Impact of dynamical collapse models on inflationary cosmology

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Cosmic microwave background radiation

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2. As the universe expanded and cooled, at a certain epoch called recombination, electrons were trapped by the protons to form Hydrogen atoms subsequently allowing photons to travel in straight lines.
3. After recombination, every point in space can be considered a source of light, and therefore also receives light from all directions.

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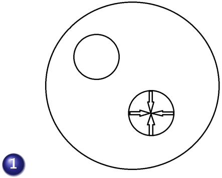

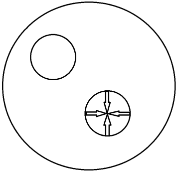

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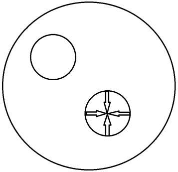
1. 

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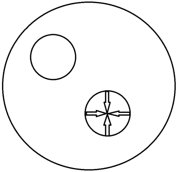
1. 

2. CMB photons that we see today were emitted in the remote past. The temperature of the radiation is almost the same in all directions.
3. These sources of the CMB photons must therefore have been in causal contact at some point before recombination.
4. To achieve this we need a finite phase of rapid accelerated expansion, such that the universe becomes extremely small very rapidly going backwards in time, allowing the different spatial points to come into causal contact.

Inflation

1. Inflationary dynamics is governed by the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{P}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \qquad (1)$$

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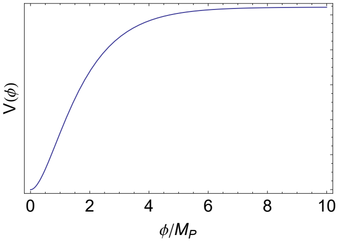
2. 

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5. The Lagrangian for the perturbations reduces to $\mathcal{L} = -\sqrt{-g}\frac{1}{2}g^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi$.

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5. Having determined the dynamics in the Heisenberg picture, assuming that initially the state of the universe was in the vacuum state $\hat{a}(\mathbf{k},\eta_0)\left|0\right\rangle = 0$, the probability for the perturbations to have a configuration $u(\mathbf{k},\eta)$ can be determined.

Quantum fluctuations and the power spectrum

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6. Modifying the Schrödinger equation as in collapse models, implies a modified time evolution of the perturbations and a modified power spectrum.

Collapse dynamics

The mass proportional CSL model is defined through the following stochastic differential equation [1]

$$d|\psi\rangle = \left[-i\hat{H}dt + \frac{\sqrt{\gamma}}{m_0}\int d\mathbf{x}\left[\hat{M}(\mathbf{x}) - \langle\hat{M}(\mathbf{x})\rangle\right]dW_t(\mathbf{x}) - \frac{\gamma}{2m_0^2}\int d\mathbf{x}d\mathbf{y}\left[\hat{M}(\mathbf{x}) - \langle\hat{M}(\mathbf{x})\rangle\right]G(\mathbf{x}-\mathbf{y})\left[\hat{M}(\mathbf{y}) - \langle\hat{M}(\mathbf{y})\rangle\right]dt\right]|\psi\rangle\,, \tag{3}$$

$$\mathbb{E}\left[\xi_t(\mathbf{x})\xi_{t'}(\mathbf{y})\right] = G(\mathbf{x}-\mathbf{y})\delta(t-t'), \qquad G(\mathbf{x}-\mathbf{y}) = \frac{e^{-\frac{(\mathbf{x}-\mathbf{y})^2}{4r_c^2}}}{(4\pi r_c^2)^{3/2}}\,. \tag{4}$$

[1] A. Bassi and G. Ghirardi, *Phys. Rep. 379, 257 (2003).*

Modifications from collapse dynamics

$$\frac{\mathrm{d}\,|\psi\rangle}{\mathrm{d}t} = -i\left[\hat{H} + \frac{\sqrt{\gamma}}{m_0}\int \mathrm{d}\mathbf{x}\,\hat{M}(\mathbf{x})\xi_t(\mathbf{x})\right]|\psi\rangle\,. \qquad (5)$$

For the purpose of computing modifications to observables due to CSL dynamics, it is sufficient to study instead the equation above since it leads to the same master equation.

Modifications from collapse dynamics

1. Treat $\hat{H}_{\mathrm{CSL}} = \frac{\sqrt{\gamma}}{m_0} \int \mathrm{d}\mathbf{x} \hat{M}(\mathbf{x}) \xi_t(\mathbf{x})$ as a perturbation,
$\langle \hat{\mathcal{O}} \rangle = \langle \hat{\mathcal{O}} \rangle_0 - \frac{1}{\hbar^2} \int_{t_0}^{t} \int_{t_0}^{t'} dt' dt'' \langle [\hat{H}_{\mathrm{CSL}}(t'), [\hat{H}_{\mathrm{CSL}}(t''), \hat{\mathcal{O}}]] \rangle + \ldots$

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2. Compute its effect on the comoving curvature perturbation
$\langle\hat{\mathcal{R}}^2\rangle \equiv \int \mathcal{P}_{\mathcal{R}}(k)d\ln k$.

Possible choices of the collapse operator

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3. In *A. Gundhi, J. L. Gaona-Reyes, M. Carlesso and A. Bassi, Phys. Rev. Lett. 127, 091302 (2021)*, we took the collapse operator to be proportional the Hamiltonian density.

Results

The correction to the inflationary spectrum was obtained to be

$$\delta\mathcal{P}_{\mathcal{R}} = -\frac{17}{36}\frac{\lambda H^3}{\pi^2 \epsilon M_{\mathrm{P}}^2 m_0^2} \ln\left(\frac{\eta_e}{\eta_0}\right) . \qquad (6)$$

The observational constraints demand $\lambda < 10^7 s^{-1}$, compatible with observational constraints.

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