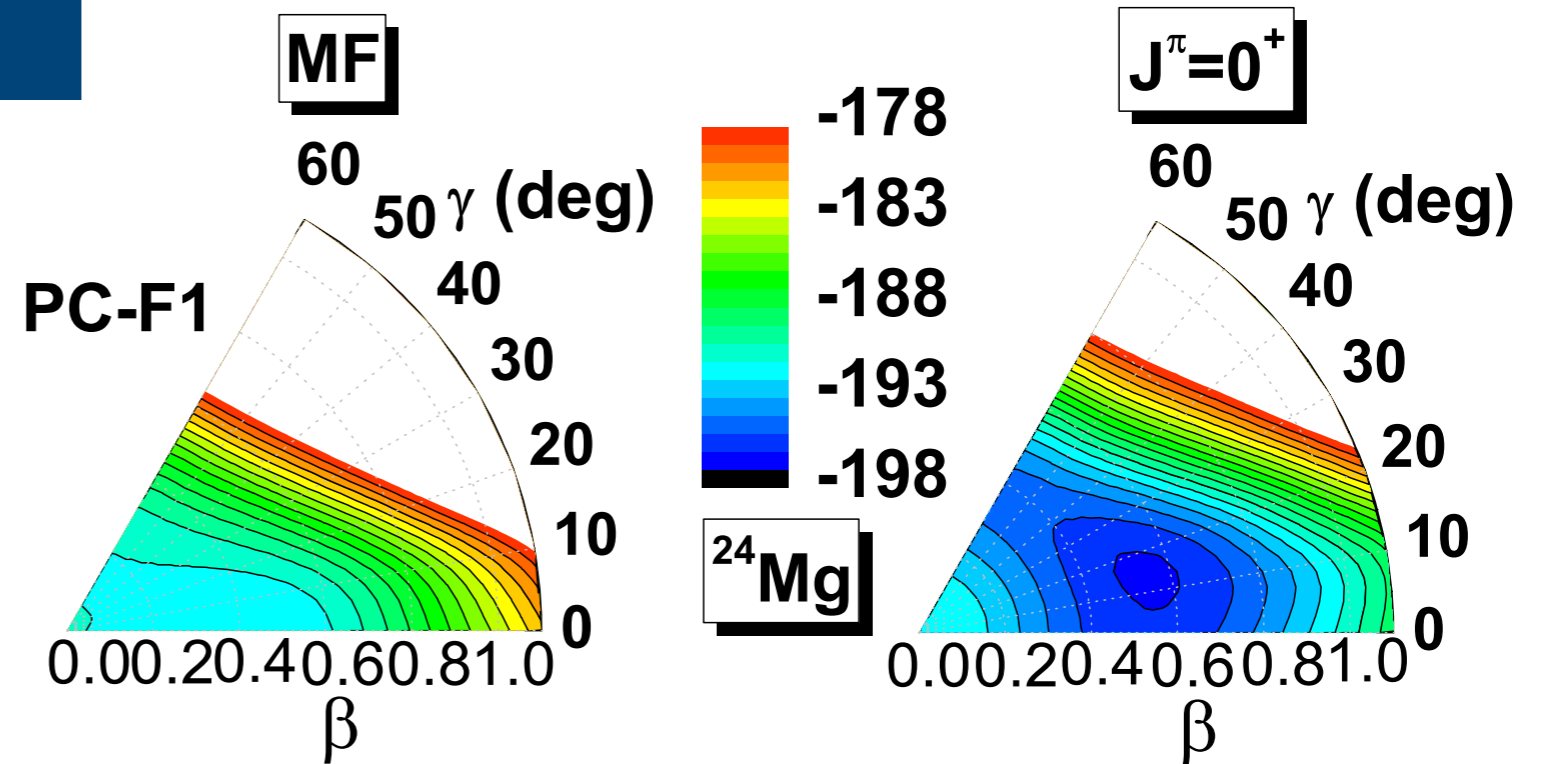
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3D AMP + GCM model

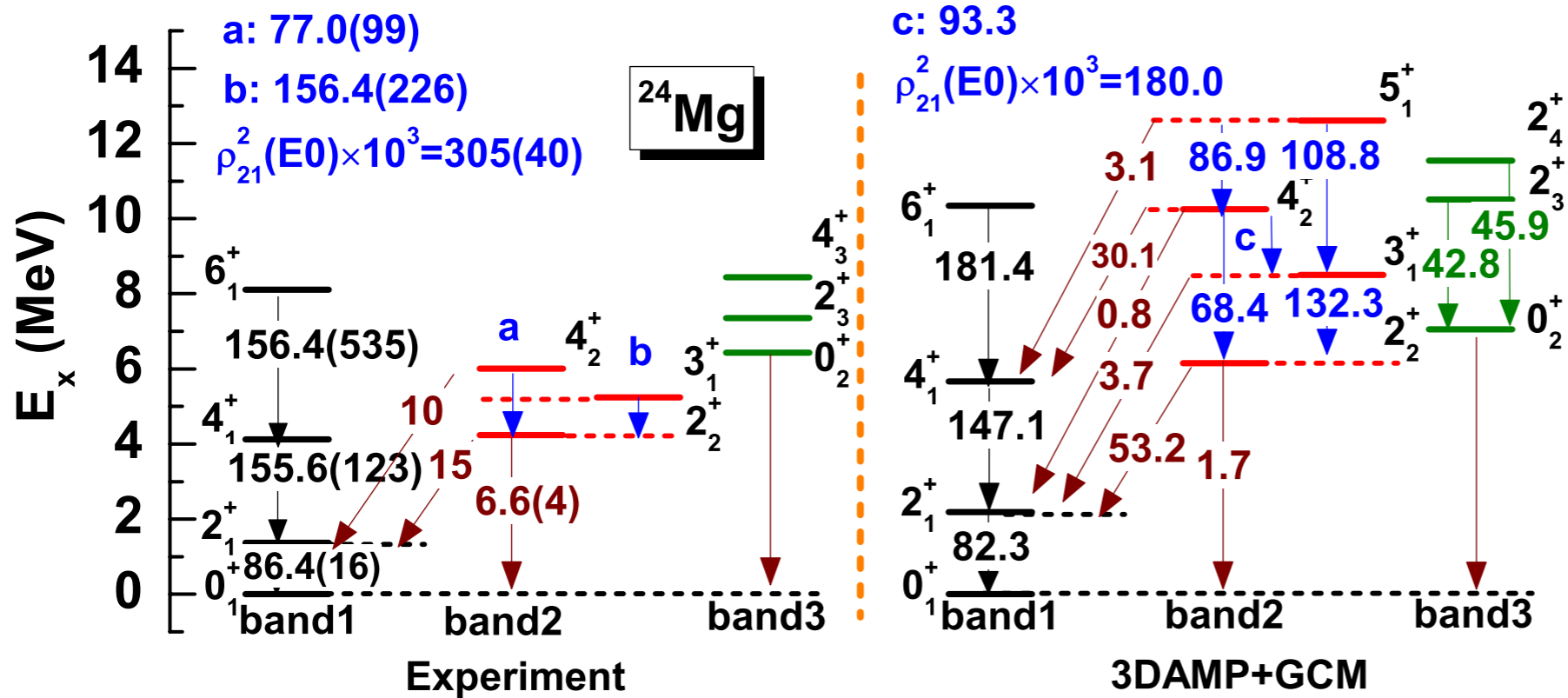
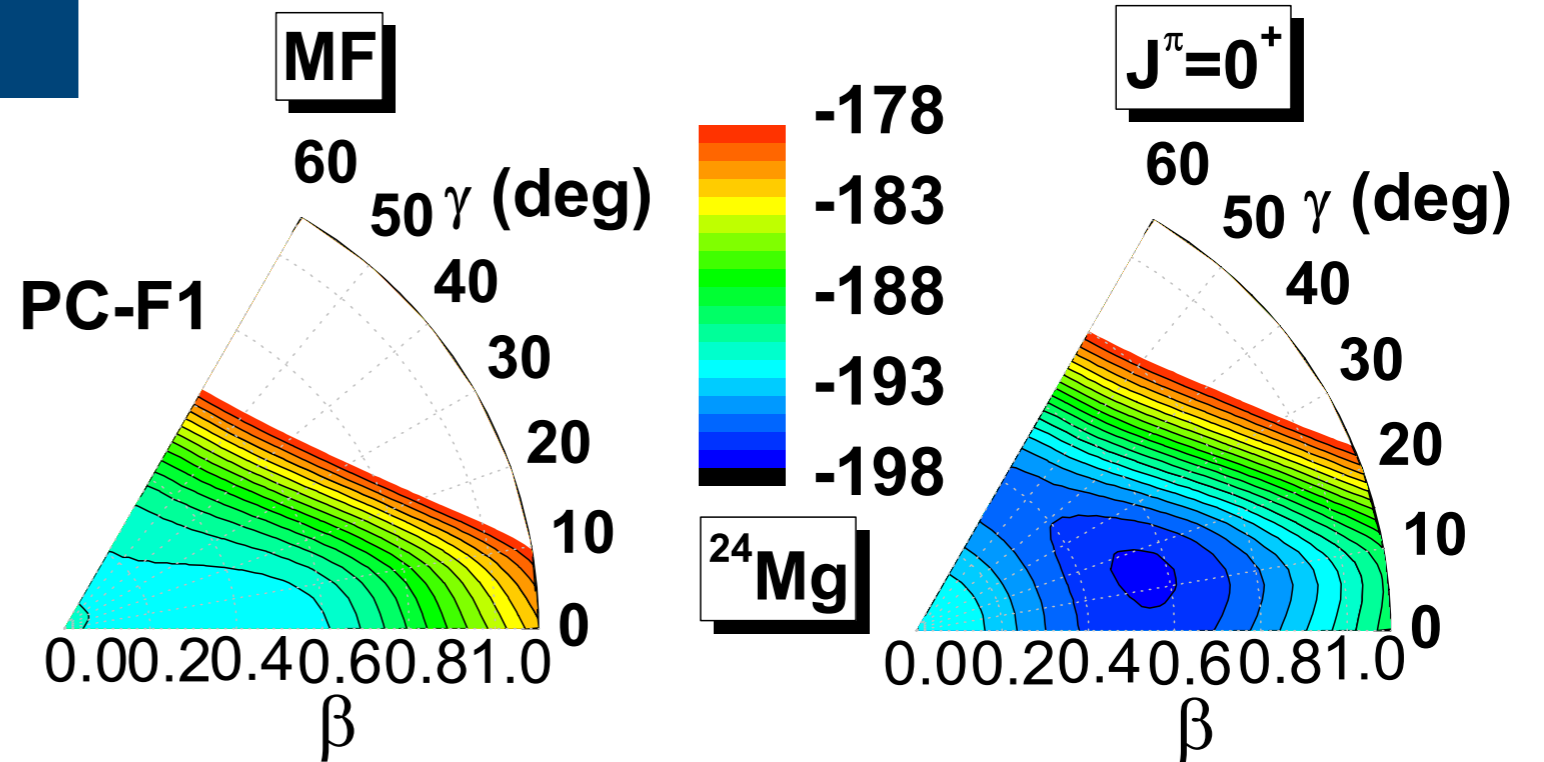
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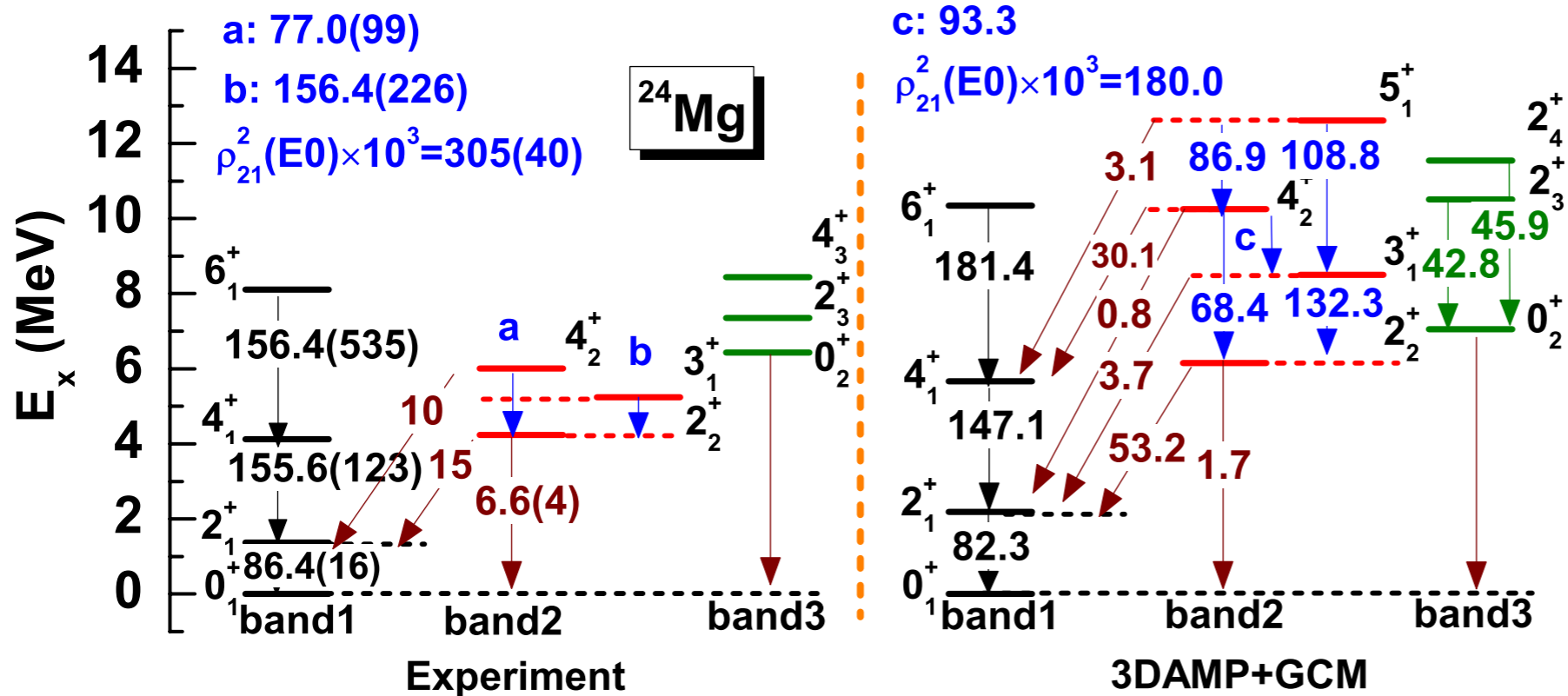
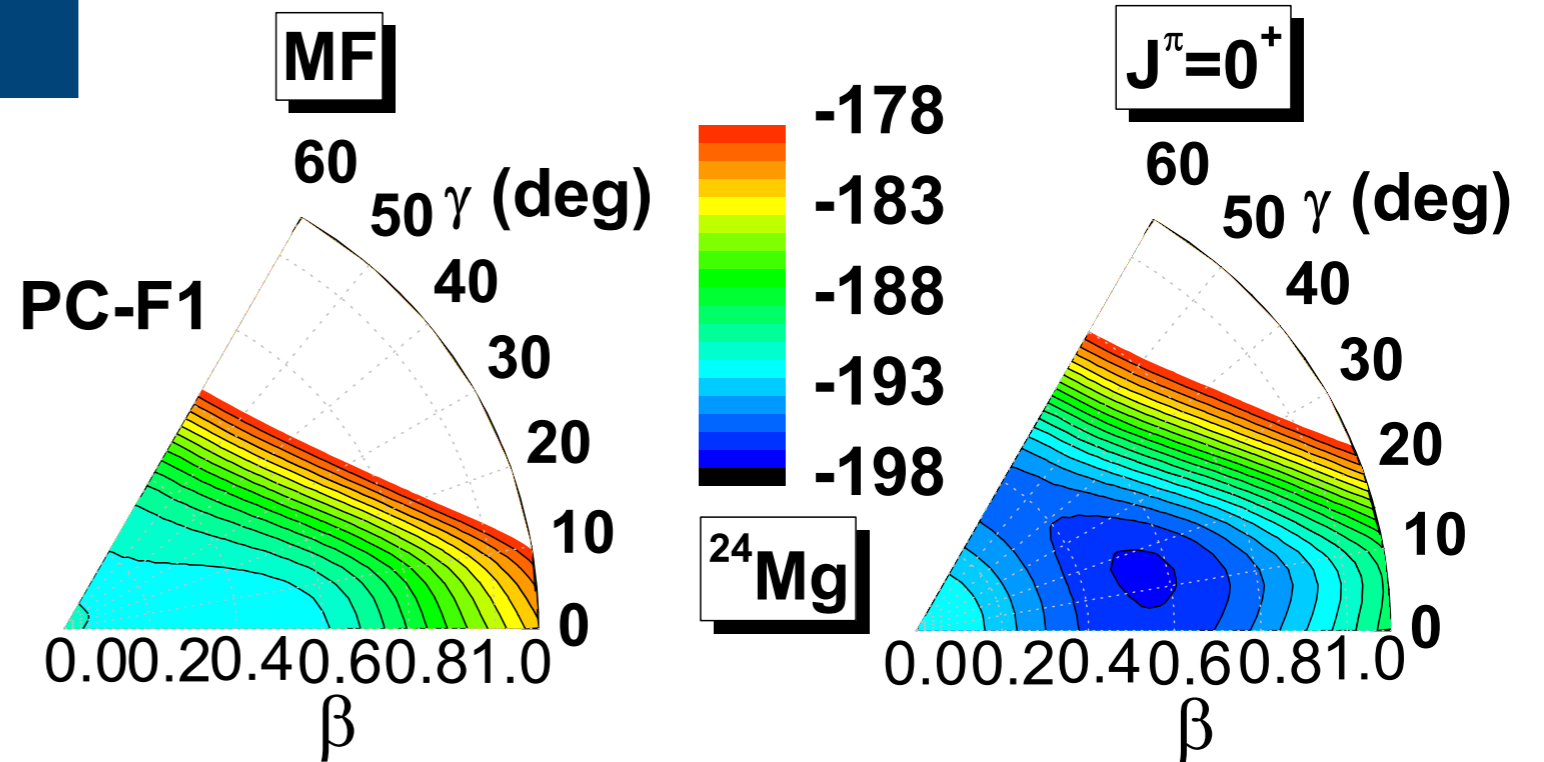
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### 3D AMP + GCM model



Yao, Meng, Ring, Vretenar,  
Phys. Rev. C **81**, 044311 (2010)

# Five-dimensional collective Hamiltonian

Nikšić, Li, Vretenar, Prochniak, Meng, Ring, Phys. Rev. C **79**, 034303 (2009)

... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom

$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta, \gamma) + \mathcal{T}_{\text{rot}}(\beta, \gamma, \Omega) + \mathcal{V}_{\text{coll}}(\beta, \gamma)$$

$$\mathcal{T}_{\text{vib}} = \frac{1}{2} B_{\beta\beta} \dot{\beta}^2 + \beta B_{\beta\gamma} \dot{\beta} \dot{\gamma} + \frac{1}{2} \beta^2 B_{\gamma\gamma} \dot{\gamma}^2$$

$$\mathcal{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{I}_k \omega_k^2$$

$$\mathcal{V}_{\text{coll}}(\beta, \gamma) = E_{\text{tot}}(\beta, \gamma) - \Delta V_{\text{vib}}(\beta, \gamma) - \Delta V_{\text{rot}}(\beta, \gamma)$$

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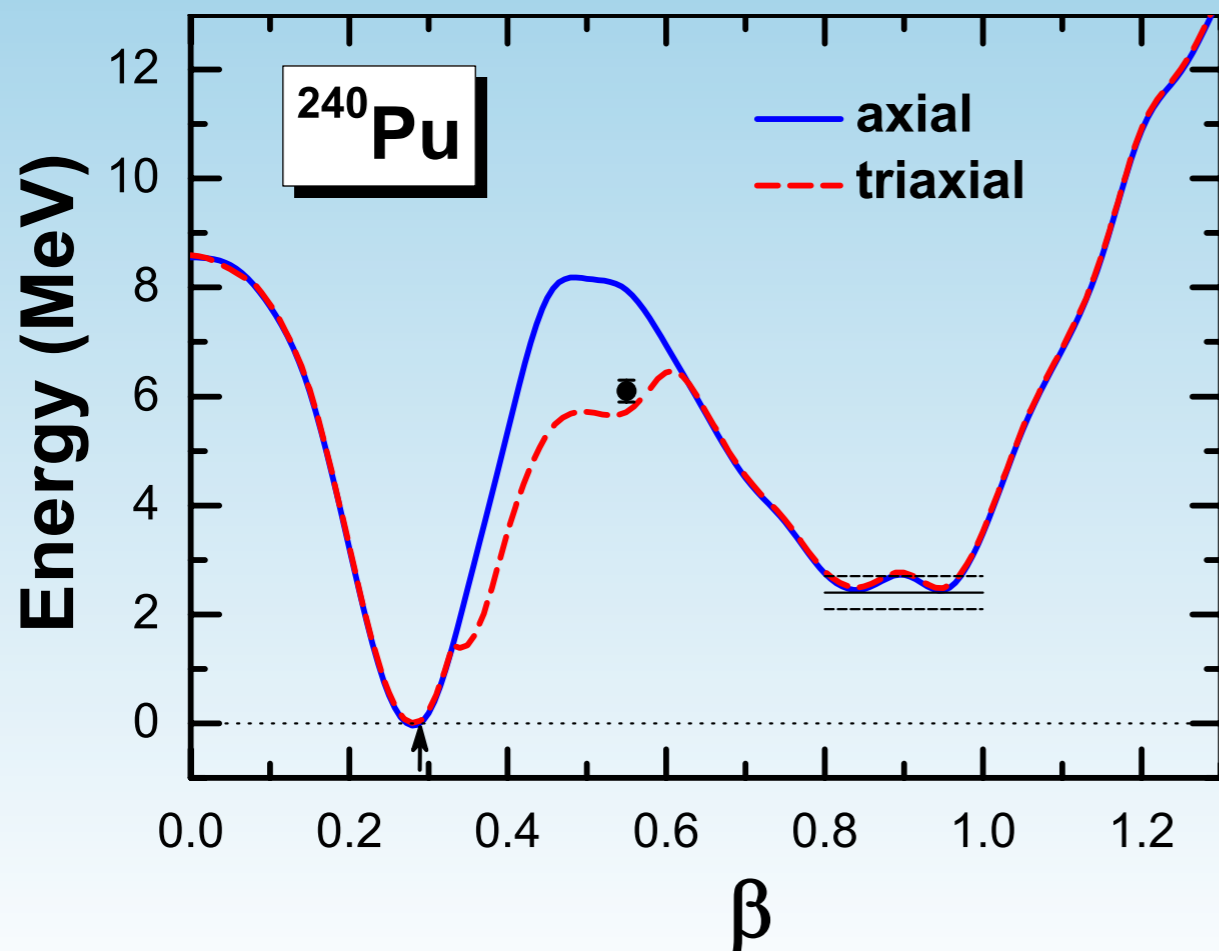
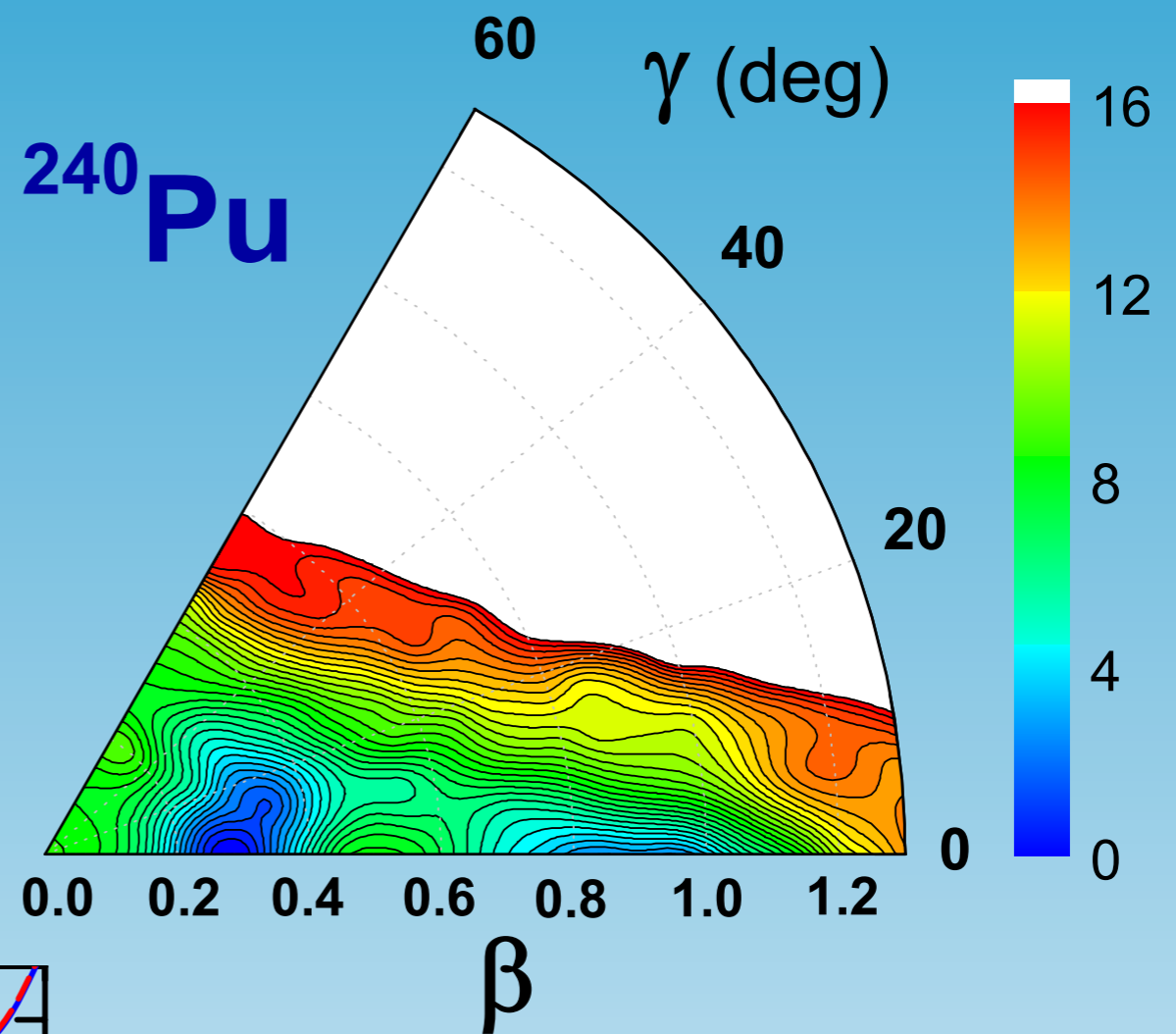
$$\mathcal{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{I}_k \omega_k^2$$

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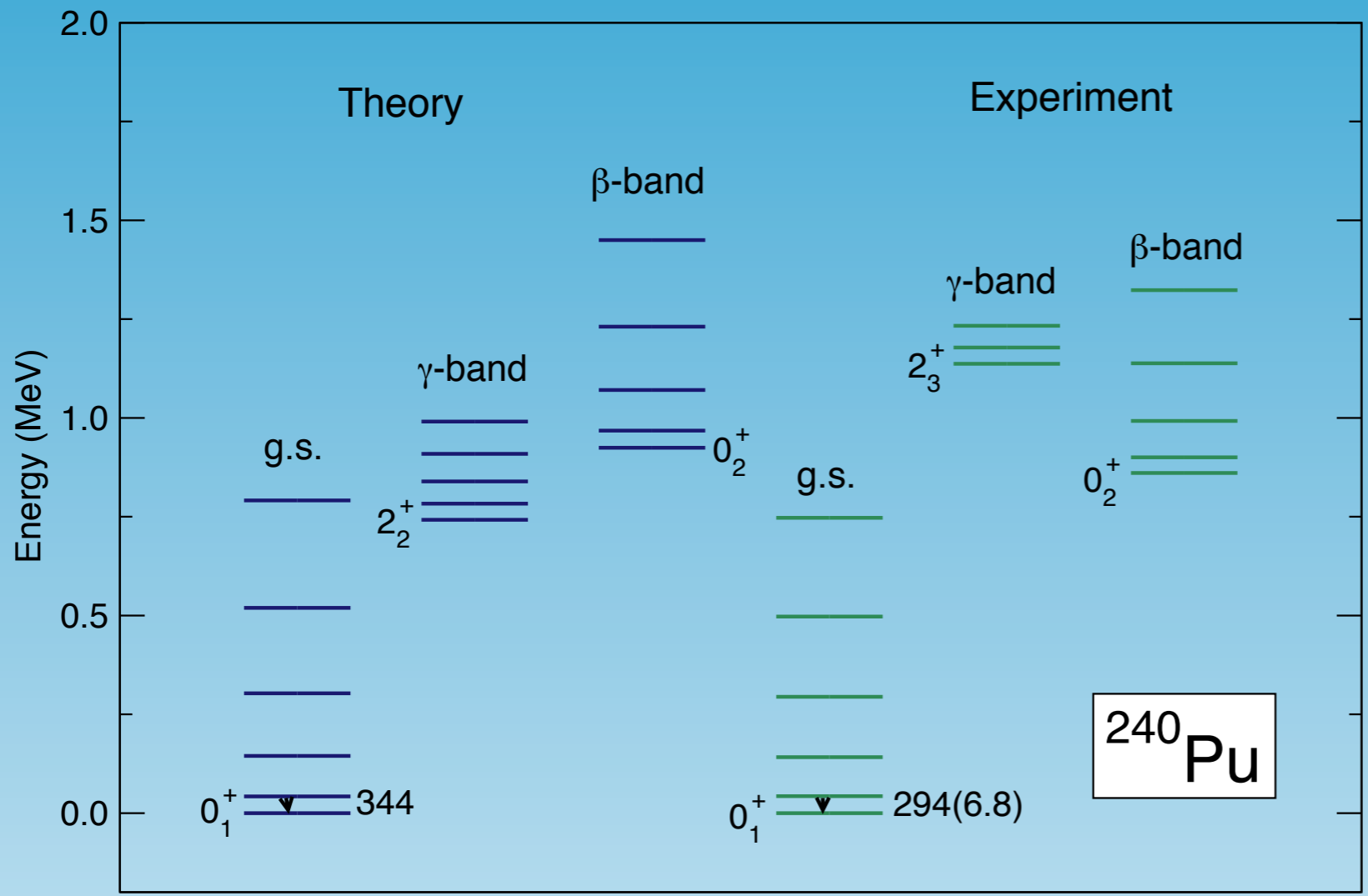
The quasiparticle wave functions and energies generated from constrained self-consistent solutions of a mean-field model, provide the microscopic input for the parameters of the collective Hamiltonian.

# Test of DD-PCI:

Fission path and barriers:



Li, Nikšić, Vretenar, Ring, Meng, Phys. Rev. C (2010)



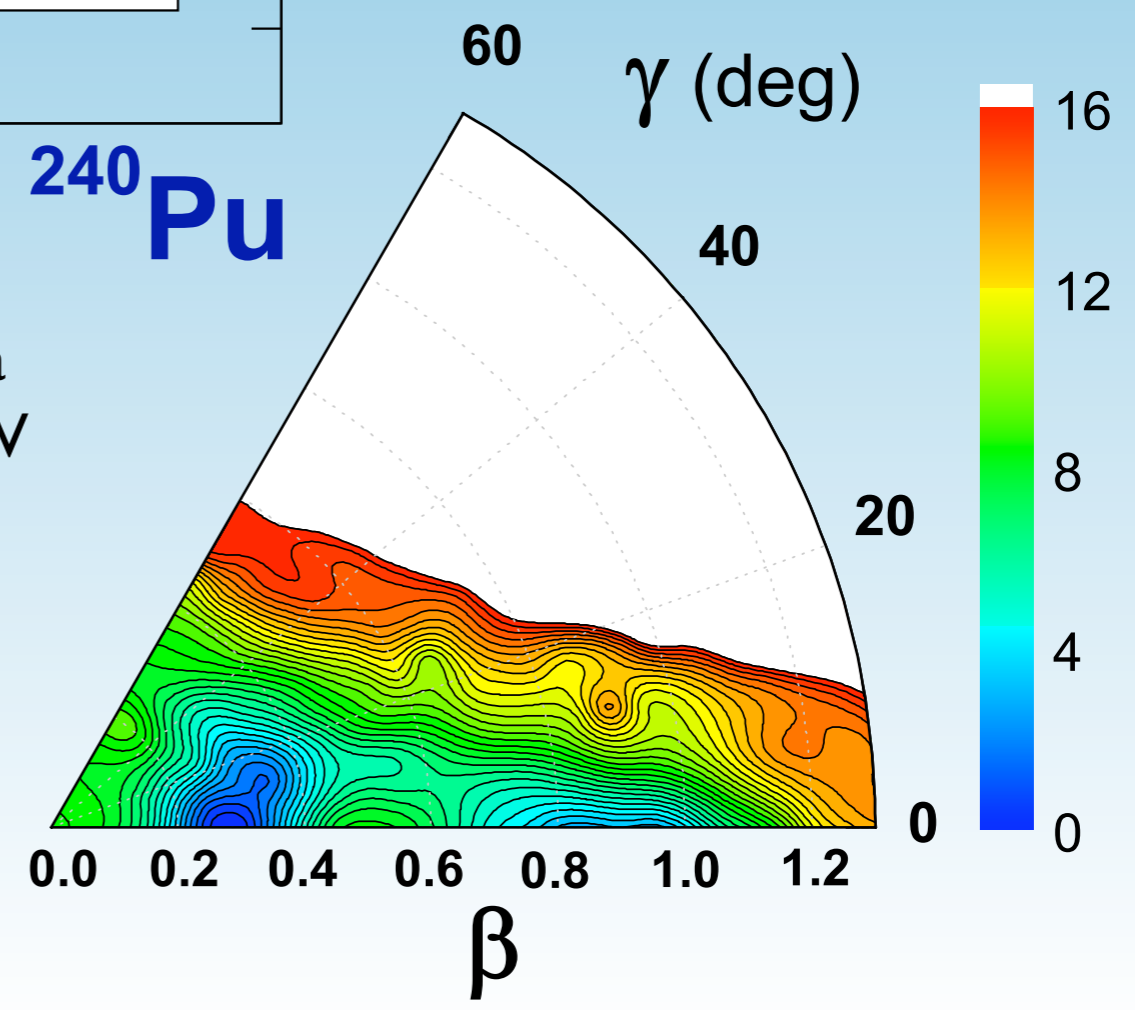
$$E_{4_1^+}^{th} / E_{2_1^+}^{th} = 3.33$$

$$E_{4_1^+}^{exp} / E_{2_1^+}^{exp} = 3.31$$

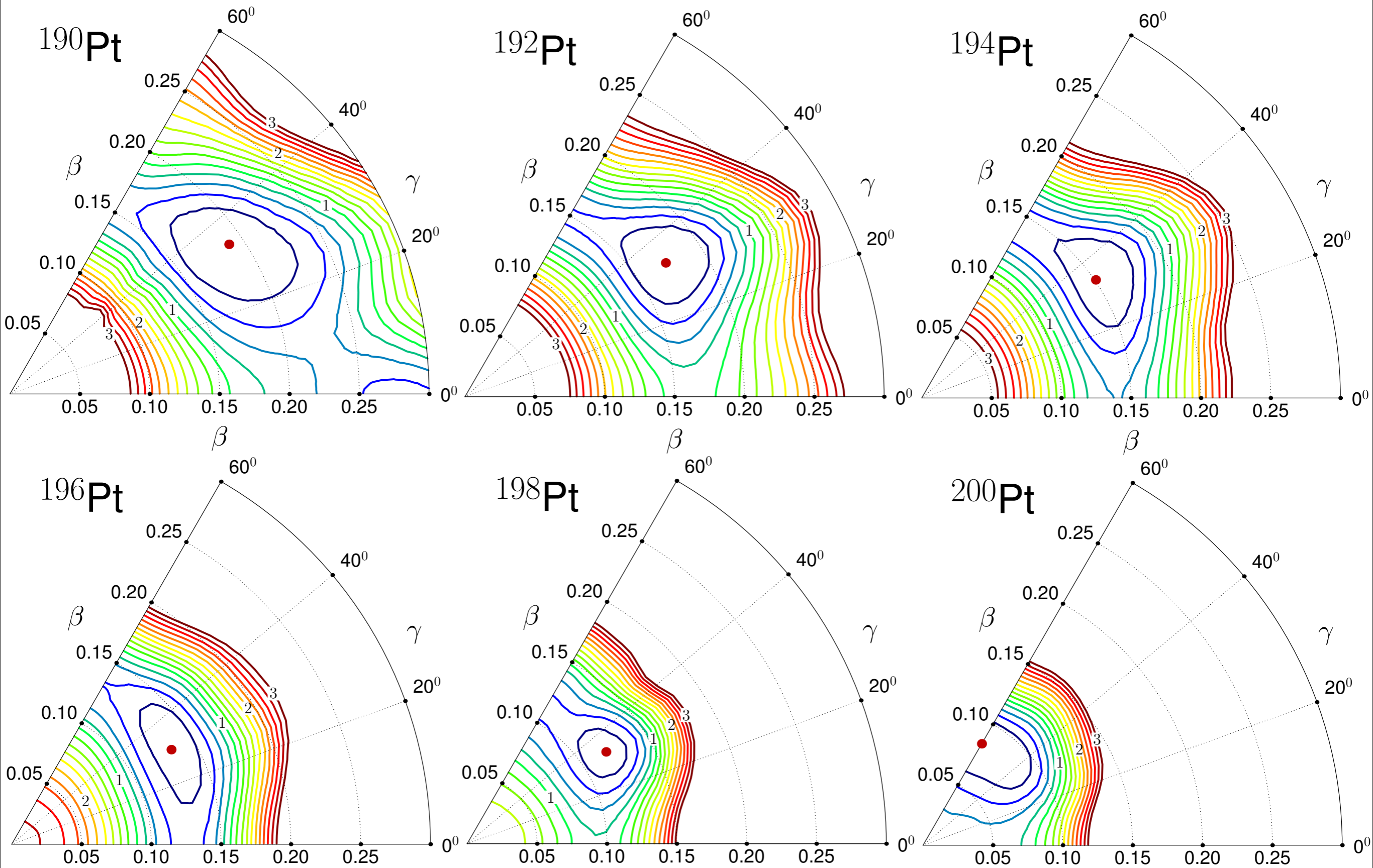
$^{240}\text{Pu}$

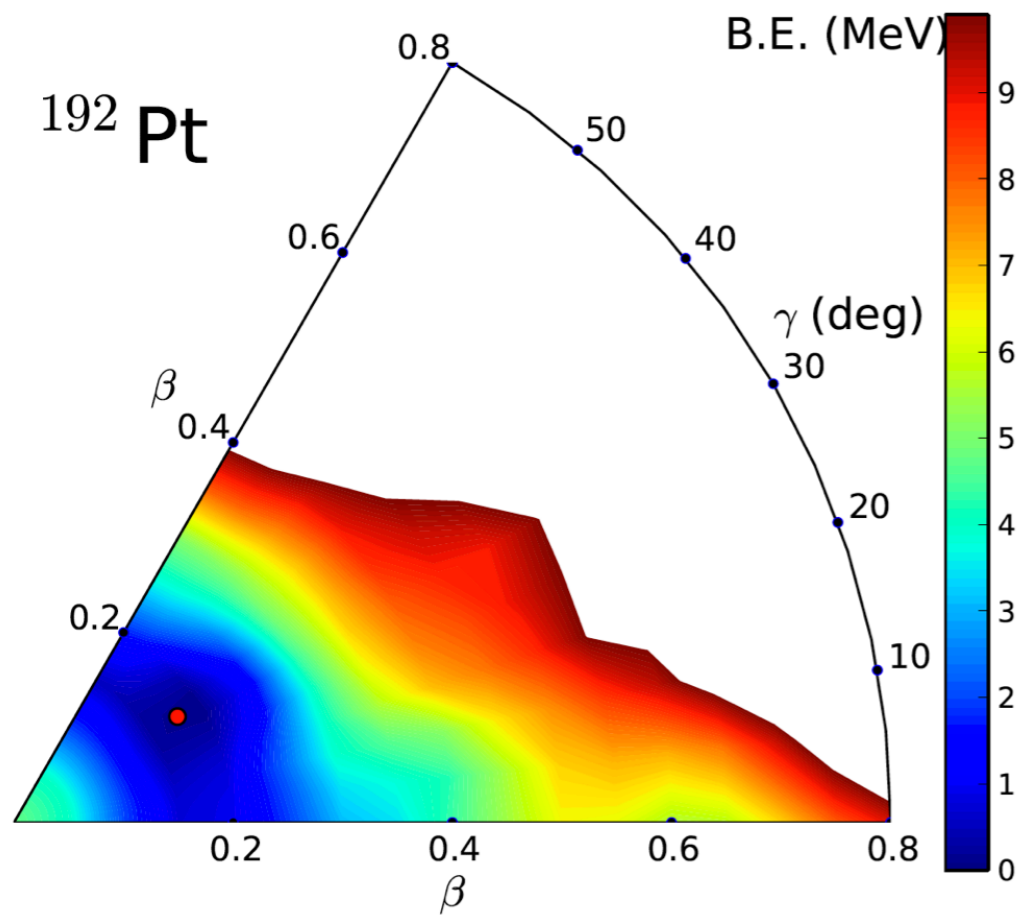
$^{240}\text{Pu}$

The moments of inertia are renormalized by a factor  $\approx 1.3$  (difference between the IB and TV moments of inertia).



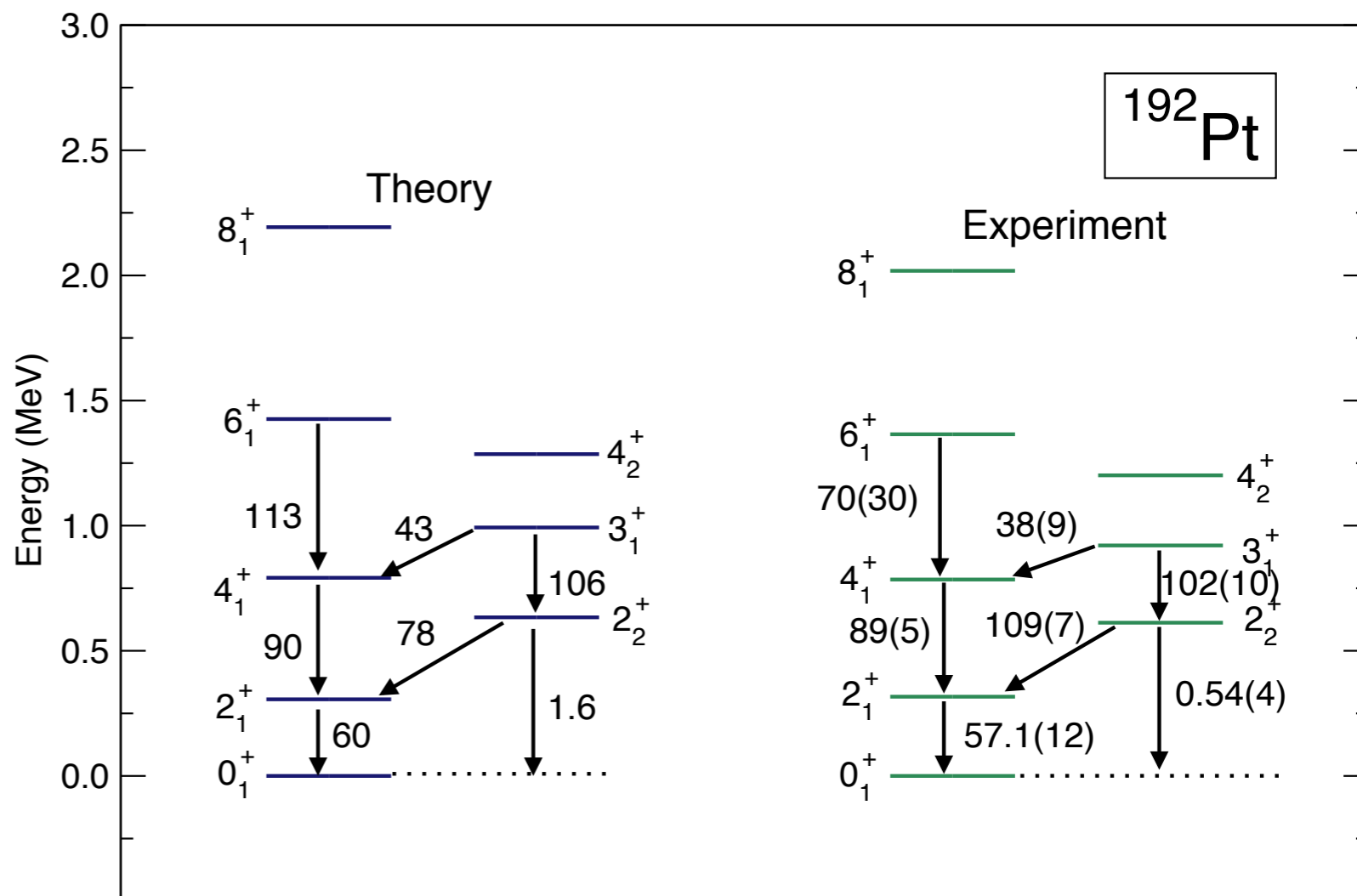
# Evolution of triaxial shapes in Pt nuclei:

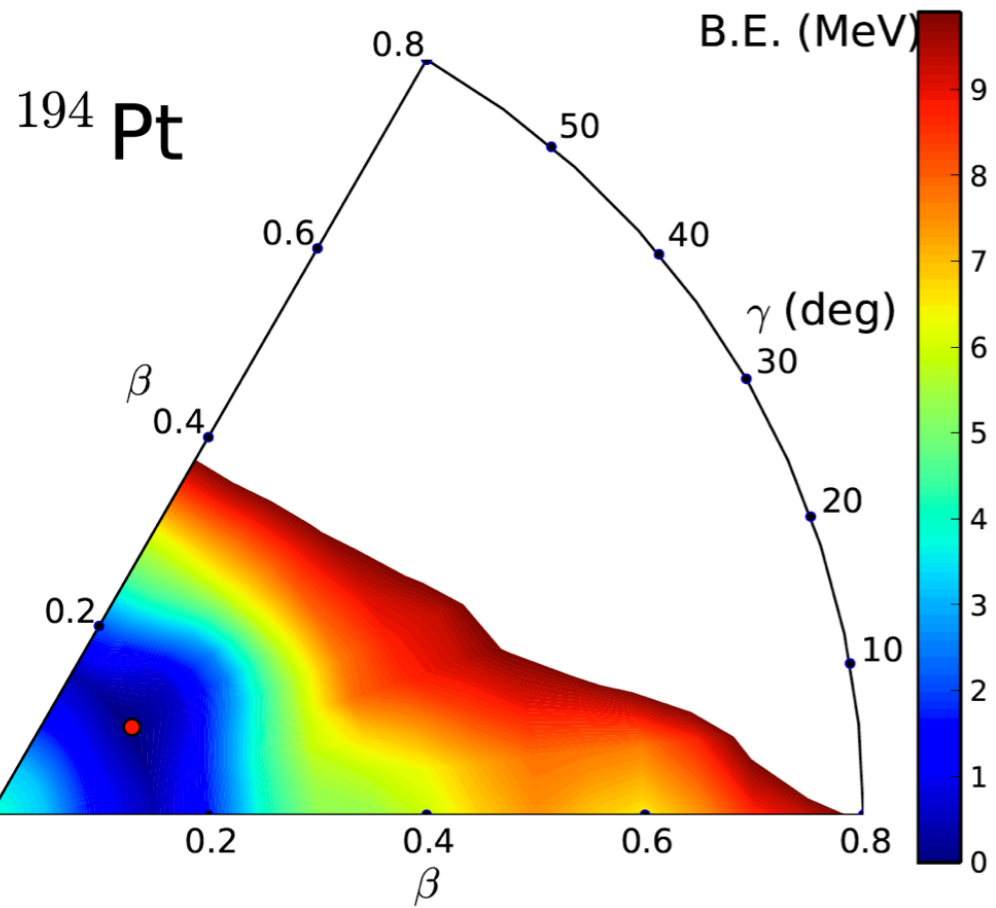




$$E_{4_1^+}^{th} / E_{2_1^+}^{th} = 2.58$$

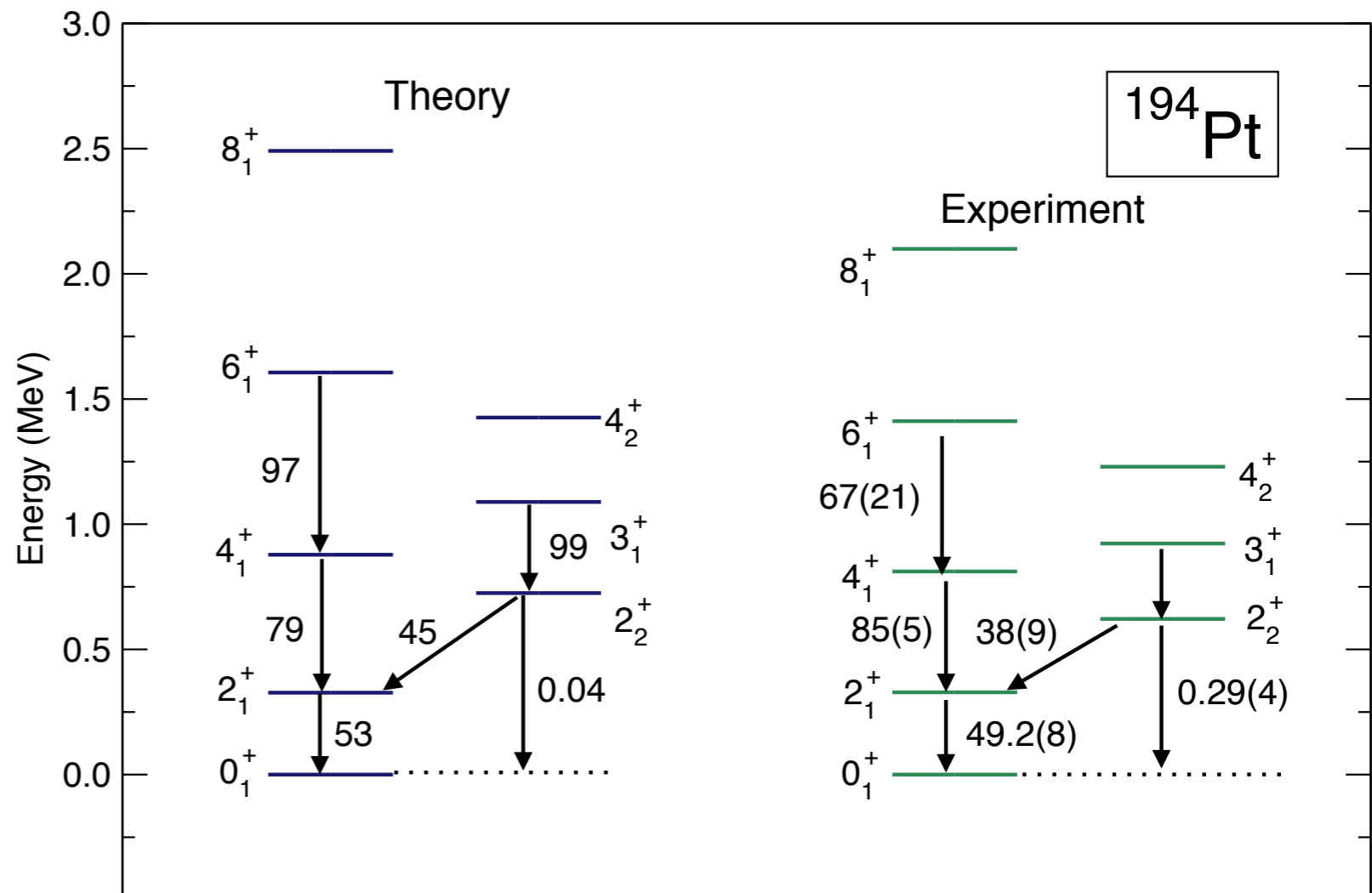
$$E_{4_1^+}^{exp} / E_{2_1^+}^{exp} = 2.48$$



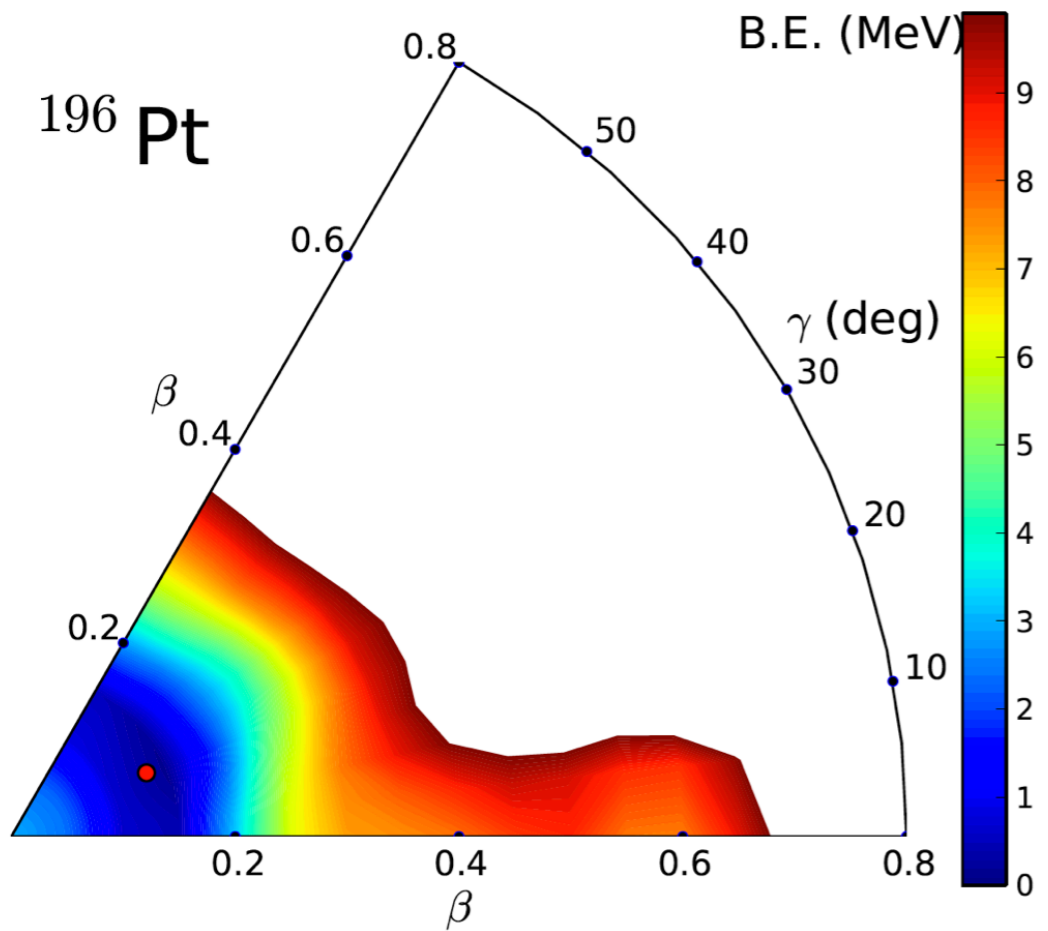


$$E_{4_1^+}^{th} / E_{2_1^+}^{th} = 2.68$$

$$E_{4_1^+}^{exp} / E_{2_1^+}^{exp} = 2.47$$

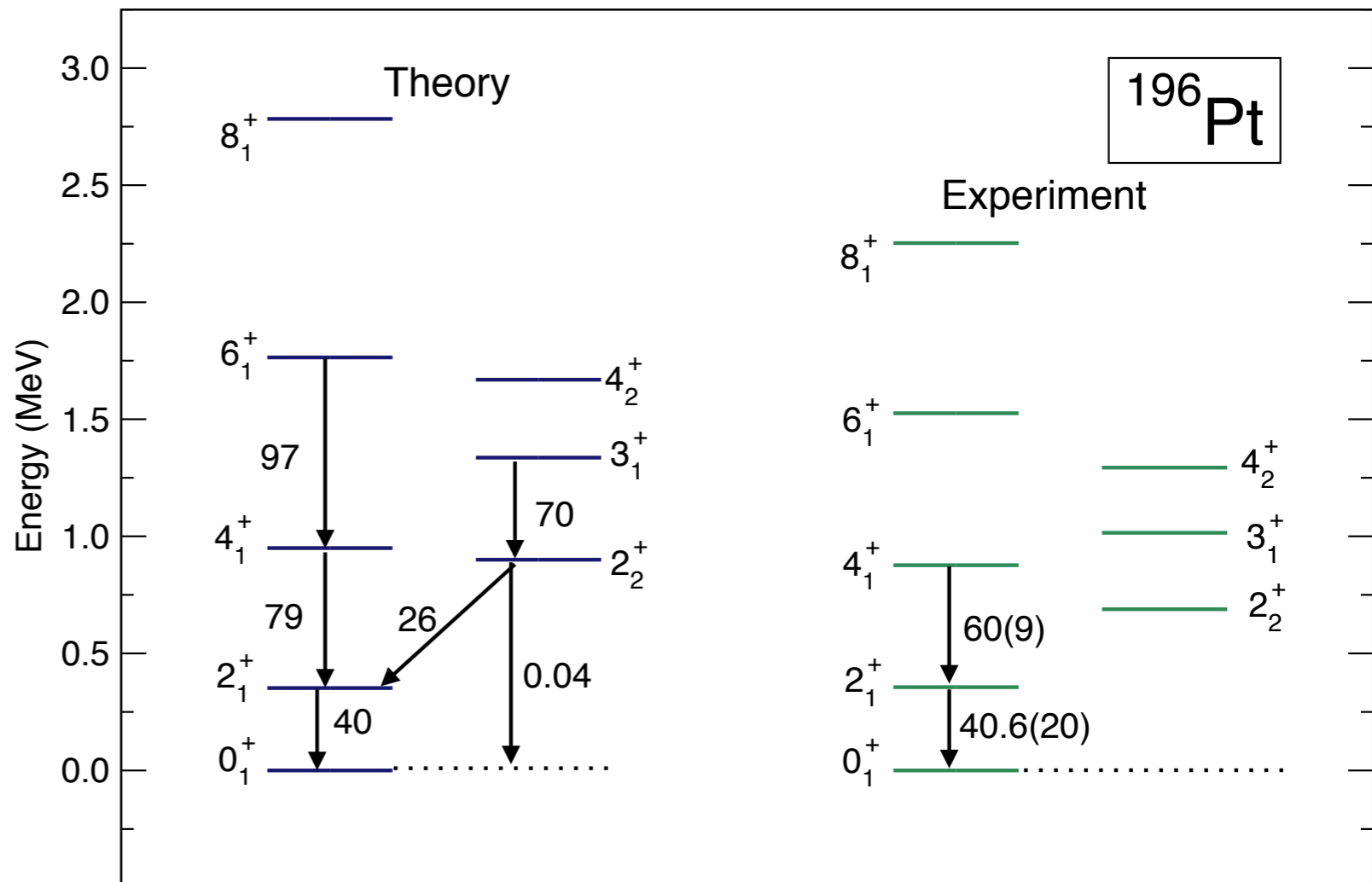


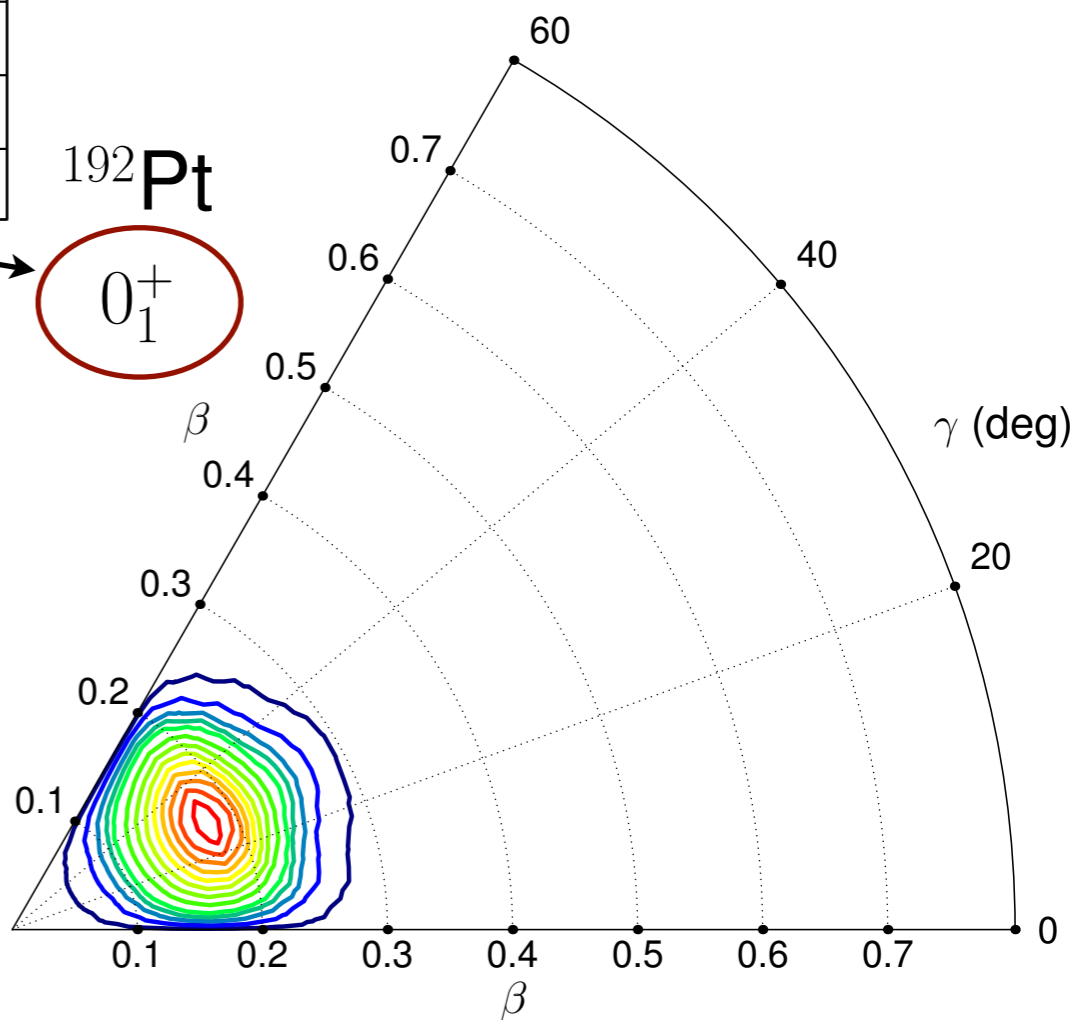
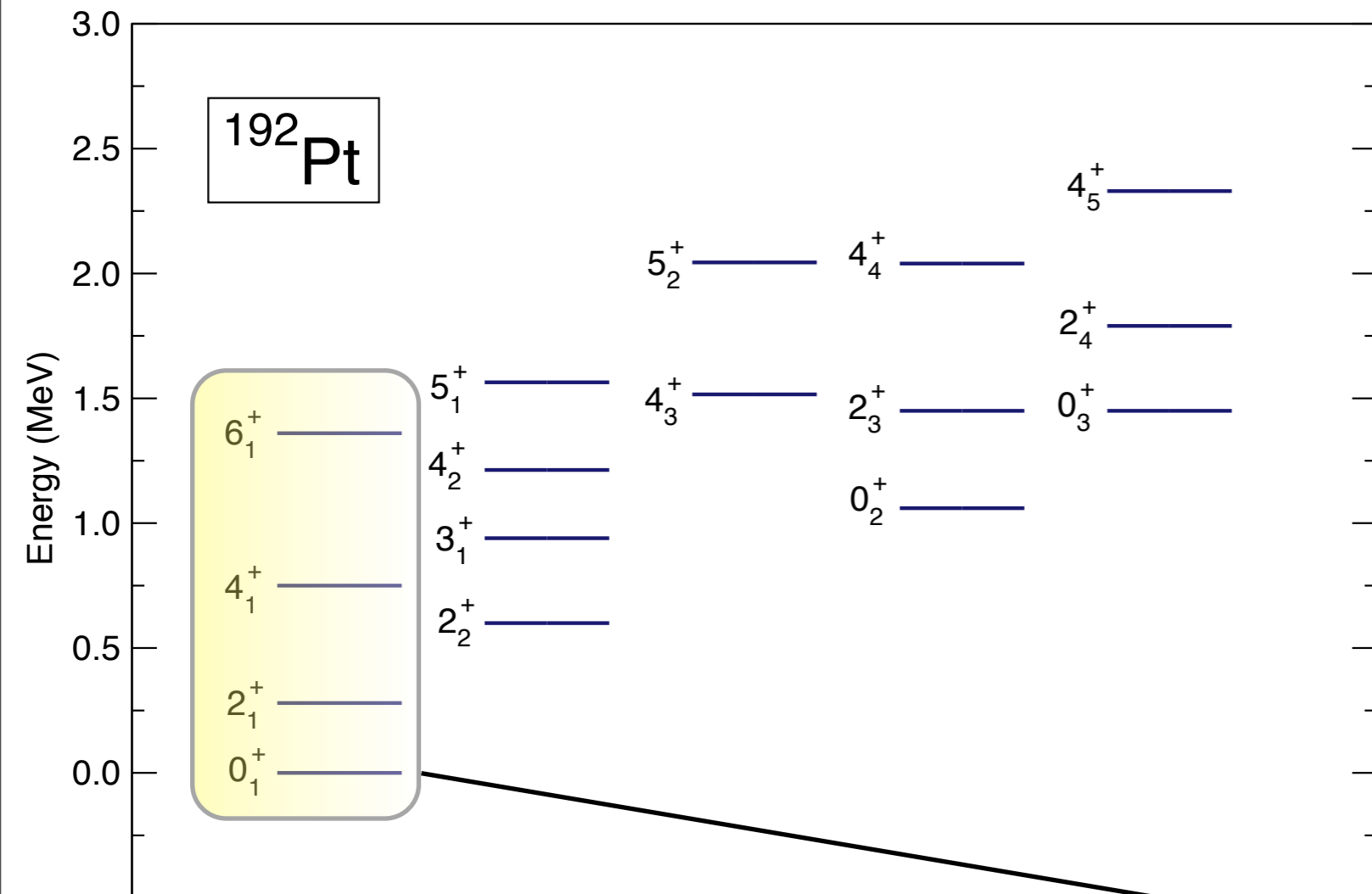




$$E_{4_1^+}^{th} / E_{2_1^+}^{th} = 2.69$$

$$E_{4_1^+}^{exp} / E_{2_1^+}^{exp} = 2.47$$



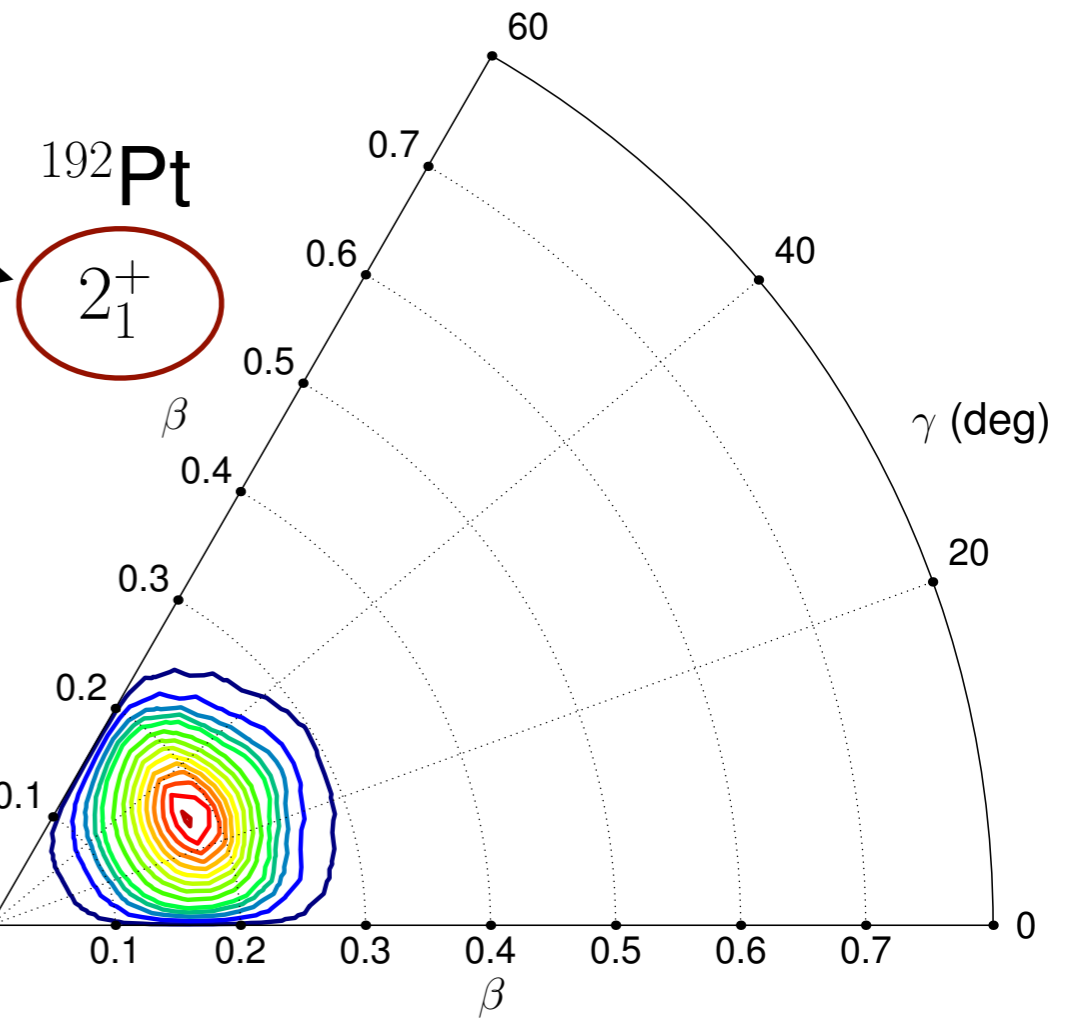
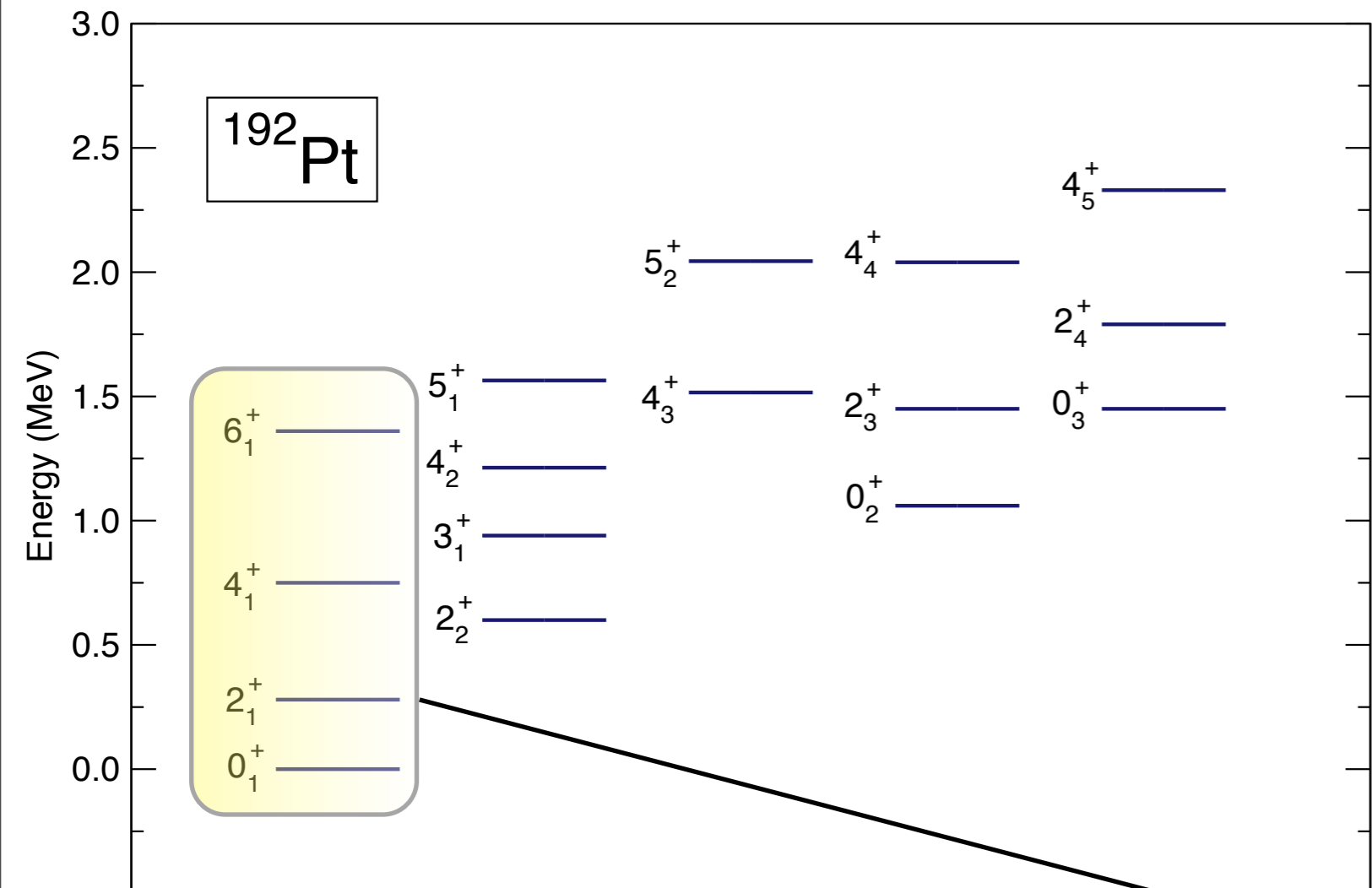


Collective wave function:

$$\Psi_{\alpha}^{IM}(\beta, \gamma, \Omega) = \sum_{K \in \Delta I} \psi_{\alpha K}^I(\beta, \gamma) \Phi_{MK}^I(\Omega)$$

Probability density distribution in the  $(\beta, \gamma)$  plane:

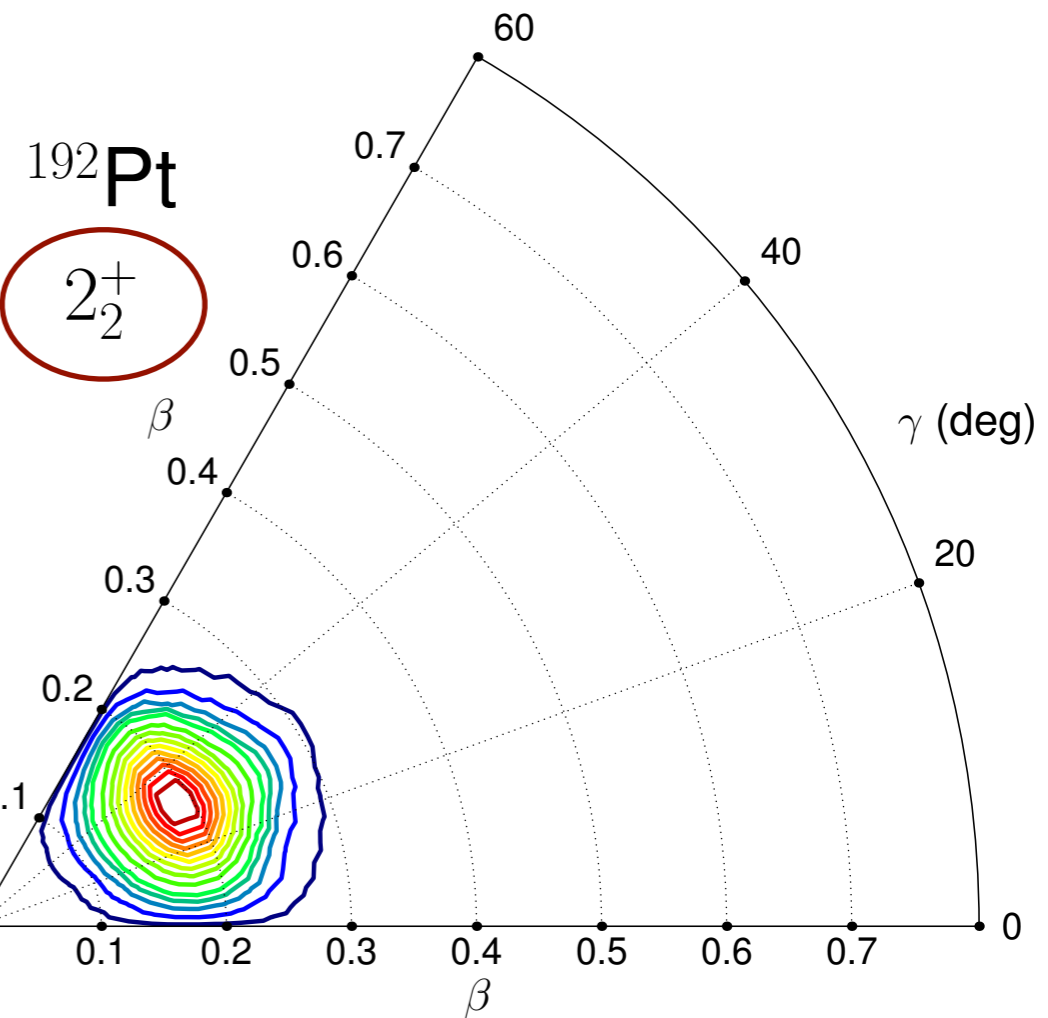
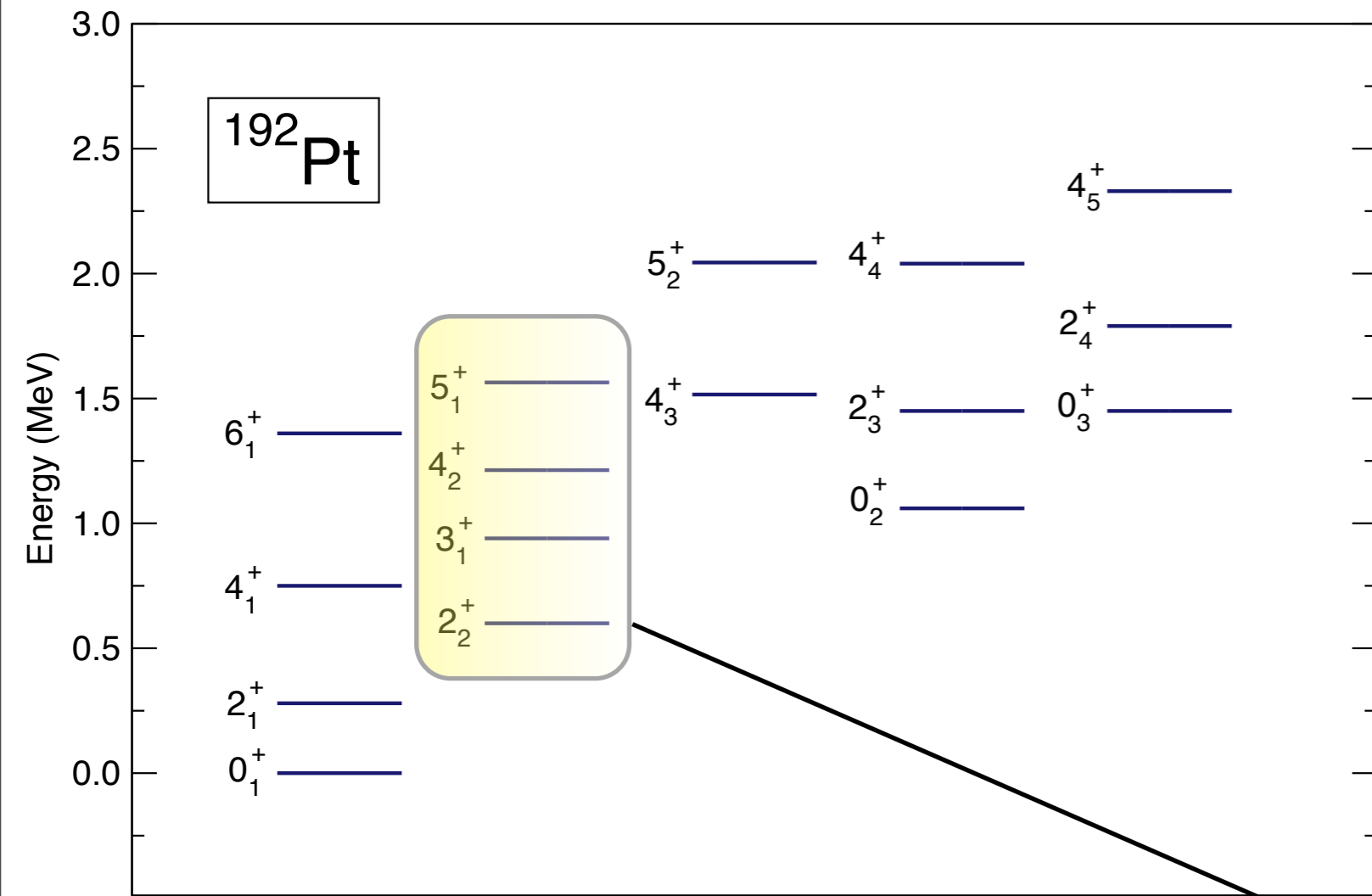
$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$



Probability density distribution in the  $(\beta, \gamma)$  plane:

$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

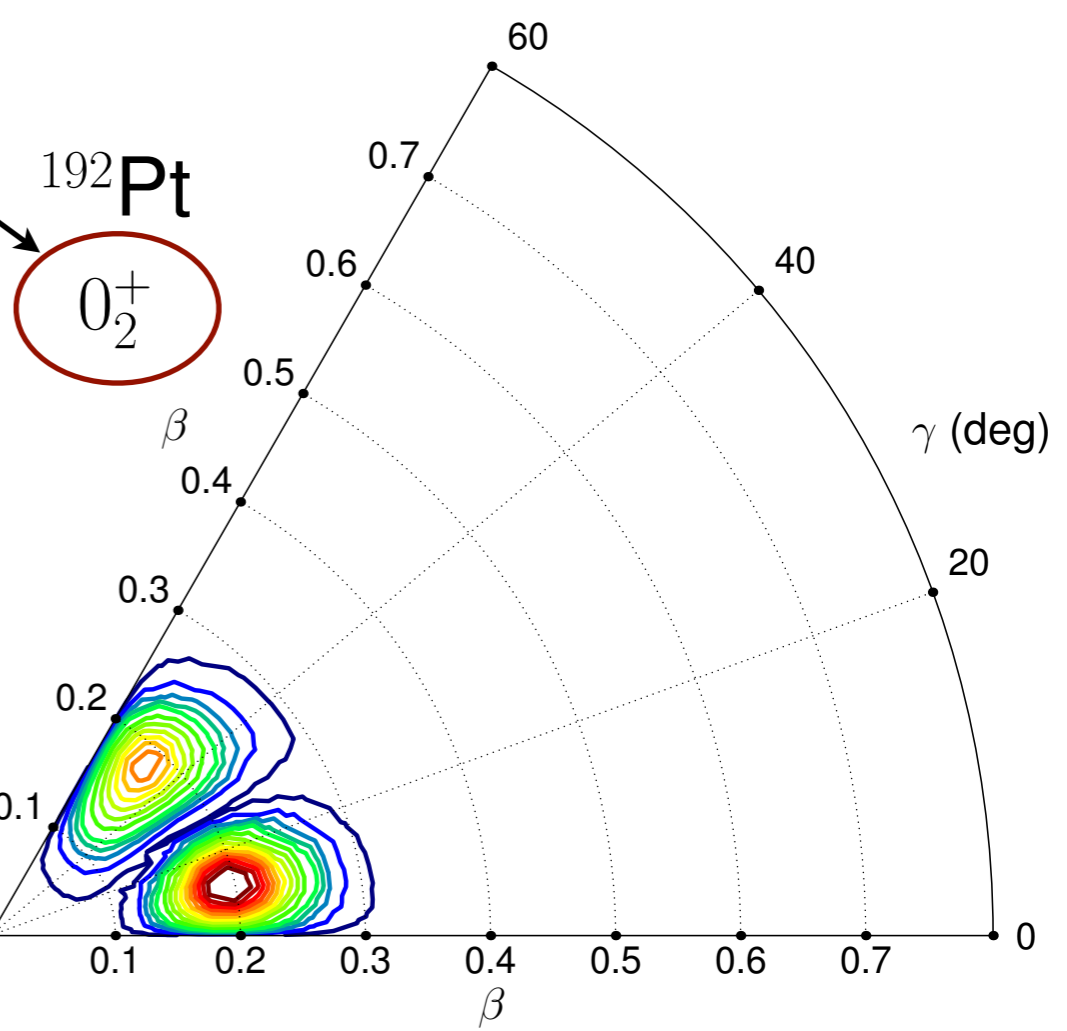
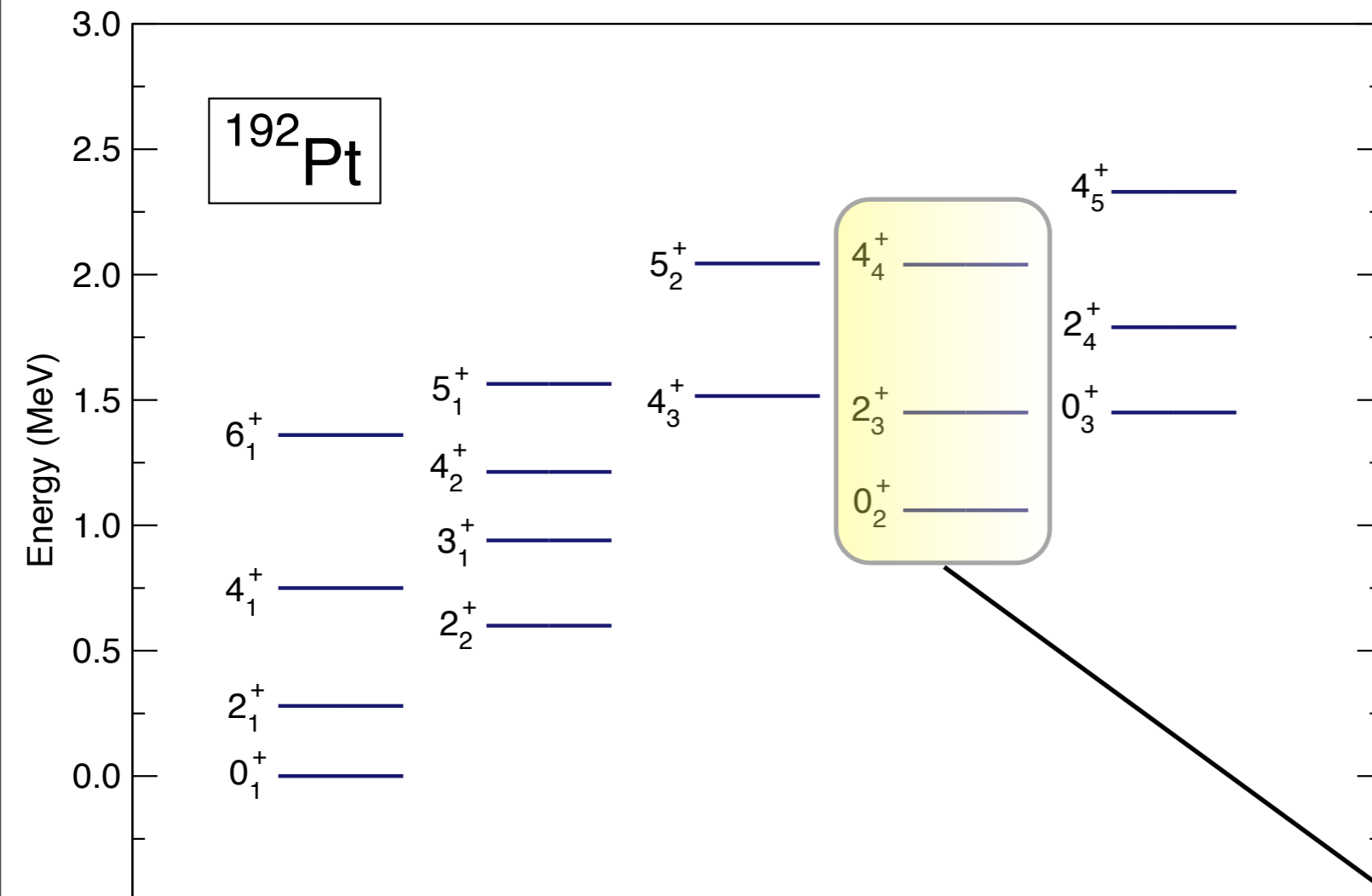
$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$



Probability density distribution in the  $(\beta, \gamma)$  plane:

$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

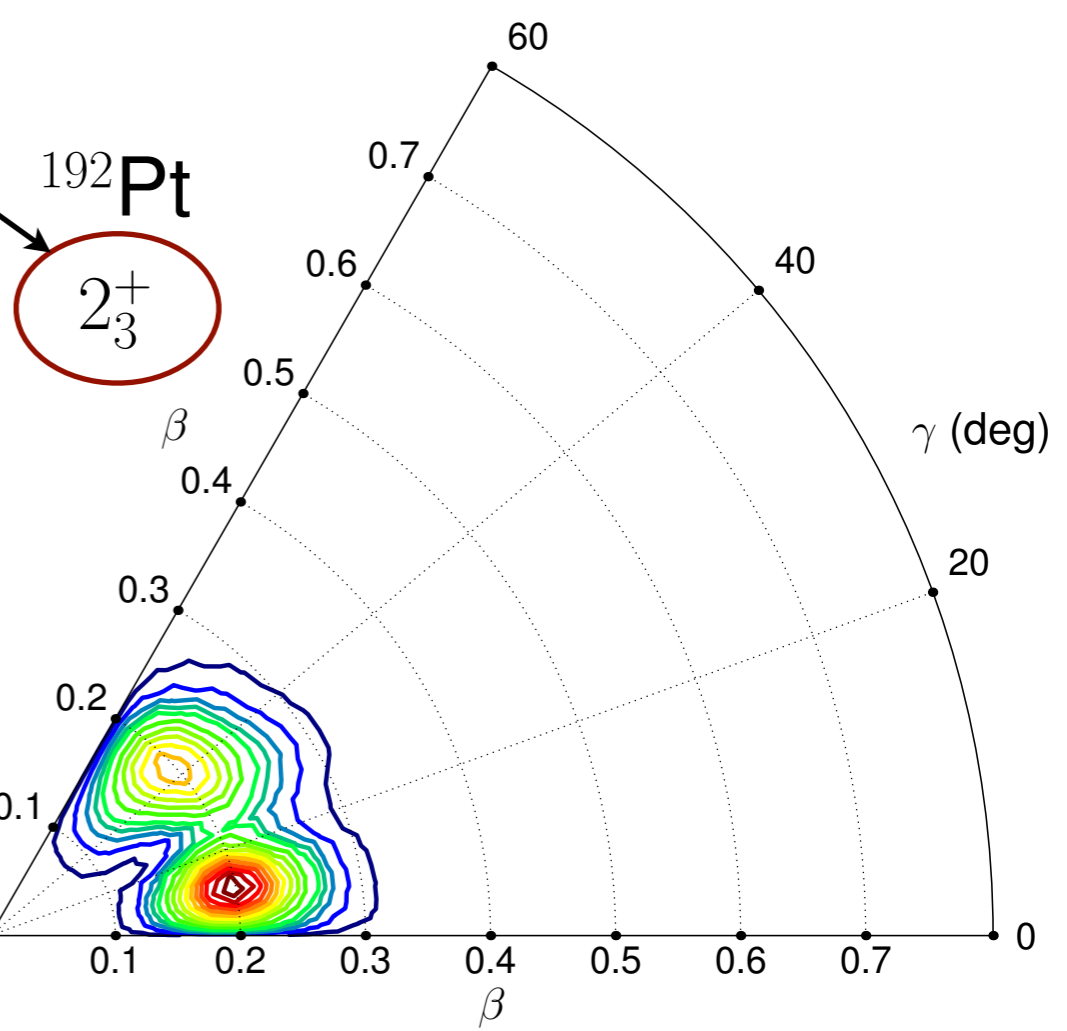
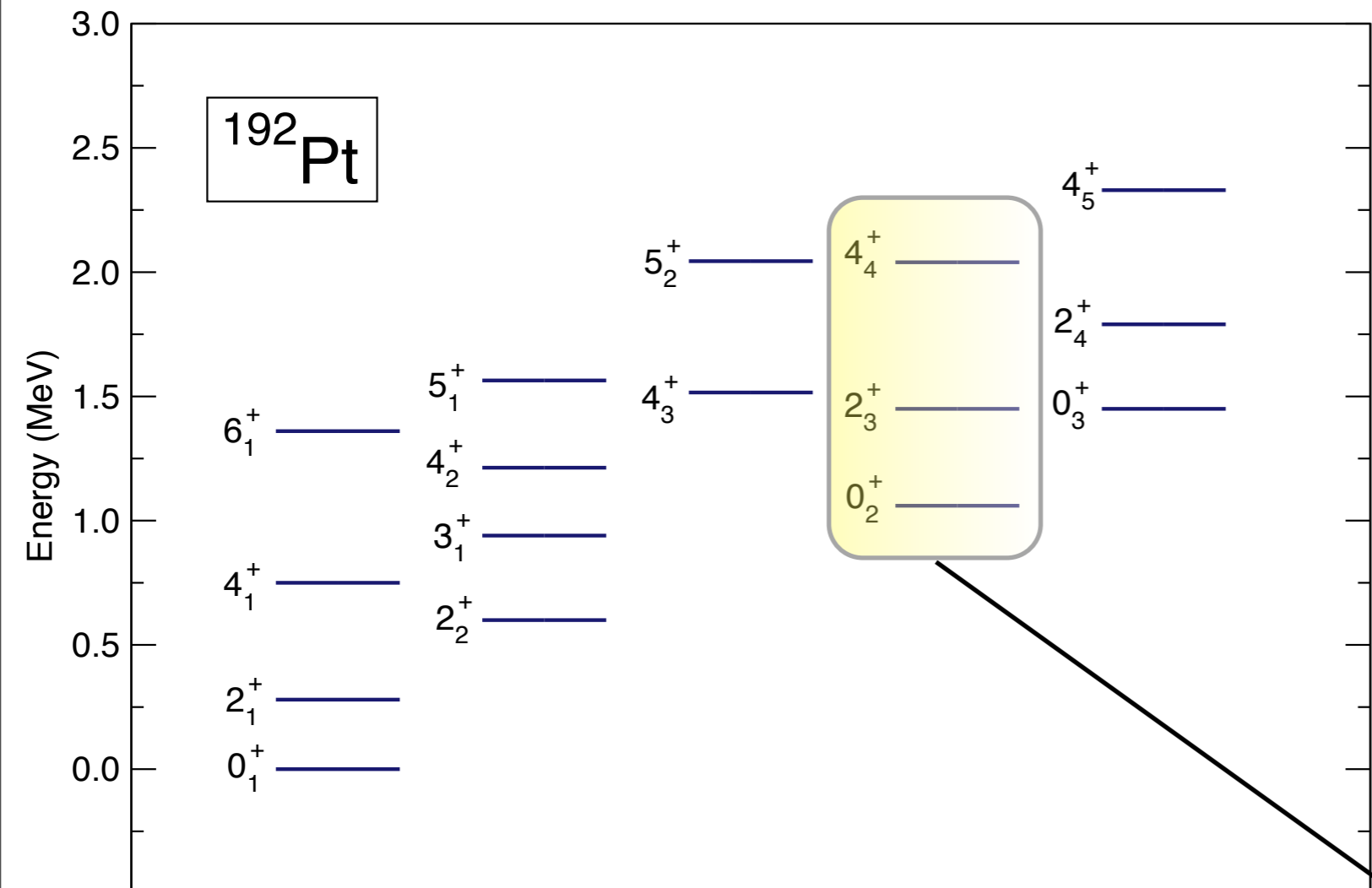
$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$



Probability density distribution in the  $(\beta, \gamma)$  plane:

$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

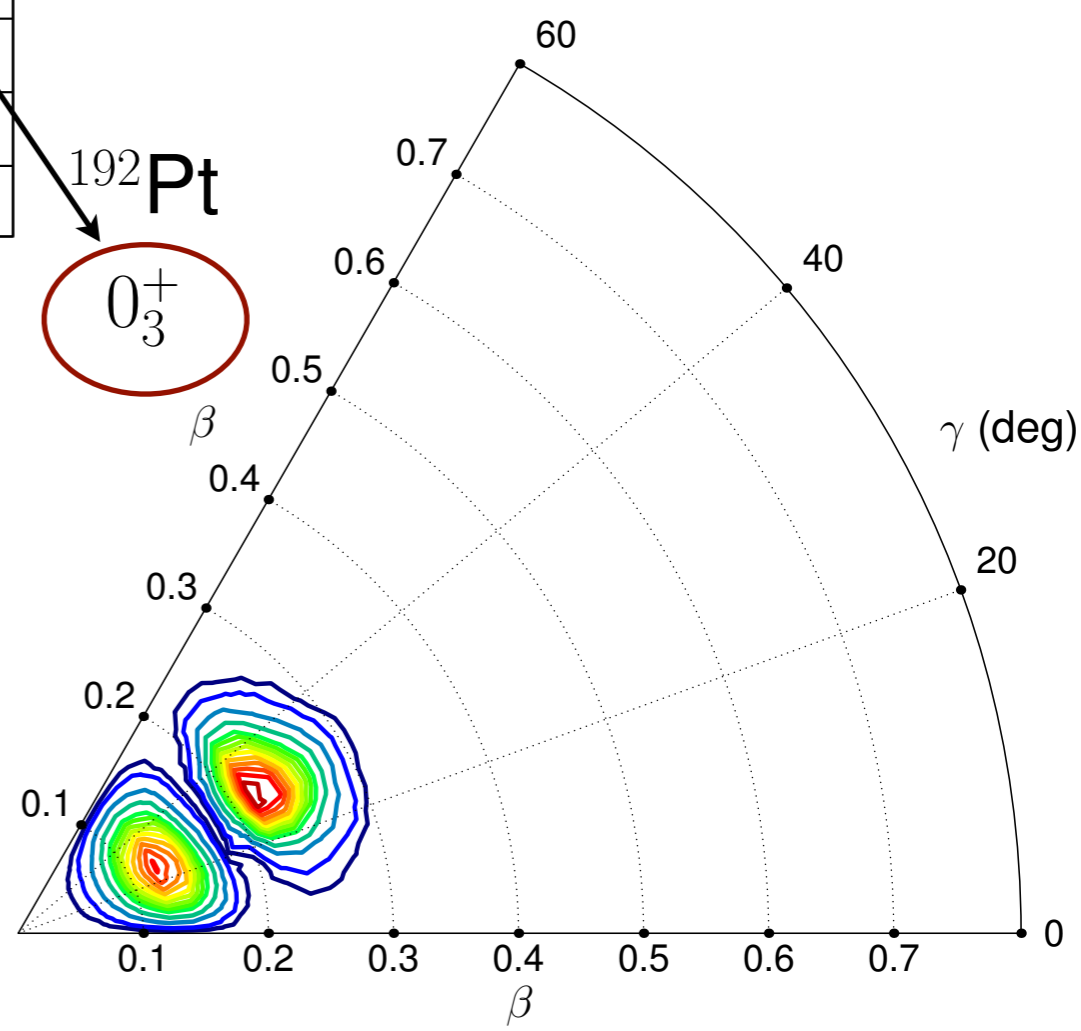
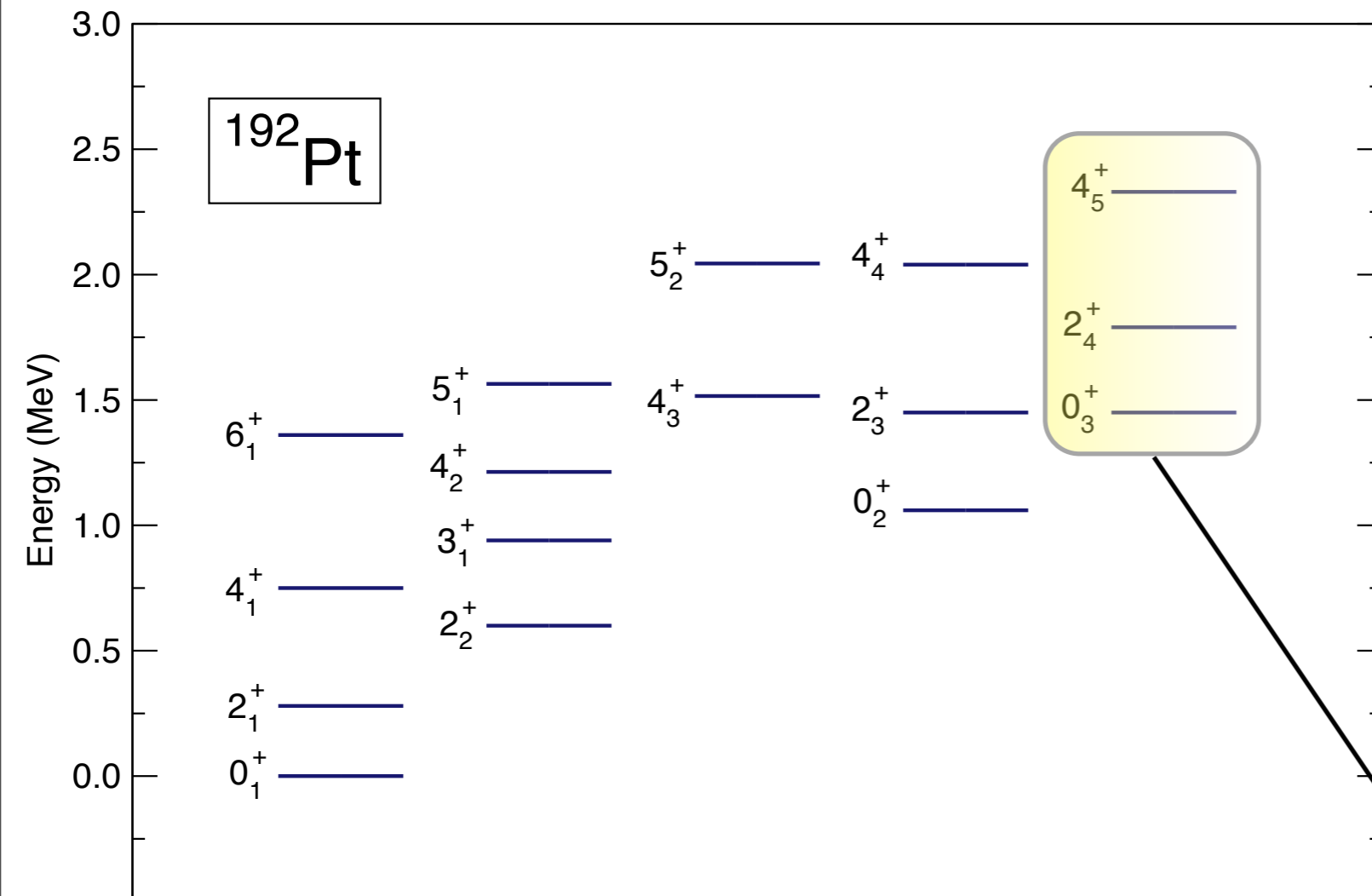
$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$



Probability density distribution in the  $(\beta, \gamma)$  plane:

$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

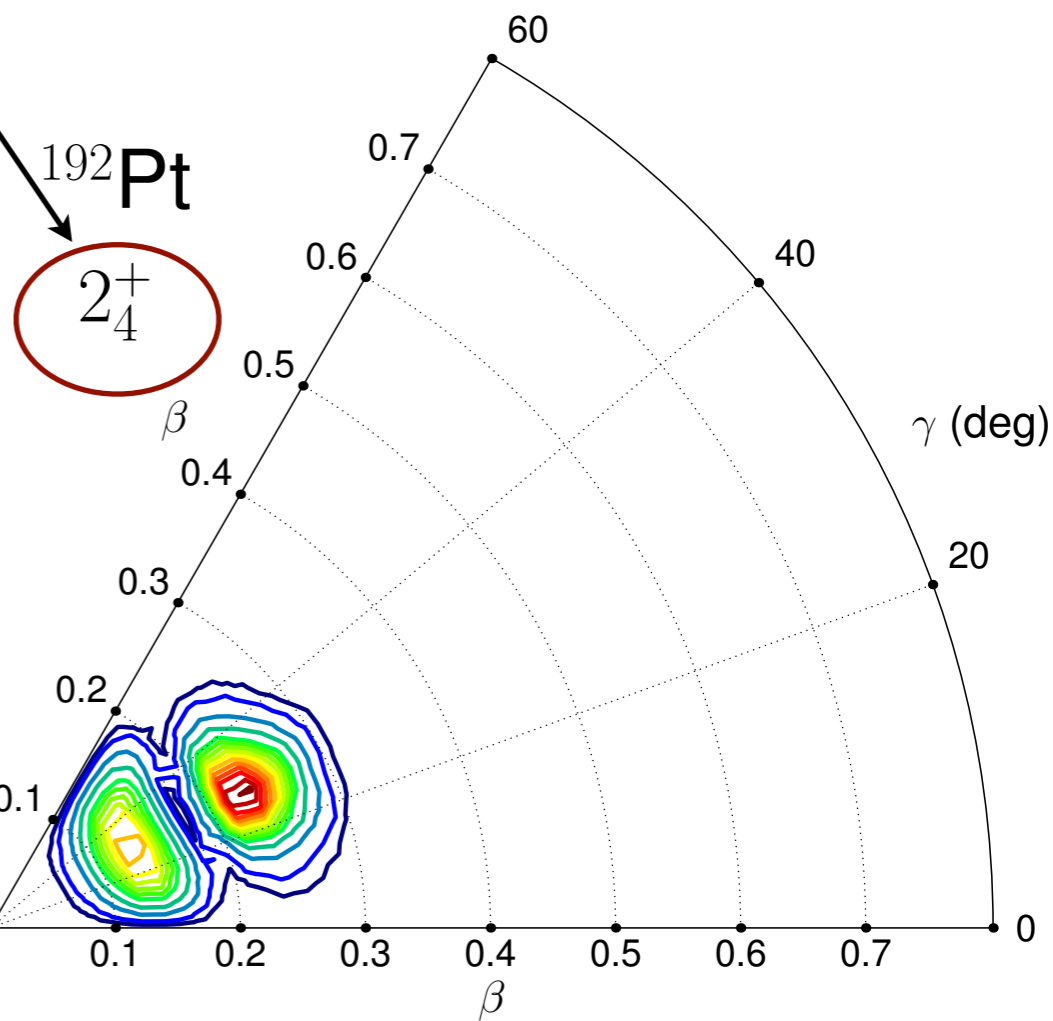
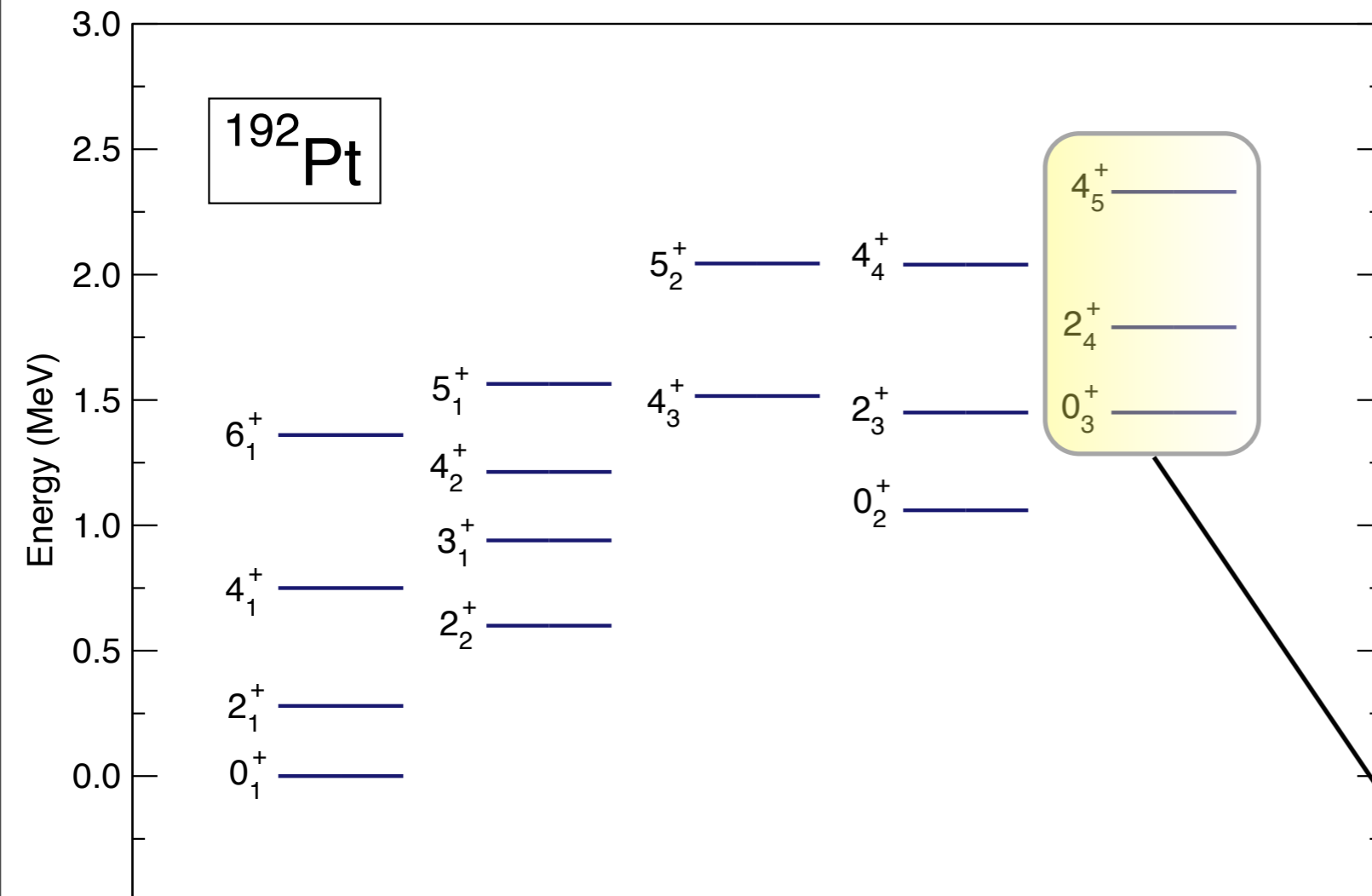
$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$



Probability density distribution in the ( $\beta, \gamma$ ) plane:

$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$



Probability density distribution in the  $(\beta, \gamma)$  plane:

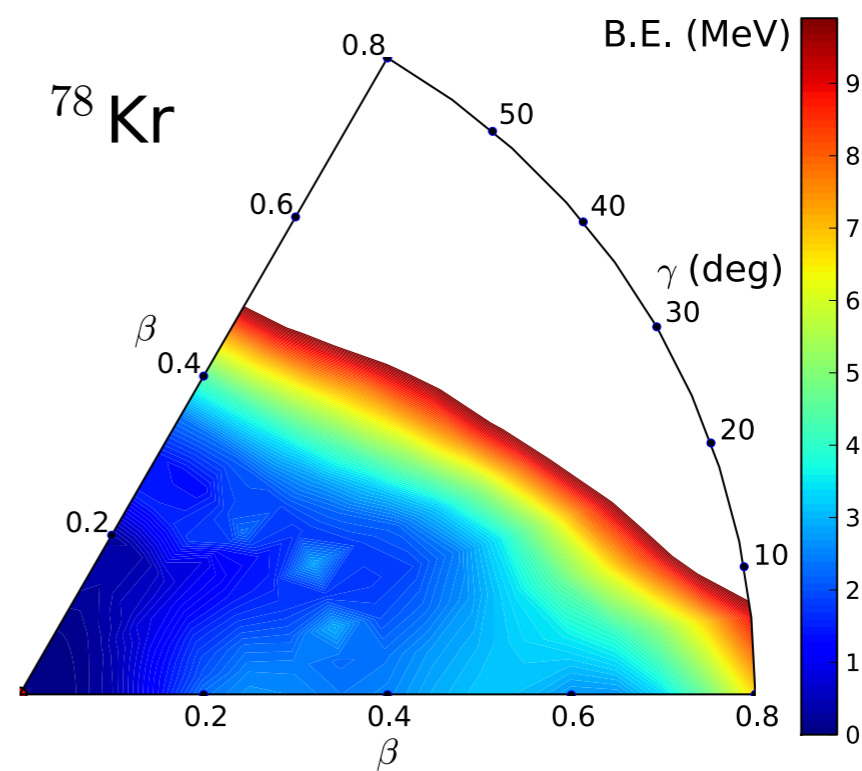
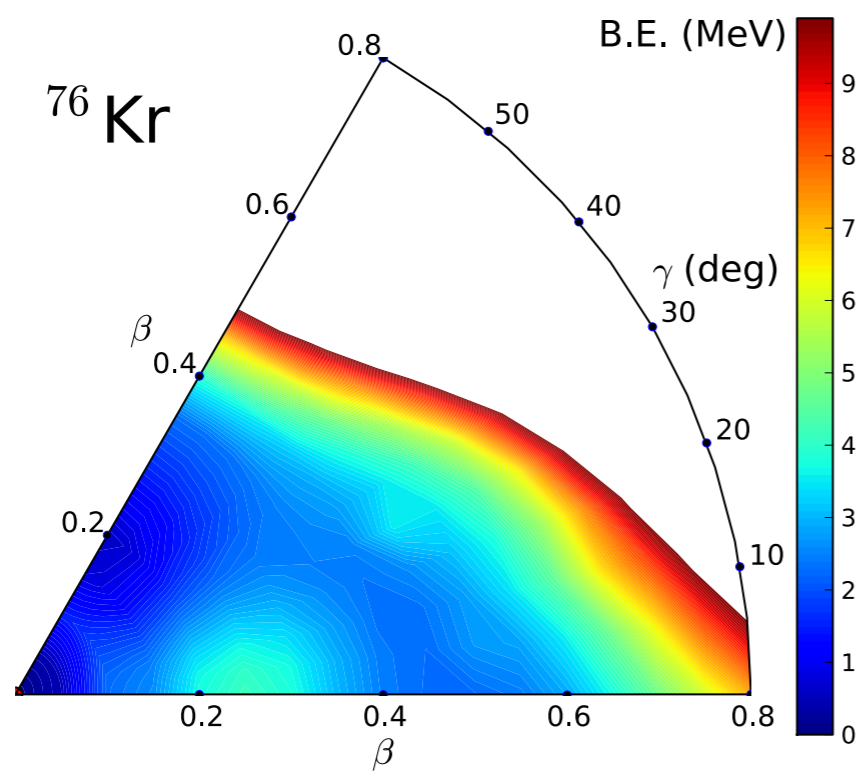
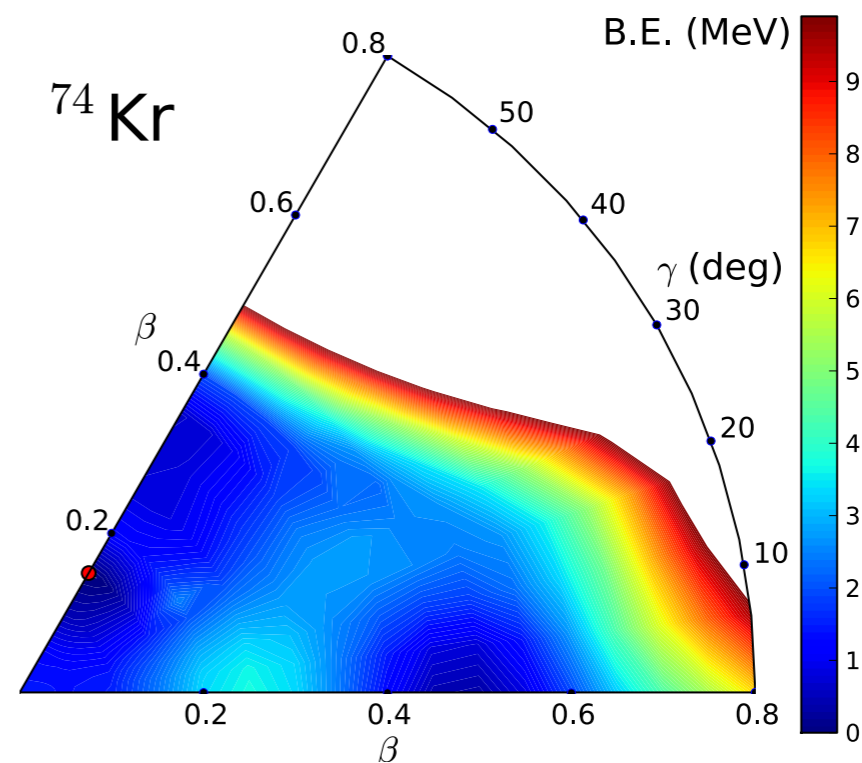
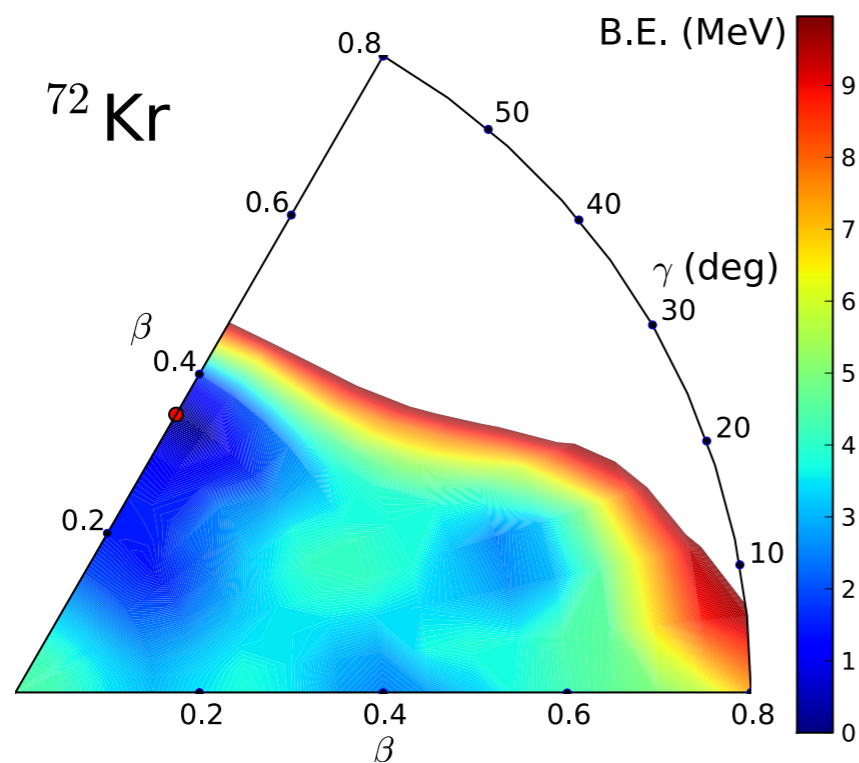
$$\rho_{I\alpha}(\beta, \gamma) = \sum_{K \in \Delta I} |\psi_{\alpha K}^I(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|$$

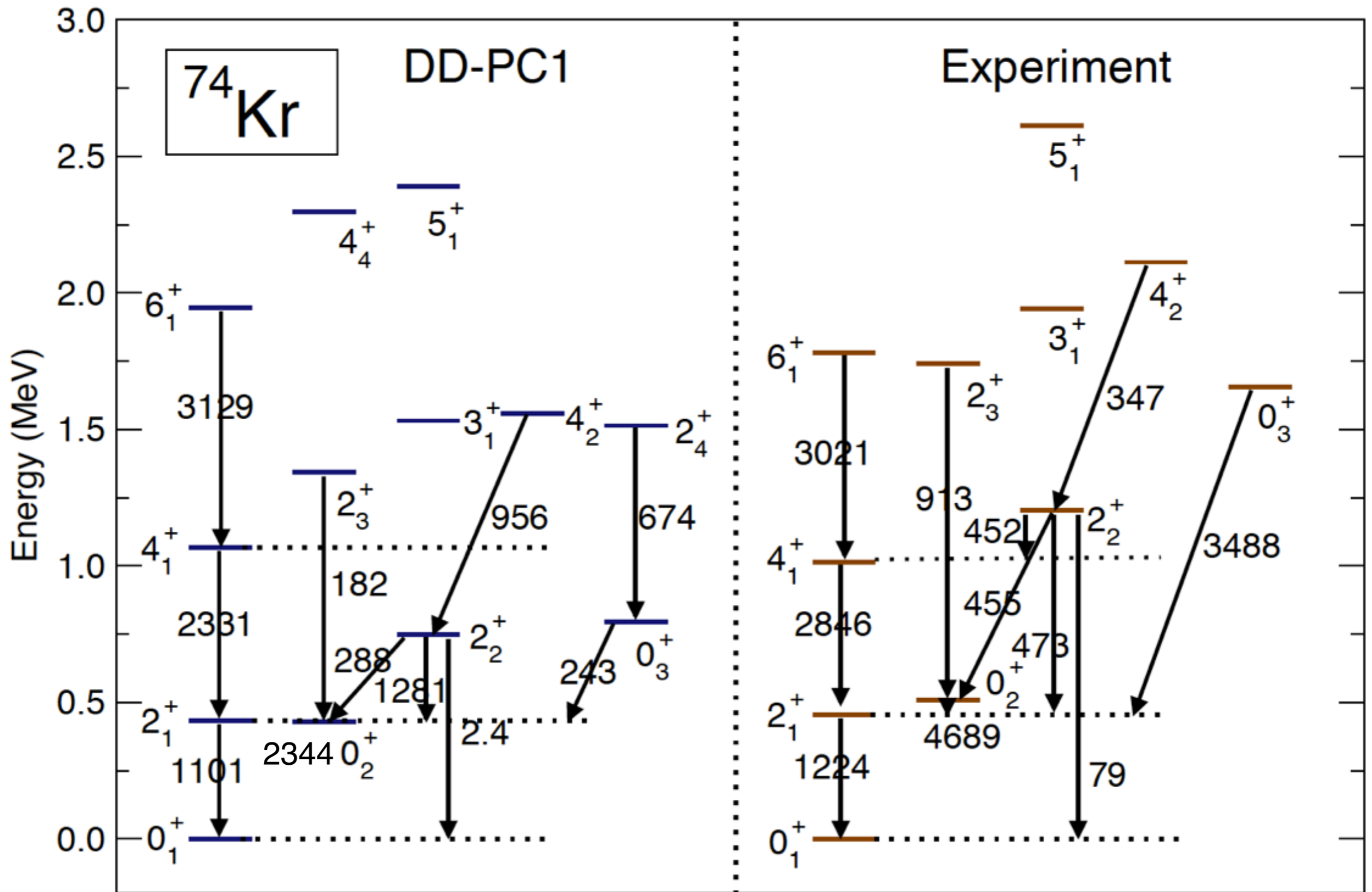
$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1$$

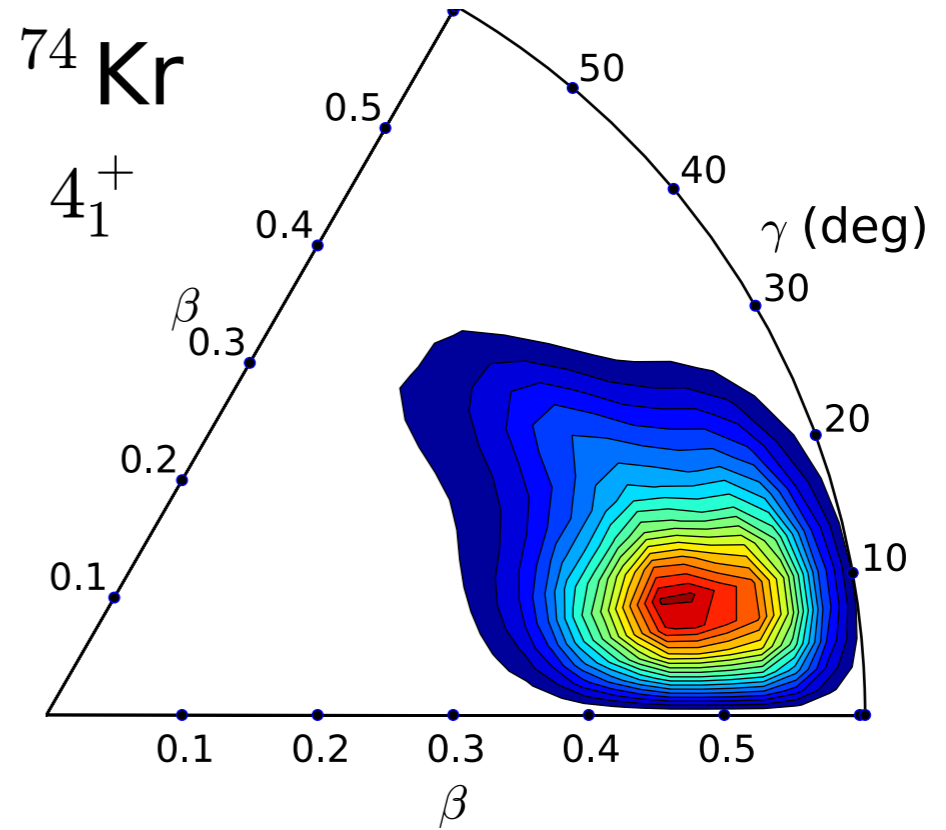
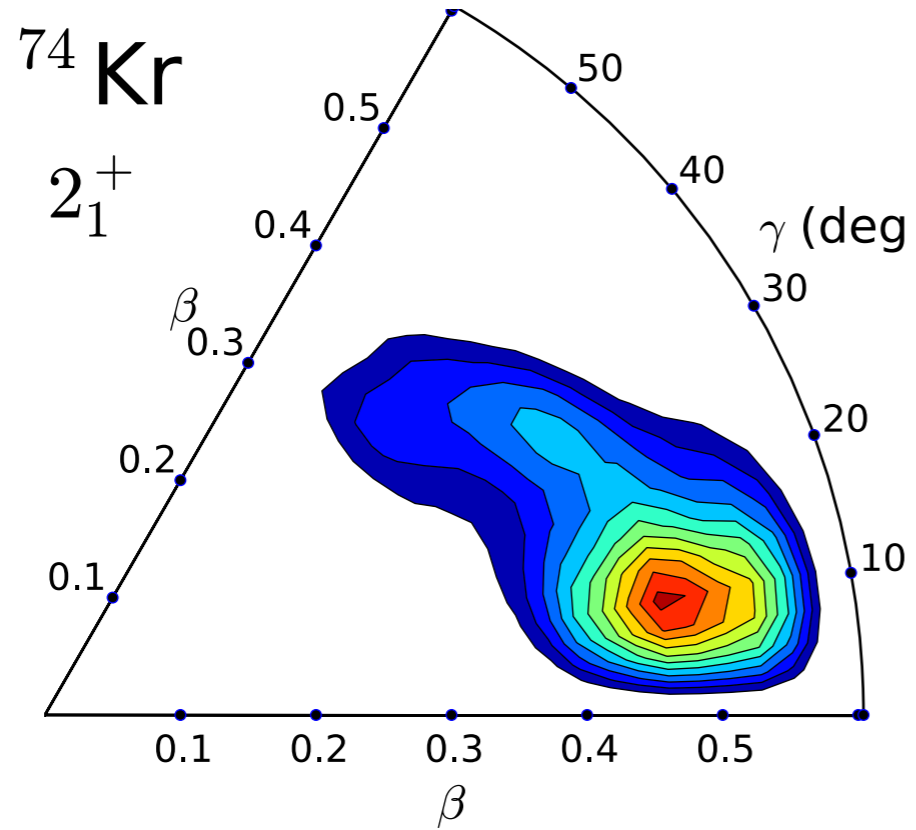
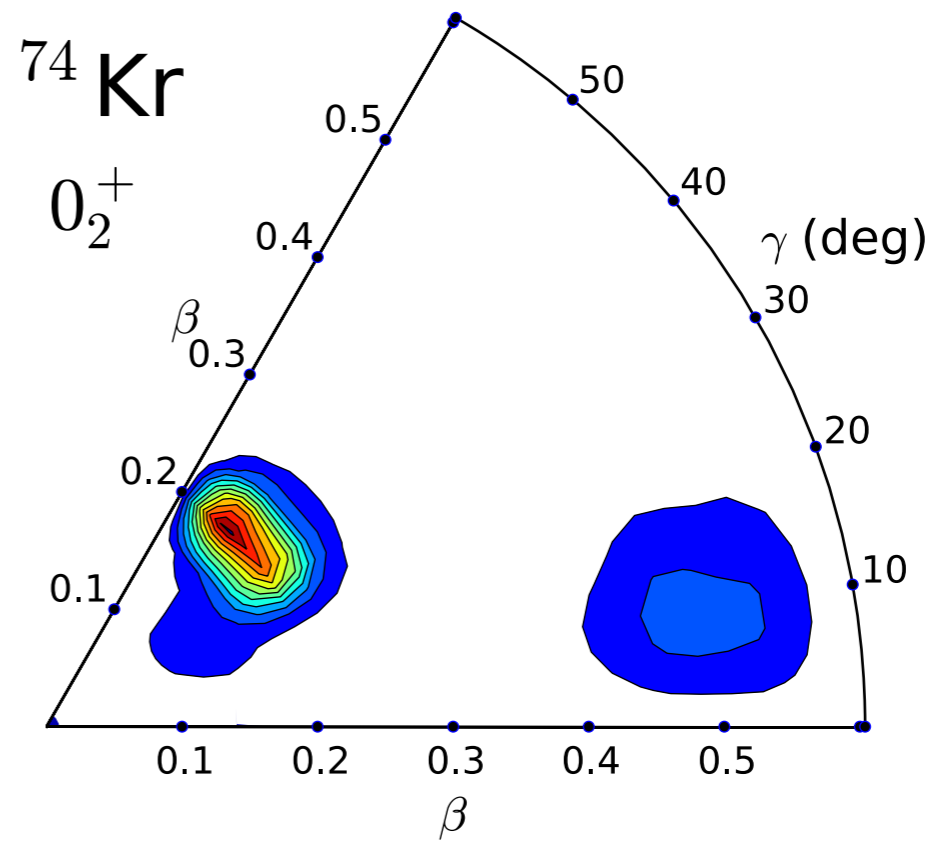
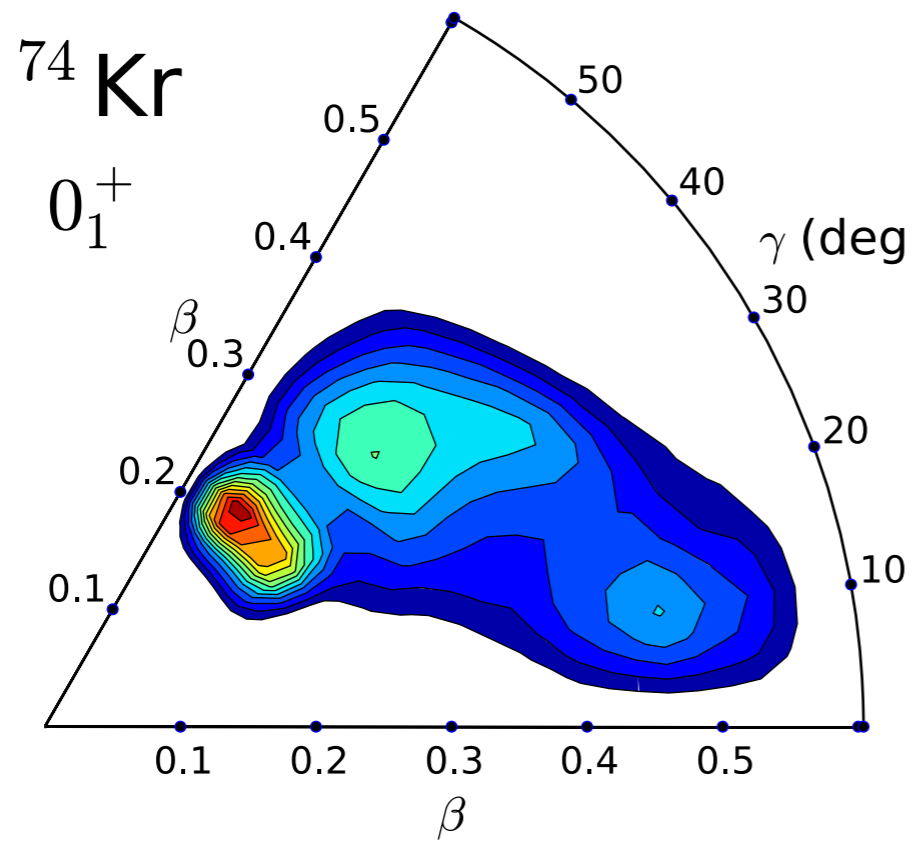


How does the functional DD-PCI extrapolate to other mass regions?

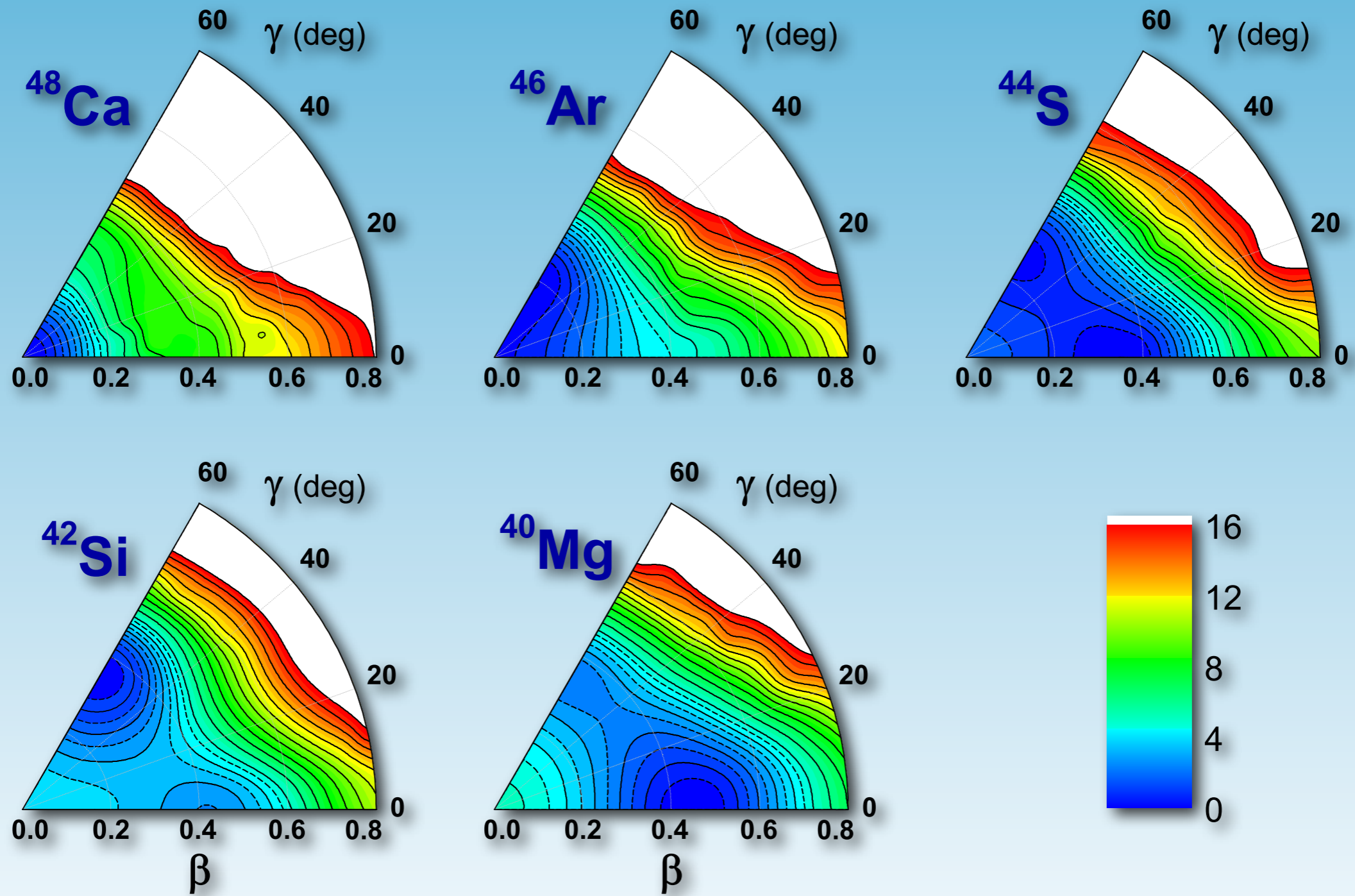
# Shape-coexistence in neutron-deficient Kr isotopes

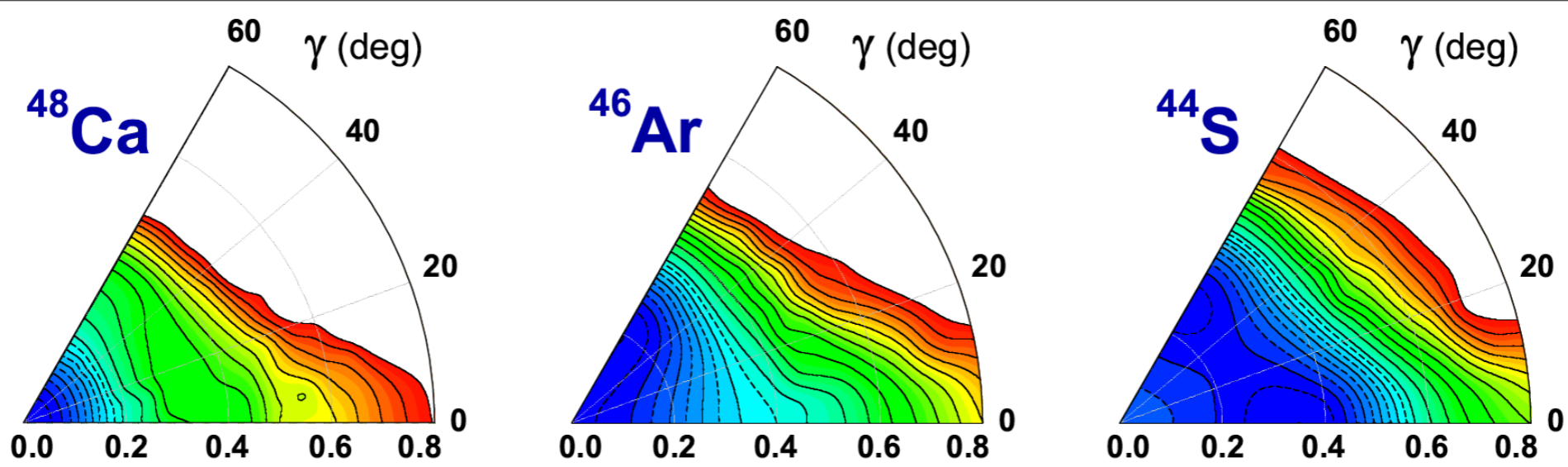






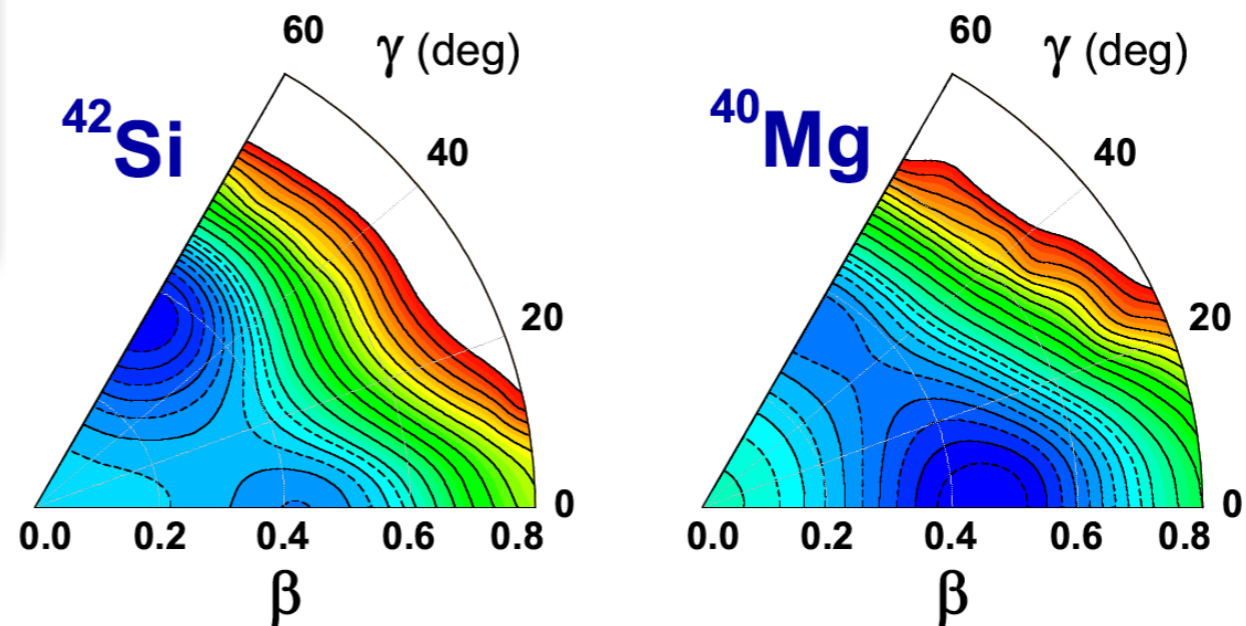
# Coexisting shapes in the N=28 isotones

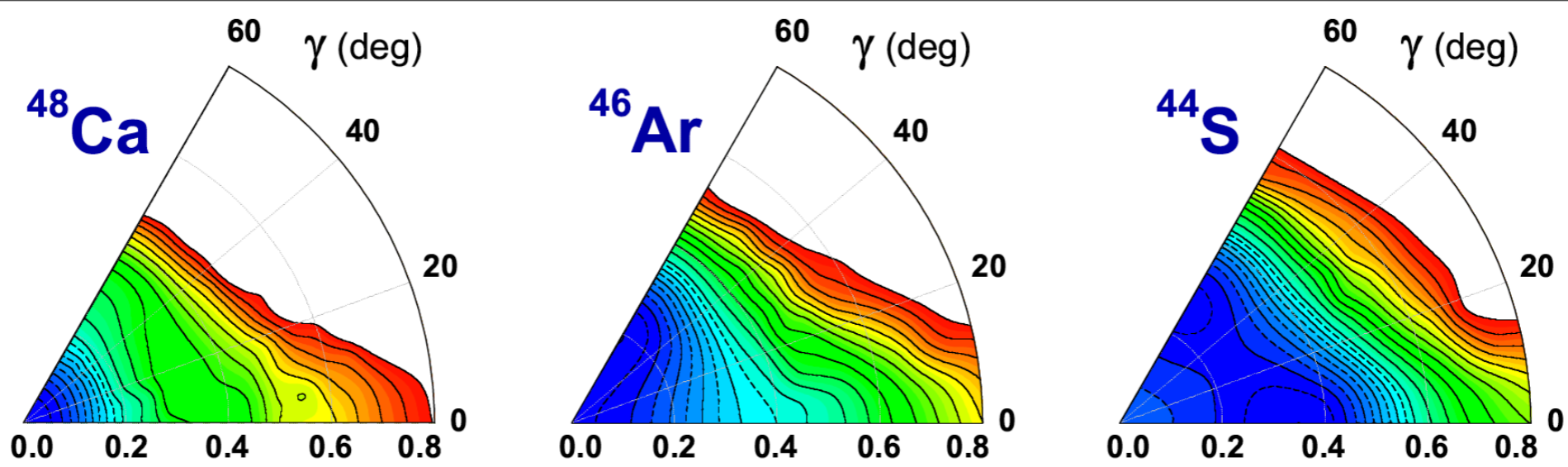




## Neutron $N=28$ spherical energy gaps

	$\Delta_{N=28}^{\text{sph.}}$	$\beta_{\text{min}}$
$^{48}\text{Ca}$	4.73	0.00
$^{46}\text{Ar}$	4.48	-0.19
$^{44}\text{S}$	3.86	0.34
$^{42}\text{Si}$	3.13	-0.35
$^{40}\text{Mg}$	2.03	0.45





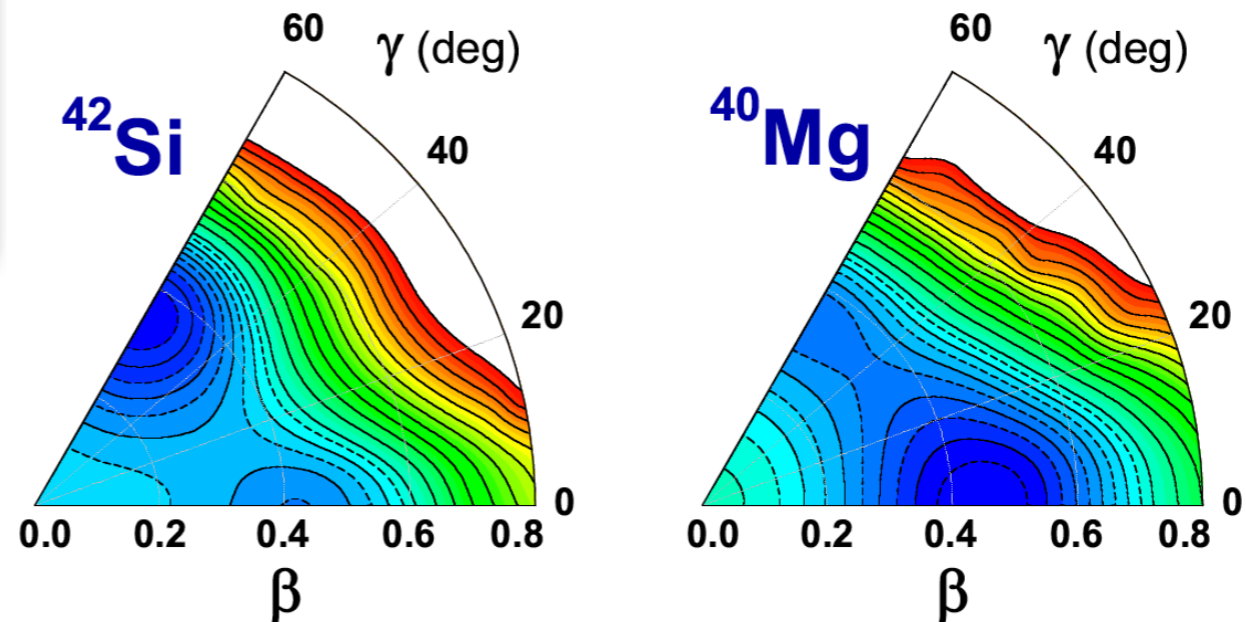
## Neutron $N=28$ spherical energy gaps

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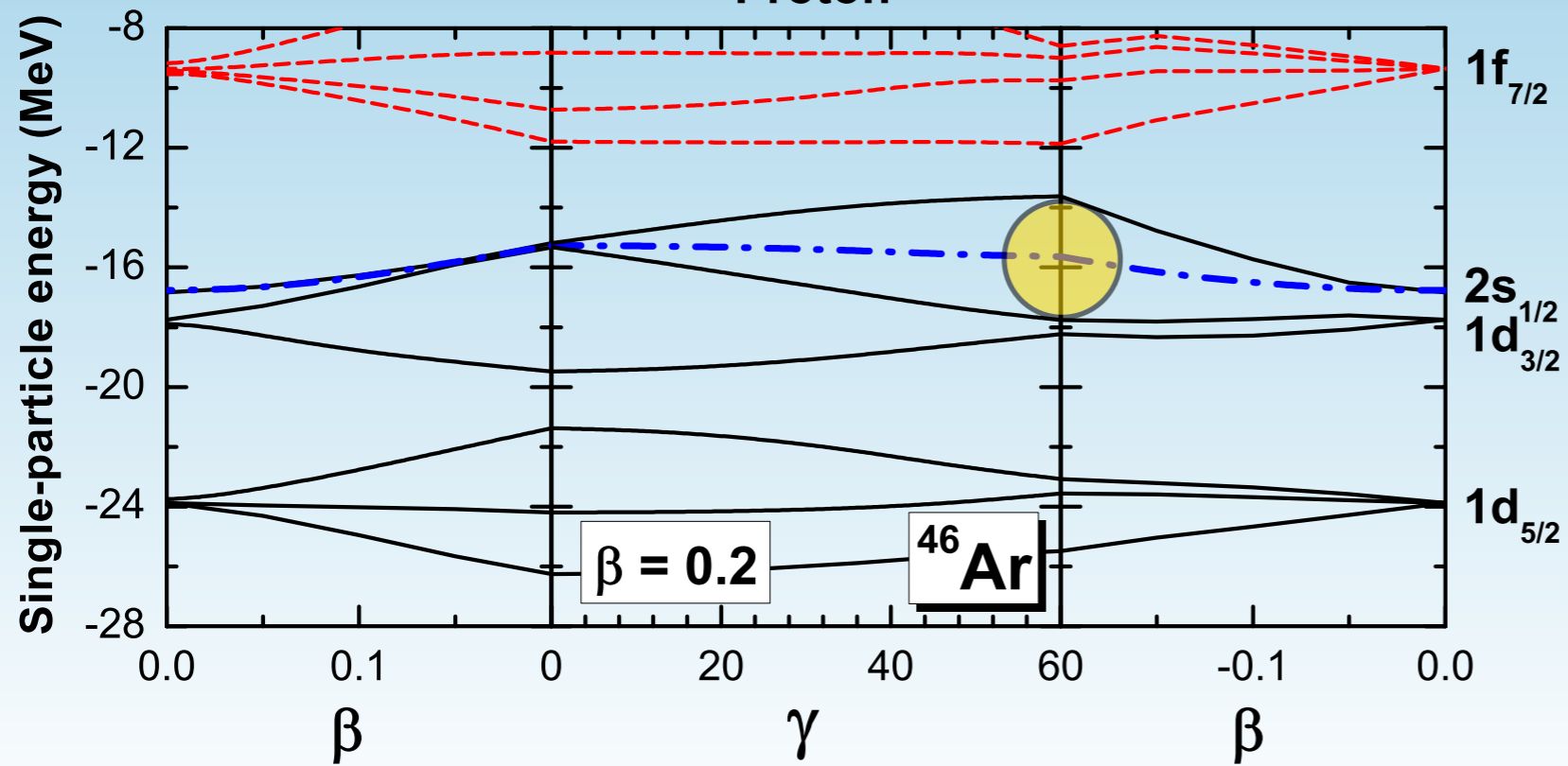
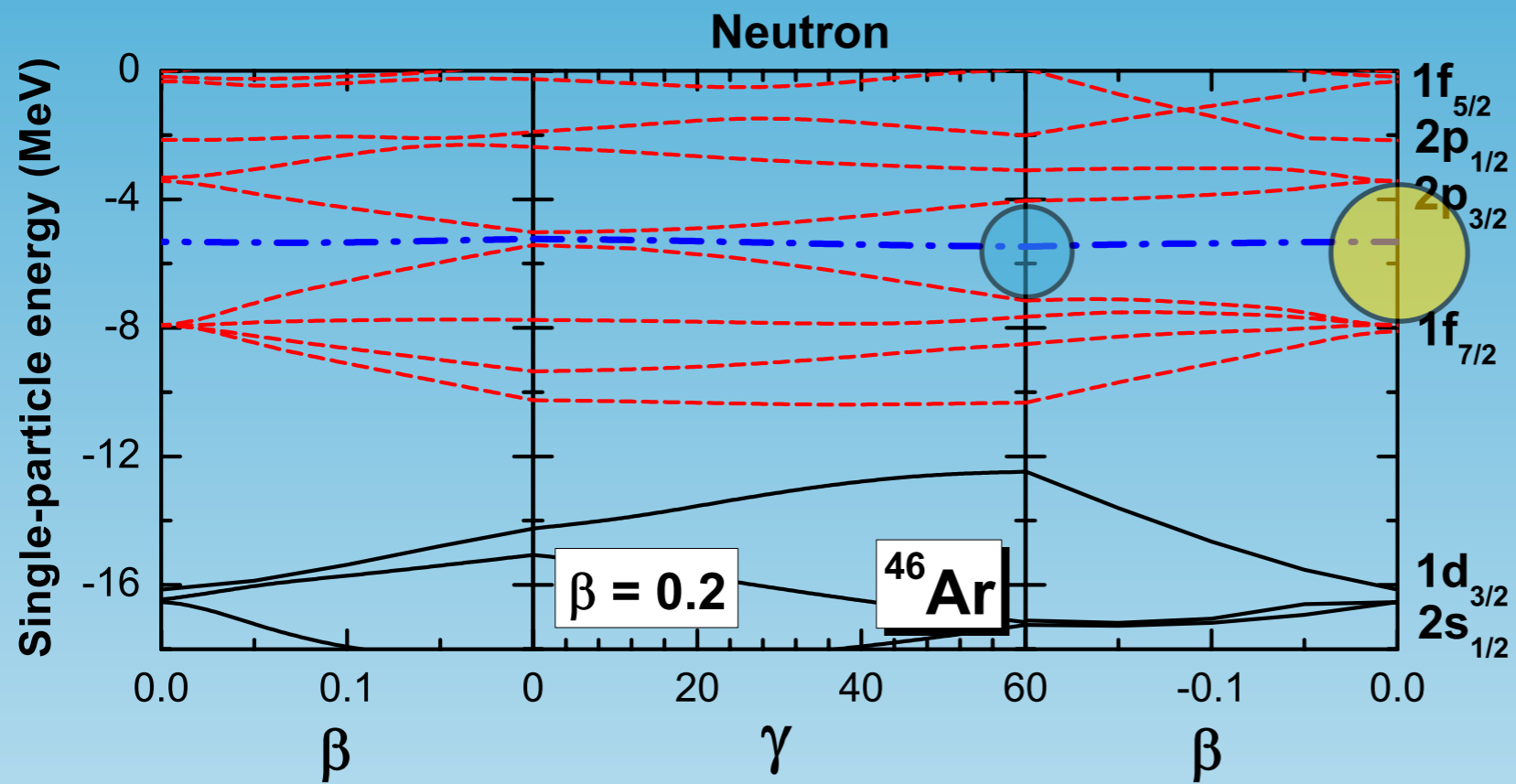
### Experimental values:

4.80 MeV

4.47 MeV

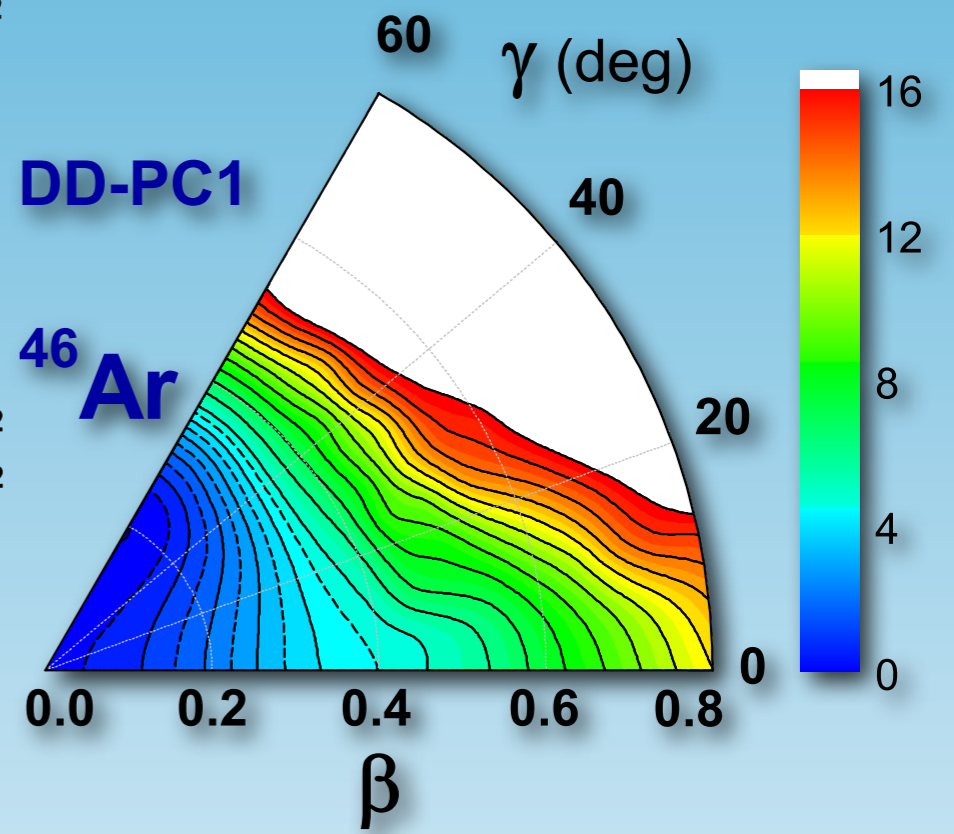
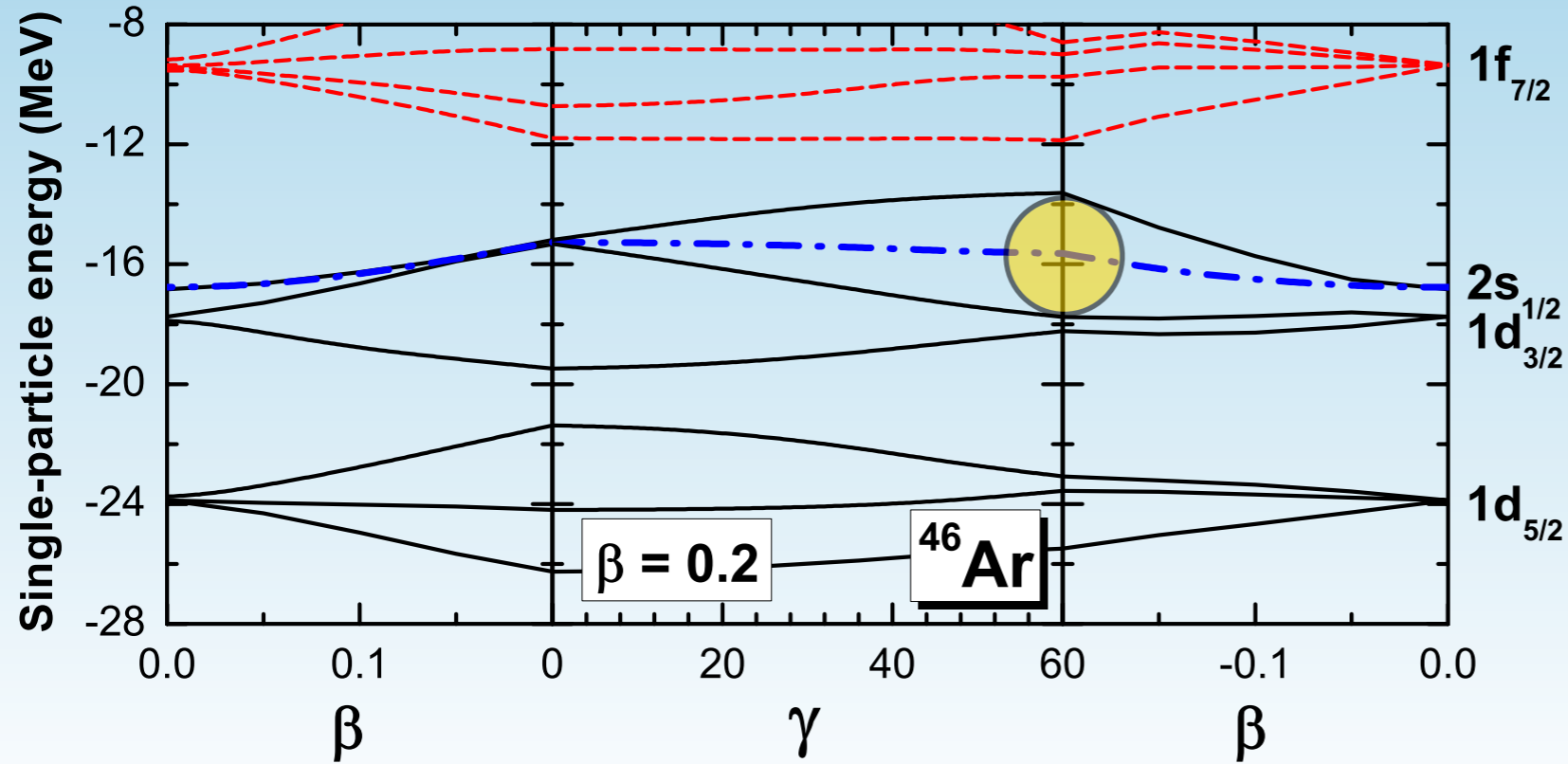
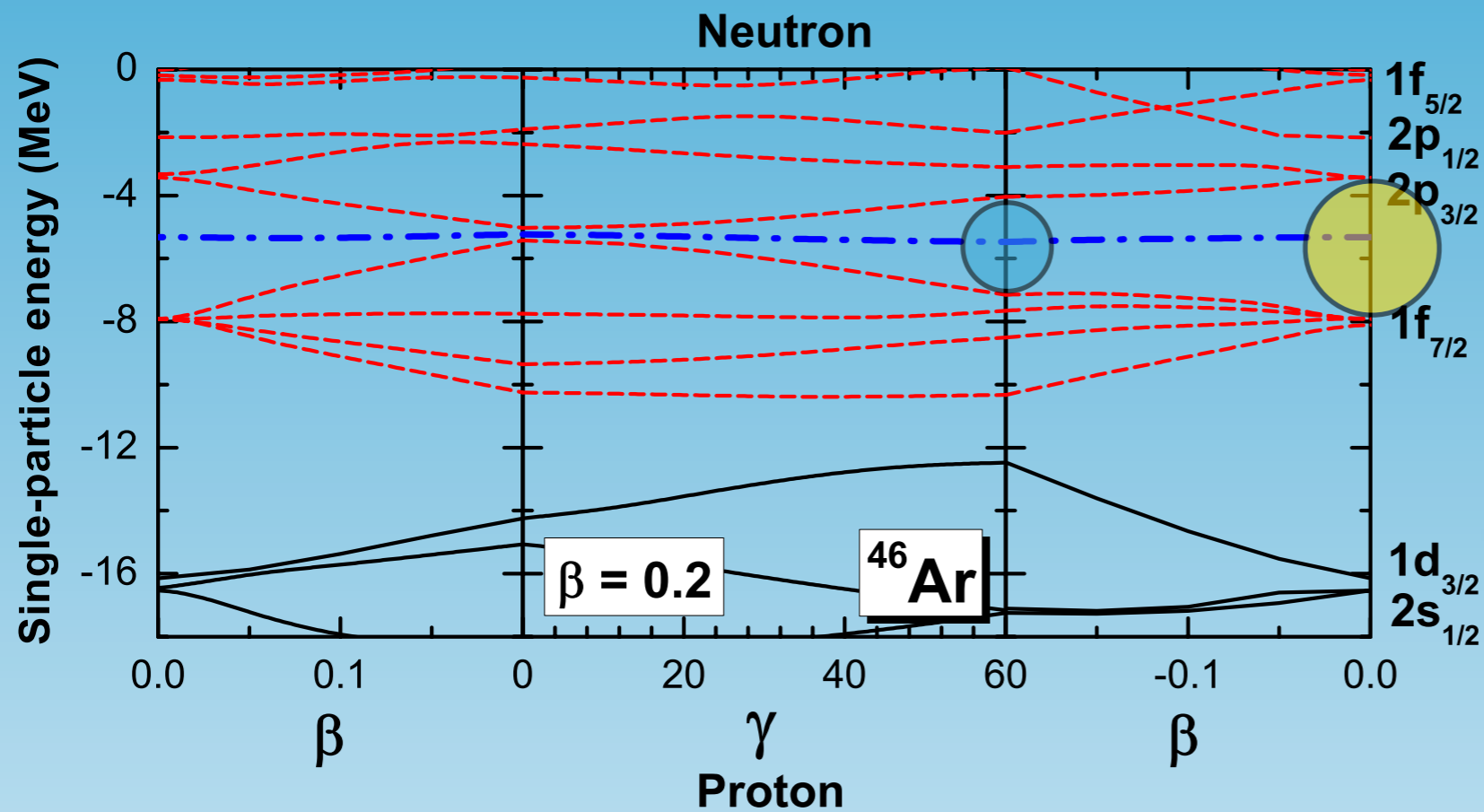


# $^{46}\text{Ar}$ : single-particle levels

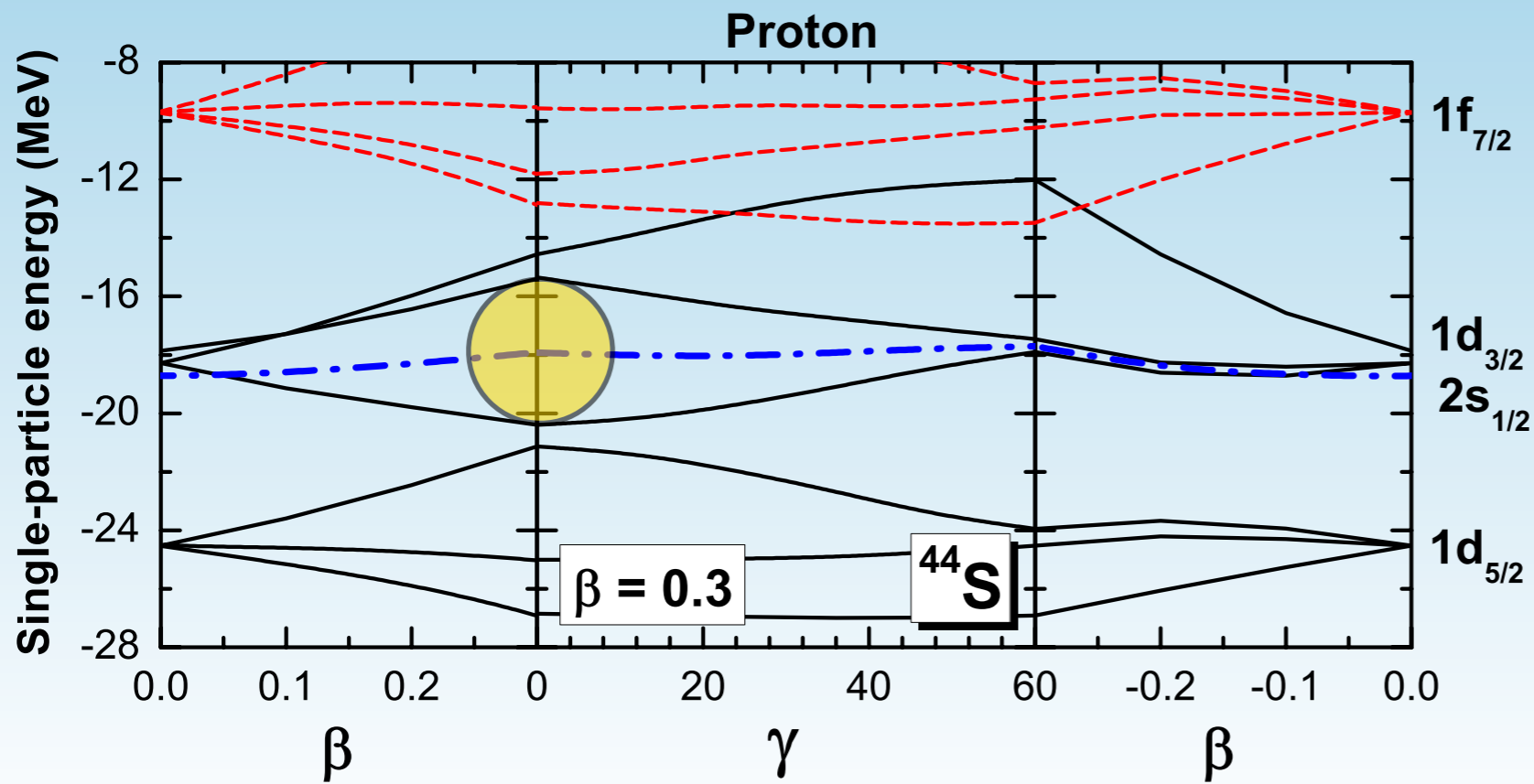
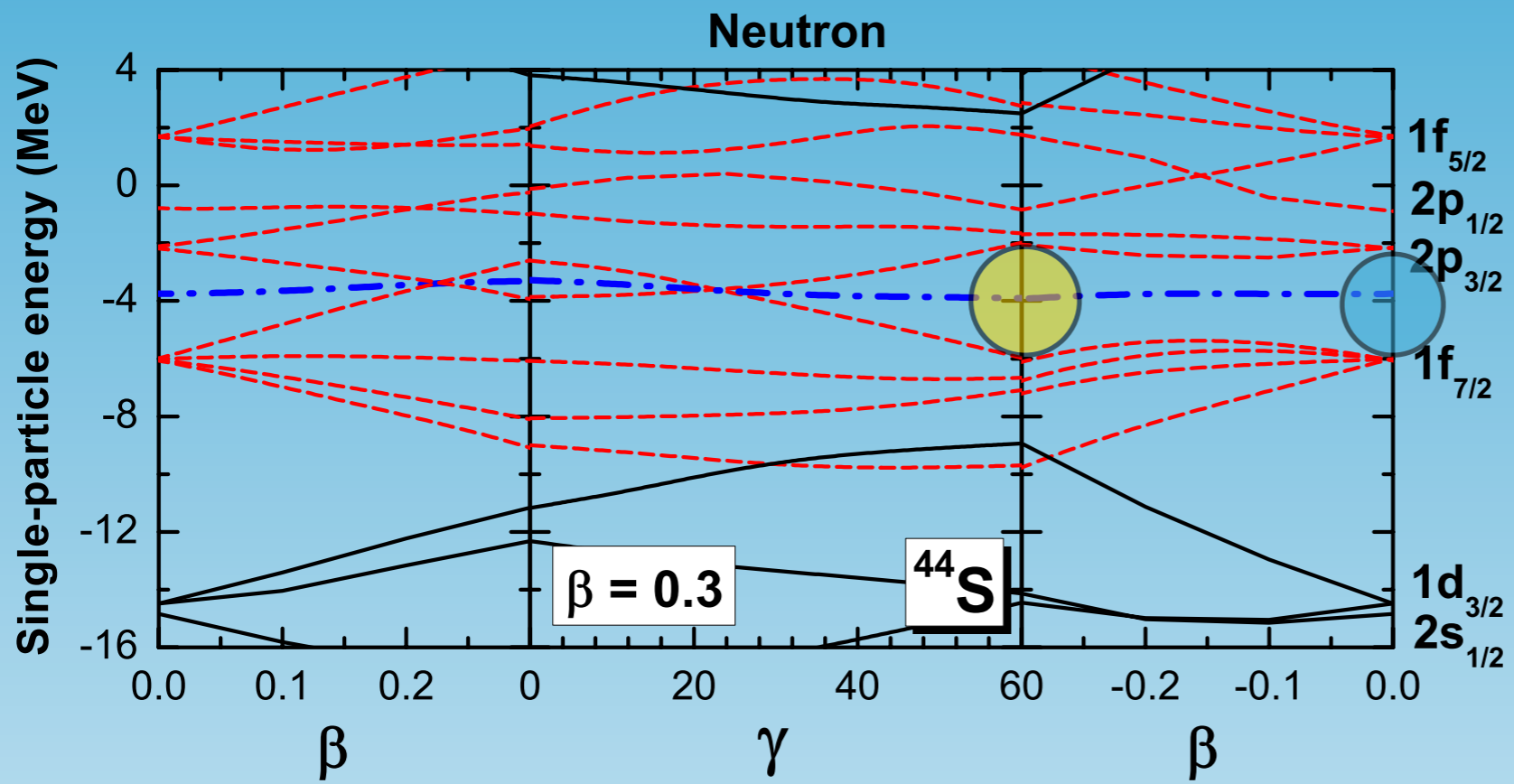




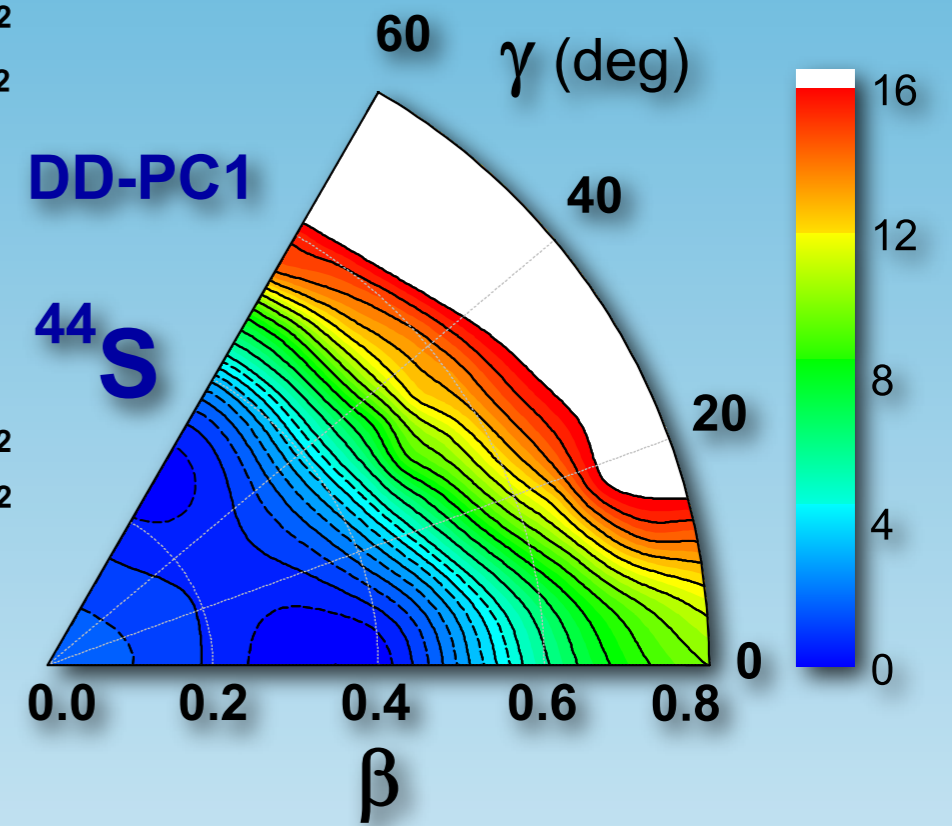
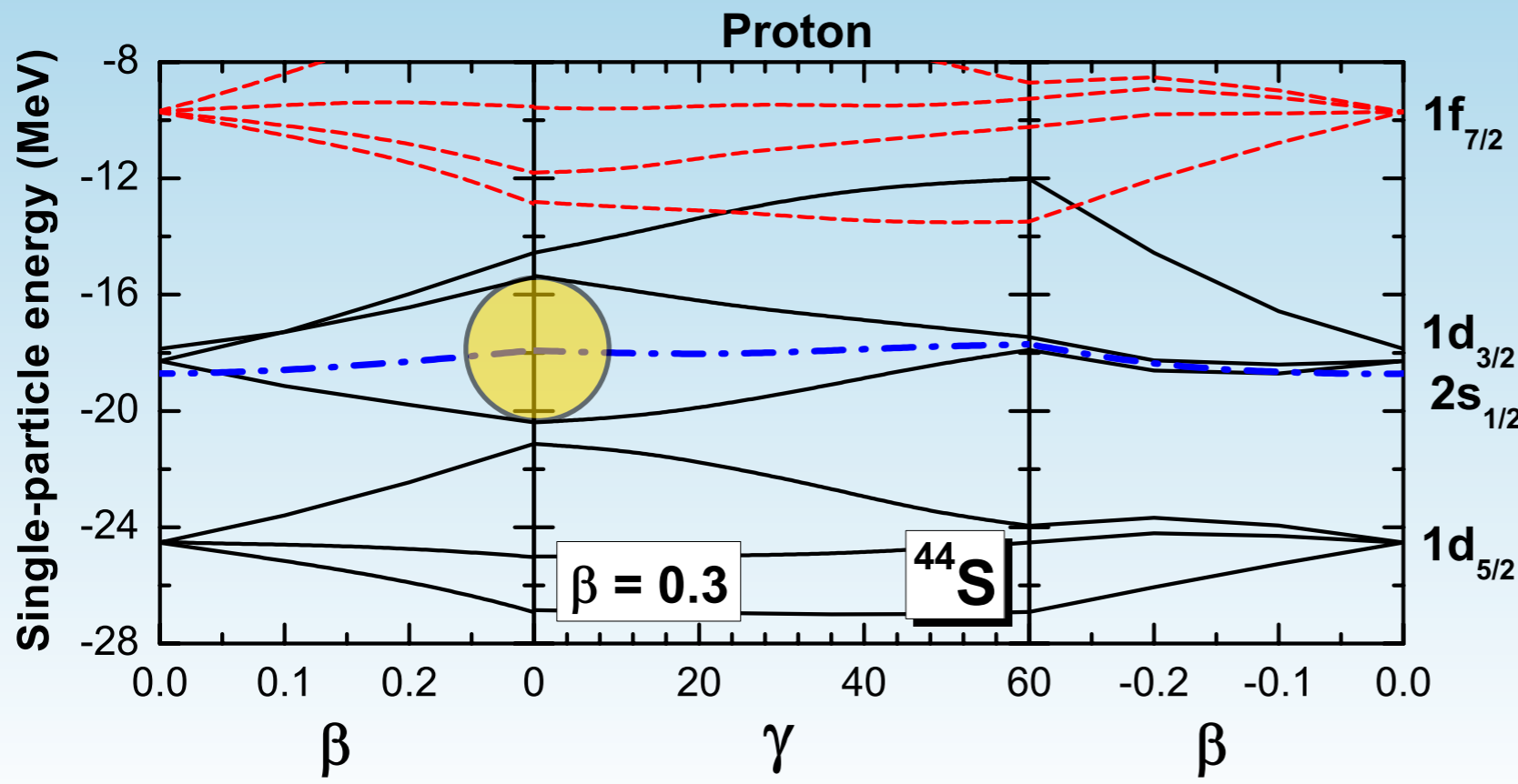
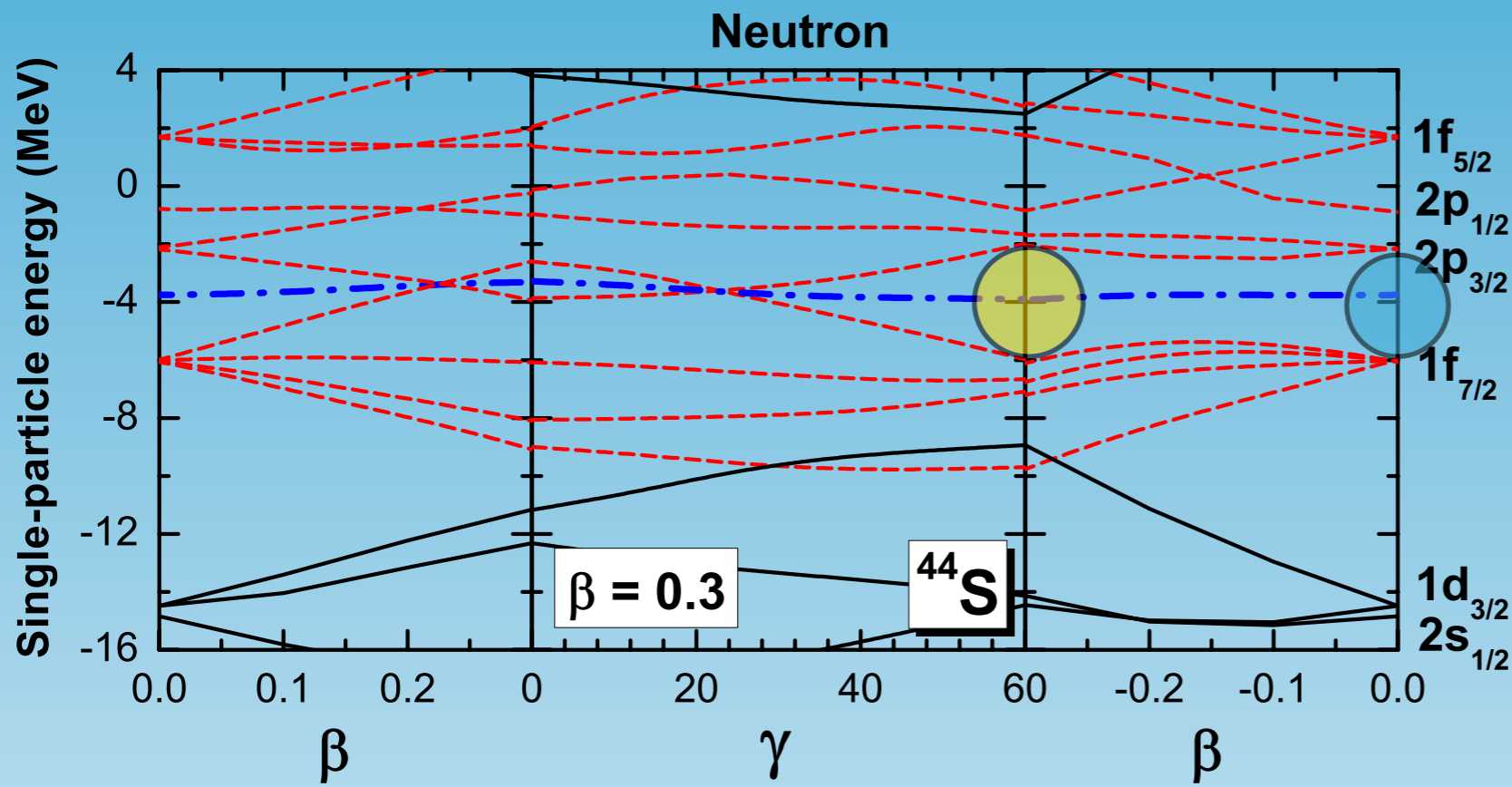
# $^{46}\text{Ar}$ : single-particle levels



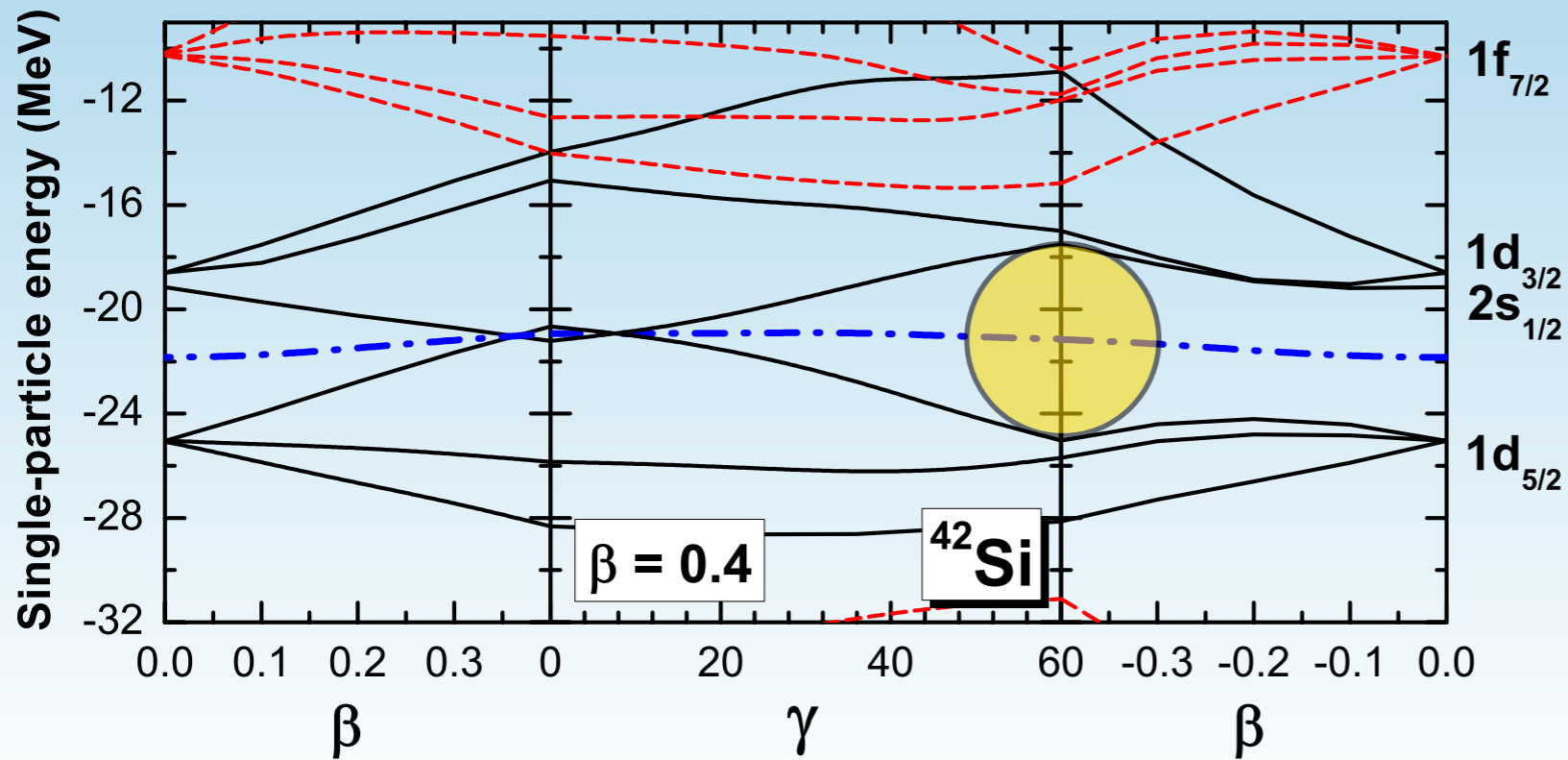
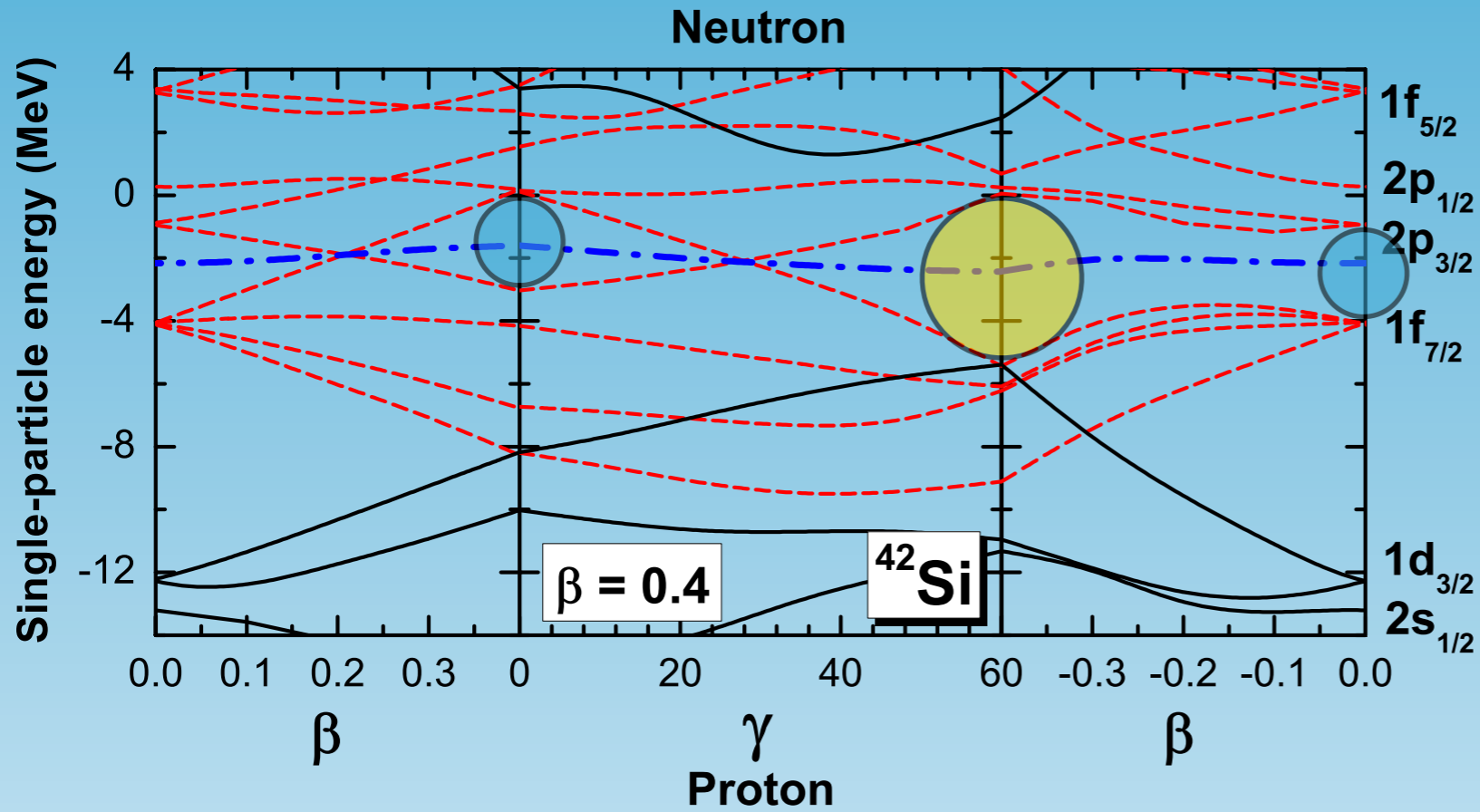
# $^{44}\text{S}$ : single-particle levels



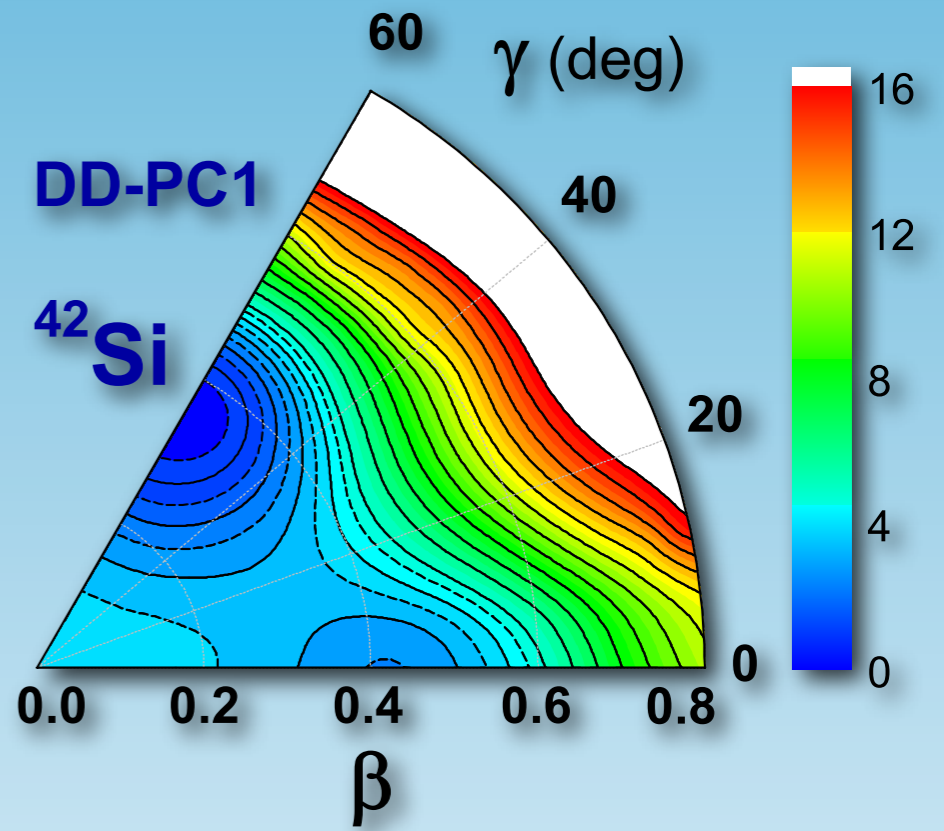
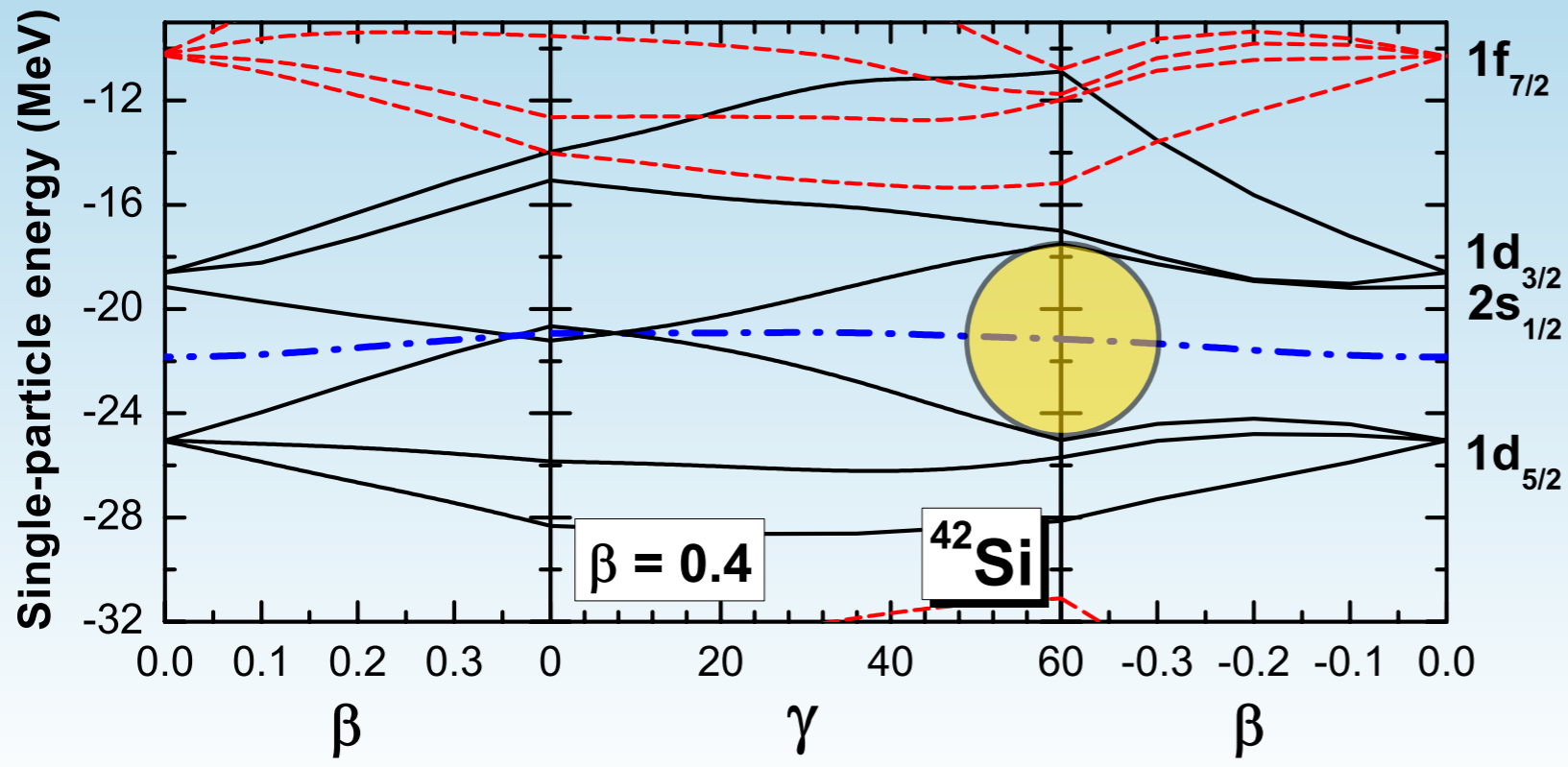
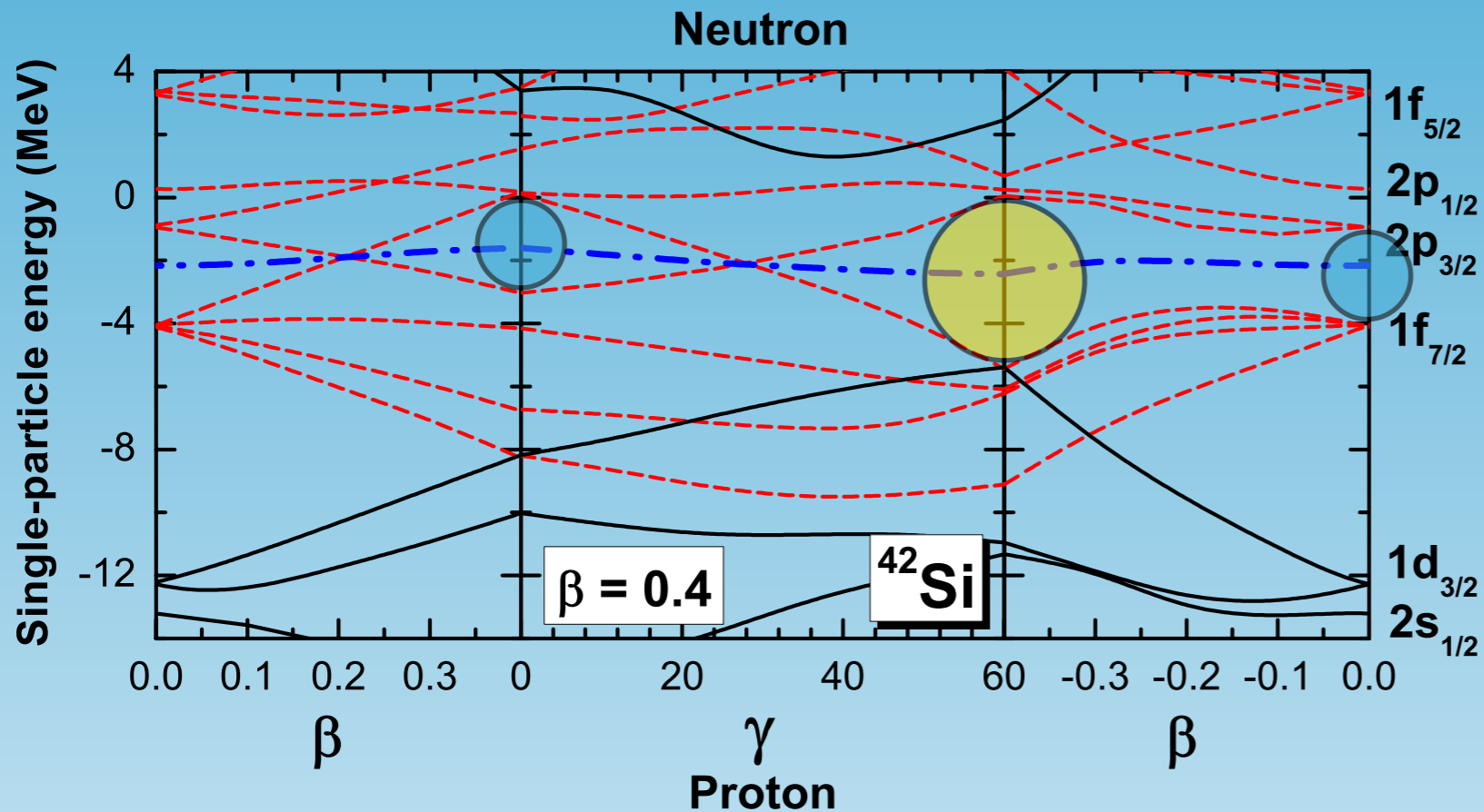
# $^{44}\text{S}$ : single-particle levels

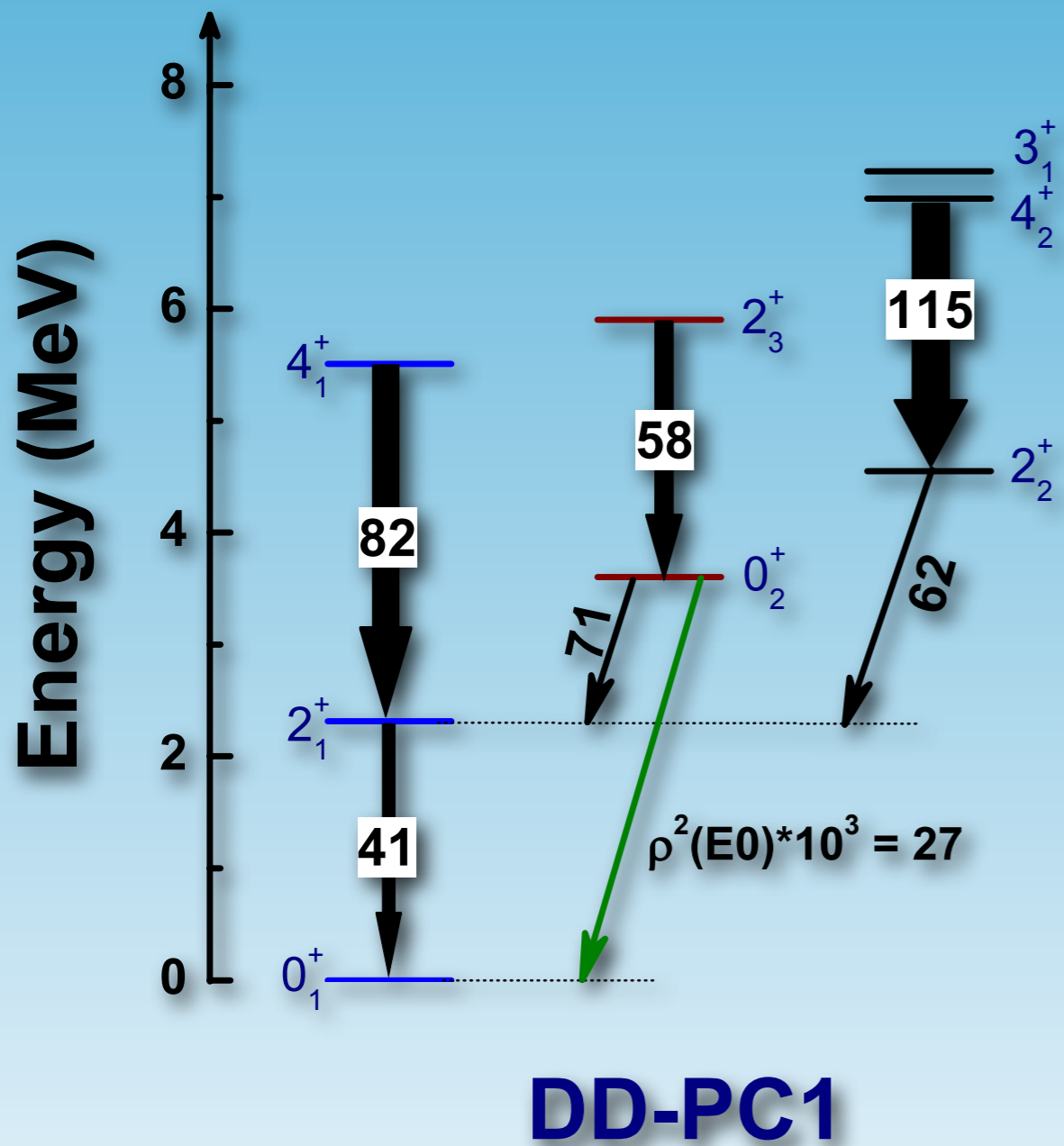


# $^{42}\text{Si}$ : single-particle levels

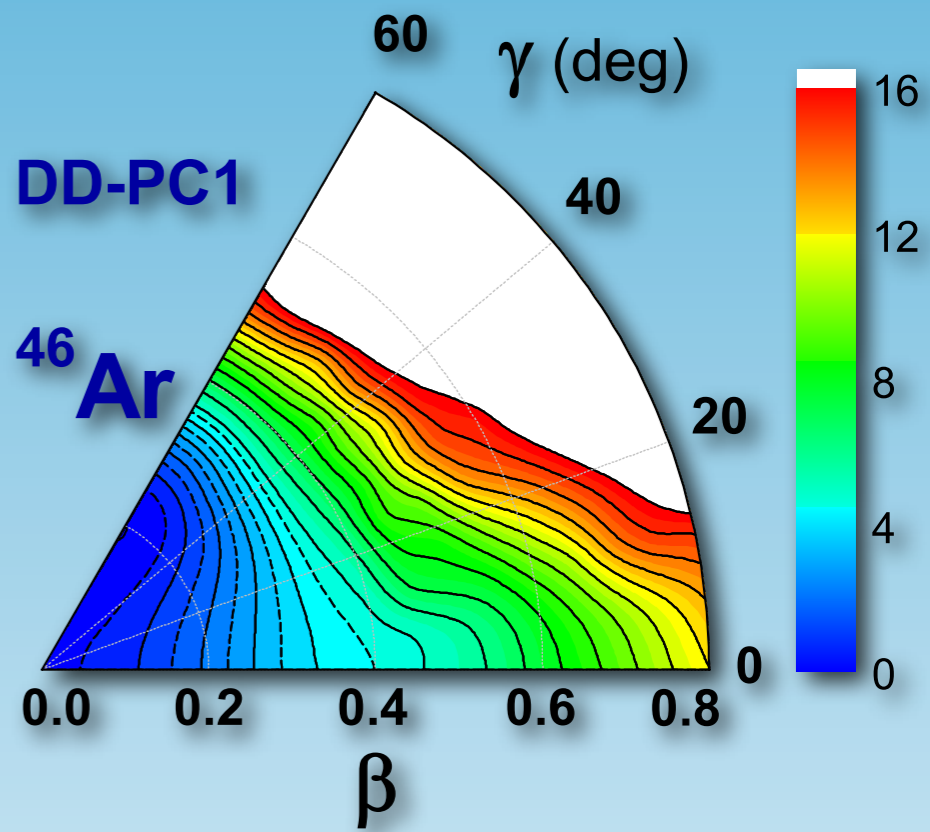
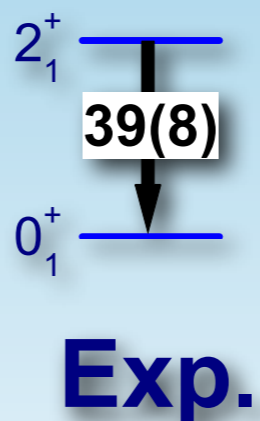


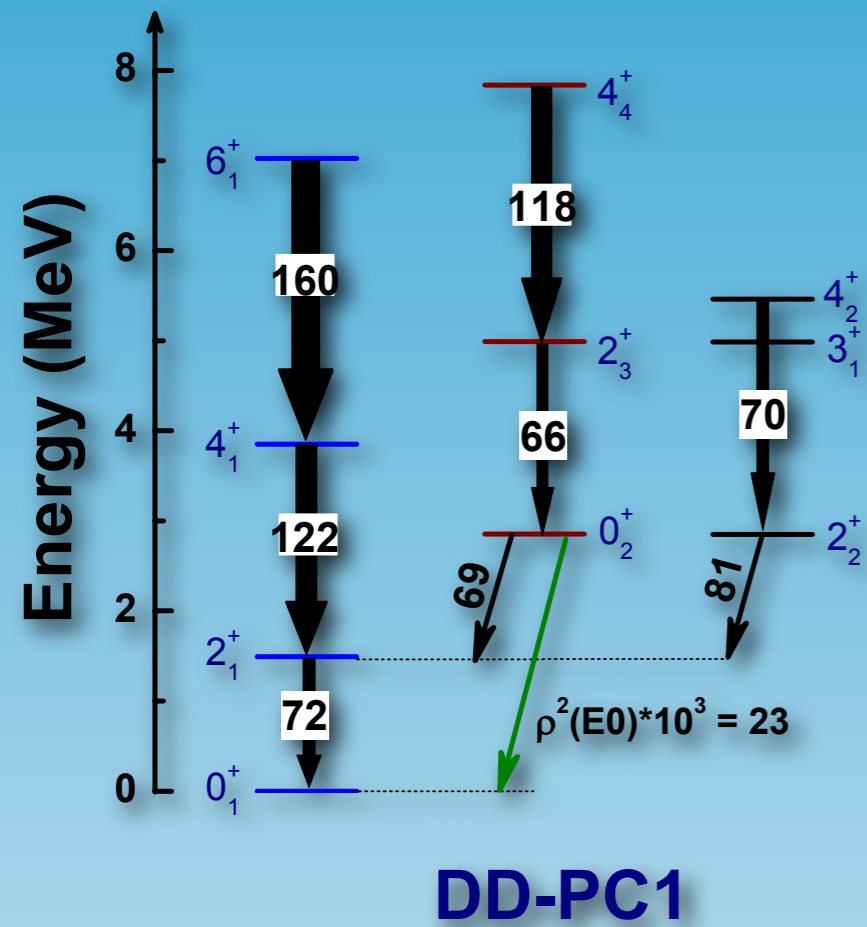
# $^{42}\text{Si}$ : single-particle levels





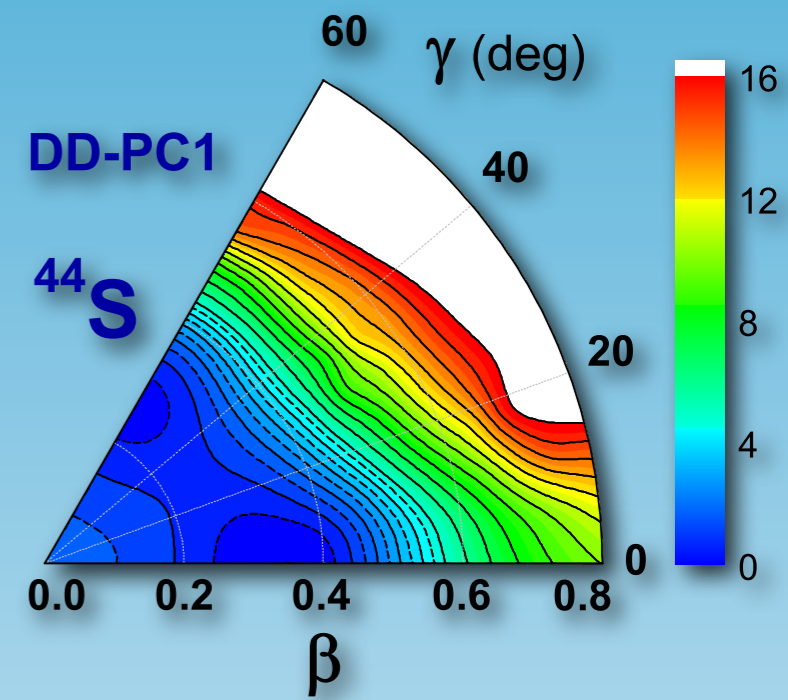
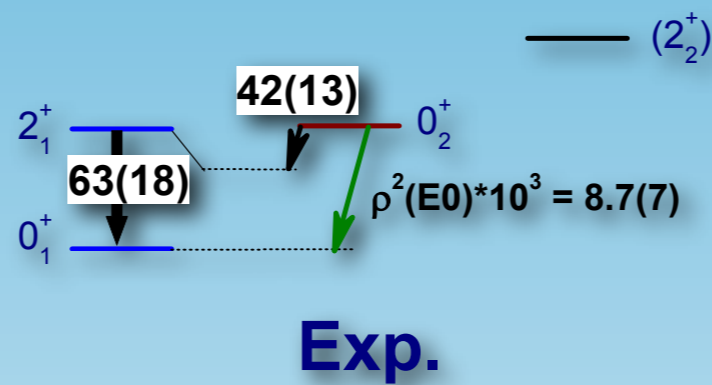
**$^{46}\text{Ar}$**

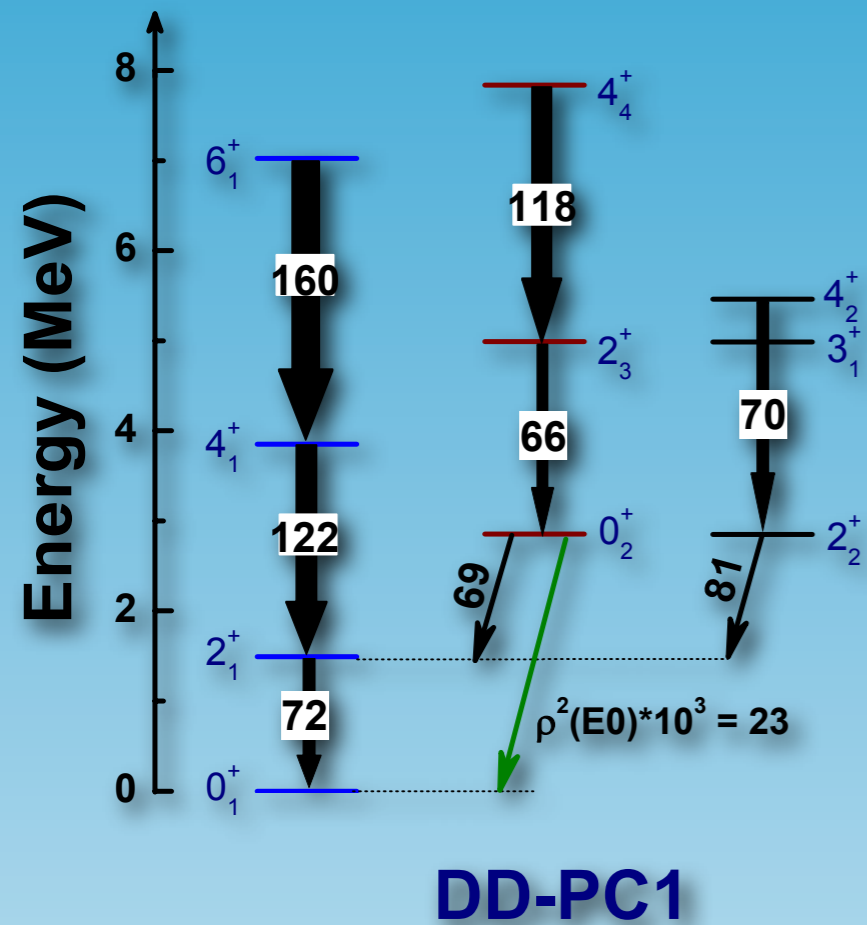




**$^{44}\text{S}$**

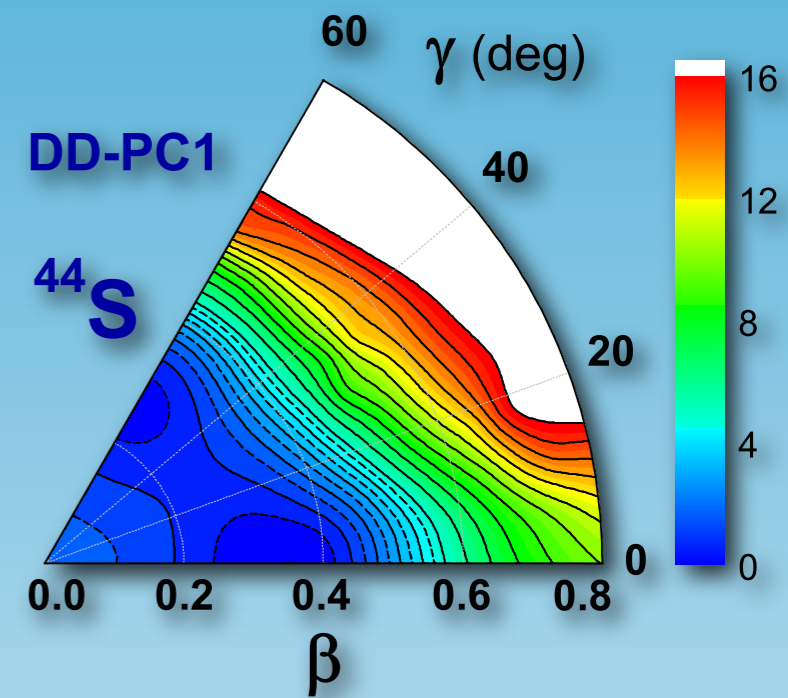
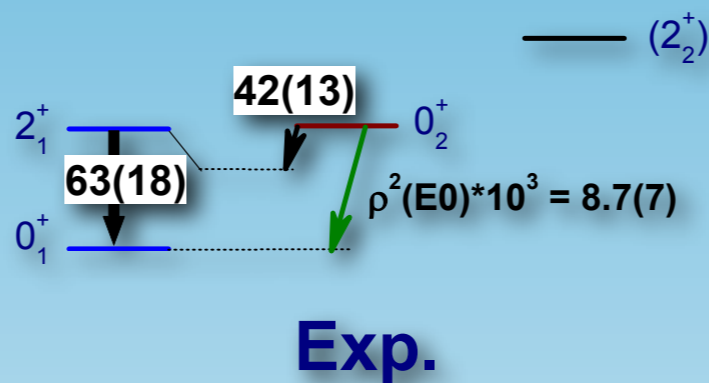
$B(E2): e^2\text{fm}^4$





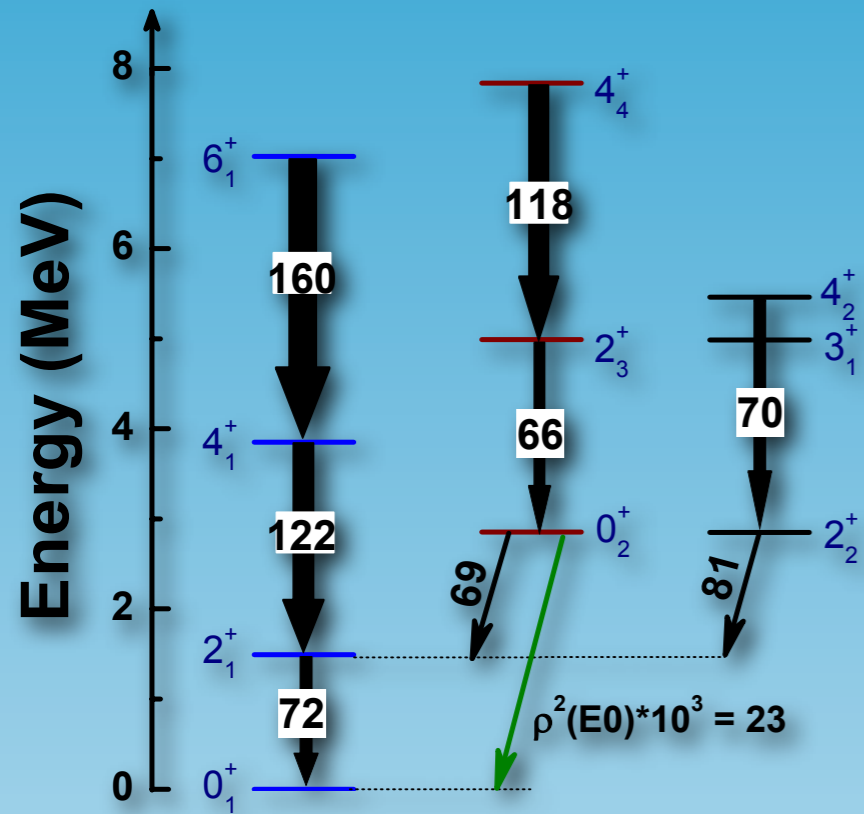
$^{44}\text{S}$

$B(E2): e^2\text{fm}^4$



Probability density distributions:

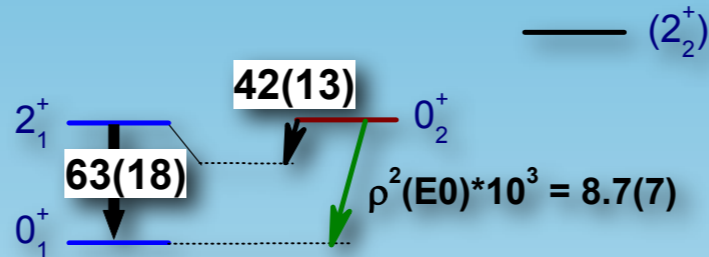




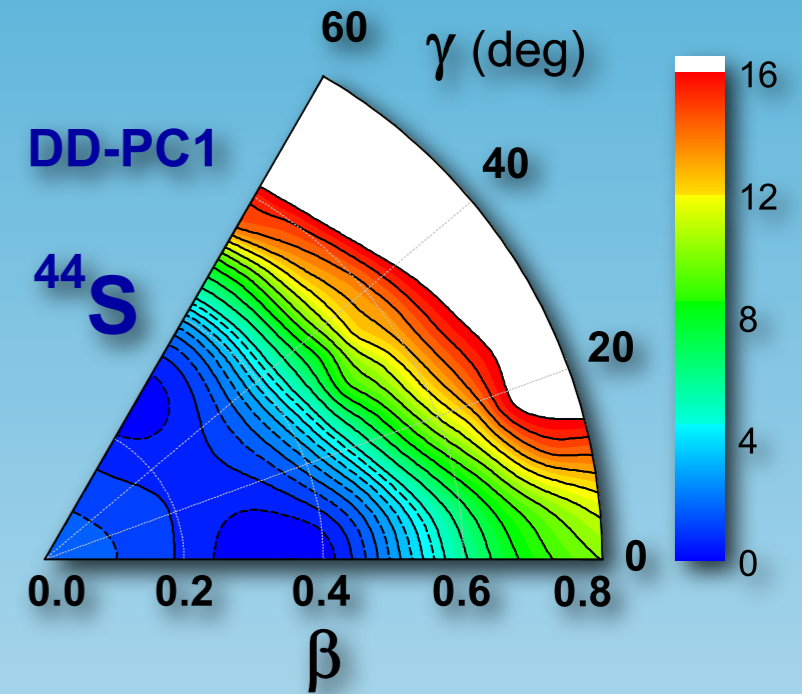
DD-PC1

$^{44}\text{S}$

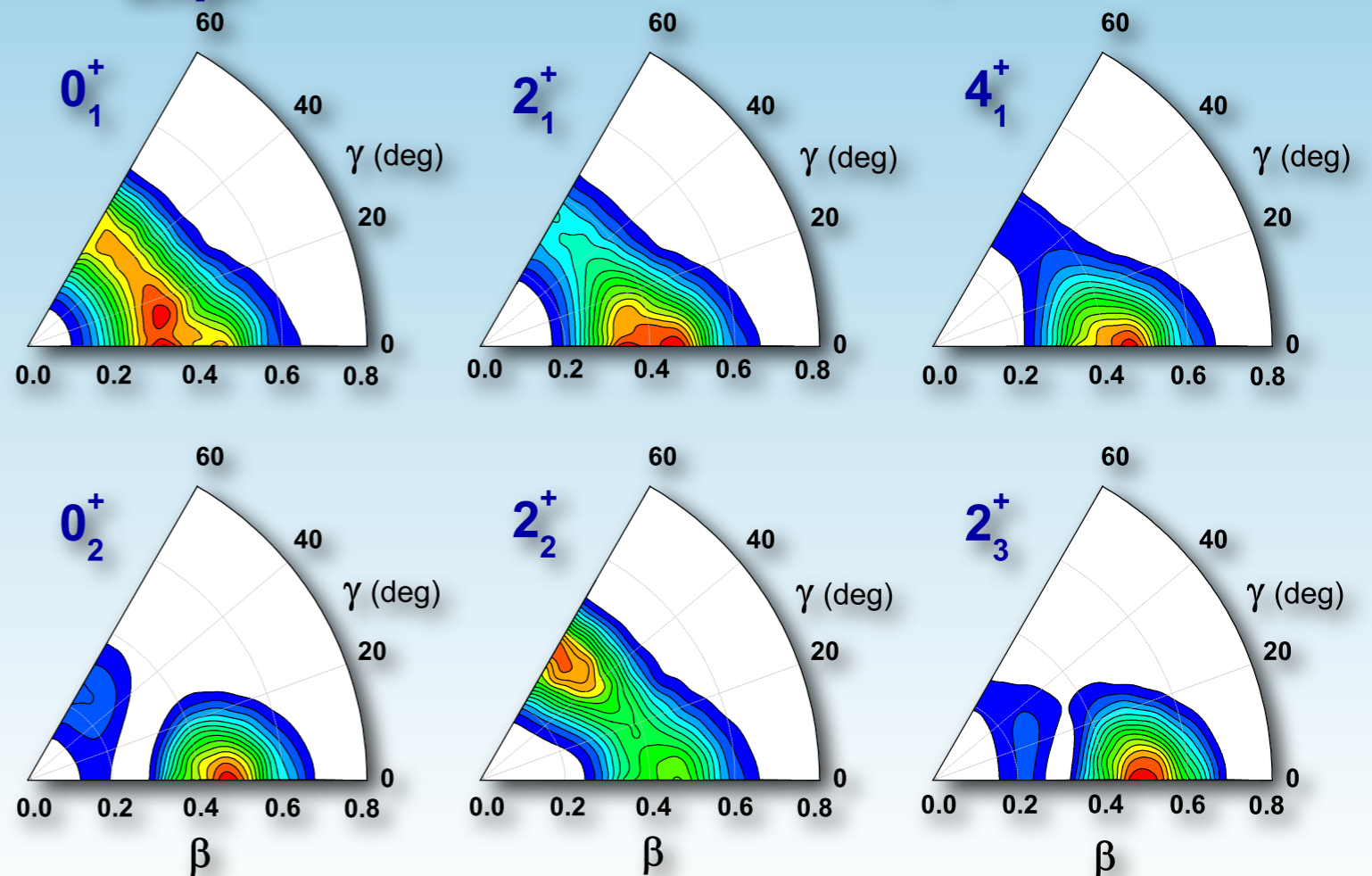
$B(E2): e^2\text{fm}^4$

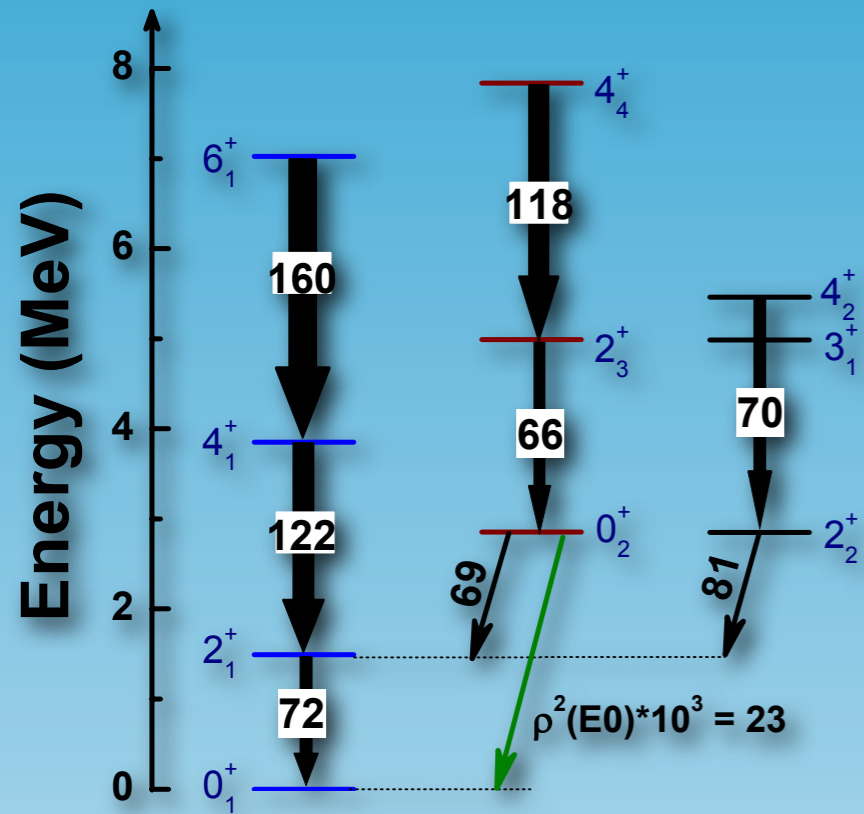


Exp.



Probability density distributions:

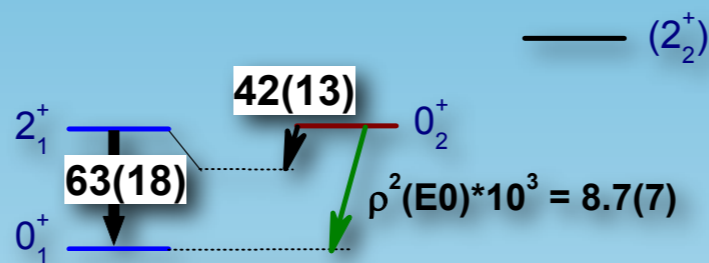




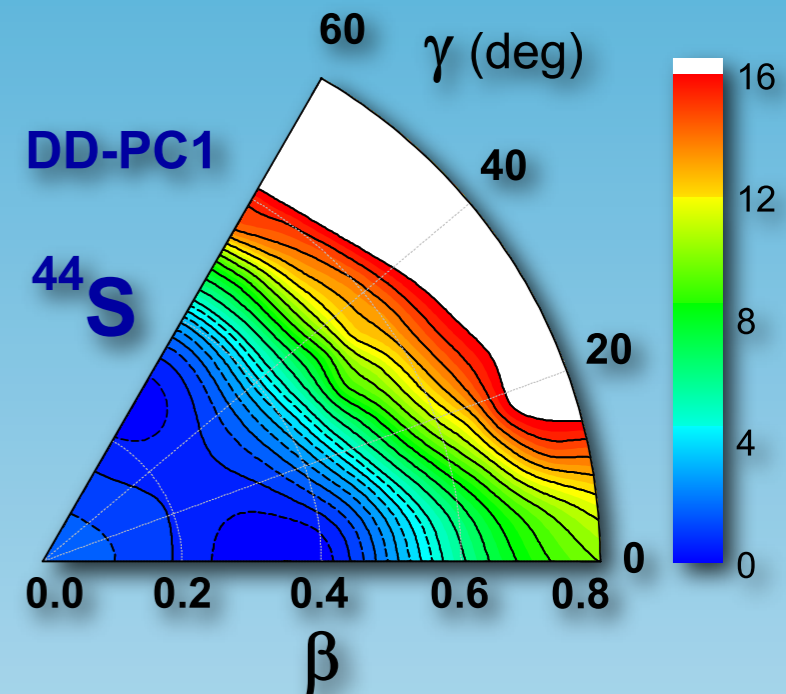
**DD-PC1**

**$^{44}\text{S}$**

$B(E2): e^2\text{fm}^4$

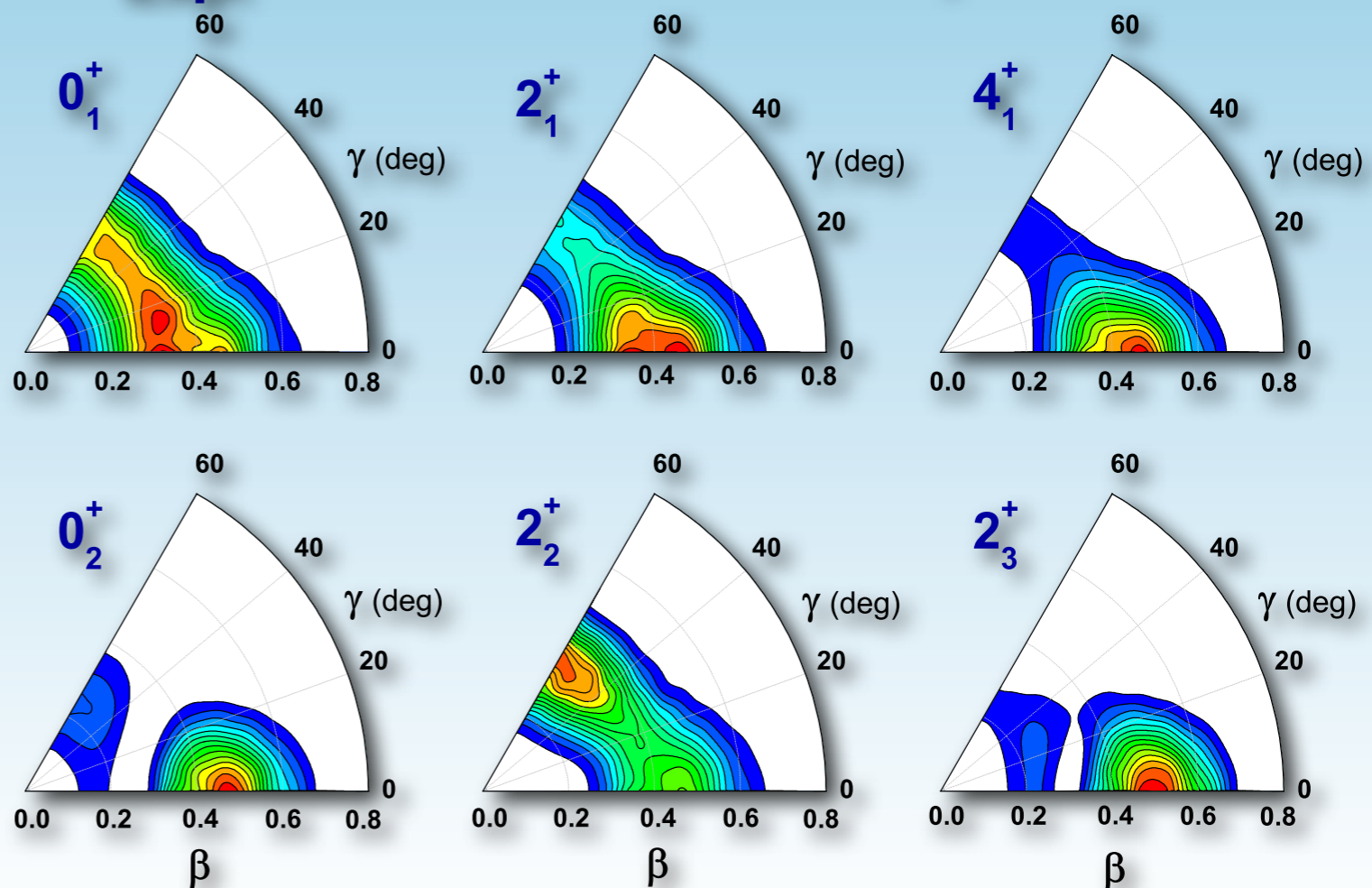


**Exp.**

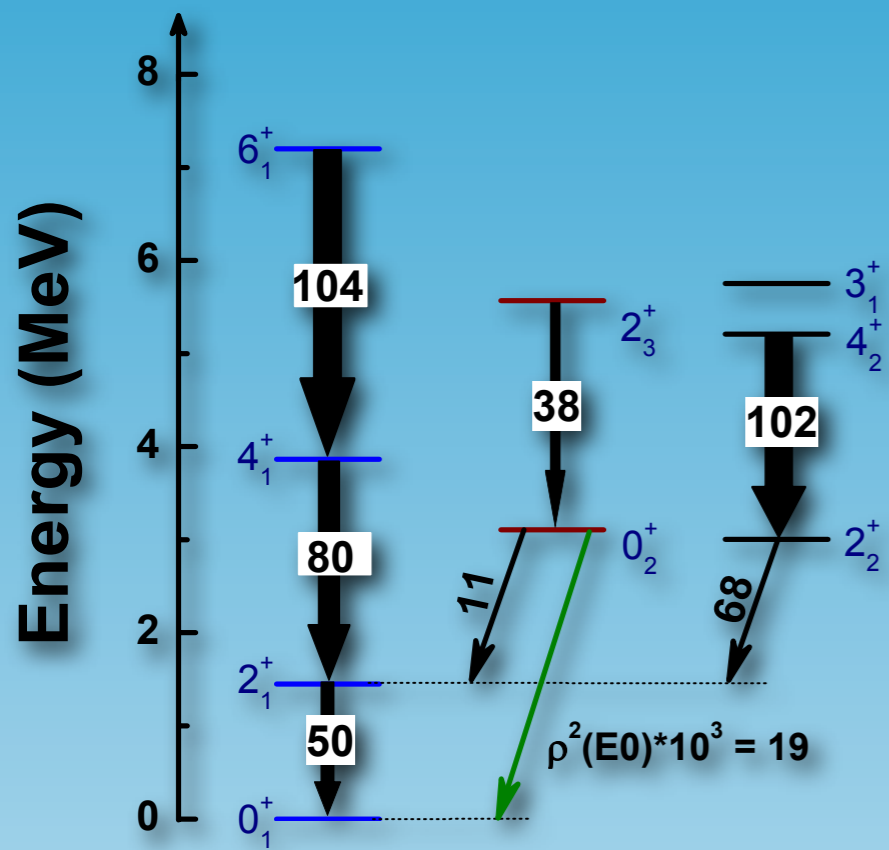


Probability density distributions:

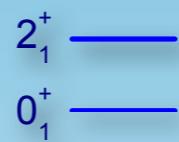
	$K = 0$	$K = 2$	$Q_{\text{spec.}}$
$2_1^+$	88.4	11.6	-10.9
$2_2^+$	21.5	78.5	7.8
$2_3^+$	80.0	20.0	-9.6



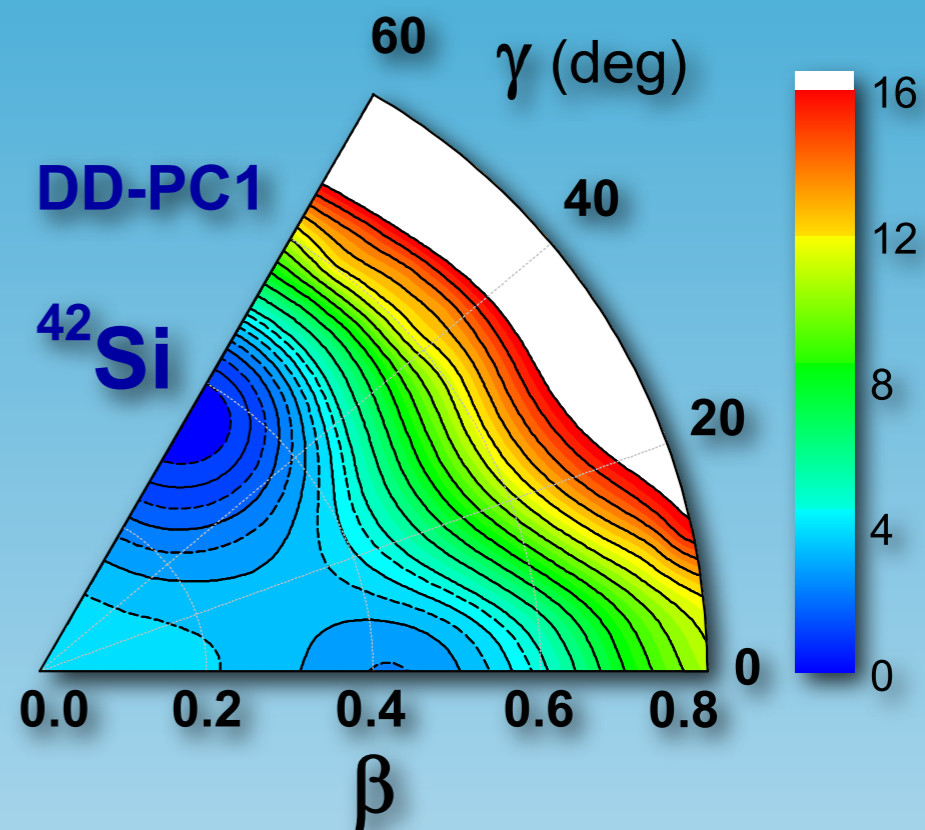
**$^{42}\text{Si}$**



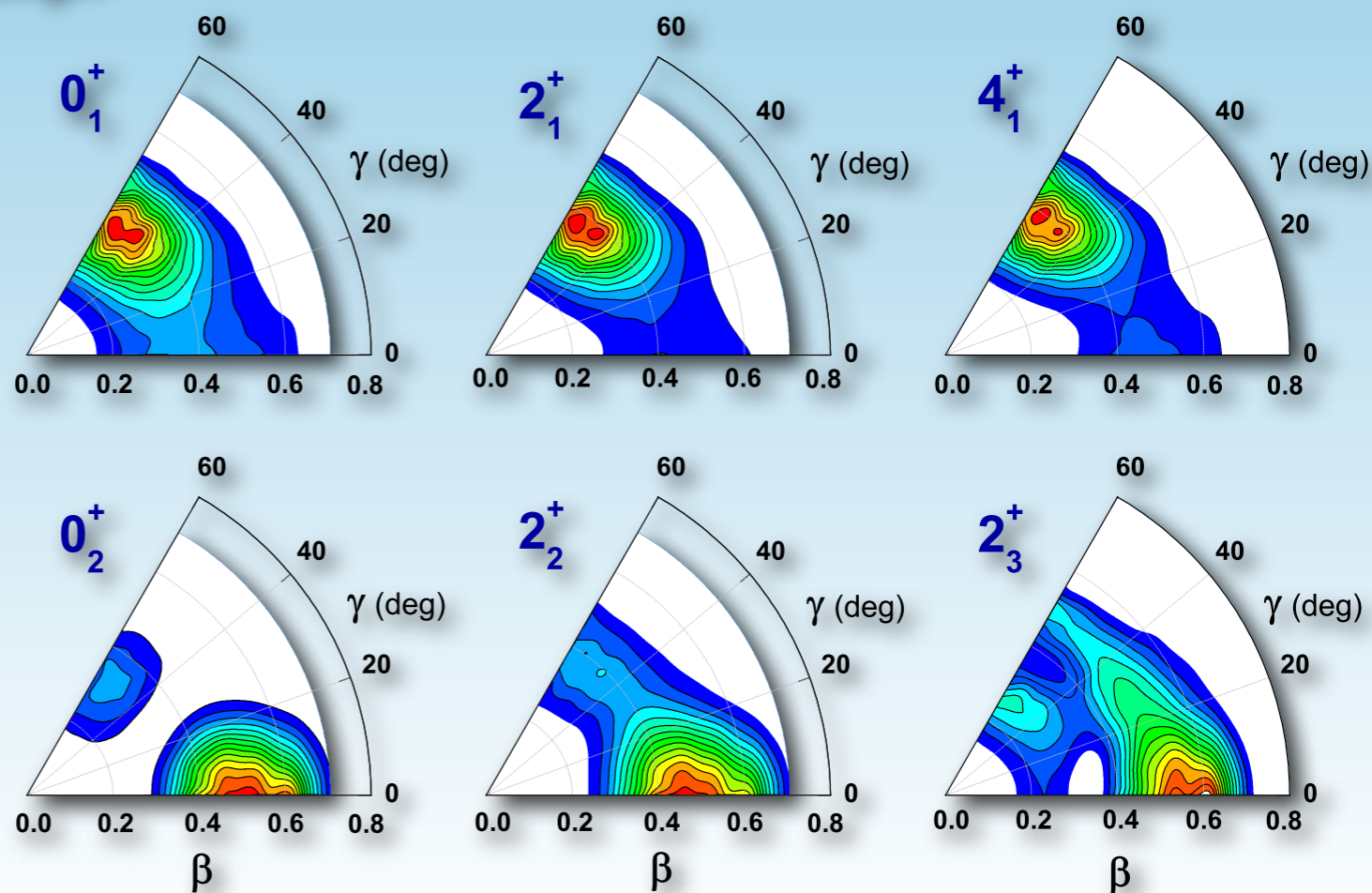
**DD-PC1**

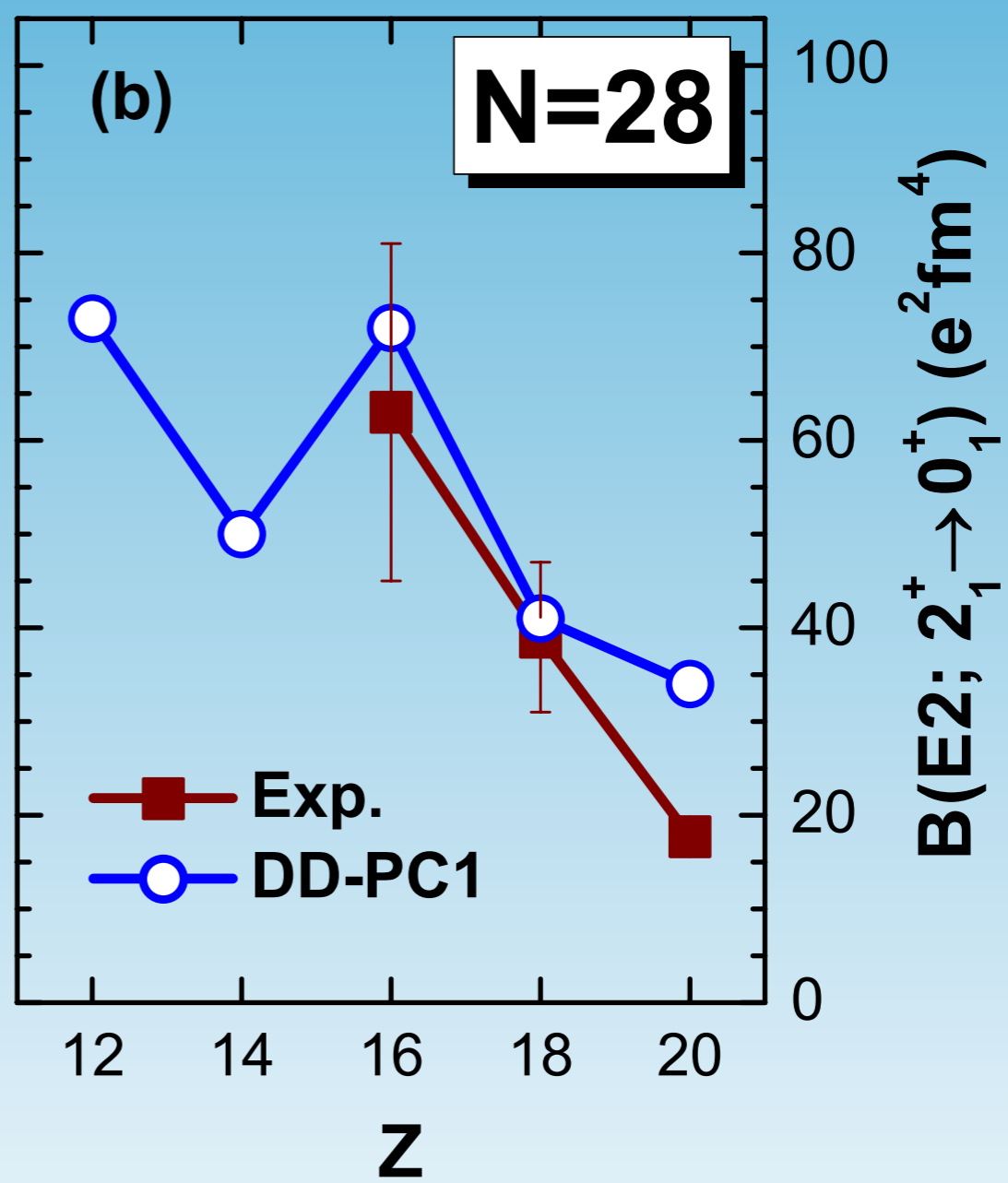
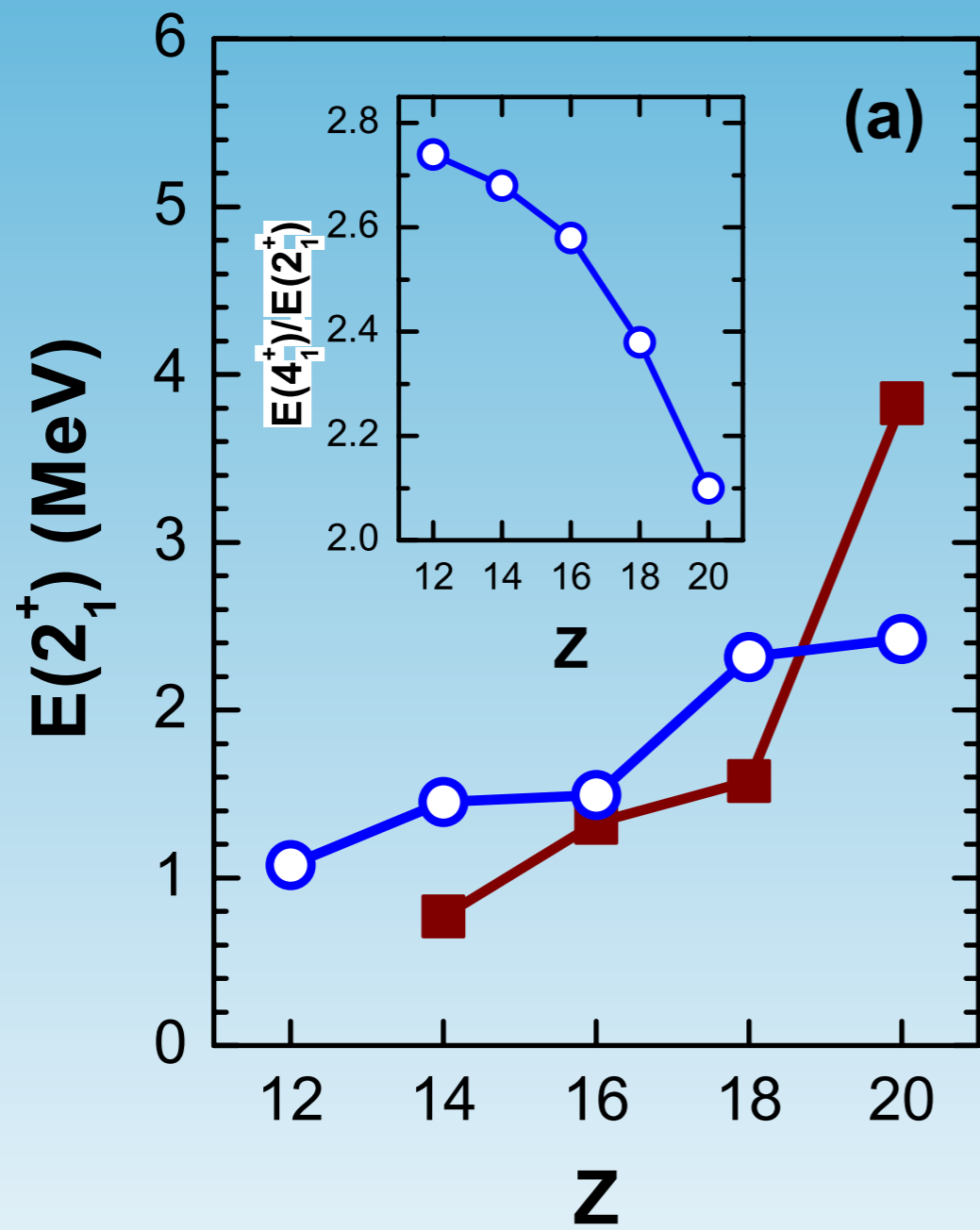


**Exp.**



Probability density distributions:





# Nuclear Energy Density Functional Framework

✓ unified microscopic description of the structure of stable and nuclei far from stability, and reliable extrapolations toward the drip lines.

✓ fully self-consistent (Q)RPA analysis of giant resonances, low-energy multipole response in weakly-bound nuclei, dynamics of exotic modes of excitation.

✓ when extended to take into account collective correlations, it describes deformations and shape-coexistence phenomena associated with shell evolution.

# NUCLEAR ENERGY DENSITY FUNCTIONALS for DUMMIES



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