A realization of de Broglie's Double Solution program: how self-induced collapse allows us to solve the measurement problem.

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Abstract

We present here solutions of a Schrödinger equation including a specific, **well-chosen, self-focusing non-linear potential***. These solutions are the product of the pilot wave with peaked solitons the velocity of which obeys the guidance equation derived by Louis de Broglie in 1926. The novelty of this result is that it is valid in presence of an* **arbitrary linear external potential***. This result constitutes thus a formal realization of* **de Broglie's Double Solution program***. It also allows us to solve the measurement problem, making use of recent results about quantum equilibrium. As we shall show it also escapes Gisin-type* **no-go theorems** *about non-linear modifications of Schrödinger equation, and circumvents criticisms usually formulated against Schrödinger-Newton equation and no collapse models.* **We discuss the possibility to test the validity of our model by implementing a humpty-dumpty Stern-Gerlach interferometer in the mesoscopic regime.**

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- de Broglie's double solution program (1927)

2. Failure of Double Solution Program for a large class of non-linear potentials

- Very general (*à la*) Ehrenfest theorem: classical dynamics instead of de Broglie-Bohm mechanics in the macroscopic regime

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- Factorisation ansatz and effective potential

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- Quantum Equilibrium
- Circumventing no-go theorems

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Appendix. Factorization ansatz and modified dB-B dynamics.

Introduction

- A first attempt to solve the wave-particle duality in favour of a pure wave picture – in fact by considering nonlinear wave equations – can be traced back to Einstein who, in 1909, wrote the following to Lorentz [1]:
  " [T]he essential thing seems to me to be not the assumption of singular points but the assumption of field equations of a kind that permit solutions in which **finite quantities of energy propagate with velocity c in a specific direction without dispersion**. *One would think that this goal could be achieved by a modification of Maxwell's theory. ... However,* **one would be forced to do the latter [i.e., introduce nonlinear or inhomogeneous equations],** *in my opinion, if one wished to manage without introducing singular points, which certainly would be the most satisfactory thing to do."*:

[1] A. Einstein. Letter from A. Einstein to H. Lorentz. *Collected papers of A. Einstein: The swiss years: correspondence 1902-1914*, 5, (2004).

de Broglie's double solution

- In general, in Quantum Physics (due to Wave-Particle duality) we must face the same problem as Einstein with the photon: waves spread but particles are sharply localized
- Born 1926: probabilistic interpretation
- 1927: de Broglie's alternative interpretation: double solution program[2]:

[2] G. Bacciagaluppi and A. Valentini. Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference. Cambridge University Press, Cambridge, 2010.

de Broglie's double solution

- Three ingredients in de Broglie's double solution program:
  1. quantum systems are (supposed by de Broglie to be) **localized in space at any time and to follow deterministic continous trajectories.**
  2. **particles**: either **singularities** (in 1927) or (in the 50's) **"humps"** of which dispersion gets compensated by non-linearity (what is called today a **soliton**).
  3. Trajectories obey de Broglie-Bohm dynamics[3]:
  $\frac{d\mathbf{x}}{dt} = \mathbf{v}_{dB-B} = \frac{\hbar}{m} \frac{Im(\Psi_L^* \nabla \Psi_L)}{|\Psi_L|^2} = \frac{\hbar}{m} \nabla \varphi_L(\mathbf{x}(t), t)$,
  where Ψ_L, the pilot-wave, is solution of the linear Schrödinger equation, and its phase is denoted φ_L.

[3] L. de Broglie, La mécanique ondulatoire et la structure atomique de la matière et du rayonnement, *J. Phys. Radium*, 8(5):225–241, (1927).

More about de Broglie's double solution

- de Broglie's double solution is WAVE MONIST (no wave-particle duality)
- BUT: there are two waves
- Ψ_L the pilot-wave, solution of the Schrödinger equation
- ϕ_{NL} the (peaked) soliton
- the (center of mass of the) soliton is guided by the pilot-wave according to the guidance equation: $\frac{d\mathbf{x}}{dt} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x}(t), t)$

Self-gravity as a possible mechanism of localization:Schrödinger-Newton equation

- 60's Möller and Rosenfeld[4]: matter is quantum, but space-time is classical[5]
- $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4} < \Psi|\hat{T}_{\mu\nu}|\Psi >, \quad (1)$$

  with $R_{\mu\nu}$ the Ricci tensor, $g_{\mu\nu}$ the space-time metric, G Newton's constant, c, the velocity of light and $\hat{T}$ the stress-energy tensor. In the non-relativistic limit, one obtains the Poisson equation

  $$\Delta V = 4\pi G m|\Psi|^2, \quad (2)$$

  where V is the gravitational potential and $m|\Psi|^2$ the density of mass, when we deal with a particle of mass m.

[4] E. Rosenfeld in the Einstein Symposium, BERLIN Paperback, 1966.

[5] R. Penrose: "On the Gravitization of Quantum Mechanics 1: Quantum State Reduction", Foundations of Physics, 2014, Vol. 44, Issue 5

Schrödinger-Newton equation

- Even in the case of an isolated, free particle, the full energy now contains a contribution from the gravitational self-energy, proportional to

$$-\frac{Gm^2}{2}\int d^3x d^3x' \frac{|\Psi(t,\mathbf{x})|^2|\Psi(t,\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}, \qquad (3)$$

- In the single particle case, we get the so-called Schrödinger-Newton (S-N) equation (Lieb, Penrose, Diosi, Carlip, Tod, Moroz, Giulini, Grossardt, Jones, van Meter and many others[6]...):

$$i\hbar\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = -\hbar^2\frac{\Delta\Psi(t,\mathbf{x})}{2m} - Gm^2\int d^3x' \frac{|\Psi(t,\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}\Psi(t,\mathbf{x}), \qquad (4)$$

[6] see S. Colin, T. Durt, and R. Willox. Would a quantum particle succumb to its own gravitational attraction? Class. Quantum Grav., 31:245003, 2014. and references therein.

Problem with Schrödinger-Newton equation: solitons obey classical dynamics and NOT de Broglie-Bohm dynamics

- One can derive a generalized Ehrenfest theorem according to which self-focused solitons obey classical, Newton dynamics[7].
- This very general theorem is valid under the condition that the solution of the non-linear Schrödinger equation is peaked, independently of the form of the function[8] g in the self-focusing potential $\int d^3x'|\Psi(t,\mathbf{x}')|^2 g(|\mathbf{x}-\mathbf{x}'|)\Psi(t,\mathbf{x})$.

[7]Hatifi, M.; Lopez-Fortin, C.; Durt, T. de Broglie's double solution: Limitations of the self-gravity approach. Ann. Fond. Louis Broglie 2018, **43**.

[8]For instance it is also valid in the case of the so-called NLS equation, when g is a delta function, see e.g. D. Fargue. Permanence of the corpuscular appearance and non linearity of the wave equation. In S. Diner et al., editor, The wave-particle dualism, pages 149-172. Reidel, (1984).

Problem with Schrödinger-Newton equation: solitons obey classical dynamics and NOT de Broglie-Bohm dynamics

- The main reason of the appearance of the classical trajectories is that the non-linear potential respects Galilean invariance, so that no self-acceleration is present, and therefore no deviation from the classical trajectories appears, as would be the the case if the guidance equation was satisfied (due to the presence of the quantum potential).

Factorization ansatz.

- Let us consider a non-linear generalisation of Schrödinger equation $i\hbar\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = -\hbar^2\frac{\Delta\Psi(t,\mathbf{x})}{2m} + V^L(t,\mathbf{x})\Psi(t,\mathbf{x}) + V^{NL}(\Psi)$,
- **Factorization ansatz**
  We impose now an ansatz solution such that
  Ψ factorizes (5) into the product of two functions Ψ_L and ϕ_{NL}:

$$\Psi(t,\mathbf{x}) = \Psi_L(t,\mathbf{x}) \cdot \phi_{NL}(t,\mathbf{x}), \tag{5}$$

- where ϕ_{NL} is a self-collapsed soliton while Ψ_L, the pilot wave, is a solution of the linear Schrödinger equation:

$$i\hbar \cdot \frac{\partial\Psi_L(t,\mathbf{x})}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi_L(t,\mathbf{x}) + V^L(t,\mathbf{x})\Psi_L(t,\mathbf{x}), \tag{6}$$

Factorization ansatz.

QUESTION: Can we derive de Broglie-Bohm dynamics from our factorization ansatz? Answer: We find a modified dB-B dynamics[9]:

- Theorem[10]: when the soliton size is small enough, its velocity obeys the generalized guidance equation

$$\mathbf{v}_{drift} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t),t) + \frac{< \phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL} >}{< \phi_{NL}|\phi_{NL} >} = \mathbf{v}_{dB-B} + \mathbf{v}_{int.}, \quad (7)$$

which contains the well-known Madelung-de Broglie-Bohm contribution ($\mathbf{v}_{dB-B} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t),t)$) plus a new contribution due to the internal structure of the soliton ($\mathbf{v}_{int.} = \frac{<\phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}$).

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[9] T. Durt, "de Broglie double solution and gravitation", Annales de la Fondation de Broglie, 42, vol.1, 2017.

[10] Proof in appendix.

Classical versus quantum.

Dynamics of the barycentre of the soliton: Newton versus de Broglie-Bohm.

- Problem: the internal velocity is not small in general and as we discussed before, it even conspires, for a large class of non-linear potentials[11], in order that the full (drift) velocity is classical: $\mathbf{v}_{drift} = \mathbf{v}_{classical}$.
- It is possible however to realize de Broglie's double solution program provided we find a well-chosen non-linearity such that $\mathbf{v}_{int.} = \frac{<\phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}=0$, which occcurs when the soliton is a purely real function, at all times.

[11] This results from the aforementioned generalized Ehrenfest theorem, valid when the self-focusing potential has the form $\int d^3x'|\Psi(t,\mathbf{x}')|^2 g(|\mathbf{x}-\mathbf{x}'|)\Psi(t,\mathbf{x})$.

Factorization ansatz and well-chosen non-linear potential: a way to fulfill the double solution program.

- We consider from now on a non-linear generalisation of Schrödinger equation
$i\hbar\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = -\hbar^2\frac{\Delta\Psi(t,\mathbf{x})}{2m} + V^L(t,\mathbf{x})\Psi(t,\mathbf{x}) + V^{NL}(\Psi)$,
- **with the following non-linear potential**[12]

$$V^{NL}(\Psi) = \frac{\hbar^2}{2m}\frac{\Delta \mid \Psi(t,\mathbf{x}) \mid}{\mid \Psi(t,\mathbf{x}) \mid} - \frac{\hbar^2}{2m}\frac{\Delta \mid \Psi_L(t,\mathbf{x}) \mid}{\mid \Psi_L(t,\mathbf{x}) \mid} \qquad (8)$$

[12] T.Durt, Testing de Broglie's double solution in the mesoscopic regime, arXiv:2201.01204 accepted for publication in Foundations of Physics, special issue: The pilot-wave and beyond: Celebrating Louis de Broglie and David Bohm quest for a quantum ontology.

Factorization ansatz and well-chosen non-linear potential: a way to fulfill the double solution program.

- Expressing $\Psi_L(t,\mathbf{x})$ in function of its modulus and its phase as $R_L(t,\mathbf{x})e^{i\varphi_L(t,\mathbf{x})}$ $(\mid \Psi_L(t,\mathbf{x}) \mid = R_L(t,\mathbf{x}))$, we get
-

$$i\hbar \cdot \frac{\partial \phi_{NL}(t,\mathbf{x})}{\partial t} = \tag{9}$$

$$-\frac{\hbar^2}{2m} \cdot (\Delta \phi_{NL}(t,\mathbf{x}) - \Delta \mid \phi_{NL}(t,\mathbf{x}) \mid \cdot \frac{\phi_{NL}(t,\mathbf{x})}{\mid \phi_{NL}(t,\mathbf{x}) \mid})$$

$$-\frac{\hbar^2}{m}(i \bigtriangledown \varphi_L(t,\mathbf{x}) \cdot \bigtriangledown \phi_{NL}(t,\mathbf{x}) + \frac{\bigtriangledown R_L(t,\mathbf{x})}{R_L(t,\mathbf{x})}(\bigtriangledown \phi_{NL}(t,\mathbf{x})$$

$$-\frac{\bigtriangledown \mid \phi_{NL}(t,\mathbf{x}) \mid}{\mid \phi_{NL}(t,\mathbf{x}) \mid} \cdot \phi_{NL}(t,\mathbf{x})).$$

Factorization ansatz and well-chosen non-linear potential: a way to fulfill the double solution program.

- Whenever $\phi_{NL}(t, \mathbf{x})$ is a real positive function, equation (9) reads

$$i\hbar \cdot \frac{\partial \phi_{NL}(t, \mathbf{x})}{\partial t} = -\frac{\hbar^2}{m} \cdot i \bigtriangledown \varphi_L(t, \mathbf{x}) \cdot \bigtriangledown \phi_{NL}(t, \mathbf{x}). \tag{10}$$

- If the size of the soliton is quite smaller than the typical size of variation of $\bigtriangledown \varphi_L(t, \mathbf{x})$, that is to say, if it is quite smaller than $\frac{||\bigtriangledown \varphi_L(t, \mathbf{x_0})||}{|\Delta \varphi_L(t, \mathbf{x_0})|}$, we may replace, in good approximation, $\bigtriangledown \varphi_L(t, \mathbf{x})$ by $\bigtriangledown \varphi_L(t, \mathbf{x_0})$.
- Then, solving equation (10), we find a solitary wave solution of the type

$$\phi_{NL}(t, \mathbf{x}) = \phi_{NL}(\mathbf{x} - \mathbf{x_0}(t)), \tag{11}$$

with $\mathbf{x_0}(t) = \mathbf{x_0}(t_0) + \int_{t_0}^{t} dt \mathbf{v}_{dB-B}(\mathbf{x_0}(t))$.

Factorization ansatz and well-chosen non-linear potential: a way to fulfill the double solution program.

- Remarkably, this solution is valid independently of the initial shape of ϕ_{NL} at time t_0, provided it is a real positive function of the position.
- It moves without deformation at all times and remains thus a real positive function, moving at the velocity $\mathbf{v}_{dB-B}$, in accordance with de Broglie's guidance equation

$$\mathbf{v}_{drift} = \mathbf{v}_{dB-B} = \frac{\hbar}{m} \nabla \varphi_L(\mathbf{x_0}(t), t) \tag{12}$$

- This result is valid in presence of an arbitrary linear potential V_L: the acceleration of the peaked solitons is the sum of Newton's force and of the force deriving from the quantum potential.
- **In other words: trajectories are quantum and not classical !**

Illustration: coherent state of a harmonic oscillator

- Let us for instance impose that $\Psi_L(t, \mathbf{x})$ is a coherent state of a 1D harmonic oscillator:
-

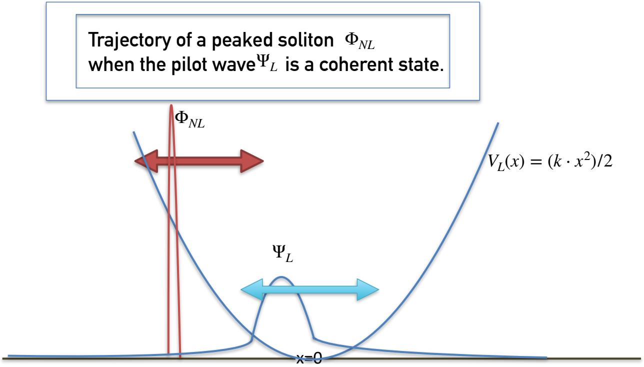
$$\Psi_L(t, \mathbf{x}) = \tag{13}$$

$$(\frac{m\omega}{\pi\hbar})^{1/4} \exp\left[-(\frac{m\omega}{2\hbar})\{(\mathbf{x} - \tilde{\mathbf{x}}_{\mathbf{0}} cos(\omega t))^2 + i(\tilde{\mathbf{x}}_{\mathbf{0}} \cdot \mathbf{x}) sin(\omega t)\} + i\theta(t)\right]$$

- Then, the non-linear equation of evolution of the soliton is gaussian and it possesses **exact analytic solutions** of gaussian shape.

Illustration: coherent state of a harmonic oscillator

- The barycentres of all solitons can be shown to follow trajectories of the type $\mathbf{x_0}(t) = \mathbf{x_0}(t = 0) - \tilde{\mathbf{x}}_\mathbf{0} + \tilde{\mathbf{x}}_\mathbf{0} cos(\omega t)$. They remain thus equidistant at all times with the peak of the pilot wave, due to the influence of the quantum potential.



Generalization to many particles.

- Our results can be generalized when N particles of masses $m_1, m_2, ...m_i...m_N$ are present. The evolution now obeys

$$i\hbar\frac{\partial\Psi(t,\mathbf{x^1},\mathbf{x^2},...,\mathbf{x^i},...,\mathbf{x^N})}{\partial t} = -\sum_{i=1...N}\hbar^2\frac{\Delta_i\Psi(t,\mathbf{x^1},\mathbf{x^2},...,\mathbf{x^i},...,\mathbf{x^N})}{2m_i}$$
$$+V^L(t,\mathbf{x})\Psi(t,\mathbf{x^1},\mathbf{x^2},...,\mathbf{x^i},...,\mathbf{x^N}) + V^{NL}(\Psi)\Psi(t,\mathbf{x^1},\mathbf{x^2},...,\mathbf{x^i},...,\mathbf{x^N}) \quad (14$$

with

$$V^{NL}(\Psi) = \sum_i \frac{\hbar^2}{2m_i}\frac{\Delta_i \mid \Psi(t,\mathbf{x^i}) \mid}{\mid \Psi(t,\mathbf{x}) \mid} - \frac{\hbar^2}{2m_i}\frac{\Delta_i \mid \Psi_L(t,\mathbf{x^i}) \mid}{\mid \Psi_L(t,\mathbf{x}) \mid}, \quad (15)$$

- where $\Psi = \Psi_L \cdot \pi_{i=1...N}\Phi^i_{NL}$

Generalization to many particles.

- For instance, privilegging gausson type solutions, we find a formal solution of equation (14) of the type

$$\Psi(t,\mathbf{x^1},\mathbf{x^2},...\mathbf{x^i}...\mathbf{x^N}) \approx \tag{16}$$

$$N'\cdot\frac{R_L(\mathbf{x^1},\mathbf{x^2},...\mathbf{x^i}...\mathbf{x^N},t)}{R_L(\mathbf{x_0^1},\mathbf{x_0^2},...\mathbf{x_0^i}...\mathbf{x_0^N},t)}e^{i\varphi_L(\mathbf{x^1},\mathbf{x^2},...\mathbf{x^i}...\mathbf{x^N},t)}\cdot e^{-(\sum_{i=1...N}\frac{A_0(i)}{2}(\mathbf{x^i}-\mathbf{x_0^i(t)})^2)}$$

- with $\frac{d\mathbf{x_0^i}}{dt}=\frac{\hbar}{m}\nabla_i\varphi_L(\mathbf{x^1},\mathbf{x^2},...\mathbf{x^i}...\mathbf{x^N},t)$
- and $N'=R_L(\mathbf{x_0^1},\mathbf{x_0^2},...\mathbf{x_0^i}...\mathbf{x_0^N},t=0)\cdot N$.
- The guidance equation in configuration space originally derived by de Broglie in 1926 is thus fulfilled, generalizing the results of the previous section to the many particles case.
- Solitons fatorize here, but the pilot wave is in general entangled, and trajectories are thus non-local in configuration space.

More about de Broglie's double solution

Remark:

- Due to EQUIVARIANCE, if at any time t_0 the Born rule satisfied, it will be so for all times.
- Valentini and coworkers[13] have shown that for a large class of Hamitonians, when trajectories obey the de Broglie-Bohm dynamics, the Born rule gets satisfied after a sufficiently long time, at least at a coarse grained level (this is called the onset of quantum equilibrium).
- Having in mind that each measurement is, ultimately, a position measurement, a solution of de Broglie's double solution program provides thus a solution of the measurement problem.

[13] A. Valentini and H.Westman. Dynamical origin of quantum probabilities.Proc. R. Soc. A, 461:253-272, (2005).

Circumventing criticisms, no-go theorems and no-go experiments.

1. Circumventing criticisms against determinism

- The deterministic nature of non-linear generalisations of Schrödinger equation such as Schrödinger-Newton equation has been criticised in the past , because it could not explain the appearance of quantum stochasticity[14].
- This criticism is not founded HERE, because of the mechanism of convergence to quantum equilibrium put into evidence by Valentini and his collaborators.
- Ultimately, deterministic dBB trajectories appear to be undeterministic FAPP due to chaoticity of the dynamics in the vicinity of zeros of the wave function[15].

[14] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht. Models of wave-function collapse, underlying theories, and experimental tests. Rev. Mod. Phys., 85:471527, (2013).

[15] C. Efthymiopoulos, G. Contopoulos and A.C. Tzemos . Chaos in de Broglie - Bohm quantum mechanics and the dynamics of quantum relaxation. *Annales de la Fondation Louis de Broglie*, **42**, 73, 2017.

Circumventing criticisms, no-go theorems and no-go experiments.

2. Circumventing Gisin no-go theorem.

- Roughly summarized, Gisin's argument[16] goes as follows: nonlinear corrections to the linear Schrödinger equation make it possible, in principle, to distinguish different realizations of the same density matrix. By performing a local measurement on a system A that is entangled with a distant system B, one is able, by collapsing the full wave function, to obtain realizations of the reduced density matrix of the system B which differ according to the choice of the measurement basis made in the region A. Therefore, in principle, nonlinearity can be a tool for sending classical information faster than light, contradicting the no-signaling property valid in the framework of linear quantum mechanics .
- **Now, at quantum equilibrium, the Born rule is satisfied and the no-signalling theorem is valid, which nullifies Gisin's argument.**

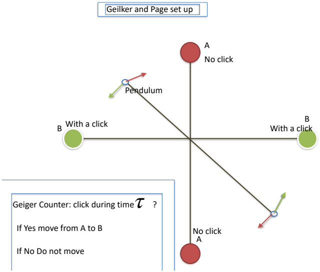
[16] N. Gisin. Weinbergs non-linear quantum mechanics and superluminal communications. Phys. Lett. A, 143(1,2):12, (1990).

3. Circumventing Geilker and Page no-go experiment.

- Roughly summarized, Geilker and Page's argument[17] goes as follows:
- A Geiger counter is placed close to a radioactive source of uranium, with a probability one halve that a click occurs after a time τ.
- Everytime τ, a pair of massive objects gets displaced from A to B if a click occured and remains in A otherwise.
- The classical gravitational torque generated by the pair of massive objects gets measured with a Cavendish pendulum.

[17] D.N. Page, C.D. Geilker. Indirect evidence for quantum gravity. Phys. Rev. Lett. 47, 979982 (1981).

3. Circumventing Geilker and Page no-go experiment.



- If no collapse occurs, the gravitational field generated by the pair of massive objects would be according to Geilker and Page the superposition of the field generated by the pair of massive objects in A and B. The pendulum does not move.
- If collapse occurs it will be generated by the pair in A when there is no click and in B when there is a click. The pendulum moves accordingly.

3. Circumventing Geilker and Page no-go experiment.

- The dBB theory is a no-collapse theory; however the pendulum moves as if there was a collapse **provided we assume that the source of the gravitational field is located where the particle is located (here the position of solitons).**
- Remark that an effective collapse[18]can be shown to occur in dBB interpretation, due to entanglement and the fact that the space is not 3D but it is the 3ND configuration space[19] .

[18]T. Norsen. Bohmian Conditional Wave Functions (and the status of the quantum state), J. Phys.: Conf. Ser., 701 012003, 2016

[19]T.Durt, Do(es the influence of) empty waves survive in configuration space? arXiv:2206.10918, accepted for publication in Foundations of Physics, special issue: The pilot-wave and beyond: Celebrating Louis de Broglie and David Bohm quest for a quantum ontology .

Circumventing criticisms, no-go theorems and no-go experiments.

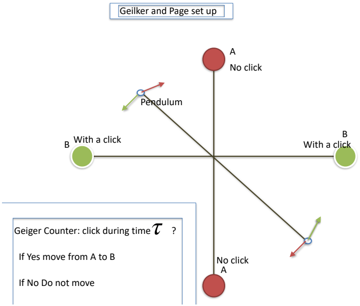
3. Circumventing Geilker and Page no-go experiment.

- Although no collapse occurs, the gravitational field generated by the pair of massive objects entangles the pendulum with the large masses and the Geiger counter: the state of the full system Geiger counter-large Masses-Pendulum is

$$\Psi^{GMP} = \frac{1}{\sqrt{2}}(|noclick >^{G} \otimes |largemassinA >^{M} |pendulumattractedbyA >$$
$$+ |withaclick >^{G} \otimes |largemassinB >^{M} |pendulumattractedbyB >^{P}) \quad (17)$$

- When the counter position indicates Yes or No, the pendulum moves accordingly due to entanglement....

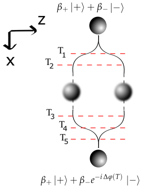
3. Circumventing Geilker and Page no-go experiment.



- Note that the reduced state of the pendulum is $(1/2)(|pendulumattractedbyA>^P< pendulumattractedbyA|^P + |pendulumattractedbyB>^P< pendulumattractedbyB|^P)$, disregarding the basis in which we measure the Geiger counter, in accordance with the no signaling theorem.

Experimental proposal: humpty dumpty Stern-Gerlach experiment with mesoscopic mass.

- Consider a SINGLE particle in a humpty-dumpty Stern-Gerlach interferometer (see figure below).



- If the particle is localized either in the right or in the left region, the spin will rotate due to a dephasing between the left and the right path[20] which could be revealed by tomography after recombining the two paths (humpty dumpty)..

[20] T.Durt, Testing de Broglie's double solution in the mesoscopic regime, arXiv:2201.01204 accepted for publication in Foundations of Physics, special issue: The pilot-wave and beyond: Celebrating Louis de Broglie and David Bohm quest for a quantum ontology.

Experimental proposal: humpty dumpty Stern-Gerlach experiment with mesoscopic mass.

- This dephasing is due to gravitational **self**-interaction
- For instance if a fifty fifty superposition of left and right paths is prepared, we expect a dephasing the order of $Gm^2T/(D\hbar)$, with a + sign in fifty percent of the cases and a - sign otherwise, where
  -G represents Newton's constant,
  -m the mass of the object (e.g. a carbon nanosphere with a NV center inside, see talk of R.Folman yesterday),
  -T the free fall time,
  -and D the size of the object.

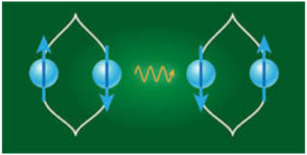
Experimental proposal: humpty dumpty Stern-Gerlach experiment with mesoscopic mass.

- This effect can be measured in principle[21] but...
- In order to be observable, the effect requires to minimize decoherence occuring inside the interferometer, but at the same time mesoscopic masses are necessary in order to observe a measurable phase-shift between the two components of the spin.
- Decoherence increases with mass which imposes to work in extreme vacuum and cryogenic conditions...

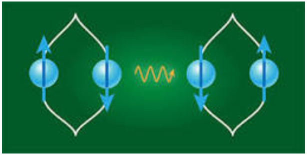
[21] Y. Margalit, O. Dobkowski, Z. Zhou, O. Amit, Y. Japha, S. Moukouri, D. Rohrlich, A. Mazumdar, S. Bose, C. Henkel and R. Folman. Realization of a complete Stern-Gerlach interferometer, Science Advances , bf 7, n^{0}22, 2020.

Experimental proposal: humpty dumpty Stern-Gerlach experiment with mesoscopic mass.

- In "standard", "orthodox" quantum mechanics (e.g. quantum gravity), no such self-interaction is expected to occur.
- However, with 2 Stern-Gerlach devices, and masses falling in parallel (Bose-Marletto-Vedral experiment), it is expected that gravitation will entangle both particles in spin, which could be revealed by tomography after recombining the two paths (humpty dumpty).



Experimental proposal: humpty dumpty Stern-Gerlach experiment with mesoscopic mass.

- 

- In our approach only one Stern-Gerlach would be enough to reveal the manifestation of gravity at the level of a single quantum object, because we assume some form of self-interaction, to the contrary of orthodox approaches...
- This self-interaction exists because we assume that
  1. particles are localized in tiny regions of space
  2. inertio-gravitational mass is localized where the object is localized (soliton)
- Actually, self-interaction can be seen as a **feedback of de Broglie-Bohm particles on the pilot wave.**

Conclusions.

- Several interpretations of the quantum theory are still in the race.
- de Broglie's double solution program is at the same time an interpretation with and without collapse: stuff localizes where the peaked solitons are present, but the linear wave function does never collapse.
- It suffers from some "problems" :
  -it is not relativistically invariant-a privileged reference frame must be chosen to express the theory, as in dBB dynamics and in the GRW model
  -elementary particles play a special role in the theory (at least in our model) so that some form of wave-particle duality is still present
- **Testing gravitational effects in the quantum regime would certainly learn us a lot about the nature of gravity, but also about possible interpretations of the quantum theory and the measurement problem.**

Appendix. Factorization ansatz and modified dB-B dynamics.

Forcing the factorisation ansatz in the non-linear evolution equation, we get

$$
\begin{aligned}
& i\hbar \cdot \left(\left(\frac{\partial \Psi_L(t,\mathbf{x})}{\partial t}\right)\phi_{NL}(t,\mathbf{x}) + \Psi_L(t,\mathbf{x}) \cdot \left(\frac{\partial \phi_{NL}(t,\mathbf{x})}{\partial t}\right)\right) = \\
& -\frac{\hbar^2}{2m}\Delta\Psi_L(t,\mathbf{x}) \cdot \phi_{NL}(t,\mathbf{x}) \\
& -\frac{\hbar^2}{2m}(2\bigtriangledown\Psi_L(t,\mathbf{x}) \cdot \bigtriangledown\phi_{NL}(t,\mathbf{x}) + \Psi_L(t,\mathbf{x}) \cdot \Delta\phi_{NL}(t,\mathbf{x})) \\
+ \quad & V^L\Psi(t,\mathbf{x}) + V^{NL}(\Psi)\Psi(t,\mathbf{x}), \qquad (18)
\end{aligned}
$$

Appendix. Factorization ansatz and modified dB-B dynamics.

Making use of the identity
$\bigtriangledown\Psi_L(t,\mathbf{x}) = (\bigtriangledown A_L(t,\mathbf{x}))e^{i\varphi_L(t,\mathbf{x})} + \Psi_L(t,\mathbf{x})i\bigtriangledown\varphi_L(t,\mathbf{x})$, we replace the non-linear equation of the previous slide by a system of two equations[22]:
-the linear Schrödinger equation

$$i\hbar\cdot\frac{\partial\Psi_L(t,\mathbf{x})}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi_L(t,\mathbf{x}) + V^L(t,\mathbf{x})\Psi_L(t,\mathbf{x}),$$

and the non-linear equation

$$\begin{aligned} i\hbar\cdot\frac{\partial\phi_{NL}(t,\mathbf{x})}{\partial t} &= -\frac{\hbar^2}{2m}\cdot\Delta\phi_{NL}(t,\mathbf{x}) \\ - &\frac{\hbar^2}{m}\cdot\left(i\bigtriangledown\varphi_L(t,\mathbf{x})\cdot\bigtriangledown\phi_{NL}(t,\mathbf{x}) + \frac{\bigtriangledown A_L(t,\mathbf{x})}{A_L(t,\mathbf{x})}\cdot\bigtriangledown\phi_{NL}(t,\mathbf{x})\right) \\ + &V^{NL}(\Psi)\phi_{NL}(t,\mathbf{x}) \end{aligned} \quad (19)$$

[22]This replacement is not one to one in the sense that there could exist solutions of equation (18) that do not fulfill the system (6,19). In any case, we focus on a particular

Appendix. Factorization ansatz and modified dB-B dynamics.

In order to solve the system of equations (6,19), it is worth noting that while the L_2 norm of the linear wave Ψ_L is preserved throughout time, because (6) is unitary, this is no longer true in the case of the non-linear wave ϕ_{NL}, because the terms mixing Ψ_L and ϕ_{NL} are not hermitian.
By a straightforward but lengthy computation that we do not reproduce here, we established the following result:
The change of norm of ϕ_{NL} obeys

$$\frac{d < \phi_{NL}|\phi_{NL} >}{dt} \approx \frac{\hbar}{m}\Delta\varphi_L(t,\mathbf{x_0})\cdot < \phi_{NL}|\phi_{NL} >$$
$$-2\frac{\nabla A_L(t,\mathbf{x_0})}{A_L(t,\mathbf{x_0})} \cdot \int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^* \frac{\hbar\nabla}{mi} \cdot \phi_{NL}(t,\mathbf{x}). \quad (20)$$

Appendix. Factorization ansatz and modified dB-B dynamics.

Let us now consider the barycentre $\mathbf{x_0}$ of the soliton: $\mathbf{x_0} \equiv \frac{<\phi_{NL}|\mathbf{x}|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}$ in order to estimate its velocity $\mathbf{v}_{drift}$:

$$\mathbf{v}_{drift} \equiv \frac{d(\frac{<\phi_{NL}|\mathbf{x}|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>})}{dt} \tag{21}$$

For instance, if we consider its z component:
$z_0 = \frac{<\phi_{NL}|z|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}$ and
$\frac{dz_0}{dt} = \frac{1}{<\phi_{NL}|\phi_{NL}>}\frac{d<\phi_{NL}|z|\phi_{NL}>}{dt} - \frac{z_0}{<\phi_{NL}|\phi_{NL}>}\frac{d<\phi_{NL}|\phi_{NL}>}{dt}$,

Appendix. Fact. ansatz and modified dB-B dynamics.

$$
\begin{aligned}
&\text{Therefore}\frac{dz_0}{dt} = \frac{1}{<\phi_{NL}|\phi_{NL}>}\int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^*\frac{\hbar\nabla_{\mathbf{z}}}{mi}\cdot\phi_{NL}(t,\mathbf{x}) \\
+ &\quad \frac{1}{<\phi_{NL}|\phi_{NL}>}\int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^*(\frac{\hbar\nabla_{\mathbf{z}}}{m}\cdot\varphi_L(t,\mathbf{x}))\phi_{NL}(t,\mathbf{x}) \\
+ &\quad \frac{1}{<\phi_{NL}|\phi_{NL}>}<\phi_{NL}|(\frac{\hbar}{m}\Delta\varphi_L(t,\mathbf{x}))\cdot z|\phi_{NL}> \\
+ &\quad \frac{1}{<\phi_{NL}|\phi_{NL}>}\frac{\hbar}{im}\int d^3\mathbf{x}\frac{\nabla A_L(t,\mathbf{x})}{A_L(t,\mathbf{x})}\cdot\nabla(\phi_{NL}(t,\mathbf{x}))^*\cdot z\cdot\phi_{NL}(t,\mathbf{x}) \\
- &\quad \frac{1}{<\phi_{NL}|\phi_{NL}>}\frac{\hbar}{im}\int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^*\cdot z\cdot\frac{\nabla A_L(t,\mathbf{x})}{A_L(t,\mathbf{x})}\cdot\nabla\phi_{NL}(t,\mathbf{x})) \\
- &\quad \frac{z_0}{<\phi_{NL}|\phi_{NL}>}\cdot(\frac{\hbar}{m})\Delta\varphi_L(t,\mathbf{x_0})\cdot<\phi_{NL}|\phi_{NL}> \qquad (22) \\
+ &\quad 2\frac{z_0}{<\phi_{NL}|\phi_{NL}>}\frac{\nabla A_L(t,\mathbf{x_0})}{A_L(t,\mathbf{x_0})}\cdot\int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^*\frac{\hbar\nabla}{mi}\cdot\phi_{NL}(t,\mathbf{x}))
\end{aligned}
$$

Appendix. Factorization ansatz and modified dB-B dynamics.

Now, $\frac{\hbar}{im}\int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^* \cdot z \cdot \frac{\nabla A_L(t,\mathbf{x})}{A_L(t,\mathbf{x})} \cdot \nabla\phi_{NL}(t,\mathbf{x})$
$\approx z_0 \frac{\nabla A_L(t,\mathbf{x_0})}{A_L(t,\mathbf{x_0})} \int d^3\mathbf{x}(\phi_{NL}(t,\mathbf{x}))^* \frac{\hbar\nabla}{mi} \cdot \phi_{NL}(t,\mathbf{x}))$,
while
$< \phi_{NL}|(\frac{\hbar}{m})\Delta\varphi_L(t,\mathbf{x}) \cdot z|\phi_{NL} > \approx z_0 \cdot (\frac{\hbar}{m})\Delta\varphi_L(t,\mathbf{x_0}) \cdot < \phi_{NL}|\phi_{NL} >$ and so on so that finally only the two first lines of (22) survive. We get thus the **Generalized dBB guidance equation** (7):

$$\mathbf{v}_{drift} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t),t) + \frac{< \phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL} >}{< \phi_{NL}|\phi_{NL} >} = \mathbf{v}_{dB-B} + \mathbf{v}_{int.}.$$

$\mathbf{v}_{drift}$ contains the de Broglie-Bohm velocity $\mathbf{v}_{dB-B} \equiv \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t),t)$ and the internal velocity $\mathbf{v}_{int.} \equiv \frac{<\phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}$; $\mathbf{v}_{int.}$ can be considered as a contribution to the average velocity originating from the internal structure of the soliton. Both contributions to the drift are evaluated at the barycentre of the soliton, $\mathbf{x_0}$.