# Bohmian mechanics:
# a(nother) quantum theory with a primitive ontology

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# Bohmian mechanics (Intro)

Some important points abouts BM:

- **Where from?** Bohmian mechanics originates from De Broglies work of 1927 and Bohms 1952 papers
- **What about?** Bohmian mechanics grounds the predictions of quantum mechanics in precise dynamical laws for point particles
- **What are its features?** Bohmian mechanics is a *deterministic*, *realistic* and *non-local hidden variables* theory

Two big advantages of Bohmian mechanics (as well as of collapse models):

1. Bohmian mechanics has no measurement problem
2. Bohmian mechanics has a primitive ontology

# Bohmian mechanics in a nutshell

N-body wave function:

$$\begin{aligned} \psi : \mathbb{R}^{3N} \times \mathbb{R} &\rightarrow \mathbb{C} \\ (\mathbf{q}, t) &\rightarrow \psi(\mathbf{q}, t) \end{aligned}$$

Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{q}, t) = -\sum_k \frac{\hbar^2}{2m_k}\Delta_k \psi(\mathbf{q}, t) + V(\mathbf{q})\psi(\mathbf{q}, t)$$

De-Broglie-Bohm guiding equation:

$$\mathbf{v}^{\psi}(\mathbf{q}, t) = \frac{\hbar}{m} \operatorname{Im} \frac{\nabla \psi}{\psi}(\mathbf{q}, t)$$

Trajectory of the system in configuration space $\mathcal{Q} \cong \mathbb{R}^{3N}$ and trajectory of the $k$-th particle in Euclidean space $\mathbb{R}^3$:

$$\text{in } \mathcal{Q} \cong \mathbb{R}^{3N} : \frac{d\mathbf{Q}(t)}{dt} = \mathbf{v}^{\psi}(\mathbf{Q}(t), t) \qquad \text{in } \mathbb{R}^3 : \frac{d\mathbf{Q}_k(t)}{dt} = \frac{\hbar}{m_k} \operatorname{Im} \frac{\nabla_k \psi}{\psi}(\mathbf{Q}(t), t)$$

# Bohmian mechanics in a nutshell

Born's rule (Born's statistical hypothesis):

*Given a system with wave function $\psi$, the positions of the particles are $\rho = |\psi|^2$-distributed.*

In BM Born's rule (also called the quantum equilibrium hypothesis) can be shown to hold by a Boltzmannian typicality argument (Dürr, Goldstein, Zanghì [1992]). The proof uses that $\rho = |\psi|^2$ is an equivariant measure.

Quantum flux equation (from Schrödinger equation):

$$\frac{\partial |\psi|^2}{\partial t} + \nabla \mathbf{j}^\psi = 0 \quad \text{with} \quad \mathbf{j}^\psi = \frac{\hbar}{m} \operatorname{Im} \psi^* \nabla \psi$$

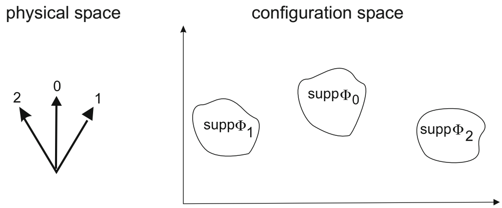
.

For $\rho = |\psi|^2$ and setting $\mathbf{v}^\psi = \frac{\hbar}{m} \operatorname{Im} \frac{\nabla \psi}{\psi}$, this becomes the continuity equation:

$$\frac{\partial |\psi|^2}{\partial t} + \nabla (|\psi|^2 \mathbf{v}^\psi) = 0.$$

# Measurement process

Pointers in $\mathbf{R}^3$ and supports of pointer wave functions in $\mathcal{Q} \cong \mathbb{R}^{3N}$:




System + apparatus coordinates:

$$\mathbf{q} = (\mathbf{x}, \mathbf{y}) \qquad \mathbf{Q} = (\mathbf{X}, \mathbf{Y})$$

System wavefunction:

$$\psi(\mathbf{x}) = \alpha_1\psi_1(\mathbf{x}) + \alpha_2\psi_2(\mathbf{x}) \qquad |\alpha_1|^2 + |\alpha_2|^2 = 1$$

Apparatus wavefunction:

$$\phi_0(\mathbf{y}),\ \phi_1(\mathbf{y}),\ \phi_2(\mathbf{y})$$

$$\mathbf{Y} \in \mathrm{supp}\phi_0 \mathrel{\widehat{=}} 0, \quad \mathbf{Y} \in \mathrm{supp}\phi_1 \mathrel{\widehat{=}} 1, \quad \mathbf{Y} \in \mathrm{supp}\phi_2 \mathrel{\widehat{=}} 2$$

# Measurement process

Initial wave function:

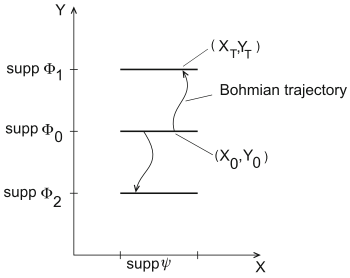
$$\Psi(\mathbf{q}) = \Psi(\mathbf{x}, \mathbf{y}) = \psi(\mathbf{x})\phi_0(\mathbf{y})$$

Schrödinger time evolution:

$$\psi_i \phi_0 \xrightarrow{t \to T} \psi_i \phi_i, \, i = 1, 2$$

$$\psi \phi_0 = \xrightarrow{t \to T} \alpha_1 \psi_1 \phi_1 + \alpha_2 \psi_2 \phi_2$$

Configuration space:



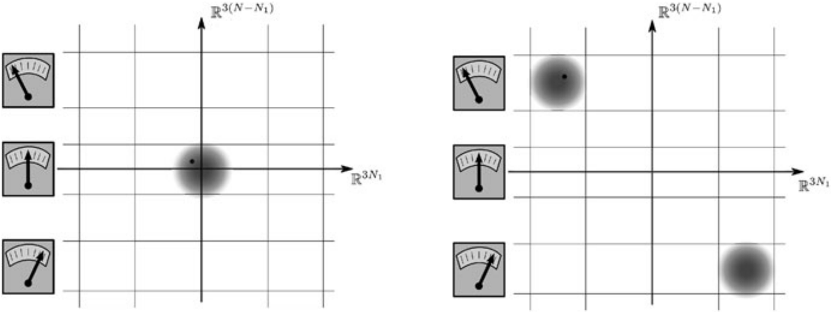
# Measurement process

Since $\mathrm{supp}\phi_1 \cap \mathrm{supp}\phi_2 = \emptyset$, the probability of pointer in position $k$ is $\mathbb{P}^{\Psi}(\mathbf{Y}(T) \widehat{=} k) \approx |\alpha_k|^2$.

Suppose $\mathbf{Y}(T) \in \mathrm{supp}\ \phi_1$ (pointer in position 1): FAPP $\phi_2(\mathbf{Y}) \equiv 0$ due to decoherence

$\Rightarrow$ Effective collapse of $\psi$

Particle is guided by $\psi_1$ with Bohmian velocity $\mathbf{v}^{\psi_1\phi_1+\psi_2\phi_2} = \mathbf{v}^{\psi_1\phi_1}$



# Universal, conditional and effective wave function

System of $N$ particles: $\mathbf{Q} = (\mathbf{Q}_1, \ldots, \mathbf{Q}_N) = (\mathbf{X}, \mathbf{Y})$

Subsystem $\mathbf{X}$ of $N_1$ particles: $\mathbf{X} = (\mathbf{Q}_1, \ldots, \mathbf{Q}_{N_1})$

Environment $\mathbf{Y}$ of $N - N_1$ particles: $\mathbf{Y} = (\mathbf{Q}_{N_1+1}, \ldots, \mathbf{Q}_N)$

**Universal** wave function:

$$\Psi(\mathbf{q}, t) = \Psi(\mathbf{x}, \mathbf{y}, t) \qquad \mathbf{x} \in \mathbb{R}^{3N_1}, \mathbf{y} \in \mathbb{R}^{3(N-N_1)}$$

**Conditional** wave function:

$$\varphi^{\mathbf{Y}}(\mathbf{x}, t) = \Psi(\mathbf{x}, \mathbf{Y(t)}, t)$$

$$\dot{\mathbf{X}}(t) = \mathbf{v}^{\Psi}_{\mathbf{x}}(\mathbf{X}(t), \mathbf{Y}(t)) \sim \mathrm{Im}\, \frac{\nabla_{\mathbf{x}} \Psi(\mathbf{x}, \mathbf{Y(t)}, t)}{\Psi(\mathbf{x}, \mathbf{Y(t)}, t)}\bigg|_{\mathbf{x}=\mathbf{X}(t)} = \mathrm{Im}\, \frac{\nabla_{\mathbf{x}} \varphi^{\mathbf{Y}}(\mathbf{x}, t)}{\varphi^{\mathbf{Y}}(\mathbf{x}, t)}$$

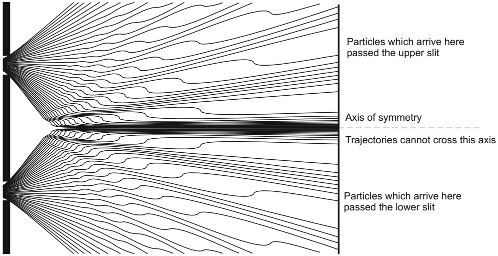
**Effective** wave function:

$$\Psi(\mathbf{x}, \mathbf{y}) = \varphi(\mathbf{x})\Phi(\mathbf{y}) + \Psi^{\perp}(\mathbf{x}, \mathbf{y})$$

with $\mathrm{supp}\Phi \cap \mathrm{supp}\Psi^{\perp} = \emptyset, \mathbf{Y} \in \mathrm{supp}\ \Phi \Rightarrow \varphi$ effective wave function

# Bohmian trajectories

Sketch of Bohmian trajectories in a double-slit experiment:

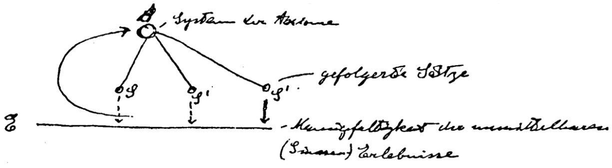



All figures were taken from:

D. Dürr, S. Teufel: *Bohmian Mechanics* (2008)
D. Dürr, D. Lazarovici: *Verständliche Quantenmechanik* (2018)

# Einstein's epistemological model

Einstein's epistemological model as sketched in a letter from Albert Einstein to Maurice Solovine in May, 1952:




Upper level: System of axioms $\mathcal{A}$ (depend psychologically on $\mathcal{E}$)

Intermediate level: propositions $\mathcal{P}$ (deduced from $\mathcal{A}$)

Lower level: Sense experiences $\mathcal{E}$ (in close intuitive, non-logical connection to the $\mathcal{P}$)

$\Rightarrow$ **Primitive ontology** (as in BM or collapse models) can provide this connection between the $\mathcal{P}$ and the $\mathcal{E}$

# Einstein's epistemological model

Einstein in his autobiographical notes [1949]:

> *I see on the one side the totality of sense-experiences, and, on the other, the totality of the concepts and propositions which are laid down in books. The relation between the concepts and propositions among themselves and each other are of a logical nature, and the business of logical thinking is strictly limited to the achievement of the connection between concepts and propositions among each other according to firmly laid down rules, which are the concern of logic. The concepts and propositions get "meaning," viz., "content," only through their connection with sense experiences. The connection of the latter with the former is purely intuitive, not itself of a logical nature. The degree of certainty with which this relation, viz., intuitive connection, can be undertaken, and nothing else, differentiates empty phantasy from scientific "truth".*

$\Rightarrow$ **Primitive ontology** (as in BM or collapse models) can provide this connection between the $\mathcal{P}$ and the $\mathcal{E}$

# The end

Thank you!